PAGE No.
DATE / /

Q2 a)

```
def find-complexity (n):
    i=1, s=1
    while  s<=n:
        i+= 1
        s+ = i
    return
```

⇒ Values of s for every iteration of i -

| | |
|---|---|
| i = 1 | s = 1 |
| i = 2 | s = 3 |
| i = 3 | s = 6 |
| i = 4 | s = 10 |
| i = 5 | s = 15 |
| ⋮ | ⋮ |

lets consider s=n and $x$ is the iterating term.

∴ for s = 1, 3, 6, 10, ...

$x$ = 1, 2, 3, 4, ...

The function can be represented as-

$$\frac{x(x+1)}{2} = n$$

$$\therefore \quad x^2 + x = 2n$$

$$\therefore \quad x^2 + x - 2n = 0$$

if $a=1$, $b=1$, $c=-2n$ is substituted in formula-

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

then, $$\frac{-1 \pm \sqrt{1+8n}}{2} = 0$$

$$\therefore \quad \sqrt{1+8n} = 1$$

Therefore, we can conclude that $x = \sqrt{1+8n} \approx \sqrt{n}$

∴ Time complexity = $O(\sqrt{n})$

Q2 b)
```
def find-complexity2(n):
    for i in range (n//2, n+1):
        for j in range(1,n-(n//2)+1):
            k = 1
            while k<=n:
            k*=2
    return
```

→ Range of i is between $\frac{n}{2}$ to $n+1$

ie $n+1-\frac{n}{2}$

$= \frac{n}{2}+1$ ... ①

Similarly, range of j is in between $1, n-(n/2)+1$

ie $n-\frac{n}{2}+1-1$

$= \frac{n}{2}$ ... ②

For while loop, the complexity is $\log n$

since $k *= 2$

$= \log n$ ... ③

combining equations ①, ②, ③

$\left(\frac{n}{2}+1\right)\frac{n}{2}\log n$

$= \left(\frac{n^2}{4}+\frac{n}{2}\right)\log n$

$= \frac{n^2}{4}\log n + \frac{n}{2}\log n$

∴ Time complexity $= O(n^2\log n)$

PAGE No.
DATE / /

Q2 c)

```
def find-complexity3 (n):
    if (n<=2):
        return 1
    else:
        return find-complexity3 (math.floor(
                    math.sqrt(n))+1)
```

⇒ For values of $n > 2$ –

~~T(n) ≠ T(√n) * log n~~

$T(n) = T(\sqrt{n}) + 1$

$= T(n^{1/2}) + 1$

For variable term $x$,

$T\left(\frac{1}{n^{2^x}}\right) + x$

'Else' will execute until $n = 2$

$\therefore \frac{1}{n^{2^x}} = 2$

Taking log on both sides

$\frac{\log 1}{\log n^{2^x}} = \log 2$

$\therefore \cancel{-2^x / \log n} \neq \cancel{1 / \log 2}$

$2^{-x} \log n = \log 2$

$\frac{1}{2^x} \log n = \log 2 \quad = \quad 2^x = \log_2 n$

$x \log 2 = \log \log n$

$x = \frac{\log(\log n)}{\log 2}$

$\therefore x = \log \log n$

$\therefore$ Time complexity $= O(\log \cdot \log n)$

| PAGE No. | |
|---|---|
| DATE | / / |

Q5 1) i) $f(n) = 5n^3 + 2n^2 + 7n + 1 \in O(n^3)$

$\Rightarrow$ for $f(n) \in O(n^3)$,

$f(n) = O(g(n))$ — To prove

ie $0 \leq f(n) \leq c \cdot g(n)$, where $n \geq n_0$ .... 1)

$\therefore 5n^3 + 2n^2 + 7n + 1 \leq 5n^3 + 2n^3 + 7n^3 + n^3$

$\therefore f(n) \leq (5+2+7+1)\, n^3$

$\therefore f(n) \leq 15n^3$

$\therefore c = 15, \; n_0 = 1$

$\therefore f(n) \in O(n^3)$ ... from 1), Hence proved.

ii) $f(n) = 5n^3 + 2n^2 + 7n + 1 \in \Theta(n^3)$

$\Rightarrow$ $f(n) = \Theta(g(n))$ — To prove

for $c_1 > 0, \; c_2 > 0, \; n_0 > 0$ —

~~f(n~~ $0 \leq c_2\, g(n) \leq f(n) \leq c_1\, g(n)$,

where $n \geq n_0$ ... 1)

$5n^3 \leq 5n^3 + 2n^2 + 7n + 1 \leq 5n^3 + 2n^3 + 7n^3 + n^3$

$5n^3 \leq f(n) \leq (5+2+7+1)\, n^3$

$5n^3 \leq f(n) \leq 15n^3$

$\therefore c_1 = 5, \; c_2 = 15, \; n_0 = 1$

$\therefore f(n) \in \Theta(n^3)$ ... from 1), Hence proved

Q5 2) i) $f(n) = 5n^3 + 2n^2 + 7n + 1 \in \omega(n^2)$

$f(n) = \omega(g(n))$ — To Prove

ie $0 \leq c \cdot g(n) < f(n)$, where $n \geq n_0$

$f(n) = 5n^3 + 2n^2 + 7n + 1$

where, $g(n) = n^2$

$\therefore cg(n) < 5n^3 + 2n^2 + 7n + 1$

$\therefore cn^2 < 5n^3 + 2n^2 + 7n + 1$ ... ①

for $c = 1, \; n = n_0 = 1$ above eqn satisfy condition.

$f(n) \in \omega(n^2)$, Hence proved.

PAGE No.
DATE / /

Q5 2) ii) $5n^3 + 2n^2 + 7n + 1 \in \Omega(n^2)$

for $c > 0$, $n_0 > 0$ and $n \geq n_0$, --- 1)

$f(n) \in \Omega(g(n)) \rightarrow$ To prove

from 1),

$$0 \leq c \cdot g(n) \leq f(n)$$

$\therefore$ $f(n) = 5n^3 + 2n^2 + 7n + 1$

$g(n) = n^2$

$c \cdot n^2 \leq 5n^3 + 2n^2 + 7n + 1$

$c \leq 5n + 2 + \frac{7}{n} + \frac{1}{n^2}$ ---( Dividing by $n^2$ )

for $c = 1$, $n = n_0 = 1$,

$1 \leq 5 + 2 + 7 + 1$

$1 \leq 15$

$f(n) \in \Omega(n^2)$, Hence proved.

PAGE No.

DATE / /

Q6 Given - steps of Elrich are increased exponentially.

Assumption - steps increased in 2 exponential.

Ex - $x, 2x, 4x \dots nx$

ie $x(1, 2, 4 \dots 2^n x)$

Elrich has to return to initial position after traversing initial path.

∴ The pattern is in geometric progression -

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$= 2\left(\frac{1(2^n - 1)}{2-1}\right), \text{ where } 2^n \approx n$$

$$= 2(n-1)$$

Similarly, for route on other half of the road -

$(2n-2) + (2n-2) \approx$

$= 4n - 4$

$= 4(n-1)$

$\approx$ ~~$O(n)$~~

∴ time complexity $= O(n)$