A Framework for Dynamic Matching in Weighted Graphs

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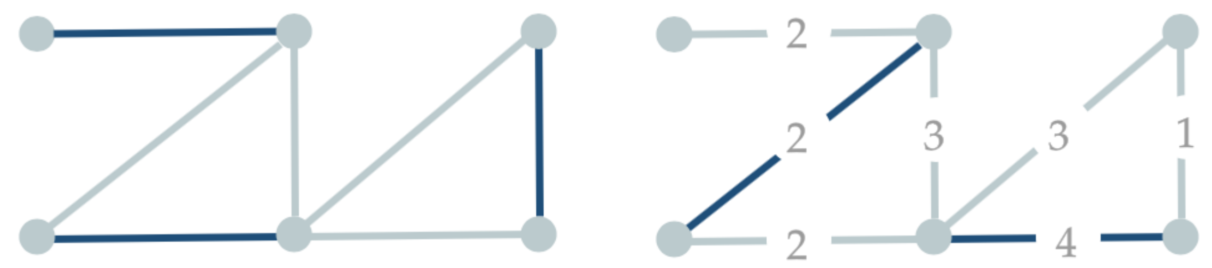
Maximum Weight Matching



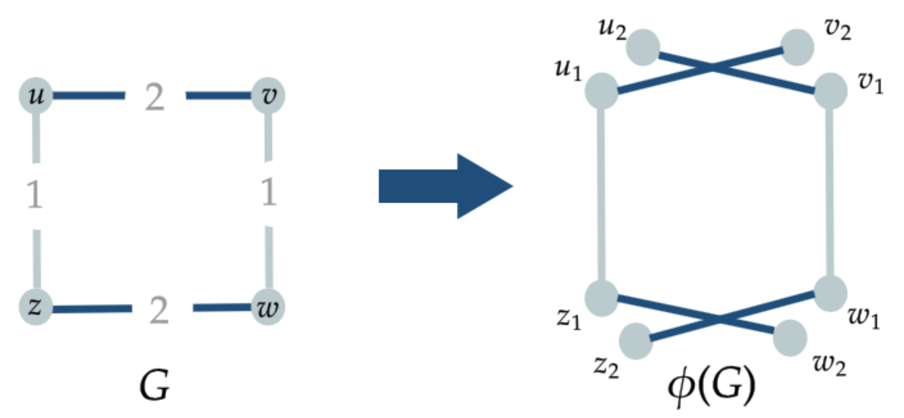
mcm(G) = 3 mwm(G) = 6

In Dynamic Setting:

1. Process a sequence of changes, maintain a large matching with a small update time.
2. Optimize for update time and approximation ratio.

Ingredient 1

Graph Unfolding:

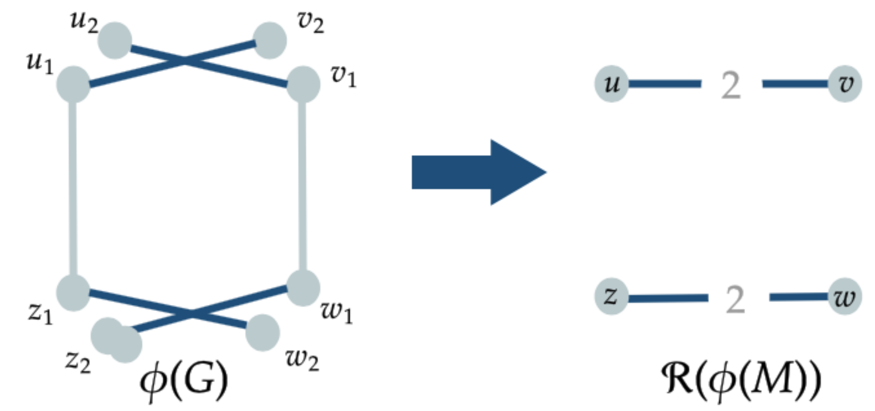


$uv \in E(G)$ iff $u^i v^{w(uv)-i+1} \in E(\phi(G))$

Bipartite graphs:

mwm(G) = mcm($\phi(G)$)

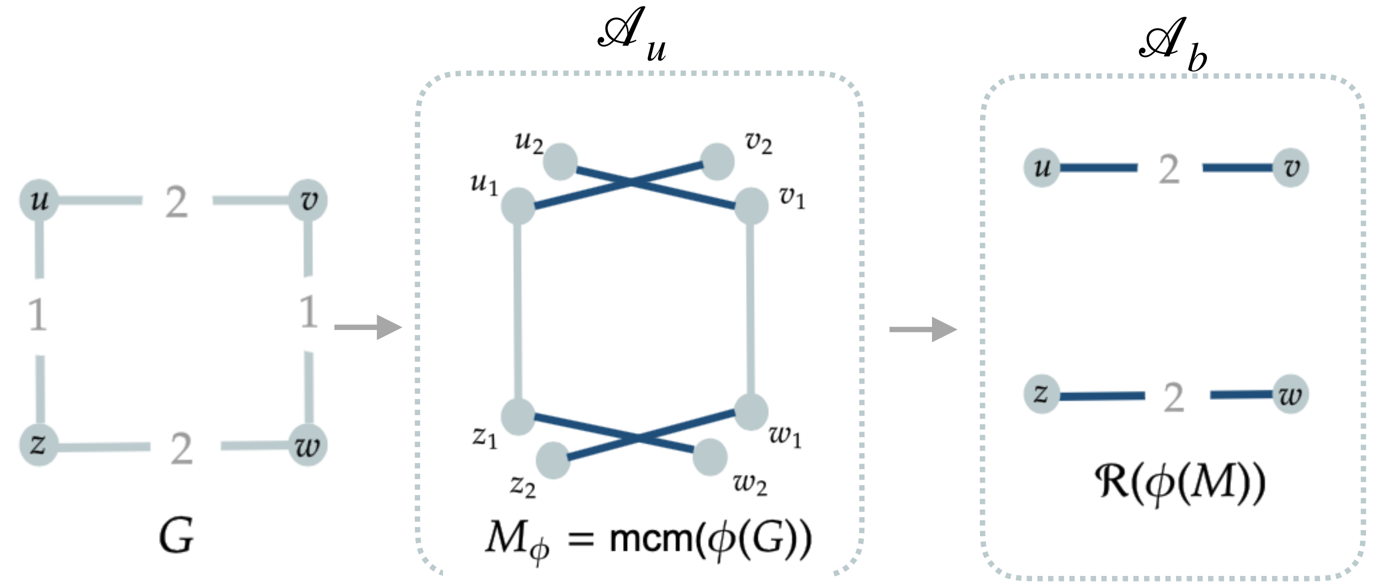
Graph Refolding:



Bipartite graphs:

mwm(G) = mwm($\mathscr{R}(\phi(G))$)

Bipartite Reduction



Update Time: $O\left(\frac{W^4 T_u}{\varepsilon^2}\right)$

Our Results

For bipartite graphs, a black box reduction:

α-approximate T-time bipartite MCM algorithm

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$(1-\epsilon)\cdot\alpha$-approximate $\widetilde{O}(T)$-time bipartite MWM algorithm

For non-bipartite graphs:

MCM (known)		MWM (known)		MWM (Our results)	
Approx	Time	Approx	Time	Approx	Time
$\frac{1}{2}-\varepsilon$	$\tilde{O}(1)$	$\frac{1}{4}-\varepsilon$	$\tilde{O}(1)$	$\frac{1}{2}-\varepsilon$	$\tilde{O}(1)$
$\frac{2}{3}-\varepsilon$	$O(m^{\frac{1}{4}})$	$\frac{1}{3}-\varepsilon$	$\tilde{O}(m^{\frac{1}{4}})$	$\frac{2}{3}-\varepsilon$	$\tilde{O}(m^{\frac{1}{4}})$
$\frac{1}{2}+\Omega(1)$	$O(\Delta^{\delta})$	$\frac{1}{4}+\Omega(1)$	$\tilde{O}(\Delta^{\delta})$	$\frac{1}{3}+\Omega(1)$	$\tilde{O}(\Delta^{\delta})$

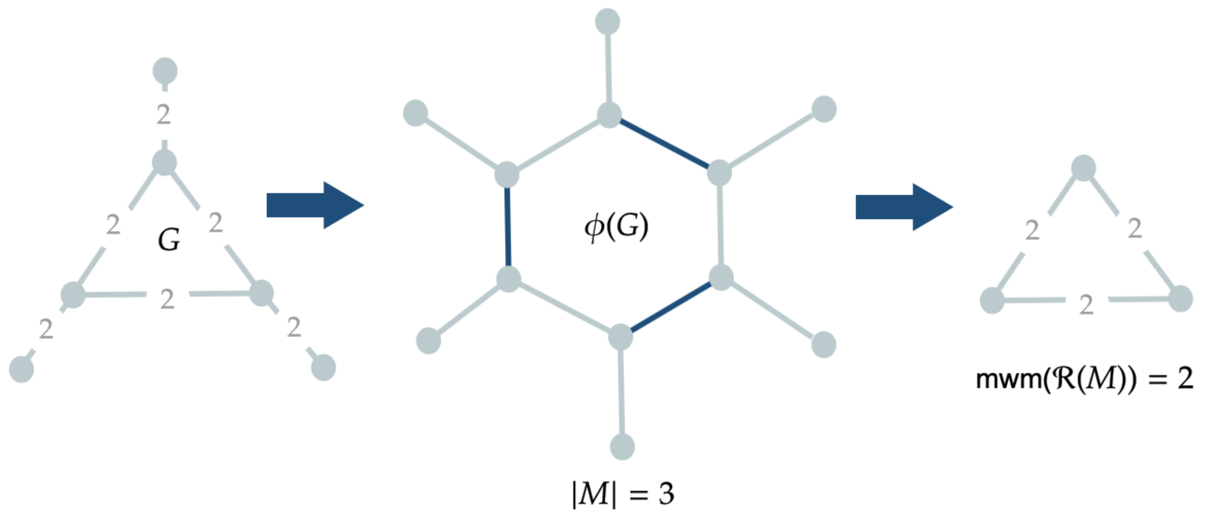
Ingredient 2

Assumption: The edge weights are in $\{1,2,3,\cdots,W\}$ where $W = \varepsilon^{-O(\varepsilon)}$.

Ingredient 3

Theorem: Algorithm $\mathscr{A}_b$ that maintains a $(1-\varepsilon)$- approximation to MWM in time $O\left(\frac{\Delta\cdot W^2}{\varepsilon^2}\right)$.

Ingredient 4: For Non-Bipartite Graphs



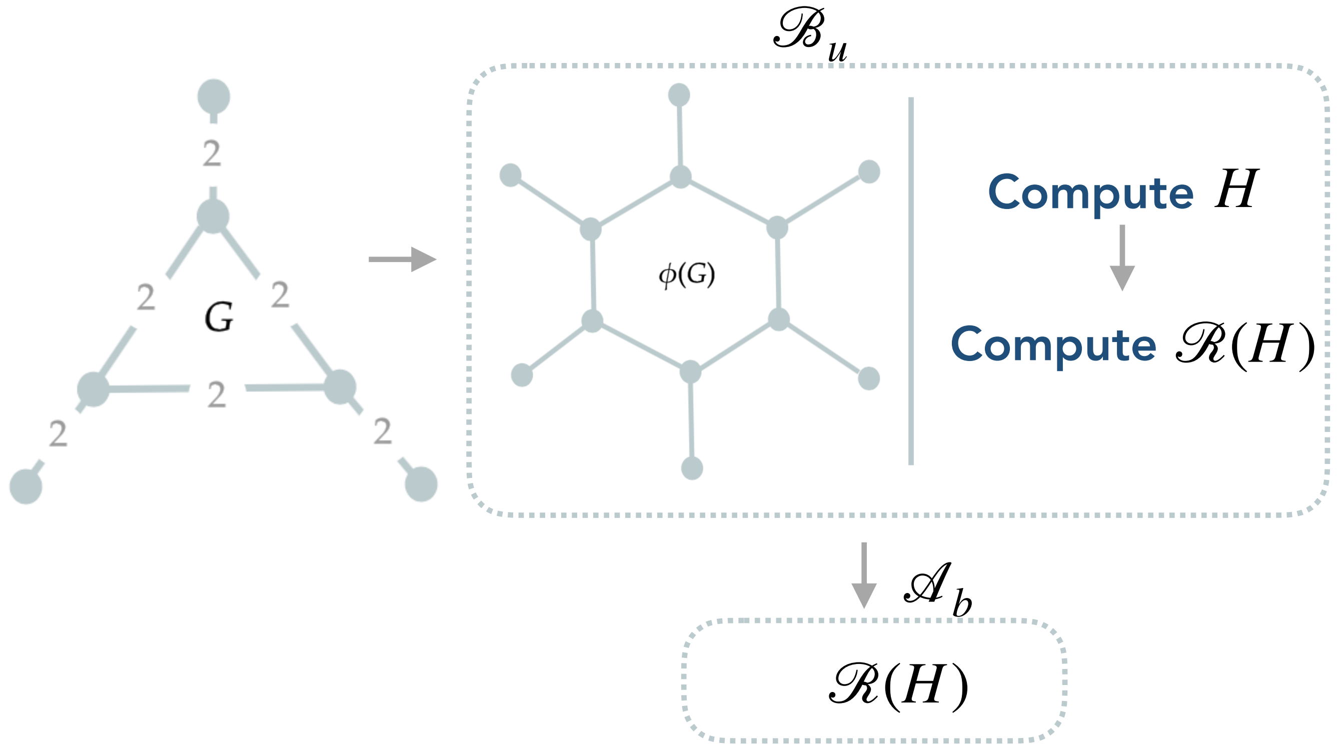
Kernels (unweighted graphs):

1. Constant degree subgraphs.
2. $\left(\frac{1}{2}-\varepsilon\right)$– approximate MCM.
3. $\tilde{O}(1)$ - update time.

Theorem: If H is a kernel of $\phi(G)$,

mwm($\mathscr{R}(H)$) $\geq \left(\frac{1}{2}-\epsilon\right)\cdot$ mwm(G).

Non-Bipartite Algorithm



Update Time: $O\left(\frac{W^4 T_u}{\varepsilon^2}\right)$

Open Questions

1. Can we improve dependence on epsilon?
2. Can kernels be maintained deterministically?