

Practice Problems :

1st order ODEs

Solve

① $y' = \frac{y}{x + \sqrt{xy}}$ ② $x^2 y \, dx - (x^3 + y^3) \, dy = 0$

③ $(y \, dx + x \, dy) \times \cos(y/x) = (x \, dy - y \, dx) y \sin(y/x)$ ④ $\frac{dy}{dx} = \frac{y+x-2}{y-x-4}$

Check for the exactness and then solve. If needed the I.F., then find

① $y^2 \, dx + (x^2 - xy - y^2) \, dy = 0$ ② $(xy \sin(xy) + \cos(xy)) y \, dx + (xy \sin(xy) - \cos(xy)) x \, dy = 0$

③ $(3xy - 2ay^2) \, dx + (x^2 - 2axy) \, dy = 0$ ④ $y(2xy + e^x) \, dx - e^x \, dy = 0$

Linear and Bernoulli's eq

① $(1-x^2) y' + xy = ax$ ② $x \cos x \, y' + (x \sin x + \cos x) y = 1$

③ $y' - \frac{2y}{1+x} = (1+x)^3$ ④ $(1+y^2) + (x - e^{\tan^{-1} y}) y' = 0$

⑤ $(x^2 y^3 + xy) y' = 1$ ⑥ $y' + yx = y^2 e^{x/2} \sin x$

Application Problems.

① S/t the system of confocal conics $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is a parameter, is self-orthogonal.

② Find the orthogonal trajectories of $x^2 + y^2 + 2gx + c = 0$, g is the parameter.

③ S/t $y^2 = 4a(x+a)$ is self-orthogonal.

④ Find the curve in which the portion of the tangent included between the coordinate axes is bisected at the point of contact.

⑤ At every point on a curve, the slope is the sum of abscissa and the product of its ordinate and abscissa. If the curve passes through $(0,1)$, find the equation of the curve.