

Q: If  $a\left(\frac{1}{b} + \frac{1}{c}\right)$ ,  $b\left(\frac{1}{c} + \frac{1}{a}\right)$ ,  $c\left(\frac{1}{a} + \frac{1}{b}\right)$  are in arithmetic progression (AP), prove that  $a, b, c$  are also in AP.  
**Solution:**

$$\begin{aligned}
 & a\left(\frac{1}{b} + \frac{1}{c}\right) + 1, b\left(\frac{1}{c} + \frac{1}{a}\right) + 1, c\left(\frac{1}{a} + \frac{1}{b}\right) + 1 \\
 \Rightarrow & a\left(\frac{1}{b} + \frac{1}{c}\right) + \frac{a}{a}, b\left(\frac{1}{c} + \frac{1}{a}\right) + \frac{b}{b}, c\left(\frac{1}{a} + \frac{1}{b}\right) + \frac{c}{c} \\
 \Rightarrow & a\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), c\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \\
 \Rightarrow & a, b, c
 \end{aligned}$$

Hence proved