

Q: If $a\left(\frac{1}{b} + \frac{1}{c}\right)$, $b\left(\frac{1}{c} + \frac{1}{a}\right)$, $c\left(\frac{1}{a} + \frac{1}{b}\right)$ are in arithmetic progression (AP), prove that a, b, c are also in AP.

Solution: Using properties of AP,

$$\begin{aligned}
 & a\left(\frac{1}{b} + \frac{1}{c}\right) + 1, b\left(\frac{1}{c} + \frac{1}{a}\right) + 1, c\left(\frac{1}{a} + \frac{1}{b}\right) + 1 \\
 \Rightarrow & a\left(\frac{1}{b} + \frac{1}{c}\right) + \frac{a}{a}, b\left(\frac{1}{c} + \frac{1}{a}\right) + \frac{b}{b}, c\left(\frac{1}{a} + \frac{1}{b}\right) + \frac{c}{c} \\
 \Rightarrow & a\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), c\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \\
 \Rightarrow & a, b, c
 \end{aligned}$$

Hence proved that a, b, c are in AP.