

Q: If $a\left(\frac{1}{b} + \frac{1}{c}\right)$, $b\left(\frac{1}{c} + \frac{1}{a}\right)$, $c\left(\frac{1}{a} + \frac{1}{b}\right)$ are in arithmetic progression (AP), prove that a, b, c are also in AP.

Solution: Common difference can be written as:

$$b\left(\frac{1}{c} + \frac{1}{a}\right) - a\left(\frac{1}{b} + \frac{1}{c}\right) = c\left(\frac{1}{a} + \frac{1}{b}\right) - b\left(\frac{1}{c} + \frac{1}{a}\right) \quad (1)$$

$$\implies (b - a)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = (c - b)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \quad (2)$$

$$\implies b - a = c - b \quad (3)$$

Hence proved that a, b, c are in AP.

parameter	value	description
$x(0)$	$a\left(\frac{1}{b} + \frac{1}{c}\right)$	First Term of given AP
d	$(b - a)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$	Common Difference of given AP
$x(n)$	$(x(0) + nd)u(n)$	General Term of given AP

TABLE I
INPUT PARAMETER TABLE

From table I

$$X(z) = x(0)\left(\frac{1}{1 - z^{-1}}\right) + d\left(\frac{z^{-1}}{(1 - z^{-1})^2}\right) \quad (4)$$

$$X(z) = a\left(\frac{1}{b} + \frac{1}{c}\right)\left(\frac{1}{1 - z^{-1}}\right) + (b - a)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)\left(\frac{z^{-1}}{(1 - z^{-1})^2}\right) \quad |z| > 1 \quad (5)$$

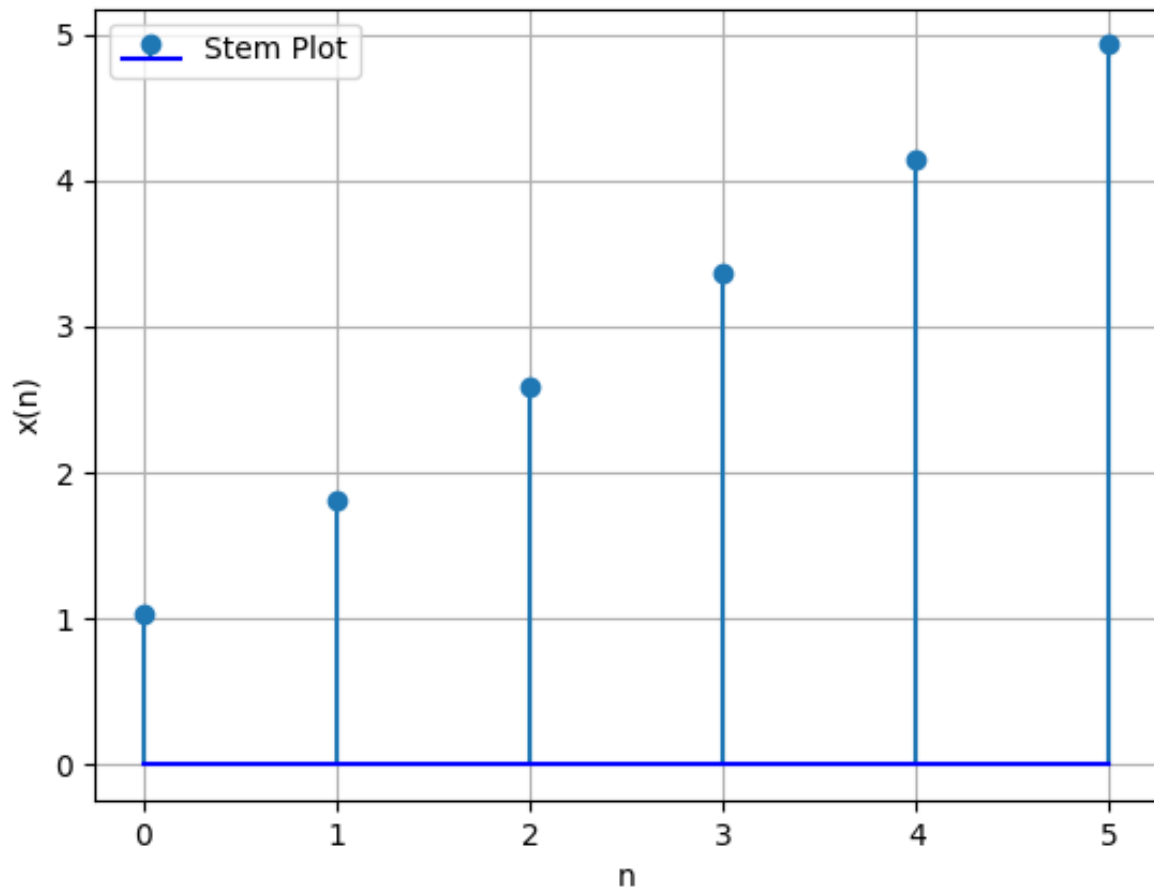


Fig. 1. graph with value of $a = 3, b = 5, c = 7$