Q: If $a\left(\frac{1}{b}+\frac{1}{c}\right)$, $b\left(\frac{1}{c}+\frac{1}{a}\right)$, $c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in arithmetic progression (AP), prove that a, b, c are also in AP.

Solution: Using properties of AP,

$$\begin{aligned}
& a\left(\frac{1}{b}+\frac{1}{c}\right)+1,\ b\left(\frac{1}{c}+\frac{1}{a}\right)+1,\ c\left(\frac{1}{a}+\frac{1}{b}\right)+1 \\
\implies & a\left(\frac{1}{b}+\frac{1}{c}\right)+\frac{a}{a},\ b\left(\frac{1}{c}+\frac{1}{a}\right)+\frac{b}{b},\ c\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{c}{c} \\
\implies & a\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right),\ b\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right),\ c\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \\
\implies & a,\ b,\ c
\end{aligned}$$

Hence proved that a, b, c are in AP.

parameter	value	description
$x(0)$	$a\left(\frac{1}{b}+\frac{1}{c}\right)$	First Term of given AP
d	$\frac{a^2(b-c)+b^2(c-a)}{abc}$	Common Difference of given AP
$x(n)$	$(x(0)+nd)u(n)$	General Term of given AP

TABLE I
INPUT PARAMETER TABLE

From table I

$$X(z) = x(0)\frac{1}{1-z^{-1}} + d\frac{z^{-1}}{(1-z^{-1})^2} \quad |z| > 1 \tag{1}$$

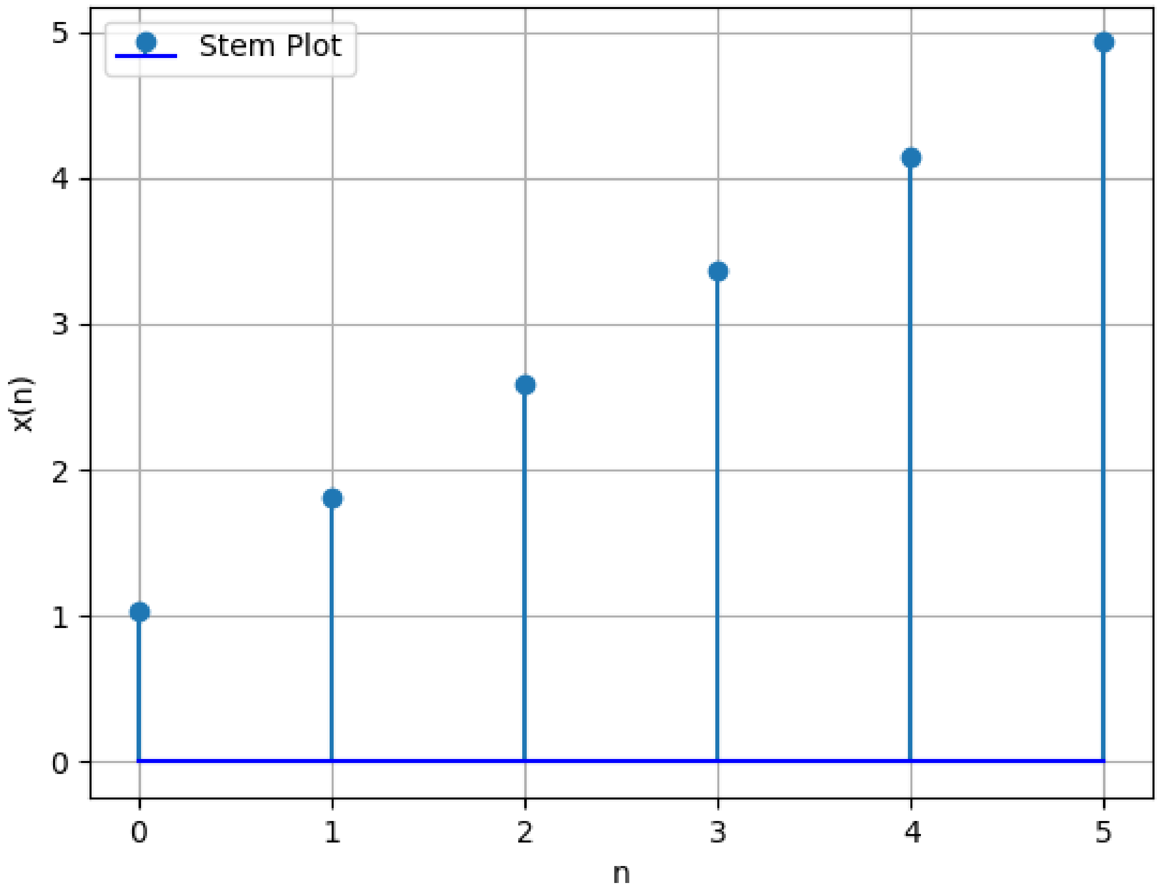

Fig. 1. graph with value of $a = 3, b = 5, c = 7$