Q: If $a\left(\frac{1}{b}+\frac{1}{c}\right)$, $b\left(\frac{1}{c}+\frac{1}{a}\right)$, $c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in arithmetic progression (AP), prove that a, b, c are also in AP.

Solution: Using properties of AP,

$$a\left(\frac{1}{b}+\frac{1}{c}\right)+1,\ b\left(\frac{1}{c}+\frac{1}{a}\right)+1,\ c\left(\frac{1}{a}+\frac{1}{b}\right)+1$$
$$\implies a\left(\frac{1}{b}+\frac{1}{c}\right)+\frac{a}{a},\ b\left(\frac{1}{c}+\frac{1}{a}\right)+\frac{b}{b},\ c\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{c}{c}$$
$$\implies a\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right),\ b\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right),\ c\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$
$$\implies a,\ b,\ c$$

Hence proved that a, b, c are in AP.

parameter	value	description
d_y	-	Common Difference of 2^{nd} AP
$y(n)$	$(y(0)+nd_y)u(n)$	General Term of another AP
$x(0)$	$\frac{2y(0)^2+3y(0)d_y}{y(0)^2+3y(0)d_y+2d_y^2}$	First Term of 1^{st} AP
d_x	$\frac{y(0)^2d_y+y(0)d_y^2+2y(0)d_y+d_y^2}{y(0)(y(0)+d_y)(y(0)+2d_y)}$	Common Difference of 1^{st} AP
$x(n)$	$(x(0)+nd_x)u(n)$	General Term of given AP

TABLE I
INPUT PARAMETER TABLE

From table I

$$X(z)=x(0)\frac{1}{1-z^{-1}}+d_x\frac{z^{-1}}{(1-z^{-1})^2} \quad |z|>1 \tag{1}$$

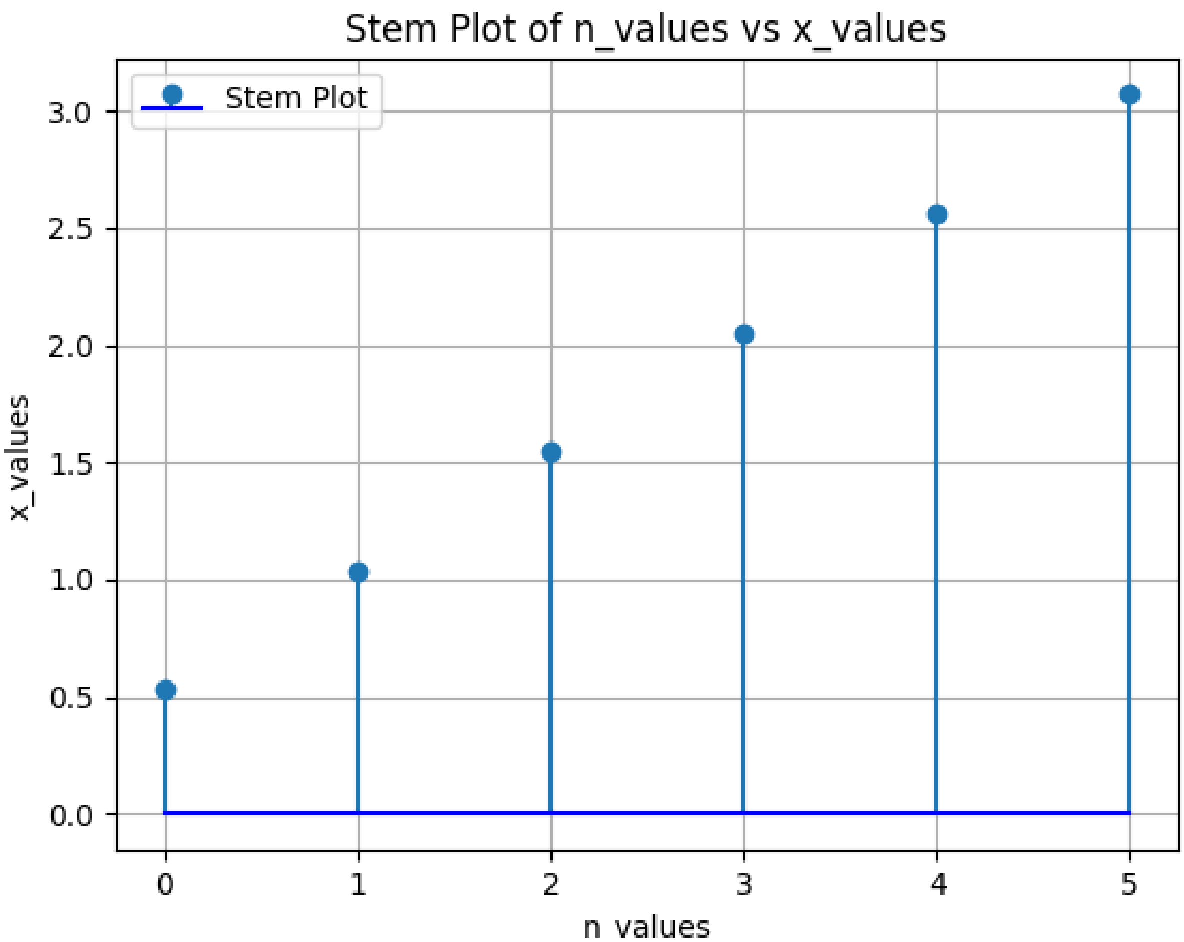
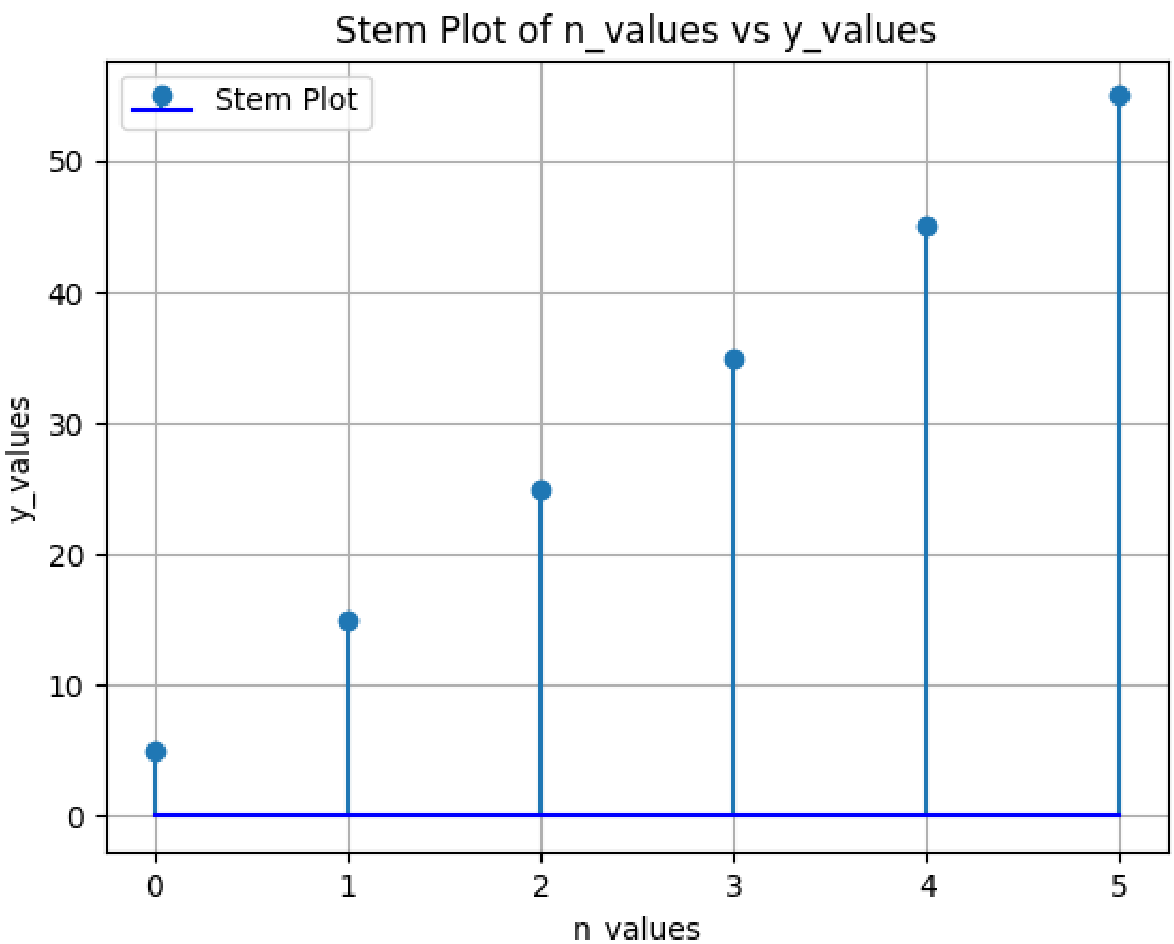


Fig. 1. graph with value of $y(0) = 5, d_y = 10$



Stem Plot of n_values vs y_values
Stem Plot
y_values
n_values
0
10
20
30
40
50
0
1
2
3
4
5