

Maths Assignment

Aditi Dure
EE23BTECH11016

January 6, 2024

Problem Statement

If $a\left(\frac{1}{b} + \frac{1}{c}\right)$, $b\left(\frac{1}{c} + \frac{1}{a}\right)$, $c\left(\frac{1}{a} + \frac{1}{b}\right)$ are in arithmetic progression (AP), prove that a, b, c are also in AP.

Solution

Given $a\left(\frac{1}{b} + \frac{1}{c}\right)$, $b\left(\frac{1}{c} + \frac{1}{a}\right)$, $c\left(\frac{1}{a} + \frac{1}{b}\right)$ are in arithmetic progression (AP)

On adding 1 to every term, using the property of AP, we get

$$\begin{aligned} & a\left(\frac{1}{b} + \frac{1}{c}\right) + 1, b\left(\frac{1}{c} + \frac{1}{a}\right) + 1, c\left(\frac{1}{a} + \frac{1}{b}\right) + 1 \\ & a\left(\frac{1}{b} + \frac{1}{c}\right) + \frac{a}{a}, b\left(\frac{1}{c} + \frac{1}{a}\right) + \frac{b}{b}, c\left(\frac{1}{a} + \frac{1}{b}\right) + \frac{c}{c} \\ & a\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), c\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \end{aligned}$$

Now cancel $\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ from all terms, using the property of AP, we get
 a, b, c are in AP

Hence proved