

Q: If $a\left(\frac{1}{b} + \frac{1}{c}\right)$, $b\left(\frac{1}{c} + \frac{1}{a}\right)$, $c\left(\frac{1}{a} + \frac{1}{b}\right)$ are in arithmetic progression (AP), prove that a, b, c are also in AP.

Solution: Using properties of AP,

$$\begin{aligned}
 & a\left(\frac{1}{b} + \frac{1}{c}\right) + 1, b\left(\frac{1}{c} + \frac{1}{a}\right) + 1, c\left(\frac{1}{a} + \frac{1}{b}\right) + 1 \\
 \Rightarrow & a\left(\frac{1}{b} + \frac{1}{c}\right) + \frac{a}{a}, b\left(\frac{1}{c} + \frac{1}{a}\right) + \frac{b}{b}, c\left(\frac{1}{a} + \frac{1}{b}\right) + \frac{c}{c} \\
 \Rightarrow & a\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right), c\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \\
 \Rightarrow & a, b, c
 \end{aligned}$$

Hence proved that a, b, c are in AP.

parameter	value	description
$x(0)$	$a\left(\frac{1}{b} + \frac{1}{c}\right)$	First Term of given AP
d	$\frac{a^2(b-c)+b^2(c-a)}{abc}$	Common Difference of given AP
$x(n)$	$(x(0) + nd)u(n)$	General Term of given AP

TABLE I
INPUT PARAMETER TABLE

From table I

$$X(z) = x(0)\frac{1}{1 - z^{-1}} + d\frac{z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1 \quad (1)$$

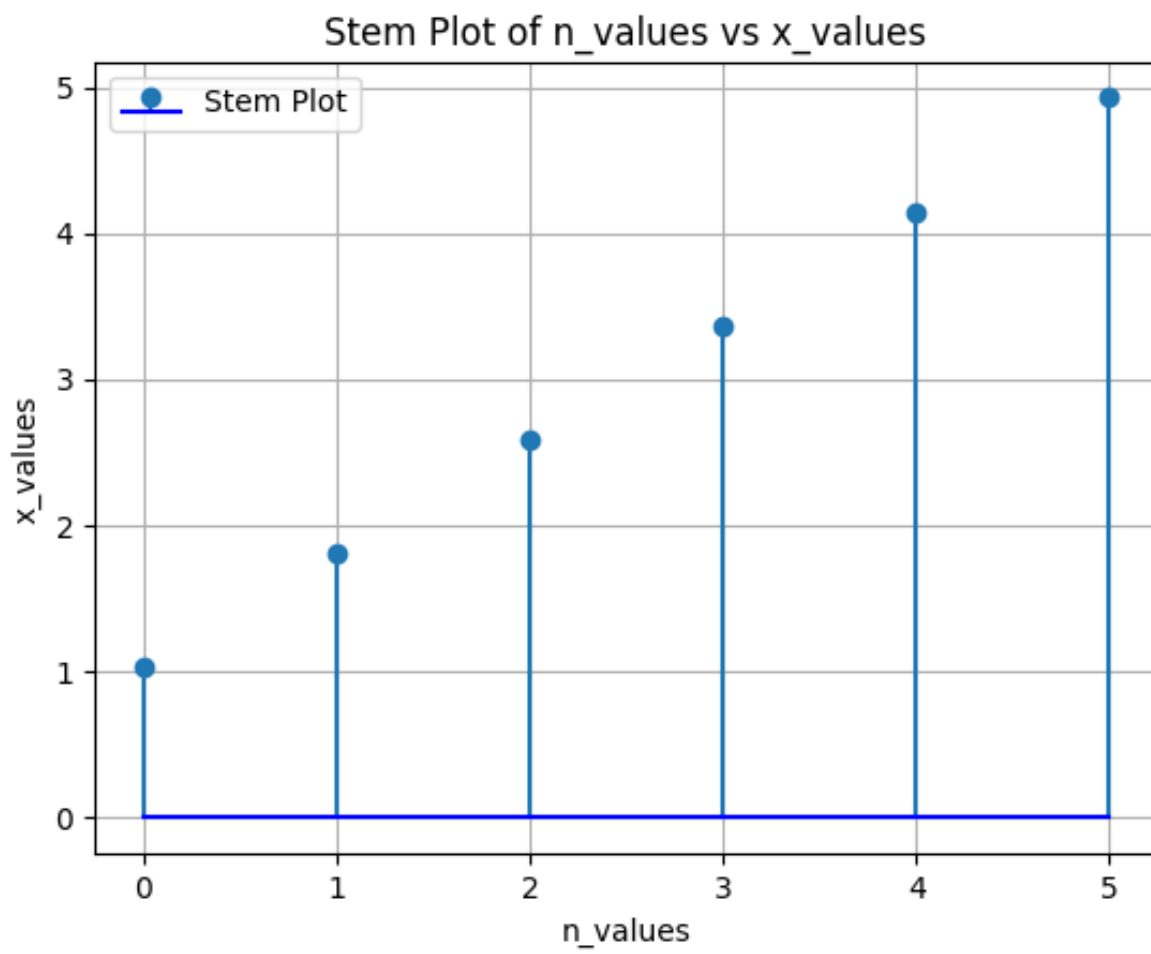


Fig. 1. graph with value of $a = 3, b = 5, c = 7$