Q: The state equation of a second order system is
$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t), \quad \boldsymbol{x}(0)$ is the initial condition.
Suppose λ_1 and λ_2 are two distinct eigenvalues of A, and v_1 and v_2 are the corresponding eigenvectors. For constants α_1 and α_2, the solution, $\boldsymbol{x}(t)$, of the state equation is

(A) $\sum_{i=1}^{2} \alpha_i e^{\lambda_i t} v_{\mathbf{i}}$
(B) $\sum_{i=1}^{2} \alpha_i e^{2\lambda_i t} v_{\mathbf{i}}$
(C) $\sum_{i=1}^{2} \alpha_i e^{3\lambda_i t} v_{\mathbf{i}}$
(D) $\sum_{i=1}^{2} \alpha_i e^{4\lambda_i t} v_{\mathbf{i}}$

Solution:

Using Laplace transform:
Given Equation:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) \tag{1}$$
$$\frac{d\boldsymbol{x}(t)}{dt} = A\boldsymbol{x}(t) \tag{2}$$

Taking Laplace Transform:

$$\mathcal{L}\left(\frac{d\boldsymbol{x}(t)}{dt}\right) = \mathcal{L}\left(A\boldsymbol{x}(t)\right) \tag{3}$$
$$sX(s) - x(0) = AX(s) \tag{4}$$
$$(sI - A)X(s) = x(0) \tag{5}$$
$$X(s) = (sI - A)^{-1}x(0) \tag{6}$$
$$= \sum_{i=1}^{2} \frac{1}{s - \lambda_i} x(0) \tag{7}$$

Now take inverse Laplace Transform

$$\mathcal{L}^{-1}\left(X(s)\right) = \mathcal{L}^{-1}\left(\sum_{i=1}^{2} \frac{1}{s - \lambda_i} x(0)\right) \tag{8}$$
$$x(t) = \sum_{i=1}^{2} e^{\lambda_i t} x(0) \tag{9}$$
$$= \sum_{i=1}^{2} \alpha_i e^{\lambda_i t} v_i \tag{10}$$

Hence the answer is option (A).