GATE-2022

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Question

The block diagram of a two-tap high-pass FIR filter is shown below. The filter transfer function is given by $H(z) = \frac{Y(z)}{X(z)}$.
If the ratio of maximum to minimum value of $H(z)$ is 2 and $|H(z)|_{max} = 1$, the coefficients β_0 and β_1 are ________________ and ________________, respectively.

Block Diagram Given in Question

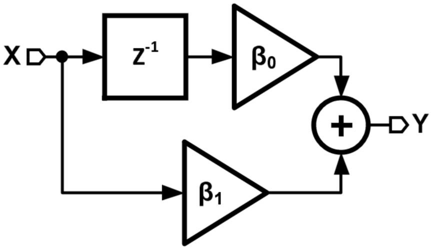

Figure: Block diagram

Options

1. 0.75, -0.25
2. 0.67, 0.33
3. 0.60, -0.40
4. -0.64, 0.36

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Solution – Input Parameter Table

Variable	Description	Value
$H(z)$	Transfer Function	$\beta_0 z^{-1} + \beta_1$
$\lvert H(z)\rvert_{max}$	Maximum value of Transfer Function	1
$\lvert H(z)\rvert_{min}$	Minimum value of Transfer Function	$\frac{1}{2}$

Table: input parameters

Solution – Continued

Since $H(z)$ is complex, on using Triangle Inequality, we get

$$|x+y| \leq |x| + |y| \tag{1}$$

And its corollary

$$||x| - |y|| \leq |x+y| \tag{2}$$

where x and y are complex numbers.

Solution – Continued

$$\left||z^{-1}\beta_0| - |\beta_1|\right| \leq \left|z^{-1}\beta_0 + \beta_1\right| \leq \left|z^{-1}\beta_0\right| + |\beta_1| \tag{3}$$

From Table 1

$$\left||z^{-1}\beta_0| - |\beta_1|\right| \leq |H(z)| \leq \left|z^{-1}\beta_0\right| + |\beta_1| \tag{4}$$

As we know that $|z| = 1$,

$$||\beta_0| - |\beta_1|| \leq |H(z)| \leq |\beta_0| + |\beta_1| \tag{5}$$

So, we can conclude that

$$|H(z)|_{max} = |\beta_0| + |\beta_1| \tag{6}$$

Now from Table 1

$$1 = |\beta_0| + |\beta_1| \tag{7}$$

Similarly,

$$\frac{1}{2} = ||\beta_0| - |\beta_1|| \tag{8}$$

Solution – Continued

On solving (7) and (8), we get

$$|\beta_0| = 0.75, |\beta_1| = 0.25 \tag{9}$$

OR

$$|\beta_0| = 0.25, |\beta_1| = 0.75 \tag{10}$$

Hence the correct answer is option (A)