Q: The block diagram of a two-tap high-pass FIR filter is shown below. The filter transfer function is given by $H(z) = Y(z)/X(z)$.
If the ratio of maximum to minimum value of $H(z)$ is 2 and $|H(z)|_{max} = 1$, the coefficients β_0 and β_1 are _______________ and _______________, respectively.

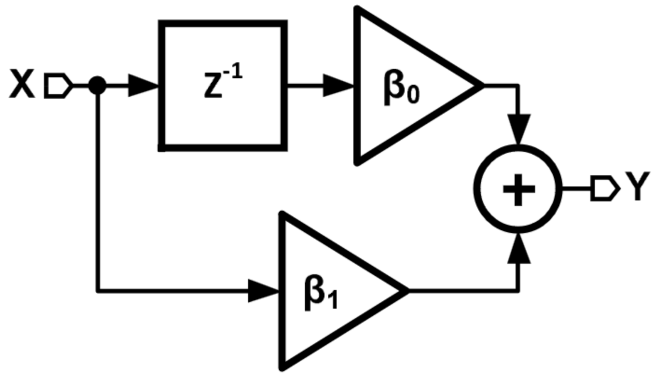

Fig. 1. Block diagram

(A) 0.75, -0.25
(B) 0.67, 0.33
(C) 0.60, -0.40
(D) -0.64, 0.36

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Solution:

Variable	Description	Value
$H(z)$	Transfer Function	$\beta_0 z^{-1} + \beta_1$
$\|H(z)\|_{max}$	Maximum value of Transfer Function	1
$\|H(z)\|_{min}$	Minimum value of Transfer Function	$\frac{1}{2}$

TABLE I
INPUT PARAMETERS

As z lies on a unit circle (in argand plane),

$$|z| = 1 \tag{1}$$
$$\implies \left|z^{-1}\right| = 1 \tag{2}$$

Since $H(z)$ is complex, on using Triangle Inequality, we get

$$|x + y| \leq |x| + |y| \tag{3}$$

And its corollary

$$||x| - |y|| \leq |x + y| \tag{4}$$

where x and y are complex numbers.

$$\left|\left|z^{-1}\beta_0\right| - |\beta_1|\right| \leq \left|z^{-1}\beta_0 + \beta_1\right| \leq \left|z^{-1}\beta_0\right| + |\beta_1| \tag{5}$$

from Table I

$$\left|\left|z^{-1}\beta_0\right| - |\beta_1|\right| \leq |H(z)| \leq \left|z^{-1}\beta_0\right| + |\beta_1| \tag{6}$$

From (2)

$$||\beta_0| - |\beta_1|| \leq |H(z)| \leq |\beta_0| + |\beta_1| \tag{7}$$

Indivisually, we have

$$|H(z)|_{max} \leq |\beta_0| + |\beta_1| \tag{8}$$

Now from Table I

$$1 \leq |\beta_0| + |\beta_1| \tag{9}$$

Similarly,

$$\frac{1}{2} \geq ||\beta_0| - |\beta_1|| \tag{10}$$

When we verify the options, only one options satisfy both (9) and (10), i.e. option (A)
Infact it satisfies the equality in corollary of triangle inequality.