

Q: The block diagram of a two-tap high-pass FIR filter is shown below. The filter transfer function is given by  $H(z) = Y(z)/X(z)$ .

If the ratio of maximum to minimum value of  $H(z)$  is 2 and  $|H(z)|_{\max} = 1$ , the coefficients  $\beta_0$  and  $\beta_1$  are \_\_\_\_\_ and \_\_\_\_\_, respectively.

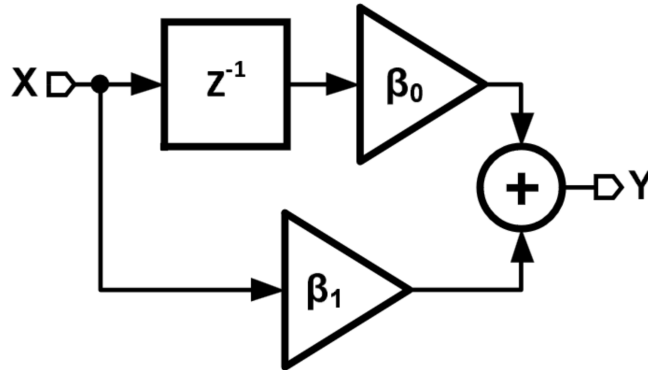


Fig. 1. Block diagram

- (A) 0.75, -0.25
- (B) 0.67, 0.33
- (C) 0.60, -0.40
- (D) -0.64, 0.36

GATE BM 2022

**Solution:**

Variable	Description	Value
$H(z)$	Transfer Function	$\beta_0 z^{-1} + \beta_1$
$ H(z) _{\max}$	Maximum value of Transfer Function	1
$ H(z) _{\min}$	Minimum value of Transfer Function	$\frac{1}{2}$

TABLE I  
INPUT PARAMETERS

As  $z$  lies on a unit circle (in argand plane),

$$|z| = 1 \quad (1)$$

$$\Rightarrow |z^{-1}| = 1 \quad (2)$$

Since  $H(z)$  is complex, on using Triangle Inequality, we get

$$|x + y| \leq |x| + |y| \quad (3)$$

And its corollary

$$||x| - |y|| \leq |x + y| \quad (4)$$

where  $x$  and  $y$  are complex numbers.

$$||z^{-1}\beta_0| - |\beta_1|| \leq |z^{-1}\beta_0 + \beta_1| \leq |z^{-1}\beta_0| + |\beta_1| \quad (5)$$

From Table I

$$||z^{-1}\beta_0| - |\beta_1|| \leq |H(z)| \leq |z^{-1}\beta_0| + |\beta_1| \quad (6)$$

From (2)

$$||\beta_0| - |\beta_1|| \leq |H(z)| \leq |\beta_0| + |\beta_1| \quad (7)$$

So, we can conclude that

$$|H(z)|_{\max} = |\beta_0| + |\beta_1| \quad (8)$$

Now from Table I

$$1 = |\beta_0| + |\beta_1| \quad (9)$$

Similarly,

$$\frac{1}{2} = ||\beta_0| - |\beta_1|| \quad (10)$$

On solving (9) and (10), we get

$$|\beta_0| = 0.75 \quad (11)$$

$$|\beta_1| = 0.25 \quad (12)$$

OR

$$|\beta_0| = 0.25 \quad (13)$$

$$|\beta_1| = 0.75 \quad (14)$$

Hence the correct answer is option (A)