GATE-2022

EE23BTECH11016 - Aditi Dure*

Question

The block diagram of a two-tap high-pass FIR filter is shown below. The filter transfer function is given by $H(z) = \frac{Y(z)}{X(z)}$.
If the ratio of maximum to minimum value of $H(z)$ is 2 and $|H(z)|_{max} = 1$, the coefficients β_0 and β_1 are ________________ and ________________, respectively.

Block Diagram Given in Question

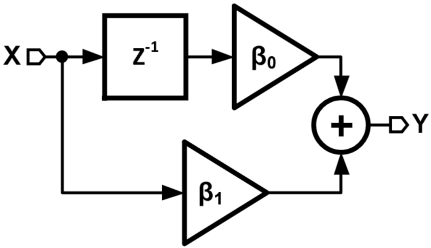

Figure: Block diagram

Options

1. 0.75, -0.25
2. 0.67, 0.33
3. 0.60, -0.40
4. -0.64, 0.36

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Results and Proofs

Time Shift Property:

$$x(n) \overset{\mathcal{Z}}{\longleftrightarrow} X(z) \tag{1}$$

$$x(n-n_0) \overset{\mathcal{Z}}{\longleftrightarrow} z^{-n_0}X(z) \tag{2}$$

Proof

Let

$$y(n) = x(n - n_0) \tag{3}$$

Taking z-transform

$$\mathcal{Z}\left(y(n)\right) = \mathcal{Z}\left(x(n - n_0)\right) \tag{4}$$

Proof – Continued

Simplifying LHS

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} \tag{5}$$

From (3)

$$Y(z) = \sum_{n=-\infty}^{\infty} x(n - n_0) z^{-n} \tag{6}$$

Let

$$n - n_0 = s \implies n = s + n_0 \tag{7}$$

From (6) and (7)

$$Y(z) = \sum_{s=-\infty}^{\infty} x(s) z^{-(s+n_0)} \tag{8}$$

$$= z^{-n_0} \sum_{s=-\infty}^{\infty} x(s) z^{-s} \tag{9}$$

Proof – Continued

As variable in Z-transform is dummy, on replacing it, we get

$$Y(z) = z^{-n_0} \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{10}$$

$$= z^{-n_0} X(z) \tag{11}$$

From (4) and (11)

$$\mathcal{Z}\left(x(n - n_0)\right) = z^{-n_0} X(z) \tag{12}$$

Hence Proved

Result

$$z^{-n_0} X(z) \overset{\mathcal{Z}^{-}}{\longleftrightarrow} x(n - n_0) \tag{13}$$

Variable	Description	Value
$H(z)$	Transfer Function	$\beta_0 z^{-1} + \beta_1$
$\lvert H(z)\rvert_{max}$	Maximum value of Transfer Function	1
$\lvert H(z)\rvert_{min}$	Minimum value of Transfer Function	$\frac{1}{2}$

Table: input parameters

Solution – Continued

In (13), put

$$n_0 = 1, \quad x(n) = \delta(n)$$

Since

$$1 \overset{\mathcal{Z}^-}{\longleftrightarrow} \delta(n)$$

$$z^{-1} \overset{\mathcal{Z}^-}{\longleftrightarrow} \delta(n-1) \tag{14}$$

This is a unit delay in discrete time and represents unit amplitude sinosoidal signal.
So,

$$z^{-1} = e^{-jw} \tag{15}$$

$$\implies \left|z^{-1}\right| = 1 \tag{16}$$

Solution – Continued

Since $H(z)$ is complex, on using Triangle Inequality, we get

$$|x+y| \leq |x| + |y| \tag{17}$$

And its corollary

$$||x| - |y|| \leq |x+y| \tag{18}$$

where x and y are complex numbers.

$$\left||z^{-1}\beta_0| - |\beta_1|\right| \leq \left|z^{-1}\beta_0 + \beta_1\right| \leq \left|z^{-1}\beta_0\right| + |\beta_1| \tag{19}$$

From Table 1

$$\left||z^{-1}\beta_0| - |\beta_1|\right| \leq |H(z)| \leq \left|z^{-1}\beta_0\right| + |\beta_1| \tag{20}$$

From (16)

$$||\beta_0| - |\beta_1|| \leq |H(z)| \leq |\beta_0| + |\beta_1| \tag{21}$$

Solution – Continued

So, we can conclude that

$$|H(z)|_{max} = |\beta_0| + |\beta_1| \tag{22}$$

Now from Table 1

$$1 = |\beta_0| + |\beta_1| \tag{23}$$

Similarly,

$$\frac{1}{2} = ||\beta_0| - |\beta_1|| \tag{24}$$

On solving (23) and (24), we get

$$|\beta_0| = 0.75, |\beta_1| = 0.25 \tag{25}$$

OR

$$|\beta_0| = 0.25, |\beta_1| = 0.75 \tag{26}$$

Hence the correct answer is option (A)