Q: The block diagram of a two-tap high-pass FIR filter is shown below. The filter transfer function is given by $H(z) = Y(z)/X(z)$.
If the ratio of maximum to minimum value of $H(z)$ is 2 and $|H(z)|_{max} = 1$, the coefficients β_0 and β_1 are _______________ and _______________, respectively.

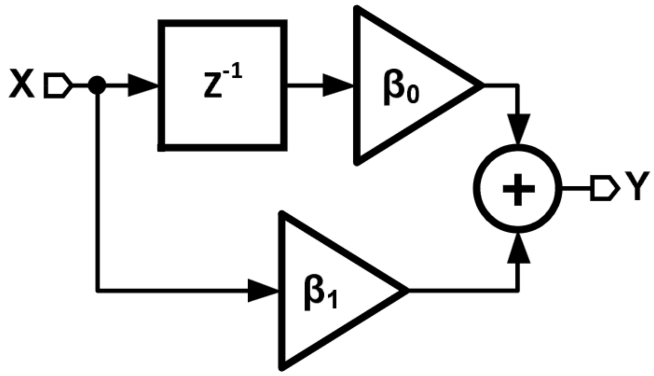

Fig. 1. Block diagram

(A) 0.75, -0.25
(B) 0.67, 0.33
(C) 0.60, -0.40
(D) -0.64, 0.36

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Solution:

Results and Proofs:

Time Shift Property:

$$x(n) \xleftrightarrow{\mathcal{Z}} X(z) \tag{1}$$

$$x(n-n_0) \xleftrightarrow{\mathcal{Z}} z^{-n_0}X(z) \tag{2}$$

Proof:
Let

$$y(n) = x(n-n_0) \tag{3}$$

Taking z-transform

$$\mathcal{Z}(y(n)) = \mathcal{Z}(x(n-n_0)) \tag{4}$$

$$\tag{5}$$

Simplifying LHS

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)z^{-n} \tag{6}$$

From (3)

$$Y(z) = \sum_{n=-\infty}^{\infty} x(n-n_0)z^{-n} \tag{7}$$

Let

$$n - n_0 = s \tag{8}$$

$$\implies n = s + n_0 \tag{9}$$

From (7) and (9)

$$Y(z) = \sum_{s=-\infty}^{\infty} x(s) z^{-(s+n_0)} \tag{10}$$

$$= z^{-n_0} \sum_{s=-\infty}^{\infty} x(s) z^{-s} \tag{11}$$

As variable in Z-transform is dummy, on replacing it, we get

$$Y(z) = z^{-n_0} \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{12}$$

$$= z^{-n_0} X(z) \tag{13}$$

From (4) and (13)

$$\mathcal{Z}\left(x(n - n_0)\right) = z^{-n_0} X(z) \tag{14}$$

Hence proved
<u>Result:</u>

$$z^{-n_0} X(z) \overset{\mathcal{Z}^{-}}{\longleftrightarrow} x(n - n_0) \tag{15}$$

Solution:

Variable	Description	Value
$H(z)$	Transfer Function	$\beta_0 z^{-1} + \beta_1$
$\lvert H(z)\rvert_{max}$	Maximum value of Transfer Function	1
$\lvert H(z)\rvert_{min}$	Minimum value of Transfer Function	$\frac{1}{2}$

TABLE I
INPUT PARAMETERS

In (15), put

$$n_0 = 1, \quad x(n) = \delta(n)$$

Since

$$1 \overset{\mathcal{Z}^{-}}{\longleftrightarrow} \delta(n)$$

$$z^{-1} \overset{\mathcal{Z}^{-}}{\longleftrightarrow} \delta(n - 1) \tag{16}$$

This is a unit delay in discrete time and represents unit amplitude sinosoidal signal.
So,

$$z^{-1} = e^{-jw} \tag{17}$$

$$\implies \left|z^{-1}\right| = 1 \tag{18}$$

Since $H(z)$ is complex, on using Triangle Inequality, we get

$$|x + y| \le |x| + |y| \tag{19}$$

And its corollary

$$||x| - |y|| \leq |x + y| \tag{20}$$

where x and y are complex numbers.

$$\left|\left|z^{-1}\beta_0\right| - |\beta_1|\right| \leq \left|z^{-1}\beta_0 + \beta_1\right| \leq \left|z^{-1}\beta_0\right| + |\beta_1| \tag{21}$$

From Table I

$$\left|\left|z^{-1}\beta_0\right| - |\beta_1|\right| \leq |H(z)| \leq \left|z^{-1}\beta_0\right| + |\beta_1| \tag{22}$$

From (18)

$$||\beta_0| - |\beta_1|| \leq |H(z)| \leq |\beta_0| + |\beta_1| \tag{23}$$

So, we can conclude that

$$|H(z)|_{max} = |\beta_0| + |\beta_1| \tag{24}$$

Now from Table I

$$1 = |\beta_0| + |\beta_1| \tag{25}$$

Similarly,

$$\frac{1}{2} = ||\beta_0| - |\beta_1|| \tag{26}$$

On solving (25) and (26), we get

$$|\beta_0| = 0.75, |\beta_1| = 0.25 \tag{27}$$

OR

$$|\beta_0| = 0.25, |\beta_1| = 0.75 \tag{28}$$

Hence the correct answer is option (A)