MVC Problem Set: Integration (Ch14) and Optimization **Name________________________**

Rules: You are to work on this problem set entirely on your own. You may not consult any internet resources at all. You may (and are expected, in fact) to use your textbook. You may ask questions of Mr. Corica. You may use a calculator (assuming you show supporting work) and you may use Maple, but if you use Maple you must submit your Maple sheets electronically.

Violation of the rules is considered academic dishonesty. As you know, violations of academic dishonesty are treated seriously, and reported to colleges (Ugh!). If you have any uncertainty at all about what the rules are, consult with Mr. Corica before doing anything that would put you at risk. **DUE DATE: Saturday, Feb. 11, noon.**

Please sign here to indicate that you have read and understand the rules: __________________________________

1. Consider the iterated integral $\int_0^9 \int_{\sqrt{x}}^3 7dy\,dx$

a) Sketch the region of integration.
b) Switch the order of integration
c) Evaluate the resulting integral from (b), showing all steps.

2. Consider the paraboloid $z = x^2 + y^2$ over the region $0 \leq x \leq 4$, $0 \leq y \leq 2$. Divide the region into four smaller rectangular regions and, using the value of the function at the middle of each region, generate an approximation for the volume of the object below the paraboloid in the first octant. Show all your work.

3. Set up, but do not evaluate, an expression giving the volume of the region bounded by the equations $z = x^2 + 2y^2$ and $z = 4y$.

4. Find the average value of the function $f(x, y) = 2xy$ over the triangular region with vertices (0,0), (5,0), (5,3).

5. A lamina is formed in the xy-plane by the region between the x-axis and a semi-circle of radius 4. The density per unit area of the lamina at any point is given by $\rho = 2y$. That is, the lamina does not have uniform density.
    a. Set up, but do not integrate, an expression in rectangular coordinates for the mass of this lamina. BE sure your setup is clear – it should be possible to enter it, as-is, into a CAS system like Maple to get the result.
    b. Set up, but do not integrate, an expression in polar coordinates for the mass of this lamina.
    c. Set up, but do not evaluate, an expression that will yield the y-coordinate of the center of mass of the lamina. You may use any coordinate system you wish (rectangular, polar, etc.).

6. In class we found equations for computing the slope and intercept of the least squares line through a set of data points. Consider the least-squares parabola through a set of data points. To simplify the problem, We'll look only for the least-squares parabola of the form $f(x) = ax^2 + bx$. That is, we are looking for a parabola that passes through the origin and has the smallest total of the squares of the deviations. Assume there are three data points, (1,1), (2,3), and (4, 12). Find the least-squares parabola, using the techniques of Ch13. A bonus point if you can find an expression for the least squares parabola through n points, (x_1, y_1), $(x_2, y_2)\ldots(x_ny_n)$.

7. The method of Lagrange Multipliers is a clever idea that Lagrange invented when he was 19 years old. Since you are all around that age, and are very capable, I expect that you would at least be able to understand it from a textbook explanation, and apply it! The method is described in section 13.10. Read this material over, particularly looking through the examples. Then submit a careful solution, using this method, to the problem of finding the minimum value of $f(x, y) = x^2 - 14x + y^2 - 10y - 8$ subject to the constraint that $x + y = 10$. Give the coordinates of the minimum point.