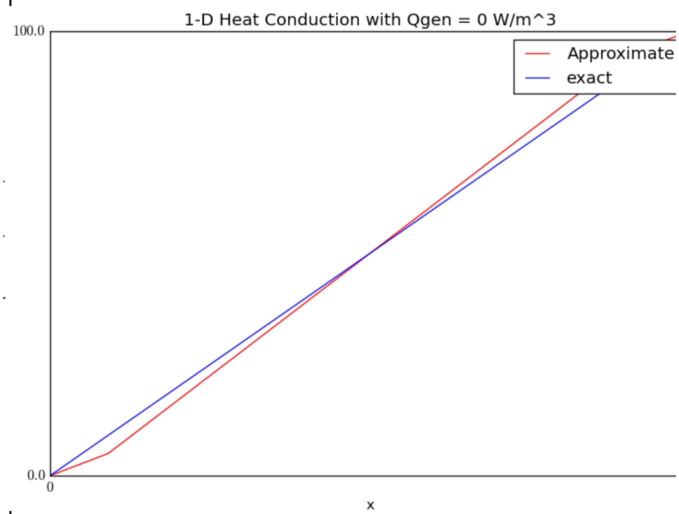
ASSIGNEMENT 1 – ME 415

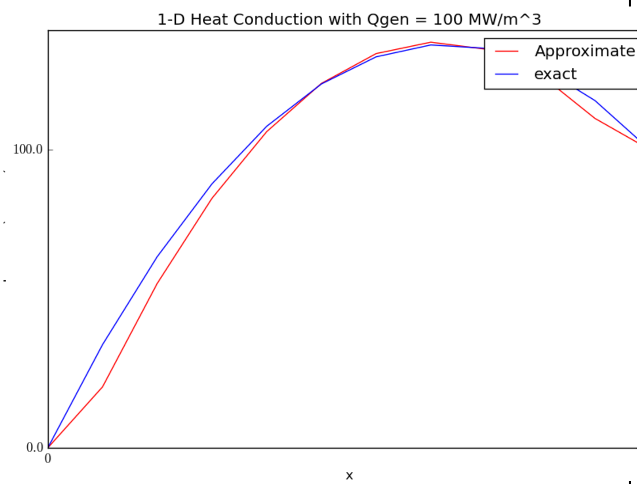
Answer Sheet

Problem # 1: Flux based methodology, with explicit method and uniform grid: 1D Conduction

Plot the steady state temperature profiles with and without volumetric heat generation and compare with the exact solution. (2 figures).



(a)



(b)

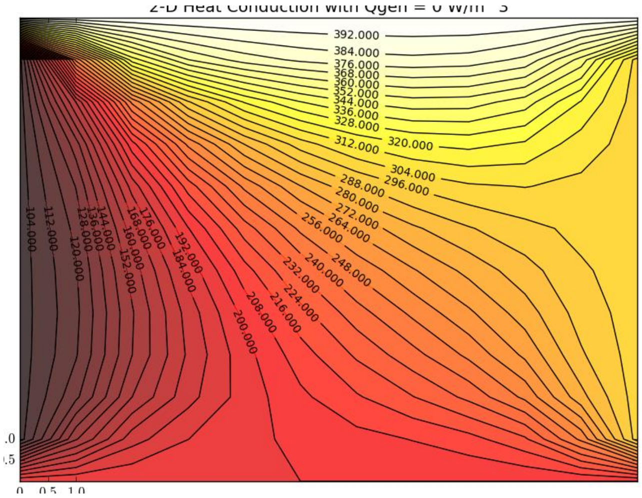
Fig. 1.1: Testing of the explicit method based 1D code on uniform Cartesian grid, for unsteady state heat conduction, (a) without and (b) with heat generation. The figure shows comparison of the numerical with the analytical solution for steady state temperature profile.

Discussion on the Fig. 1.1: Write your answer, limited inside this text box only

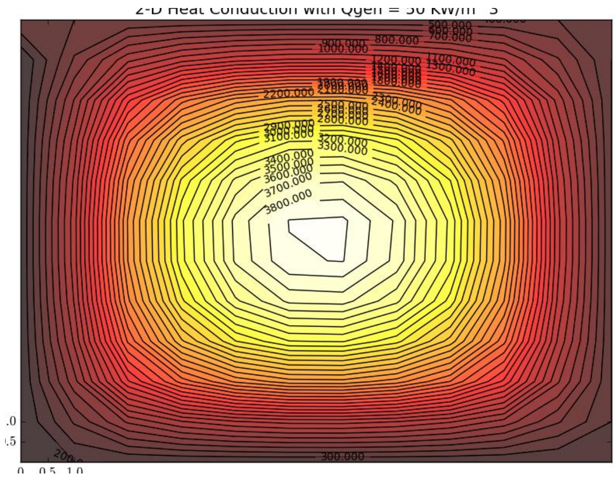
For 1-Dimensional steady state conduction with no heat generation, the equation follows as d2T/dx2 = 0 , which is an equation of a straight line. The linear equation which relates Temperature and distance from the left wall is given by : (T - T_wb) /(T_eb - T_wb)= x/L , where T_wb and T_eb are constant temperatures at left and right wall respectively.
The deviation from the actual solution occurs because we have taken small number of grid points, although epsilon_st is taken to be small. As we increase imax(grid-points in x-direction), or decrease epsilon_st, the numerical solution approaches more towards the exact solution.
If constant heat generation takes place inside the 1-D sheet, the equation for steady state heat conduction is given by d2t/dx2 = -q/k , where q is the volumetric heat generation.
On integrating, the equation for temperature can be given as-
T = -q/(2*k)*(x**2) + (T_eb - T_wb + (q/(2*k)*L*L))*x/L + T_wb
where, q is the q is the volumetric heat generation, T_wb and T_eb are constant temperatures at left and right wall respectively.

Problem # 2: Flux based methodology, with explicit method and uniform grid: 2D Conduction

Plot the steady state temperature contours with and without volumetric heat generation. (2 figures).



(a)



(b)

Fig. 1.2: CFD Application of the explicit method based 2D code on uniform Cartesian grid: Steady state temperature contour for unsteady state heat conduction, (a) without and (b) with heat generation.

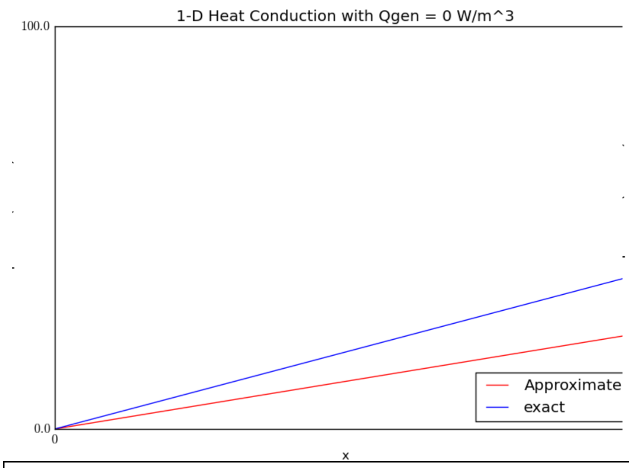
Discussion on the Fig. 1.2: Write your answer, limited inside this text box only

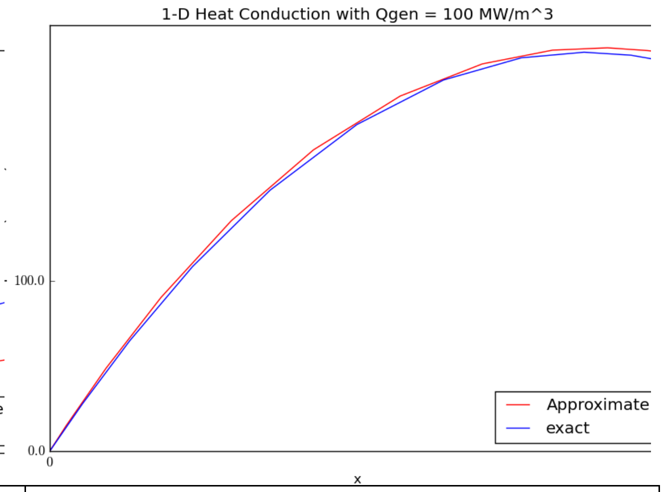
The first figure shows steady state temperature contours for a 2D plate for unsteady state heat conduction without heat generation. The figure is developed using flux based implicit method.
Because of less number of grid points, the contours are not smooth. As can be seen from the figure, The contours become parallel at the edges since temperature is constant at the walls. (100degees at left wall, 200 degrees at bottom wall, 300 degrees at right wall and 400 degrees at the top wall.

The second figure shows steady state temperature contours for a 2D plate for unsteady state heat conduction with heat generation of 50kW/m2. The temperature achieved is higher in this case because of heat generation and it can also be seen that highest temperature occurs at centre of the plate. In this case too, the contours become parallel at the edges due to constant wall temperature along the 4 sides.
As jmax and imax are increased(i.e. the numbers of grid points) , the contours become more smooth but the computation time also increases.

Problem # 3: Coefficient of LAEs based methodology, with implicit method and non-uniform grid: 1D conduction

Plot the steady state temperature profiles with and without volumetric heat generation and compare with the exact solution. (2 figures).





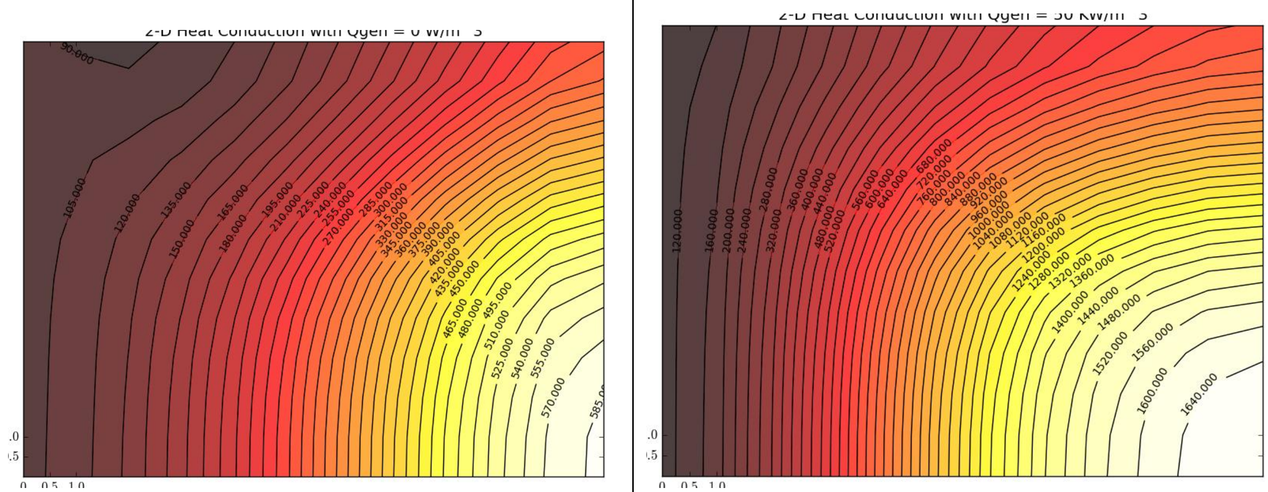
Discussion on the Fig. 1.3: Write your answer, limited inside this text box only

The first plot depicts the steady state of 1D plate for unsteady heat conduction without heat generation on a non uniform grid. The left wall is kept at constant temperature while the right wall is exposed to convective heat transfer with h=1000 W/m2-K and T_inf = 100 degrees.
The deviation in the solution from numerical method and analytical solution occurs because of the approximation that we have taken. Also the number of grid points are smaller.
However, more number of grid points will increase the computation time.

The second plot depicts the steady state of 1D plate for unsteady heat conduction with heat generation of 100 MW/m2 on a non uniform grid. The left wall is kept at constant temperature while the right wall is exposed to convection with h=1000 W/m2-K and Tinf = 100 degrees. As can be seen, the numerical solution fits the analytical solution closely with some deviation.
Temperature rises near the centre due to heat generation, attains a maximum at some value and then drops again to 100 degrees at the right wall because of constant temperature boundary condition.

Problem # 4: Coefficient of LAEs based methodology, with implicit method and non-uniform grid: 2D conduction

Plot the steady state temperature profiles with and without volumetric heat generation. (2 figures).



(a) (b)

Fig. 1.4: CFD Application of the implicit method based 2D code on non-uniform Cartesian grid: Steady state temperature contour for unsteady state heat conduction, (a) without and (b) with heat generation.

Discussion on the Fig. 1.4: Write your answer, limited inside this text box only

The first figure shows the contour plots for temperature at the steady state for unsteady heat conduction without heat generation on a non-uniform grid. The left wall is at constant temperature, therefore the contour lines are parallel to the wall. The bottom wall is insulated, thus the contour lines fall perpendicular to the wall (dT/dx = 0). The right wall is given a constant flux of 10kW/m2 and thus dT/dx = constant = 10kW/m2 . The top wall is exposed to convective heat transfer with h=100 W/m2 -K and T_inf = 30 degrees, hence the slope of contour is a linear function of temperature.

The second figure shows the contour plots for temperature at the steady state for unsteady heat conduction with heat generation of 50kW/m2 on a non-uniform grid with same boundary conditions as above. Thus the pattern of contour lines is similar. However, higher temperatures are achieved for this case in the centre of the plate because of heat generation.
Plot can be made more accurate by reducing time step, reducing epsilon or increasing number of grid points.