AE 622 - Computing of high speed flows
Assignment 2: Report
Numerical viscocity and Shock capturing

Vinod Kumar Metla - 130010048
Aditi Taneja - 13D100026

Introduction

Normal shock tube is simulated in octave with initial conditions as -

$$\begin{bmatrix}\rho_l\\ u_l\\ p_l\\ T_l\end{bmatrix} = \begin{bmatrix}1.0\\ 10.0\\ 0.7148\\ 1.0\end{bmatrix}, \begin{bmatrix}\rho_r\\ u_r\\ p_r\\ T_r\end{bmatrix} = \begin{bmatrix}5.7143\\ 1.75\\ 83.273\\ 20.387\end{bmatrix}$$

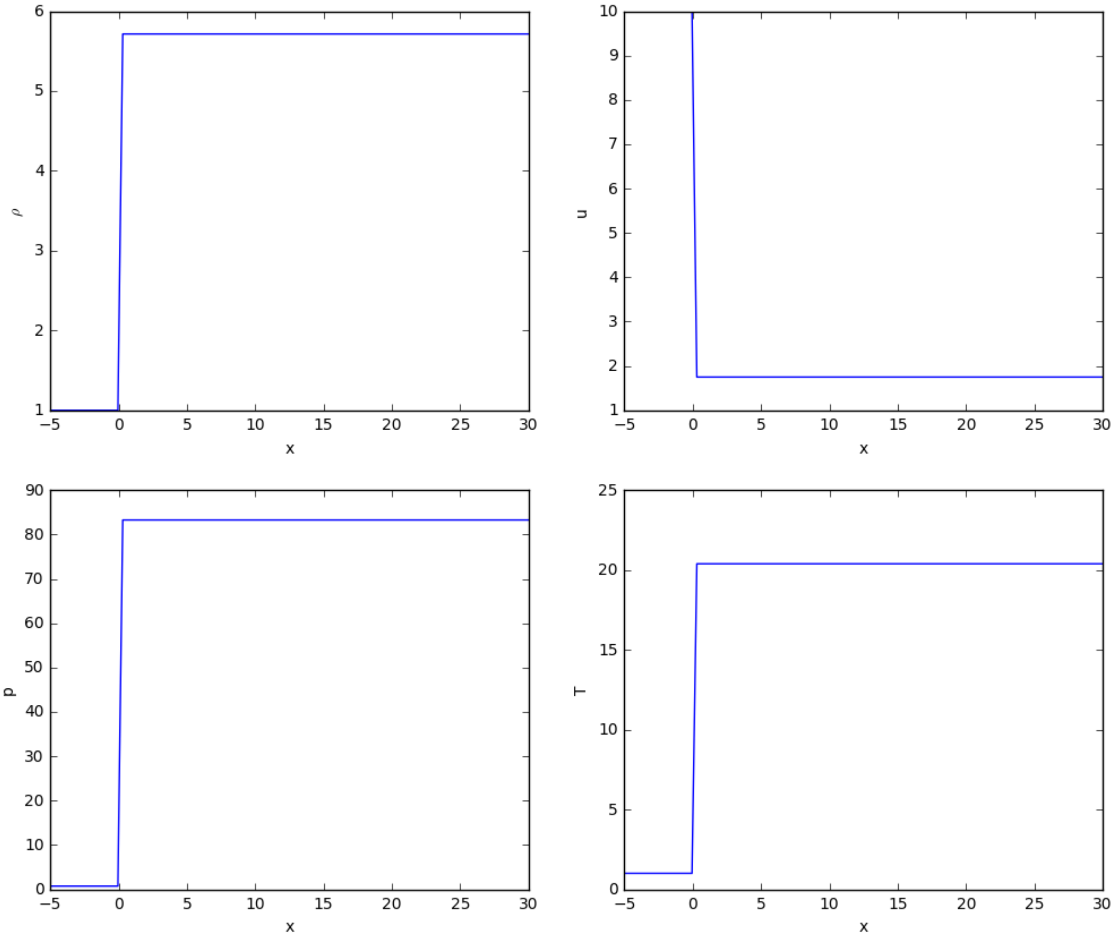
Simulations are carried out by varying alpha, Grid size, and CFL number. Their effects on ρ, u, p, T are shown in the graphs with corresponding L2 errors plots in u. All values are non-dimensional.
Exact solution is plotted below for the same initial conditions.

Dependencies

1. Pdflatex
2. Octave
3. Numpy
4. Matplotlib

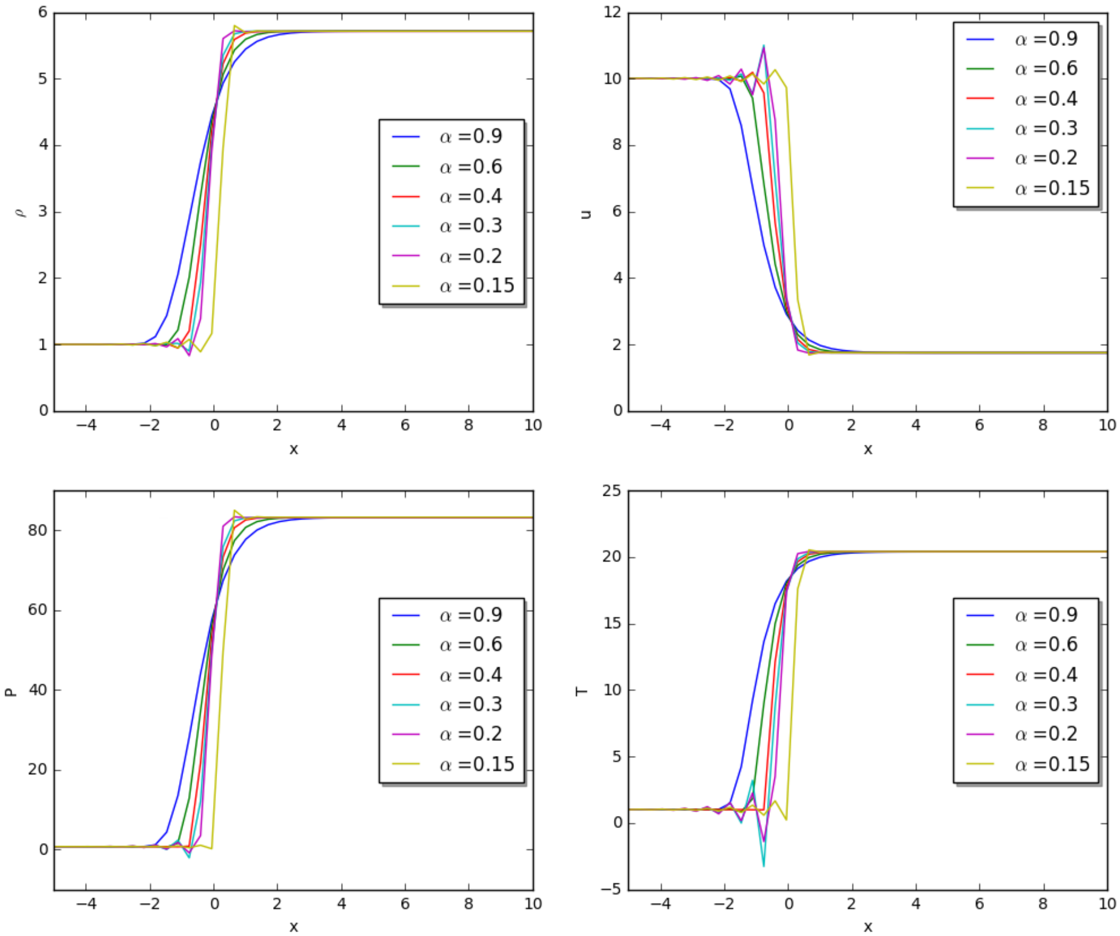
Exact Solution

The exact solution for a normal shock for the initial conditions given above:



Varying α

Following Plots were obtained for $\alpha = [0.9, 0.6, 0.4, 0.3, 0.2, 0.15]$
CFL number was kept constant at 0.2
Grid Size = 100
Mach Number = 10.0 (Hypersonic)



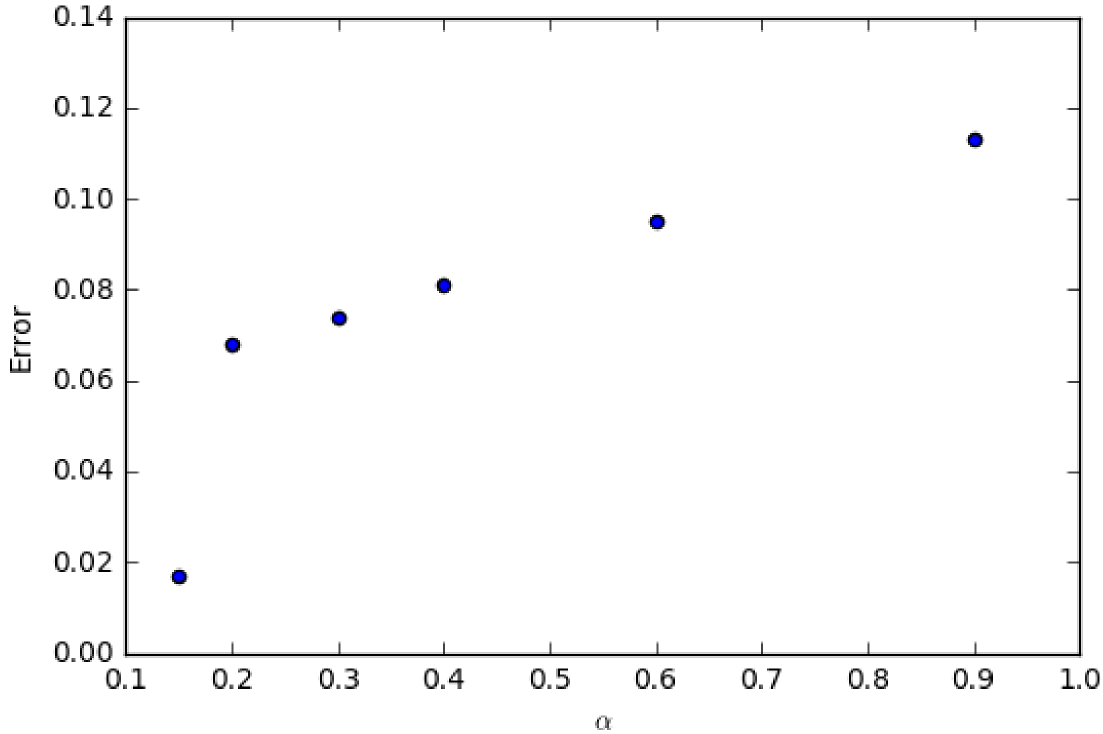


Figure 1: L2 error in u with varying α

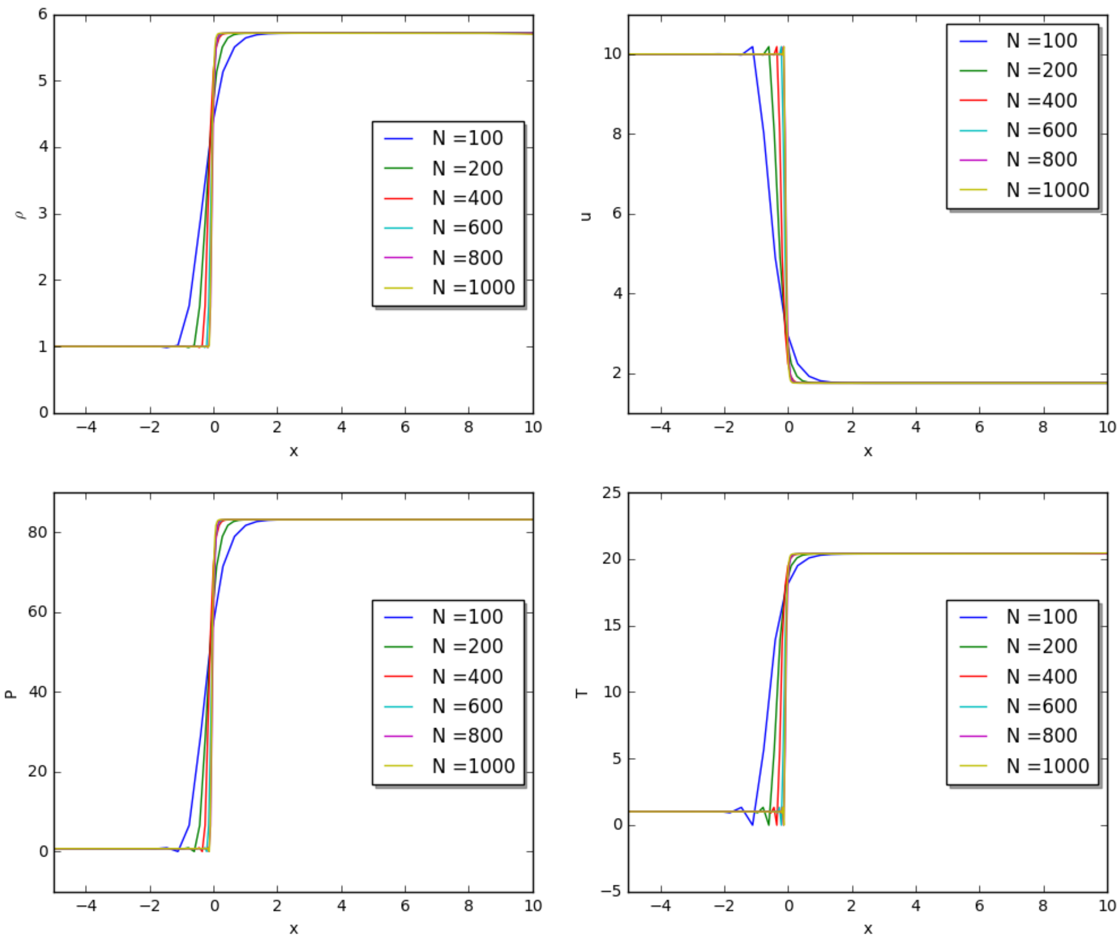
As α is decreased, approximate solution becomes closer to the exact solution, L2 error in u decreases.

As α is decreased, the effect of diffusion decreases and dispersion increases. This happens because the second order derivative term in U contributes to diffusion (even order derivatives), and the first order derivative contributes to dispersion (odd order derivatives).So, when alpha is decreased, the contribution of dissipative/diffusion term decreases and in turn, effect of dispersion term increases.

Minimum alpha for given CFL, Mach number and Grid size lies between 0.145-0.15.

Varying Grid size(n)

Following Plots were obtained for n = $[100, 200, 400, 600, 800, 1000]$
CFL number was kept constant at 0.2
$\alpha = 0.5$
Mach Number = 10.0 (Hypersonic)



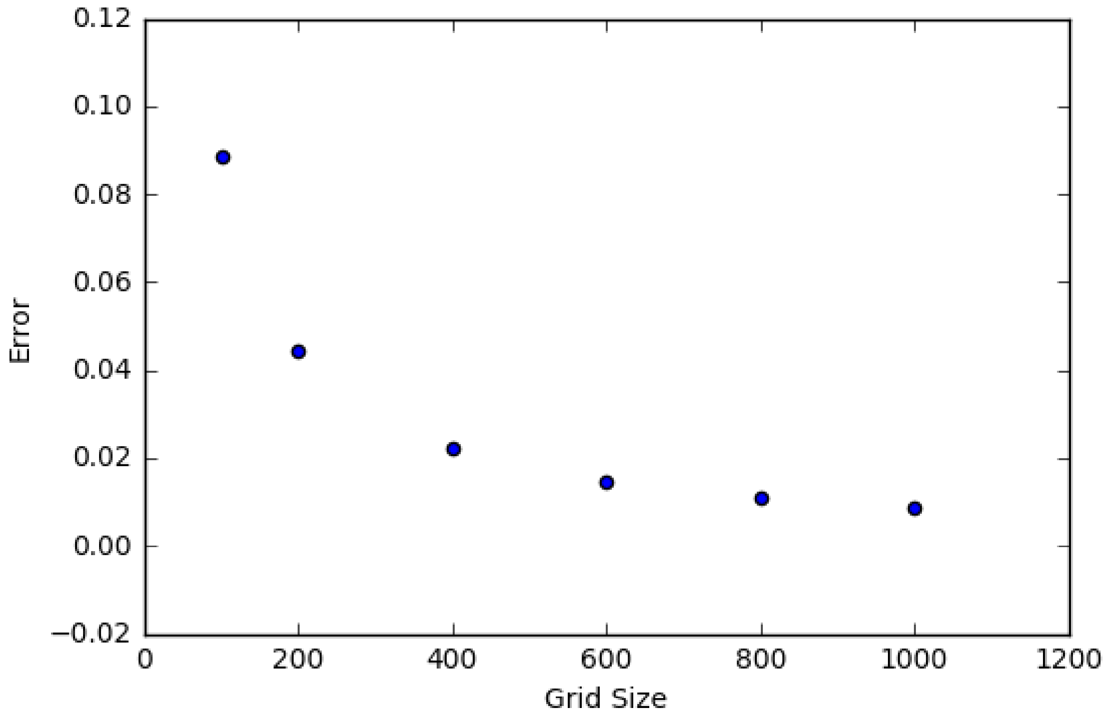


Figure 2: L2 error in u for varying grid size

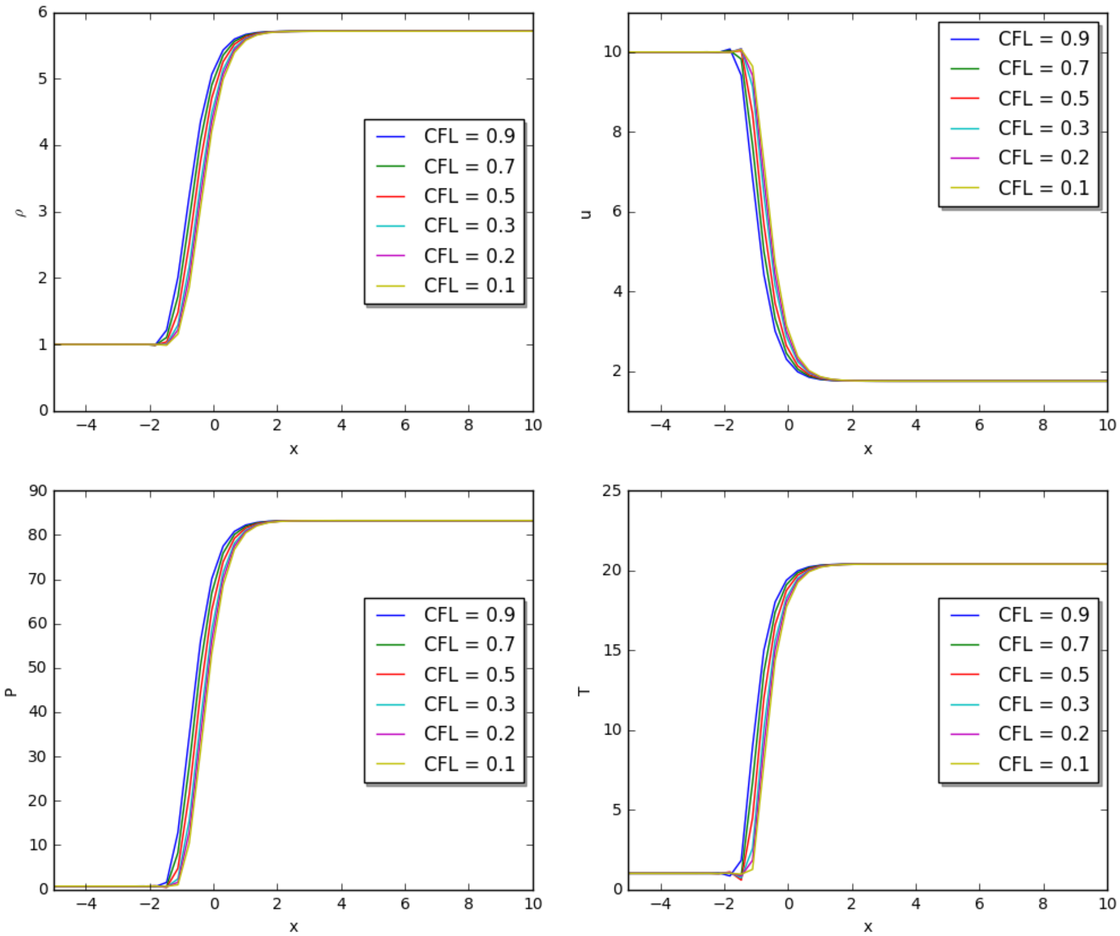
As grid size is increased, approximate solution obtained for normal shock tends more towards exact solution. Dissipation as well as dispersion effects are less significant.

As grid size is increased, cost of computation increases. Thus, there is a trade off between accuracy and cost of comptation. Optimum value of Grid size can be obtained from the L2 error plot for u.

From our analysis, optimum value of grid size should be close to 600 because error does not decrease much as the grid size is increased from 600 to 800.

Varying CFL number

Following Plots were obtained for CFL = $[0.9, 0.7, 0.5, 0.3, 0.2, 0.1]$
Grid Size = 100
$\alpha = 0.6$
Mach Number = 10.0 (Hypersonic)



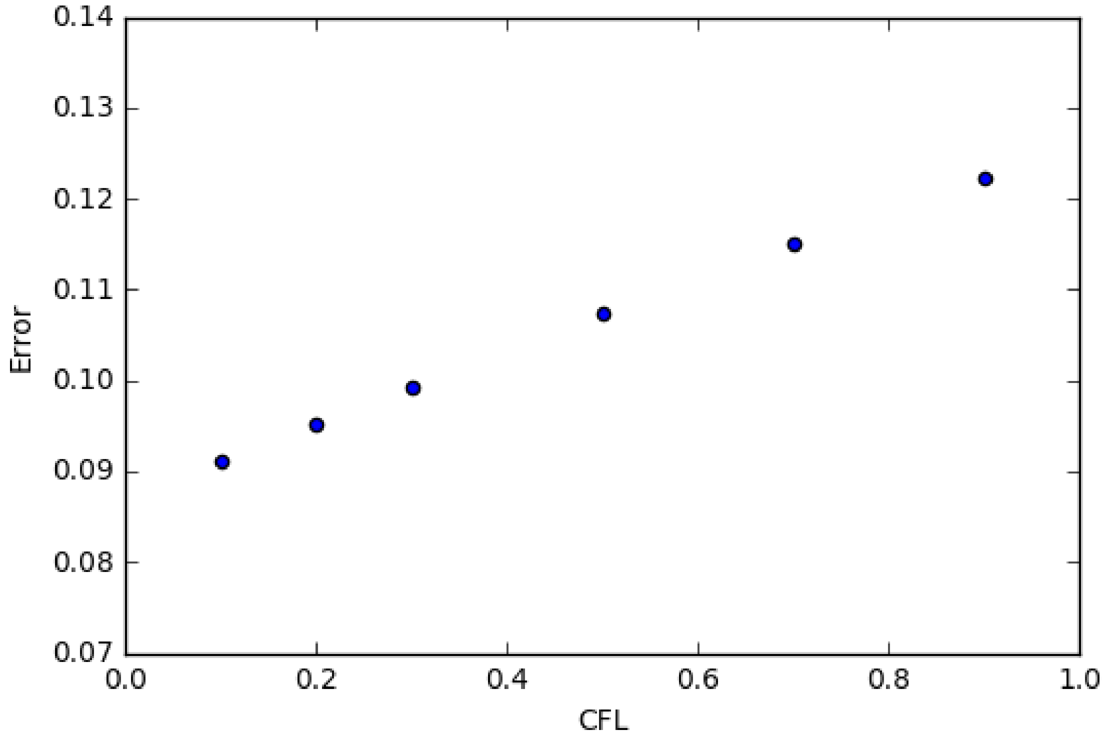


Figure 3: L2 error in u with varying CFL

As CFL number is decreased, relative error decreases and therefore, the approximate solution tends more towards exact solution but, number of computations increase with decreasing δt if final time remains. However, here number of time steps are kept constant = 10000.

With decreasing CFL number, diffusion effect also decreases.

As CFL is increased, minimum value of alpha for which the code does not diverge increases.

Additional Questions

1. **There are no boundary conditions specified in the code. Why?**
   Boundary conditions are not specified because values for ρ, u, p, T were specified in the initial conditions and do not change at the boundaries. Computations are only carried out at grid points from 2 to n-1. Hence, no need to specify boundary conditions.

2. **What happens if you have a non-uniform grid?**
   Fine grid near/around the shock and coarser grid away from the shock will be beneficial, since it will give better solutions near the shock and minimize redundant calculations.

3. **Are there any effect of initial conditions?**
   Yes, if initial conditions are changed, Mach number upstream the shock will change, due to which all properties (u, ρ, p, T) upstream and downstream of the shock will change. Thus, different initial conditions will lead to different shock strengths.