# Bachelor Term Project

# Final Report

*Configuring Passive Intelligent Surfaces for maximizing data-rates in wireless communication*

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May 26, 2022

# Acknowledgements

I am profoundly grateful to **Dr. Salil Kashyap** for his expert guidance and continuous encouragement throughout to see that this project rights its target since its commencement to its completion.

I am also extremely grateful to **Dr. Sarvendranath Rimalapudi** for his expert knowledge and support to help disseminate the state of the art and help come up with novel ideas.

I would like to express deepest appreciation towards my friends and family whose invaluable support supported me in completing this project.

At last I must express my sincere heartfelt gratitude to all the members of EEE Department who helped me directly or indirectly during this course of work.

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Batch of 2018-2020

EEE

**Abstract**

Wireless Communication is ever improving with 5G based wireless communication now being deployed in a lot of geographies. With the advent of beamforming technology and MIMO based communication, concerns over power consumption, security and logistical issues are bound to be raised. Many solutions have come forward for what is labelled as "5G and beyond" stack, one such solution is the use of reflecting surfaces which have reflecting elements which can inflict a phase and magnitude shift on the incoming wave and reflect it without the use of any active components. Passive Intelligent Surfaces (PIS) or Intelligent Reflecting Surfaces (IRS) as they are sometimes called are a state of the art advancement to the ever improving field of wireless communication.

PIS have a wide variety of applications including but not limited to unclogging dead zones, enhancing physical layer security and enhancing the power of the received signal. There are a wide variety of problems related to the development, deployment and operation of such a surface but this thesis concerns with the specific class of problems related to setting the reflection coefficients of the IRS elements such that the data rates for information users are maximised. Algorithms are suggested for beamforming for OFDM and Multi User environments with a focus on both computational complexity and training overhead.

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# Chapter 1

# Publications from Thesis

1. A. Jain, R. Gowda, S. Kashyap and R. Sarvendranath, “Low Complexity Passive Beamforming Algorithms for Intelligent Reflecting Surfaces with Discrete Phase-Shifts over OFDM Systems” *Accepted in National Conference on Communications (NCC), 2022*
2. A. Jain, S. Kashyap and R. Sarvendranath, “Low Complexity Passive Beamforming and Antenna Selection for Practical Intelligent Reflecting Surfaces in Multiuser Environment” *Manuscript in Progress for submission as IEEE Communications Letter*

# Chapter 2

# Introduction

## 2.1 Introduction

5G and beyond networks come with more demands than their predecessor, like improving energy efficiency, decreasing monetary costs, higher reliability and domineeringly lower latency. However, emerging solutions to 5G services (e.g. ultra-reliable and low latency communication (URLLC)) include an increasing number of active nodes, packing more antennas and migrating to higher frequencies involves increasing energy, hardware and cost requirements. Hence this need for cheaper energy-efficient smart solutions has led to the conceptualization of Passive Intelligent Surfaces(PISs) (also known as Intelligent Reflecting Surfaces (IRSs). These are surfaces (not necessarily flat) which reflect signals from base station to the user and vice-versa. In addition to reflecting they are also capable of changing the phase and magnitude of the signal and are configurable by a controller.

Formally, PIS is a re-configurable environment for Beyond 5G wireless communication systems constituting passive elements which reflect incoming signals and additionally inflict a controllable phase and magnitude change to them [1].

PIS reflection surface is realizable using the existing programmable meta surfaces [2]. These reflection elements are tuned by the help of a controller connected to the base station, and Micro Electro Mechanical Switches (MEMS) are used to control the reflection coefficients. This thesis does not delve into the physical implementation any more than this but details can be found in [3].

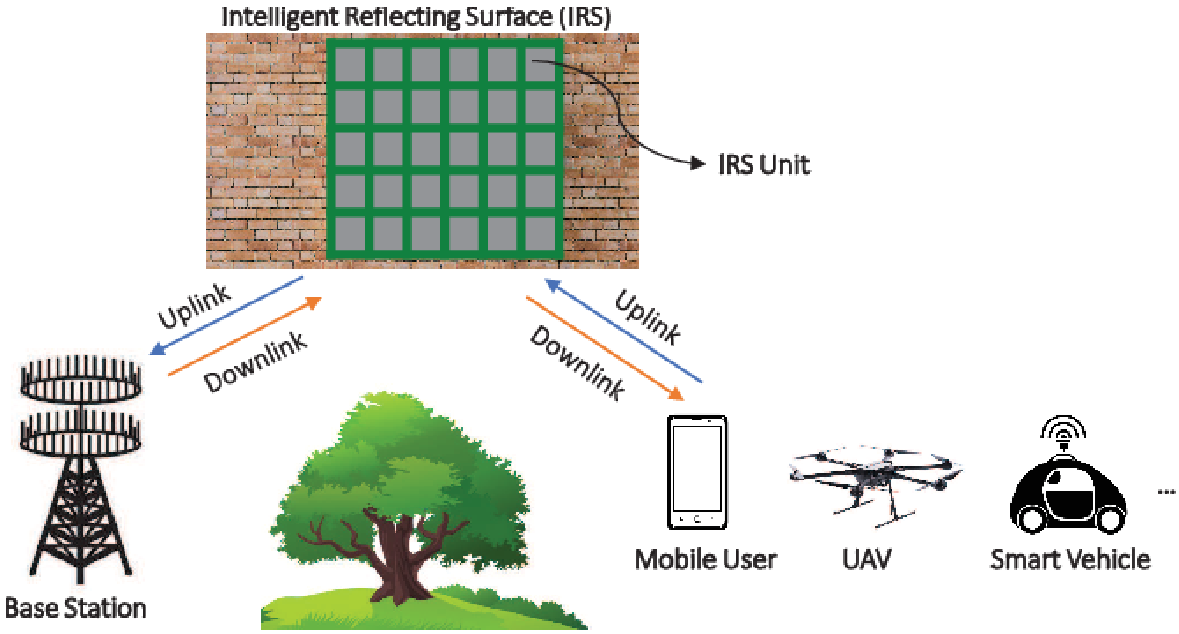


Figure 2.1: Basic setup of how Passive Intelligent Surfaces or Intelligent Reflecting Surfaces are deployed and work. Image Credits: Figure 1 of [4]

## 2.2 Literature Review

PIS has a variety of different use cases been explored in the areas of Wireless Communication and Energy Transfer as surveyed in [5]. Configuring a PIS to maximize energy recieved or data-rates requires information about the channel coefficients [6]. Estimation can be done in both the frequency and time domain, and using different sequences of PIS elements is required, as was studied in the previous phase of this project and also in [7].

Once estimated, the reflection coefficients need to be tuned to meet the optimization objective. This could be done as a joint beamforming optimization where both active beamforming at the Base Station and passive beamforming by setting coefficients of PIS, similar to [8]. The phase shifts of the elements need not be discrete and could be based in a practical setting as studied in [9] and [10]. [10] suggested iterative alternating optimization algorithms to optimize the SINR of the recieved signal.

There has been much research on channel estimation, passive beamforming, and placement of IRS in a network in recent years, as is summarized in [11]. This current work is concerned with low complexity passive beamforming design in practical IRS. Passive beamforming refers to programming the reflection coefficients of the IRS elements to obtain constructive or destructive interference at the user and meet a particular objective. The objective can be to optimize data rates, to cancel out signals for eavesdroppers or to optimally transfer energy etc., [12] [13] [6]. In this work, the objective is to optimize the achievable sum data rate. There has been extensive research on passive beamforming for optimizing data rates over frequency-flat channels, on both continuous and discrete phase shifts of IRS elements [14] [15]. Different methods have been suggested for passive beamforming in order to achieve optimal data rates. Research also deals with jointly designing active and passive beamforming using a Semidefinite Relaxation (SDR) based approach [16]. There has also been research on jointly optimizing the IRS coefficients, and the power allocation to different users [13]. More recently, gradient projection and cross entropy-based algorithms have been suggested as more efficient solutions than directly using conventional optimization methods [17] [18] [19].

# Chapter 3

# Passive Beamfroming in OFDM

What has been a relatively nascent research area is optimizing phase shifts in an OFDM based setup. While some techniques have been looked at in a continous phase shift based IRS [16] [20], a few recent papers have looked at discrete phase shifts [21], [22] as well. But these techniques suffer on two counts, the continous phase shifts don't capture the practicalities of deployment and for methods on discrete phase shifts, the methods are either based on only binary reflection elements or are computationally complex for very large number of IRS [23].

***State of the Art:*** Practical models involving IRS aided MIMO-OFDM based communication have been explored recently, which goes on to show that solving the optimization problem of maximizing the data rates has non-convex constraints, which makes it tough to solve [22]. Some techniques like Successive Convex Approximation (SCA), Block Coordinate Descent have been used to find a sub-optimal solution, however, such approaches have non-linear time complexity [11] [22]. A low complexity (log-linear) approach has been suggested but this has not been studied under IRS with discrete phase shifts to evaluate how having discrete phase shifts at the IRS affects performance [24].

While these methods help achieve the best possible data rates, they have a high computational penalty, which is not desirable since IRS would be used at the edge of the network where computing power might not be abundant. It is also desirable that the time taken to configure the phase shifts at IRS be as low as possible since the total time for training and beamforming might be limited. Having low time complexity is also desirable for situations where the user is mobile. Certain other situations and applications with strict delay constraints may also arise where a linear time complexity would be more desirable than attaining the best possible data rate.

## 3.1 System Model

As shown in Figure 4.1, a single antenna base station serves a single antenna user assisted with an IRS. There are $N$ reflection elements in the IRS setup which take indices from the set $\mathcal{N} = \{1, 2, \ldots, N\}$. There are $M$ discrete phase shifts or $W = \lceil \log_2 M \rceil$ bits available to configure each IRS element. That is, the $j^{\text{th}}$ element can take a phase shift $\theta_j$ from the

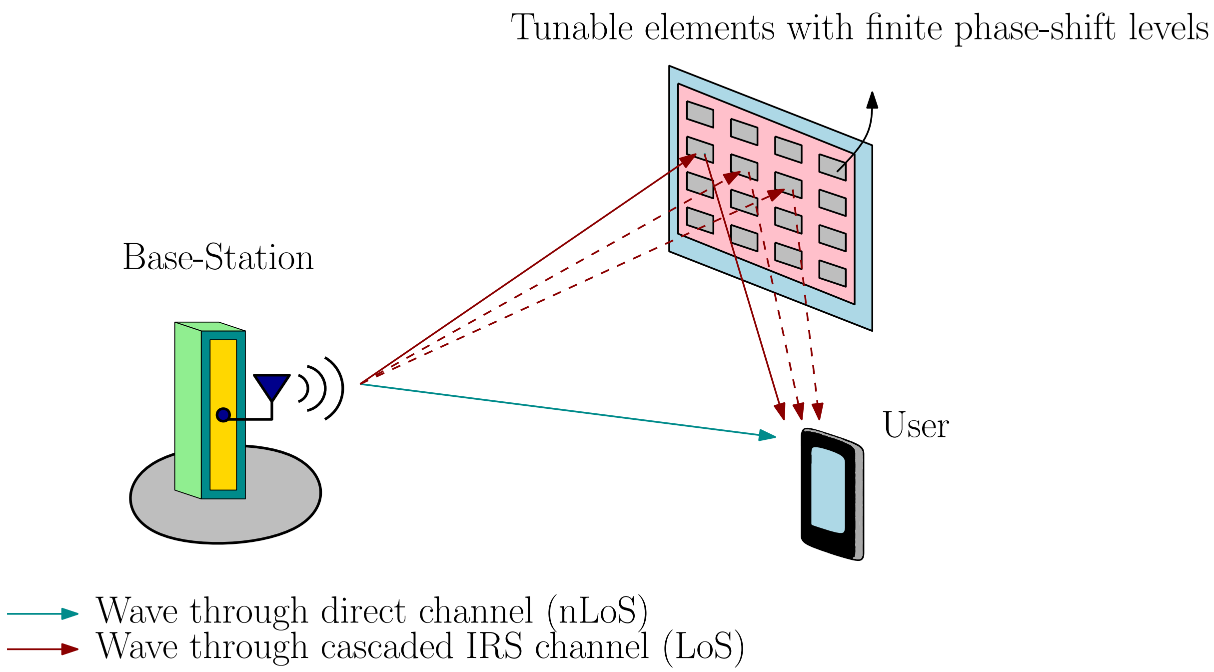


Figure 3.1: This is a representation of the system model considered in this paper.

set $\mathbb{A} = \{0, 2\pi\frac{1}{M}, 2\pi\frac{2}{M}, \ldots, 2\pi\frac{M-1}{M}\}$. The reflection gains of the IRS elements are taken to be 1. While there is a non Line-of-Sight (nLoS) channel between Base Station (BS) and User, each of the links from BS to IRS and IRS to user is Rician faded and has a LoS component as well. The signal transmitted by the BS on the $k^{\text{th}}$ subcarrier is given by $x_k = \sqrt{p_k} s_k$, where $p_k$ is the power allocated to $k^{\text{th}}$ subcarrier and the information symbols $s_k$ is chosen such that $|s_k|^2 = 1$. Note that $\sum_{k=1}^{K} p_k = P$ is the total power available at the BS and $K$ is the total number of subcarriers. The received symbol $y_k$ at the user on the $k^{\text{th}}$ subcarrier can be written as

$$y_k = (h_k^{su*} + ({h_k^{si}}^H \odot h_k^{iu})\ \Theta)\ x_k + z_k \tag{3.1}$$

where ${h_k^{si}}^H \in \mathbb{C}^{1\times N}$, $h_k^{iu} \in \mathbb{C}^{1\times N}$ and $h_k^{su} \in \mathbb{C}$ denote the channel coefficients for the BS-IRS channel, the IRS-User channel and the direct channel between BS-User for the $k^{\text{th}}$ subcarrier. Furthermore, $z_k$ is complex additive white Gaussian noise with power spectral density $N_0$. $\Theta = [e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_N}]^T \in \mathbb{C}^{N\times 1}$ denotes the IRS reflection coefficients where $\theta_j \in \mathbb{A}$ is the phase shift induced by the $j^{\text{th}}$ IRS element. We introduce

$$h_\Theta[k] = h_k^{su*} + ({h_k^{si}}^H \odot h_k^{iu})\ \Theta \tag{3.2}$$

$$h_\Theta[k] = h_k^{su*} + (h_k^{siu})\ \Theta \tag{3.3}$$

for the remaining paper as the channel coefficient for the cascaded plus the direct channel, where $h_k^{siu} \in \mathbb{C}^{1\times N}$ is a compact representation of the cascaded channel response over the $k^{\text{th}}$ subcarrier. The term *link* is used to describe the direct channel or a channel through a single IRS element.

## 3.2 Problem Description

The sum data rate over all subcarriers for a particular IRS configuration in our setup is expressed as

$$R = \frac{B}{K+L-1} \sum_{k=0}^{K-1} \log_2 \left( 1 + \frac{p_k |h_\Theta[k]|^2}{BN_o} \right) \tag{3.4}$$

where $h_\theta[k] \in \mathbb{C}$ denotes the effective channel (direct link and the $N$ links through IRS) for $k^{\text{th}}$ subcarrier, $B$ is bandwidth and $L$ is the number of taps.

We study the problem of maximizing the sum data rate of an OFDM system enabled by IRS subject to the constraint that phases at the IRS are allowed to take values from a finite set of discrete phase shifts over spatially correlated channels. Considering equal power allocation across all subcarriers [1] the data rate expression in (4.3) gets modified to:

$$R = \frac{B}{K+L-1} \sum_{k=0}^{K-1} \log_2 \left( 1 + \frac{P |h_\Theta[k]|^2}{KBN_o} \right) \tag{3.5}$$

Mathematically, the optimization problem can be stated as

$$\begin{aligned} \mathcal{P}_1 : &\max_{\Theta} \; R \\ &\text{s.t. } \theta_j \in \mathbb{A} \; \forall j \; \in \; \mathcal{N} \end{aligned} \tag{3.6}$$

Here, the set $\mathbb{A} = \{0, 2\pi \frac{1}{M}, 2\pi \frac{2}{M}, \ldots, 2\pi \frac{M-1}{M}\}$ denotes the discrete phase shifts that an IRS element can potentially induce. To make the problem more tractable, instead of (3.6) we can consider to optimize the sum of the absolute square of the effective channel gain over different subcarriers, with the modified objective function as

$$R_{\text{aprx}} = \sum_{k=0}^{K-1} |h_\Theta[k]|^2 \tag{3.7}$$

$$R_{\text{aprx}} = \sum_{k=0}^{K-1} |h_k^{su*} + h_k^{siu} \Theta|^2 \tag{3.8}$$

$$\tag{3.9}$$

And the optimization problem $\mathcal{P}_1$ gets modified as,

$$\mathcal{P}_2 : \max_{\Theta} \quad R_{\text{aprx}} \tag{3.10}$$

$$\text{s.t.} \quad \theta_j \in \mathbb{A} \; \forall j \; \in \; \mathcal{N} \tag{3.11}$$

An exhaustive search could be performed to evaluate the IRS configuration which maximizes the sum data rates but this would take $O(M^N)$ and turns out to be infeasible in practical scenarios. We propose two heuristic solutions aimed at efficiently solving this and evaluate their performance.

[1] The power can be distributed in the different subcarriers with different $p_k$ as well, water-filling can be used in an alternating optimization setup with passive beamforming [25].

## 3.3 Optimizing the Passive Beamforming

Our main objective is to maximize the average achievable rate of $\mathcal{P}_1$ but we instead optimize the absolute square of the effective channel gain summed over all subcarriers in $\mathcal{P}_2$. $\mathcal{P}_1$ is difficult to solve optimally even with the continuous phase shifts of IRS reflection elements [12]. A suboptimal solution could be to solve using a single statistic created using channel coefficients of the different subcarriers. We first highlight our underlying approach by considering the channel obtained by taking the mean across all subcarriers.

We then improve this algorithm by optimizing over the strongest tap in the time domain. We benchmark both the techniques against the theoretical upper bound and discuss computational complexity with the existing algorithm of Successive Convex Approximation.

#### Vector Heuristic (VH)

One representation of the channel via the $j^{\text{th}}$ IRS element (and the direct channel) is the average over all the frequency coefficients in that channel, i.e.,

$$h_{j\ \text{mean}}^{siu} = \frac{1}{K}\sum_{k=0}^{K-1} h_{j\ k}^{siu}\ \forall\ j \in\ \mathcal{N} \tag{3.12}$$

$$h_{\text{mean}}^{su} = \frac{1}{K}\sum_{k=0}^{K-1} h_{k}^{su} \tag{3.13}$$

We take these average $N+1$ channel coefficients and use them to calculate the reflection coefficients required.

Intuitively these $N+1$ complex coefficients can be considered as $N+1$ two dimensional vectors. The approximate $R_{\text{aprx}}$ term then reduces to

$$R_{\text{aprx}} = |{h_{\text{mean}}^{su}}^* + h_{\text{mean}}^{siu}\Theta|^2 \tag{3.14}$$

Defining $h_{\text{ref}} = max(max(h_{\text{mean}}^{siu}), {h_{\text{mean}}^{su}}^*)$. Note that $h_{\text{ref}}$ denotes the complex coefficient with the highest absolute value among the $N+1$ average values computed over $K$ subcarriers for the $N$ reflected links and the direct link. And as mentioned above, $max(a)$ is a function which returns the entry with the highest absolute value. If $|\text{Angle}(\frac{h_{\text{ref}}}{{h_{\text{mean}}^{su}}^*})| > \frac{\pi}{2}$, we assign $h_{\text{ref}} = -max(max(h_{\text{mean}}^{siu}), {h_{\text{mean}}^{su}}^*)$, the reasoning being that since the direct channel can't be configured, we instead align along the reference direction, which is in the same half as the direct channel. Since even a 1 bit IRS element has a phase shift of $-\pi$ possible, this alignment is always possible and will result in a better phase shift.

Further, let $\Delta_j = \text{Angle}\left(\frac{h_{j\ \text{mean}}^{siu}}{h_{\text{ref}}}\right)$ where $h_{j\ \text{mean}}^{siu}$ is the $j^{\text{th}}$ element of $h_{\text{mean}}^{siu}$. This delta is simply the angle between complex numbers $h_{j\ \text{mean}}^{siu}$ and $h_{\text{ref}}$ and we are trying to align all elements along the $h_{\text{ref}}$. For all $j$, $\theta_j$ is set to $\theta_j = \text{RoundOff}(-\Delta_j, \mathbb{A})$. The reasoning behind this comes from the passive beamforming rule of continuous phase shifts where the IRS element exactly compensates the phase difference between direct and cascaded channel. RoundOff simply rounds off the $-\Delta_j$ to the nearest value in set $\mathbb{A}$.

Simply put, all IRS elements would be aligned along the strongest mean across all subcarrier links. The main reason why we don't simply align everything along the direct channel is that for a case with very less phase shifts possible and the direct channel being weak (which is often the case with nLOS direct channel), the direct channel might not be the best possible choice to align along.

The time complexity of this algorithm is $O(NK)$. This makes this algorithm a linear algorithm in terms of the number of IRS elements and hence extremely scalable. This algorithm can be deployed in situations where the number of IRS elements are very high and when there is a time constraint on passive beamforming or in applications that are delay-sensitive.

**Algorithm 1** Low Complexity Vector Heuristic Algorithm for Passive Beamforming

**Input:** $h_{j\ k}^{siu}, h_k^{su}\ \forall\ j,k$
**Output:** $\Theta$
  $h_{j\ \text{mean}}^{siu} \leftarrow \frac{1}{K}\sum_{k=0}^{K} h_{jk}^{siu}\ \forall i$
  $h_{\text{mean}}^{siu} \leftarrow [h_{1\ \text{mean}}^{siu}, h_{2\ \text{mean}}^{siu} \dots h_{N\ \text{mean}}^{siu}]$
  $h_{\text{ref}} \leftarrow max(max(h_{\text{mean}}^{siu}), h_{\text{mean}}^{su\ *})$
  **if** $\left|\text{Angle}\left(\frac{h_{\text{ref}}}{h_{\text{mean}}^{su}}\right)\right| > \frac{\pi}{2}$ **then**
    $h_{\text{ref}} \leftarrow -max(max(h_{\text{mean}}^{siu\ H}), h_{\text{mean}}^{su\ *})$
  **end if**
  $\Theta \leftarrow [0, 0 \dots 0]_{1\times N}$
  **for all** $j$ in $1, 2 \dots N$ **do**
    $\Delta_j = \text{Angle}\left(\frac{h_{j\ \text{mean}}^{siu}}{h_{\text{ref}}}\right)$
    $\Theta[j] \leftarrow \text{RoundOff}(-\Delta_j, \mathbb{A})$
  **end for**

**Strongest-Tap Selection (STS)**

To further improve performance, we could consider beamforming along the strongest tap of all the $N$ links via the IRS elements and the direct channel. As mentioned in footnote 1, a similar kind of algorithm has been suggested using strongest taps, but on continuous phase shifts [16] [24] [26]. The motivation behind this time domain transformation is that in wireless setting, the number of taps $L$ is usually much smaller than the number of subcarriers $K$. Therefore, it easier to design reflection coefficients based on the time-domain representation. To obtain the time domain representation, the Inverse Discrete Fourier Transform (IDFT) for the $K$ channel coefficients for the $N+1$ links is taken and trimmed to first $L$ entries. Therefore for each link, we define:

$$g_j^{siu} = \text{IDFT}(h_j^{siu})[1:L]\ \forall\ j\ \in\ \mathcal{N} \tag{3.15}$$
$$g^{su} = \text{IDFT}(h^{suH})[1:L] \tag{3.16}$$

The strongest entry out of these $L$ taps is considered for each of the $N$ IRS elements and these are used and processed to optimize for the passive beamforming.

$$g_{j\ \text{strong}}^{siu} = max(g_j^{siu})\ \forall\ j\ \in\ \mathcal{N} \tag{3.17}$$

$$g_{\text{strong}}^{su} = max(g^{su}) \tag{3.18}$$

where $max(a)$ is a function which returns the maximum of the vector $a$. In case entries of $a$ are complex, it returns the entry with the highest absolute value. We use these $N+1$ coefficients, i.e., $g_{\text{strong}}^{siu}$ and $g_{\text{strong}}^{su}$ to obtain the $N$ IRS configuration in a similar fashion as Algorithm 1. We find out the reference vector by taking the maximum along all $N+1$ coefficients, $g_{\text{ref}} = max(max(g_{\text{strong}}^{siu}), g^{su})$.

Finally, similar to our approach in VH we set the IRS reflection coefficients by considering $\Delta_j = \text{Angle}\left(\frac{g_{j\,\text{strong}}^{siu}}{g_{\text{ref}}}\right)$. For all $j$, $\theta_j$ is set to $\theta_j = \text{RoundOff}(-\Delta_j, \mathbb{A})$. This ensures that the IRS elements discretely align the strongest taps of all links along the reference strongest tap. Note that since we only select the strongest tap, which is a scalar for all the links, we don't have to take a Discrete Fourier Transform and can directly align along the strongest tap.

The time complexity of the resulting algorithm 2 comes out to be $O(NK\log K + NL)$. There are $N+1$ DFT computations of complexity $O(K\log K)$ each. Then for the direct channel and each IRS element's cascaded channel we select the strongest tap which leads to an additional $O(NL)$ complexity.

**Algorithm 2** Log-Linear Strongest Tap Selection Algorithm for Passive Beamforming

**Input:** $h_j^{siu}, h^{su}\ \forall\ j$
**Output:** $\Theta$
  $g_j^{siu} \leftarrow \text{IDFT}(h_j^{siu^H})[1:L]\ \forall\ j\ \in\ \mathcal{N}$
  $g^{su} \leftarrow \text{IDFT}(h^{su^H})[1:L]$
  $g_{j\ \text{strong}}^{siu} \leftarrow \max(g_j^{siu})\ \forall\ j\ \in\ \mathcal{N}$
  $g_{\text{strong}}^{su} \leftarrow \max(g^{su})$
  $g_{\text{strong}}^{siu} \leftarrow [g_{1\ \text{strong}}^{siu}, g_{2\ \text{strong}}^{siu} \cdots g_{N\ \text{strong}}^{siu}]$
  $g_{\text{ref}} \leftarrow max(max(g_{\text{strong}}^{siu}), g^{su})$
  $\Theta \leftarrow [0, 0 \ldots 0]_{1\times N}$
  **for all** $j$ in $1, 2 \ldots N$ **do**
    $\Delta_j = \text{Angle}\left(\frac{g_{j\,\text{strong}}^{siu}}{g_{\text{ref}}}\right)$
    $\Theta[j] \leftarrow \text{RoundOff}(-\Delta_j, \mathbb{A})$
  **end for**
  **return** $\Theta$

## 3.4 Numerical Results and Discussion

In this section, the algorithms proposed are benchmarked against the existing state of the art solutions both in terms of date rate maximization and computation time performance or complexity.

### 3.4.1 Setup

As highlighted above, this is a single antenna BS and single user setup with an IRS stationed closer to the user. The IRS is stationed at the origin (0,0,0) and the BS is at (50,50,-5), making the distance between IRS and BS to be fixed at 70.88 m. The user's $x$ and $y$ coordinates vary uniformly between 0 to 7 m and $z$ coordinate varies between 0 m to $-4$ m. There is nLoS channel between BS and User, but there is an LoS component in the channel between BS and IRS, and IRS and User, i.e., each of the cascaded links via the IRS which undergoes Rician fading. The rice factor is calculated as $K_f = 13 - 0.03d$ dB where $d$ is the distance of the link [21]. The system model is illustrated in Figure 4.1. Unless stated otherwise for our simulations, we are considering a system with number of subcarriers $K$ as 500, number of channel taps $L$ between different links as 20, and the number of bits used to program the IRS phase shifts vary between 1 to 4 bits (i.e. 2 to 16 available phase shifts). The total power $P$ is taken to be 30 dBm, the bandwidth $B$ is taken as 100 MHz and $N_0$ is taken as $-174$ dBm/Hz [22]. The path loss exponent for the direct link is taken to be 3.6 and for the cascaded channels via the IRS is taken to be 2.2 [27].

The carrier frequency is taken to be 3 GHz. The separation between two IRS elements is taken to be $\lambda/2$. Our simulations also capture spatially correlated cascaded IRS channels based on the sinc correlation model as discussed in [27]. This adds to the practicality of the results. For the purpose of illustration the simulations are run assuming perfect CSI [21].

### 3.4.2 Benchmarking

By configuring reflection coefficients at the IRS separately for each of the subcarriers, an upper bound on the sum data rate in (3.6) can be obtained. The optimal reflection coefficient for each subcarrier would then be the difference in the phases of the cascaded link and the direct link on that subcarrier. The final expression for the upper bound for the sum data rate in such a scenario can be expressed as: [26]

$$R \leqslant R_{\text{UB}} = \frac{B}{K+L-1} \sum_{k=0}^{K-1} \log_2 \left( 1 + \frac{p_k (h_{\text{optim}}[k])^2}{BN_o} \right) \tag{3.19}$$

where $h_{\text{optim}}[k] = |h_k^{su*}| + \sum_{j=0}^{N-1} |h_{j\ k}^{siu}|$ is the optimally tuned channel coefficient for $k^{\text{th}}$ sub carrier. The upper bound for the discrete case will certainly be lesser than or equal to this bound since the search space is now more restricted. Since, this is the best possible sum data rate that can be obtained, we use this bound for the continuous case to benchmark our results under different conditions.

### 3.4.3 Results and Discussion

There are two primary results that are obtained, firstly the effect of the number of bits used to program the phase shifts at the IRS on both the proposed algorithms. Secondly,

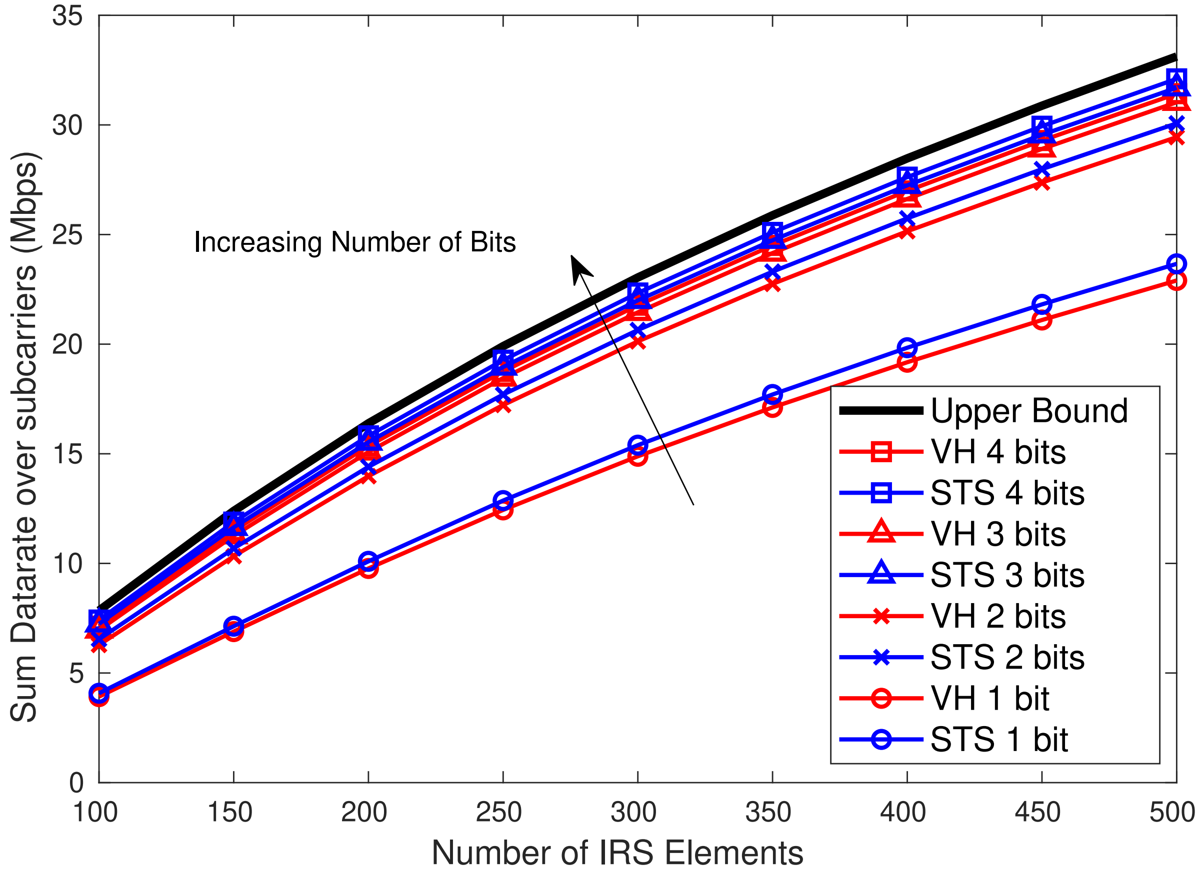


Figure 3.2: Benchmarking VH and STS algorithms against the upper bound and studying the effect of increasing the number of bits.

by studying the time complexity we analyse the benefits of our methods over the SCA technique and study the tradeoff for the vector based heuristic method between the time complexity and performance.

Figure 3.2 plots the sum data rate achieved based on the two proposed algorithms, namely, VH and STS as a function of the number of IRS elements for different number of bits used to configure the IRS. As a benchmark, we also plot the theoretical upper bound (based on continuous phase shifts at IRS) to assess how well the proposed algorithms perform. As expected, we observe that the sum data rate improves as we increase the number of bits, since the reflected signals align better with the direct signal as the number of levels into which we quantize the phase-shifts at IRS increases. We observe that 4 bits are sufficient to reach the best possible achievable sum data rate with the STS Technique. We also note that STS performs consistently better than VH, and STS with 3 bits of phase shifts is better than VH with 4 bits of phase shifts.

Figure 3.3 shows the effect of varying the number of IRS elements and subcarriers on the proportional number of operations required for the algorithm to optimize for SCA, STS and VH as derived from their theoretical worst case complexities. Looking at the corresponding time performance we see a stark improvement in the number of complex computations in comparison to the benchmark. The computational complexity of our STS algorithm increases with the order of $O(NK \log K + NL)$ and VH with order of $O(NK)$

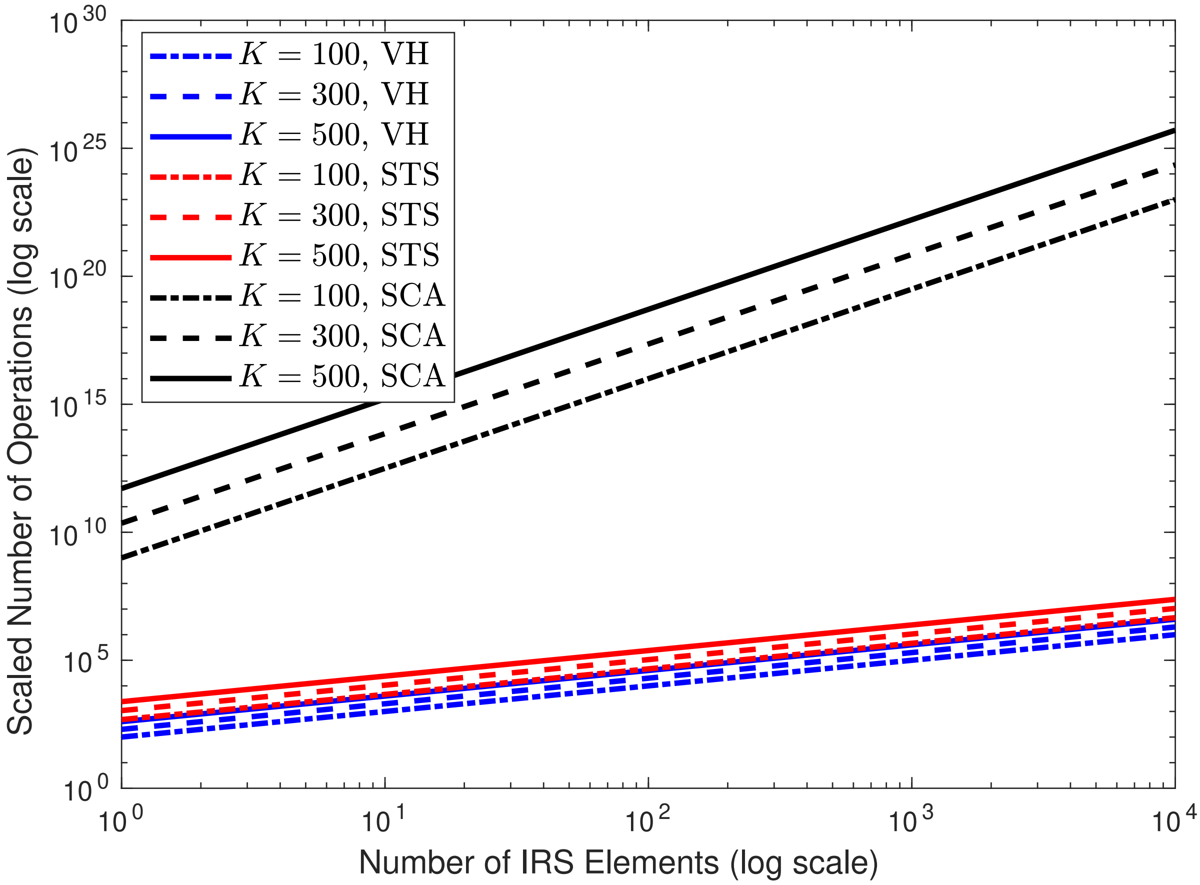


Figure 3.3: Theoretical worst case complexity of the STS and the VH benchmarked against SCA algorithm.

with increase in IRS elements and subcarriers and SCA based algorithm increases with $O(N^{3.5}K^{4.5})$ [12]. Our algorithms are certainly more scalable and deployable with both number of subcarriers and IRS elements than the SCA approach.[2] We also note that for a very large $N$, the VH algorithm can be applied with a tradeoff on the data rate.

[2]Although we have not benchmarked the sum data rate performance of our algorithms against SCA approach, our methods achieve near optimal performance (theoretical upper bound in (3.19)) with 3 to 4 phase shift bits, and the gain in time performance is order of magnitudes higher than the SCA which make our algorithms a more scalable and practical choice. Furthermore, even with SCA the sum data rate can never exceed the theoretical upper bound in (3.19).

# Chapter 4

# Passive Beamforming and Antenna Selection In Multi User Environment

***Related Literature and State of the Art:*** Passive beamforming has been studied extensively, and as the research matures, more and more focus is put on practical considerations. One such consideration is that of a multiuser environment. There has been work in the multiuser setting as well, using Fractional Programming (FP), Successive Convex Approximation (SCA), Block Coordinate Descent (BCD) and Unsupervised learning based techniques [28] [29] [30]. Although the work in this direction has been promising, there are certain practical aspects that provide scope of improvement in this direction. For instance, very little research has gone into looking into the pilot considerations and the training overhead involved in a multiuser environment.

Another aspect is that of antenna selection is part of ongoing wireless research as the next generation of communications would involve a lot of antennas and the number of RF Chains might not be desirable in as high as a number as the antennas since each of these RF Chains have high power requirements [31]. Selecting a subset therefore helps reduce the power requirement while still achieving diversity that having multiple antennas offer [32]. This has been studied in the context of IRS as well with works developing greedy algorithms and discrete cuckoo algorithm for selecting the antennas that will be used for transmission [33]. Recent works have also used deeplearning based models to select a subset of antennas [34]. There are works which look at the problem of jointy optimizing antenna selection in a multiuser environment but still requires complete Channel State Information (CSI) and hence a training overhead [35].

***Novel Contributions:*** Our contribution in this paper are on both the modelling and signal processing side. Our contributions are as follows:

1. We model our objective function with a pre-log factor which accounts for the training overhead incurred in estimating the channel coefficients. This pre-log factor is fixed for most proposed algorithms since they require complete CSI. We however propose an algorithm which require only the sum CSI over the different users.
2. We propose two approaches to configure the IRS elements and optimize the sum datarates. First one is using Manifold Optimization (MO) and optimizing the ex-

act expression for the sum datarates. The second algorithm is the Vector Heuristic (VH) which is based on optimizing an approximate version of the objective function and turns out to be sub-optimal but extremely scalable with both the number of users and IRS elements. We show that both of these can be coupled with single antenna selection by iterating over the different antennas and calculating the optimum datarate using these algorithms.

3. We also include several practical considerations making our results deployable and more realistic. We consider that only a single RF chain is available at the Access Point (AP), and the CSI is imperfect. The results are presented with discrete phase shift available at the IRS elements and coupled IRS elements [27].

***Notations:*** Lowercase boldface letters e.g. $\mathbf{v}$ denotes a vector, uppercase boldface letters e.g. $\mathbf{A}$ denote a matrix and lowercase normal face letters e.g. $x$ denote a scalar. $\mathbb{C}$ denotes the set of complex numbers.

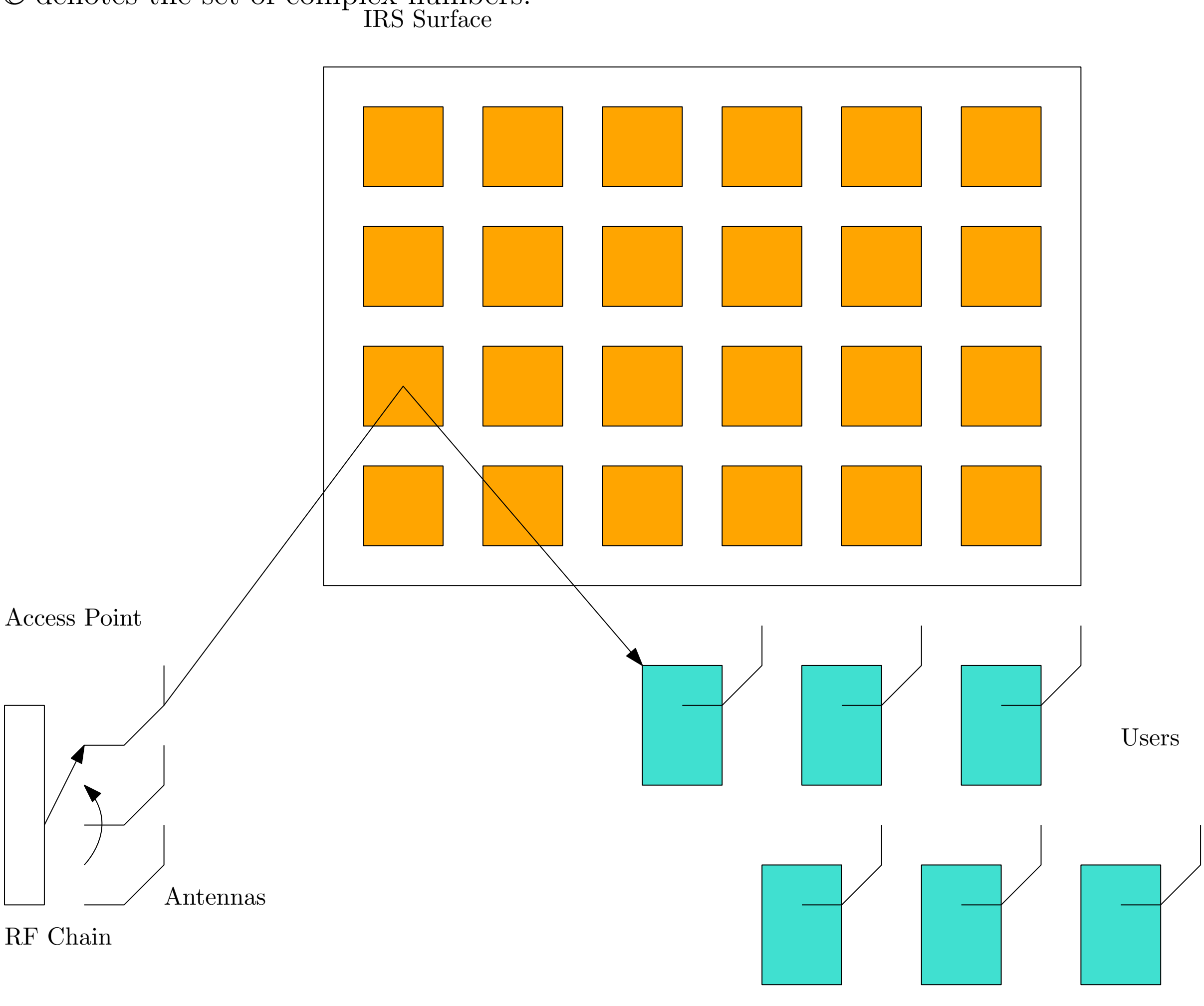


Figure 4.1: This is a representation of the system model considered in this chapter.

## 4.1 System Model

The system model is taken to be similar to that of Figure 1. We consider an IRS with $N$ reflecting elements assisted wireless system. There are $K$ users that are served by a single AP with $N_t$ antennas and a single RF Chain. The IRS elements form the set $\mathcal{N} = \{1, 2, \ldots, N\}$. All the IRS elements have unity gain and offer phase shifts $\{\theta_1, \theta_2, \ldots \theta_N\}$, hence can be represented by a reflection gain vector $\mathbf{\Theta} = [e^{j\theta_1}, e^{j\theta_2}, \ldots, e^{j\theta_N}] \in \mathbb{C}^{N\times 1}$ The Line of Sight component is absent from the direct channel between AP and each user. The AP can select an antenna out of the $N_t$ antennaes which form a set $\mathcal{A} = \{1, 2, \ldots, N_t\}$ and transmit using that antenna by connecting it to the RF Chain. The selected antennae is indexed by $a \in \mathcal{A}$. Since there is only a single RF Chain, there is a single symbol $s \in \mathbb{C}$ that is broadcasted to all users. The symbol is transmitted with power $P$ and the transmitted symbol is given by $x = \sqrt{P}s$. The symbol $y^{(k)}$ recieved at the user $k$ is given by

$$y^{(k)} = (h_{d,a}^{(k)} + \mathbf{g}_a \; diag(\mathbf{h}_r^{(k)})\mathbf{\Theta})x + z_k \tag{4.1}$$

where the direct channel coefficient between antenna $a$ and user $k$ is given by $h_{d,a}^{(k)} \in \mathbb{C}^{1\times 1}$, the channel coefficient between antenna $a$ and the IRS is given by $g_a \in \mathbb{C}^{1\times N}$, and between the IRS and user $k$ by $h_r^{(k)} \in \mathbb{C}^{1\times N}$. $z_k$ is Complex White Gaussian Noise (CWGN) with noise spectral density $N_0$.

The cascaded channel $\mathbf{g_a} \; diag(\mathbf{h_r}^{(k)})$ can be represented by $\mathbf{h_{c,a}}^{(k)} \in \mathbb{C}^{1\times N}$ making equation 4.1 as,

$$y^{(k)} = (h_{d,a}^{(k)} + \mathbf{h}_{c,a}^{(k)}\mathbf{\Theta})x + z_k \tag{4.2}$$

## 4.2 Problem Description

The objective that we aim to optimize is the sum datarates over all users. The sum datarates can be expressed as follows,

$$R = \frac{T_c - N_p}{T_c} \sum_{k=1}^{K} \log\left(1 + \frac{P|h_{d,a}^{(k)} + \mathbf{h}_{c,a}^{(k)}\mathbf{\Theta}|^2}{N_0}\right) \tag{4.3}$$

The optimization problem is hence given by,

$$\mathcal{P}_1 : \max_{a,\mathbf{\Theta}} R \tag{4.4}$$

$$|\theta_i| = 1 \; \forall i \in \mathcal{N} \tag{4.5}$$

$$a \in \mathcal{A} \tag{4.6}$$

where $T_c$ is the coherence interval and $N_p$ is the number of pilots used for estimating the CSI. Problem $\mathcal{P}_1$ is fundamentally a difficult problem to solve [cite]. There are locally optimum ways explored to solve this like the one suggested in the next section. However the complexity of the problem can be reduced and an heuristic method can be described if we approximate the rate function with a sum of absolute channel coefficients.

$$R' = \sum_{k=1}^{K} |h_{d,a}^{(k)} + \mathbf{h}_{c,a}^{(k)}\mathbf{\Theta}| \tag{4.7}$$

Using the triangular inequality on (4.7), we can find a lower bound on $R'$ as,

$$\sum_{k=1}^{K} |h_{d,a}^{(k)} + \mathbf{h}_{c,a}^{(k)}\mathbf{\Theta}| \geq \left| \sum_{k=1}^{K} \left( h_{d,a}^{(k)} + \mathbf{h}_{c,a}^{(k)}\mathbf{\Theta} \right) \right| \tag{4.8}$$

$$= \left| \sum_{k=1}^{K} h_{d,a}^{(k)} + \left( \sum_{k=1}^{K} \mathbf{h}_{c,a}^{(k)} \right) \mathbf{\Theta} \right| \tag{4.9}$$

$$= K \left| h_{d,a}^{(\text{mean})} + \mathbf{h}_{c,a}^{(\text{mean})}\mathbf{\Theta} \right| = R'' \tag{4.10}$$

where we define $h_{d,a}^{(\text{mean})} = \frac{1}{K}\sum_{k=1}^{K} h_{d,a}^{(k)}$ and $\mathbf{h}_{c,a}^{(\text{mean})} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{h}_{c,a}^{(k)}$ are the mean directed and the cascaded channel coefficients over different users. Since $R''$ is a lower bound on $R'$, maximizing $R''$ would inturn maximize $R'$. Hence to optimize the approximate sum of absolute channel coefficients we construct the second optimization problem as,

$$\mathcal{P}_2 : \max_{a,\mathbf{\Theta}} R'' \tag{4.11}$$

$$|\theta_i| = 1 \ \forall i \in \mathcal{N} \tag{4.12}$$

$$a \in \mathcal{A} \tag{4.13}$$

## 4.3 Optimization

In this section we propose two optimization schemes to solve $\mathcal{P}_1$ and $\mathcal{P}_2$. We iterate over all antennas one by one and then optimize the data rate fixing the antenna to get reflection coefficients. Finally we select the antenna with which the optimized refection coefficients give the maximum datarate. For the passive beamforming we first propose a MO based approach which exploits the unit modulus constraint to solve $\mathcal{P}_1$. Note that $\mathcal{P}_1$ is a non-convex problem to solve and conventional convex optimization techniques are not applicable here. We solve $\mathcal{P}_2$ optimally and presents it solution in the proposed VH algorithm.

### 4.3.1 Antenna Selection (AS)

Since there is only a single RF chain at the AP, to optimize $\mathcal{P}_1$ and $\mathcal{P}_2$ we iterate over the different antennas one by one and optimize and finally select the antenna which gives the best data rate out of the $N_t$ antennas. Since the antenna have uncorrelated channels it's not possible to do any better than this for the single antenna selection, however novel greedy algorithms have been suggested for subset antenna selection.

### 4.3.2 Manifold Optimization (MO)

There have been papers in the IRS literature shown to find locally optimal solutions to non convex problems using manifold optimization technique [36] [32]. For the selected antenna we solve $\mathcal{P}_1$ using a trust regions based manifold optimization technique. This is possible because the unit modulus constraints defines a Riemannian Manifold. Optimizing over a Riemannian Manifold is shown to be locally similar to optimizing over an Euclidean space [37].

#### Training Overhead

MO requires complete CSI of the direct channel and the cascaded channel. The number of pilots required to acquire the CSI of the direct channel for a single antennae requires $K$ pilots, and since we select the antennae out of all possible antennas the total number of pilots required for the direct channel is $N_t K$. The cascaded channel is estimated for each IRS, each antenna and each user combination which requires a single pilot. Hence through a single antenna the number of pilots is $NK$ and again for doing antennae selection the total number of pilots is $N_t NK$. This makes the total number of pilots required for MO (and infact any algorithm requiring complete CSI) to be $N_t K(N+1)$. Since the number of pilots scale linearly with the number of users, more pilots would be required for a higher number of users.

#### Time Complexity

MO typically has a complexity $\mathcal{O}\left(K^{1.5}N^{1.5}\right)$ [cite paper] and since MO is performed one by one for each antenna the total complexity comes out to be $\mathcal{O}\left(N_t K^{1.5} N^{1.5}\right)$. This is much less than the computational complexity of FP and SCA.

### 4.3.3 Vector Heuristic (VH)

Although MO locally finds an optimal solution to $\mathcal{P}_1$ it is computationally complex and requires complete CSI increasing the training overhead much like the existing techniques in the literature. Hence we propose the vector heuristic which solves the approximate rate expression $\mathcal{P}_2$ but requires substantially less training overhead.

For a fixed antenna $a$, the expression of (4.10) can be optimized by aligning all the IRS elements such that the corresponding mean cascaded links, $\mathbf{h}_{c,a}^{(\text{mean})}$ are aligned along the mean direct channel, $h_{d,a}^{(\text{mean})}$. The $i^{\text{th}}$ reflection coefficient is given by,

$$\theta_i = \angle\left(h_{d,a}^{(\text{mean})} - h_{c,a}^{i\ (\text{mean})}\right)\ \forall i \in \mathcal{N} \tag{4.14}$$

where $h_{c,a}^{i\ (\text{mean})}$ is the $i^{\text{th}}$ element of $\mathbf{h}_{c,a}^{(\text{mean})}$. The VH sets the angle of each IRS element to that of (4.14) which is computed based on the phase difference between the mean direct link and the mean cascaded link through that IRS element. $\mathbf{h}_{c,a}^{(\text{mean})}$ and $\mathbf{h}_{c,a}^{(\text{mean})}$ can be obtained by estimating the complete CSI first and then taking a mean over all the users but instead we propose a training scheme in which only the sum CSI is estimated hence saving on the training pilots. One way to estimate the sum CSI of a link is by letting the users broadcast the same symbol to the AP.

#### Training Overhead

For estimating the direct link, for each antenna selected we only need a single pilot transmission, making the total pilot transmissions to be $N_t$. And for estimating the cascaded link, for each antenna selected there only $N$ pilot transmission required, one for link through each IRS element. This makes the total pilots required for the cascaded channel to be $N_tN$. The total number of pilots required for the VH turns out to be $N_t(N + 1)$. The number of pilots hence do not depend on the number of users and therefore VH scales exceptionally well with the number of users in terms of the training overhead.

#### Time Complexity

The time complexity of the VH comes out to be $\mathcal{O}(N)$ since only the computed reflection coefficients need to be set and the computations themselves take constant time complexity. Hence VH is a linear time algorithm with respect to the number of IRS elements and a constant time algorithm with respect to the number of users.

## 4.4 Numerical Results and Discussion

In this section we compare our two proposed algorithms against each other and also see the effect of number of users on each of their performances.

### 4.4.1 Simulation Setup

We conduct our simulations with $N = 100$ and $N_t = 10$. $K$ varies between 5 to 50. The bandwidth is taken to be 10 MHz with a carrier frequency of 3 Ghz. The power $P$ is taken to be 30 dBm. The imperfection in CSI estimation is modelled according to a similar approach used in [cite], with the estimation error coefficient $\sigma$ as $10^{-10}$. The noise power spectral density is taken to be $-174$ dBm/Hz. Coherence interval $T_c$ is taken to be $3 \times 10^5$.

The channels between AP and users are taken to be nLoS and are modelled as Rayleigh Fading and the channels between AP and IRS, and IRS and Users are LoS with Rice Factor calculated as $13 - 0.03d$, where $d$ is the distance between the channels [21]. The path loss exponent for the direct channel is taken as 3.6 and for channels to and from the IRS is taken as 2.4 The distance between the IRS elements are taken as $\frac{\lambda}{2}$ and the channel correlation is modelled as sinc correlation [27]. There are 1000 different channel realizations taken unless otherwise specified.

### 4.4.2 Results

There are two main results that are discussed, the first being the effect of the number of users on the locally optimal MO and the suboptimal VH. Second we study the effect of the practical considerations, namely, discrete phase shifts, single antenna selection, and the number of IRS elements.

Figure 2 plots the effect of the number of users on the sum datarates for both the algorithm under perfect and imperfect CSI. The sum datarates of (4.3) achieved reaches a maximum and then decreases with increasing number of users. This is because although the sum of log is optimized by the MO and increases monotonically with the number of users, the pre-log factor in (4.3) decreases due to increase in the number of pilots with increasing number of users. Thus VH scales more efficiently with the number of users than any technique which would require complete CSI. Also increasing the difference between the imperfect CSI and the perfect CSI conditions are less evident in VH in comparision to MO. This is primarily due to the fact that VH requires $K$ times less pilot sequences than MO. This makes VH more robust to errors during estimation.

Figure 3 plots the effect of a) increasing the number of IRS elements and b) introducing discrete phase shifts in the IRS elements. The numbers of users is fixed to 25 and it can be seen that the VH and MO perform comparably close. We see that increasing the number of elements has a gain in terms of the sum datarate achieved and the We also see that with 3 bits or 8 phase shifts at the IRS elements we achieve a performance close enough to the case with continuous phaseshifts.

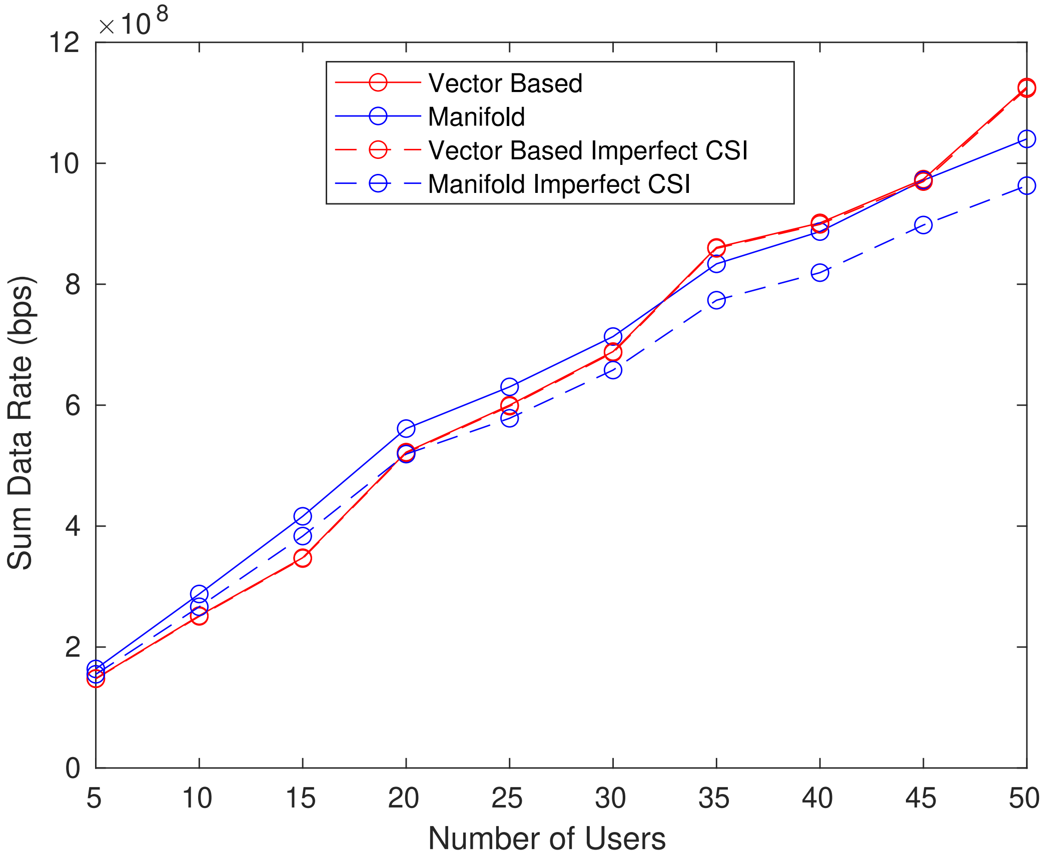


Figure 4.2: Performance of the suggested algorithms with increasing number of users under both Perfect and Imperfect CSI

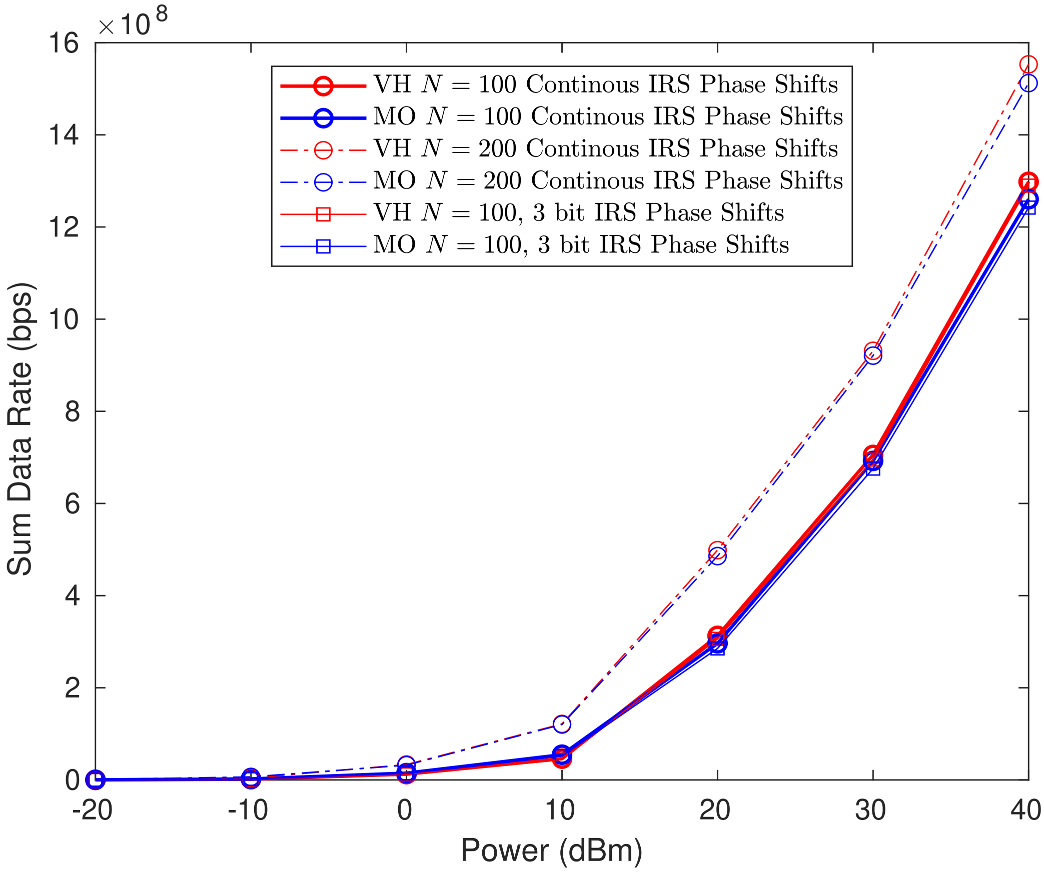


Figure 4.3: Performance of the suggested algorithms with increasing number of users and discrete phase shifts for 25 users.

# Chapter 5

# Conclusion and Future Work

Results show that the proposed heuristic algorithms for passive beamforming using discrete phase shifts in an OFDM based IRS aided wireless setup achieve near-optimal data rate performance even with 4 phase shift bits and have a significant advantage in terms of the time performance. This gain in time performance makes them a better choice in delay sensitive applications, in scenarios when more time for training is needed and when the user is mobile and optimal IRS phase shifts need to be determined again and again. This particular setting with discrete phase-shift programmable IRS elements of different number of bits has not been studied earlier and is of practical consequence, since during deployment IRS elements will end up having discrete phase shifts. Our method is also not dependent on any initialization unlike many methods like the SCA, where number of iterations depend on the initialization strategy.

Extension to a joint optimization related to power allocation across different subcarriers is straightforward as highlighted in previous research using alternating optimization with waterfilling based approach for power allocation. Although these results give an optimistic outlook, further work is needed to make this applicable, including looking into a multiuser based setup and MIMO based transmission.These additions would definitely make the optimization involved but the strongest tap and the vector heuristic technique still might prove useful in lowering the complexity of designing the phase-shifts.

This report also investigated the problem of optimizing the sum datarates in a multiuser environment with the constraint of a single RF chain. We proposed two approaches to solve the problem, an MO based approach which locally optimizes the exact sum of log expression and a VH algorithm which optimizes the sum of absolute channel coefficients. The VH algorithm has much training overhead which is independent of the number of users. We show that increasing the number of users with a fixed coherence interval platues and eventually decreases the performance of MO algorithm and infact any algorithm which depends on complete CSI. We also show that our VH algorithm is more robust to training errors because of the less number of estimations required. Finally we study both the algorithm under practical considerations and see that our algorithms are both scalable and deployable.

Further research is required to study the effect of having multiple antenna selection, and if possible extending the VH to a multiple antenna case. The problem can be further complicated by considering an OFDM based setup. With that being said, the treatment that this work presents by studying the effect of training pilots in existing algorithms for passive beamforming in IRS should be helpful.

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