Naive Bayes

- Bayes thm

* Conditional Probability

A, B

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{s.t. } P(B) \neq 0$$

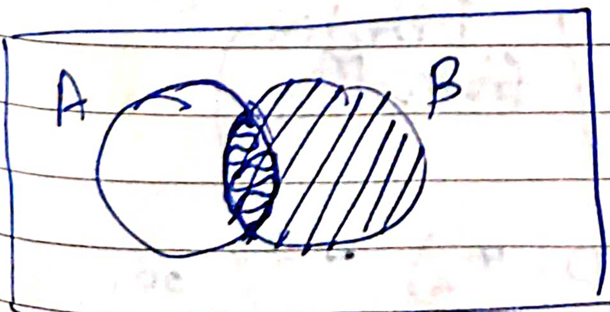
e.g. Throwing of 2 dice

* Independent Events

$$P(A \cap B) = P(A) * P(B)$$

$$P(A) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(B)}{n(S)}}$$

$$P(A|B) = \frac{n(A \cap B)}{n(B)} \quad \text{I}$$



$$\boxed{P(A) = P(A|B)}$$

Mutually Exclusive Events

$$P(A \cap B) = 0$$

$$P(A|B) = 0$$

Bayes Thm

$$\underbrace{P(A|B)}_{\text{posterior probability}} = \frac{\overbrace{P(B|A)}^{\text{likelihood}} \cdot \overbrace{P(A)}^{\text{Prior}}}{\underbrace{P(B)}_{\text{Evidence}}}$$

given $p(B) \neq 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(B|A) * P(A)$$ Replace

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

	M_1	M_2	M_3
	20	30	50
detect	5%	3%	1%

Given
$P(M_1) = 1/5$
$P(M_2) = 3/10$
$P(M_3) = 1/2$

Probability of (~~detective~~ M_3 / ~~from M3~~ Detective)

$P(D|M_1) = 1/20$ $\quad P(D|M_2) = 3/100$

$P(D|M_3) = 1/100$

$$P(M_3 | D) = \frac{P(D | M_3)\, P(M_3)}{P(D)}$$

$$= \frac{(1/100)(1/2)}{6/250} = \frac{1}{\cancel{200}_{4}} \times \frac{\cancel{250}^{5}}{6}$$

$$= 5/24$$

$$P(D) = P(D \cap M_1) + P(D \cap M_2) + P(D \cap M_3)$$

$$\cancel{= \frac{1}{20} + \frac{3}{100} + \frac{1}{100} = \frac{9}{100}}$$

$$= P(D|M_1)P(M_1) + P(D|M_2)P(M_2) + P(D|M_3)P(M_3)$$

$$= \frac{1}{20} \times \frac{1}{5} + \frac{3}{100} \times \frac{3}{10} + \frac{1}{100} \times \frac{1}{2}$$

SUNDAY 11

$$= \frac{1}{100} + \frac{9}{1000} + \frac{1}{200}$$

$$= \frac{10 + 9 + 5}{1000}$$

$$= \frac{24}{1000} = \frac{6}{250}$$

Toss	Venue	Outlook	Result
won	Mumbai	overcast	Won
Lost	Chennai	sunny	Won
won	Kolkata	Sunny	Won
won	C.	Sunny	Won
lost	M	sunny	Lost
won	C	Overcast	Lost
won	K	Overcast	Lost
won	M	Sunny	Won

Will CSK win ~~for~~ for following condition
{ lost , Mumbai , Sunny }

$P(W \mid Lost \cap Mumbai \cap Sunny)$, $P(L \mid Lost \cap M \cap Sunny)$

$$= \frac{P(lost, M, S \mid W)\, P(W)}{P(lost, Mumbai, Sunny)}$$

Only consider Numerators

$= 0 \times 5/8 = 0$

$P(l/W)\, P(M/W)\, P(S/W)\, P(W)$

$\frac{1}{5} \times \frac{2}{5} \times \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{8}_{\cancel{2}}} = \frac{1}{25}$

$$= \frac{P(l, M, S \mid L)\, P(L)}{P(l, M, S)}$$

$P(l|L)\, P(M|L)\, P(S|L)\, P(L)$

$= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{\cancel{3}} \times \frac{\cancel{3}}{8}$

$= \frac{1}{72}$

$X = \{x_1, x_2, x_3, \ldots x_n\}$ features (Attributes)

$C_k = \{C_1, C_2, C_3, \ldots C_k\}$ no. of classes

$$P(C_k|X) = P(X|C_k)\, P(C_k)$$

$$P(A|B) = P(A \cap B) / P(B)$$

$$P(A \cap B) = P(A|B)\, P(B)$$

$$\Rightarrow P(C_k|X) = P(X \cap C_k)$$

$$= P(X, C_k)$$

$$= P(\overset{A}{\boxed{x_1}}, \overset{\cap \quad B}{\boxed{x_2, x_3 \ldots x_n, C_k}})$$

$$= P(\underbrace{x_1 | x_2, x_3 \ldots x_n, C_k}_{a})\, P(x_2, x_3 \ldots x_n, C_k)$$

$$P(C_k|X) = a \times P(x_2, x_3, \ldots x_n, C_k)$$

$$P(\overset{A}{x_2}, \overset{\cap \quad B}{\boxed{x_3, \ldots x_n, C_k}}) = P(\underbrace{x_2 | x_3, x_4 \ldots x_n, C_k}_{b})\, P(x_3, x_4 \ldots x_n, C_k)$$

$$\Rightarrow P(C_k|X) = a\, b\, P(x_3, x_4, \ldots x_n, C_k)$$

Chain Rule for conditional Probability...

9 ~~$P(x_n | C_k)$~~.

10 $P(C_k | X) = P(x_1 | x_2, x_3 \dots x_n, C_k)\, P(x_2 | x_3, x_4 \dots x_n, C_k)$
$P(x_3 | x_4, x_5 \dots x_n, C_k) \dots \dots$
11 $P(x_{n-1} | x_n, C_k)\, P(x_n | C_k)\, P(C_k)$

12 Assumption → x_1 doesn't depend on $x_2, x_3 \dots x_n$
1 x_1 depends only on C_k.

2 Conditional Independance

3 $P(A|B) = P(A)$ $P(A|B,C) = P(A|C)$

4 $P(C_k | X) =$
$P(x_1 | C_k)\, P(x_2 | C_k)\, P(x_3 | C_k) \dots P(x_n | C_k)\, P(C_k)$

5

6 $$\boxed{P(C_k | X) = P(C_k) \prod_{i=1}^{n} P(x_i | C_k)}$$

product

$$\hat{y} = \underset{k \in \{1,2,\dots k\}}{\operatorname{argmax}} P(C_k) \prod_{i=1}^{n} P(x_i | C_k)$$

maximum aposterion Rule.