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Class - SYCSE

Roll no. - 7

Batch - S1

Sub. - DMCAT

Assignment-6

Q1. For each of the following, determine whether $*$ is a binary operation.

a. on \mathbb{Z} , where $a * b = a^b$.

b. on \mathbb{R} , where $a * b = a \times |b|$.

Soln:

a. On \mathbb{Z} , where $a * b = a^b$.

=> No, since $2 * (-1) = 2^{-1} = \frac{1}{2} \notin \mathbb{Z}$.

b. On \mathbb{R} , where $a * b = a \times |b|$.

=> Yes, since $*$ is a function, with $a \times |b| \in \mathbb{R}$.

Q2. For each of the following determine whether the binary operation $*$ is commutative or associative.

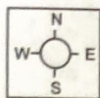
a. On \mathbb{N} , where $a * b = \min(a, b)$.

b. On \mathbb{N} , where $a * b = ab + 2b$.

Soln:

a. On \mathbb{N} , where $a * b = \min(a, b)$.

=> $*$ is commutative as well as associative.



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b. On \mathbb{N} , where $a * b = ab + 2b$

=)

 $*$ is not commutative since

$$2 * 3 = 6 + 6 = 12, \text{ while}$$

$$3 * 2 = 6 + 4 = 10$$

Q3. Let $(A, *)$ be an algebraic system such that for all a, b, c, d ,

$$a * a = a,$$

$$(a * b) * (c * d) = (a * c) * (b * d)$$

Show that $a * (b * c) = (a * b) * (a * c)$ Solⁿ:

$$\text{Since } a * a = a$$

$$a * (b * c) = (a * a) * (b * c)$$

$$= (a * b) * (a * c)$$

Q4. The following table, of a binary operation $*$ is given. Is $*$ commutative?

$*$	a	b	c
a	b	c	a
b	c	b	a
c	a	b	c

Solⁿ:

From the table we observe the following:

$$a * b = c, \quad b * a = c$$

$$a * c = a, \quad c * a = a$$

$$b * c = a, \quad c * b = b, \quad \text{and } a \neq b.$$

[\therefore Hence $*$ is not commutative.]



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Q.5. Define -

a. Monoid

b. Group

c. Semigroup

d. Field

e. Integral Domain

f. Ring

Soln:

a. Monoid

=> A monoid is a semigroup $(A, *)$ that has an identity element.

b. Group

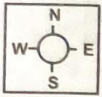
=> A group $(G, *)$ is a monoid, with identity e , such that every element $a \in G$ there exists an element $a^{-1} \in G$, called as the inverse of a , such that $a * a^{-1} = a^{-1} * a = e$.

c. Semigroup

=> Let $(A, *)$ be an algebraic system, with a binary operation $*$ on A . Then $(A, *)$ is called a semigroup if $*$ is associative.

d. Field

=> If every non-zero element has a multiplicative inverse, then R is called a field. A field is an integral domain, since if $a, b \in R$.



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E) Integral Domain

\Rightarrow

Let R be a commutative ring. Then R is called an integral domain if it has no zero divisors.

F) Ring

\Rightarrow

A ring R is said to be a ring with unit element if there exists an element, denoted by the symbol 1 such that $a \cdot 1 = 1 \cdot a = a$, for all $a \in R$.