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Assignment - 1

Q1] Find $L\left\{\frac{\sin t \cdot \sin 5t}{t}\right\}$

Soln:

$$\sin A \cdot \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin t \cdot \sin 5t = \frac{1}{2}\left[\cos(-4t) - \cos(6t)\right]$$

$$= \frac{1}{2}\left[\cos 4t - \cos 6t\right]$$

$$L\{\sin t \cdot \sin 5t\} = \frac{1}{2}\left[L\{\cos 4t - \cos 6t\}\right]$$

$$= \frac{1}{2}\left[\frac{s}{s^2+16} - \frac{s}{s^2+36}\right]$$

$$L\left\{\frac{\sin t \cdot \sin 5t}{t}\right\} = \frac{1}{2} \times \frac{1}{2}\int_s^\infty \left[\frac{2s}{s^2+16} - \frac{2s}{s^2+36}\right] ds$$

$$= \frac{1}{4}\left[\log(s^2+16) - \log(s^2+36)\right]_s^\infty$$

$$= \frac{1}{4}\log\left[\frac{s^2+16}{s^2+36}\right]_s^\infty$$

$$= \frac{1}{4}\left[0 - \log\left(\frac{s^2+16}{s^2+36}\right)\right]$$

$$\boxed{L\left\{\frac{\sin t \cdot \sin 5t}{t}\right\} = \frac{1}{4}\log\left(\frac{s^2+36}{s^2+16}\right)}$$

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Q2] Find $L\left\{\int_0^t te^{-3t}\sin 2t\,dt\right\}$

Soln:

$f(t) = \sin 2t$

$L\{f(t)\} = L\{\sin 2t\} = \frac{2}{s^2+4}$

We know that,

$L\{f(t)\} = f(s) = \frac{2}{s^2+4}$

By using multiplication by t property

$L\{t\sin 2t\} = (-1)^1 \frac{d}{ds}\left(\frac{2}{s^2+4}\right)$

$L\{t\sin 2t\} = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$

By using $\frac{d}{ds}\left(\frac{u}{v}\right) = \frac{u \cdot v' - v \cdot u'}{v^2}$

Hence,

$\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = 2\frac{d}{ds}\frac{(s^2+4) - (s^2+4)\frac{d}{ds}}{(s^2+4)^2}$

Now by using first shifting theorem

$L\{e^{-at}.f(t)\} = f(s+a)$

$f(s) = \frac{-4s}{(s^2+4)^2}$ $(t\sin 2t)$

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$$L\{e^{-3t} \cdot t\sin 2t\} = \frac{-4(s+3)}{((s+3)^2+4)^2}$$

$$= \frac{-4(s+3)}{[s^2+6s+9+4]^2}$$

$$= \frac{-4(s+3)}{[s^2+6s+13]^2}$$

$$\therefore L\{e^{-3t} \cdot t\sin 2t\} = \frac{-4(s+3)}{[s^2+6s+13]^2}$$

Now we know that,

$$L\left\{\int_0^t f(t)\,dt\right\} = \left\{\frac{1}{s} F(s)\right\}$$

$$\therefore L\left\{\int_0^t t \cdot e^{-3t}\sin 2t\,dt\right\} = \frac{1}{s}\left[\frac{-4(s+3)}{[s^2+6s+13]^2}\right]$$

$$\boxed{\therefore L\left\{\int_0^t t e^{-3t}\sin 2t\,dt\right\} = \frac{1}{s}\left[\frac{-4(s+3)}{[s^2+6s+13]^2}\right]}$$

Q3] Evaluate $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$

Soln:-

Here,

$$\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$$

$$\int_0^\infty e^{-t} \left(\frac{1-e^{-2t}}{t}\right) dt \quad - ①$$

Now we have to find $L\left\{\frac{1-e^{-2t}}{t}\right\}$

$\therefore L\{1-e^{-2t}\} = L\{1\} - L\{e^{-2t}\}$

$$= \frac{1}{s} - \frac{1}{s+2}$$

$$L\left\{\frac{1-e^{-2t}}{t}\right\} = \int_s^\infty \frac{1}{s} - \frac{1}{s+2} \, ds$$

using $\int \frac{f'(x)}{f(x)} = \log f(x)$

Here $f(x) = s$ & $f(x) = s+2$

$f'(x) = 1$ $\quad f'(x) = 2$

$$\therefore L\left\{\frac{1-e^{-2t}}{t}\right\} = [\log(s) - \log(s+2)]_s^\infty$$

$$= \left[\log\left(\frac{s}{s+2}\right)\right]_s^\infty$$

$$= \left[0 - \log\left(\frac{s}{s+2}\right)\right]$$

$$\therefore L\left\{\frac{1-e^{-2t}}{t}\right\} = \log\left(\frac{s+2}{s}\right) \quad - ②$$

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Now from eq^n ① put value of $s=1$ in eq^n ②.

put $s=1$

$$= \log\left(\frac{1+2}{1}\right)$$

$$= \log 3$$

$$\boxed{\therefore \int_0^\infty \frac{e^{-t}-e^{-3t}}{t}\,dt = \log 3}$$