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Assignment – 6

Q1. For each of the following, determine whether * is a binary operation.

a. on Z, where $a * b = a^b$.

b. on R, where $a * b = a \times |b|$.

Soln:

a. On Z, where $a * b = a^b$.

⇒ No, since $2 * (-1) = 2^{-1} = \frac{1}{2} \notin Z$.

b. on R, where $a * b = a \times |b|$

⇒ Yes, since * is a function, with $a \times |b| \in R$.

Q2. For each of the following determine whether the binary operation * is commutative or associative.

a. On N, where $a * b = \min(a, b)$

b. On N, where $a * b = ab + 2b$.

Soln:

a. On N, where $a * b = \min(a, b)$.

⇒

* is commutative as well as associative.

b. On N, where $a * b = ab + 2b$

$\Rightarrow$

$*$ is not commutative since

$2 * 3 = 6 + 6 = 12$, while

$3 * 2 = 6 + 4 = 10$

Q3. Let $(A, *)$ be an algebraic system such that for all $a, b, c, d \in B$

$a * a = a,$

$(a * b) * (c * d) = (a * c) * (b * d)$

Show that $a * (b * c) = (a * b) * (a * c)$

Soln:

Since $a * a = a$

$a * (b * c) = (a * a) * (b * c)$

$= (a * b) * (a * c)$

Q4. The following table, of a binary operation $*$ is given. Is $*$ commutative?

*	a	b	c
a	b	c	a
b	c	b	a
c	a	b	c

Soln:

from the table we observe the following:

$a * b = c, \quad b * a = c$

$a * c = a, \quad c * a = a$

$b * c = a, \quad c * b = b,$ and $a \neq b.$

[$\therefore$ Hence $*$ is not commutative]

Q5. Define-
a. Monoid
b. Group
c. Semigroup
d. field
e. Integral Domain
f. Ring

Soln:-

a. Monoid

=) A monoid is a semigroup $(A, *)$ that has an identity element.

b. Group

=) A group $(G, *)$ is a monoid, with identity e, such that every element $a \in G$ there exists an element $a^{-1} \in G$, called as the inverse of a, such that $a * a^{-1} = a^{-1} * a = e$.

c. Semigroup

=) Let $(A, *)$ be an algebraic system, with a binary operation $*$ on A. Then $(A, *)$ is called a semigroup if $*$ is associative.

d. field

=) If every non-zero element has a multiplicative inverse, then R is called a field. A field is an integral domain, since if $a, b \in R$.

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e) Integral Domain

=)

Let R be a commutative ring. Then R is called an integral domain if it has no zero divisors.

F) Ring

=)

A ring R is said to be a ring with unit element if there exists an element, denoted by the symbol 1 such that $a \cdot 1 = 1 \cdot a = a$, for all $a \in R$.