UNIT 4: Greedy Technique

Greedy Technique:

Dijkstra’s algorithm

Variants of the shortest path problem:

- Single- source shortest - paths problem
- Single- destination shortest - paths problem
- Single - pair shortest - path problem
- All - pairs shortest - paths problem

Algorithms to solve shortest path problem:

- **Dijkstra's algorithm** solves the single-source shortest paths problem with non-negative edge weight.
- **Bellman–Ford algorithm** solves the single-source shortest paths if edge weights may be negative. *But do not solve if there is a negative cycle.*
- **Floyd–Warshall algorithm** solves all pairs shortest paths.
- **Johnson's algorithm** solves all pairs shortest paths, and may be faster than Floyd–Warshall on sparse graphs.
- **A* search algorithm** solves for single-pair shortest path using heuristics to try to speed up the search.

Shortest paths applications:

- Driving directions on web mapping
- If a nondeterministic abstract machine is represented as a graph where vertices describe states and edges describe possible transitions, shortest path algorithms can be used to find an optimal sequence of choices to reach a certain goal state, or to establish lower bounds on the time needed to reach a given state

  if vertices represent the states of a puzzle like a Rubik's Cube and each directed edge corresponds to a single move or turn, shortest path algorithms can be used to find a solution that uses the minimum possible number of moves.

- networking or telecommunications
- operations research: plant and facility layout planning , robotics, transportation
- VLSI design

Dijkstra’s algorithm to find Single Source Shortest Paths

Single Source Shortest Paths Problem

- For a given vertex called the source in a weighted connected graph, find shortest paths to all its other vertices.
- The single-source shortest-paths problem asks for a family of paths, each leading from the source to a different vertex in the graph, though some paths may, of course, have edges in common.

NOTE:

BFS algorithm finds shortest path; works on unweighted graphs; each edge is considered to have unit weight.

Dijkstra's algorithm is usually the working principle behind

- link-state routing protocols,
- Open Shortest Path First (OSPF) and
- Intermediate System to Intermediate System(IS-IS)

```
ALGORITHM  Dijkstra(G, s)
    //Dijkstra's algorithm for single-source shortest paths
    //Input: A weighted connected graph G = ⟨V, E⟩ with nonnegative weights
    //          and its vertex s
    //Output: The length d_v of a shortest path from s to v
    //           and its penultimate vertex p_v for every vertex v in V
    Initialize(Q)   //initialize vertex priority queue to empty
    for every vertex v in V do
        d_v ← ∞;   p_v ← null
        Insert(Q, v, d_v)   //initialize vertex priority in the priority queue
    d_s ← 0;   Decrease(Q, s, d_s)   //update priority of s with d_s
    V_T ← ∅
    for i ← 0 to |V| − 1 do
        u* ← DeleteMin(Q)   //delete the minimum priority element
        V_T ← V_T ∪ {u*}
        for every vertex u in V − V_T that is adjacent to u* do
            if d_u* + w(u*, u) < d_u
                d_u ← d_u* + w(u*, u);   p_u ← u*
                Decrease(Q, u, d_u)
```

Time efficiency of Dijkstra's algorithm

- Depends on the data structures used for implementing the priority queue and for representing an input graph itself.

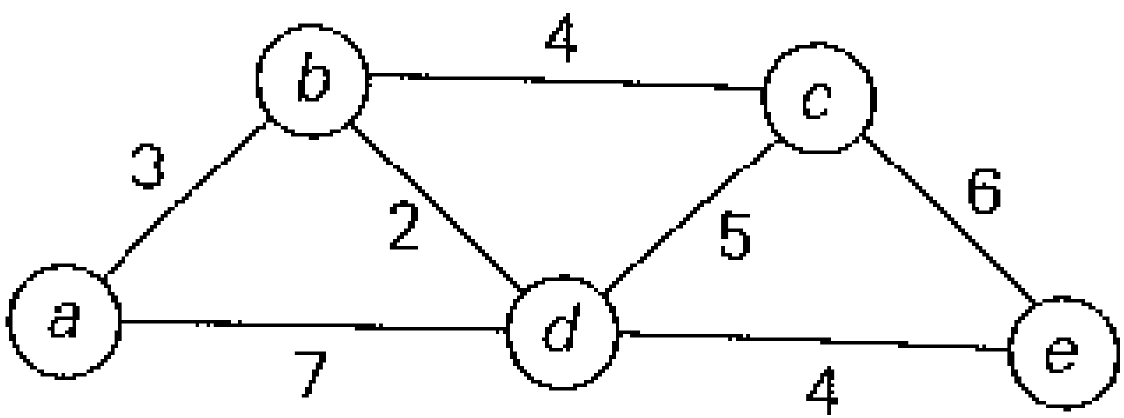
- Weight matrix + priority queue implemented as an unordered array: $\mathbf{\Theta(|v|^2)}$
- Adjacency list + priority queue implemented as a min-heap: $\mathbf{O(|E| \log|V|)}$

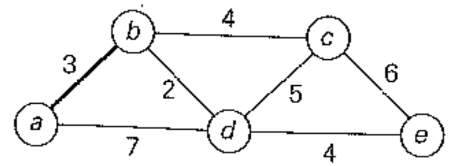
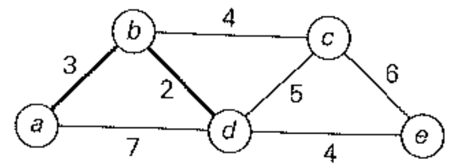
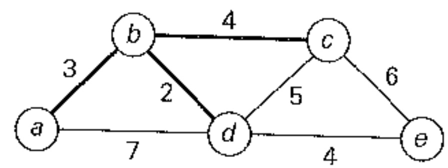
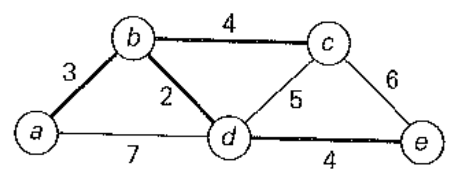
Dijkstra’s algorithm visualization

https://www.cs.usfca.edu/~galles/visualization/Dijkstra.html

https://visualgo.net/en/sssp

Apply Dijkstra's algorithm to find single source shortest paths from source vertex 'a'



<table>
<tr><th>Tree vertices</th><th>Remaining vertices</th><th>Illustration</th></tr>
<tr><td>a(−, 0)</td><td>b(a, 3) c(−, ∞) d(a, 7) e(−, ∞)</td><td>
</td></tr>
<tr><td>b(a, 3)</td><td>c(b, 3 + 4) d(b, 3 + 2) e(−, ∞)</td><td>
</td></tr>
<tr><td>d(b, 5)</td><td>c(b, 7) e(d, 5 + 4)</td><td>
</td></tr>
<tr><td>c(b, 7)</td><td>e(d, 9)</td><td>
</td></tr>
<tr><td>e(d, 9)</td><td></td><td></td></tr>
</table>

from a to b :	$a - b$	of length 3
from a to d :	$a - b - d$	of length 5
from a to c :	$a - b - c$	of length 7
from a to e :	$a - b - d - e$	of length 9

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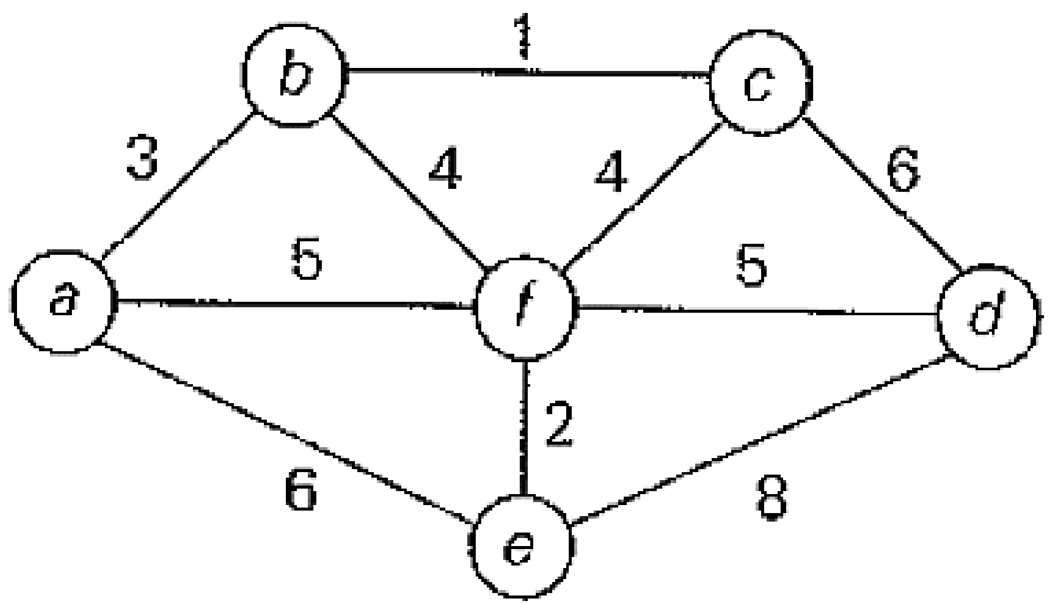
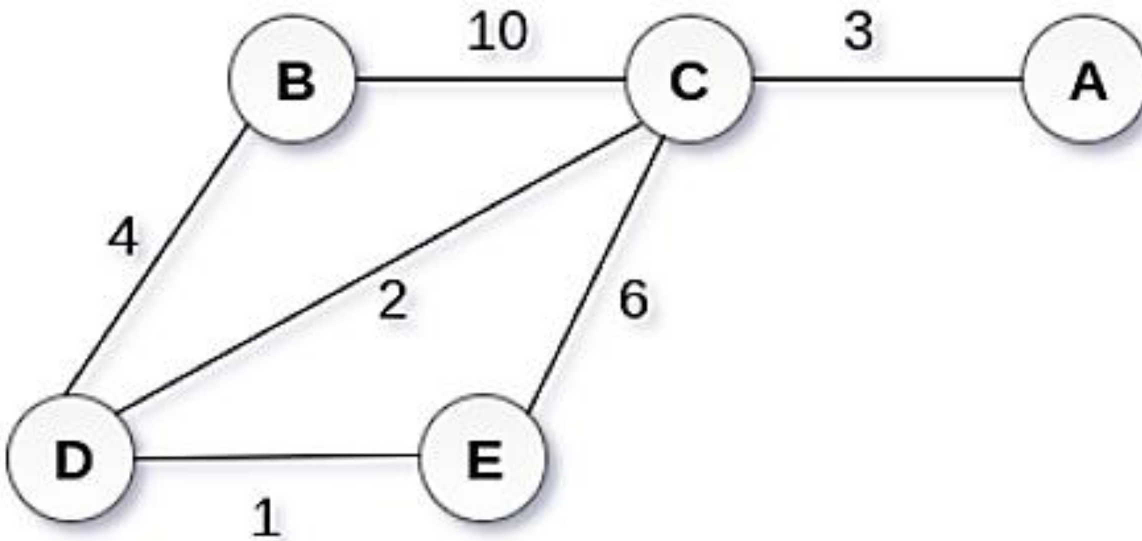


Let’s check our understanding



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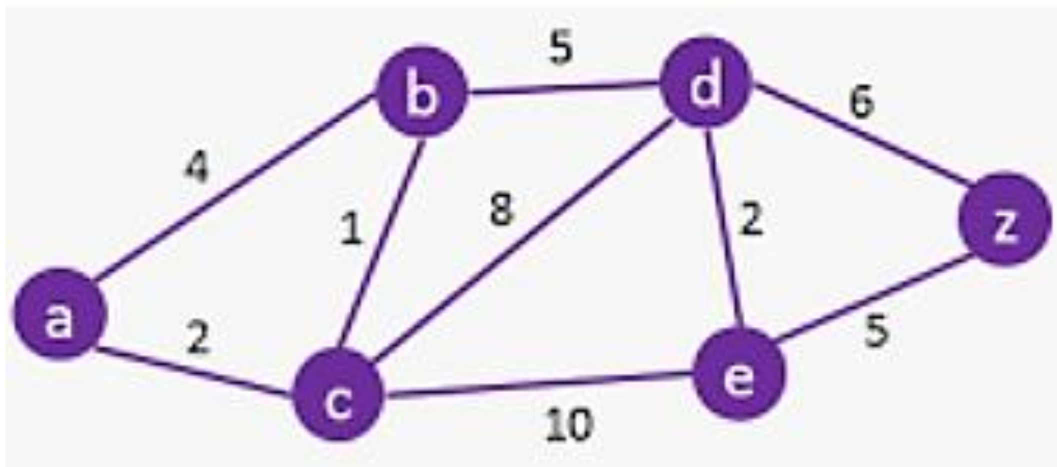

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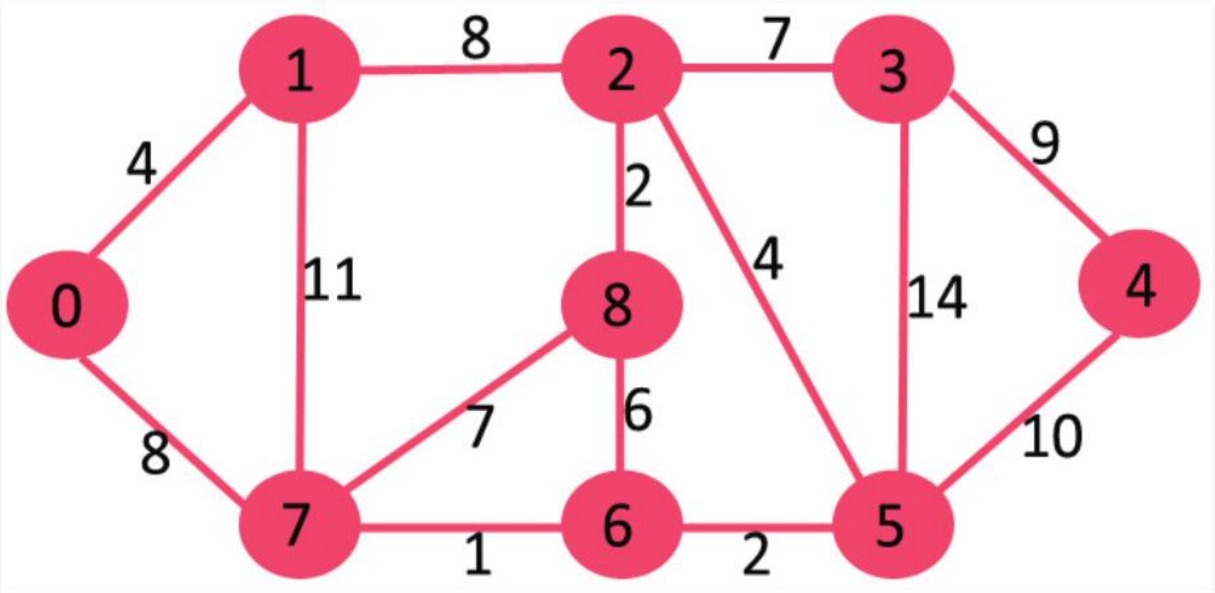

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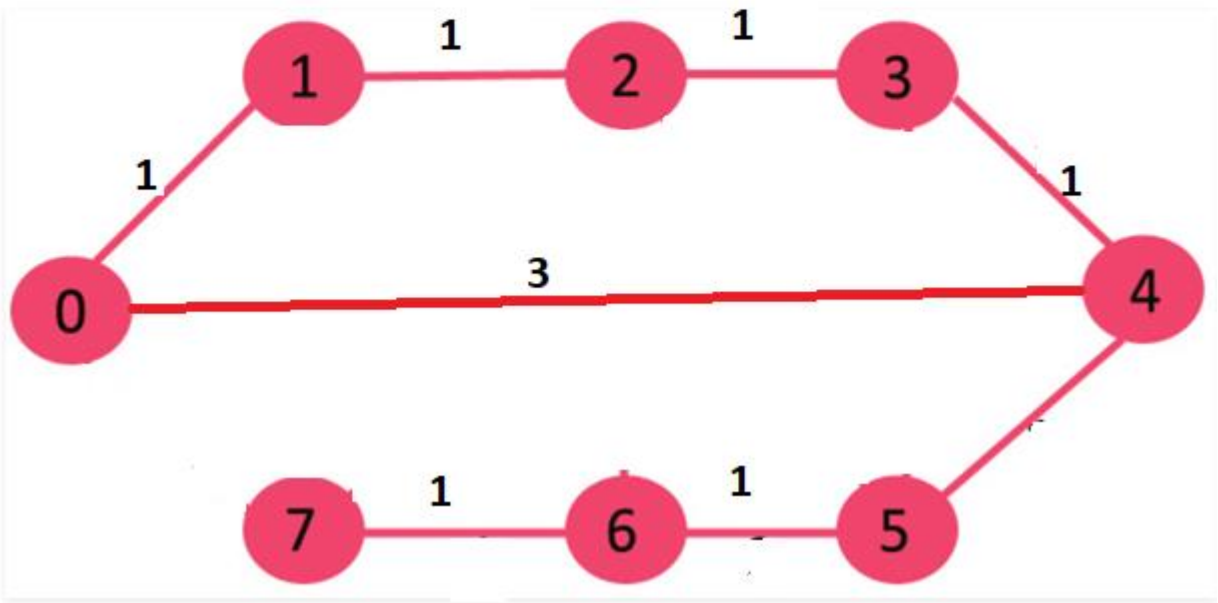
MST vs Shortest Paths



Find MST, Single source shortest paths : consider 0 as the source vertex

Extra Miles…

Bellman Ford Algorithm