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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

III Semester B. E. Regular / Supplementary Examinations Jan / Feb-2025

Artificial Intelligence & Machine Learning

MATHEMATICS FOR ARTIFICIAL INTELLIGENCE & MACHINE LEARNING

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of third semester handbook to be provided.

PART-A

			M	BT	CO
1	1.1	An e-mail filter is planned to separate valid e-mails from spam. The word free occurs in 70% of the spam messages and only 5% of the valid messages. Also, 15% of the messages are spam. The probability that the message contains the word free is ________.	02	2	2
	1.2	Suppose the number of heads in tossing a biased coin has expectation $\mu = 2$ and a variance of 1.5. According to Chebyshev's inequality, the probability of getting more than 5 heads is ________.	02	4	4
	1.3	Assume that each of your calls to a popular radio station has a probability of 0.06 of connecting. Assume that your calls are independent. The probability that it requires more than 6 calls for you to connect is ________.	02	2	1
	1.4	The thickness of a flange on an aircraft component is uniformly distributed between 0.96 and 1.04 millimeters. What thickness is exceeded by 90 % of the flanges?	02	4	4
	1.5	Two ballpoint pens are selected at random from a box that contains 2 blue pens, 1 red pen, and 3 green pens. If X is the number of red pens selected and Y is the number of blue pens selected, construct the joint probability distribution table.	02	3	3
	1.6	If the joint density function of X and Y is $f(x,y) = \frac{1}{2},\ 0 < x < y < 2$ and the marginal density function of X is $P_1(x) = \frac{2-x}{2}, 0 < x < 2$, then the conditional probability $P(y < 1 \mid x = \frac{1}{2})$ is ________.	02	2	1
	1.7	The value of k such that the vectors $2x - 3x^2,\ 3 + 2x,\ 2 + kx^2$ are linearly dependent is ________.	02	2	2
	1.8	The vector (1,3) is reflected about the line y=x. The reflection matrix and the resultant vector are ________ and ________ respectively.	02	1	1
	1.9	Given $y = (2,-1)$ and $u = (2,2)$, the orthogonal projection of y onto u and the vector orthogonal to u are ________ and ________ respectively.	02	1	1
	1.10	The singular values of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & -1 \end{bmatrix}$ are ________.	02	3	3

PART-B

2	a	For the given probability mass function $f(x) = c(x^2+1)$, for $x = 1,2,3,4$ find: i) the constant c, ii) $P(X \leq 3)$, iii) $P(X > 2)$, iv) $P(1 \leq X < 3)$, v) the cumulative distribution vi) mean and variance of X.	08	3	2
	b	The diameter of a particle of contamination (in micrometers) is modeled with the probability density function $f(x) = \frac{2}{x^3}$ for $x > 1$. Determine: i) $P(X < 3)$, ii) $P(X > 4)$, iii) $P(X < 6 \text{ or } X > 8)$ iv) Determine k such that $P(X < k) = 0.90$.	08	4	3
3	a	A software package consists of 10 programs, 4 of which must be upgraded. If 4 programs are randomly chosen for testing, i) What is the probability that at least two of them must be upgraded? ii) What is the expected number of programs, out of the chosen four, that mat be upgraded? (Use binomial distribution)	04	3	2
	b	Messages arrive at an electronic message center at random times, with an average of 5 messages per 30 minutes. Using Poisson distribution compute the probability of receiving : i) more than 2 messages in the next hour, ii) less than 3 messages in the next hour, iii) exactly 4 messages in the next hour.	06	3	3
	c	The speed of a file transfer from a server on campus to a personal computer at a student's home on a weekday evening is normally distributed with a mean of 60 kilobits per second and a standard deviation of 4 kilobits per second. What is the probability that: i) the file will transfer at a speed of 65 kilobits per second or more? ii) Below what values should the speed be such that the probability is 0.75.	06	4	4
		OR			
4	a	After a computer virus entered the system, a computer manager checks the condition of all important files. She knows that each file has probability 0.2 to be damaged by the virus, independently of other files. Compute the probability that: i) at least 2 of the first 10 files are damaged, ii) exactly 2 of the first 10 files are damaged.	04	3	2
	b	On an average, 1 computer in 1000 crashes during a severe thunderstorm. A certain company has 5,000 working computers when the area was hit by a severe thunderstorm. Compute the probability that: i) less than 3 computers crashed, ii) more than 3 computes crashed.	06	3	3

	c	The time between calls to a corporate office is exponentially distributed with a mean of 10 minutes: i) What is the probability that there are no calls within half an hour? ii) What is the probability that there is a call within half an hour? iii) Determine x such that the probability that there are no calls within x hours is 0.10.	06	4	4
5	a	For a mobile website, let X denote the number of bars of service, and Y denote the response time (to the nearest second) for a particular user and site. The joint distribution of X and Y is given by $P(X,Y)$, where $P(1,1) = 0.01$, $P(1,2) = 0.02$, $P(1,3) = 0.02$, $P(1,4) = 0.05$, $P(2,1) = 0.10$, $P(2,2) = 0.05$, $P(2,3) = 0.03$, $P(2,4) = 0.02$, $P(3,1) = 0.25, P(3,2) = 0.20, P(3,3) = 0.15, P(3,4) = 0.10$. Construct the joint distribution table and hence find: i) the marginal probability distribution of X and Y ii) the probability that the number of bars of the service is at least 2, iii) the response time is less than 2 seconds, iv) the conditional probability of Y, given $X = 1$.	08	2	2
	b	The joint density for the random variables (X,Y) is given as: $f(x,y) = \begin{cases} cxy^2, & 0 < x < 1, 0 < y < 1 \\ 0, & elsewhere \end{cases}$. Compute the constant c, the expected values of X and Y.	08	3	3
		OR			
6	a	Determine the value of c that makes the function $f(x,y) = 2c(x+y)$ a joint probability mass function over the nine points with $x = 1,2,3$ and $y = 1,2,3$. Determine: i) the marginal distributions of X and Y, ii) $E(X), E(Y), E(XY), Cov(X,Y)$.	08	2	2
	b	Let the random variable X denote the time until a compute server connects to your machine (in milliseconds), and let Y denote the time until the server authorizes you as a valid user (in milliseconds). Each of these random variables measures the wait from a common starting time and $Y < X$. Assume that the joint probability density function for X and Y is $f(x,y) = 6e^{-2x-y}$ for $0 < y < x < \infty$. i) Verify that $f(x,y)$ is a valid joint density function, ii) the marginal density function of Y, iii) the conditional probability density function for X, given that $Y = y$.	08	3	3
7	a	Prove that the set of vectors P_1 = set of polynomials of degree less than or equal to one, is a vector space over the field $\mathbb{R}$.	06	3	2
	b	Find the bases and dimension of the Four fundamental subspaces of the matrix $A = \begin{bmatrix} 1 & -2 & 1 & -5 \\ 2 & 0 & -6 & -2 \\ -3 & 1 & 7 & 5 \end{bmatrix}$.	10	4	4
		OR			
8	a	Verify if the sets i) $W1$ =set of 2×2 symmetric matrices, ii) $W2$ =set of 2×2 non-singular matrices are subspaces of $M_{2\times2}$, the set of 2×2 matrices.	06	3	2

	b	Obtain the linear transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that $T(1,1,1) = (3,0,3), T(2,2,1) = (6,2,4), T(1,-1,1) = (-1,-4,3)$. Also find the rangespace, nullspace and verify the rank-nullity theorem.	10	4	4
9	a	Obtain the QR factorization of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \\ 0 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$.	06	3	3
	b	Decompose the matrix $A = \begin{bmatrix} 1 & 5 & 5 \\ 5 & 5 & 1 \\ -5 & 1 & 5 \end{bmatrix}$ as PDP^{-1} by the process of diagonalization.	10	4	4
		OR			
10	a	Obtain the orthonormal basis for the column space of the matrix $\begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$.	06	3	3
	b	Obtain the Singular Value Decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ -1 & 2 \end{bmatrix}$.	10	4	4