DEPARTMENT OF MATHEMATICS

Discrete Mathematical Structures and Combinatorics (CS241AT)
Practice Problems

Set 1

1. In how many ways can Mr. Rahul distribute ten distinct books to his students (one book to each student) and then collect and redistribute the books so that each child has the opportunity to read two different books?
2. There are 30 participants in a running race and each participant has a chance to win a different sized trophies that are to be awarded to first 8 runners who finish.
   a) In how many ways can the trophies be awarded?
   b) If Ram and Shyam are two participants in the race, in how many ways can the trophies be awarded with these two runners among the top three?
3. How many positive integers n can be formed using 3,4,4,5,5,6,7 if we want n to exceed 50,00,000.
4. Obtain the number of proper divisors of 10800.
5. Find the values of n in each the following
   (a) $P(n,2) = 90$, (b) $P(n,3) = 3P(n,2)$ and (c) $2P(n,2) + 50 = P(2n,2)$.
6. How many different paths in the xy plane are there from $(0,0)$ to (7,7) if a path proceeds one step at a time by going either one space to the right (R) or one space upwards(U)? How many such paths are there from $(2,7)$ to (9,14)?
7. Determine the number of six-digit positive integers (no leading zeros) in which
   (a) No digit may be repeated
   (b) Digits may be repeated
   (c) How many 6-digit positive integer which is (i) even, (ii) divisible by 5, (iii) divisible by 4 be formed without repeating the digits?
8. In how many ways 7 people can be arranged about a circular table? If two people insist to sit next to each other, how any arrangements are possible?
9. A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carried out if (a)there are no restrictions? (b) there must be 6 men and 6 women? (c) there must be an even number of women? (d) there must be more women than men? (d) there must be atleast 8 men?
10. In how many ways can 12 different books be distributed among 4 children so that (a) each child gets three books? (b) the two oldest children get four books each and two youngest get two books each?
11. How many ways are there to select four pieces of fruit from a bowl containing apples, oranges, and pears if the order in which the pieces are selected does not matter, only the type of fruit and not the individual piece matters, and there are at least four pieces of each type of fruit in the bowl?
12. In the complete expansion of
    $(a+b+c+d)(e+f+g+h)(u+v+w+x+y+z)$ how many terms such as agw, cfv and dgz?
13. Determine the coefficient of x^9y^3 in the expansions of (a) $(x+y)^{12}$ (b) $(x+2y)^{12}$ (c) $(2x-3y)^{12}$
14. Find the coefficient of xyz^2 in $(2x-y-z)^4$.
15. In how many ways can 10 (identical) chocolates be distributed among 5 if (a) there are no restrictions?
    (a) each child gets at least one chocolate? (c) the oldest child gets at least two?
16. In how many ways can a teacher distribute 8 chocolate donuts and 7 jelly donuts among three students if each student wants at least one of each kind?
17. Determine the number of positive integers n, where $1 \le n \le 100$ and n is not divisible by 2, 3 or 5.

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Set 2

1. Solve the recurrence relation $a_n = 5a_{n-1} + 6a_{n-2}$, $a \geq 2$, $a_0 = 1, a_1 = 3$.
2. Solve the recurrence relation $a_n + 4a_{n-2} = 0$, $a \geq 2$, $a_0 = 1, a_1 = 1$.
3. If $a_0 = 0, a_1 = 1, a_2 = 4$ and $a_3 = 37$ satisfy the recurrence relation $a_n + ba_{n-1} + ca_{n-2} = 0$, where $n \geq 2$, and b, c are constants, solve for a_n.
4. An electrical circuit has a current sequence defined by the recurrence relation: $I_n = 5I_{n-1} - 6I_{n-2}$ for $n \geq 2$ with initial conditions $I_0 = 2, I_1 = 5$. Find the general formula for I_n.
5. A city's population follows the recurrence relation (Discrete population growth model): $P_n = 1.3P_{n-1} - 0.3P_{n-2} + 500$ with the initial values $P_0 = 10000, P_1 = 11000$. Find the general formula for P_n.
6. Determine the constants b and c if $a_n = c_1 + c_2\, 7^n$, $n \geq 0$ is the general solution of the relation $a_n + ba_{n-1} + ca_{n-2} = 0$.
7. Obtain the recurrence relation to solve the Tower of Hanoi problem and hence solve.
8. Solve the recurrence relation $a_n - a_{n-1} = 2n + 3, n \geq 1$, $a_0 = 1$.
9. Solve $a_n - 2a_{n-1} + a_{n-2} = 2^n, n \geq 2$, $a_0 = 1, a_1 = 2$ using generating functions.
10. Solve $a_n - 3a_{n-1} = 5^{n-1}, n \geq 1$, $a_0 = 1$ using generating functions.
11. A person invests some amount at the rate of 10% annual compound interest. Determine the period for principal amount to get doubled.
12. Write a recurrence relation for the sequence $0, 2, 6, 12, 20, 30, 42, \ldots$ .....