

DEPARTMENT OF MATHEMATICS



MANUAL FOR EXPERIENTIAL LEARNING USING MATLAB

LINEAR ALGEBRA AND PROBABILITY THEORY (MA231TC)

III - SEMESTER

B.E. Programs: CS, CS - AI, CS - CD, CS - CY, IS



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DEPARTMENT OF MATHEMATICS

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Introduction

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- 1. Linear Algebra I
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1 LINEAR ALGEBRA - I

Topic learning outcomes:

Student will be able to:

- 1. Find Four Fundamental Subspaces,
- 2. Basis for the Range and null space of a Linear Transformation.

Syntax and description:

- sym() -Stores the values in symbolic math toolbox notation.
- null (A) Finds the basis for the nullspace of the matrix A.
- colspace (A) Finds the basis for the columnspace of the matrix A.
- null (A') Finds the basis for the left nullspace of the matrix A.
- colspace (A') Finds the basis for the rowspace of the matrix A.
- B*inv(A) Finds the matrix representation of Linear transformation, where A is the
 matrix having the basis vectors of the domain as its columns, B is the matrix having the
 images of the basis vectors as its columns.
- null (LT) Finds the basis for the nullspace of the Linear transformation, where LT is the matrix representation of the Linear transformation.
- colspace (B) Finds the basis for the columnspace of the Linear transformation, where B is the matrix having the images of the Linear transformation as its columns.
- rank (colspace (B)) Finds the rank of the Linear transformation
- rank(null(LT)) Finds the nullity of the Linear transformation.

Example 1.1: Obtain the bases for the Four Fundamental Subspaces of the matrix A, by storing the matrix using symbolic math toolbox notation.

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

A= sym([1,2,3,4;5,6,7,8;9,10,11,12;13,14,15,16])

nsA=null(A)

csA=colspace(A)

$$\begin{array}{ccc}
\mathsf{CSA} & = \\
\begin{pmatrix}
1 & 0 \\
0 & 1 \\
-1 & 2 \\
-2 & 3
\end{pmatrix}$$

lnsA=null(A')

rsA=colspace(A')

$$rsA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \\ -2 & 3 \end{pmatrix}$$

Note:

One can also find the Four Fundamental Subspaces using .m file.

(i) Create a function file with the name ffss.

```
function[fourfundamentalsubspaces]=ffss(A)
nsA=null(A)
csA=colspace(A)
lnsA=null(A')
rsA=colspace(A')
end
```

(ii)Enter the matrix A in command window using symbolic math toolbox notation as:

```
A= sym([1,2,3,4;5,6,7,8;9,10,11,12;13,14,15,16])
```

$$A = \begin{pmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & 12 \\
13 & 14 & 15 & 16
\end{pmatrix}$$

(iii) call the m file in the command window as ffss(A)

ffss(A)

Example 1.2: Find the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ such that T(1,1) = (0,1,2), T(-1,1) = (2,1,0)

A=[1,-1;1,1]

$$A = 2 \times 2$$
 $1 \quad -1$
 $1 \quad 1$

B=[0,2;1,1;2,0]

LT=B*inv(A)

Example 1.3: Find the range space, null space, rank and nullity of T, where $T: V_3(R) \rightarrow V_4(R)$, defined by $T(e_1) = (0,1,0,2)$, $T(e_2) = (0,1,1,0)$, $T(e_3) = (0,1,-1,4)$

$$T(e_1) = (0,1,0,2), T(e_2) = (0,1,1,0), T(e_3) = (0,1,-1,4)$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & 4 \end{pmatrix}$$

LT=B*inv(A)

$$LT = \begin{pmatrix}
0 & 0 & 0 \\
1 & 1 & 1 \\
0 & 1 & -1 \\
2 & 0 & 4
\end{pmatrix}$$

rsLT=colspace(B)

rLT=rank(rsLT)

$$rLT = 2$$

nLT=rank(nsLT)

$$nLT = 1$$

Exercise:

1. Compute the bases for the four fundamental subspace of the following matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 9 & 10 & 11 \\ 13 & 14 & 15 \end{pmatrix}$$

2. Compute the bases for the four fundamental subspace of the following matrix

$$B = \begin{pmatrix} 3 & 4 & -2 & -5 \\ 4 & 3 & 2 & 4 \\ 2 & 5 & -6 & -14 \end{pmatrix}.$$

- 3. Find the bases for the range space and null space of the transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by T(x) = Ax, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 1 & 3 \end{pmatrix}$.
- 4. Find the bases for the range space and null space of the transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by T(x) = Ax, where $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ -2 & 1 & -3 \end{bmatrix}$.

2. LINEAR ALGEBRA II

Topic learning outcomes:

Student will be able to:

- 1. Obtain orthonormal bases for vectors. Eigen values, Eigen vectors & diagonalization of a matrix.
- 2. Form QR-factorization and singular value decomposition (SVD) of a matrix.

Syntax and description:

- [Q,R]=qr (A)-returns an upper triangular matrix R and a unitary matrix Q such that A =
 Q*R.
- The orthogonal or QR, factorization expresses any rectangular matrix as the product of an orthogonal or unitary matrix and an upper triangular matrix. An orthogonal matrix or a matrix with orthonormal columns, is a real matrix whose columns all have unit length and are perpendicular to each other. If Q is orthogonal then $Q^TQ = I$
- e=eig (A) -returns a column vector containing the eigenvalues of square matrix A.
- [V, D] = eig (A) -returns diagonal matrix D of eigenvalues and matrix V whose columns are the corresponding right eigenvectors, so that A * V = V * D.
- The eigenvalue problem is to determine the solution to the equation $Av = \lambda v$, where A is an $n \times n$ matrix, v is a column vector of length n and λ is a scalar. The values of λ that satisfy the equation are the eigenvalues. The corresponding values of v that satisfy the equation are the right eigenvectors.
- s=svd (A) -returns the singular values of matrix A in descending order.
- [U, S, V] = svd (A) -performs a singular value decomposition of matrix A such that A = U * S * V.

Singular value decomposition is a process through which any $m \times n$ matrix A can be factored into A = U * S * V = (orthonormal matrix) (diagonal matrix) (orthonormal matrix). The columns of U ($m \times m$) are eigenvectors of AA^T and the columns of V ($n \times n$) are eigenvectors of A^TA . The r singular values on the diagonal of S ($m \times n$) are the square roots of the non-zero eigenvalues of both AA^T and A^TA .

Example 2.1: For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$ find Q and R matrices representing the QR decomposition of A.

A=[1,1,1;1,2,3;1,3,6]

[Q,R]=qr(A)

Example 2.2: Find the characteristic equation, the eigenvalues and the eigenvectors of

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

$$A = [8 -6 2; -6 7 -4; 2 -4 3]$$

p=poly(A)

$$p = 1 \times 4$$

1.0000 -18.0000 45.0000 -0.0000

e=eig(A)

$$e = 3 \times 1$$
0.0000
3.0000
15.0000

[V,D]=eig(A)

Example 2.3: Compute the singular values of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix}$.

$A = [1 \ 0 \ 1; -1 \ -2 \ 0; \ 0 \ 1 \ -1]$

$$s = svd(A)$$

Example 2.4: Find the singular value decomposition of a rectangular matrix $\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$.

$$A = [-1; 2; 2]$$

$$A = 3 \times 1$$

$$-1$$

$$2$$

$$2$$

[U,S,V] = svd(A)

Example 2.5: Obtain an orthonormal basis of the range of the matrix $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$.

$$A = sym([2 -3 -1; 1 1 -1; 0 1 -1])$$

 $B = orth(A)$

$$\begin{pmatrix} \frac{2\sqrt{5}}{5} & -\frac{\sqrt{6}}{6} & -\frac{\sqrt{2}\sqrt{15}}{30} \\ \frac{\sqrt{5}}{5} & \frac{\sqrt{6}}{3} & \frac{\sqrt{2}\sqrt{15}}{15} \\ 0 & \frac{\sqrt{6}}{6} & -\frac{\sqrt{2}\sqrt{15}}{6} \end{pmatrix}$$

Exercise:

- 1. If $v_1 = (0, 1, 2)$, $v_2 = (1, 1, 2)$, $v_3 = (1, 0, 1)$ construct an orthonormal basis. 2. Find the QR factorization of $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$.
- 3. Consider the matrix $D = \begin{pmatrix} 1 & 4 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$, find a factorization D = QR.
- 4. Find the characteristic equation, eigenvalues of the matrix $B = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Also diagonalize it.
- 5. Diagonalize the matrix $A = \begin{pmatrix} 5 & 0 & 4 \\ 0 & 5 & 4 \\ 4 & 4 & 9 \end{pmatrix}$. Also find its characteristic equation and its eigenvalues.
- 6. Obtain the SVD of matrix $A = \begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}$.

3. Probability Distributions

Topic learning outcomes:

Student will be able to:

- 1. Generate probability distributions based on given parameters.
- 2. Plot the probability distribution functions.
- 3. Fit a probability distribution.

Syntax and description:

A Binomial Distribution object consists of parameters, a model description, and sample data for a binomial probability distribution

The binomial distribution models the total number of successes in repeated trials from an infinite population under the following conditions:

- Only two outcomes are possible for each of *n* trials.
- The probability of success for each trial is constant.
- All trials are independent of each other.

The binomial distribution uses the following parameters.

Parameter	Description	Support	
N	Number of trials	positive integer	
P	Probability of success	$0 \le p \le 1$	

Distribution Parameters

N — Number of trials positive integer value p — Probability of success positive scalar value in the range [0,1]

Creation

There are several ways to create a Binomial Distribution probability distribution object.

- Create a distribution with specified parameter values using makedist.
- Fit a distribution to data using fitdist.
- Interactively fit a distribution to data using the **Distribution Fitter** app.

'Name'	Distribution Input Parameter A		Input Parameter	
'Binomial'	Binomial Distribution	n number of	p probability of success for each trial	
'Chisquare'	Chi-Square Distribution	v degrees of freedom		
'Exponential'	Exponential Distribution	μ mean		
'Gamma'	Gamma Distribution	a shape parameter	b scale parameter	
'Normal'	Normal Distribution	μ mean	σ standard deviation	
'Poisson'	Poisson Distribution	λ mean		

Example 2.1: The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men now aged 60,

```
(i) at most 6,
```

(ii) at least 7

will live to be 70?

```
pd = makedist('Binomial','N',10,'p',0.65)
x = [6];
y = cdf(pd,x)
z=1-y
```

```
\begin{array}{ll} \text{pd = Binomial Distribution} \\ \text{Binomial distribution} \\ \text{N = } 10 \\ \text{p = 0.65} \end{array}
```

```
y = 0.4862
z = 0.513
```

Example 2.2: Create a binomial distribution object by specifying the parameter values. Also compute the mean of the distribution.

```
pd = makedist('Binomial','N',30,'p',0.25)

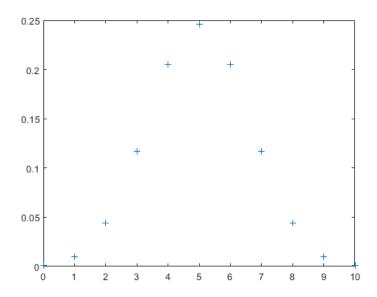
m = mean(pd)

pd =
    Binomial distribution
    N = 30
    p = 0.25

m =
7.5000
```

Example 2.3: Generate a plot of the binomial pdf for n = 10 and p = 1/2.

```
x = 0:10;
y = binopdf(x,10,0.5);
plot(x,y,'+')
```

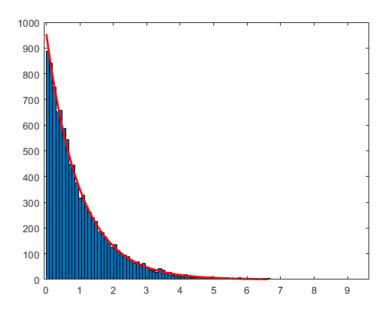


Example 2.4: Create a normal probability distribution object with mean 50 and SD 30. Generate a 2-by-3-by-2 array of random numbers from the distribution.

```
pd = makedist('Normal','mu',50,'sigma',30)
r = random(pd,[2,3,2])
```

Example 2.5: Create an exponential probability distribution object using the default parameter values, generate random numbers from the distribution. Construct a histogram using 100 bins with Exponential distribution fit.

```
pd = makedist('Exponential')
rng('default') % For reproducibility
r = random(pd, 10000, 1)
histfit(r,100,'Exponential')
pd =
Exponential Distribution
  Exponential distribution
   mu = 1
r = 10000 \times 1
    0.2049
    0.0989
    2.0637
    0.0906
    0.4583
    2.3275
    1.2783
    0.6035
    0.0434
    0.0357
```



Example 2.6: Create a Poisson distribution object with the rate parameter, λ , equal to 2. Compute the pdf and cdf values for the Poisson distribution at the values in the input vector $\mathbf{x} = [0,1,2,3,4]$.

```
lambda = 2;
pd = makedist('Poisson','lambda',lambda);
x = [0,1,2,3,4];
y = cdf(pd,x)
z = pdf(pd,x)
y = 1 \times 5
    0.1353
               0.4060
                           0.6767
                                       0.8571
                                                  0.9473
z = 1 \times 5
    0.1353
               0.2707
                           0.2707
                                      0.1804
                                                  0.0902
```

Exercise:

- 1. Execute the example questions for different distributions with different parameter values.
- 2. The probability that an individual will suffer a bad reaction from an injection of a given serum is 0.001. Determine the probability that out of 2000 individuals
- (i) exactly 3
- (ii) more than 2

individuals will suffer a bad reaction. (use lambda = np)

3. The length of a telephone conversation on a cell phone has been an exponential distribution and found on an average to be 3 minutes. Find the probability that a random call made from this phone ends in less than 3 minutes.

- 4. In a test of 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for
- (i) more than 1950 hours
- (ii) more than 1920 and less than 2160 hours.

4. Sampling Distributions

Syntax and description:

- t = (xb-mu)/(s/sqrt(n)) returns the t score for a sample of size n, mean xb, standard deviation s, coming from a normal population of mean mu.
- chi2=(n-1)*s^2/sig^2 returns the chi square value of a sample of size n and standard deviation s, drawn from a population of standard deviation sig
- h=ttest(x,mu,'alpha',0.01) return a value of either '0' or '1' for a sample x, drawn from a population of mean mu, for a t test under 1% level of significance. A score of 0 indicates to accept the hypothesis, and a score td 1 indicates to reject the hypothesis.
- h=ttest(x,mu,'alpha',0.05) return a value of either '0' or '1' for a sample x, drawn from a population of mean mu, for a t test under 5% level of significance. A score of 0 indicates to accept the hypothesis, and a score td 1 indicates to reject the hypothesis.
- [h,p,st] = chi2gof(bins,'Ctrs',bins,...

'Frequency',o, ...

'Expected',e,...

'Alpha',0.01)

returns the h value of either '0' or '1', p-the p value, chi2stat-the chi square value, O-the observed frequencies, E-the expected frequencies, at 1% level of significance. A score of 0 indicates to accept the hypothesis, and a score td 1 indicates to reject the hypothesis.

• [h,p,st] = chi2gof(bins,'Ctrs',bins,...

'Frequency',o, ...

'Expected',e,...

'Alpha',0.05)

returns the h value of either '0' or '1', p-the p value, chi2stat-the chi square value, O-the observed frequencies, E-the expected frequencies, at 5% level of significance. A score of 0 indicates to accept the hypothesis, and a score td 1 indicates to reject the hypothesis.

Example 1: Find the student's t for a sample of size 10, sample mean 0.742, sample standard standard deviation 0.04 and population mean 0.7.

```
n=10;
xb=0.742;
s=0.04;
mu=0.7;
t = (xb-mu)/(s/sqrt(n))
```

Example 2: The following are the IQs of a randomly chosen sample of 10 boys: 70, 120, 110, 101,88, 83, 95, 98, 107, 100. Find the student's t for the given sample taking the mean of population to be 100.

```
x=[70 120 110 101 88 83 95 98 107 100];
n=length(x);
xb=mean(x);
s=std(x);
mu=100;
t = (xb-mu)/(s/sqrt(n))
t=-0.6203
```

Example 3: Find the chi square value of the sample 1.9, 2.4, 3.0, 3.5 and 4.2 chosen from a normal population with mean 3 and standard deviation 1.

```
x=[1.9 2.4 3.0 3.5 4.2];
n=length(x);
xb=mean(x);
s=std(x);
sig=1;
chi2=(n-1)*s^2/sig^2
```

chi2=3.26

Example 4: A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that, on the whole, the stimulus will change the blood pressure at 1% and 5% level of significance.

```
x=[5,2,8,-1,3,0,6,-2,1,5,0,4];
n=length(x);
xb=mean(x);
```

```
s=std(x);
mu=0;
t = (xb-mu)/(s/sqrt(n));
h1=ttest(x,mu,'alpha',0.01)
if h1==1
     'reject the hypothesis at 1% level of significance'
else 'accept the hypothesis at 1% level of significance'
end
h5=ttest(x,mu,'alpha',0.05)
if h5==1
     'reject the hypothesis at 5% level of significance'
else 'accept the hypothesis at 5% level of significance'
end
```

h1 = 0

ans = 'accept the hypothesis at 1% level of significance.

h5 = 1

ans = 'reject the hypothesis at 5% level of significance.

Example 5: The observed frequencies and expected frequencies are given as follows:

O	37	101	84	18
Е	30	90	90	30

Using chi square test, test the hypothesis that the observed frequencies are consistent with the expected frequencies, at 1% and 5% level of significance.

```
bins = 0:3;
o = [37 101 84 18];
n = sum(o);
e = [30 90 90 30];
[h1,p,st] = chi2gof(bins, 'Ctrs', bins,...
                         'Frequency', o, ...
                         'Expected', e, ...
                         'Alpha',0.01)
if h1==1
    'reject the hypothesis at 1% level of significance'
else 'accept the hypothesis at 1% level of significance'
end
[h5,p,st] = chi2gof(bins, 'Ctrs', bins,...
                         'Frequency',o, ...
                         'Expected',e,...
                         'Alpha',0.05)
if h5==1
    'reject the hypothesis at 5% level of significance'
else 'accept the hypothesis at 5% level of significance'
```

end

chi2stat:8.1778

h1 = 0

ans = 'accept the hypothesis at 1% level of significance.

h5=1

ans = 'reject the hypothesis at 5% level of significance.

DEPARTMENT OF MATHEMATICS

VISION

Impart outcome-based education, focusing on quality teaching with an emphasis on Mathematical modeling and sustainable interdisciplinary research

MISSION

- To create infrastructure and resources for quality teaching
- To train graduates with mathematical background for solving complex engineering problems
- Enable to acquire latest skills of experiential learning for problem analysis and develop solutions
- Create an ambience for sustainable interdisciplinary research



