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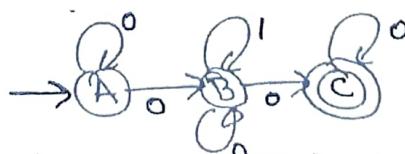
RV COLLEGE OF ENGINEERINGTM
 (An Autonomous Institution affiliated to VTU)
III Semester B. E. Additional Examinations Dec-2020
Computer Science and Engineering
DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours**Maximum Marks: 100****Instructions to candidates:**

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

PART-A

1	1.1	How many bit strings of length 8 either start with a 1 bit or end with the two bits 00?	01
	1.2	How many different strings can be made from the letters in ABRACADABRA with all As must be consecutive.	01
	1.3	Give the recursive definition of the sequence $a_n, n = 1, 2, 3, \dots$ if $a_n = n(n+1)$.	01
	1.4	Find $f(5)$ if f is defined recursively by $f(0) = f(1) = 1$ and for $n = 1, 2, \dots, f(n+1) = f(n)f(n-1)$.	01
	1.5	Let $p(x, y)$ denote the sentence "x divides y". What is the truth value of $\forall x \exists y P(x, y)$ where the domain of x, y is the set $\{1, 2, 4, 6, 12\}$?	01
	1.6	Express the negation of the statement so that all negation symbols immediately precede predicates. $\forall x \exists y (P(x, y) \wedge \exists z R(x, y, z))$.	01
	1.7	Let $A = \{1, 2, 3, 4\}$. The relation R on A is given by the matrix M_R . Determine the properties of R . $M_R = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$.	02
	1.8	If $\{\{a, c, e\}, \{b, d, f\}\}$ is a partition of the set $A = \{a, b, c, d, e, f\}$. Determine the corresponding equivalence relation induced by this partition.	01
	1.9	Find $GLB(\{15, 45\})$ and $LUB(\{3, 5\})$ in the $POSET(\{3, 5, 9, 15, 24, 45\}, 1)$.	02
	1.10	Determine the nature of the function $f: R \rightarrow R$, also determine the range of $f(R)$. $f = x^2 + x$.	01
	1.11	For the NFA given below, find the language of the given NFA.	02



1.12 Find $\delta^*(A, 00122)$ in the NFA below.



1.13 If the binary operation * is associative then find X and Y in the following table.

*	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	X	Y	d
d	d	c	c	d

1.14 Let $E: 2^3 \rightarrow 2^9$ be the encoding function for the (9,3) triple repetition code. If $D: 2_2^9 \rightarrow 2_2^3$ is the corresponding decoding function by applying D, decode the received word 111101100.

1.15 The encoding function $E: z_2^2 \rightarrow z_2^5$ is given by the generator matrix G as below:

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}. \text{ Find the associated parity check matrix } H.$$

02

01

01

02

PART-B

2 a How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$, where $x_i; i = 1, 2, \dots, 6$ is a non-negative integer such that.

- i) $x_i > 1$ for $i = 1, 2, 3, 4, 5, 6$?
- ii) $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4, x_5 \geq 6, x \geq 6$?
- iii) $x_1 \leq 5$?
- iv) $x_1 < 8$ & $x_2 > 8$?

04

b The sequence of Lucas numbers is defined by $l_0 = 2, l_1 = 1$ and $l_n = l_{n-1} + l_{n-2}$ for $n = 2, 3, 4, \dots$. Prove that $f_n + f_{n+2} = l_{n+1}$ whenever n is a positive integer, where f_i and l_i are the i^{th} Fibonacci and i^{th} Lucas number respectively.

06

c Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$, where $n \geq 0$ and $a_0 = 1$ and $a_1 = 4$.

06

3 a State and prove the following rules of inference.

- i) Law of syllogism
- ii) Rule of destructive dilemma

06

b Let $P(x), q(x)$ and $r(x)$ be the following open statements: $P(x): x^2 - 7x + 10 = 0, q(x): x^2 - 2x - 3 = 0, r(x): x < 0$. Determine the truth or falsity of the following statements where the universe is all integers. If the statement is false provide a counter example or explanation.

- i) $\forall x (P(x) \rightarrow \neg r(x))$
- ii) $\exists x (P(x) \rightarrow r(x))$
- iii) $\exists x (q(x) \rightarrow r(x))$
- iv) $\forall x (q(x) \rightarrow r(x))$

04

Show that the argument $(j \vee \neg j) \rightarrow (i \rightarrow h) \vdash j \vee k$. OR $h \rightarrow i, (h \wedge I) \rightarrow j, \neg k \rightarrow (h \vee i)$, 06

Establish the validity of the following argument.

$$\forall x [p(x) \vee q(x)]$$

$$\exists x \sim p(x)$$

$$\forall x [\sim q(x) \vee r(x)]$$

$$\forall x [s(x) \rightarrow \sim r(x)]$$

$$\therefore \forall x \sim s(x)$$

b Let $p(x), q(x)$ and $r(x)$ be the following open statements $p(x): |x| > 3, q(x): x > 3$ and $r(x): x < -3$. Write the converse inverse and contrapositive of the implication $\forall x (p(x) \rightarrow (r(x) \vee q(x)))$. Express each of the following statements in equivalent predicate logic formula. What is their negation? 06

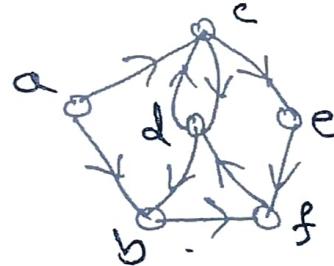
- i) All humming birds are richly colored.
- ii) No large birds live on honey.
- iii) Birds that do not live on honey are dull in color
- iv) Humming birds are small.

06

5 a Let R be a relation on Z such that aRb iff $ab > 0$ for all $a, b \in Z$. Verify whether R is an equivalence relation. 04

b Let R be the relation whose digraph is given below.

- i) Draw digraph of R^3 .
- ii) Write M_R^6 .



c Let $f, g, h : z \rightarrow z$ defined by $f(x) = x - 1, g(x) = 3x, h(x) = \begin{cases} 0, x \text{ even} \\ 1, x \text{ odd} \end{cases}$. 06

Determine fog , hog , $fo(goh)$ and h^3 . 06

OR

6 a Define POSET. Draw the Hasse diagram representing the partial ordering $\{(a, b) | a \text{ divides } b\}$ on the set $\{1, 2, 3, 6, 12, 24, 36, 48\}$. Find the lower bounds, upper bounds, GLB, LUB for the sets $\{2, 6, 12\}$ and $\{12, 36, 48\}$. 05

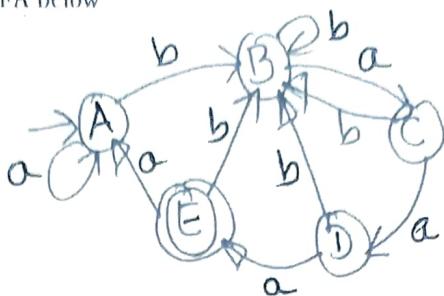
b Let $f: R \rightarrow R$ determine whether f is invertible and if so determine f^{-1} where $f_1 = \{(x, y) / 2x + 3y = 7\}$. 05

c Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{v, w, x, y, z\}$. Determine the number of functions $f: A \rightarrow B$ where

- i) $|f(A)| = 2$
- ii) $f(A) = \{w, x, y\}$
- iii) $|f(A)| = 4$

06

a Consider the DFA below



- i) What is the language of DFA?
ii) Find $\delta^*(\Lambda, ababaaa)$ & $\delta^*(\Lambda, ababaaba)$.

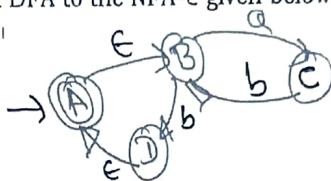
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b Construct NFA which accepts the following language over $\Sigma = \{0,1\}$.

- i) Set of all strings ends with 01 or 10
ii) Set of all strings consists of 101 as a substring.

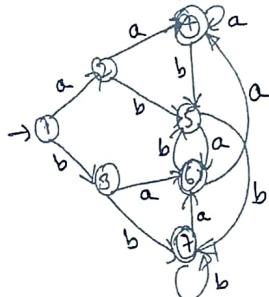
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c Find the equivalent DFA to the NFA- ϵ given below



04

d If possible to minimize the states , find the minimum state DFA for the given DFA.



04

8 a Define the binary operation * on Z by $x * y = x + y + 1$. Verify that $(Z, *)$ is an abelian group.

04

b Find all subgroups of $(Z_{18}, +)$.

04

c Prove that if f is an isomorphism from G_1 to G_2 , then f^{-1} is an isomorphism from G_2 to G_1 .

04

d The encoding function $E: Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$. Determine all the code words. Find the associated parity check matrix H. Use H to decode the received words 00111, 00110.

04

SCHEME AND SOLUTION

SUBJECT CODE: 12CS36.

SUBJECT: Discrete Mathematical Structures (12CS36,

Question No		Marks																				
1.1	$P(8,5) = 8! / (8-5)! = 8! / 3! = 6720$	01																				
1.2	<p>Let $x = 2a$, $y = -3b \therefore (2a-3b)^7 = (x+y)^7$.</p> <p>$\therefore$ coefficient of x^5y^2 in $(x+y)^7$ is $\binom{7}{5}x^5y^2$</p> $\Rightarrow \binom{7}{5}(2a)^5(-3b)^2 \Rightarrow \binom{7}{5}(2^5 - (-3)^2)a^5b^2 = 6048a^5b^2$ <p>\therefore co-efficient of $a^5b^2 = a^5b^2 = 6048$</p>	02																				
1.3	<p>Definitions: given a set A, construct the set $P(A)$ consisting of all subsets of A. The obtained set is called as power set of A and denoted by $P(A)$</p> <p>$P(A) = \{\emptyset, \{0\}, \{1\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$</p> <p>$A = 2^3 = 8$ elements. or subsets.</p>	02																				
1.4	<p>Given $P = \{(A \cap B) \cup \emptyset\} \cup \{(C \cap D) \cup \emptyset\}$</p> <p>Duality: $P_d = \{(A \cup B) \cap \emptyset\} \cap \{(C \cup D) \cap \emptyset\}$.</p> <p>'$\cap$' replaced by '$\cup$', '$\emptyset$' replaced by '$\cap$' '$\cup$' " "$\cap$" "$\emptyset$" "</p>	01																				
1.5	Ans: $a_1 = 3$ and $a_{n+1} = a_n + 2n + 3$ for $n \geq 1$	02																				
1.6	<table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">P</td> <td style="padding: 5px;">q</td> <td style="padding: 5px;">r</td> <td style="padding: 5px;">$(q \vee r)$</td> <td style="padding: 5px;">$p + (q \vee r)$</td> </tr> <tr> <td style="padding: 5px;">1</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">1</td> </tr> </table> <p>$p + (q \vee r)$ is true in all three cases.</p>	P	q	r	$(q \vee r)$	$p + (q \vee r)$	1	1	1	1	1	1	0	1	1	1	1	0	0	1	1	<p>01=for truths table.</p> <p>01=for checking the truths values.</p>
P	q	r	$(q \vee r)$	$p + (q \vee r)$																		
1	1	1	1	1																		
1	0	1	1	1																		
1	0	0	1	1																		

SCHEME AND SOLUTION

SUBJECT:

SUBJECT CODE:

Question No

Marks

1.1 The law of negation of a conditional is as follows:
 $\neg(P \rightarrow Q) \Leftrightarrow [P \wedge \neg Q]$

01

$$\neg(P \rightarrow Q) \Leftrightarrow [P \wedge \neg Q]$$

Vertices	Indegree	Outdegree	Diagraph
1	0	1	1 → 2
2	2	1	2 → 1
3	1	2	3 → 1, 3 → 2
4	1	0	4 ↔ 3
	0 0 0 0	1 1 2 0	0 2

$$1.9 \text{ Let } M_P = \begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

01+

1.10 Identity element

$$x \cdot e = e \cdot x = x \quad \text{for all } x \in Z$$

$$\therefore x \cdot (-1) = (-1) \cdot x = x$$

$$\therefore x + (-1) + 1 = (-1) + x + 1 = x$$

$\therefore x = x = x \therefore -1 \text{ is the identity element.}$

Inverse element.

$$x \cdot (x^{-1}) = (x^{-1}) \cdot x = e \quad \text{for all } x \in Z.$$

$$\therefore x + x^{-1} + 1 = x^{-1} + x + 1 = e$$

$$\therefore x + x^{-1} + 1 = e \quad \& \quad x^{-1} + x + 1 = e$$

$$\therefore x + x^{-1} + 1 = e \quad \& \quad x^{-1} + x + 1 = e$$

$$\therefore x^{-1} = -1 - x \quad x^{-1} = -1 - x - 1$$

$$\therefore x^{-1} = -(x+2) \quad \& \quad x^{-1} = -(x+2)$$

$\therefore -(x+2) \text{ is inverse element}$

Question No		Marks
1.11	$S(S, 3) = 25$. it is Stirling number of second kind Solution: $P(S, 3) = \sum_{k=0}^3 (-1)^k 3 C_{3-k} (3-k)^S$ $= (-1)^0 \cdot 3 C_3 \cdot 3^5 + (-1)^1 \cdot 3 C_2 \cdot 2^5 + (-1)^2 3 C_1 \cdot 1^5$ $= 243 - 96 + 3$ $= 150$ $\therefore S(m, n) = P(m, n) / n! = 150 / 6 = \underline{\underline{25}}$	02
1.12	Given $c = 1010110$, $e = 0101101$ $r = c+e = (1, 0, 1, 0, 1, 1, 0) + (0, 1, 0, 1, 1, 0, 1)$ $= (1+0, 0+1, 1+0, 0+1, 1+1, 1+0, 0+1)$ $= (1, 1, 1, 1, 0, 1, 1)$ using addition Rule in Z_2 . $\therefore r = \underline{\underline{1111011}}$	01
1.13	11100 , weight is 03 01101 , " " 03.	01
2 a.	PART B. Possible arrangements of letters in MASSASAVEA = $\frac{10!}{4!3!1!1!1!}$ $= 25200$ ways. \therefore Number of A's = $\frac{7!}{4!3!1!1!1!} = 840$.	2.5
b.	$ V = 52$, $ A = 30$, $ B = 28$, $ A \cap B = 13$ \therefore By addition Principle $ A \cup B = A + B - A \cap B = 30 + 28 - 13 = \underline{\underline{45}}$ $\& A \bar{U} B = V - A \cup B = 52 - 45 = \underline{\underline{7}}$ US students study at least one of the two languages & 7 " " neither of these languages.	05
c.	Binomial theorem: - Theorems should be written & explained. (6 marks)	03

SCHEME AND SOLUTION

SUBJECT CODE: 12CS36 . SUBJECT: Discrete Mathematical Structures

Question No		Marks
3.a.	Root polynomial is $1+6x+10x^2+6x^3$	05
b.	obtain the formula-theory & steps should be written to get the formula.	05
c	$d_n=9$, & $4!-9=15$ permutations of $1, 2, 3, 4$ are not derangements.	05
4a.	<p>let $s(n) : a_n \leq 3^n$.</p> <p><u>Basis step:</u> $a_0 = 1 \leq 3^0$, $a_1 = 2 \leq 3^1$, $a_2 = 3 \leq 3^2 \therefore (s(n))$ is true for $n=0, 1, 2$.</p> <p><u>Induction step:</u> Assume $s(n)$ is true for $n=0, 1, 2, \dots, k$ where $k \geq 2$, i.e. we note that.</p> $ \begin{aligned} a_{k+1} &= a_k + a_{k-1} + a_{k-2} \\ &\leq 3^k + 3^{k-1} + 3^{k-2} \quad \because s(k), s(k-1), s(k-2) \in s(k-3) \text{ are true by assumption.} \\ &\leq 3^k + 3^k + 3^k \\ &= 3 \times 3^k = 3^{k+1}, \therefore s(k+1) \text{ is true. Hence Proved} \end{aligned} $	06
b.	$L_2=3$, $L_3=4$, $L_4=7$, $L_5=11$, $L_6=18$, $L_7=29$, $L_8=47$, $L_9=76$	05
c.	<p>Find explicit formula for i) $a_1=5$, $a_{n+1}=a_n+2$ for $n \geq 1$</p> <p>ii) $a_1=3$, $a_n=a_{n-1}+3$ for $n \geq 2$</p>	05
	OR	
5a	generating function $= f(x) = (x^3+x^4+\dots+x^8)^4$. final answer is <u>125 ways</u> .	06
5b	Ackermann's numbers are defined recursively for $m, n \in \mathbb{N}$ as follows: $A_{0,n} = n+1$ for $n \geq 0$ $A_{m,0} = A_{m-1,1}$ for $m > 0$ $A_{m,n} = A_{m-1, p}$ for $p = A_{m,n-1}$ ($m, n > 0$)	05

Q. NO.

S. 6

The relation is recurrence relation, & can be solved by substituting the varying values of $n \geq 0$. i.e. $n = 0, 1, 2, 3, 4, \dots$

Marks.

05-

6a.

Truth table must be constructed & for all the ~~true~~ values of P, Q, R , the truth value of the given compound proposition

05

$$[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow (P \rightarrow R), \text{ should be } 1.$$

Then it's a tautology

6 b

Proof must be provided using laws of logic ~~or~~ and also using truth tables.

05
(2.5+2.5)

6 c

$$(i) P + (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$$

~~Q5~~ 5
2.5+2.5

$$\begin{aligned} LHS &= P + (Q \rightarrow R) \Leftrightarrow \neg P \vee (\neg Q \vee R) \\ &\Leftrightarrow (\neg P \vee \neg Q) \vee R \\ &\Leftrightarrow \neg (P \wedge Q) \vee R \\ &\Leftrightarrow (P \wedge Q) \rightarrow R - RHS. \end{aligned}$$

$$(ii) [\neg P \wedge (\neg Q \wedge R)] \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$$

~~Q5~~

$$\Leftrightarrow \{ R \wedge [\neg (P \vee Q)] \} \vee \{ R \wedge (P \vee Q) \}$$

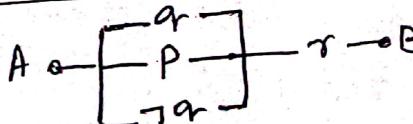
$$\Leftrightarrow R \wedge \{ [\neg (P \vee Q)] \vee (P \vee Q) \}$$

$$\Leftrightarrow R \wedge T_0$$

$$\Leftrightarrow R.$$

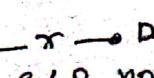
05
(1+2+1)

7a.

switching network: A $\xrightarrow{\quad}$  $\xrightarrow{\quad}$ B

$$\begin{aligned} \text{simplification: } (Q \vee P \vee \neg Q) \wedge R &\Leftrightarrow \{ (Q \vee \neg Q) \vee P \} \wedge R \\ &\Leftrightarrow (T_0 \vee P) \wedge R \text{ by inverse law} \\ &\Leftrightarrow T_0 \wedge R \text{ by domination law} \\ &\Leftrightarrow R \text{ by identity law.} \end{aligned}$$

simplified switching circuit



Ques. no. 11
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exists. e.g. R not

(Q) NO.

7.b. Let $P(x)$: x has equal sides $r(x)$: x has two equal angles. $q(x)$: x is isosceles. Let c is triangle ABC. 06

$$\begin{aligned} \text{i. The argument is. } & \forall x, [P(x) \rightarrow r(x)] \\ & \forall x, [q(x) \rightarrow r(x)] \\ & \frac{\neg r(c)}{\therefore \neg P(c)} \end{aligned}$$

we note that.

$$\begin{aligned} & \frac{? \forall x, [P(x) \rightarrow q(x)]}{? \wedge ? \forall x, [q(x) \rightarrow r(x)]} ? \wedge \neg r(c) \\ & \Rightarrow ? \forall x, [P(x) \rightarrow r(x)] ? \wedge \neg r(c) \\ & \Rightarrow ? P(c) + r(c) ? \wedge \neg r(c) \text{ by Rule of Universal Specification} \\ & \Rightarrow \neg P(c) \text{ by Modus Tollens Rule.} \end{aligned}$$

Hence the argument is proved.

7.c. (i) For any integer x , if x is a perfect square then $x \geq 0$. 05

= false.

(ii) For some integer x , x is divisible by 3 and x is not even. = true

(iii) For any integer x , x is not a perfect square - false

(iv) For any integer x , x is a perfect square or x is divisible by 7 - false

8a. $f(A_1) = \{x_0\}$, $f(A_2) = \{x_0, x_1\}$, $f(A_3) = \{x_0, x_2\}$ 05

$f(A_4) = \{x_2\}$, $f(A_5) = \{x_1, x_2\}$

b. $f(0) = 1$, $f(-1) = 4$, $f(5/3) = 0$, $f^{-1}(-3) = \emptyset$, $f^{-1}(-6) = \emptyset$ 05

Mark:

8c. $f \circ (g \circ h)(x) = f\{g\{h(x)\}\}$

(06)
(3+3)

$$= f\{3h(x)\}$$

$$= 3h(x) - 1$$

$$= \begin{cases} 2-1 & \text{if } x \text{ is even} \\ 2 & \text{if } x \text{ is odd.} \end{cases}$$

$\therefore (f \circ g)(x) = f\{g(x)\} = g(x) - 1 = 3x - 1$

$$\therefore (f \circ g) \circ h(x) = (f \circ g)\{h(x)\}$$

$$= 3h(x) - 1$$

$$= \begin{cases} 2-1 & \text{if } x \text{ is even} \\ 2 & \text{if } x \text{ is odd.} \end{cases}$$

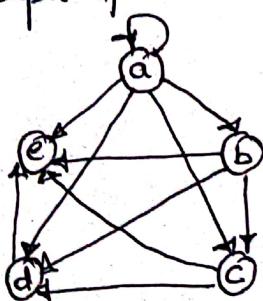
\therefore Hence Proved.

OR

9a

(06)
2+2+2

$$R^{ab} = \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,c), (b,d), (b,e), (c,d), (c,e), (d,e)\}$$

Digraph of R^{ab} 

$$M_{R^{ab}} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- b. R is not Reflexive, hence not equivalence relation
not a partial ordered relation, for nature of relations, the answer should consists of all the six properties.

(5)
2+1+1

- c. The answer should consists of only extremal elements related to subset B_1 .

(5)
1+1+1+11) Upper bound of B_1 = 3, 4, 5, 6, 7, 8.2) $LUB(B_1) = 3$.3) Lower bound of B_1 = None $GLB(B_1) = \text{None}$.

The max. or min. or GLB not exists. Page NO 7

QNO

10 a)

Let $\omega_4 = \{1, -1, i, -i\}$ is a set of fourth root of unity.

The operation table is

\times	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

b) $(\alpha \beta)^{-1} = \beta^{-1} \alpha^{-1}$ - to be proved
 $\alpha \beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 3 \end{pmatrix} \alpha \beta^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$
 $\alpha^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}, \beta^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 3 \end{pmatrix}$
 $\therefore (\alpha \beta)^{-1} = \beta^{-1} \alpha^{-1}$.

c) From multiplication table.

$$b^2 = b * b = c, \quad b^3 = b^2 * b = c * b = d$$

$$b^4 = b^3 * b = d * b = e, \quad b^5 = b^4 * b = e * b = f.$$

$$b^6 = b^5 * b = f * b = a.$$

Every element of \mathbb{G} is an integral power of b \therefore (Q, *)
 is a cyclic group with b as generator

OR

Probability that c is received as r is.

$$P^2(1-P)^{7-2} = (0.02)^2 (1-0.02)^5 = 0.00036.$$

The error pattern is $r = c + e$, by component wise

addition, the pattern is $e = e_1 e_2 e_3 e_4 e_5 e_6 e_7 = 0001001$

b) complete definition & explanation of decoding & encoding of message is to be written, with relevant notations

c) Decoded words are as follows.

$$D(C11101100) = 101 \quad D(C010011111) = 011$$

$$D(C000100011) = 000 \quad D(C0011110011) = 011$$