RV College of Engineering®
Mysore Road, RV Vidyaniketan Post,
Bengaluru - 560059, Karnataka, India

DEPARTMENT OF MATHEMATICS
Academic year 2024-2025 (Odd Semester)

Date	21/10/2024	Time	09:30 AM to 11:30 AM	
TEST	CIE - I	Maximum Marks	10+50	
Course Title	LINEAR ALGEBRA AND PROBABILITY THEORY		Course Code	MA231TC
Semester	III	Programs	CS, CD, CY, IS	

Instructions
1. **Answer all questions**
2. **Mathematics handbook of Second-year B.E. programme is allowed**

Sl. No.	Quiz	M	CO	BTL	
1	If the column vectors of a matrix A are linearly independent, then the basis of null space of A is ______.	1	1	1	
2	The dimension of the subspace of all vectors in $\mathbb{R}^4$ whose components add to zero is ______.	1	2	2	3
3	Let $W = \{[x \quad y \quad z \quad t]' \in \mathbb{R}^4 \mid x = 2y, 3y = 5z\}$. Then $\dim(W) =$______.	1	2	2	2
4	Let $W = \{f \in P_2 \mid f(0) = 0\}$, where P_2 is the vector space of all polynomials of degree at most two with real coefficients. Then $\dim(W) =$ ______.	1	2	2	2
5	If A is a 5×3 matrix with real entries, then ______ $\leq \dim(Col(A)) \leq$ ______. (Write sharp bounds)	2	3	3	0 3
6	Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T([1 \quad 0]') = [2 \quad -1]'$ and $T([0 \quad 1]') = [1 \quad 1]'$. Then $T([1 \quad 3]') =$ ______.	2	2	2	5 2
7	The orthogonal projection matrix for projecting a vector $[x \quad y]'$ onto the vector $[\sqrt{3} \quad -1]'$ is ______.	2	3	2	

Sl. No.	Test	M	CO	BTL
1	Verify which of the following are subspaces of the corresponding vector spaces. (i) $W_1 = \{[x \quad y]' \in \mathbb{R}^2 \mid e^{x-y} = 0\}$ (ii) $W_2 = \{f \in P_2 \mid f(0) = 1\}$ (iii) $W_3 = \left\{\begin{bmatrix} 0 & b \\ a & 0 \end{bmatrix} \in \mathbb{R}^{2\times 2} \middle\vert a + b = 0\right\}$ (iv) $W_4 = \{A \in \mathbb{R}^{2\times 2} \mid \det(A) \neq 0\}$ (v) $W_5 = \{[x \quad y \quad z]' \in \mathbb{R}^3 \mid x = -y\}$	10	3	3
2a	Let $a_1 = [1 \quad 0 \quad 3]'$, $a_2 = [4 \quad 2 \quad 14]'$, $a_3 = [3 \quad 6 \quad 10]'$, and $b = [-1 \quad 8 \quad -5]'$. Determine whether b is in span$\{a_1, a_2, a_3\}$.	4	2	2

2b	Determine a basis and the dimension of the subspace spanned by the subset S of the vector space P_3, of all polynomials of degree less than or equal to 3, where $S = \{1 - x,\ 1 - x^2,\ 1 - x^3,\ x - x^2,\ x - x^3,\ x^2 - x^3\}$	6	3	3
3	Determine a basis and the dimension of the row space, the column space and the null space of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -4 & 8 \\ -1 & 1 & 3 & -5 \\ -1 & 2 & 5 & -6 \\ -1 & -2 & -3 & 1 \end{bmatrix}$. Hence verify rank-nullity theorem.	10	1	2
4a	Let $W = \{A \in \mathbb{R}^{2\times 2} \mid A^T = A\}$. Show that W is a subspace of $\mathbb{R}^{2\times 2}$ and determine a basis and the dimension of W.	5	4	3
4b	Determine the matrix which rotates a vector by 30° in clockwise direction, followed by reflection about the line $y = x$. Also find the image of the vector $[1 \quad \sqrt{3}]'$ under this transformation.	5	3	3
5a	Verify which of the following functions are linear transformations (i) $T_1: \mathbb{R}^{2\times 2} \to \mathbb{R}^2$ defined by $T_1\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = [a + b \quad \max(c, d)]'$. (ii) $T_2: P_2 \to \mathbb{R}^2$ defined by $T_2(ax^2 + bx + c) = [2a + b \quad a - c \quad b + c]'$, where P_2 is the vector space of all polynomials of degree at most two with real coefficients.	6	2	2
5b	Obtain the matrix of the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ such that $T([1 \quad 2]') = [1 \quad 2 \quad 0]'$, $T([1 \quad -1]') = [1 \quad 0 \quad 3]'$.	4	2	2

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Department of Mathematics
Academic year 2024-2025 (Odd Semester)

Date	**21/10/2024**	Time	**09:30 AM to 11:30 AM**	
TEST	**CIE - I**	Maximum Marks	**10+50**	
Course Title	**LINEAR ALGEBRA AND PROBABILITY THEORY**		Course Code	**MA231TC**
Semester	**III**	Programs	**CS, CD, CY, IS**	

SCHEME AND SOLUTION

Sl. No.	**Quiz**	M
1	{ } or basis does not exist	1
2	3	1
3	2	1
4	2	1
5	$0 \leq \dim\,(Col(A)) \leq 3.$	1+1
6	$[5 \quad 2]$	1+1
7	$\theta = \pi/3, \begin{bmatrix} 3/4 & \sqrt{3}/4 \\ \sqrt{3}/4 & 1/4 \end{bmatrix}$	1+1

Sl. No.	**Test**	M
1	a) $W_1 = \{\}$, W_1 is not a subspace b) $0 \notin W_2$, W_2 is not a subspace c) Verification, W_3 is a subspace d) Justification, W_4 is not a subspace e) Verification, W_5 is a subspace	1+1 1+1 1+1 1+1 1+1
2(a)	$c_1[1 \quad 0 \quad 3]' + c_2[4 \quad 2 \quad 14]' + c_3[3 \quad 6 \quad 10]' = [-1 \quad 8 \quad -5]'$ $\left[\begin{array}{ccc\|c} 1 & 4 & 3 & -1 \\ 0 & 2 & 6 & 8 \\ 3 & 14 & 10 & -5 \end{array}\right] \sim \left[\begin{array}{ccc\|c} 1 & 4 & 3 & -1 \\ 0 & 2 & 6 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right]$ As the system is consistent b ca be expresser as linear combination of a_1, a_2 and a_3	1 2 1
2(b)	$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Basis: $\{1-x,\ \ 1-x^2,\ \ 1-x^3\}$ or $\{1-x, x-x^2, x^2-x^3\}$ Dimension: 3	1+3 1 1
3	$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & -1 & -4 & 8 \\ -1 & 1 & 3 & -5 \\ -1 & 2 & 5 & -6 \\ -1 & -2 & -3 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Basis for row space $\{R_1, R_2, R_4\}$ or $\{(1,2,3,-1), (0,1,2,-2), (0,0,0,1)\}$ dim: 3 Basis for column space $\{C_1, C_2, C_4\}$ dim: 3	2 2 2

P.T.O.

	Null space $AX = 0$ $X = x_3[1 \quad -2 \quad 1 \quad 0]'$ Basis for Null space $\{(1,-2,1,0)\}$ dim:1 $Rank + nullity = n$ $3+1=4$	1 2 1
4a	Let $A, B \in W$. $(A+B)^T = A^T + B^T = A + B$. Therefore $A + B \in W$ Let $A \in W$ and $\alpha \in \mathbb{R}$. $(\alpha A)^T = \alpha A^T = \alpha A$. Therefore $\alpha A \in W$. $W = \{A \in \mathbb{R}^{2\times 2} \| A^T = A\} = \left\{\begin{bmatrix} a & b \\ b & c \end{bmatrix} \middle\| a,b,c \in \mathbb{R}\right\}$ $= \left\{a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \middle\| a,b,c \in \mathbb{R}\right\}$ $= span\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$ Basis $\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$ and dimension 3.	1 1 1 1+1
4b	Rotation matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$ Reflection matrix $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$ $\begin{bmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$	1 1 1 1+1
5a	(i) $T_1\left(-2\begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}\right) = T_1\left(\begin{bmatrix} 0 & 0 \\ -2 & -4 \end{bmatrix}\right) = (0+0, \max(-2,-4)) = (0,-2)$ $-2\,T_1\left(\begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}\right) = -2(0+0, \max(1,2)) = (0,-4) \neq T_1\left(-2\begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}\right)$ T_1 is not a linear transformation (ii) Verifying $T(u+v) = T(u) + T(v)$ Verifying $T(\alpha u) = \alpha T(u)$ T_2 is a linear transformation	1 1 1 1 1 1
5b	$[x \quad y] = c_1[1 \quad 2] + c_2[1 \quad -1]$, $c_1 = \frac{x+y}{3},\ c_2 = \frac{2x-y}{3}$ $T([x \quad y]) = \frac{x+y}{3}T([1 \quad 2]) + \frac{2x-y}{3}T([1 \quad -1]) = \frac{x+y}{3}[1 \quad 2 \quad 0] + \frac{2x-y}{3}[1 \quad 0 \quad 3]$ $= \left[x \quad \frac{2x+2y}{3} \quad 2x-y\right]$ Matrix of the transformation is $\begin{bmatrix} 1 & 0 \\ 2/3 & 2/3 \\ 2 & -1 \end{bmatrix}$	1 1 1 1

Note: Appropriate marks maybe awarded for the alternative methods.

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Department of Mathematics
Academic Year 2023-2024 (Even Semester 2024)

Date	02/12/2024	Time	09:30 AM to 11:30 AM	
TEST	CIE - II	Maximum Marks	10+50	
Course Title	LINEAR ALGEBRA AND PROBABILITY THEORY		Course Code	MA231TC
Semester	III	Programs	CS, CD, CY, IS	

Instructions
1. **Answer all questions.**
2. **Mathematics handbook of Second-year B.E. programme is allowed.**

PART - A

Sl. no.	Questions	M	BT	CO
1	Given $A = [2 \quad 1 \quad 3]$, then singular values of A is/are ________.	1	1	1
2	If 0 is an eigenvalue of a symmetric matrix $A_{3\times3}$ repeated twice, then $Nullity(A) =$______.	1	2	2
3	If X is a discrete random variable with $E[X] = 2$ and $E[X^2] = 5$, then $Var(X+1) =$______.	1	2	2
4	Let $F(x) = \frac{x^2+k}{25}$, $x = 0, 1, 2, 3$ be the cumulative distribution of a random variable X. The value of k is _______.	1	2	2
5	Let $C\left[0, \frac{\pi}{4}\right]$ be the inner product space of continuous functions over $\left[0, \frac{\pi}{4}\right]$ with respect to the inner product defined by $\langle f, g\rangle = \int_0^{\pi/4} f(x)g(x)dx,\ \forall f,\ g \in C\left[0, \frac{\pi}{4}\right]$. Determine $\lVert f \rVert$ where $f(x) = \cos(x)$.	2	3	3
6	If $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ are the eigen values of a symmetric matrix $A_{2\times2}$ with respect to the eigen value 1 and 2 respectively, then $A =$________	2	2	2
7	Determine c that renders $f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < c \\ 0, & \text{elsewhere} \end{cases}$ a valid density function. (pdf)	2	3	2

PART – B

Sl. no.	Questions	M	BT	CO
1	Given the matrix A such that the column vectors are linearly independent and for $X \in \mathbb{R}^4$: $$A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix},\ X = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$ i) Obtain an orthonormal basis for column space of A. ii) Find the shortest distance from X to $Col(A)$.	10	3	3
2a	Let $u_1 = \begin{bmatrix} 2/3 \\ 1/3 \\ -2/3 \end{bmatrix},\ u_2 = \begin{bmatrix} -1/\sqrt{5} \\ 2/\sqrt{5} \\ 0 \end{bmatrix},\ u_3 = \begin{bmatrix} 4/\sqrt{45} \\ 2/\sqrt{45} \\ 5/\sqrt{45} \end{bmatrix}$ and $X = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ i) Show that $B = \{u_1, u_2, u_3\}$ form orthonormal basis for $\mathbb{R}^3$. ii) Obtain the coordinate vector of X with respect to the basis B.	4	3	3

Department of Mathematics
Academic Year 2023-2024 (Even Semester 2024)

2b	Given $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ i) Obtain a diagonal matrix D and an orthogonal matrix X such that $A = XDX^T$. ii) Hence determine the eigenvalues and eigenvectors of $B = A^2 - 2I$.	6	3	3
3	Obtain the singular value decomposition of the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$.	10	4	4
4a	Two balls are chosen randomly from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win Rs. 200 for each black ball selected and we lose Rs. 100 for each white ball selected. Let X denotes our winnings. Determine i) The possible values of X, and the probabilities associated with each value. ii) The expected winning amount. iii) The cumulative distribution of X.	6	2	2
4b	Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error, and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function $f_X(x) = \begin{cases} k(3 - x^2), & -1 \le x \le 1 \\ 0, & \text{Otherwise} \end{cases}$ i) Determine k that renders $f_X(x)$ a valid density function. ii) Find the probability that a random error in measurement is less than 1/2.	4	2	2
5a	The probability density function of a random variable X is given by $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 \le x \le 2. \\ 0, & \text{Otherwise.} \end{cases}$ i) Determine the cumulative density function $F(x)$ ii) Find $P(X \ge 1.5)$.	5	2	2
5b	Given the function $p(x, y) = cxy$ for $x = 1, 2, 3$; $y = 1, 2, 3$. i) Determine c such that $p(x, y)$ is the joint probability mass function of random variables X and Y. ii) Find the marginal distribution of X and Y. iii) Obtain $E[XY]$.	5	1	1

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BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

	Particulars	CO1	CO2	CO3	CO4	L1	L2	L3	L4	L5	L6
Marks Distribution	Test Max Marks	5	15	20	10	5	15	20	10	--	--
	Quiz Max Marks	1	7	2	-	1	5	4	—	-	-

Scheme and Solutions : CIE – 2 02/12/2024

Linear Algebra and Probability theory (MA231TC)

Sl. No.	Quiz	M
1	$\sqrt{14}$	1
2	2	1
3	1	1
4	16	1
5	$\langle f,f\rangle = \int_0^{\pi/4}\cos^2 x\,dx = \frac{1}{2}\int_0^{\pi/4}(1+\cos 2x)dx = \frac{1}{2}\left(\frac{\pi}{4}+\frac{1}{2}\right)$ $\lVert f\rVert = \sqrt{\frac{1}{2}\left(\frac{\pi}{4}+\frac{1}{4}\right)} = 0.72$	1 1
6	$P = \frac{1}{\sqrt{13}}\begin{bmatrix}2 & 3\\ 3 & -2\end{bmatrix}, D = \begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\ A = PDP^T = \frac{1}{13}\begin{bmatrix}22 & -6\\ -6 & 17\end{bmatrix}$	1+1
7	$(c^3+1)/9 = 1,\ c = 2$	1+1

TEST

1) Let $w_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, w_2 = \begin{bmatrix}-1\\4\\4\\-1\end{bmatrix}, w_3 = \begin{bmatrix}4\\-2\\2\\0\end{bmatrix}$ w_1, w_2 and w_3 are LI, hence form a basis for 3-dimensional subspace of $\mathbb{R}^4$.

We use Gram-Schmidt process to construct orthonormal basis $\{u_1, u_2, u_3\}$

$$u_1 = \frac{w_1}{\lVert w_1\rVert} = \left(\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\right)^T \quad (1)$$

Let $y_2 = w_2 - \langle w_2, u_1\rangle u_1$, $\quad u_2 = \frac{y_2}{\lVert y_2\rVert}$

where $\langle w_2, u_1\rangle = w_2^T u_1 = -\frac{1}{2} + \frac{4}{2} + \frac{4}{2} - \frac{1}{2} = 3$

$\therefore\ y_2 = w_2 - 3u_1 \left(-1-\tfrac{3}{2}\ \ 4-\tfrac{3}{2}\ \ 4-\tfrac{3}{2}\ \ -1-\tfrac{3}{2}\right)^T = \left(\frac{-5}{2}\ \frac{5}{2}\ \frac{5}{2}\ -\frac{5}{2}\right)^T$

$\lVert y_2\rVert = \sqrt{\frac{25}{4}+\frac{25}{4}+\frac{25}{4}+\frac{25}{4}} = \sqrt{\frac{100}{4}} = 5\ \therefore u_2 = \frac{y_2}{\lVert y_2\rVert} = \left(-\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\ -\tfrac{1}{2}\right)^T \quad (2)$

Let $y_3 = w_3 - \langle w_3, u_1\rangle u_1 - \langle w_3, u_2\rangle u_2$ and $u_3 = \frac{y_3}{\lVert y_3\rVert}$

Where $\langle w_3, u_1\rangle = w_3^T u_1 = \frac{4}{2} - \frac{2}{2} + \frac{2}{2} + 0 = 2$

$\langle w_3, u_2\rangle = w_3^T u = -\frac{4}{2} - \frac{2}{2} + \frac{2}{2} + 0 = -2$

$\therefore\ y_3 = w_3 - 2u_1 + 2u_2 = (4\ -2\ 2\ 0)^T - 2\left(\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\right)^T + 2\left(-\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\right)^T$

$$y_3 = (2\ \ -2\ \ 2\ \ -2)^T \Rightarrow \|y_3\| = \sqrt{4+4+4+4} = \sqrt{16} = 4$$

$$\therefore\ u_3 = \frac{y_3}{\|y_3\|} = \left(\tfrac{1}{2}\ \ -\tfrac{1}{2}\ \ \tfrac{1}{2}\ \ -\tfrac{1}{2}\right)^T$$ 3

Thus $\{u_1, u_2, u_3\}$ forms orthonormal basis for col(A)

2

$$\text{Proj}_{C(A)}(x) = QQ^Tx = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \\ 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & -1/2 \end{bmatrix}\begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 & -1/2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/4 & -1/4 & 1/4 & 1/4 \\ -1/4 & 3/4 & 1/4 & 1/4 \\ 1/4 & 1/4 & 3/4 & -1/4 \\ 1/4 & 1/4 & -1/4 & 3/4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 7/4 \\ 5/4 \\ 5/4 \end{bmatrix}$$

$$y = x - \text{Proj}_{C(A)}(x) = (1\ \ 2\ \ 1\ \ 1)^T - (3/4\ \ 7/4\ \ 5/4\ \ 5/4)^T = (1/4\ \ 1/4\ \ -1/4\ \ -1/4)^T$$ 1

shortest distance

$$\|y\| = \sqrt{(1/4)^2 + (1/4)^2 + (-1/4)^2 + (-1/4)^2} = 1/2$$ 1

2 a) (i) $u_i^T u_j = 0 \quad \forall\, i \neq j$ and $u_i^T u_j = 1 \quad \forall\, i = j$ 2

(ii) $$[x]_B = \begin{bmatrix} \langle x, u_1 \rangle \\ \langle x, u_2 \rangle \\ \langle x, u_3 \rangle \end{bmatrix} = \begin{bmatrix} 4/3 + 1 + 2/3 \\ -2/\sqrt{5} + 6/\sqrt{5} + 0 \\ 8/\sqrt{45} + 6/\sqrt{45} - \frac{5}{\sqrt{45}} \end{bmatrix} = \begin{bmatrix} 3 \\ 4/\sqrt{5} \\ 9/\sqrt{45} \end{bmatrix} = \begin{bmatrix} 3 \\ 4/\sqrt{5} \\ 3/\sqrt{5} \end{bmatrix}$$ 2

2 b) i) $|A - \lambda I| = 0 \Rightarrow \lambda^2 - 8\lambda + 12 = 0 \Rightarrow \lambda = 2, 6$ 1

For $\lambda = 2$, $[A - \lambda I]x = 0 \Rightarrow \begin{bmatrix} 4-2 & 2 \\ 2 & 4-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \Rightarrow x_1 = -x_2 \Rightarrow x = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} = x_2\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

For $\lambda = 6$, $[A - \lambda I]x = 0 \Rightarrow \begin{bmatrix} 4-6 & 2 \\ 2 & 4-6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \Rightarrow x_1 = x_2 \Rightarrow x = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix} = x_2\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\therefore X = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix},\quad X^{-1} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix},\quad D_\lambda = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$$ 1

(ii) 2

$$B = A^2 - 2I \Rightarrow (2^2 - 2),\ (6^2 - 2)$$

1

$= 2,\ 34$ are the eigenvalues

eigenvectors are same A. 1

3) $$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 5 \end{bmatrix} \Rightarrow \begin{aligned} &|AA^T - \lambda I| = 0 \\ &\lambda^3 - 7\lambda^2 + 6\lambda = 0 \end{aligned}$$ 2

$$\Rightarrow \lambda(\lambda^2 - 7\lambda + 6) = 0,\ \lambda = 0, 1, 6$$

for $\lambda = 6$, $[AA^T - \lambda I]x = 0 \Rightarrow \begin{bmatrix} -5 & 0 & 1 \\ 0 & -5 & 2 \\ 1 & 2 & -1 \end{bmatrix} \Rightarrow \frac{x_1}{1} = \frac{-x_2}{-2} = \frac{x_3}{5},\ x = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

for $\lambda = 1$, $[AA^T - \lambda I]\,x = 0 \Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 1 & 2 & 4 \end{bmatrix} \Rightarrow \frac{x_1}{-4} = \frac{-x_2}{-2} = \frac{x_3}{0},\ x = \begin{bmatrix} -4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

for $\lambda = 0$, $[AA^T - \lambda I]x = 0 \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 5 \end{bmatrix} \Rightarrow \frac{x_1}{1} = \frac{-x_2}{-2} = \frac{x_3}{-1}, x = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ 3

$$\therefore U = \begin{bmatrix} 1/\sqrt{30} & -2/\sqrt{5} & 1/\sqrt{6} \\ 2/\sqrt{30} & 1/\sqrt{5} & 2/\sqrt{6} \\ 5/\sqrt{30} & 0 & -1/\sqrt{6} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \therefore V^T = \begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ 2/\sqrt{5} & -1/\sqrt{5} \end{bmatrix}$$

1 1 1

Now consider

$$A^TA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}, \quad |A - \lambda I| = 0 \Rightarrow \lambda^2 - 7\lambda - 6 = 0$$
$$\Rightarrow \lambda = 6, 1$$

for $\lambda = 6$

$$[A^TA - \lambda I]x = 0 \Rightarrow \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \Rightarrow \frac{x_1}{-1} = -\frac{x_2}{2} \Rightarrow x = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow v_1 = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$$

for $\lambda = 1$

$$[A^TA - \lambda I]x = 0 \Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \Rightarrow \frac{x_4}{4} = -\frac{x_2}{2} \Rightarrow x = \begin{bmatrix} 4 \\ -2 \end{bmatrix} \Rightarrow v_2 = \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \end{bmatrix}$$

2

4 a)

x :	-200	-100	0	100	200	400	1
P(x) :	0.3077	0.1758	0.0110	0.3516	0.0879	0.0659	3
F(x) :	0.3077	0.4835	0.4945	0.8462	0.9341	1	1

$E[x] = 0$ 1

O	O	W	W	B	O
O	W	W	B	B	B
0	-100	-200	100	400	200
$\frac{2C_2}{14C_2}$	$8C_1 2C_1$	$8C_2$	$8C_1 4C_1$	$4C_2$	$2C_1 4C_1$

4 b) (i) $\int_{-1}^{1} k(3 - x^2)\,dx = 1$

$$\Rightarrow k\left[3x - \frac{x^3}{3}\right]_{-1}^{1} = 1 \Rightarrow k\left[3(1+1) - \frac{1}{3}(1+1)\right] = 1$$

$$\Rightarrow k = \frac{3}{16}$$ 2

(ii) $P(x < \frac{1}{2}) = \int_{-1}^{1/2} \frac{3}{16}(3 - x^2)\,dx = \frac{3}{16}\left[3x - \frac{x^3}{3}\right]\Big|_{-1}^{1/2} = \frac{99}{128}$ 2

$$5\text{ a) } F(x) = \begin{cases} 0 & ; -\infty \le x \le 0 \\ \int_0^x x\,dx & ; 0 \le x \le 1 \\ \int_0^1 x\,dx + \int_1^x (2-x)\,dx & ; 1 \le x \le 2 \\ 1 & ; x > 2 \end{cases} = \begin{cases} 0 & ; -\infty \le x \le 0 \\ x^2/2 & ; 0 \le x \le 1 \\ \frac{1}{2} + \left(2x - \frac{x^2}{2} - \frac{3}{2}\right) & ; 1 \le x \le 2 \\ 1 & ; x > 2 \end{cases}$$

1 2 1 1

$$\therefore P(X \ge 1.5) = 1 - P(X < 1.5) = 1 - F(1.5) = 1 - 2(1.5) - \frac{(1.5)^2}{2} - 1 = 1 - 0.8750$$
$$= 0.125$$

5 b) 2

x|y		Y: 1	2	3	
X	1	C	2C	3C	6C
	2	2C	4C	6C	12C
	3	3C	6C	9C	18C
		6C	12C	18C	1

$\sum_i \sum_j P_{ij} = 36C = 1 \Rightarrow C = 1/36$ 1

X :	1	2	3	Y :	1	2	3
P(X) :	6/36	12/36	18/36	P(Y) :	6/36	12/36	18/36
:	1/6	1/3	1/2				

1

$$E[XY] = \sum_i \sum_j x_i y_j P_{ij} = 1(C) + 2(2C) + 3(3C) + 2(2C) + 4(4C) + 6(6C) + 3(3C) + 6(6C) + 9(9C)$$ 1
$$= (1 + 4 + 9 + 4 + 16 + 36 + 9 + 36 + 81)C = 196\frac{1}{36} = \frac{49}{9} = 5.44$$

USN | 1 | R | V | 2 | 3 | C | D | 0 | 0 | 3 |

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

III Semester B. E. Regular / Supplementary Examinations Jan / Feb-2025

LINEAR ALGEBRA AND PROBABILITY THEORY

Common to CS / CD / CY / IS

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. 2nd year Hand book of mathematics to be provided

PART-A

			M	BT	CO
1	1.1	The value of k such that the vectors $2+t+2t^2, -2t+kt^2, 2-t+3t^2$ are linearly dependent is ________.	02	3	2
	1.2	The 2×2 reflection matrix about the line $y=x$ is ________ and the vector (2,1) reflected about the line $y=x$ is ________.	02	2	2
	1.3	The orthogonal projection of y onto u and the vector z orthogonal to u, where $y=(4,2)$ and $u=(1,1)$ are ________ and ________ respectively.	02	2	2
	1.4	If the sum and product of the eigen values of $A=\begin{bmatrix} a & 2 \\ 0 & b \end{bmatrix}$ are 2 and -3 respectively, then the matrix is ________.	02	1	1
	1.5	A shipment of 20 similar laptop computers to a retail outlet contains 4 that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives.	02	2	3
	1.6	The installation time, in hours, for a certain software module has a probability density function $f(x)=(4/3)(1-x^3)$ for $0<x<1$. Compute the cumulative distribution function.	02	2	3
	1.7	In a Poisson distribution, if $P(0)=\frac{8}{9}P(2)$, then the mean $\mu =$ ________.	02	2	3
	1.8	Suppose the mean and variance of a finite population of size 4 are 9 and 20 respectively, then the mean and variance of a sample of size 2 are ________ and ________ respectively.	02	2	2
	1.9	A publisher of college textbooks claims that the average price of all hardbound college textbooks is Rs. 1200. A student group believes that the actual mean is lower and wishes to test their belief. State the relevant null and alternate hypothesis.	02	1	1
	1.10	Suppose that it is known that the population is normally distributed with standard deviation $\sigma = 0.25$ and suppose that the test of hypothesis $H_0=\mu=12$ versus $H_a=\mu\neq 12$ will be performed with a sample of size 31. Construct the rejection region for the test at 5% level of significance.	02	1	1

PART-B

2	a	Show that the subset $W =$ the set of all 2×2 symmetric matrices is a subspace of $M_{2\times 2}$, the set of all 2×2 matrices over the Field $\mathbb{R}$.	04	2	2
	b	Find the bases and dimension of row space and null space of the matrix $A = \begin{bmatrix} 2 & 1 & 2 & 4 \\ 1 & -2 & 0 & 2 \\ 5 & -5 & 2 & 10 \\ 1 & 3 & 2 & 2 \end{bmatrix}$.	06	3	3
	c	Find the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$, such that $T(1,1,1) = (2,2,0), T(1,2,1) = (4,3,1), T(2,1,0) = (4,1,3)$. Also find the basis of the range space.	06	4	4
3	a	Applying the Gram-Schmidt process, obtain the QR factorization of the matrix $A = \begin{bmatrix} 1 & 5 \\ -1 & -4 \\ -1 & -3 \\ 1 & 7 \\ 1 & 1 \end{bmatrix}$	06	3	3
	b	Using the process of diagonalization decompose the matrix A as PDP^{-1}, where $A = \begin{bmatrix} 4 & -2 & 2 \\ -1 & 5 & 1 \\ -1 & -1 & 7 \end{bmatrix}$.	10	4	3
		OR			
4	a	Obtain the orthogonal basis for the column space of the matrix $A = \begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$.	06	3	3
	b	Decompose the matrix A as $U\Sigma V^T$, using the singular value decomposition process, where $A = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$.	10	4	3
5	a	The response time is the speed of page downloads, and it is critical for a mobile website. Let X denote the number of bars of service, and let Y denote the response time (to the nearest second) for a particular user and site. The joint probability mass function is defined as $p(1,4) = 0.15, p(1,3) = 0.02, p(1,2) = 0.02, p(1,1) = 0.01, p(2,4) = 0.1, p(2,3) = 0.1, p(2,2) = 0.03, p(2,1) = 0.02, p(3,4) = 0.05, p(3,3) = 0.05, p(3,2) = 0.2, p(3,1) = 0.25$. Construct the joint distribution table and hence find: i) the marginal distributions of X and Y, ii) the probability that the number of bars of the service is at most 2, iii) the probability that the response time is more than 2 seconds iv) the conditional probability of Y such that $X = 3$.	08	3	3
	b	The joint density function for the random variables (X, Y) is given as: $f(x,y) = \begin{cases} 6xy^2, 0 \le x \le 1, 0 \le y \le 1 \\ 0, \ otherwise \end{cases}$. Determine: i) $E(X)$ ii) $E(Y)$, iii) $E(XY)$ and iv) $Cov(X,Y)$.	08	4	4
		OR			

6	a	If the joint probability mass function of X and Y is given by $f(x,y) = \frac{x+y}{36}$ over the nine points with $x = 1,2,3$ and $y = 1,2,3$ construct the joint distribution table and hence determine : i) $E(X)$ ii) $E(Y)$, iii) $E(XY)$, iv) $Cov(X,Y)$	08	3	3
	b	The random variable X denotes the time until a computer server connects to your machine (in milliseconds) and Y denotes the time until the server authorizes you as a valid user (in milliseconds). Each of these random variables measures the wait from a common starting time and $X < Y$. The joint probability density functions for X and Y is given by $f(x,y) = 6 \times 10^{-6}\, e^{-0.001x-0.002y}$ for $0 < x < y < \infty$. Determine: i) the marginal density function of X, ii) the marginal density function of Y, iii) the conditional probability density function for Y given that $X = x$.	08	4	4
7	a	A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers. Using Binomial distribution, find: i) the mean and variance of the situation, ii) the probability that the virus entered at least 3 computers.	04	2	2
	b	Suppose X has an exponential distribution with mean equal to 10. Determine: i) $P(X > 10)$, ii) $P(20 < X < 30)$, iii) the value of x such that $P(X < x) = 0.95$.	06	3	3
	c	An infinite population has a mean 48.4 and standard deviation 6.3. Find: i) the mean and standard deviation of $\bar{x}$ for a sample of size 64, ii) the probability that the mean of a sample of size 64 is less than 46.7, iii) the probability that the mean of sample of size 64 is greater than 49.	06	4	4
		OR			
8	a	On an average, one computer in 800 crashes during a severe thunderstorm. A certain company had 4000 working computers when the area was hit by a severe thunderstorm. Using Poisson distribution, compute the probability that: i) exactly 4 computers crashed, ii) less than 4 computers crashed.	04	2	2
	b	Given the normally distributed variable X with mean 18 and standard deviation 2.5, find: i) $P(X < 15)$, ii) $P(17 < X < 21)$, iii) the value of k such that $P(X < k) = 0.2236$.	06	3	3

	c	Two kinds of thread are being compared for strength. Brand A has an average tensile strength of 78.3 kilograms with a standard deviation of 5.6 kilograms, while brand B has an average tensile strength of 87.2 kilograms with a standard deviation of 6.3 kilograms. Fifty pieces of each types of thread are tested under similar conditions. What is the probability that brand B has an average tensile strength which is at least 10 kilograms more than that of brand A.	06	4	4
9	a	A die was thrown 8000 times and a throw of 1,3 or 5 was obtained 4100 times. On the assumption of random throwing, does the data indicate an unbiased die at 0.01 level of significance?	06	3	3
	b	The useful lifetime of a battery is normally distributed with mean 40 hours and standard deviation of 4 hours. A sample of 100 batteries were found to have a useful lifetime of 38.5 hours. Determine the test statistics z and the P-value to test the null hypothesis $H_0: \mu = 40$ against the alternate hypotehesis $H_a: \mu \leq 40$.	04	2	2
	c	Based on field experiments, a new variety of green gram is expected to give a yield of 12 quintals per hectare. The variety was tested on 10 randomly selected farmer fields. The yield (quintals/hectare) were recorded as: 14.3, 12.6, 13.5, 10.9, 13.6, 12.0, 11.4, 12.0, 12.6, 13.1. Do the results confirm the expectation? Use t-test. Given $t_{0.05}(9) = 2.26$.	06	4	4

OR

10	a	To test the hypothesis that a coin is fair, the following rule of decision is adopted. Accept the hypothesis if the number of heads in a sample of 200 tosses is between 90 and 110, reject the hypothesis otherwise. Find the probability of Type-I error.	06	3	3
	b	Two random samples taken from two normal populations yielded the following information: $n_1 = 9, n_2 = 31, s_1^2 = 123, s_2^2 = 543$. Find the statistic F. Perform the test of hypotheses $H_0 = \sigma_1^2 = \sigma_2^2$ versus $H_a = \sigma_1^2 \neq \sigma_2^2$ at the 5% level of significance. Given $F_{0.975,8,30} = 0.26$, $F_{0.025,8,30} = 2.65$.	04	2	2
	c	A data sample is sorted into four categories with an assumed probability distribution (see table below). Find the expected number E of observations for each level, if the sampled population has a probability distribution as assumed. Find the chi-squared test statistic χ^2. Test the goodness of fit at 5% and 1% level of significance. Given $\chi^2_{0.05}(3) = 7.815$, $\chi^2_{0.01}(3) = 11.345$.	06	4	4

Factor Levels	1	2	3	4
Assumed distribution	0.3	0.3	0.2	0.2
Observed Frequency	23	30	19	18