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2020-21 (b)

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution affiliated to VTU)

III Semester B. E. Examinations March-2021

Common to CS / IS

DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, 7 and 8 are compulsory. Answer any one full question from 3 and 4 & one full question from 5 and 6

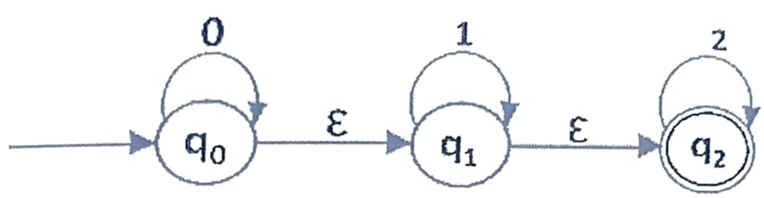
PART-A

1	1.1	Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed? 210 X120 = 25200	02
	1.2	Determine the coefficient of $x^9\,y^3$ in the expansion of $(2x-3y)^{12}$. -3041280 ~~[illegible]~~	02
	1.3	State inverse and contrapositive of the conditional statement: "If the quadrilateral is a parallelogram, then its diagonal bisects each other".	02
	1.4	Let $A = \{a, b, c\}$ and $B = \{0,1\}$, and $R = \{(a, 0), (b, 0), (c, 1)\}$ be the relation from A to B. Write down the matrix of this relation.	02
	1.5	Evaluate $S(8,7)$, given that $S(7,6) = 21$. 28	02
	1.6	Show that $(Z, X)$is not a group.	02
	1.7	Obtain a DFA to accept string of a's & b's having exactly two a's.	02
	1.8	Check the validity of the following statement: If Sachin hits a century, he gets a free car. Modus Tollens Sachin does not get a free car. ----------------------- ∴Sachin has not hit a century.	02
	1.9	Define the extended transition function for DFA.	02
	1.10	A binary symmetric channel has probability p=0.05 of incorrect transmission. If the word $c = 011011101$ is transmitted, what is the probability that single error occurs. 0.2985	02

PART-B

2	a	A computer science professor has seven different programming books on a bookshelf. Three of the books deal with C++, the other four with Java. In how many ways can the professor arrange these books on the shelf i) If there are no restrictions? 5040 = 7! ii) If the languages should alternate? 144 = 4!3! iii) If all the C++ books must be next to each other? 720 = 5!3! iv) If all the C++ books must be next to each other and all the Java books must be next to each other? 2! 3! 4! = 288	06

	b	By Mathematical Induction, prove that 11n-4n is divisible by 7, for $n \geq 1$.	04
	c	If $a_0 = 0, a_1 = 1$ *and* $a_2 = 4, a_3 = 37$ satisfy the recurrence relation $a_{n+2} + ba_{n+1} + ca_n = 0$ for $n \geq 0$. Determine the constants b and c and then solve the relation for a_n.	06
3	a	Write down the following proposition in symbolic form and find its negation: "If all triangles are right – angled, then no triangle is equiangular."	04
	b	Let p(x) be the open statement "$x^2 = 2x$" and q(x) be the open statement "$x^3 = 4x$" with the set of all integers as the universe. Write down the truth values of the following quantified statements: i) $\forall x, p(x) \wedge q(x)$ ii) $\exists x, p(x) \wedge q(x)$ iii) $\forall x, p(x) \vee q(x)$	06
	c	Prove the following logical equivalences: $\exists x, [p(x) \wedge q(x)] \Rightarrow \exists x, p(x) \wedge \exists x, q(x)$ Is the converse true?	06
		OR	
4	a	Prove the validity of the following argument Invalid. p = 0 q = 0 r = 1 s = 1 $p \to q, \neg r \vee s, p \vee r \ \therefore \neg q \to \neg s$	06
	b	Establish the validity of the following argument No engineering student of first or second semester studies logic. Anil is an engineering student who studies Logic. Therefore Anil is not in second semester.	06
	c	Write down the following proposition in symbolic form: i) An equilateral triangle has three angles of 60 degree, and conversely. ii) Every rational number is a real number and not every real number is a rational number.	04
5	a	Let $f: A \to B$ *and* $g: B \to C$ be any two functions. Then the following are true: i) If f and g are one-to-one, so is gof. ii) If gof is one-to-one, then f is one-to-one.	06
	b	Consider the function f and g defined by $f(x) = x^3$ and $g(x) = x^2 + 1$, $\forall\, x \in R$. Find gof, fog, f^2 *and* g^2.	04
	c	On the set of all integers, Z, the relation R is defined by $(a, b) \in R$ if and only if $a^2 - b^2$ is an even integer. Show that R is an equivalence relation.	06
		OR	
6	a	Let $f: R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 \ for\ x > 0 \\ -3x + 1 for\ x \leq 0 \end{cases}$ Determine: i) $f(-5/3) and\ f(5/3)$ 6, 0 ii) $f^{-1}(-3)\ and\ f^{-1}(-6)$ 2/3, ϕ iii) What are $f^{-1}([-5,5])\ and\ f^{-1}([-6,5])$?	06

	b	Let A = {1,2,3,4} and let R be the relation on A defined by x R y if and only if y = 2x. i) Write down R as a set of ordered pairs. ii) Draw the digraph of R. iii) Determine the in-degree and out-degree of the vertices in the digraph.	06
	c	Prove that the set of all positive integers is not totally ordered by the relation of divisibility.	04
7	a	Convert the following ε −NFA to DFA. By first converting it into its equivalent NFA. 	08
	b	Draw a DFA to accept i) Even no. of a's and b's ii) Even no. of a's and odd no of b's iii) Odd no. of a's and even no of b's iv) Odd no. of a's and odd no of b's	08
8	a	The encoding function $E: Z_2{}^3 \rightarrow Z_2{}^6$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ 110101 010011 i) Determine the code word assigned to 110 and 010. ii) Find the associated parity-check matrix. iii) Use H to decode the received words: 110110, 111101. 100, 110 iv) Show that decoding of 111111 is not possible by using H.	08
	b	State and prove Lagrange's theorem.	04
	c	Let G be a group and H be a subgroup of G. For $a \varepsilon G$, Prove that aH=H if and only if $a \varepsilon H$.	04

Part-A

1.1) The no of ways to choose 3 consonants out of 7

$^7C_3 = 35$

The no of ways to chose 2 vowels out of 4

$^4C_2 = 6$

The no of ways to arrange 5 letters $= 5! = 120$

$35 \times 6 \times 120 = 25200$

1.2) $x^9y^3, (2x-3y)^{12}$

$\sum \binom{12}{k} (2x)^k (-3y)^{12-k}$

$\binom{12}{9} (2x)^9 (-3y)^3$

$220 \times 2^9 \times (-3)^3 x^9 y^3$

$-3041280x^9y^3$

1.3) "If the quadrilateral is a parellogram then its diagonals bisect each other".

$p \rightarrow$ "the quadrilateral is parellelogram."

$q \rightarrow$ "diagonals bisect each other

inverse: $\neg p \rightarrow \neg q$

"If the quadrilateral is not a parellelogram then its diagonals do not bisect each other".

contrapositive: $\neg q \rightarrow \neg p$

"If the diagonals do not bisect each other, then the quadrilateral is not a parellelogram.

1.4)
$$\begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

1.5) $S(7,6) = 21$

$S(8,7) = S(n,k)\,k + S(n,k-1)$

$S(8,7) = 7 \times S(7,7) + S(7,6)$

$S(8,7) = 7 \times 1 + 21 = \underline{28}$

1.6> Given $(Z, \times)$

① closure : $\forall a,b \in Z \quad a \times b \in Z$

② Associative: $a,b,c \quad (a \times b) \times c = a \times (b \times c) \in Z$

③ Identity : $a \in Z$, $a \times e = e \times a = a \quad 1 \in Z$

④ Inverse : $a \in Z$, $a^{-1} \in Z$ such that $a \times a^{-1} = a^{-1} \times a = 1$

for $a = 2$, $a^{-1} = 1/2 \notin Z$

So $(Z, \times)$ is not group.

1.8> "Sachin hits a century, he gets a free car"

"sachin does not get a free car"

∴ Sachin has not hit a century

p: sachin hits centurary

q: sachin gets free car

$$\begin{array}{l} p \to q \\ \underline{\neg q} \\ \underline{\neg p} \end{array}$$

Modus tollens

1.10> $p = 0.05$

$C = 1110\ 11101$

${}^9C_1 \times (0.05)^1 (0.95)^8$

$\underline{0.2985391}$

Part B

2a) (i) no restriction

$7! = 7\times6\times5\times4\times3\times2\times1 = 5040$

(ii) If the languages are alternate

$3! \times 4! = 144$

(iii) If all C++ books must be next to each other

5 block $5! = 120$

3 C++ books in $3!$ ways = 6 ways

$5! \times 3! = 720$ ways

(iv) If all the C++ books must be next to each and all the Java books must next to each other.

$2! = 2$ ways

3 C++ books = $3! = 6$ ways

4 Java books = $4! = 24$ ways

$2\times6\times24 = 288$

2c) if $a_0=0, a_1=1$ and $a_2=4, a_3=37$,

$a_{n+2}+b\,a_{n+1}+c\,a_n=0$

$a_0=0, a_1=1, a_2=4, a_3=37$

$n=0$, $a_2+ba_1+c.a_0=0$

$4+b.1+c.0=0$

$b=-4$

$n=1$

$a_3+ba_2+ca_1=0$

$37+b4+c.1=0$

$b=-4$

$37+(-4).4+c=0$

$37-16+c=0$

$c=-21$

$a_{n+2}-4a_{n+1}-21a_n=0$

solving,

Characteristic equation $r^2-4r-21=0$

$$r^2+3r-7r-21=0$$

$$r(r+3)-7(r+3)=0$$

$$(r-7)(r+3)=0$$

$$r=7,\ r=-3$$

G.S of recurrance relation

$$a_n = A7^n + B(-3)^n$$

$$a_0 = A7^0+B(-3)^0 = A+B=0$$

$$A=-B$$

$a_1=1,$

$$a_1 = A7^1+B(-3)^1 = 7A-3B=1$$

$$A=-B$$

$$7(-B)-3B=1$$

$$-7B-3B=1$$

$$B=-1/10$$

$$A=1/10$$

$$\therefore a_n = 1/10\, 7^n - 1/10\,(-3)^n$$

3a) "If all triangles are right angled, then no triangle is equiangular".

$p \rightarrow$ All triangles are right angled

$q \rightarrow$ No triangle is equiangular

$$p \rightarrow q$$

$$\neg(\neg p \vee q)$$

$$(p \wedge \neg q)$$

"All triangles are right angled and there exist at least one triangle that is equiangular.

3b) ⓐ $\forall x\ p(x) \wedge q(x)$

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$p(x)$ is true only when $x=0$, $q(x)$ is true when $x=0$; $x=2$ and $x=\underline{-2}$.

for $x=0$, both $p(0)$ and $q(0)$ are true, so $p(0) \wedge q(0)$ is true. However for all other integer x $p(x)$ is false which makes $p(x) \wedge q(x)$ false.

⑥ $\exists x \cdot p(x) \wedge q(x)$

$\exists x$ means "there exist an integer x such that $p(x) \wedge q(x)$ is true

© $\forall x \; p(x) \rightarrow q(x)$

$p(x) \rightarrow q(x)$ if $p(x)$ is true, then $q(x)$ must be true, for all other values x, $p(x)$ is false, in logic an implication $p(x) \rightarrow q(x)$ is true.

$\therefore \forall x \; p(x) \rightarrow q(x)$ is true.

3 c) $\exists x \, [p(x) \wedge q(x)] \longrightarrow (\exists x, p(x) \wedge \exists x, q(x))$

Slno	Statement	Reason
①	$\exists x \, [p(x) \wedge q(x)]$	premesis
②	$p(x_0) \wedge q(x_0)$	Universal law of specification

$\exists x \, (p(x) \wedge q(x))$ is true, there is at least one element c in the universe such that $p(c) \wedge q(c)$ is true.

By rule of conjunctive simplification $[p(c) \wedge q(c) \rightarrow p(c)]$ such that. $p(c)$ is true for $\exists x \, p(x)$

$|||^{ly}$ we obtain $\exists x \, q(x)$, another true statement

$\exists x \, p(x) \wedge \exists x \, q(x)$ is true statement.

since $\exists x \, p(x) \wedge \exists x \, q(x)$ is true

$\exists x \, (p(x) \wedge q(x))$ is true

$\therefore \exists x \, [p(x) \wedge q(x)] \rightarrow (\exists x \, p(x) \wedge \exists x \, q(x))$

4a) $p \to q$

$\neg r \vee s$

$p \vee r$

$\therefore \neg q \to \neg s$

sl no	statement	reason
1	$p \vee r$.	premesis
2	$\neg r \vee s$	premesis

Assuming ① p is true

② r is true

for p is true, $p \to q$, q must be true.

for r is true, $\neg r$ is false, $\neg r \vee s$, s must be true

since q is true, s is true $q \to \neg s$ is false as it will not leads to $\neg q \to \neg s$, hence the statement is invalid,

or

for $p=0$, $q=0$ $r=1$, $s=1$

$p \to q \to$ true

$\neg r \vee s \to$ true

$p \vee r \to$ true

$\neg q \to \neg s \to$ false

4b) "No engineering student of first or second semester studies logic"
"Anil is an engineering student who studies logic"
"There fore Anil is not in second semester."

$p(x)$: x is An engineering student

$q(x)$: x is in first semester

$r(x)$: x is in the second semester.
$t(x)$: ~~t~~ x studies logic

$$(p(x) \wedge (q(x) \vee r(x))) \rightarrow \neg t(x)$$

$x_0 \in x$

$p(x_0) \wedge t(x_0)$

Sl no	Statement	Reason
①	~~$p(x) \wedge q(x)$~~ $(p(x) \wedge (q(x) \vee (r(x)))) \rightarrow \neg t(x)$	premesis
②	$p(x_0) \wedge (q(x_0) \vee r(x_0)) \rightarrow \neg t(x_0)$	universal law of specification
③	$p(x_0) \wedge t(x_0)$	universal law of specification
④	$p(x_0)$	Rule of Conjunction
⑤	$(q \cancel{(x)} \vee r)$	Rule of conjunction by ④ and ②
⑥	$\neg r$	Rule of disjunctive syllollis m

4c) (i) "A equilateral triangle has three angles of 60 degree, and Conversely".

$e(t)$: Triangle t is equilateral
$a(t)$: Triangle t has 3 angles of $60°$

then $\forall t \; (e(t) \leftrightarrow a(t))$

(ii) "Every rational number is a real number and not every real number is raticonal number".

$p(x)$: x is rational number
$q(x)$: x is real number.

$\forall x\,(p(x) \to q(x)) \land \neg\forall y\,(q(x) \to p(x))$

5a) (a) To prove $g \circ f : A \to C$ is one to one, let $a_1, a_2 \in A$ with $(g \circ f)(a_1) = (g \circ f)(a_2)$ then $(g \circ f)(a_1) = g \circ f(a_2)$

$$g(f(a_1)) = g(f(a_2))$$

$$f(a_1) = f(a_2)$$

because g is one to one

$$f(a_1) = f(a_2) \Rightarrow a_1 = a_2$$

$\therefore$ $g \circ f$ is one to one

(b) For $g \circ f : A \to C$, let $z \in C$,

since g is onto, there exists $y \in B$ with $g(y) = z$ with f onto, there exist $x \in A$ with $f(x) = y$,

$$z = g(y) = g(f(x)) = g \circ f(x)$$

so, the range of $g \circ f = C =$ the codomain of $g \circ f$ and $g \circ f$ is onto.

5b) $f(x) = x^3 \quad g(x) = x^2 + 1$

$$g \circ f(x) = g(f(x)) = g(x^3) = (x^3)^2 + 1$$
$$= x^5 + 1$$

$$f \circ g(x) = f(g(x)) = f(x^2+1) = (x^2+1)^3$$

$$f^2 = f \circ f = f(f(x)) = f(x^3) = (x^3)^3 = x^9$$

$$g^2 = g \circ g = g(g(x)) = g(x^2+1) = (x^2+1)^2 + 1$$

5c) Z defined by $(a, b) \in R$

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reflexive: there exist $a \in Z$ $(a,a) \in R$ such that

$$a^2 - a^2 = 0$$

Since 0 is an even integer, therefore $(a,a) \in R$

symmetric :- let $(a,b) \in R$ then $b,a \in R$

$$a^2 - b^2 = -(b^2 - a^2) \in R$$

$\therefore R$ is symmetric

Transitive: let $a,b \in R$ and $(b,c) \in R$

$a^2 - b^2$ is even

$b^2 - c^2$ is even adding

$a^2 - c^2$ is also even

R is transitive

6a) $f : R \to R$

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$$

(i) $f(-5/3)$ and $f(5/3)$

$f(-5/3) = +\cancel{3}(+5/\cancel{3}) + 1$

$= 6$

$f(5/3) = \cancel{3} \times 5/\cancel{3} - 5 = 0$

(ii) $f^{-1}(-3)$ and $f^{-1}(-6)$

$-3x + 1 = -3$; $3x - 5 = -3$

$-3x = -4$; $3x = 2$

$x = 4/3$ & $x \le 0$; $x = 2/3$ $x \ge 0$

$-3x + 1 = -6$ & $x \le 0$; $+3x - 5 = -6$ $x > 0$

$-3x = -7$; $3x = -1$

$x = 7/3$ & $x \le 0$; $x = -1/3$ $x > 0$

(iii) $f^{-1}([-5,5])$ and $f^{-1}([-6,5])$

$-5 < 3x - 5 < 5 \quad x \geq 0$

$0 < 3x < 10$

$0 < x < 10/3 \quad x \geq 0$

$\boxed{[0, 10/3]}$

$-5 < -3x + 1 < 5 \quad x < 0$

$-6 < -3x < 4$

$2 < x < -4$

$2 > x > -4 \quad x < 0$

$-6 < 3x - 5 < 5 \quad x > 0$

$-1 < 3x < 10$

$-1/3 < x < 10/3 \quad x > 0$

$[0, 10/3]$

$-6 < -3x + 1 < 5 \quad x < 0$

$-7 < -3x < 4$

$7 > 3x > -4$

$7/3 > x > -4/3 \quad x < 0$

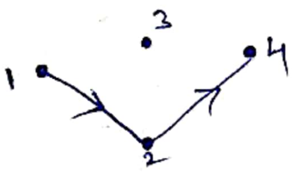
$[-4/3, 0]$

66) $A = \{1,2,3,4\}$

xRy, $y = 2x$

(i) $R = \{(1,2), (2,4)\}$

(ii)



(iii) indegree (1) = 0 outdegree (1) = 1
indegree (2) = 1 outdegree (2) = 1
indegree (3) = 0 outdegree (3) = 0
indegree (4) = 1 outdegree (4) = 0

6c) To prove Z^+ is not totally ordered by divisibility, we need to find a pair of elements $a, b \in Z^+$ such that $a | b$ nor $b | a$ holds eg: let $2, 3 \in Z^+$

$2|3, \Rightarrow 3 = 2k$

$\therefore$ there is no integer k satisfies above equation.

also

if $3|2 \Rightarrow 2 = 3k$.

but $2/3$ is not integer.

$\therefore$ since $(2,3) \in Z^+$ but not compatarable under the divisibility relation. Hence it is not totally ordered by the relation of divisibility.

8a)
$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$E: Z_2^3 \to Z_2^6$

(i) Code for 110

$$[1\ 1\ 0] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = [1\ 1\ 0\ 1\ 0\ 1]$$

Code for 010

$$[0\ 1\ 0] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = [0\ 1\ 0\ 0\ 1\ 1]$$

(ii)
$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(iii)
$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

fliping 3rd bit decoded message $\underline{\underline{100}}$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

flipping 3rd bit we get decoded message $\underline{\underline{110}}$

(iv)
$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

as [000] is not corresponding to any column it is not possible to decode

86) Proof: if $H = G$ the result follows, otherwise $m < n$ and there exists an element $a \in G - H$, since $a \notin H$ it follows that $aH \neq H$, so $aH \neq H$, so $aH \cap H = \phi$. If $G = aH \cup H$, then $|G| = |aH| + |H| = 2|H|$ and theorem follows. If not, there is an element $b \in G - (H \cup aH)$, with $bH \cap H = \phi = bH \cap aH$ and $|bH| = |H|$. If $G = bH \cup aH \cup H$, we have $|G| = 3|H|$.

other wise $c \in G$ with $c \notin bH \cup aH \cup H$.

The group G is finite, so this process terminates and we find that

$G = a_1H \cup a_2H \cup \ldots \cup a_kH$. Therefore

$|G| = k|H|$ and m divides n

8c) Let G be group, H is subgroup of G, let $a \in G$,

$aH = H$ iff $a \in H$

$aH = \{ah \mid h \in H\}$

since $a \in H$ and H is subgroup, the product of any element H will a will still be in H.

$\therefore h \in H$, $ah \in H$ so $aH \subseteq H$ also, $ae = a \in H$ $H \subseteq aH$

Since $aH \subseteq H$ and $H \subseteq aH$ it follows $aH = H$

(ii) if $aH = H$ then $a \in H$

$aH = H$

for $e \in aH$, there exists some $h_0 \in H$ such that $a = ah_0$ ~~where~~, because $aH = H$ implies that a can be expressed as $a = ah_0$ $h_0 \in H$

$h_0 \in H$ follows $a \in H$

$aH = H$ implies $a \in H$ if $a \in H$ $aH = H$