21CS36

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RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

III Semester B. E. Examinations April/May-2023

Common to CS / IS /AIML

DISCRETE MATHEMATICAL STRUCTURES

Time: 03 Hours *Maximum Marks: 100*

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.

PART-A

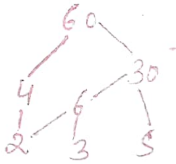
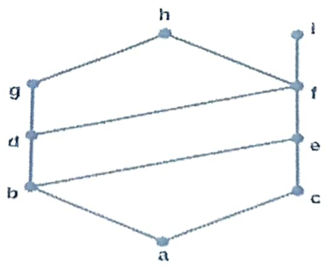
1	1.1	Find the number of ways of travelling from $(2,1)$ to $(5,6)$ considering moves as one unit UP Move or $RIGHT$ move.	02
	1.2	Write the recurrence relation for the integer sequence $3, 15, 75, \ldots$ and solve.	02
	1.3	Negate and simplify the compound statement $(p \wedge q) \rightarrow r$.	02
	1.4	Show that $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$ by writing truth table is tautology.	02
	1.5	Let R be a relation on set A with 3 elements. Find the number of reflexive and antisymmetric relations on set A.	02
	1.6	Let $A = \{1,2,3,4,5,6,7,8\}$ R is an equivalence relation on A which includes the partition $\{1,4,8\} \cup \{3\} \cup \{5,6\} \cup \{2,7\}$. Determine R.	02
	1.7	Write any two applications of Finite Automata.	02
	1.8	Write the DFA to accept all strings of $0's$ and $1's$ which contain at least two $0's$.	02
	1.9	Define homomorphism. Give an example.	02
	1.10	Find the probability of transmitting $c = 10110$ and receiving with 2 bit error with $p = 0.05$ as the probability of incorrect transmission.	02

PART-B

$^{24}P_5 \times 20! \times 2$

2	a	In how many ways can the letters of the English alphabet be arranged so that there are exactly 5 letters between the letters a and b?	04
	b	A student is to answer 7 out of 10 questions. In how many ways can he/she make his selection if i) there are no restrictions? $^{10}C_7$ ii) must answer the first two questions? $^{8}C_5$ iii) must answer at least four of the first 6 questions? $1\,\overline{oc}$	04
	c	Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$, where i) $x_1 \geq 0, 1 \leq i \leq 4$. $^{35}C_{32}$ ii) $x_i > 0, 1 \leq i \leq 4$ $^{31}C_{28}$ iii) $x_1, x_2 \geq 5, x_3, x_4 \geq 7$. $^{11}C_8$ iv) $x_i > -2, 1 \leq i \leq 4$. $^{43}C_{40}$	04
	d	A bank pays 6% (annual) interest in savings, compounding the interest monthly. If Boonie deposits \$1000 on the first day of May, how much will this deposit be worth a year later? Write the recurrence relation.	04

$P_n = 1.005 P_n \Rightarrow P_n = P_0(1.005)^n$ — \$1061.65

3	a	Simplify using Laws of Logic: $(p \vee q) \wedge \neg(\neg p \wedge q)$.	
	b	Write the symbolic representation of the statement "If Joan goes to Lake George, then Mary will pay for Joan's shopping spree". Also write its negation in words as well as symbolic form. p $p \to q$ negation - $\neg(p \to q)$ $\equiv p \wedge \neg q$ p: Joan goes to Lake George q: Mary will pay for Joan's shopping spree	04
	c	Prove the validity of the following argument: $\neg r(c)$ $\forall t[p(t) \to q(t)]$ $\forall t[q(t) \to r(t)]$ $\therefore \neg p(c)$	04 08
		OR	
4	a	Write inverse, converse and contrapositive of the statement: *If you brush with brite, then your teeth will be pearly white.*	
	b	Show that argument is invalid $[(p \wedge \neg q) \wedge (p \to (q \to r))] \to \neg r$.	04
	c	Find the negation of the following: $\forall x \exists y\, [(p(x,y) \wedge q(x,y) \to r(x,y)]$ $\exists x\, \forall y\, [p(x,y) \wedge q(x,y) \wedge \neg r(x,y)]$	04 04
	d	Write the symbolic representation of the statement if x is odd, then x^2 -1 is even. Also write its negation in words as well as symbolic form.	04
5	a	Let $f: R \to R$ and $g: R \to R$, given by $f(x) = x^2$ and $g(x) = x + 5$ find $g \circ f$ and $f \circ g$. $gof(x) = x^2+5$, $fog(x) = (x+5)^2$	04
	b	Given that the set $A = \{a, b, c\}$ with the relations $R = \{(a,a), (a,c), (b,a), (c,b)\}$ and $S = \{(a,b), (b,c), (c,c)\}$. Find the i) Converse of R $\{(a,a), (c,a), (a,b), (b,c)\}$ ii) Complement of S $\{(a,a), (a,c), (b,a), (b,b), (c,a)$ iii) The composition $R \circ S$ $\{(a,b), (a,c), (b,b), (c,$ iv) Show that $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$	05
	c	i) Draw a Hasse diagram for set A with divisibility relation, where $A = \{2, 3, 4, 5, 6, 30, 60\}$. ii) Find the greatest and least elements. 60, ϕ iii) Find LB, UB, LUB and GLB for $\{6, 30\}$. $\{2,3,6\}$, $\{30,60\}$, $\{30\}$, $\{6\}$ 	07
		Q1M 4M	
		OR	
6	a	Find minimal, maximal and least elements for the following Hasse diagram in Fig.6a. $\{a\}$ $\{h,i\}$ $\{a\}$  Fig.6a Also, find the upper bound, lower bound, LUB and GLB for the set $\{b, c\}$. $\{e,f,h,i\}$ $\{a\}$ $\{e\}$ $\{a\}$	08
	b	Let the function $f: R \to R$ be defined by $f(x,y) = \{(x,y) \mid y = mx + b$, where $m, b \in R$. Then find f^{-1}. $(x-b)/m$	04
	c	Prove that the composition of binary relations is associative.	04
7	a	Define DFA and design DFA for $L = \{w \mid \Sigma = \{a,b\}, N_a(w) \text{ is odd and } N_b(w) \text{ is divisible by } 3\}$.	06

b Convert the following NFA shown in Fig. 7b to DFA. Represent equivalent DFA using the Transition table also.

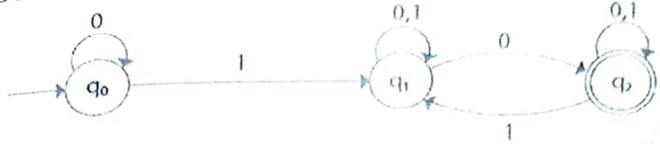


Fig. 7b — 06

q_1

q_1

$q_1 q_2$

q_0	q_0
q_1	$q_1 q_2$
$q_1 q_2$	$q_1 q_2$

c List any four differences DFA and NFA. — 04

OR

8 a Define language accepted by epsilon NFA and construct an epsilon NFA to accept any number of zeros followed by any number of ones followed by any number of twos. Also construct an equivalent NFA without epsilon. — 10

b Construct a DFA which accepts all ternary strings divisible by 4. — 06

9 a With $m = 3, W \in Z_2^3 : E: Z_2^3 \to Z_2^9$ and decoding function $D: Z_2^9 \to Z_2^3$ of triple repetition code, find — 1.5 X 4 M

i) $E(10110111)$ = 101101111011011110110111

ii) $D(111101100)$ = 101

iii) three different received words r for which $D(r) = 001$ 001 001 001

iv) For each $W \in Z_2^3$, what is $|D^{-1}(W)|$? 26 — 06

b Show that a group of complex numbers is an abelian group under multiplication. Is it a cyclic group? Table + abelian + generator {i} — 06 — 1+4+1 M

c State and prove Lagrage's theorem. — 04

OR

000000, 100110, 010011, 001101, 110101, 011110,

10 a With $E: Z_2^3 \to Z_2^6$ and $G = \begin{matrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{matrix}$ $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

101000

111000

Find

i) All possible code words

ii) Error detection and correction capability 2, 1

iii) H matrix

iv) Decoded word for received words $r = 110110$ and $r = 000111$. — 10

100, not possible

b Show $(Z_5, +)$ is a cyclic group. — 06

4+2M generators = {1, 2, 3, 4}

p(x): x is odd, q(x): x^2-1 is even. $\boxed{\forall x [p(x) \to q(x)]}$

negation: $\neg[\forall x [p(x) \to q(x)]] \iff \exists x, [\neg(p(x) \to q(x))]$

$\iff \boxed{\exists x [p(x) \wedge \neg q(x)]}$

There exist an integer such that x is odd & x^2-1 is odd.

Converse: If you do not brush with white, then your teeth will be pearly white.

If your teeth is pearly white then you brush with white.

If your teeth are not pearly white then you do not brush with white

21-Scheme

Part-A

1.1 Total number of moves to move up = 5
Total number of moves to move right = 3

$$\therefore \frac{8!}{3!\,5!} = 56$$

1.2 Recurrence relation $a_{n+1} = 5a_n$, $(a_0 = 3 \quad d = 5)$

Solving the recurrence relation,

$$a_{n+1} = a_0 5^n$$
$$= 3 \times 5^n$$

1.3 $\neg(p \wedge q) \rightarrow r \quad = \neg(\neg(p \wedge q) \vee r)$ $[\because (p \rightarrow q \Leftrightarrow \neg p \vee q)]$

$$= (p \wedge q) \wedge \neg r$$ (De Morgan's law and law of Double negation)

1.4

p	q	r	$q\to r$	$p\to(q\to r)$	$p\wedge q$	$(p\wedge q)\to r$	$p\to(q\to r)\equiv(p\wedge q)\to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

Since all values in final column is true the statement is a tautology.

1.5 Reflexive relations $= 2^{n^2-n} = 2^{9-2} = 2^7 = 64$

Antisymmetric relations $= 3^{\frac{n(n-1)}{2}} = 3^{\frac{3(2)}{2}} = 27$

1.6 $R = \{(1,1), (1,4), (1,8), (4,1), (4,4), (4,8), (8,1), (8,4), (8,8), (3,3), (5,5), (5,6), (6,5), (6,6), (2,2), (2,7), (7,2), (7,7)\}$

1.7 Pattern Matching: Finited Automata are used in pattern matching algorithms, such as those used in text editors and search engines to find occurrences of a substring within a string.

Lexical Analysis: In the compilation process, Finite Automata are used in lexical analysis to tokenize the input source code.

1.8

1.9 If (G, o) and $(H, *)$ are groups and $f: G \to H$, then f is called a group homomorphism if for all $a, b \in G$ $f(a \circ b) = f(a) * f(b)$

Ex: $(\mathbb{Z}, +)$ and $(\mathbb{Z}/n\mathbb{Z}, +)$. $f(x) = x \bmod n$

1.10. $^5C_2 \times (0.05)^2 \times (0.95)^3 = 0.02143$

Part - B

2a. * Position for ~~that~~ the 7 letters can be selected from 20 positions $(26-7+1=20)$

* $2!$ ways to arrange a and b.

* The 5 letters can be arranged in $5!$ ways

* Remaining 19 in $19!$ ways

$\therefore$ $20 \times 2 \times 5! \times 19!$

2b. i) $^{10}C_7 = 120$

ii) $^{8}C_5 = 56$

iii) $(^6C_4 \times {^4C_3}) + (^6C_5 \times {^4C_2}) + (^6C_6 \times {^4C_1})$

$= 60 + 36 + 4 = 100$

2c i) $x_i \geq 0,\ 1 \leq i \leq 4$

$(^{35}C_3) = 6545$

ii) $x_i > 0,\ 1 \leq i \leq 4$

$\binom{28+4-1}{C_{4-1}} = {^{31}C_3} = 4495$

iii) $\binom{8+4-1}{C_{4-1}} = ({}^{11}C_3) = 165$

iv) $\binom{40+4-1}{C_{4-1}} = ({}^{43}C_3) = 12341$

2d, $P = \$1000$ $r = 0.06$ $n = 12$ long period $t = 1$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$= 1000\left(1 + \frac{0.06}{12}\right)^{12}$$

Deposit $= \$1061.65$

3

3a, $(p \vee q) \wedge \neg(\neg p \wedge q) = (p \vee q) \wedge (p \vee \neg q)$ (∴ DeMorgan's law)

$= p \vee (q \wedge \neg q)$ (Distribution law)

$= p \vee (\text{false}) = \underline{p}$

3b. p: Joan goes to Lake George

q: Mary will pay for Joan's shopping spree.

$p \to q$: (Symbolic statement

Negation : $\neg(p \to q) \Leftrightarrow p \wedge \neg q$

∴ "Joan goes to Lake George and Mary will not pay for Joan's shopping spree."

No	Steps	Reasons
1.	$\neg r(c)$	Premise
2.	$\forall t\,[p(t) \to q(t)]$	Premise
3.	$\forall t\,[q(t) \to r(t)]$	Premise
4.	$p(c)$ is true	(assumption)
5.	$p(c) \to q(c)$	From (2), Law of universal specification
6.	$q(c) \to r(c)$	From (3), Law of universal
7.	$r(c)$ is true	From (6)
8.	But $\neg r(c)$	From (1)
∴ 9.	$p(c)$ is false	Contradiction.
	∴ $\neg p(c)$ is true	

4a. p: you brush with Brite

q: your teeth will be pearly white.

$p \to q$

i) Inverse: $\neg p \to \neg q$

If you do not brush with Brite, then your teeth will not be pearly white.

ii) Converse: $q \to p$

If your teeth are pearly white, then you Brush with Brite.

iii) Contrapositive: $\neg q \to \neg p$

If your teeth are not pearly white, then you do not brush with Brite.

4b. Sol

	Steps	Reason
1.	$[(p \land q) \to q] \land [p \to (q \to r)] \to \neg r$	Premise
2.	$(\neg(p \land q) \lor q) \land [\neg p \lor (\neg q \lor r)] \to \neg r$	[$\because p \to q \Leftrightarrow \neg p \lor q$]
3.	$[(\neg p \lor \neg q \lor q) \land [\neg p \lor (\neg q \lor r)] \to \neg r$	Demorgan's law
4.	$\neg[(\neg p \lor q) \land (\neg p \lor \neg q \lor r)] \to \neg r$	($\because p \to q \Leftrightarrow \neg p \lor q$)

No	Steps	Reason
5.	$(p \wedge \neg q) \vee (p \wedge q \wedge \neg r) \vee \neg r$	De Morgan's law and law of Double Negation.
6.	p is true, q is true, r is false	Assumption
7.	Then $(p \wedge \neg q)$ is false, $(p \wedge q \wedge \neg r)$ is false, $\neg r$ is true.	from 6
8.	But this contradicts premise	

So the statement is invalid

(c) $\neg (\forall x \exists y [(p(x,y) \wedge q(x,y)) \rightarrow r(x,y)]$

$= \exists x \forall y [\neg ((p(x,y) \wedge q(x,y)) \rightarrow r(x,y))]$

$= \exists x \forall y [\neg (\neg (p(x,y) \wedge q(x,y)) \vee r(x,y))]$ (De Morgan's law)

$= \exists x \forall y [(p(x,y) \wedge q(x,y)) \wedge \neg r(x,y)]$ (Law of negation and double negation)

4d. Let:
$p(x)$: x is odd

$q(x)$: x^2-1 is even

$\therefore p(x) \rightarrow q(x)$

Negation : $\exists x[p(x) \wedge \neg q(x)]$

"There exists an odd number x such that x^2-1 is not even."

5a. $g \circ f = g(f(x))$

$= g(x^2) = x^2+5$

$f \circ g = f(g(x)) = f(x+5)$

$= (x+5)^2 = x^2+10x+25$

5b. i) Converse of R :

$R^{-1} = \{(a,a), (c,a), (a,b), (b,c)\}$

ii) Compliment of S : $(R-S)$

$\bar{S} = \{(a,a), (a,c), (b,a), (b,b),$
$(c,a), (c,b)\}$

iii) Composition ROS

$$ROS = \{(a,a), (a,b), (a,c), (b,a), (c,a), (c,b)\}$$

iv) $(ROS)^{-1} = S^{-1} o R^{-1}$

Proof: $(ROS)^{-1} = \{(a,a), (c,a), (b,b), (b,c)\}$ — ①

$S^{-1} = \{(b,a), (c,b), (c,c)\}$

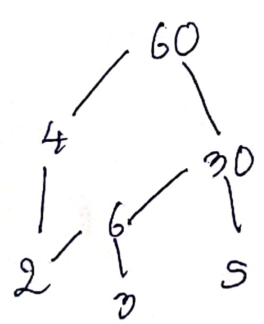
$R^{-1} = \{(a,a), (c,a), (a,b), (b,c)\}$

$S^{-1} o R^{-1} = \{(a,a), (c,a), (b,b), (b,c)\}$ ②

① = = ②

Hence proved.

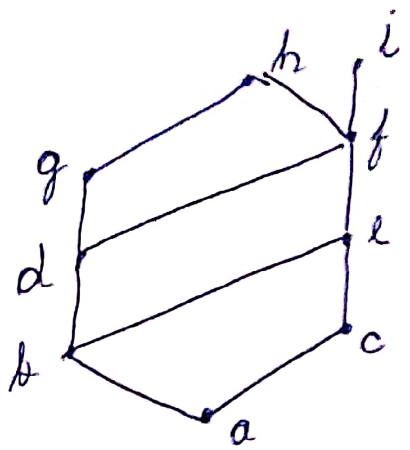
5c. i)



ii) 60, ϕ

iii) LB = {2, 3, 6} UB = {30, 60} LUB = {30}
GLB = {6}

2,



minimal : {a}   maximal : {h, i}   least : {a}

UP : {e, f, h, i}   LB : {a}   LUB : {e}

GLB : {a}

b.

$$y = f(x) = mx + b$$

$$y - b = mx$$

$$x = \frac{y-b}{m}$$

Replacing x with y

$$f^{-1}(x) = \frac{x-b}{m}$$

6c. Let $R \subseteq A \times B$, $S \subseteq B \times C$ and $T \subseteq C \times D$.
(Binary relation is subset of cartesian product $A \times B$)

i) $LHS = R \circ (S \circ T)$

Consider $S \circ T$,

$S \circ T = \{ (b, d) \mid \exists c \in C$ such that $(b, c) \in S$ and $(c, d) \in T \}$ — ①

Now,

$R \circ (S \circ T) = \{ (a, d) \mid \exists b \in B$ such $(a, b) \in R$ and $(b, d) \in (S \circ T) \}$

WKT $(b, d) \in (S \circ T)$ (from ①)

$\therefore R \circ (S \circ T) = \{ (a, d) \mid \exists b \in B, \exists c \in C$ such that $(a, b) \in R, (b, c) \in S, (c, d) \in T \}$

ii) $RHS = (R \circ S) \circ T$

Consider $R \circ S$;

$R \circ S = \{ (a, c) \mid \exists b \in B$ such that $(a, b) \in R$ and $(b, c) \in S \}$ — ②

Now,

$(R \circ S) \circ T = \{ (a, d) \mid \exists c \in C$ such that $(a, c) \in (R \circ S)$ and $(c, d) \in T \}$

WKT $(a, c) \in (R \circ S)$ (from ②)

$(R \circ S) \circ T = \{(a, d) \mid \exists c \in C, \exists b \in B$ such that
$(a, b) \in R, (b, c) \in S$, and $(c, d) \in T\}$

Since LHS = RHS,
composition of binary relations is associative.

9a. i) $E(101\,101\,111) = 101\,101\,111\,101\,101\,111\,101\,101\,111$

ii) $D(111\,101\,100)$

$$\begin{array}{ccc} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ \hline \end{array}$$

$$= \underline{\underline{1\;0\;1}}$$

iii) $r_1 = 000\,001\,001$ $\quad r_2 = 001\,000\,001$

$r_3 = 001\,001\,001$

iv) $|D^{-1}(w)| = 2^6$ (Since each bit has two choices and 6 bits are majority)

9b, Let $C = \{1, -1, i, -i\}$

i) Table :-

$\times$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

ii) * Since table contains only the elements in C, closure is satisfied.

* For any three elements of C, $a \times (b \times c) = (a \times b) \times c$, so associativity is satisfied.

* Since the row with 1 is same as top row, 1 is the identity element.

* Since each element $1, -1, i, -i$ has inverse $1, -1, -i, i$ respectively, inverse property is satisfied.

Since $(C, \times)$ satisfies closure, associativity, identity and inverse, it is a group

* Table is symmetric along the main diagonal so $a \times b = b \times a$. So commutative property is satisfied.

$\therefore$ $(C, \times)$ is an abelian group.

iii) $[i]^1 = i$ $[i]^2 = -1$ $[i]^3 = -i$ $[i]^4 = 1$

$[-i]^1 = -i$ $[-i]^2 = 1$ $[-i]^3 = i$ $[-i]^4 = -1$

Here $i, -i$ are generators.

So, $(C, \times)$ is a cyclic group.

9c. Lagrange's Theorem:- If G is a finite group of order n with H a subgroup of order m, then m divides n.

Proof:- If $H = G$ the result follows. Otherwise $m < n$ and there exists an element $a \in G - H$. Since $a \notin H$, it follows that $aH \neq H$, so $aH \cap H = \phi$. If $G = aH \cup H$, then $|G| = |aH| + |H| = 2|H|$ and the theorem follows. If not, there is an element $b \in G - (H \cup aH)$, with $bH \cap H = \phi = bH \cap aH$ and $|bH| = |H|$. If $G = bH \cup aH \cup H$, we have $|G| = 3|H|$, Otherwise we're back to an element $c \in G$ with $c \in bH \cup aH \cup H$, the group G is finite, so this process terminates and we find that $G = a_1H \cup a_2H \ldots \cup a_kH$. therefore, $|G| = k|H|$ and m divides n.

10.a. $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$

$W = \{000, 001, 010, 011, 100, 101, 110, 111\}$

i) $E(000) = [000] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = [000\ 000]$

$E(001) = [001] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = [001101]$

$E(010) = [010] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = [010011]$

$E(011) = [011] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = [011110]$

$E[100] = [100] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = [100110]$

$$E(101) = [101]\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = [1\,0\,1\,0\,1\,1]$$

$$E(110) = [110]\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = [1\,1\,0\,1\,0\,1]$$

$$E[111] = [111]\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} = [1\,1\,1\,0\,0\,0]$$

$\therefore C = \{000000,\ 001101,\ 010011,\ 011110,\ 100110,\ 101011,\ 110101,\ 111000\}$

ii) Minimum bit change is 3

$k+1 = 3$

$k = 2$

$\therefore$ There can be $\underline{2}$ bit error detection

$2k+1 = 3$

$k = 1$

$\therefore$ There can be $\underline{1}$ bit correction.

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(iii) $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

iv) $r_1 = 110110$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \cancel{[110110]} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

there is a syndrome match in 2nd column.

$\therefore$ flipping 2nd bit of r_1, $r_1 = 100110$.

$\therefore$ Decoded word is 100

$r_2 = 000111$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Not possible since more that 2 bit error.

10 b. i) $(Z_5, +) = \{0, 1, 2, 3, 4, ~~5~~\}$

+	0	1	2	3	4	~~5~~
0	0	1	2	3	4	~~5~~
1	1	2	3	4	0	~~0~~
2	2	3	4	0	1	~~1~~
3	3	4	0	1	2	~~2~~
4	4	0	1	2	3	~~3~~
~~5~~	~~5~~	~~0~~	~~1~~	~~2~~	~~3~~	~~4~~

ii) * Since table ~~or~~ contains only elements of Z_5, closure is satisfied.

* For any three elements of Z_5, $a+(b+c) = (a+b)+c$ so associativity is satisfied.

* Since the row with 0 is same as top row, 0 is the identity element.

* Each element in Z_5 0, 1, 2, 3, 4, ~~5~~ has inverse 0, 4, 3, 2, 1 respectively. So inverse rule satisfied.

* Since table is symmetric about main diagonal, commutative rule is satisfied.

∴ Since $(Z_5, +)$ satisfies closure, associativity, identity, inverse, commutative it is an abelian group.

iii) $[1]^1 = 1$ $[1]^2 = 2$ $[1]^3 = 3$ $[1]^4 = 4$ $[1]^5 = 0$ $[3]^1 = 3$ $[3]^2 = 1$ $[3]^3 = 4$ $[3]^4 = 2$ $[3]^5 = 0$

$[2]^1 = 2$ $[2]^2 = 4$ $[2]^3 = 1$ $[2]^4 = 3$ $[2]^5 =$ $[4]^1 = 4$ $[4]^2 = 3$

∴ 1, 2, 3, 4 are generators. So $(Z_5, +)$ is cyclic group