RV College of Engineering
Mysore Road, RV Vidyaniketan Post,
Bengaluru - 560059, Karnataka, India

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DEPARTMENT OF MATHEMATICS

Academic year 2024-2025 (Even Semester)

CIE -I

Date	01 April 2025	Maximum Marks	10 +50
Time	09:30 AM to 11 :30 AM	Duration	120 Mins
Semester	IV Semester B. E. **(AIML, CS, CY, CD, IS)**		
CS241AT- Discrete Mathematical Structures and Combinatorics			

Q. No.	Quiz	M	BT	CO
1	Let p: Ravi fails in exam; q: Ravi practices at home; and r: Ravi is attentive in class; Express $\sim(r \wedge q) \rightarrow \sim p$ in words.	2	1	1
2	Let p and q be primitive statements for which the implication $p \rightarrow q$ is false, then truth value of $(p \rightarrow \sim q) \vee (q \rightarrow \sim p)$ is ______ and $(p \rightarrow q) \vee \sim(p \leftrightarrow \sim q)$ is ______.	2	2	1
3	The coefficient of $x^3y^2z^5$ in $(x+2y+z)^{10}$ is ______.	2	1	2
4	If $a_n = c_1 + c_2\, 5^n, n \geq 0$ is the general solution of the relation $a_n + ba_{n-1} + ca_{n-2} = 0$ then $b =$ ______ and $c =$ ______.	2	2	2
5	The number of ways seven bananas and six oranges can be distributed among four children so that each child receives at least one banana is ______.	2	1	1
	Test			
1a	A group of 5 students is assigned to 5 desks in a classroom. How many ways can the students are seated such that exactly two students are sitting at their originally assigned desk.	5	2	2
1b	Evaluate the number of integer solutions to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$ where $x_i \geq 0$ for $i = 1,2,3,4,5,6$ such that $x_1 \leq 5$.	5	3	3
2a	A person invests some amount at the rate of 12% annual compound interest. Determine the period for the principal amount to get tripled.	5	4	4
2b	Solve the recurrence relation $a_n = 4a_{n-1} - 5a_{n-2}, n \geq 2$, with the initial condition $a_0 = 1$ and $a_1 = 3$.	5	3	3
3	Use generating function to solve the recurrence relation $a_n - a_{n-1} = 2^n, n \geq 1$, with the initial condition $a_0 = 1$.	10	3	3
4a	Find the number of positive integers not exceeding 300 and not divisible by any of 2, 3, and 5.	5	2	2
4b	Consider the following statement: If $\sqrt{2}$ is rational, then $\sqrt{2}+1$ is rational and 27 is a prime. Write the converse, inverse and contrapositive of the statement and determine their truth values.	5	2	2
5a	Check if the compound proposition $[(p \wedge q) \vee (\sim r)] \leftrightarrow p$ is a tautology, contradiction or a contingency using truth table.	5	1	1
5b	Using laws of logic, prove that $(\sim p \wedge \sim q \wedge r) \vee [(q \wedge r) \vee (p \wedge r)] \equiv r$.	5	3	3

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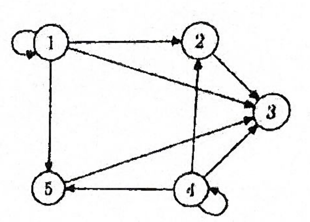
Date: 05/05/2025	CIE-2	Max. Marks: 10+50
Semester: *IV*	UG (CS, CD, CY, AIML)	Duration: 120 Mins
Course Title: Discrete Mathematical Structures and Combinatorics		Course Code: CS241AT

Department of Mathematics

Note: (i) Answer all the questions. (ii) Answer the quiz within first two pages of the answer booklet. (iii) Marks will not be awarded for direct answers without supporting steps.

Q. No	Quiz	M	BT	CO
1.	Write the conclusion from the premises and identify the rule of inference so that a valid argument is presented: "If Janice has trouble starting her car, then her daughter Angela will check the spark plugs, Janice has trouble starting her car".	2	2	1
2.	If the domain consists of all real numbers, then the truth value of $\forall x(x^2 - x + 4 = 0)$ is ________ and $\exists x(e^x = -1)$ is ________.	2	2	1
3.	Let $A = \{1,2,3,4,5,6\}$ and the relation $R = \{(x,y) : x + y \text{ is even}\}$. Then $[1] =$ ___ and $[2] =$ ___.	2	2	1
4.	Compute the number of ways to partition the set $A = \{1,2,3,4,5,6\}$ into two non-empty subsets.	2	2	2
5.	$A = \{1,2,3,4\}$ and $B = \{1,2,3,4,5,6\}$ then number of functions from A to B are ___ and number of one-to-one functions from A to B are ________.	2	1	2

Q. No	Test	M	BT	CO
1a	For the following argument, explain which rule of inference is used for each step. "Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Doug, a student in this class, can use a word processing program".	5	3	3
1b	For the universe of all integers, let $p(x), q(x), s(x), t(x)$ be the following open statements. $p(x): x > 0$ $q(x): x$ is even $s(x): x$ is a perfect square $t(x): x$ is divisible by 5 i. Translate "If x is even, then x is not divisible by 5" to logical expression. ii. Determine the truth value of the above statement (i). iii. Express the quantified statement $\forall x[s(x) \rightarrow p(x)]$ in words. iv. Provide a counter example if the above statement (iii) is false.	5	2	2
2a	Let $M(x,y)$ be "x has sent y an e-mail message" and $T(x,y)$ be "x has telephoned y", where the domain consists of all students in your class. i. Use quantifiers and translate the statement "There are two different students in your class who have sent each other e-mail messages" to logical expression. ii. Write the negation of the above statement in logical expression.	5	2	2

2b	Use proof by contraposition and contradiction to show that if $x > 1$, then $x^2 > x$ where $x \in \mathbb{R}$.	5	2	2
3a	Show that the relation R on the set of all bit strings such that $(s,t) \in R$ iff s and t contain the same number of 1's is an equivalence relation. Also determine the equivalence class of the bit string 011 for the equivalence relation R with string length three.	5	3	4
3b	If $A = \{1,3,5,7\}$ and R, S are relations on A defined by $R = \{(1,3),(1,5),(3,7),(7,7)\}$, $S = \{(1,1),(1,3),(1,5),(1,7),(3,5),(3,7)\}$. Find $S \circ (S \circ R)$, R^3 and S^2.	5	3	3
4a	Discuss the nature (Reflexive, symmetric, transitive, antisymmetric, irreflexive) of the following relations. $a)\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $b)$ 	4	2	2
4b	Let $A = \{1,2,3,4,6,12\}$. If relation R on A defined by $(a, b) \in R$ if and only if a divides b. Prove that R is a partial order relation on A. Draw the Hasse diagram for the relation R.	6	2	2
5a	Consider the function $f: \{\mathbb{R} - \{3\}\} \to \{\mathbb{R} - \{1\}\}$ defined as $f(x) = \frac{x-2}{x-3}$. Verify whether the function $f(x)$ is bijective or not, if it is bijective, then find $f^{-1}(x)$.	5	2	3
5b	Let f, g and h are functions from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = x^2,\ g(x) = x + 5,\ h(x) = \sqrt{x^2 + 2}$. Verify that $(h \circ g) \circ f = h \circ (g \circ f)$.	5	2	3

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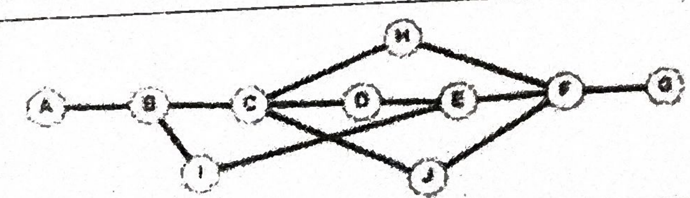
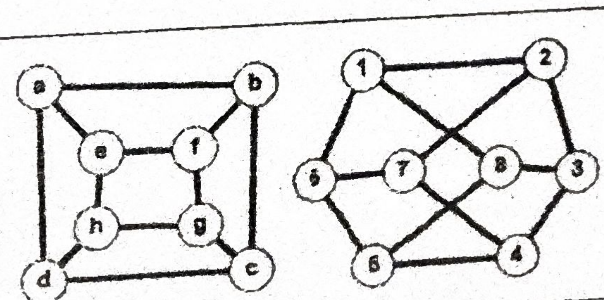
Date: 02-06-25	Improvement Test and Quiz	Max. Marks:10+50
Semester: IV	UG(CSE, CD, CY, ISE, AIML)	Duration: 2 Hrs
Course Title: Discrete Mathematical Structures and Combinatorics		Course Code: CS241AT

Department of Mathematics

Sl. No.	Quiz	M	BT	CO
1	In the group $(\mathbb{Q}^+, \times)$, the identity element of $\mathbb{Q}^+$ and the inverse of $a \in \mathbb{Q}^+$ are ______ and ______ respectively.	02	2	1
2	The order of the elements 2 and 3 of the group $(\mathbb{Z}_5, +_5)$ are ______ and ______ respectively.	02	2	1
3	If $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}, g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ are elements of the symmetric group S_3, then f^{-1} and $f^{-1} \circ g$ are respectively ______ and ______.	02	2	2
4	Examine if there exists a simple and connected graph with 10 vertices and 16 edges, if the degree of each vertex is either 2 or 3.	02	3	2
5	The number of zeros in the adjacency matrix of $K_{4,7}$ is ______.	02	3	2
	Test	**M**	**BT**	**CO**
1a	In the set $G = \mathbb{Q} - \{1\}$, the binary operation $*$ is defined as $a * b = a + b - ab$. Show that $(G,*)$ is a group.	05	3	2
1b	Prove that the identity element of a group is unique.	05	3	2
2a	Show that the multiplicative group of cube roots of unity is a cyclic group and hence abelian. List all its generators.	05	3	3
2b	Find (i) all the subgroups of the group $(\mathbb{Z}_{15}, +_{15})$ and (ii) all distinct left cosets of $H = \{0, 5, 10\}$ in $\mathbb{Z}_{15}$.	05	3	3
3a	The mapping $f: \langle \mathbb{Z}, + \rangle \to \langle \mathbb{Z}_n, +_n \rangle$ is defined by $f(a) = r, \forall a \in \mathbb{Z}$, where r is the unique least non negative remainder when a is divided by n. Show that f is a homomorphism and hence find $ker(f)$. Is f an isomorphism? Justify your answer.	05	3	3
3b	The $(5m, m)$ five times repetition code has encoding function $E: \mathbb{Z}_2^m \to \mathbb{Z}_2^{5m}$, where $E(w) = wwwww$. The decoding function $D: \mathbb{Z}_2^{5m} \to \mathbb{Z}_2^m$ is accomplished by the majority rule. Assume the probability of incorrect transmission as $p = 0.02$. (i) With $m = 3$, what is the probability for the transmission and correct decoding of the signal 101? (ii) With $m = 2$ decode the received word $r = 1001010111$, (iii) With $m = 1$, find two received words r, where $D(r) = 1$.	05	3	4
4a	The encoding function $E: \mathbb{Z}_2^3 \to \mathbb{Z}_2^6$ is given by the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ (i) Encode the words 100, 010, 001, 000, (ii) What is the error correction capability of this code?, (iii) Find the associated parity-check matrix and hence decode the received words 101011, 010111.	05	3	4
4b	Prove that in a simple graph, number of odd degree vertices is always even.	05	3	2
5a	Determine the eccentricity of all vertices of the given graph and hence the diameter and radius of the graph. 	05	3	2
5b	Give the conditions on which two graphs can be said to be isomorphic to each other. Hence show that the following graphs are isomorphic. 	05	3	3

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RV College of Engineering

R V College of Engineering

R V Vidyanikethan Post
Mysuru Road Bengaluru - 560 059

IV Semester BE Regular / Supplementary Examinations June/July -2025
(Common to CS, IS, CD, CY and AIML)
Course : Discrete Mathematical Structures and Combinatorics-CS241AT

Time : 3 Hours

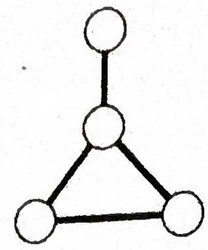
Maximum Marks : 100

Instructions to the students

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2, is compulsory. Answer any one full question from 3 & 4, 5 & 6, 7 & 8, and 9 & 10.

Part A

Question No	Question	M	CO	BT
1.1	If $G(x)$ is the generating function of the sequence $(a_n) = (a_0,\ a_1,\ a_2,\ a_3, \ldots)$ then the generating function for the sequence $(a_0,\ 0,\ a_1,\ 0,\ a_2,\ 0, \ldots)$ is ____.	02	1	2
1.2	Number of derangements of $\{A,\ B,\ C,\ D\}$ when A is in the second position is ____________.	02	1	1
1.3	Translate the statement “Some drivers do not obey the speed limit” into logical expression using quantifiers and express the negation of the statement in terms of quantifiers.	02	1	2
1.4	Without using truth table, prove that $\neg(p \vee (q \wedge r)) \iff \neg p \wedge (\neg q \vee \neg r)$.	02	2	2
1.5	If $R = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,3),(3,3),(4,4)\}$ is defined on the set $A = \{1,2,3,4\}$ determine the partition induced by R.	02	1	1
1.6	If there are 60 one-one functions from A to B and $\lvert A\rvert = 3$, then $\lvert B\rvert =$ ____.	02	1	1
1.7	For $n = 3$ and $x = 101 \in \mathbb{Z}_2^3$, the sphere $S(x, 2)$ is ________.	02	2	2
1.8	Verify if $f : \langle \mathbb{R}, + \rangle \longrightarrow \langle \mathbb{R}, + \rangle$ defined by $f(x) = 3x + 1, \forall x \in \mathbb{R}$ is a homomorphism.	02	2	2

1.9	The vertex and edge chromatic number of the following graph are _____ and _____ respectively. 	02	2	1
1.10	Number of zeros in the incidence matrix of K_{17} is ___________.	02	2	2

Part B

Question No	Question	M	CO	BT
2a	Determine the number of derangements of $\{1, 2, 3, 4, 5, 6\}$ which (i) begin with integers 1, 2, and 3 in some order. (ii) end with integers 1, 2, and 3 in some order.	05	3	3
2b	A bank pays a certain annual interest rate on deposits, with the interest compounded quarterly (i.e., once every 3 months). A deposit doubles in 6 years and 6 months. Obtain the recurrence relation for the value of the deposit at the end of n^{th} quarter and hence determine the annual percentage of interest rate paid by the bank.	06	3	4
2c	Determine the number of six- digit positive integers (without repetition of digits) which are (i) even (ii) divisible by 5.	05	2	2
3a	Verify whether $(p \land q) \lor r$ is logically equivalent to $(p \lor r) \land (q \lor r)$ using a truth table.	05	1	1
3b	Let $M(x, y)$ be "x has sent y an e-mail message" and $T(x, y)$ be "x has telephoned y" where the domain consists of all students in your class. i) Use quantifiers and translate the statement "There is a student in your class who has not received an e-mail message from anyone else in the class and who has not been called by any other student in the class" to logical expression. ii) Write the negation of the above statement in logical expression and in words.	05	3	3

Write the following argument in symbolic form. Verify the validity of the argument using rules of inference.

3c "All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners." 06 3 3

OR

4a Write the converse, inverse and contrapositive of the statement: If $x > 0$ then $x^2 > 0$. Also, write the truth value of all the three statements. 06 2 2

4b Use a direct proof and proof by contraposition to show that if m is an even integer then $m + 7$ is odd. 05 4 3

4c Prove that $\neg\left[\neg((p \vee q) \wedge r) \vee \neg q\right] \iff q \wedge r$ using laws of logic. 05 3 3

5a Prove that the $f : \mathbb{R} \to \mathbb{R}^+$ defined as $f(x) = e^{2x+5}$ is one-one and onto function. Also compute $f^{-1}(x)$. 06 3 4

5b Draw the Hasse diagram for the partial order R on the set $A = \{1, 2, 3, 4, 5\}$ whose matrix is given below. Also, write the relation R in each case. 05 2 2

i) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

5c Let R be a relation on the set of all bit strings of length three or more, where two bit strings x and y are related if and only if their first three bits are equal. Show that R is an equivalence relation. Describe the equivalence classes of this relation. In particular, determine the equivalence class of the bit strings: (i) 11111, (ii) 01010101. 05 4 4

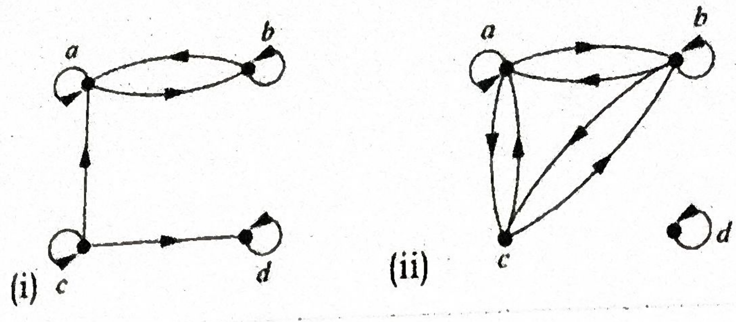
OR

6a Consider the functions $f, g, h : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x) = x - 1,\ g(x) = 3x,$ 06 3 3

$$h(x) = \begin{cases} 0 \text{ if } x \text{ is even} \\ 1 \text{ if } x \text{ is odd.} \end{cases}$$

Then compute $f \circ g,\ g \circ f,\ g \circ h,\ h \circ g,\ f \circ (g \circ h),\ (f \circ g) \circ h$.

6b Determine whether the relations represented by the directed graphs shown in (i) and (ii) are reflexive, irreflexive, symmetric, antisymmetric, and transitive. Justify your answer. 05 2 3



6c	Let R and S be two relations on a set of positive integers, where $R = \{(x, 2x) \mid x \in Z^+\}$, $S = \{(x, 7x) \mid x \in Z^+\}$. Find $R \circ S$, $S \circ R$, R^2.	05	2	3
7a	Let G be a cyclic groups. Prove that If $\lvert G\rvert$ is finite , then G is isomorphic to $(\mathbb{Z}_n, +_n)$.	05	3	3
7b	Show that $\langle \mathbb{Z}, * \rangle$, where $\mathbb{Z}$ is the set of integers and $a * b = a + b - 1 \,\forall\, a, b \in \mathbb{Z}$ is a group.	05	3	3
7c	The $(5m, m)$ five times repetition code has encoding function $E : \mathbb{Z}_2^m \to \mathbb{Z}_2^{5m}$, where $E(w) = wwwww$. The decoding function $D : \mathbb{Z}_2^{5m} \to \mathbb{Z}_2^m$ is accomplished by the majority rule. Assume the probability of incorrect transmission as $p = 0.03$. With $m = 3$ (i) what is the probability for the transmission and correct decoding of the signal 011, (ii) decode the received word 001011111101110, (iii) find two received words r, where $D(r) = 101$.	06	4	4

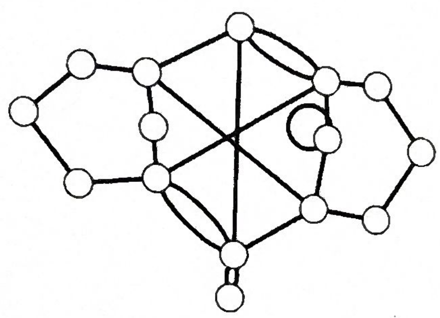
OR

8a	The $(5m, m)$ five times repetition code has encoding function $E : \mathbb{Z}_2^m \to \mathbb{Z}_2^{5m}$, where $E(w) = wwwww$. The decoding function $D : \mathbb{Z}_2^{5m} \to \mathbb{Z}_2^m$ is accomplished by the majority rule. Assume the probability of incorrect transmission as $p = 0.01$. With $m = 2$ (i) what is the probability for the transmission and correct decoding of the signal 11, (ii) decode the received word 1010100001, (iii) find two received words r, where $D(r) = 11$.	06	4	4
8b	Show that $\langle \mathbb{Z}_5, +_5 \rangle$, where $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ is a group.	05	3	3
8c	Find the index of H in $\langle G, \times_{13} \rangle$ and all the distinct right cosets of $H = \{1, 3, 9\}$ in the group G where $G = \mathbb{Z}_{13}^*$.	05	3	3
9a	Eight cities A, B, C, D, E, F, G, and H are required to be connected by a new road network. The possible roads and the cost involved to lay them (in crores of rupees) are summarized in the following table:	06	3	3

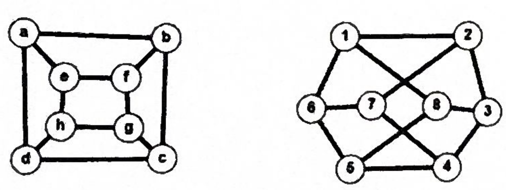
Between	Cost	Between	Cost
A and B	140	D and F	120
A and D	135	E and F	140
A and G	120	F and G	150
B and C	135	F and H	140
C and D	140	G and H	150
C and E	85	D and F	80

(i) Draw a weighted graph which represents the new road network.

(ii) Further determine a road network of minimal cost that connects all the cities using Kruskal's algorithm. Also mention the minimum cost.

9b	Briefly outline the procedure for detection of planarity of a graph using elementary reduction. Hence verify whether the following graph is planar or not? (Justify your answer) 	05	3	4
9c	If T is a tree with n vertices and m edges then prove that $m = n - 1$.	05	2	2

OR

10a	Give the conditions on which two graphs can be said to be isomorphic to each other. Establish a one-one correspondence between the vertices and edges to show that the following graphs are isomorphic. 	06	2	4
10b	Seven volleyball teams from Andhra Pradesh (AP), Goa (GO), Tamilnadu (TA), Delhi (DE), Kerala (KE), Maharashtra (MA) and Orissa (OR) have been invited to participate in tournaments, where each team is scheduled to play a certain number of other teams (given below). No team is to play more than one game each day. Using graph theory set up a schedule of games over the smallest number of days. **AP:** GO, TA, MA, OR ; **GO:** AP, TA, OR ; **TA:** AP, GO, DE, KE ; **DE:** TA, KE, MA, OR; **KE:** TA, DE, MA ; **MA:** AP, DE, KE, OR ; **OR:** AP, GO, DE, MA .	05	4	3
10c	If G is a graph with n vertices, m edges and f regions, then prove that $n - m + f = 2$.	05	2	2