Test of hypothesis for means and proportions:

Testing a hypothesis:

Some information about a characteristic of the population is known. We wish to know whether this information can be accepted. We choose a random sample and obtain information about this characteristic. Based on this information, we conclude whether the available information of the characteristic of the population can be accepted or rejected. we also wish to know that if it can be accepted then to what degree of confidence it can be accepted. this is called the problem of testing of hypothesis.

Errors:

If a hypothesis is rejected while it should have been accepted is known as Type-I error.

If a hypothesis is accepted, while it should have been rejected is known as type-II error.

Standard error:

The standard deviation of the sampling distribution is called the standard error.

The standard error is used to assess the difference between the expected and observed values. The reciprocal of the standard error is called precision.

If $n \geq 30$ a sample is called large, otherwise small.

The statistical testing of hypothesis aims at limiting the type-I error to a pre assigned value say 5% or 1% and to minimize the Type-II error.

The only way to reduce both types of errors is to increase the sample size, if possible.

Null hypothesis and Alternate hypothesis.

A population is given to us and we wish to have information about a characteristic of the population. We start with the assumption that there is not significant difference between the two sample statistics or between the sample statistic and the corresponding population parameters. This assumption that there is no significant difference is called a null hypothesis and is denoted by H_0. A hypothesis that is different from null hypothesis is called an alternate hypothesis and is denoted by H_1. The methods that are used to decide whether to accept or reject a null hypothesis or alternate hypothesis are called tests of hypothesis.

Consider the following example:

Let the null hypothesis be defined as:

H_0 : the population has an assumed value of mean μ_0

i.e $\mu = \mu_0$

The alternate hypothesis can be defined as any of the following:

(i) $H_1 : \mu \neq \mu_0$ i.e $\mu > \mu_0$ or $\mu < \mu_0$

(ii) $H_1 : \mu > \mu_0$

(iii) $H_1 : \mu < \mu_0$

The alternate hypothesis (i) is called a two tailed alternative (ii) is called right tailed alternative and (iii) is called the left tailed alternative.

The alternatives (ii) and (iii) are also called Single tailed tests where as (i) is called a two tailed test.

Critical region:

A region corresponding to a statistic t, in the sample space S which amounts to rejection of the null hypothesis H_0 is called as critical region or region of rejection.

The region of the sample space S which amounts to the acceptance of H_0 is called acceptance region.

Level of significance:

The probability of the value of the variate falling in the critical region is known as level of significance.

Depending on the nature of the problem we use a single tail test or double-tail test to estimate the significance of a result. Let a level of significance α be prescribed. We denote it by Z_α, the critical value of the test statistic Z at the given level of significance α.

Significance level:

The probability level, below which leads to the rejection of the hypothesis is known as the significance level. This probability conventionally fixed at 0.05 or 0.01 i.e 5% or 1%. These are called significance levels.

We feel confident in rejecting a hypothesis at 1% level of significance than 5% level of significance.

Let us suppose that we have a normal population with mean μ and SD σ. If $\bar{x}$ is the sample mean of a random sample of size n the quantity z defined by $z = \dfrac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ is called standard normal variate.

From the table of normal areas we find that 95% of the area lies b/w $z=-1.96$ and $z=1.96$. In other words we can say with 95% confidence that z lies b/w -1.96 and 1.96. Further 5% level of significance is denoted by $z_{0.05}$.

Thus
$$-1.96 \leq \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \leq 1.96$$

$$\Rightarrow \quad -\frac{\sigma}{\sqrt{n}}(1.96) \leq \bar{x}-\mu \leq \frac{\sigma}{\sqrt{n}}1.96$$

$$\Rightarrow \quad \mu \leq \bar{x}+\frac{\sigma}{\sqrt{n}}1.96 \text{ and } \bar{x}-\frac{\sigma}{\sqrt{n}}1.96 \leq \mu$$

combining the two results,

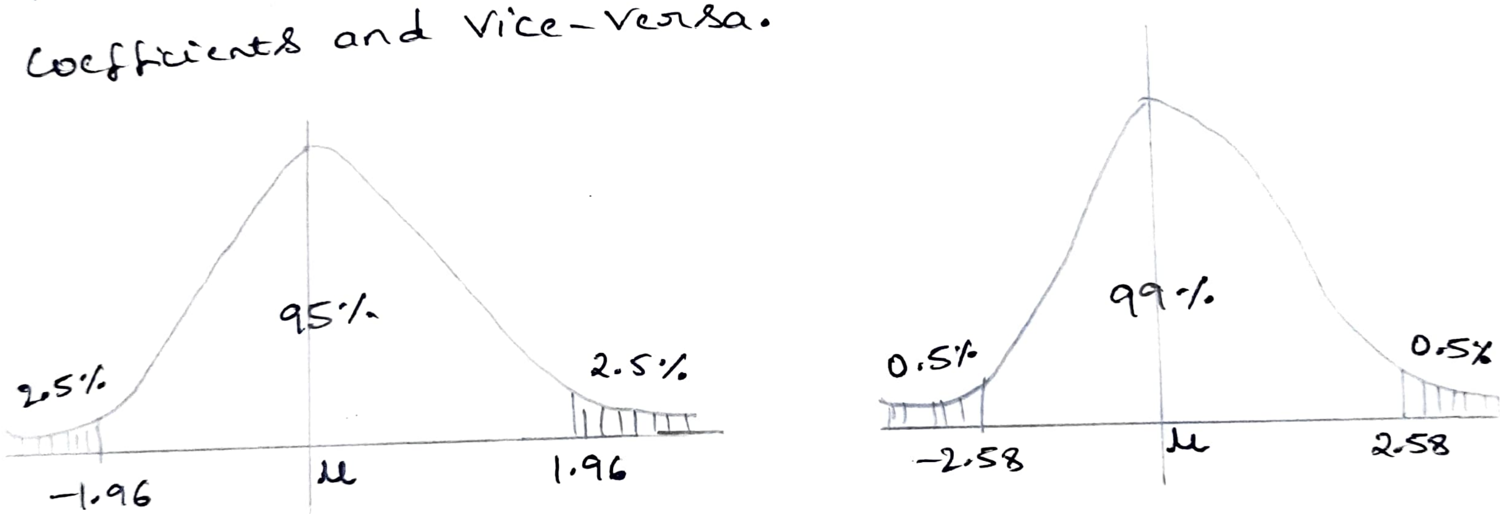
$$\bar{x}-1.96\left(\frac{\sigma}{\sqrt{n}}\right) \leq \mu \leq \bar{x}+1.96\left(\frac{\sigma}{\sqrt{n}}\right) \rightarrow (1)$$

Similarly, from the table of normal areas 99% of the area lies b/w −2.58 and 2.58. This is equivalent to the form

$$\bar{x} - 2.58 \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + 2.58 \frac{\sigma}{\sqrt{n}} \rightarrow (2)$$

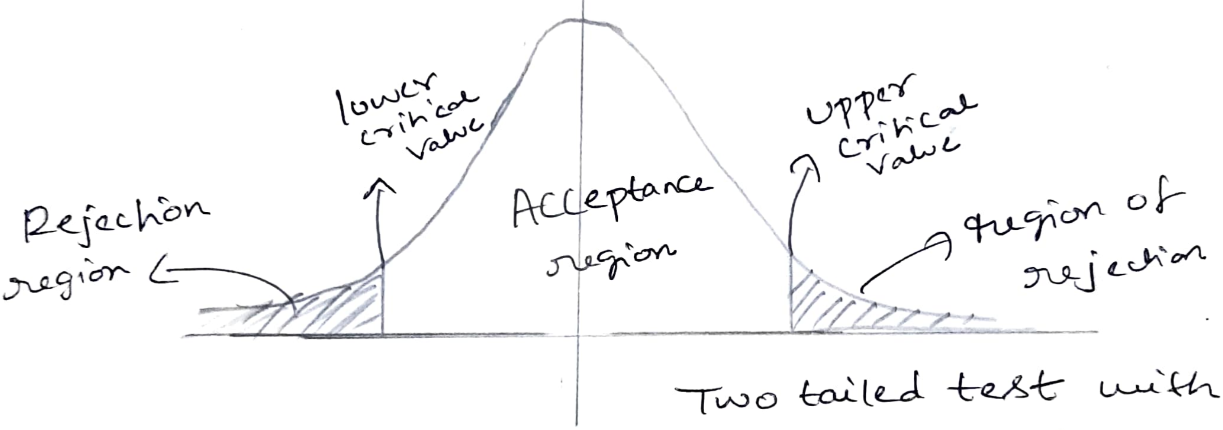
Thus we can say that (1) is the 95% confidence interval and (2) is the 99% confidence interval.

The constants 1.96, 2.58 etc in the confidence limits are called confidence coefficients denoted by Z_c. From confidence levels we can find confidence coefficients and vice-versa.



As reflected in the figure, we can say with 95% confidence that if the hypothesis is true, the value of z for an actual sample lies between −1.96 to 1.96 since the area under the normal curve between these values is 0.95. However if the value of z for random sample lies outside the range we can conclude that the probability of the happening of such an event is only 0.05 if the given hypothesis is true.

The total shaded area 0.05 being the level of significance of the test, represent the probability of making type-I error (rejecting the hypothesis when it should have been accepted). The set of values of Z outside the range −1.96, 1.96 constitutes the critical (significant) region or the region of rejecting the hypothesis whereas the values of Z within the same range constitutes the insignificant region or the region of acceptance of the hypothesis.



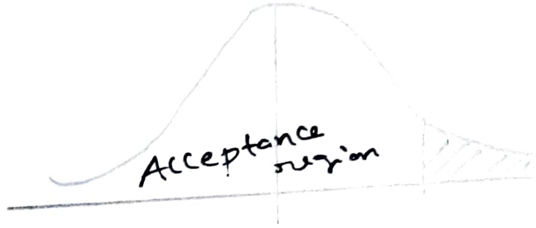
Two tailed test with Significance

One tailed test

Sometimes we will be interested in extreme values to only one side of the mean in which case the region of significance will be a region to one side of the distribution. The area of such a region will be equal to the level of significance itself. Such a test is called a One tailed test.

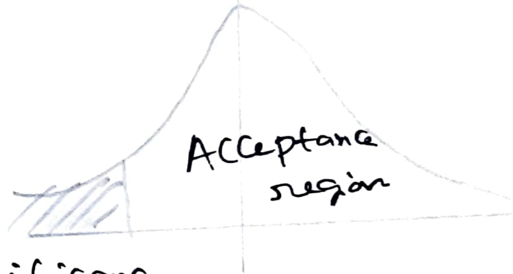
Right tailed test

The critical value Z_α is obtained from the equation $P(Z > Z_\alpha) = \alpha$. The total area of the critical region α is the area of the right tail under the probability curve.



Left tailed test

The critical value Z_α is obtained from the equation $P(Z < -Z_\alpha) = \alpha$. The total area of the critical region α, is the area of the left tail under the probability curve.



Test	level of Significance 1% (0.01)	5% (0.05)
Two tailed test	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 1.96$
Right tailed test	$Z_\alpha = 2.33$	$Z_\alpha = 1.645$
left tailed test	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$

Test of Significance for large Samples.

The sample size is taken as large if the sample size $n > 30$. For such sample we apply normal test as binomial distribution tends to normal for large "n".

Under ~~the~~ large sample test, the following are the important test to test the significance.

1) Testing of significance for single proportion
2) Testing of significance for difference of proportion
3) Testing of significance for single mean
4) Testing of significance for difference of means.

Test of Significance of proportions

Here Mean proportion of success $= \frac{np}{n} = p$

Standard deviation or Standard error proportion of success $= \frac{\sqrt{npq}}{n} = \sqrt{\frac{pq}{n}}$

Let x be the observed number of success in a sample size of n and $\mu = np$ be expected number of success. The associated standard normal variate z be defined as $z = \frac{x-\mu}{\sigma} = \frac{x-np}{\sqrt{npq}}$

If $|z| > 2.58$ we conclude that the difference is highly significant and reject the hypothesis.

Since p is the probability of success and $\sqrt{\frac{pq}{n}}$ is the Standard error or Standard deviation proportion of success, $p \pm 2.58\sqrt{\frac{pq}{n}}$ are the probable limits.

① A coin was tossed 400 times and head turned up 216 times. Test the hypothesis that the coin is unbaised at 5% level of significance.

Solution:- Suppose the coin is unbaised then the Probability of getting the head in a toss is $= \frac{1}{2} = p$

$$p + q = 1 \Rightarrow q = \frac{1}{2}$$

Expected number of heads in 400 tosses $= \frac{1}{2} \times 400$

$= 200 = np$

In the data observed number of success (heads)

$= 216 = x$

Standard deviation $= \sqrt{npq} = \sqrt{400 \times \frac{1}{2} \times \frac{1}{2}} = 10$

$$\therefore \quad z = \frac{x - np}{\sqrt{npq}} = \frac{216 - 200}{10} = 1.6 < 1.96$$

Since $z < 1.96$ the hypothesis is accepted at 5% level of significance. Hence we conclude that the coin is unbaised at 5% level of significance.

(2). A die is thrown 9000 times and a throw of 5 or 6 was observed 3240 times. On the assumption of random throwing, do the data indicate an unbiased die.

Soln:. Probability of getting 5 or 6 in a single throw is $p = \frac{2}{6} = \frac{1}{3}$

$q = 1 - p = \frac{2}{3}$

Expected number of success $= \frac{1}{3} \times 9000 = 3000 = np$

Observed value of success $= 3240 = x$

Standard deviation of sampling $= \sqrt{npq} = \sqrt{9000 \times \frac{1}{3} \times \frac{2}{3}}$

$= 44.7213$

Hence $Z = \frac{x - np}{\sqrt{npq}} = \frac{240}{44.7213} = 5.3665$

Since $Z = 5.3665 > 2.58$ the hypothesis has to be rejected at 1% level of significance and we conclude that the die is biased.

(3) If in a random sample of 600 cars making a right turn at a certain traffic junction 157 drove into the wrong lane, test whether actually 30% of all drivers make this mistake or not at this given junction. Use (a) 0.05 (b) 0.01 level of significance.

Soln:- Let X be discrete random variable denoting the number of cars driving into the wrong lane at a junction.

Null hypothesis: $H_0 = p = 0.3$

Alternate hypothesis: $H_1 = p \neq 0.3$

level of significance: a) 0.05   b) 0.01

Acceptance region: a) $-1.96 < Z < 1.96$

b) $-2.58 < Z < 2.58$

($p = 0.3, q = 1-p = 0.7$)

Here $\mu = np = 600(0.3) = 180$

$\sigma = \sqrt{npq} = \sqrt{600 \times (0.3) \times (0.7)} = \sqrt{126} = 11.2249$

observed value of success $= x = 157$

$\therefore Z = \dfrac{x - np}{\sqrt{npq}} = \dfrac{157 - 180}{11.2249} = -2.0490$

Since $-2.0490 < -1.96$ reject the Null hypothesis

Since $-2.0490 > -2.58$ accept the Null hypothesis.

④ Test the claim of manufacturing that 95% of his stabilizers confirm ISI specifications if out of a random sample of 200 stabilizers produced by this manufacturing 18 were faulty use a) 0.01 b) 0.05 level of significance.

Soln:- Let p = probability that the stabilizers are of ISI standard i.e good

Null hypothesis: $H_0: p = 0.95$

Alternate hypothesis $H_1: p < 0.95$

$\mu = np = 200\left(\frac{95}{100}\right) = 190$

$\sigma = \sqrt{npq} = \sqrt{200 \times 0.95 \times 0.05} = 3.0822$

observed value of success $= 200 - 18 = 182 = x$

$\therefore Z = \frac{x - np}{\sqrt{npq}} = \frac{182 - 190}{3.0822} = -2.5955$

Since $Z = -2.5955 < -2.33$ reject the null hypothesis

$Z = -2.5955 < -1.645$ reject the null hypothesis

reject the claim of the manufacturer at both levels using one tailed test.

5) Result extracts revealed that in a certain school over a period of five years 725 students had passed and 615 students had failed. Test the hypothesis that success and failure are in equal proportions.

Solution:- Total number of students $= 725 + 615 = 1340$

observed proportion of success $= \frac{725}{1340} = 0.54$

Suppose that success and failure are in equal proportion, then $p = \frac{1}{2}$

$\therefore$ Difference in proportion $= 0.54 - 0.5 = 0.04$

Standard deviation proportion of success $= \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.5 \times 0.5}{1340}}$

$= 0.0136$

$\therefore$ In terms of proportion $Z = \frac{x - \mu}{\sigma} = \frac{0.04}{0.0136} = 2.9284$

$Z = 2.9284 > 2.58$

Thus the hypothesis that success and failure are in equal proportion is rejected.

OR

Observed number of success $= 725 = x$

Expected number of success $= \frac{1340}{2} = 670$

$$Z = \frac{x - np}{\sqrt{npq}} = \frac{725 - 670}{\sqrt{1340 \times \frac{1}{2} \times \frac{1}{2}}} = 3.0049 > 2.58$$

Hence the hypothesis is rejected.

6) A random sample of 500 apples was taken from a large consignment and 65 were found to be bad. Estimate the proportion of bad apples in the consignment as well as the standard error of the estimate. Also find the percentage of bad apples in the consignment.

Soln:- proportion of bad apples in the sample : $p = \frac{65}{500}$

$p = 0.13$

$q = 1 - p = 0.87$

Standard error proportion of bad apples $= \sqrt{\frac{pq}{n}}$

$$= \sqrt{\frac{0.13 \times 0.87}{500}} = 0.0150$$

Probable limits of bad apples in the consignment

$$= p \pm 2.58 \sqrt{\frac{pq}{n}}$$

$$= 0.13 \pm 2.58\ (0.0150)$$

$$= 0.13 \pm 0.0388$$

$$= 0.0912,\ 0.1688$$

i.e 9.12% and 16.88%.

Thus the required percentage of bad apples in the consignment lies between 9.13 and 16.87.

7) A sample of 100 days is taken from meteorological records of a certain district and 10 of them are found to be foggy. What are the probable limits of the percentage of foggy days in the district.

Soln:- p = proportion of foggy days in a sample of 100 days is given by $\frac{10}{100} = 0.1$

$q = 1 - p = 0.9$

$\therefore$ probable limits of foggy days

$$= p \pm 2.58\sqrt{\frac{pq}{n}}$$

$$= 0.1 \pm 2.58\sqrt{\frac{0.1 \times 0.9}{100}}$$

$$= 0.1 \pm 0.0774$$

$$= 0.0226 \text{ and } 0.1774$$

Thus the percentage of foggy days lies b/w 2.26 and 17.74

8) To know the mean weights of all 10 year old boys in certain city a sample of 225 was taken. The mean weight of the sample was found to be 67 pounds with S.D of 12 pounds. What can we infer about the mean weight of the population?

Soln: Given $\bar{x} = 67$, $n = 225$, $\sigma = 12$

95% confidence limits for the mean of the population corresponding to a given sample is $\bar{x} \pm 1.96\left(\frac{\sigma}{\sqrt{n}}\right)$

and 99% confidence limits for the mean is

$$\bar{x} \pm 2.58\frac{\sigma}{\sqrt{n}}.$$

we have $\frac{\sigma}{\sqrt{n}} = \frac{12}{15} = 0.8$

We get 95% confidence limits : $67 \pm 1.96 (0.8)$

$= 65.432$ and 68.568

99% confidence limits ; $67 \pm 2.58 (0.8)$

$= 64.936$ and 69.064

We can say with 95% confidence that the mean weight of the population lies between 65.4 pounds and 68.6 pounds.

With 99% confidence we can say that the mean weight lies between 64.9 pounds to 69.1 pounds.

9) The mean and S.D of the maximum loads supported by 60 cables are 11.09 tonnes and 0.73 tonnes respectively. Find (a) 95% (b) 99% confidence limits for mean of the maximum loads of all cables produced by the company.

Soln: By data $\bar{x} = 11.09$, $\sigma = 0.73$

(a) 95% confidence limits for the mean of maximum loads are given by

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} = 11.09 \pm 1.96 \left(\frac{0.73}{\sqrt{60}}\right)$$

$$= 11.09 \pm 0.1847$$

Thus 10.9053 tonnes to 11.2747 tonnes are the 95% confidence limits for the mean of maximum loads.

(b) 99% confidence limits for the mean of maximum loads are given by $\bar{x} \pm 2.58 \left(\frac{\sigma}{\sqrt{n}}\right)$

$$= 11.09 \pm 258 \left(\frac{0.73}{\sqrt{60}}\right)$$

$$= 11.09 \pm 0.2431$$

Thus 10.8469 tonnes to 11.3331 tonnes are the 99% confidence limits for the mean of maximum loads.

Testing of Significance for difference of proportions

To test the significance of the difference between the sample proportions, the test statistic under the null hypothesis H_0 that there is no significant difference between the two sample proportions

we have $$z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$ where $$p = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

$$q = 1 - p$$

P_1, P_2 are the sample proportions in respect of an attribute corresponding to two large samples of size n_1 and n_2 drawn from two populations.

① One type of aircraft is found to develop engine trouble in 5 flights out of a total of 100 and another type in 7 flights out of a total of 200 flights. Is there a significant difference in the two types of aircrafts so far as engine defects are concerned?

Solution:- Let P_1 and P_2 be the proportion of defects in two types of aircrafts

$$\therefore P_1 = \frac{5}{100} = 0.05, \quad P_2 = \frac{7}{200} = 0.035$$

H_0 is the null hypothesis that there is no significant difference between the two types of aircrafts

Population proportion $$p = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{100 \times 0.05 + 200 \times 0.035}{100 + 200}$$

$$p = 0.04$$

$q = 1 - p = 0.96$

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.05 - 0.035}{\sqrt{0.04 \times 0.96\left(\frac{1}{100}+\frac{1}{200}\right)}} = 0.625$$

$$Z = 0.625 \begin{cases} < Z_{0.05} = 1.96 \text{ (two tailed test)} \\ < Z_{0.01} = 2.58 \text{ (two tailed test)} \end{cases}$$

Thus the null hypothesis is accepted both at 5% and 1% levels of significance.

② In an exit poll enquiry it was revealed that 600 voters in one locality and 400 voters from an other locality favoured 55% and 48% respectively a particular party to come to power. Test the hypothesis that there is a difference in the locality in respect of the opinion.

Soln:- By data $p_1 = \frac{55}{100} = 0.55$

$p_2 = \frac{48}{100} = 0.48$

Ho is the null hypothesis that there is no difference in the locality.

Population proportion $p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{600(0.55) + 400(0.48)}{600 + 400}$

$= 0.522$

$\therefore q = 1 - p = 0.478$

$$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.55 - 0.48}{\sqrt{0.522 \times 0.478\left(\frac{1}{600}+\frac{1}{400}\right)}} = 2.171$$

$Z = 2.171 > Z_{0.05} = 1.96$

$Z = 2.171 < Z_{0.01} = 2.58$

Thus the null hypothesis that there is no significant difference between the localities is rejected at 5% level but not at 1% level of significance.

3) A machine produced 16 defective articles in a batch of 500. After over hauling it produced 3 defectives in a batch of 100. Has the machine improved?

Soln: By data $n_1 = 500$, $n_2 = 100$

P_1 = proportion of defectives in the first sample $= \frac{16}{500}$

$P_1 = 0.032$

P_2 = proportion of defectives in the second sample (after overhauling of machine)

$= \frac{3}{100} = 0.03$

Null hypothesis: $H_0 : P_1 = P_2$: the machine has not improved after overhauling.

Alternate hypothesis: $H_1 : P_2 < P_1$ OR $P_1 > P_2$ (Right tail test)

level of significance: $\alpha = 0.05$

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$P = \frac{n_1P_1 + n_2P_2}{n_1 + n_2} = \frac{500 \times 0.032 + (100 \times 0.03)}{500 + 100} = 0.03166$$

$$q = 1 - P = 0.9683$$

$$\therefore Z = \frac{0.032 - 0.03}{\sqrt{(0.0316)(0.9683)\left(\frac{1}{500} + \frac{1}{100}\right)}} = 0.1043$$

Since $Z = 0.1043 < 1.645$ (Right tailed test) it is not significant at 5% level of significance.
H_0 is accepted. i.e the machine has not improved after overhauling.

4) In a city A, 20% of a random sample of 900 school boys had a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant.

Soln: $n_1 = 900$, $n_2 = 1600$, $P_1 = 0.2$, $P_2 = 0.185$

Null hypothesis: H_0: there is no significant difference between the two. $H_0: P_1 = P_2$

Alternate hypothesis $H_1: P_1 \neq P_2$ (two tailed)

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{900 \times 0.2 + 1600 \times 0.185}{900 + 1600}$$

$$= 0.1904$$

$$q = 1 - P = 0.8096$$

$$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.20 - 0.185}{\sqrt{(0.1904)(0.8096)\left(\frac{1}{900} + \frac{1}{1600}\right)}} = 0.9169$$

$$z = 0.9169 < 1.96$$

$\therefore$ H_0 may be accepted at 5% level of significance and we may conclude that there is no significant difference between proportions.

Test of Significance for Single mean

To test whether the difference between the sample mean $\bar{x}$ and population mean μ is significant or not.

Under the null hypothesis that there is no difference between the sample mean and population mean.

The test statistic is $Z = \dfrac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ where σ is the Standard deviation of the population

If σ is not known, we use the test statistic

$$Z = \frac{\bar{x}-\mu}{\frac{S}{\sqrt{n}}}$$ where S is the Standard deviation of the sample.

① A normal population has a mean of 6.8 and Standard deviation of 1.5. A sample of 400 members gave a mean of 6.75. Is the difference Significant.

Solution: H_0: there is no significant difference between $\bar{x}$ and μ

By data $\mu = 6.8$, $\sigma = 1.5$, $\bar{x} = 6.75$, $n = 400$

$$|Z| = \left|\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right| = \left|\frac{6.75-6.8}{\frac{1.5}{\sqrt{400}}}\right| = |-0.67|$$

$$Z = 0.67 < 1.96$$

Hence H_0 is accepted. i.e there is no Significant difference between $\bar{x}$ and μ.

② A sample of 400 male students is found to have a mean height of 160cms. Can it reasonably regarded as a sample from a large population with mean height 162.5c[m] and Standard deviation 4.5cms.

Solution: H_0: there is no significant difference between $\bar{x}$ and μ

By data $\mu = 162.5$, $\sigma = 4.5$, $\bar{x} = 160$, $n = 400$

$$|Z| = \left| \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \right| = \left| \frac{160 - 162.5}{\frac{4.5}{\sqrt{400}}} \right| = |-11.11|$$

$$= 11.1 > 1.96$$

$\therefore$ H_0 is rejected. i.e the sample is not drawn from the population with mean 162.5.

3) It has been found from experience that the mean breaking strength of a particular brand of thread is 275.6 gms with S.D of 39.7gms. Recently a sample of 36 pieces of thread showed a mean breaking strength of 253.2gms. Can one conclude at a significance level of (a) 0.05 (b) 0.01 that the thread has become inferior?

Soln: We have to decide between the two hypothesis

H_0: $\mu = 275.6$gms, mean breaking strength

H_1: $\mu < 275.6$ gms, Inferior in breaking strength

We choose one tailed test

$\mu = 275.6$, $\bar{x} = 253.2$, $n = 36$, $\sigma = 39.7$

$$Z = \left| \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \right| = \left| \frac{-22.4}{39.7/6} \right| = 3.38$$

$Z = 3.38$ is greater than the critical value of $Z = 1.645$ at 5% level and 2.33 at 1% level of significan[ce]

Under the hypothesis H_1 that the thread has become inferior is accepted at both 0.05 and 0.01 levels with one tail test.

Test of Significance for difference of means

Let μ_1 and μ_2 be the mean of two populations

Let $\bar{x}_1, \sigma_1$ and $\bar{x}_2, \sigma_2$ be the mean and S.D of two large samples of size n_1 and n_2 respectively.

We wish to test the null hypothesis H_0 that there is no difference between the population means.

i.e $H_0 : \mu_1 = \mu_2$

The test statistic for this is given by

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Also confidence limits for the difference of means of the population are

$$(\bar{x}_1 - \bar{x}_2) \pm Z\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

① Intelligent test were given to two groups of boys and girls

	Mean	S.D	Size
Girls	75	8	60
Boys	73	10	100

Find out if the two mean significantly differ at 5% level of significance.

Soln:- Null hypothesis: H_0: there is no significant difference between the mean scores. i.e $\bar{x}_1 = \bar{x}_2$

Alternate hypothesis: $\bar{x}_1 \neq \bar{x}_2$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} \mp \frac{\sigma_2^2}{n_2}}} = \frac{75-73}{\sqrt{\frac{64}{60}+\frac{100}{100}}} = \frac{2}{\sqrt{1.066+1}} = 1.3912$$

$Z = 1.3912 < 1.96$.

∴ H_0 is accepted. i.e there is no significant difference between the mean scores.

(2) A Sample of 100 bulbs produced by a Company A showed a mean life of 1190 hours and a S.D of 90 hours. Also a sample of 75 bulbs produced by a Company B showed a mean life of 1230 hours and a SD of 120 hours. Is there a difference between the mean life of bulbs produced by the two Companies at (a) 5% levele of significance (b) 1% level of significance.

Soln: By data $\bar{x}_1 = 1190$, $\sigma_1 = 90$, $n_1 = 100$ (company A)

$\bar{x}_2 = 1230$, $\sigma_2 = 120$, $n_2 = 75$ (company B)

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{1190-1230}{\sqrt{\frac{90^2}{100}+\frac{120^2}{75}}} = -2.4209$$

$|Z| = 2.4209$

$Z = 2.4209 > Z_{0.05} = 1.96$

$Z = 2.4209 < Z_{0.01} = 2.58$

The null hypothesis that there is no difference between the mean life time of bulbs is rejected at 5% level but not at 1% level of significance.

③ The mean yield of wheat from a district A was 210lbs with SD = 10lbs per acre from a sample of 100 plots. In another district B the mean yield was 220 lbs with SD = 12 lbs from a sample of 150 plots. Assuming that the standard deviation of the yield in the entire state was 11lbs, test whether there is any significant difference between the mean yield of crops in the two districts.

Soln: given $n_1 = 100$, $\bar{x}_1 = 210$, $\sigma_1 = 10$

$n_2 = 150$, $\bar{x}_2 = 220$, $\sigma_2 = 12$

We are also given that the standard deviation of the yield in the entire state is 11lbs. i.e the common standard deviation of the two populations of yields from district A and B is 11

$\therefore \quad \sigma_1 = \sigma_2 = \sigma = 11$

Null hypothesis: $H_0 : \bar{x}_1 = \bar{x}_2$ i.e the mean yields of crops in two districts do not differ significantly.

Alternate hypothesis: $H_1 : \bar{x}_1 \neq \bar{x}_2$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{210 - 220}{\sqrt{\frac{11^2}{100} + \frac{11^2}{150}}} = -7.04$$

$|Z| = 7.04 > 2.58$. Hence the null hypothesis is rejecte