RV College of Engineering®
Mysore Road, RV Vidyaniketan Post,
Bengaluru - 560059, Karnataka, India

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Academic year 2023-2024 (Even Sem)
DEPARTMENT OF

COMPUTER SCIENCE & ENGINEERING

Date	June 2024	Maximum Marks	50
Course Code	CS241AT	Duration	90 Min
Sem-IV	**Test-1**	Staff: HKK/ASP/SMS/SGR/MNV	

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS

(Common to CSE, ISE & AIML)

		Marks	BT	CO
1a.	Determine if the expansion of $\left(x^2 - \frac{2}{x}\right)^{18}$ will contain a term containing x^{10}. $r = 28/3$, NO.	5	4	2
1b.	If a person places 6 letters into 6 addressed envelopes, what is the probability that exactly two of them are placed correctly.	5	3	2
2a.	Find the number of non negative integer solutions of i. $x_1 + x_2 + x_3 + x_4 + x_5 = 40$ $44c_{40} = 135751$ ii. $x_1 + x_2 + x_3 + x_4 + x_5 \le 40$ $45c_{40} = 1221759$ iii. $x_1 + x_2 + x_3 + x_4 + x_5 = 40$ with $x_1 \ge 1$, $x_2 \ge 2$, $x_3 \ge 3$, $x_4 \ge 4$, $x_5 \ge 5$ $29c_4 = 23751$ iv. $x_1 + x_2 + x_3 + x_4 + x_5 = 40$ with $x_1 < 20$ $44c_{40} - 24c_{20} = 125,125$	6	3	2
2b.	Simplify using the laws of logic: $\neg[\neg\{(p \vee q) \wedge r\} \vee \neg q]$ $q \wedge r$	4	3	1
3a.	Write the recurrence relation to solve the Tower of Hanoi problem. Also solve that recurrence relation using the generating function.	6	4	4
3b.	~~Draw the circuit diagram to represent the following statement:~~ $[p \vee (p \wedge q) \vee (p \wedge q \wedge \neg r)] \wedge [p \wedge r \wedge t) \vee t]$	~~4~~	~~2~~	~~3~~
4a.	If a person invests ₹ 25,000 at at 9% annual interest, find the amount he will get at the end of 5 years if • interest compounded half yearly $(1.045)^n P_0$ 38824.2 • interest compounded monthly $(1.0075)^n P_0$ 39142.4 Write the recurrence relation and solve.	6	3	4
4b.	Determine the truth values of p, q, r, s, t when $[p \wedge (q \wedge r)] \rightarrow (s \veebar t)$ is false.	4	2	1
5a.	Show the validity of the argument: 4.b) $p=q=r=1$, $s=t=0$ or $p=q=r=1$, $s=t=1$ $(\neg p \wedge \neg q) \rightarrow (r \wedge s)$ $r \rightarrow t$ $\neg t$ valid ----------------------- $\therefore p$	6	3	3
5b.	Find the number of ways in which 5 people A, B, C, D, and E can be seated at a round table, such that • C and D always sit together $3! * 2 = 12$ • C and D never sit together $4! - 3! = 12$	4	1	1

1.b) $\frac{6C_2 \times d_4}{6!}$

$\frac{135}{720} = \frac{3}{16}$

$= 0.1875$

$a_n = 2a_{n-1} + 1$

$\boxed{a_0 = 0}$

$a_n = 2^n - 1$

~~$a_n = 1.045 a_{n-1}$~~

$u_n = 1.0075 a_{n-1}$

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	CO5	L1	L2	L3	L4	L5	L6
	Max Marks	12	16	10	12	-	4	8	27	11	-	-

1.a) $x^{10} = x^{38-3r}$ so $r = 28/3$ not an integer so there will not be a term containing x^{10} in the expansion

1.b Total arrangements $= 6! = \underline{\underline{720}}$ $\Rightarrow {}^6C_2 \times d_4 = {}^6C_2 \times 9$
$= \underline{\underline{135}}$

required probability $= 135/720 = 3/16$

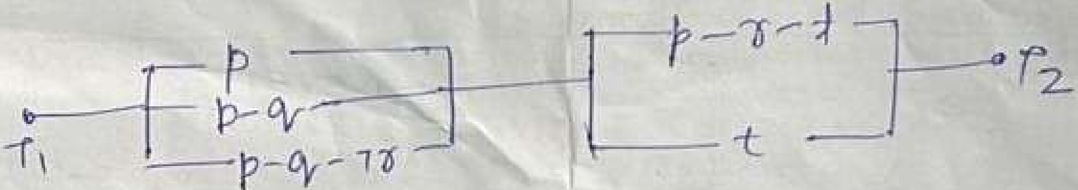
2.a) i) $^{44}C_{40}$ (ii) $^{45}C_{40}$ (iii) $^{29}C_4$ (iv) $^{44}C_{40} - {}^{24}C_{20}$

2.b) $q \wedge r$

3.a) Recurrence relation is $a_n = 2a_{n-1} + 1$, $a_0 = 0$

$a_n = 2^n - 1$, $n \geq 0$

3.b)



4.a) i) $P_n = (1.045)^n P_0 = 38824.2$

ii) $P_n = (1.0075)^n P_0 = 39142.4$

4.b) $p = q = r = 1$, $s = t = 0$ or

$p = q = r = 1$, $s = t = 1$

5.a) valid

5.b) i) $3! * 2 = 6 * 2 = 12$

ii) $4! - 3! = 24 - 12 = 12$

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Academic year 2023-2024 (Even Sem)
DEPARTMENT OF
COMPUTER SCIENCE & ENGINEERING

Date	22^{nd} July 2024	Maximum Marks	60
Course Code	CS241AT	Duration	120 Min
Sem-IV	Test-II	Staff: HKK/ASP/SMS/SGR/MNV	

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS
(Common to CSE, ISE & AIML)

	PART-A	Marks	BT	CO
1.1	Let A={1, 2, 3, 4}. How many relations on A which are antisymmetric? How many relations on A which are neither reflexive nor irreflexive?	2	1	2
1.2	Let R be the relation on the set A={1, 2, 3, 4, 5} containing the ordered pairs R={(1, 3), (2, 4), (3, 1), (3, 5), (4, 3), (5, 1), (5, 2), (5, 4)}. Find R^4.	2	2	1
1.3	For the POSET (A, |) where A={2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72} find the upper bounds, lower bounds, LUB and GLB of {2, 6, 9, 18}.	2	2	2
1.4	How many ways can one distribute 4 distinct objects among 3 identical containers?	1	1	1
1.5	If A={1, 2, 3, 4, 5} and there are 6720 injective functions f: A→B, what is |B|?	1	1	1
1.6	Let P(x, y) denote the sentence: x divides y. What are the truth values of $\forall x \exists y P(x, y)$, $\forall x \forall y P(x, y)$, where the domain of x, y is the set {1, 2, 4, 6, 12}?	1	1	1
1.7	Express the negation of the below statement so that all negation symbols immediately precede predicates. $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$	1	1	1
	PART-B			
2a.	For the following statement state the *converse, inverse,* and *contrapositive*. Also determine the truth value for the given statement, as well as the truth value for its *converse, inverse*, and *contrapositive*. "For all real numbers x, if $x^2+4x-21>0$, then x>3 or x<-7.	05	3	2

		M	BT	CO
2b.	Test the validity of the following argument: *Some rational numbers are powers of 7.* *All integers are rational numbers.* ――――――――――――― *Some integers are power of 7.*	05	4	3
3a.	Let A={1, 2, 3, 4} and R={(1, 2), (2, 3), (3, 4), (2, 1)}. Write the matrix for R. Find the R^{∞} by computing matrices for R^2, R^3,....	04	2	2
3b.	Suppose A is a set, R is an equivalence relation on A, and a and b are elements of A. Prove the following. i. If aRb, then [a]=[b]. ii. [a]=[b] or $[a]\cap[b] = \emptyset$. iii. Distinct equivalence classes of R form a partition of A.	06	4	1
4a.	Define POSET. Show that the set A={1, 2, 3, 6, 12, 15, 24, 36, 48} under the divisibility (|) operation forms a POSET. Draw the Hasse diagram for (A, |)	05	2	2
4b.	Let U={1, 2, 3, 4, 5, 6, 7}, with A=P(U) (power set of U), and R be the subset relation on A. For B={{1}, {2}, {2, 3}}$\subseteq$ A, determine each of the following. a) The number of upper bounds of B that contains 4 elements. b) The number of upper bounds that exists for B. c) The lub of B d) The number of lower bounds that exists for B. e) The glb of B.	05	3	2
5a.	i. Let $f(x) = x^3$ and $g(x) = x-1$ for all real numbers x. Find g o f and f o g. Verify whether g o f equals f o g. ii. Let f, g: R→R, where $g(x) = 1-x+x^2$ and $f(x) = ax+b$. If $(g \circ f)(x)=9x^2-9x+3$, determine a and b.	04	3	2
5b.	If f: A→B and g: B→C are invertible functions, then g o f: A→C is an invertible function and $(g \circ f)^{-1}=f^{-1} \circ g^{-1}$. Prove this.	06	4	3
6a.	Let f: A→B, g: B→C and h: C→D. Prove the following. i. If f and g are one-to-one, then g o f is one-to-one. ii. If f and g are onto, then g o f is onto. iii. (h o g) o f=h o (g o f).	06	4	3
6b.	Let A={1, 2, 3, 4, 5} and B={6, 7, 8, 9, 10, 11, 12}. i. How many functions f: A→B are there? ii. How many functions are one-to-one? iii. How many functions f: A→B are such that $f^{-1}(\{6, 7, 8\})=\{1, 2\}$?	04	3	2

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	CO5	L1	L2	L3	L4	L5	L6
	Max Marks	12	31	17	-	-	6	13	18	23	-	-

CIE-II

Scheme and Solutions

Date: 22-07-24

Duration: 120 min

Sub: Discrete Mathematical Structures and Combinatorics

Sub. Code: CS241AT

PART-A

1.1 No. of antisymmetric relations $= 2^4 \cdot 3^{\frac{1}{2}(4^2-4)}$

$= 11,664$ — 1 Mark

No. of relations which are neither reflexive nor irreflexive $= 2^{16} - 2 \cdot 2^{(16-3)}$

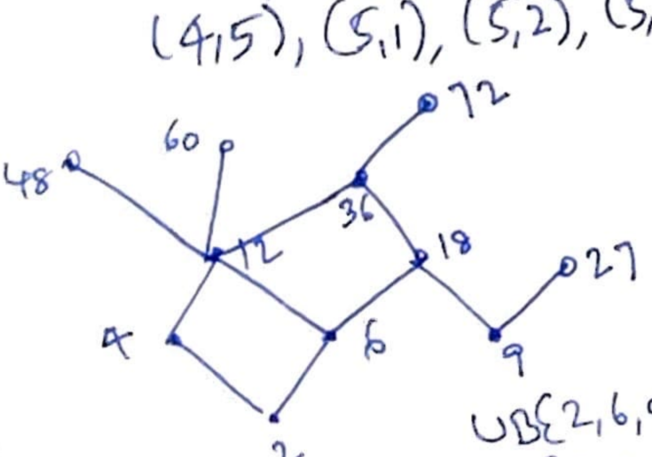
$= 57,344$ — 1 Mark

1.2 $R^4 = R \circ R \circ R \circ R$

$= \{(1,1), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,3), (4,4), (4,5), (5,1), (5,2), (5,3), (5,4), (5,5)\}$.

— 2 Marks.

1.3



UB$\{2,6,9,18\} = \{18, 36, 72\}$

LB$\{2,6,9,18\} =$ Nil

LUB$\{2,6,9,18\} = 18$ — $\frac{1}{2} \times 4 = 2$ Marks

GLB$\{2,6,9,18\} =$ Nil

1.4 $\sum_{i=1}^{3} S(4,i)$

$= 1 + 7 + 6 = 14$ ways — 1 Mark.

1.5 Let $|A| = m = 5$

$|B| = n$

$\therefore$ ^nP_m possible injective functions from $A \to B$

$^nP_5 = 6720$ $\quad \frac{n!}{(n-5)!} = 6720$

$n = 8$ $\quad \frac{8!}{3!} = \frac{40320}{3!} = \frac{40320}{6} = 6720$

$\therefore |B| = 8$ 1 mark

1.6 Truth Value of $\forall x \exists y\, P(x,y)$ is TRUE

Truth Value of $\forall x \forall y\, P(x,y)$ is FALSE

$1/2 + 1/2 = 1$ mark

1.7 $\exists x \forall y\, (P(x,y) \wedge \sim Q(x,y))$ 1 mark

PART-B

2.a) Statement:

$\forall x\, [(x^2+4x-21>0) \to [(x>3) \vee (x<-7)]]$

$\Rightarrow$ TRUE

Converse:

$\forall x\, [[(x>3) \vee (x<-7)] \to (x^2+4x-21>0)]$

$\Rightarrow$ TRUE

Inverse:

$\forall x\, [(x^2+4x-21 \leq 0) \to [(x \leq 3) \wedge (x \geq -7)]]$

$\Rightarrow$ TRUE

Contrapositive:

$\forall x\, [[(x \leq 3) \wedge (x \geq -7)] \to (x^2+4x-21 \leq 0)]$

$\Rightarrow$ TRUE

$1+1+1+1 = 4$

Truth Value $= 1$

2b) Let $P(x)$: x is an integer
$Q(x)$: x is a rational number — 1 Mark
$R(x)$: x is a power of 7

Therefore the given argument is translated into the following.

$$\begin{array}{l} \exists x\,(Q(x) \rightarrow R(x)) \\ \forall x\,(P(x) \rightarrow Q(x)) \\ \hline \therefore \exists x\,(P(x) \rightarrow R(x)) \end{array}$$ — 2 Marks

To verify the validity of this proced as follows

1. $\exists x\,(Q(x) \rightarrow R(x))$; premise
2. $Q(a) \rightarrow R(a)$; Rule of ES - 1
3. $\forall x\,(P(x) \rightarrow Q(x))$; premise
4. $P(a) \rightarrow Q(a)$; Rule of US - 3
5. $P(a) \rightarrow R(a)$; Rule of syllogism 4, 2.
6. $\therefore \exists x\,(P(x) \rightarrow R(x))$; Rule of EG - 5.

— 2 Marks

3.a)

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R^2 = M_R \odot M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ 1 - Mark

$$M_{R^3} = M_{R^2} \odot M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R^3 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ 1-Mark

$$M_{R^4} = M_R^3 \odot M_R = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \odot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R^4 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ 1- Mark

$$M_{R^\infty} = M_R \vee M_{R^2} \vee M_{R^3} \vee M_{R^4} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\therefore R^\infty = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4) (3,4)\}$

1- Mark.

3.b)(i) If aRb then $[a] = [b]$.

Proof: Let $x \in [a]$. So xRa, by definition of class. But aRb, by hypothesis. Thus by transitivity of R xRb $\therefore x \in [b]$, by definition of class. $\therefore [a] \subseteq [b]$ 2-Marks

But $x \in [b]$, then xRb, by definition of class. Now aRb by hypothesis. Since R is symmetric bRa also. Then, since R is transitive xRb, & bRa $\therefore xRa \Leftrightarrow x \in [a]$ $\therefore [b] \subseteq [a]$. Hence $[a] = [b]$

(ii) $[a] \cap [b] = \phi$ or $[a] = [b]$

Proof: If A is a set, R is an equivalence relation on A, and a and b are elements of A, then $[a] \cap [b] = \phi$.

Let $[a] \cap [b] \neq \phi$, Then there exists an element $x \in A$ such that $x \in [a] \cap [b]$.

By definition of intersection

$x \in [a]$ and $x \in [b]$ and so

xRa and xRb, by definition of class

Since R is symmetric and xRa then aRx.

But R is transitive, and so, aRx, xRb,

aRb

Thus a, b satisfies the proof in (i) $[a] = [b]$.

— 2 Marks

(iii) Let $x \in A$. By reflexivity of R, xRx

$x \in [x]$ and x must be in one of the distinct equivalence classes $A_1, A_2, \ldots A_n$

$$\therefore x \in \bigcup_{i=1}^{n} A_i$$

$$A \subseteq \bigcup_{i=1}^{n} A_i \quad — (1)$$

Now, let $x \in \bigcup_{i=1}^{n} A_i$, $x \in A_i$ for some $i = 1, 2, \ldots n$

but each A_i is an equivalence class of A.

$\therefore A_i \subseteq A$ and so $x \in A$

$$\therefore \bigcup_{i=1}^{n} A_i \subseteq A \quad — (2)$$

From (1) & (2) we get

— 2 Marks

$$A = \bigcup_{i=1}^{n} A_i$$

$||^{ly}$ $A_i \cap A_j = \phi$ from (ii)

4.a) Definition of POSET.

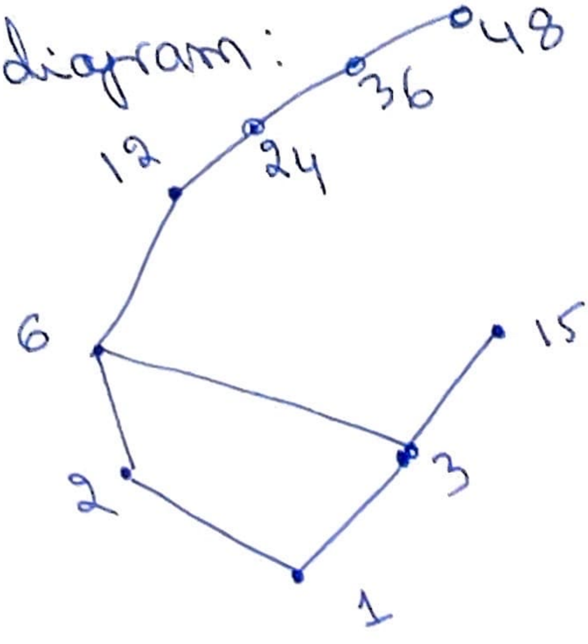
If Any set A under the given relation R where R is a partial ordered relation, then (A, R) is called as POSET. — 1 Mark

$(A, |) = \{(1,1), (1,2), (1,3), (1,6), (1,12), (1,15), (1,24), (1,36), (1,48), (2,2), (2,6), (2,12), (2,24), (2,36), (2,48), (3,3), (3,6), (3,12), (3,15), (3,24), (3,36), (3,48), (6,6), (6,12), (6,24), (6,36), (6,48), (12,12), (12,24), (12,36), (12,48), (15,15), (24,24), (24,48), (36,36), (48,48)\}$.

This is reflexive, antisymmetric & transitive — 2 Marks

$\therefore$ It is a POSET.

Hasse diagram:



— 2 Marks

4.b) a) 4 number of upperbounds of B
b) In total 15 upperbounds of B
c) LUB(B) = {1, 2, 3}
d) Only one lowerbound of B
e) glb(B) = $\emptyset$ $1 \times 5 = 5$ Marks.

5.a) i) $(g \circ f)(x) = g(f(x)) = g(x^3)$
$= x^3 - 1 \quad \forall x \in R$
$(f \circ g)(x) = f(g(x)) = f(x-1) = (x-1)^3 \quad \forall x \in R$
— 1 Mark
— 1 Mark

$(g \circ f)(x) \neq (f \circ g)(x)$
For instance, let $x = 2$
$(g \circ f)(2) = 2^3 - 1 = 8 - 1 = 7$
$(f \circ g)(2) = (2-1)^3 = 1$

ii) $(g \circ f)(x) = g(f(x))$
$= g(ax+b)$ — 1 Mark
$= 1 - (ax+b) + (ax+b)^2$
$= a^2x^2 + (2ab - a)x + (b^2 - b + 1)$ — ①

It is given that
$(g \circ f)(x) = 9x^2 - 9x + 3$ — ②

Comparing eqn ① and ②, we get
$a^2 = 9$ — 1 Mark
$2ab - a = -9$
$b^2 - b + 1 = 3$
$\therefore a = 3, \; b = 1$

7

5.b) Since f and g are invertible functions, both are bijective. Concequently $g \circ f$ is bijective.
$\therefore g \circ f$ is invertible. — 1 Mark

Now, the inverse f^{-1} of f is a function from B to A and the inverse g^{-1} of g is a function from C to B.
$\therefore h = f^{-1} \circ g^{-1}$ then h is a function from C to A.
— 1 Mark

we find that

$$
\begin{aligned}
(g \circ f) \circ h &= (g \circ f) \circ (f^{-1} \circ g^{-1}) \\
&= g \circ (f \circ f^{-1}) \circ g^{-1} \\
&= g \circ I_B \circ g^{-1} \\
&= g \circ g^{-1} = I_C
\end{aligned}
$$

— 1½ Mark

and

$$
\begin{aligned}
h \circ (g \circ f) &= (f^{-1} \circ g^{-1}) \circ (g \circ f) \\
&= f^{-1} \circ (g^{-1} \circ g) \circ f \\
&= f^{-1} \circ I_B \circ f \\
&= f^{-1} \circ f = I_A
\end{aligned}
$$

— 1½ Mark

Therefore, h is the inverse of $g \circ f$
i.e $h = (g \circ f)^{-1}$ — 1 Mark
$\therefore (g \circ f)^{-1} = f^{-1} \circ g^{-1}$

6.a) Proof (i):

Let $a_1, a_2 \in A$ with $(g \circ f)(a_1) = (g \circ f)(a_2)$, then

$(g \circ f)(a_1) = (g \circ f)(a_2) \Rightarrow g(f(a_1)) = g(f(a_2))$

$\Rightarrow f(a_1) = f(a_2)$.

because g is one to one, $a_1 = a_2$ Consequently

$g \circ f$ is one to one. — 2 Marks

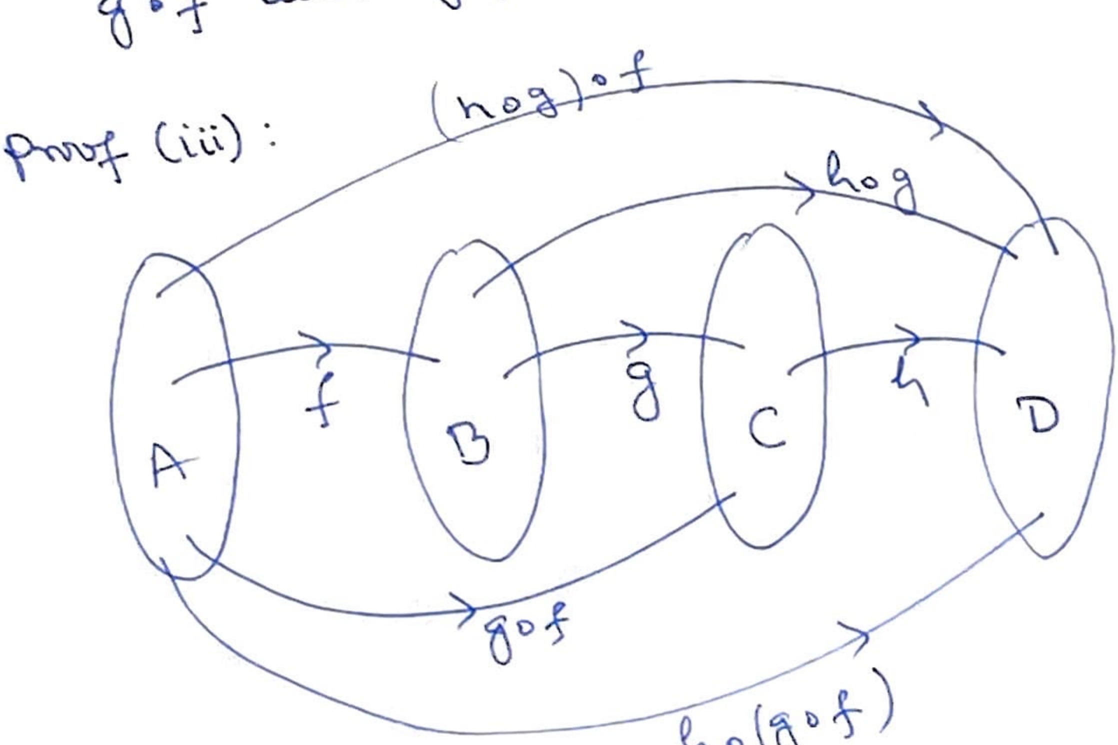
Proof (ii):

For $g \circ f : A \to C$, let $z \in C$.

Since g is onto, there exits $y \in B$ with $g(y) = z$. with f onto, there exits $x \in A$ with $f(x) = y$. Hence $z = g(y) = g(f(x)) = (g \circ f)(x)$,

so the range of $g \circ f = C$ = Co-domain of $g \circ f$ and $g \circ f$ is onto. — 2 Marks

Proof (iii):



$\therefore (h \circ g) \circ f = h \circ (g \circ f)$ is a fn from A to D.

— 2 Marks

9

6.b) i) $|A| = 5$, $|B| = 7$

$|B|^{|A|} = 7^5$ —— 1 Mark

$= 16,807$ functions from A to B

iii) $P(|B|, |A|)$

$= (7, 5) = \frac{7!}{(7-5)!} = \frac{7!}{2!}$ —— 1 Mark

$= 2520$ one to one functions from A to B

iii) $3^2 \cdot 4^3 = 576$ functions. —— 2 Marks

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Academic year 2023-2024 (Even Sem)
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COMPUTER SCIENCE & ENGINEERING

Date	August 2024	Maximum Marks	60
Course Code	CS241AT	Duration	120 Min
Sem-IV	Improvement Test	Staff: HKK/ASP/SMS/SGR/MNV	

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS

(Common to CSE, ISE & AIML)

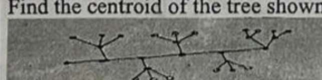
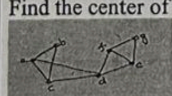
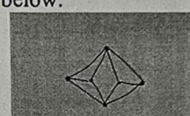
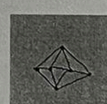
	PART-A	Marks	BT	CO
1.1	Let G be the set of real numbers not containing -1 and * be the binary operation defined by a*b=a+b+ab. What is the inverse of any number a ∈ G.	1	1	1
1.2	If the binary operation * is associative, then complete the following table. (table below)	1	1	1
1.3	Let G= $(Z_{12}, +)$ and H= {[0], [4], [8]}. What is the partition of G induced by the subgroup H.	1	3	2
1.4	A binary symmetric channel has probability p=0.05. What is the probability of sending the code word 110101101 and making at most 2 errors in the transmission?	1	2	2
1.5	Let E: $Z_2^3 \rightarrow Z_2^9$ be the encoding function for (9, 3) triple repetition code and D: $Z_2^9 \rightarrow Z_2^3$ is the corresponding decoding function. Find three different received words r for which D(r) =000.	1	2	1
1.6	Let G be the Peterson graph. Find $\chi(G)$.	1	2	2
1.7	What is the value of $\chi'(G)$ where G is $K_{3,2}$.?	1	2	3
1.8	If 5 colors are available, how many proper colorings are possible to color the graph $K_{3,3}$.	1	3	2
1.9	Find the centroid of the tree shown below. 	1	2	2
1.10	Find the center of the graph shown below. 	1	2	2

Table for 1.2:

*	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	d	c	d
d			c	d

	PART-B			
~~2a.~~	In a group (G, *), prove that $(a * b)^{-1} = b^{-1} * a^{-1}$ for all a, b $\in$ G.	05	3	2
2b.	Show that $(Z_{12}, +)$ is a cyclic group and find all its generators.	05	4	3
3a.	Let G be a group and let a be any fixed element of G. Show that the function f: G→G defined by $f(x) = axa^{-1}$, for x $\in$ G, is an isomorphism.	05	3	2
3b.	Let E: W□C be an encoding function with the set of messages $W \subseteq Z_2^m$ and the set of code words $E(W) = C \subseteq Z_2^n$, where m<n. For $k \in Z^+$, we can detect transmission errors of weight $\leq$ k iff the minimum distance between code words is at least k+1. Prove this.	05	2	3
~~4a.~~	Define the encoding function $E: Z_2^3 \rightarrow Z_2^6$ by means of the parity check matrix $H = \begin{pmatrix} 1&0&1&1&0&0 \\ 1&1&0&0&1&0 \\ 1&0&1&0&0&1 \end{pmatrix}$ ~~i.~~ Determine all code words. ~~ii.~~ What is the error detection and correction capability? ①,⓪ ~~iii.~~ Decode the received words 000011, 111100. 011, 100	06	2	2
4b.	i. If $x \in Z_2^{10}$, determine $\lvert S(x, 1)\rvert$, $\lvert S(x, 2)\rvert$, $\lvert S(x, 3)\rvert$. ii. For n, k $\in Z^+$ with $1 \leq k \leq n$, if $x \in Z_2^n$, what is $\lvert S(x, k)\rvert$?	04	3	2
5a.	Find $P(G, \lambda)$ for the graph shown below. 	06	3	4
~~5b.~~	Give an example of a connected graph that has ~~i.~~ Neither an Euler circuit nor a Hamilton cycle. ~~ii.~~ An Euler circuit but no Hamilton cycle. ~~iii.~~ No Euler circuit but has Hamilton cycle. ~~iv.~~ Both Euler circuit and a Hamilton cycle.	04	1	1
~~6a.~~	Prove that in every tree $\lvert V\rvert = \lvert E\rvert + 1$.	05	2	3
6b.	By applying the decomposition theorem, find the number of spanning trees for the graph shown below. 	05	3	4

BT-Blooms Taxonomy, CO-Course Outcomes, M-Marks

Marks Distribution	Particulars	CO1	CO2	CO3	CO4	CO5	L1	L2	L3	L4	L5	L6
	Max Marks	7	26	16	11	-	6	22	27	5	-	-

Dept. of C.S.E

Subject: DMS

Sub. Code: CS24IAT Improvement CIE

Date: Aug-2024

Scheme and Solutions

Part-A

1.1 $-a/(1+a)$ —— 1

1.2 $d*a = d$, $d*b = c$ —— 1

1.3 $[0]+H = \{0,4,8\}$
$[1]+H = \{1,5,9\}$
$[2]+H = \{2,6,10\}$
$[3]+H = \{3,7,11\}$
$\therefore$ partition of $G = \{\{0,4,8\}, \{1,5,9\}, \{2,6,10\}, \{3,7,11\}\}$ —— 1

—— 1

1.4 $(0.95)^9 + \binom{9}{2}(0.05)(0.95)^7$

1.5 000000000, 000 000 001, 100 000 000, 000 100 000, 000 010 000 Etc any three words —— 1

—— 1

1.6 $\chi(G) = 4$ —— 1

1.7 $\chi'(K_{3,2}) = 4$

1.8 1440 proper colorings are possible —— 1

1.9 Centroid $= \{12\}$ i-e the node with weight $= 12$. —— 1

1.10 Center $= \{d\}$, the radius of G is 3 —— 1

Part - B

2.a) **Proof**: Let $a, b \in G$, then $a^{-1}, b^{-1}, a*b, b^{-1}*a^{-1}$ all belogs to G. Let us consider

$$(b^{-1}*a^{-1})*(a*b)$$

Since $*$ is associative

$$(b^{-1}*a^{-1})*(a*b) = [b^{-1}*(a^{-1}*a)]*b$$ —— 2

$$= (b^{-1}*e)*b$$

$$= b^{-1}*b = e$$

Next, consider $(a*b)\circ(b^{-1}*a^{-1})$, since $*$ is associative

$$(a*b)*(b^{-1}*a^{-1}) = a*(b*b^{-1})*a^{-1}$$ —— 2

$$= (a*e)*a^{-1}$$

$$= a*a^{-1} = e$$

Therefore, $(b^{-1}*a^{-1})*(a*b) = (a*b)*(b^{-1}*a^{-1}) = e$ —— 1

This proves that $(b^{-1}*a^{-1})$ is the unique inverse of $a*b$.

2.b) To construct the multiplication table for $(Z_{12}, +)$ —— 1

To show $(Z_{12}, +)$ is a cyclic group. —— 2

To list all the generators —— 2

$\langle 1\rangle, \langle 5\rangle, \langle 7\rangle, \langle 11\rangle$ are the generators.

3.a) Let $x, y \in G$,

$$f(xy) = axya^{-1} = axa^{-1}aya^{-1} = f(x)f(y)$$ —— 1

$\therefore f$ is homomorphism.

Suppose $x \in G$, then

$$f(a^{-1}xa) = aa^{-1}xaa^{-1} = x,$$ so f is onto. —— 2

Suppose $f(x) = f(y) \Rightarrow axa^{-1} = aya^{-1}$

$$\Rightarrow a^{-1}(axa^{-1})a = a^{-1}(aya^{-1})a$$

$\therefore x = y$. So f is one to one i.e isomorphism —— 2

3.b) **Proof**: The set C is known to both sender and receiver, so if $w \in W$ is the message and $c = E(w)$ is transmitted. Let $c \neq T(c) = r$.
If the minimum distance b/w codewords is at least $k+1$, then the transmission of c can result in as many as k errors and r will not be listed in C. Hence we can detect all errors e where $wt(e) \leq k$. Conversely, let c_1, c_2 are code words with $d(c_1, c_2) < k+1$. Then $c_2 = c_1 + e$ where $wt(e) \leq k$. If we send c_1 and $T(c_1) = c_2$, then we feel that c_2 had been sent hence failing to detect an errors of weight $\leq k$. — 5

4.a) Given

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore G = [I_3 | A] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$ — 1

i) Code words are

$[000][G] = [000\ 000]$, $[001][G] = [001\ 101]$,
$[010][G] = [010\ 010]$, $[011][G] = [011\ 111]$, — 2
$[100][G] = [100\ 111]$, $[101][G] = [101\ 010]$,
$[110][G] = [110\ 101]$, $[111][G] = 111\ 000]$.

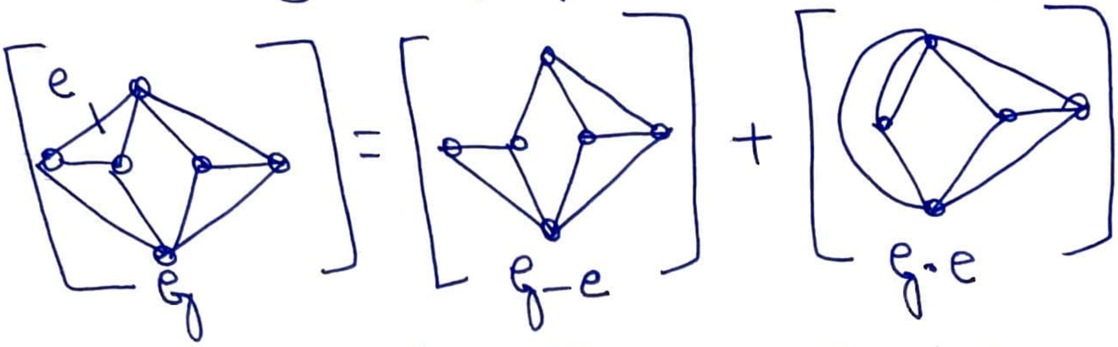
ii) Minimum distance between the codewords is 2
$\therefore$ All errors of single bit are detectable — 1
And No errors can be corrected. — 2

iii) $[H]\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $[H]\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

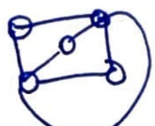
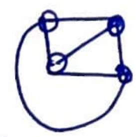
4.b) i) $|S(x,1)| = 11$, $|S(x,2)| = 56$, $|S(x,3)| = 176$ $\rightarrow 1\times 3 = 3$

ii) $|S(x,k)| = \sum_{i=0}^{k} \binom{n}{i}$ — 1

5.a) To show the working of decomposition theorem on the given graph — 4



$P(G-e,\lambda) = 2\lambda^6 - 3\lambda^5 - 2\lambda^4 + 2\lambda^3 + 16\lambda^2 - 15\lambda$ —
$P(G.e,\lambda) = \lambda^6 - 2\lambda^5 + 2\lambda^4 - 5\lambda^3 + 14\lambda^2 - 10\lambda$. — } 2
$P(G,\lambda) = 3\lambda^6 - 5\lambda^5 - 3\lambda^3 + 30\lambda^2 - 25\lambda$ —

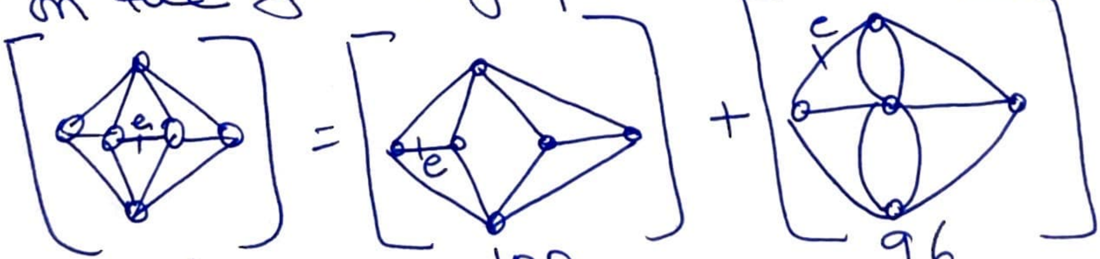
5.b) (i)  ii)  iii)  iv) 

$1\times 4 = 4$

OR Any example graphs

6.a) Using mathematical induction on $|E|$ this is to be proved. — 1
For Basis — 1
for Hypothesis — 3
for Inductive proof

6.b) To show the working of decomposition theorem on the given graph — 3



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USN 1RV22CD059

RV COLLEGE OF ENGINEERING®

(An Autonomous Institution Affiliated to VTU)

IV Semester B. E. Regular Examinations SEP/OCT – 2024

Computer Science and Engineering

DISCRETE MATHEMATICAL STRUCTURES AND COMBINATORICS

Time: 03 Hours ***Maximum Marks: 100***

Instructions to candidates:

1. Answer all questions from Part A. Part A questions should be answered in first three pages of the answer book only.
2. Answer FIVE full questions from Part B. In Part B question number 2 is compulsory. Answer any one full question from 3 and 4, 5 and 6, 7 and 8, 9 and 10.
3. Use of statistical tables and formula handbook permitted.

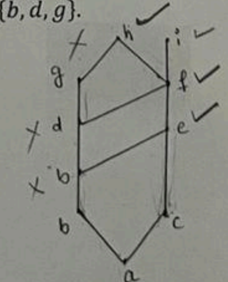
PART-A

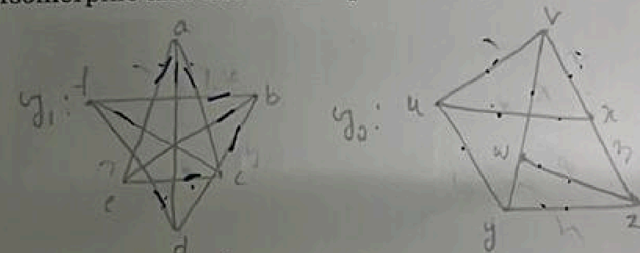
			M	BT	CO
1	1.1	Fins the number of ways that the alphabets A, B, C, D, E, F, G are arranged such that A is not first position, B is not in second position, ... G is not in seventh position.	02	2	2
	1.2	Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}, n \geq 2$ subject to the initial conditions $a_1 = 1, a_2 = 3$.	02	2	2
	1.3	Simply using law of logic: $[(p \vee q)\, n(p \vee \sim q)] \vee q \Leftrightarrow p \vee q$	02	1	1
	1.4	Write down the converse, inverse of the following compound proposition. "A person is successful in life if he puts sincere efforts"	02	2	2
	1.5	Let $f: z \to z$ and $g: z \to z$, given by $f(x) = x - 1$ $g(x) = 2x$ find $f \circ g$ and $g \circ f$.	02	2	2
	1.6	If $R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)\}$ is defined on the set $A = \{1,2,3,4\}$ determine the partition induced.	02	2	2
	1.7	Let $\boldsymbol{G} = \{q \in \theta \mid q \neq -1\}$. Identify the identify element of $\{G, 0\}$ where $xoy = x + y + xy$ for all $x, y \in G$.	02	2	2
	1.8	What is the hamming distance between the codes '11001011' and '10000111'.	02	1	1
	1.9	Let G be a simple graph of order n. If the size of G is 56 and size of $\bar{G}$ is 80, what is n?	02	2	2
	1.10	Given $V = \{(1,2,3,4,5,6)\}$ and $E = \{12, 13, 23, 35, 61, 66\}$ draw undirected and directed graph $G = (V, E)$. Also write down the order and size of G.	02	2	2

PART-B

2	a	Find the number of proper divisors of 38808.	04	2	2
	b	A person inverse some amount at the rate of 11% annual compound interest. Determine the period for principal amount to get doubled.	04	2	3
	c	Find a generating function for the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 0$ for $n \geq 0, a_0 = 5, a_1 = 12$.	04	2	2
	d	In how many ways can 10 identical dimes be distributed among 5 children if i) There are no restrictions ii) Each child gets at least one dime.	04		

21

3	a	Let p, q and r be the propositions $P: I$ Study; $q: I$ will fail in the examination $r: I$ watch TV in the evening Express each of these proposition as an English sentence (i) $p \rightarrow nq$ (ii) $q \rightarrow r$ (iii) $(p \rightarrow \sim r) \cup (q \rightarrow \sim r)$ (iv) $\sim p \rightarrow (q \cup r)$	04	1	1
	b	Prove that $[(\neg p \vee q) \wedge \{p \wedge (p \wedge q)\}] \Leftrightarrow p \wedge q$ using laws of logic.	06	2	2
	c	Write the following proposition in the symbolic form and find its negation. "If all triangles are right angles then no triangle is equiangular"	06	3	2
		OR			
4	a	For the following statement, state the converse inverse and contrapositive. The universe consist of all integers "If m divides n and n divides p, then m divides p"	04	2	2
	b	Prove the validity of the following argument $p \rightarrow (q \wedge g)$ $r \rightarrow s$ $\underline{\sim(g \wedge s)}$ $\therefore \sim p$	06	2	2
	c	Define open statement and find whether the following variable is valid. No engineering students of 1st or 2nd sem studies logic. Anil is an engineering student who studies logic. $\therefore$ Anil is not in second semester.	06	3	3
5	a	If $f: A \rightarrow B, g: B \rightarrow C, h: C \rightarrow D$ then show that $(h \circ g) o f = h \circ (g \circ f)$.	04	2	2
	b	If $A = \{1,2,3,4\}$ and R, S are relations on A defined by $R = \{(1,2)(1,3)(2,4)(4,4)\}$ $S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,3)(2,4)\}$. Find $R \circ S,\ S \circ R,\ R^2 and\ S^2$.	05	2	2
	c	Draw the Hasse diagram for all positive integer divisors of 72. Also write R.	07	3	3
		OR			
6	a	Find the lower and upper bounds of the subsets $\{a, b, c\}; \{i, h\}$ and $\{a, c, d, f\}$ in the poset with Hasse diagram shown in Fig 6a. Also find the glb and lub of $\{b, d, g\}$.  Fig 6a	08	3	2
	b	Let $f: R \rightarrow R$ be define by $f(x) = \begin{cases} 3x - 5 & if\ x > 0 \\ -3x + 1 & if\ x \leq 0 \end{cases}$. Find $f^{-1}(1)$ and $f^{-1}(3)$.	04	3	3
	c	If R is a relation on $A = \{1,2,3,4\}$ define by xRy if x divides y. Prove that (A, R) is a poset.	04	2	2
7	a	If $\circ$ is an operation on Z define by $x \circ y = x + y + 1$. Prove that $(G.\circ)$ is an Abelian group.	05	3	3
	b	Prove that i) Identity element in a group is unique ii) Inverse of each element in a group is unique	04	2	2

	c	The encoding function $E = Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix $G = \left[\begin{array}{cc\|ccc} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right]$ i) Find associated parity check matrix H ii) Determine all code words iii) Find decoded word for received msq $[1 \quad 1 \quad 1 \quad 0 \quad 1]$ iv) What is error detection and correction capability.	07	4	4
		OR			
8	a	If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ Find $\alpha.\beta$ *and* α^{-1}.	04	2	2
	b	Define the encoding function $E: Z_2^3 \rightarrow Z_2^6$ by means of the parity – check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ Determine all code words.	05	3	3
	c	State and prove Lagrange's theorem. Find the right cosets of $H = \{1, -1\}$ in multiplicative group of fourth root of unity.	07	2	3
9	a	Define Isomorphic and show that G_1 is isomorphic to G_2. 	08	3	3
	b	Explain the Konigsberg – Bridge problem.	08	4	4
		OR			
10	a	If a tree has four vertices of degree 2, one vertex of degree 3, two of degree 4 and one of degree 5, how many pendant vertices does it have?	08	2	2
	b	Prove the following for the graph $G = (V, E)$ i) $\sum_{v \epsilon V} \deg(V) = 2\|E\|$ ii) The number of vertices of odd degree must be even.	08	2	2