Glide in wind shear

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Contents

Chapter 1

Problem statement

Consider an aircraft that is gliding (you may take climb too) in three different scenarios:

1. No wind condition. Determine the C_L required as function of time or height for aircraft to descent along a straight line, in this case same as constant flight path (descend) angle .

2. For a wind shear model Uw = 0.1 hm/s, h in m, with the above determined control input calculate the trajectory as well as flight path angle variation.

3. Determine the C_L for above wind field condition for constant flight path angle corresponding to that in part 1. Compare the C_L values as well as trajectory with that obtained in part 1.

4. Determine the control input C_L required for the aircraft to follow same trajectory in wind shear as the case of no wind shear.

Flight Parameters:
V_0 = 30 m/s
x_0 = 0
z_0 = 1000 m
AR = 10
K = $1/\pi * AR$
C_{D0} = 0.1
ρ = 1.225 kg/m^3
g = 9.81 m/s^2
S = 0.6 m^2

Chapter 2

Study of Flight in Wind Shear

2.1 Motivation

Glide in wind shear is a very critical study towards Dynamic soaring. It serves as a base to understand the concepts behind the motion of an object through the wind and how wind can affect it's motion. With the series of problems we go from very basic level of no wind case to study Differential flatness equations to observe the behaviour of the different control inputs on the flat variables.

For our analysis we are considering "Descend/Glide". In all the cases we will be seeing how the trajectory takes the shape with different conditions of wind, varying Cl, varying flight path angle γ, etc

2.2 No wind case

2.2.1 Theory

In this case, there is no wind present. We need to determine the trajectory of the non-powered aircraft. Considering aircraft to be point object, we can draw its free body diagram and see the different forces acting on it and also write the kinematic equations.

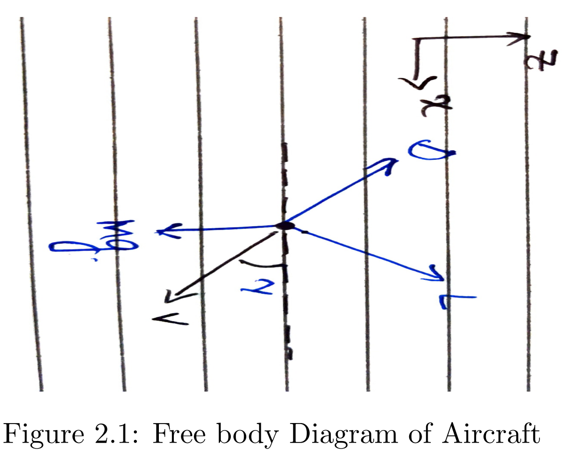


Figure 2.1: Free body Diagram of Aircraft

Note that the flight path angle is constant and $\gamma < 0$ for descend, the velocity in the x-direction and z-direction can be written as

$$\dot{x} = V cos(\gamma) \tag{2.1}$$

$$\dot{z} = V sin(\gamma) \tag{2.2}$$

From the force balance equation

$$L = mgcos(\gamma) \tag{2.3}$$

$$m\dot{v} = -D - mgsin(\gamma) \tag{2.4}$$

where the drag force D is given as

$$D = \frac{1}{2}\rho v^2 S C_d \tag{2.5}$$

Therefore the differential equation for $\dot{v}$ is given as

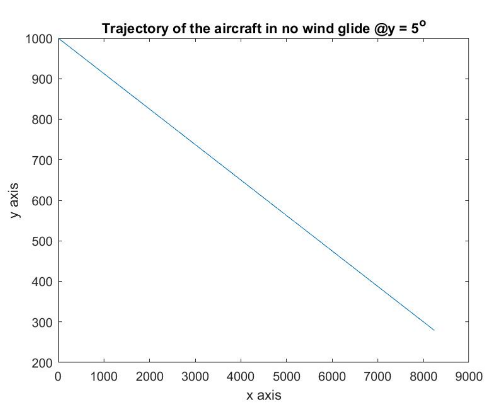
$$\dot{v} = -gsin(\gamma) - \frac{\rho v^2 S C_d}{2m} \tag{2.6}$$

Considering drag polar $C_d = C_{d_0} + kC_l^2$ where C_{d_0} is the profile drag and kC_l^2 is the induced drag. The C_l can be obtained from eqn2.3 as

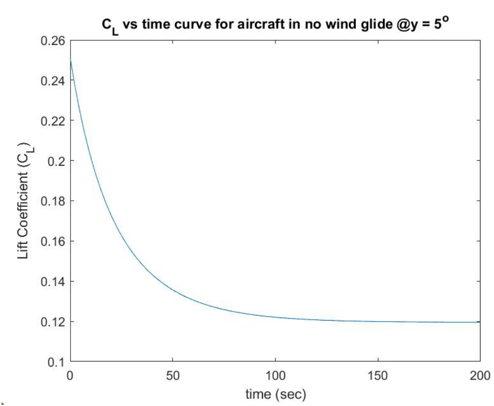
$$Cl = \frac{2mgcos(\gamma)}{\rho v^2 S} \tag{2.7}$$

2.2.2 Analysis

Clearly the three differential equation 2.1, 2.2 and 2.6 are coupled and can be solved numerically to obtain the variations. We are using ode45 function from MATLAB which is based out of Runge-kutta method to solve these differential equations. from these we get the data of variation of horizontal distance, vertical distance and velocity. By plotting the vertical distance against horizontal distance we get the Trajectory followed by the aircraft which can be seen in following figures.



(a) Trajectory



(b) Variation of C_L with time

From the eqn 2.7 we can find the set of data for C_l for various values of velocities and plot it against the time as shown in fig above.

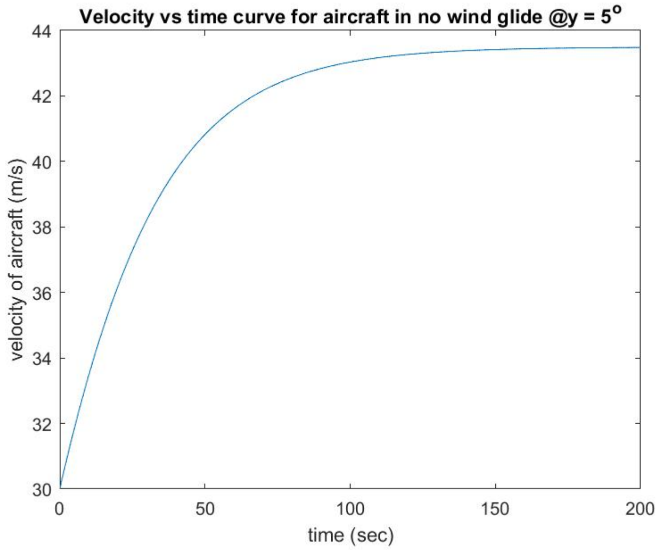


Figure 2.2: Variation of Velocity of Aircraft with time

2.3 Linear wind with almost similar C_l as that of the No wind case

2.3.1 Theory

Now in this section we consider the linear wind profile along x-direction varying along z direction given as

$$u_w = 0.1z \tag{2.8}$$

we have to find the gamma variation for the same control input C_l as that of the no wind case. So for that we need to get the C_l data from the previous case and plug it in the equations governing gamma. So the next question is what are those equations. And the answer is we have a set of differential equations which are obtained from the force balance and the kinematic equations in the wind relative frame(we can do it in any frame, but doing it in wind frame gives us advantage in different things such as reducing the complexity of the equations and also aerodynamic forces considers relative velocity and not the ground velocity). Now by updating the equations from first part, the governing differential equations needed to solve for the trajectory and the gamma variation are as follows:

$$\dot{x} = Vcos(\gamma) + u_w \tag{2.9}$$

$$\dot{z} = Vsin(\gamma) \tag{2.10}$$

$$\dot{v} = -gsin(\gamma) - \dot{u_w}cos(\gamma) - D \tag{2.11}$$

where again $D = \frac{1}{2}\rho v^2 SC_d$ and considering parabolic drag polar $C_d = C_{d_0} + kC_l^2$ the equation 2.11 can be modified as

$$\dot{v} = -gsin(\gamma) - \dot{u_w}cos(\gamma) - \frac{1}{2}\rho v^2 SC_{d_0} - \frac{1}{2}\rho v^2 SkC_l^2 \tag{2.12}$$

And the C_l is obtained from the data from no wind case. Now the governing differential equation for γ is as follows:

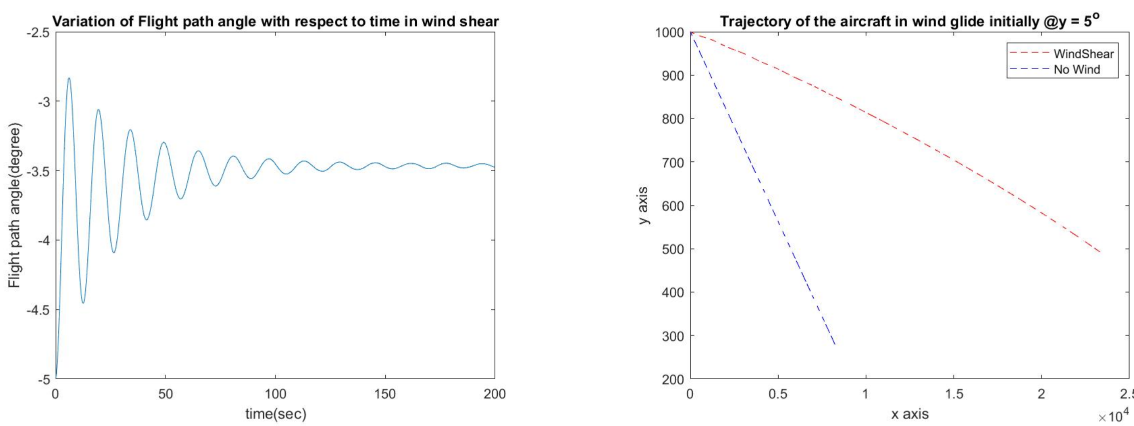
$$\dot{\gamma} = \frac{-gcos(\gamma)}{v} + \frac{\dot{u_w} sin(\gamma)}{v} - \frac{L}{mv} \tag{2.13}$$

Where the lift force L is given as $L = \frac{1}{2}\rho v^2 SC_l$. Substituting it in equation 2.13 we get the updated equation for $\dot{\gamma}$ as

$$\dot{\gamma} = \frac{-gcos(\gamma)}{v} + \frac{\dot{u_w} sin(\gamma)}{v} - \frac{\rho v SC_l}{2m} \tag{2.14}$$

2.3.2 Analysis

The coupled differential equations given by equations 2.9, 2.10, 2.12 and 2.14 are the governing differential equations and by solving them we can get the dataset for horizontal distance, vertical distance, velocity and γ. Again by using ode45 we have plotted the data and obtained the γ variation and trajectory of aircraft which can be seen in figure below (Here we have taken initial condition of aircraft gliding at $\gamma_0 = -5°$) .

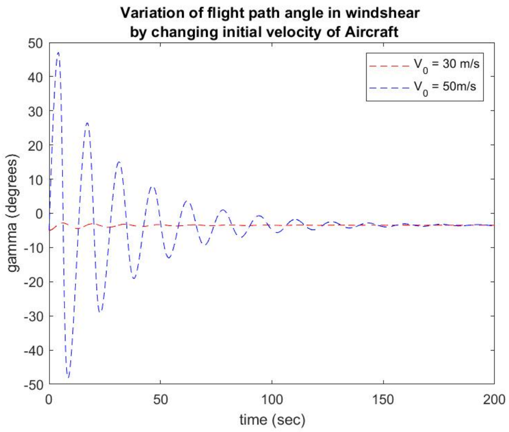


(a) (b)

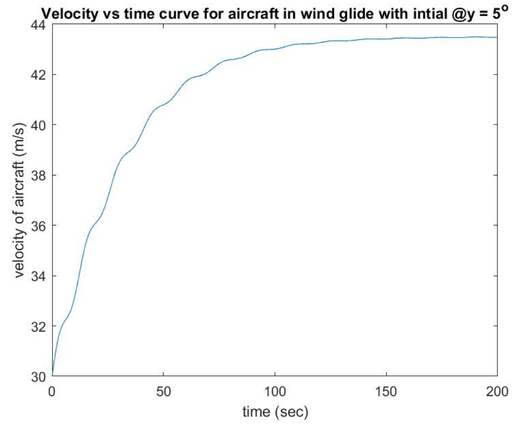
Inference on γ: The perturbations in γ suggests that the aircraft is not under equilibrium. We can observe that due to Wind shear effects γ is undergoes damped oscillations which look to settle at about $\gamma = -3.5°$ at around $t = 200$sec.

Inference on Trajectory: In case of Wind Shear, the Aircraft travels more distance Horizontally for the same Altitude descend in comparison to No wind Case.

To get an idea of how γ variations change based on initial flight velocity, we plot variation of γ with respect to time for 2 given initial velocities ($V_0 = 30m/s$ and $V_0 = 50m/s$) and found the behaviour as shown in figure below. The velocity variation with time of the Aircraft in Wind Shear condition is also shown below.



(a)



(b)

Inference: In Wind Shear condition, the amplitude of perturbations in γ increase with increase in initial velocity of the aircraft. With same $\gamma_0 = -5°$, if the aircraft has 50m/s initial velocity it could undergo violent oscillations of about 50°amplitude while for lower velocity the amplitude decreases.

2.4 Linear wind with γ held constant

2.4.1 Theory

In this section the γ is held constant same as that of the no wind case and we have to find the cl variation which can actually satisfy this condition. So the equations will pretty much be the same with key component as $\dot{\gamma} = 0$. Therefore from equation 2.14

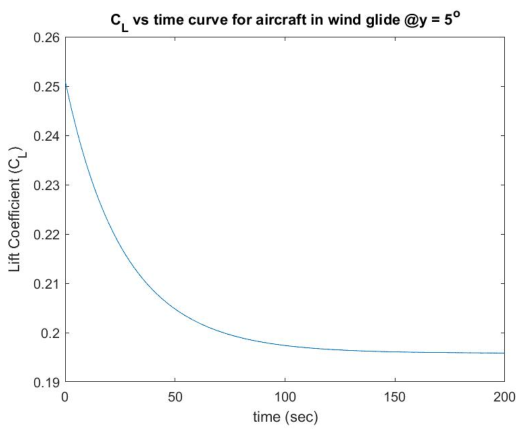
$$0 = \frac{-gcos(\gamma)}{v} + \frac{\dot{u_w}sin(\gamma)}{v} - \frac{\rho v S C_l}{2m} \tag{2.15}$$

which will give

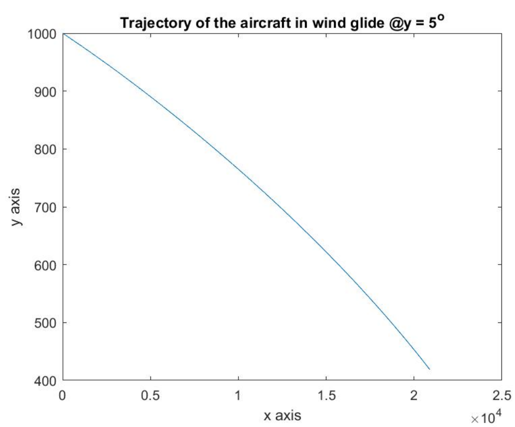
$$C_l = \frac{-2mgcos(\gamma)}{\rho v^2 S} + \frac{2m\dot{u_w}sin(\gamma)}{\rho v^2 S} \tag{2.16}$$

2.4.2 Analysis

By solving equations 2.9, 2.10 and 2.12 through ode45 we can get the data for horizontal distance, height and velocity similar to what we obtained in previous cases. The Trajectory can be obtained by plotting height against horizontal distance. from eqn 2.16 we can find the dataset for C_l which we can plot against time as seen in figure below.

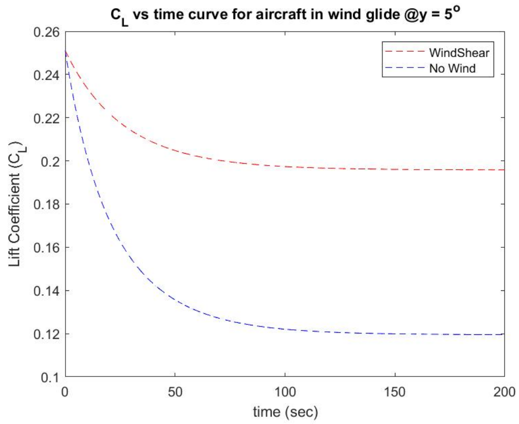


(a)

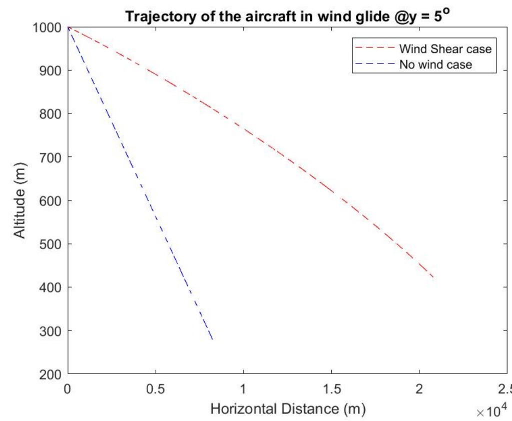


(b)

We can now compare the C_L variation of case3 (wind Shear with fixed γ condition) to case1 (No wind with fixed γ condition. Also, we can see the comparison of trajectories given on the next page.



(a)



(b)

Inferences from the plots above:

From the C_L variation with time plots, we can observe that for No wind case the value of C_L decreases steeply upto around 0.12 when flight path angle was kept constant. While in case of wind, the value of C_L decreases slowly and settles at around 0.2 for constant γ flight.

While it is evident from the trajectories that an Aircraft flying with Constant γ can fly larger distance (horizontal) for same altitude descend when moving in Wind in comparison to No wind Flight.

2.5 Linear wind with trajectory coordinates same as that of no wind case

2.5.1 Theory

In this case we have to find the control inputs from the given trajectory (x and z coordinates same as that of the no wind case). It could be done with the differential flatness equations but here we are doing with our conventional governing differential equations. So from equation 2.1 and 2.2 we can get the data of horizontal distance x and vertical distance z. From this data we can find their derivative by using forward difference or backward difference method (we have used forward difference over here). Similarly, we can get $\dot{\gamma}$ values. Important point to note over here is, in a first look γ would seem to be constant as we are assuming the coordinates of the profile with respect to no wind case. But we have to be careful over here as flight path angle considers the quantities(velocities) in relative frame and not the ground frame. Therefore

$$\gamma = atan(\frac{\dot{z}}{\dot{x} - u_w}) \tag{2.17}$$

The velocity can be found as follows:

$$v = \frac{\dot{x} - u_w}{cos(\gamma)} \tag{2.18}$$

here we can see that it is an iterative process tosolve for v and γ with known initial values of both of them. $\dot{\gamma}$ can be obtained through forward difference method. Now that we have $\dot{\gamma}$ and we have eqn 2.14, we can find C_l and it can be given as

$$C_l = \frac{2m\dot{\gamma}}{\rho v S} - \frac{2mgcos(\gamma)}{\rho v^2 S} + \frac{2m\dot{u_w}sin(\gamma)}{\rho v^2 S} \tag{2.19}$$

2.5.2 Analysis

Note that here we have used forward differences to find the derivatives which cause slight deviation from the actual plots because of the error and error could be reduced if take smaller intervals or higher order differentials. Obviously the trajectory will be same as that of the no wind case. Eqn 2.18 will give us the control input C_l which we can plot against time as seen in the figure below.Also, we can plot the C_L from No wind case alongside it to compare.

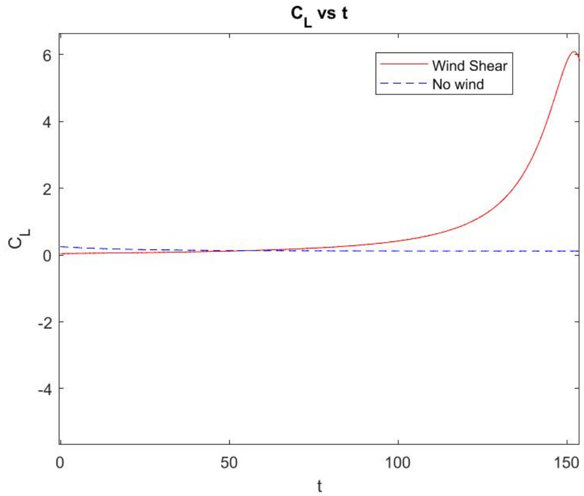
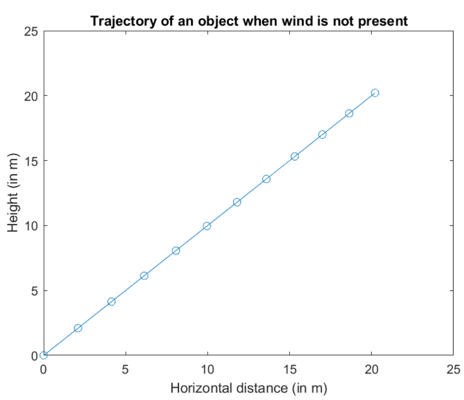


Figure 2.3: Comparison of C_L for Case1 and Case4

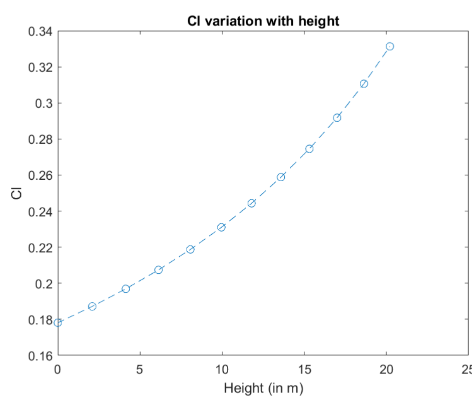
2.6 Observations and Review of Climb

We Also attempted all the above exercises with aircraft in climb(The analysis discussed above was for descend case but have did the analysis for climb case as well) and found the Results as Follows:

Case1: No Wind

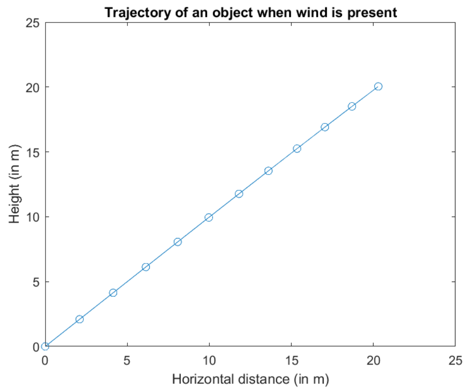


(a) Trajectory

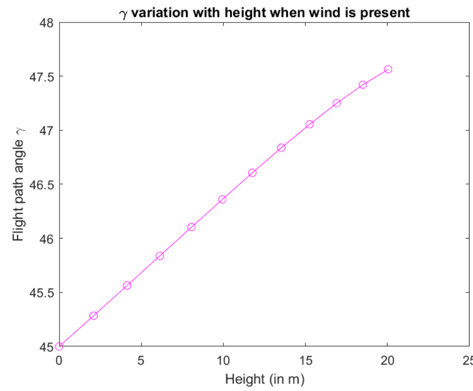


(b) Variation of C_L with time

Case2: Wind with same C_L input

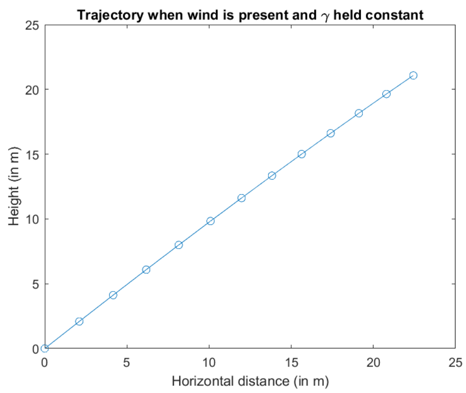


(a) Trajectory

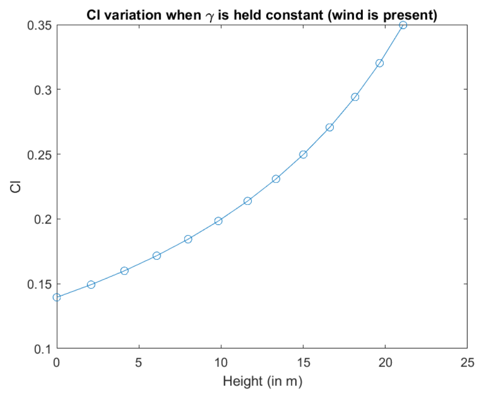


(b) Variation of γ with time

Case3: Wind with constant γ input

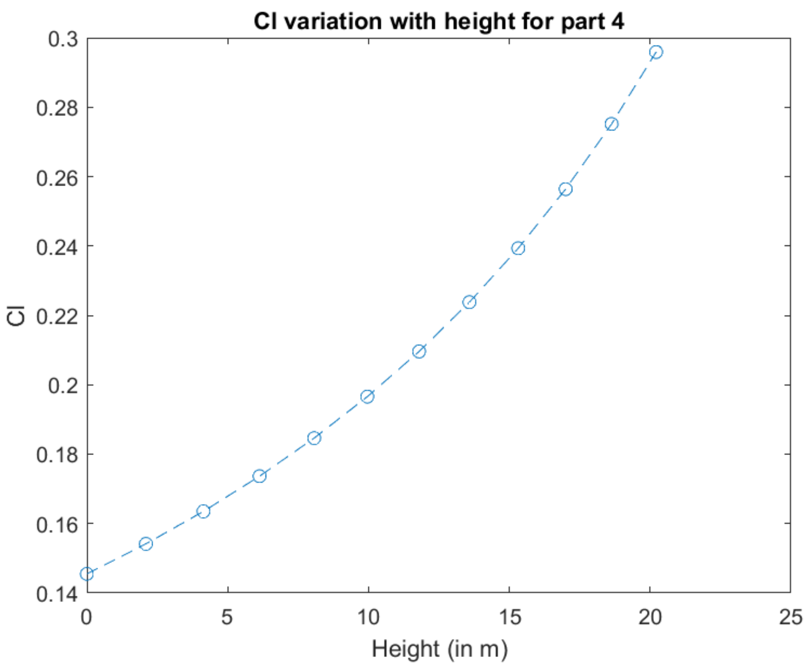


(a) Trajectory



(b) Variation of C_L with time

Case4: Wind with with same trajectory as in no wind case



(a) Variation of C_L Variation with time

Appendix A

Codes used for glide

A.1 Case-1: No Wind

```
%%No wind Condition
%% Aircraft is gliding gamma = 5deg
clc;clear;close all;
%Flight parameters
AR = 10; %Aspect Ratio
K = 1/(pi*AR);
Cd_0 = 0.01;
rho = 1.225; %density
g =9.81; % acceleration due to gravity
m = 8.5; % mass of UAV
w = m*g; % Weight of UAV
gamma = -pi/36; %flight path angle
S = 0.6; %Reference Area
v0 = 30;%initial velocity
x0 = 0;
y0 = 1000; %initial altitude
tspan = linspace(0,200,1000);

%solving Differential Equation
[tsol,Vsol] = ode45(@(t,v)noWind(t,v),tspan,[x0,y0,v0]);


Cl = zeros(size(Vsol(:,3)));
for i= 1:size(Cl)
    Cl(i) = 2*m*g*cos(gamma)/(rho*Vsol(i,3).^2*S);
end


%differential equation function handle
function dYdt = noWind(t,Y)
AR = 10; %Aspect Ratio
K = 1/(pi*AR);
Cd_0 = 0.01;
rho = 1.225; %density
g =9.81; % acceleration due to gravity
```

```
m = 8.5; % mass of UAV
w = m*g; % Weight of UAV
S = 0.6; %Reference Area
gamma = -pi/36; %flight path angle
x = Y(1);
y = Y(2);
v = Y(3);
dxdt = Y(3)*cos(gamma);
dydt = Y(3)*sin(gamma);
dvdt = -(0.5*rho*S*Cd_0.*v^2)/m - (2*K*m*g^2*cos(gamma)^2)/(rho.*v^2*S) ...
    -g*sin(gamma);
dYdt = [dxdt;
    dydt;
    dvdt];
% CL = (w*cos(gamma))/(0.5*S*rho.*V^2);
% CD = Cd_0+K.*CL^2;
% % D = 0.5*rho*S*CD.*V^2;
% dVdt = -D-w*sin(gamma);

end
```

A.2 Case-2

```
% wind U_w = 0.1h m/s Condition
% Aircraft is initially GLIDING gamma = 5deg
% initial parameters
clc;clear;close all;
v0 = 30; %initial Velocity
x0 = 0;
y0 = 1000; %initial Altitude
A = importdata("Cl.mat");
Vsol_nw = A.Vsol;
x_nw = Vsol_nw(:,1);
y_nw = Vsol_nw(:,2);
tsol1 = A.tsol;
Cl = A.Cl;
gamma0 = -pi/36; %initial flight path angle= 5 degree

tspan = linspace(0,200,1000);

[tsol,Vsol] = ode45(@(t,v)

Wind2(t,v,tsol1,Cl),tspan,[x0,y0,v0,gamma0]);




%differential equation function handle
function dYdt = Wind2(t,Y,tsol,Cl)
AR = 10; %Aspect Ratio
K = 1/(pi*AR);
Cd_0 = 0.01;
```

```
rho = 1.225; %density
g =9.81; % acceleration due to gravity
m = 8.5; % mass of UAV
w = m*g; % Weight of UAV
S = 0.6; %Reference Area
x = Y(1);
y = Y(2);
v = Y(3);
gamma = Y(4);
C = interp1(tsol,Cl,t);
dxdt = Y(3)*cos(Y(4))+0.1*Y(2);
dydt = Y(3)*sin(Y(4));
dvdt = -(0.5*rho*S*Cd_0*Y(3).^2)./m - 0.1*Y(3).*sin(Y(4)).*cos(Y(4)) ...
    -(0.5*S*rho*K*Y(3).^2*C.^2)/(m) -g*sin(Y(4));
dgammadt = (-g*cos(Y(4))/Y(3)) + 0.1*dydt*sin(Y(4))/Y(3) + ...
    0.5*rho*Y(3).^2*S*C/(Y(3)*m);

dYdt = [dxdt;
    dydt;
    dvdt;
    dgammadt];
end
```

A.3 Case 3

```
%%wind Condition U_w = 0.1h m/s
%% Aircraft is gliding with constant gamma = 5deg
%%initial parameters
clc;clear;close all;
AR = 10; %Aspect Ratio
K = 1/(pi*AR);
Cd_0 = 0.01;
rho = 1.225; %density
g =9.81; % acceleration due to gravity
m = 8.5; % mass of UAV
w = m*g; % Weight of UAV
gamma = -pi/36; %flight path angle
S = 0.6; %Reference Area
v0 = 30;%initial Velocity
x0 = 0;
y0 = 1000;%initial Alitude
tspan = linspace(0,200,1000);
[tsol,Vsol] = ode45(@(t,v)Wind3(t,v),tspan,[x0,y0,v0]);
%import Data from noWind file

A = importdata("Cl.mat");
x_nw = A.Vsol(:,1);
y_nw = A.Vsol(:,2);
Cl_nw = A.Cl;
tsol_nw=A.tsol;


```

```

Cl = zeros(size(Vsol(:,3)));
for i= 1:size(Cl)
    Cl(i) = 2*m*g*cos(gamma)/(rho*Vsol(i,3).^2*S);
end

function dYdt = Wind3(t,Y)
AR = 10; %Aspect Ratio
K = 1/(pi*AR);
Cd_0 = 0.01;
rho = 1.225; %density
g =9.81; % acceleration due to gravity
m = 8.5; % mass of UAV
w = m*g; % Weight of UAV
S = 0.6; %Reference Area
gamma = -pi/36; %flight path angle
x = Y(1);
y = Y(2);
v = Y(3);

dxdt = Y(3)*cos(gamma)+ 0.1*Y(2);
dydt = Y(3)*sin(gamma);
dvdt = -(0.5*rho*S*Cd_0.*Y(3)^2)/m - (2*K*m*g^2*cos(gamma)^2)/(rho.*Y(3)^2*S) ...
    -g*sin(gamma)+ 0.1*dydt*cos(gamma);
dYdt = [dxdt;
    dydt;
    dvdt];
```

A.4 Case 4

```
% finding control input Cl such that the trajectory in absence of wind and
% wind shear are same.
clc;close all;clear;

%import Data from noWind file
A = importdata("Cl.mat");
Vsol_nw = A.Vsol;
x = Vsol_nw(:,1);
z = Vsol_nw(:,2);
tsol_nw=A.tsol;
Cl_nw = A.Cl;

%input data
AR = 10; %Aspect Ratio
K = 1/(pi*AR);
Cd_0 = 0.01;
rho = 1.225; %density
g =9.81; % acceleration due to gravity
m = 8.5; % mass of UAV
w = m*g; % Weight of UAV
S = 0.6; %Reference Area
%finding x_dot and z_dot using forward difference scheme
```

```
x_dot = zeros(size(x));
z_dot = zeros(size(z));
for i = 1:size(x) - 1
    x_dot(i)=(x(i+1)-x(i))./(tsol_nw(i+1)-tsol_nw(i));
    z_dot(i)=(z(i+1)-z(i))./(tsol_nw(i+1)-tsol_nw(i));
end
%finding gamma
gamma = atan(z_dot./(x_dot-0.1*z));

%finding Velocity
v = (x_dot - 0.1*z)./cos(gamma);
%using forward difference scheme find gamma_dot
gamma_dot =zeros(size(gamma));
for i = 1:size(gamma) - 1
    gamma_dot(i)=(gamma(i+1)-gamma(i))./(tsol_nw(i+1)-tsol_nw(i));
end

%finding Cl
Cl = zeros(size(tsol_nw));
for i = 1:size(Cl)
    Cl(i) = (2*m/(rho*v(i).^2*S)).*(v(i).*gamma_dot(i) + g*cos(gamma(i)) - ...
        0.1*z_dot(i).*sin(gamma(i)));
end


plot(tsol_nw, Cl, 'r');
hold on
plot(tsol_nw,Cl_nw,"b--")
xlabel('t');
ylabel('C_L');
legend("Wind Shear","No wind")
title('C_L vs t');
```