Question 6

Prove or disprove that (P → (Q ˅ R)) and ((P ˄ ¬R) → Q) are logically equivalent.

Truth table:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | Q | R | ¬R | (Q ˅ R) | (P ˄ ¬R) | (P → (Q ˅ R)) | ((P ˄ ¬R) → Q) |
| T | T | T | F | T | F | T | T |
| T | T | F | T | T | T | T | T |
| T | F | T | F | T | F | T | T |
| T | F | F | T | F | T | F | F |
| F | T | T | F | T | F | T | T |
| F | T | F | T | T | F | T | T |
| F | F | T | F | T | F | T | T |
| F | F | F | T | F | F | T | T |

Since both (P → (Q ˅ R)) and ((P ˄ ¬R) → Q) have the same truth values, they are logically equivalent.

Question 7

Let p be the proposition “k is an integer and 5k + 4 is odd”, and q be the proposition “k is odd”. Now suppose that p is true and ¬q is true. That is, that k is an integer and 5k + 4 is odd, and that k is even. If k is even, then it can be written as k = 2n, where n is some integer. This means that:

5k + 4 = 5(2n) + 4

= 10n + 4

= 2(5n + 2)

This makes 5k + 4 an even number, and causes a contradiction with proposition p, which states that it is odd. Therefore, p ˄ ¬q creates a contradictory statement p ˄ ¬p, which is impossible. Therefore, q must be true, and k must be odd. This completes our proof by contradiction.

Question 8

Let p be the proposition “x2(y2 – 2y) is odd” and q be the proposition “x and y are both odd”. Now suppose that ¬q is true, meaning that x is even and y is even. Then x and y can be rewritten as x = 2n and y = 2m where m and n are both integers. Then x2(y2 – 2y) can be rewritten as:

x2(y2 – 2y) = (2n)2((2m)2 – 2(2m))

= 16n2m2 – 16n2m

= 2(8n2m2 – 8n2m)

This means that x2(y2 – 2y) is even, which means p is false and ¬p is true. Therefore we can say that ¬q → ¬p is true, and by contraposition, p → q must be true, and that if x2(y2 – 2y) is odd, then x and y are both odd.

Question 9

1) )

2) )

Question 10

Prove that for any positive integer n,

BASE CASE: P(0). If n = 0, then

.

This is true.

INDUCTIVE STEP: Suppose P(k) is true. This means that for any k,

. We must prove that

P(k+1) can be written as

This means that for any positive integer n, is true. This completes our proof by induction.

Question 11

Prove that any postage of n cents(n ≥ 18) can be formed using only 3-cent and 10-cent stamps.

This can be re-written as: for any n ≥ 18,

n = 3x + 10y.

BASE CASE: P(18). n = 18 = 3(6) + 10(0).

This is true.

INDUCTIVE STEP: Suppose that P(j) is true. This means that any integer P(k) will be true such that k ≥ 18 and k < j. In this case, P(k+1) must also be true, meaning that P(k+1) = 3x + 10y. This means that for any n ≥ 18, n = 3x + 10y. This completes our proof by strong induction.