aj563_sf3_code

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1 Task 1

1.1 Task 1.1

```
[48]: import numpy as np
      from CartPole import *
      from sklearn import linear_model
[49]:
          def start_the_cart(initial_values1, initial_values2=None,_
       initial_values3=None, steps=10, remap_angle=False, visual=False, u
       →display_plots=True, variable = None):
              cp = CartPole(visual=visual)
              cp.cart_location, cp.cart_velocity, cp.pole_angle, cp.pole_velocity = __
       \hookrightarrowinitial_values1
              for step in range(steps):
                  if visual:
                      cp.drawPlot()
                  cp.performAction()
                  if remap_angle:
                      cp.remap_angle()
                  inter= [cp.cart_location, cp.cart_velocity, cp.pole_angle, cp.
       →pole_velocity]
                  try:
                      x_history = np.vstack((x_history, np.array(inter)))
                  except:
                       x_history = np.vstack((np.array(initial_values1),np.
       →array(inter)))
              x_axis=range(len(x_history))
```

if initial_values2:

```
cp.cart_location, cp.cart_velocity, cp.pole_angle, cp.pole_velocityu
→= initial_values2
           for step in range(steps):
               if visual: cp.drawPlot()
              cp.performAction()
               if remap_angle: cp.remap_angle()
               inter= [cp.cart_location, cp.cart_velocity, cp.pole_angle, cp.
→pole_velocity]
              try:
                  y_history = np.vstack((y_history, np.array(inter)))
              except:
                  y_history = np.vstack((np.array(initial_values2),np.
→array(inter)))
       if initial_values3:
           cp.cart_location, cp.cart_velocity, cp.pole_angle, cp.pole_velocityu
→= initial_values3
           for step in range(steps):
              if visual: cp.drawPlot()
               cp.performAction()
               if remap_angle: cp.remap_angle()
              inter= [cp.cart_location, cp.cart_velocity, cp.pole_angle, cp.
→pole_velocity]
              try:
                  z_history = np.vstack((z_history, np.array(inter)))
               except:
                  z history = np.vstack((np.array(initial values3),np.
→array(inter)))
       if display_plots:
           fig, axs = plt.subplots(2, 2, figsize=(10, 7))
           axs[0,0].plot(x_axis, [x[0] for x in x_history],label='First')
           if initial_values2: axs[0,0].plot(x_axis, [x[0] for x in_
if initial_values3: axs[0,0].plot(x_axis, [x[0] for x in_
→z_history],label='Third')
           axs[0,1].plot(x_axis, [x[1] for x in x_history])
           if initial_values2: axs[0,1].plot(x_axis, [x[1] for x in y_history])
```

```
if initial_values3: axs[0,1].plot(x_axis, [x[1] for x in z_history])
           axs[1,0].plot(x_axis, [x[2] for x in x_history])
           if initial_values2: axs[1,0].plot(x_axis, [x[2] for x in y_history])
           if initial_values3: axs[1,0].plot(x_axis, [x[2] for x in z_history])
           axs[1,1].plot(x_axis, [x[3] for x in x_history])
           if initial_values2: axs[1,1].plot(x_axis, [x[3] for x in y_history])
           if initial_values3: axs[1,1].plot(x_axis, [x[3] for x in z_history])
           #Set titles
           axs[0,0].set_title('Cart location')
           axs[0,0].set_xlabel('Steps')
           axs[0,0].set_ylabel('x')
           axs[0,1].set_title('Cart velocity')
           axs[0,1].set_xlabel('Steps')
           axs[0,1].set_ylabel('x_dot')
           axs[1,0].set_title('Pole angle')
           axs[1,0].set_xlabel('Steps')
           axs[1,0].set_ylabel('theta')
           axs[1,1].set_title('Pole velocity')
           axs[1,1].set xlabel('Steps')
           axs[1,1].set_ylabel('theta_dot')
           if variable: fig.suptitle(('Effect of different initial {} on cart⊔
→dynamics').format(variable),fontsize=16)
           fig.legend()
           fig.tight_layout()
      return x_history[-1]
```

1.1.1 Stable equilibrium

1.1.2 Complete rotation of pendulum

```
[51]: \begin{tabular}{ll} \#history = start\_the\_cart([0,0,np.pi,15],visual=False,remap\_angle=False) \\ \hline \end{tabular}
```

1.2 Task 1.2

```
[52]: initialize = np.array([np.random.uniform(-5,5), np.random.uniform(-10, 10), np.
      →random.uniform(-np.pi,np.pi), np.random.uniform(-15,15)])
      #print(initialize)
[53]: variable number={0:'Cart location',1:'Cart velocity',2:'Pole angle',3:'Pole__
       →velocity'}
[54]: def one_step(variable, x_axis_range, x_axis_intervals):
          x = initialize.copy()
          x_axis = np.linspace(x_axis_range[0],x_axis_range[1], x_axis_intervals)
          steps=1
          for i in x_axis:
             x[variable] = i
              y = start_the_cart(x, steps=steps, display_plots=False)
              try:
                  final_y = np.vstack((final_y, np.array(y)))
              except:
                  final_y = np.array(y)
          fig, axs = plt.subplots(2, 2, figsize=(10, 7))
          axs[0,0].plot(x_axis, [y[0] for y in final_y])
          axs[0,1].plot(x_axis, [y[1] for y in final_y])
          axs[1,0].plot(x_axis, [y[2] for y in final_y])
          axs[1,1].plot(x_axis, [y[3] for y in final_y])
          #Set titles
          axs[0,0].set_xlabel('Initial value of {}'.format(variable_number[variable]))
          axs[0,0].set_ylabel('Cart location')
          axs[0,1].set_xlabel('Initial value of {}'.format(variable_number[variable]))
          axs[0,1].set_ylabel('Cart velocity')
          axs[1,0].set_xlabel('Initial value of {}'.format(variable_number[variable]))
          axs[1,0].set_ylabel('Pole angle')
```

```
axs[1,1].set_xlabel('Initial value of {}'.format(variable_number[variable]))
axs[1,1].set_ylabel('Pole velocity')

fig.suptitle(('Effect of initial {} on cart dynamics after {} step'.

format(variable_number[variable], steps)), fontsize=16)

fig.tight_layout()
```

1.2.1 Vary cart location

```
[55]: #one_step(0,[-5,5],15)
```

1.2.2 Vary cart velocity

```
[56]: #one_step(1,[-10,10],15)
```

1.2.3 Vary pole angle

```
[57]: #one_step(2,[-np.pi,np.pi],15)
```

1.2.4 Vary pole velocity

```
[58]: #one_step(3,[-15,15],15)
```

1.2.5 Creating a variable "y", the difference between x

```
[59]: def one_step_difference(variable, x_axis_range, x_axis_intervals):
    x = initialize.copy()
    x_axis = np.linspace(x_axis_range[0],x_axis_range[1], x_axis_intervals)
    steps=1
    for i in x_axis:
        x[variable] = i
        x_t = start_the_cart(x, steps=steps, display_plots=False)
        y = x_t-x

    try:
        final_y = np.vstack((final_y, np.array(y)))
        except:
        final_y = np.array(y)

fig, axs = plt.subplots(2, 2, figsize=(10, 7))
```

```
axs[0,0].plot(x_axis, [y[0] for y in final_y])
  axs[0,1].plot(x_axis, [y[1] for y in final_y])
  axs[1,0].plot(x_axis, [y[2] for y in final_y])
  axs[1,1].plot(x_axis, [y[3] for y in final_y])
   #Set titles
  axs[0,0].set_xlabel('Initial value of {}'.format(variable_number[variable]))
  axs[0,0].set_ylabel('Cart location change')
  axs[0,1].set_xlabel('Initial value of {}'.format(variable_number[variable]))
  axs[0,1].set_ylabel('Cart velocity change')
  axs[1,0].set_xlabel('Initial value of {}'.format(variable_number[variable]))
  axs[1,0].set_ylabel('Pole angle change')
  axs[1,1].set_xlabel('Initial value of {}'.format(variable_number[variable]))
  axs[1,1].set_ylabel('Pole velocity change')
  fig.suptitle(('Effect of initial {} on cart dynamics after {} step'.
→format(variable_number[variable],steps)),fontsize=16)
  fig.tight_layout()
  return final_y
```

```
final_y = np.array(y)

return x_axis, final_y
```

```
[61]: def together_plot_1_2():
          x_axis1, c_1_real = one_step_difference_2(0, [-5,5], 15)
          x_axis2, c_v_real = one_step_difference_2(1, [-10, 10], 15)
          x_axis3,p_a_real = one_step_difference_2(2,[-np.pi,np.pi],15)
          x_axis4,p_v_real = one_step_difference_2(3,[-15,15],15)
          fig, axs = plt.subplots(2, 2, figsize=(15, 12))
          for i in range(4):
              axs[0,0].plot(x_axis1, [y[i] for y in c_l_real], label='{}'.
       →format(variable_number[i]))
              axs[0,1].plot(x_axis2, [y[i] for y in c_v_real],label='Real c_v')
              axs[1,0].plot(x_axis3, [y[i] for y in p_a_real],label='Real p_a')
              axs[1,1].plot(x_axis4, [y[i] for y in p_v_real],label='Real p_v')
              axs[0,0].legend()
          axs[0,0].set_xlabel('Initial value of cart location')
          axs[0,0].set_ylabel('Cart dynamics changes')
          axs[0,1].set_xlabel('Initial value of cart velocity')
          axs[0,1].set_ylabel('Cart dynamics changes')
          axs[1,0].set_xlabel('Initial value of pole angle')
          axs[1,0].set_ylabel('Cart dynamics changes')
          axs[1,1].set_xlabel('Initial value of pole velocity')
          axs[1,1].set_ylabel('Cart dynamics changes')
          fig.tight_layout()
          plt.show()
```

1.2.6 (i) Scans of single relationships

```
[62]: #together_plot_1_2()
```

1.2.7 (ii) Contour plots

```
[63]: different_pairs = [[0,1],[0,2],[0,3],[1,2],[1,3],[2,3]]
      axes_ranges = \{0 : np.linspace(-5,5,10), 1 : np.linspace(-10,10,10), 2 : np.
       \rightarrowlinspace(-np.pi,np.pi,10), 3 : np.linspace(-15,15,10)}
      def axes_for_pairs(index_pair):
          range_of_variables = []
          for index in index_pair:
              range_of_variables.append(axes_ranges[index])
          return range_of_variables
      def contours_of_pairs(index_pair, range_of_variables):
          index_1, index_2 = index_pair
          range_1, range_2 = range_of_variables
          initial_grid = np.zeros((len(range_1),len(range_2),4))
          final_grid = np.zeros((len(range_1),len(range_2),4))
          for i,value_1 in enumerate(range_1):
              for j, value_2 in enumerate(range_2):
                  x = initialize.copy()
                  x[index_1] = value_1
                  x[index_2] = value_2
                  initial\_grid[i,j] = x
                  final_grid[i,j] = np.array(start_the_cart(x, steps=1,__

→display_plots=False))
          y_grid = final_grid - initial_grid
          y_grid = np.moveaxis(y_grid, -1, 0)
          fig, axs = plt.subplots(2, 2, figsize=(12, 9))
          axs[0,0].contourf(range_1, range_2, y_grid[0].T, vmin=y_grid.min(),_
       →vmax=y_grid.max())
          axs[0,0].set_title('cart_location')
          axs[0,0].set_xlabel('{} initial value'.format(variable_number[index_1]))
          axs[0,0].set_ylabel('{} initial value'.format(variable_number[index_2]))
          axs[0,1].contourf(range_1, range_2, y_grid[1].T, vmin=y_grid.min(),_
       →vmax=y grid.max())
```

```
[94]:

| for indices in different_pairs:
| print('Plots of {} and {}'.
| → format(variable_number[indices[0]], variable_number[indices[1]]))
| contours_of_pairs(indices, axes_for_pairs(indices))
| plt.show()
| """
```

[94]: "\nfor indices in different_pairs:\n print('Plots of {} and {}'.format(variable_number[indices[0]],variable_number[indices[1]]))\n contours_of_pairs(indices, axes_for_pairs(indices))\n plt.show()\n"

1.3 Task 1.3

```
return final_x,final_y-final_x
[66]: x,y= get xy pairs(500)
[67]: #Create train and test sets
      proportion = 0.95
      number_of_samples = 500
      cutoff = int(proportion*number_of_samples)
      train_x= x[:cutoff]
      test x=x[cutoff:]
      train_y=y[:cutoff]
      test_y=y[cutoff:]
[68]: model = linear_model.LinearRegression()
      model.fit(train_x,train_y)
      n=model.predict(test_x)
[69]: def predict(train_x, test_x, train_y):
          W = np.matmul(np.linalg.pinv(train x),train y)
          prediction = np.matmul(test_x,W)
          return prediction, W
[70]: m,W = predict(train_x, test_x, train_y)
```

1.3.1 First plot. Real and predicted plotted against initial

```
axs[1,0].scatter([x[2] for x in input],[y[2] for y in next_step])
          axs[1,0].scatter([x[2] for x in input],[y[2] for y in pred_next_step])
          axs[1,0].set_xlabel('Pole angle initial value')
          axs[1,0].set_ylabel('Pole angle final value')
          axs[1,1].scatter([x[3] for x in input],[y[3] for y in next_step])
          axs[1,1].scatter([x[3] for x in input],[y[3] for y in pred_next_step])
          axs[1,1].set_xlabel('Pole velocity initial value')
          axs[1,1].set_ylabel('Pole velocity final value')
          fig.suptitle('Predictions vs Real values after 1 step plotted against,
       ⇔various initial values')
          fig.tight_layout()
[72]: \#vertical\_plot(test\_x, test\_y + test\_x, m + test\_x)
[73]: def rmse_calc(A,B):
          squared = (A-B)**2
          cl_mse =0
          cv_mse=0
          pa mse=0
          pv_mse=0
          for row in squared:
              cl_mse+=row[0]
              cv mse+=row[1]
              pa_mse+=row[2]
              pv_mse+=row[3]
          cl_mse=(cl_mse/len(squared))**0.5
          cv_mse=(cv_mse/len(squared))**0.5
          pa_mse=(pa_mse/len(squared))**0.5
          pv_mse=(pv_mse/len(squared))**0.5
          return cl_mse,cv_mse,pa_mse,pv_mse
[74]: rmse_calc(test_y,n)
[74]: (0.11861650064178725, 1.3658805420008469, 2.119815738456754, 3.232319734005139)
[75]: rmse_calc(test_y,m)
[75]: (0.12023238299058406,
       1.386829387917551,
       2.0989986937181055,
```

3.2341823869798754)

1.3.2 second plot. This one is real vs predicted only

```
[76]: def real vs predicted(real, predicted):
          fig, axs = plt.subplots(2, 2, figsize=(10, 7))
          axs[0,0].scatter([x[0] for x in real],[y[0] for y in predicted])
          axs[0,0].set_xlabel('Cart location real value')
          axs[0,0].set_ylabel('Cart location predicted value')
          axs[0,1].scatter([x[1] for x in real],[y[1] for y in predicted])
          axs[0,1].set_xlabel('Cart velocity real value')
          axs[0,1].set_ylabel('Cart velocity predicted value')
          axs[1,0].scatter([x[2] for x in real],[y[2] for y in predicted])
          axs[1,0].set_xlabel('Pole angle real value')
          axs[1,0].set_ylabel('Pole angle predicted value')
          axs[1,1].scatter([x[3] for x in real],[y[3] for y in predicted])
          axs[1,1].set_xlabel('Pole velocity real value')
          axs[1,1].set_ylabel('Pole velocity predicted value')
          fig.suptitle('Predictions vs Real values after 1 step')
          fig.tight_layout()
```

```
[77]: \#real\_vs\_predicted(test\_y+test\_x,m+test\_x)
```

1.3.3 Scans with varying parameters

```
[78]: def one_step_difference_with_predictions_2(variable, x_axis_range,_
       \rightarrowx_axis_intervals):
          x = initialize.copy()
          x_axis = np.linspace(x_axis_range[0],x_axis_range[1], x_axis_intervals)
          steps=1
          for i in x_axis:
              x[variable] = i
              x t = start the cart(x, steps=steps, ...
       →display_plots=False,remap_angle=True)
              y = x_t-x
              pred = model.predict([x])
              try:
                  final_y = np.vstack((final_y, np.array(y)))
                  final_pred = np.vstack((final_pred,np.array(pred)))
              except:
                  final_y = np.array(y)
```

```
final_pred = np.array(pred)

return x_axis,final_y,final_pred
```

```
[79]: def together_plot():
          x_axis1, c_l_real,c_l_pred =_
       →one_step_difference_with_predictions_2(0,[-5,5],15)
          x_axis2,c_v_real,c_v_pred =_
       →one_step_difference_with_predictions_2(1,[-10,10],15)
          x_axis3,p_a_real,p_a_pred = one_step_difference_with_predictions_2(2,[-np.
       \rightarrowpi,np.pi],15)
          x_axis4,p_v_real,p_v_pred =_
       →one_step_difference_with_predictions_2(3,[-15,15],15)
          fig, axs = plt.subplots(2, 2, figsize=(17, 11))
          for i in range(4):
              axs[0,0].plot(x_axis1, [y[i] for y in c_l_real],label='Real {}'.
       →format(variable_number[i]))
              axs[0,0].scatter(x_axis1, [y[i] for y in c_l_pred],label='Predicted {}'.
       →format(variable number[i]))
              axs[0,1].plot(x_axis2, [y[i] for y in c_v_real],label='Real c_v')
              axs[0,1].scatter(x_axis2, [y[i] for y in c_v_pred],label='Predicted_u

c_v')

              axs[1,0].plot(x_axis3, [y[i] for y in p_a_real],label='Real p_a')
              axs[1,0].scatter(x_axis3, [y[i] for y in p_a_pred],label='Predicted_u
       →p_a')
              axs[1,1].plot(x_axis4, [y[i] for y in p_v_real],label='Real p_v')
              axs[1,1].scatter(x_axis4, [y[i] for y in p_v_pred],label='Predicted_u
       \hookrightarrow p_v'
              axs[0,0].legend()
          axs[0,0].set_xlabel('Initial value of cart location')
          axs[0,0].set_ylabel('Cart dynamics changes')
          axs[0,1].set_xlabel('Initial value of cart velocity')
          axs[0,1].set_ylabel('Cart dynamics changes')
```

```
axs[1,0].set_xlabel('Initial value of pole angle')
axs[1,0].set_ylabel('Cart dynamics changes')

axs[1,1].set_xlabel('Initial value of pole velocity')
axs[1,1].set_ylabel('Cart dynamics changes')

fig.tight_layout()

plt.show()
```

[80]: | #together_plot()

1.4 Task 1.4

```
[81]: def future_predictions_from_predictions(initial_conditions,_
       →time_steps,remap_angle=True):
          final_y=np.array(initial_conditions)
          final_pred=np.array(initial_conditions)
          for i in range(time steps):
              real= start_the_cart(final_y[i], steps=1,__

→display_plots=False,remap_angle=remap_angle)
              pred = np.matmul(final_pred[i],W)
              pred+=final pred[i]
              final_y = np.vstack((final_y, np.array(real)))
              final_pred = np.vstack((final_pred,np.array(pred)))
          x_axis=np.linspace(0,time_steps,time_steps+1)
          fig, axs = plt.subplots(2, 2, figsize=(12, 10))
          axs[0,0].plot(x_axis, [y[0] for y in final_y],label='Real value')
          axs[0,0].plot(x_axis, [y[0] for y in final_pred],label='Predicted value')
          axs[0,0].legend()
          axs[0,1].plot(x_axis, [y[1] for y in final_y])
          axs[0,1].plot(x_axis, [y[1] for y in final_pred])
          axs[1,0].plot(x_axis, [y[2] for y in final_y])
          axs[1,0].plot(x_axis, [y[2] for y in final_pred])
          axs[1,1].plot(x_axis, [y[3] for y in final_y])
          axs[1,1].plot(x_axis, [y[3] for y in final_pred])
```

```
#Set titles
          axs[0,0].set_xlabel('Time steps')
          axs[0,0].set_ylabel('Cart location')
          axs[0,1].set_xlabel('Time steps')
          axs[0,1].set_ylabel('Cart velocity')
          axs[1,0].set_xlabel('Time steps')
          axs[1,0].set_ylabel('Pole angle')
          axs[1,1].set_xlabel('Time steps')
          axs[1,1].set_ylabel('Pole velocity')
          fig.suptitle('Predicted (from predictions) vs Real change in cart dynamics⊔
       →over time with initial conditions {}'.format(initial_conditions),fontsize=16)
          fig.tight_layout()
[82]: | #future_predictions_from_predictions([[10,5,2,6]],10,remap_angle=True)
[83]: #future predictions from predictions([[0,0,np.pi,15]],10,remap_angle=True)
[84]: def future_predictions_from_real(initial_conditions,__
       →time_steps,remap_angle=True):
          final_y=np.array(initial_conditions)
          final_pred=np.array(initial_conditions)
          for i in range(time_steps):
              real= start_the_cart(final_y[i], steps=1,__
       →display_plots=False,remap_angle=remap_angle)
              pred = np.matmul(final_y[i],W)
              pred+=final_y[i]
              final_y = np.vstack((final_y, np.array(real)))
              final_pred = np.vstack((final_pred,np.array(pred)))
          x_axis=np.linspace(0,time_steps,time_steps+1)
          fig, axs = plt.subplots(2, 2, figsize=(12, 10))
```

```
axs[0,0].plot(x_axis, [y[0] for y in final_y],label='Real value')
  axs[0,0].plot(x_axis, [y[0] for y in final_pred],label='Predicted value')
  axs[0,0].legend()
  axs[0,1].plot(x_axis, [y[1] for y in final_y])
  axs[0,1].plot(x_axis, [y[1] for y in final_pred])
  axs[1,0].plot(x_axis, [y[2] for y in final_y])
  axs[1,0].plot(x_axis, [y[2] for y in final_pred])
  axs[1,1].plot(x_axis, [y[3] for y in final_y])
  axs[1,1].plot(x_axis, [y[3] for y in final_pred])
   #Set titles
  axs[0,0].set_xlabel('Time steps')
  axs[0,0].set_ylabel('Cart location')
  axs[0,1].set_xlabel('Time steps')
  axs[0,1].set_ylabel('Cart velocity')
  axs[1,0].set_xlabel('Time steps')
  axs[1,0].set ylabel('Pole angle')
  axs[1,1].set_xlabel('Time steps')
  axs[1,1].set_ylabel('Pole velocity')
  fig.suptitle('Predicted (from recent dynamics) vs Real change in cart⊔

→dynamics over time with initial conditions {}'.
→format(initial_conditions),fontsize=16)
  fig.tight_layout()
```

```
[85]: #future_predictions_from_real([[10,5,2,6]],10,remap_angle=True)

[86]: #future_predictions_from_real([[0,0,np.pi,15]],10,remap_angle=True)
```

1.4.1 Here I will train the model on non-remapped angle to see the effect

```
[87]: def get_xy_pairs_2(n):
    for iteration in range(n):
```

```
random_point= np.array([np.random.uniform(-5,5), np.random.uniform(-10,_u
      →10), np.random.uniform(-np.pi,np.pi), np.random.uniform(-15,15)])
             y = start_the_cart(random_point, steps=1,__
      →remap_angle=False,display_plots=False)
             try:
                 final_y = np.vstack((final_y, np.array(y)))
                 final_x = np.vstack((final_x, np.array(random_point)))
             except:
                 final_y = np.array(y)
                 final_x = np.array(random_point)
         return final_x,final_y-final_x
[88]: x2,y2= get_xy_pairs_2(500)
[89]: #Create train and test sets
     proportion = 0.95
     number_of_samples = 500
     cutoff = int(proportion*number_of_samples)
     train x2= x2[:cutoff]
     test_x2=x2[cutoff:]
     train_y2=y2[:cutoff]
     test_y2=y2[cutoff:]
[90]: r,B = predict(train_x2, test_x2, train_y2)
[91]: print(B)
     [ 2.00738062e-01 2.75503518e-03 4.36027474e-03 2.88063797e-02]
      [ 4.49557298e-02 2.36752739e-01 1.63781782e-01 1.19756697e+00]
      [ 2.37050721e-03  2.53349442e-02  1.97746279e-01  -4.52693684e-02]]
[92]: np.linalg.eigvals(W)
[92]: array([ 0.04308479, -0.04819714, -0.15212995, -0.86964171])
[93]: np.linalg.eigvals(B)
[93]: array([ 0.56116959,  0.06452381, -0.06584885, -0.43700644])
 []:
```