

1) Count number of nodes

$$\sum_{i=1}^n 1 + \sum_{i=1}^l 2$$

Where  $n$ : number of nodes

$l$ : number of leaf nodes

2) quicksort

$$C(n) = 2C(n/2) + n \text{ for } n > 1, C(1) = 0$$

Master Theorem:

$$T(n) = aT(n/b) + f(n)$$

$$a = 2, b = 2, f(n) = n$$

$$f(n) \in \Theta(n^d) = \Theta(n')$$

$$\therefore d = 1$$

$$a = b^d \therefore 2 = 2^1$$

$$\therefore T(n) \in \Theta(n \log n)$$

$$C(n) \in \Theta(n \log n)$$

3) merge sort

$$C(n) = 2C(n/2) + C_{\text{merge}}(n) \text{ for } n > 1,$$

$$C(1) = 0$$

in the worst case,  $C_{\text{merge}}(n) = n-1$

$$\therefore C_{\text{worst}}(n) = 2C_{\text{worst}}(n/2) + n - 1$$

Master Theorem:

$$T(n) = aT(n/b) + f(n)$$

$$a = 2, b = 2, f(n) = n - 1$$

$$f(n) \in \Theta(n^d) = \Theta(n')$$

$$\therefore d = 1$$

$$a = b^d \therefore 2 = 2^1$$

$$\therefore T(n) \in \Theta(n \log n)$$

$$C_{\text{worst}}(n) = \Theta(n \log n)$$