Date
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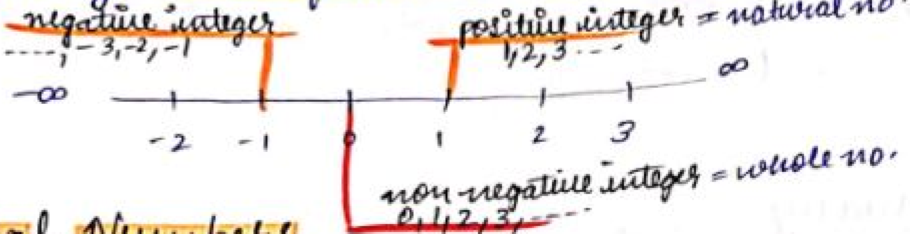
★ Real Numbers

Any number that can be represented on an imaginary line from $-\infty$ to $+\infty$ is known as Real no.



★ Natural Numbers

Set of all positive integers is known as Natural numbers.

★ Whole Number

Set of all non negative integers is known as whole no.

★ Prime Number

Any natural number having exactly two different factors i.e., 1 and N number itself is known as Prime number.

highest prime no. under 100

only Even Prime number

$\boxed{2}, \underbrace{3, 5, 7, 11, 13, 17, 19, 23, 29, \ldots 97}_{\text{odd}} \ldots \infty$

RULE

Except (2) and (3) all other Prime numbers belong to the family of $\boxed{6K-1}$ or $\boxed{6K+1}$ where

[K ∈ 1, 2, 3, 4 ...... (positive integer)]

NOTE This is a necessary condition but not sufficient.

Q: How many primes numbers?

(1-100) ⇒ (25)

(100-200) ⇒ (21)

★ Composite ~~factor~~ Number

Any natural number having atleast 3 distinct factors is known as Composite number.

first composite number → (4), 6, 8, 9, 10, 12, ...

4	6	8
1	1	1
2	2	2
4	3	4
	6	8

NOTE (1) is neither prime nor composite.

★ Even Number

Any positive integer

2

★ Odd Number

no. that is not divisible by 2.

Factorials

$n!$ or $\lfloor n$ $\boxed{0! = 1}$

$n! = n(n-1)(n-2)(n-3) \ldots\ldots\ldots 3.2.1$

$\boxed{\text{Factorials}}$

① Number of zero in the end

② Highest power

① Number of zero in the end.

Ex ①

$5 \times 12 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45$

$5 \times 2 \times 5 \times 3 \times 5 \times 2^2 \times 5 \times \underset{\text{less}}{5^2} \times 2 \times 3 \times 5 \times 5 \times 7 \times 2^3 \times 5 \times 3^2 \times 5$

$\Rightarrow 3^4 \times 7 \times 2^{7} \times 5^{10}$

$\Rightarrow \underline{\quad\quad} \underbrace{0000000}_{7 \text{ zeroes}}$

Ex ②

$12! = 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$12! = 2^{10} \times 3^5 \times 5^{2} \times 7^1 \times 11^1 \Rightarrow \underline{\quad} \underbrace{00}_{2 \text{ zeroes}}$

Shortcut method { no. diya ho factorial Toh prime number factors nikalne ho Toh }

$\frac{12}{2} = 6$ {larger than 2} repeat

$\frac{6}{2} = 3$ (repeat)

$\frac{3}{2} = 1$ (stop)

$\overline{10} \Rightarrow 2^{10}$

$\frac{12}{3} = 4$

$\frac{4}{3} = 1$

$\overline{5} \Rightarrow 3^5$

$$50! = 2^{a \to 47} \times 3^{b \to 22} \times 5^{c \to 12} \times 7^{d \to 8} \times \cdots\cdots (47)^1$$

2	3	5	7
25	16	10	7
12	5	2	1
6	1	12	8
3	22		
1			
47			

12 zeroes

- hum akele 5 ke count karenge kyunki obvious hai 2 ke zyada factors honge 5 se toh pair toh 5 hi decide karega

$$100! = 2^{a \to 97} \times 3^{b \to 48} \times 5^{c \to 24} \times 7^{d \to 16} \times \cdots\cdots (97)^1$$

2	3	5	7
50	33	20	14
25	11	4	2
12	3	24	16
6	1		
3	48		
1			
97			

24 zeroes

$1012! = 5^{251}$

$1000 + 12$

$200 + 2 \to 202$

40

8

1

251 zeroes

$(6!)^{9!}$

$(6!)^{8!}$

$(5^1)^{8!} \Rightarrow 5^{8!} \Rightarrow 8!$ zeroes

$(8!)^{9!} \times (10!)^{11!}$

$8! \; (5^1)^{9!} \times (5^2)^{11!}$

$5^{1\times 9!} \times 5^{2\times 11!}$

$5^{9!} \times 5^{(2\times 11!)}$

$5^{\boxed{9! + 2(11!)}}$

$18! + 19!$

a000

b000

_000

deciding

$18! + 19 \times (18!)$

$18! \, (1+19)$

$18! \times 20$

$5^3 \times 5 \times 2^2$

5^4

____0000

4 zeroes

Highest Power

Ex: ① highest power of 2 that can divide 100! ?

$$\frac{100!}{2^x} = \frac{2^{97} \times 3^{48} \times 5^{24} \times 7^{16} \times \dots 97^1}{2^{97}}$$

Q

97 Ans

② highest power of 3 that can divide 100! ?

$$\frac{100!}{3^x} = \frac{3^{\boxed{48}}(Q)}{3^{\boxed{48}}}$$

48 Ans

③ $$\frac{100!}{(4)^x} = \frac{2^{97} \times Q}{(2^2)^x}$$

$$\frac{(2^2)^{48} \times 2^1 \times Q}{}$$

$$= \frac{(4)^{48} \times Q}{(4)^{48}}$$

or

$\frac{97}{2} = 48$

$\Rightarrow$ 48 Ans

④ $$\frac{100!}{(9)^x} = \frac{3^{48} \times Q}{(3^2)^x} = (9)^{24}$$

24 Ans

$9 = 3^2$

$\frac{48}{2} = 24$

⑤ $$\frac{100!}{(8)^x} = \frac{2^{97} \times Q}{(2^3)^x}$$

32 Ans

$(2^3)^x$

$\frac{97}{3} = 32$

$$\frac{100!}{(6)^x} = \frac{2^{97} \times 3^{48} \times \boxed{Q}}{2^{(9)} \times 3^{(6)}}$$

(97) less (48)

$6 = 2 \times 3$

$$\Rightarrow 6^{(48)} = (48) \text{ Ans}$$

$$\frac{100!}{(72)^x} = \frac{2^{97} \times 3^{48} \times \boxed{Q}}{(2^3)^9 \times (3^2)^6}$$

$$\frac{97}{3} = (32) \qquad \frac{48}{2} = (24)$$

$$(2^3)^{32} \qquad (3^2)^{24}$$

less

$$(8)^{32} \times (9)^{24}$$

$$(72)^{24}$$

$$\Rightarrow (24) \text{ Ans}$$

Trailing zeroes and highest power

Q Find the max value of n such that 50! is perfectly divisible by 2520^n.

$2520 = 7 \times 3^2 \times 2^3 \times 5$

The value of n would be given by the value of the number of 7s in 50!

$\frac{50!}{7} \Rightarrow 7 + 1 = 8$

Q: max value of n such that 50! is perfectly divisible by 12600^n.

$12600 = 7 \times 3^2 \times 2^3 \times 5^2$

The value of 'n' would depend on which of no. of 7's and number of 5^2's is lower in 50!

No. of 7's in 50! = 8

No. of 5's in 50! = 12

No. of 5^2's in 50! = $\left[\frac{12}{2}\right] = 6$

NOTE

If we check for 7's we do not need to check for 3^2s as there would be atleast two 3's before a 7 comes in every factorial's value. Similarly, there would always be atleast three 2's before a 7 comes in any factorial value. Thus, the no. of 3^2s and no of 2^3's can never be lower than 7's in any factorial value.

$(5^2)^6$ $(7)^8$ → 6 Ans

Q: Max value of n such that 77! is perfectly divisible by 720^n.

$720 = 2^4 \times 5^1 \times 3^2$

In 77! = contains $2^{73} \Rightarrow \frac{73}{4} \Rightarrow (2^4)^{18}$

$3^{35} \Rightarrow \frac{35}{2} \Rightarrow (3^2)^{17}$ → 17 Ans

$5^{18} \Rightarrow (5^1)^{18}$

Rational Number

Any simplest form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

$\frac{4\sqrt{3}}{7\sqrt{3}} = \frac{4}{7}$ → Reduced form → Simplest form

$\frac{p}{q}$ — p, q Integers; $q \neq 0$

Ex: $\frac{1}{5}, \frac{5}{7}, \frac{6}{1}, \frac{4\sqrt{3}}{7\sqrt{3}} = \frac{4}{7}$

Irrational Number

Either or both are not integer.

$\frac{p}{q}$ Ex: $\frac{4\sqrt{3}}{7}, \frac{4}{7\sqrt{3}}, \frac{4\sqrt{3}}{7\sqrt{2}}$

- π.
- Log value (Exponential 10^n form)
- e^x, e^1, e^2 — Exponential
- Surds $\left\{ \begin{matrix} \sqrt{2} \\ \sqrt{3} \end{matrix} \right\}$

Rational number → Terminating $\frac{1}{2} = 0.5$ $\frac{1}{4} = 0.25$

Rational number → Non Terminating Recurring

$\frac{1}{3} = 0.333333 \ldots \infty$

$\frac{22}{7} = 3.\boxed{142857}\boxed{142857}\boxed{142857} \ldots \infty$

$\frac{1}{6} = 0.166666 \ldots \infty$

Re-occur

non terminating

Irrational number → Non Terminating Non Recurring

$\pi = 3.1428516781 3456 \ldots \infty$

Q4 Simplest form

$\frac{1}{2468097\text{-}53}$ is $= \frac{1}{2^n \times 5^m}$

(x) (a) Terminating

(b) Non Term, Recurring

(x) (c) Non Term, Non-Recurring

(d) None of these.

☆ Rules of Terminating

(a) $\frac{1}{2^n}$ $\quad \frac{1}{2} = 0.5 \quad \frac{1}{4} = 0.25 \quad \frac{1}{8} = 0.125$

(b) $\frac{1}{5^n}$ $\quad \frac{1}{5} = 0.2 \quad \frac{1}{25} = 0.04$

(c) $\frac{1}{2^a \times 5^b}$ $\quad \frac{1}{100} = 0.01$

$2^2 \times 5^2$

☆ Converting Non Terminating Recurring no. into $\frac{p}{q}$

$x = 2.\overline{7}$

$$10x = 27.777777\ldots\ldots\infty$$
$$-\,x = 2.777777\text{----}\infty$$

$$9x = 27 - 2$$

$$\boxed{x = \frac{25}{9}}$$

Shortcut

① $12.\overline{345}$

3 no's under the bar

$$\frac{12345-12}{999}$$

② $123.\overline{45}$

$$\frac{12345-123}{99}$$

③ $1.23\overline{456}$

without bar

$$\frac{123456-123}{99900}$$

④ $123.4\overline{56}$

$$\frac{123456-1234}{990}.$$

ESE Q1

Rational no.

$x = 0.abcabcabc\ldots\ldots\infty$ (a, b, c diff no's)

$\underbrace{x \times y(\text{natural no})}= z^{(\text{positive integer})}$

which among the option is the probable value of (y)?

(a) 9999

(b) 99999

(c) 1998

(d) none of these

$$\frac{abc}{999} \times (\quad) = \text{integer}$$

999 or multiple of 999

$999\overline{)9999}(1$
999
9

$999\overline{)99999}(1$
999
99

$999\overline{)1998}(2$
1998
X

Divisibility Rules

① Rule of ② → $\frac{N}{2}$ → Last digit of N is [0] even number.

Ex: 5128

② Rule of ③ → $\frac{N}{3}$ → Sum of digits of N is divisible by 3

Ex: 6123 ⇒ 6+1+2+3 = $\frac{12}{3}$

③ Rule of ④ → $\frac{N}{4}$ → Last ② digit of N is divisible by 4

Ex: 7128 = 7100 + 28
= 71×100+28
= ~~71×25×4+28~~
$\frac{\quad}{4}$

④ Rule of ⑤ → $\frac{N}{5}$ → Last digit → 0, 5

⑤ Rule of ⑥ → $\frac{N}{2}$ and $\frac{N}{3}$ $\boxed{\frac{N}{6}}$

⑥ Rule of ⑦ → Oscillation no. ②

34[3

34
−6
28/7

NOTE

* 1001 is always divisible by 7, 11, 13

$\boxed{\star\ 1001 = 7\times11\times13}$

$\boxed{\star\ 111 = 37\times3}$

NOTE

* Any 6 digit number of the form

abcabc

is always divisible by 7, 11, 13

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Ex:
① 111111
② 123123
③ 212212

① if 7 is (111111)1 ⇒ 1111110 + 1

$\Rightarrow \frac{111111 \times 10 + 1}{7} \Rightarrow 1$

② if 8 1's $11111111 \div 7 \Rightarrow 11111100 + 11$

$\Rightarrow \frac{111111 \times 100 + 11}{7}$

$\Rightarrow 4$

③ $\frac{\overbrace{111\text{-------}}^{100 \text{ times } (96)}|1111}{7}$ 7)1111(158 ⇒ 5

```
7)1111(158
  7
  41
  35
   61
   56
    5
```

⑦ Rule of 8 →

$\frac{N}{8}$ → Last ③ digit is divisible by 8.

Ex: 7512 ⇒ 7000 + 512 ⇒ $\frac{\cancel{7 \times 125 \times 8} + 512}{8}$

⇒ 7×1000+512

(8) Rule of (9) $\rightarrow \frac{N}{9} \rightarrow$ Sum of digits of N is divisible by 9.

Ex: 6723 $\rightarrow$

$$
\begin{array}{llll}
6000 & = 6\times1000 & = 6\times(999+1) & = 6\times999+6 \\
+700 & +7\times100 & +7(99+1) & = 7\times99+7 \\
+20 & +2\times10 & +2(9+1) & = 2\times9+2 \\
+3 & +3 & +3 & +3 \\
\hline
 & & & \cancel{9}
\end{array}
$$

(2)

(9) Rule of (11)

Sum of digits at even places − Sum of digits at odd places

first place

$abc\boxed{d} \div 11$

$$(d+b)-(c+a) = 0$$
$$= \boxed{11K}$$

Ex: 6 7 3 (2)

$$(2+7)-(3+6)=0$$

$$
\begin{array}{ll}
= 6\times(1001-1) & = 6\times1001-6 \\
= +7\times(99+1) & = 7\times99+7 \\
= +3\times(11-1) & = 3\times11-3 \\
= +2 & = +2 \\
\hline
 & \cancel{11}
\end{array}
$$

Q. $x = ?$

$$97215x6 \div 11$$

$$(6+5+2+9)-(x+1+7)=0$$
$$=11K$$

$$22-(8+x)=0$$
$$x=14 \text{ (Not possible)}$$

$$22-(8+x)=11$$
$$\boxed{x=3}$$

Q. N = 123456789

(N) is divided by following no's. Find remainder

(a) 3 (b) 4 (c) 8 (d) 9 (e) 11

$1+2+\dots+9 \Rightarrow \frac{45}{3} = 0$; $\frac{9\times10}{2}$

(b) $\frac{489}{4} = 1$

(c) $\frac{789}{8} = 5$

(d) $\frac{45}{9} = 0$

(e) $25-20 \Rightarrow \frac{5}{11} \Rightarrow 5$

Q. $1568x35y$ is divisible by 88. Find remainder.

$88 \to 8\times11$

$8 \overline{)35y}$ (4
32
$3y \to y=2$

$\Rightarrow (16+y)-(12+x)$

$\Rightarrow 4+y-x$

$\Rightarrow 6-x$ $\quad x=6$

Q. $8A5146B \div 88$ what is $B^A = ?$

$\Rightarrow 8\overline{)46B}$ 5
40
6B $\quad$ $B=4$

$(17+B)-(7+A)$

$10+B-A$

$14-A=11$

$14-11=A$

$\boxed{A=3}$

$4^3 \Rightarrow 64$

8
16
24
32
40
48
56
64

Q. $5x8146y \div 88$ what is $x=?$ and $y=?$

$88 \to 8\times11$

$8\overline{)46y}$ 5
40
$6y \to y=8$

$(17+y)-(7+x)$

$10+y-x$

$18-x = 0$ (x)

$18-x=11$

$18-11=x$

$\boxed{x=7}$

Condition of Divisibility for Algebric Function

1. a^n+b^n is exactly divisible by $(a+b)$ only when n is odd.

   <u>Ex</u> → $a^3+b^3=(a+b)(a^2-ab+b^2)$ is divisible by $a+b$, also a^5+b^5 is divisible by $a+b$.

2. a^n+b^n is never divisible by $(a-b)$ (whether n is odd or even).

   <u>Ex</u> → $a^3+b^3=(a+b)(a^2-ab+b^2)$ is not divisible by $(a-b)$

3. a^n-b^n is always divisible by $(a-b)$ (whether n is odd or even).

   <u>Ex</u> → a^9-b^9 is exactly divisible by $(a-b)$. also. $a^{12}-b^{12}$ is exactly divisible by $(a-b)$.

4. a^n-b^n is divisible by $a+b$ when 'n' is even natural number.

   <u>Ex</u> → $a^4-b^4=(a^2-b^2)(a^2+b^2)=(a-b)(a+b)(a^2+b^2)$.

   ↳ always divisible by $(a+b)$ but ~~not~~ a^3-b^3 will not be.

Unit Digit

$(\_\_\_\_ x)^P$

Step① Divide (P) by ④ and get the remainder

$\frac{P}{4} \rightarrow$ Remainder | Step② Raise the remainder over x | Step③ solve ↓ Last Digit (Ans)

Remainder	
1	x^1
2	x^2
3	x^3
0	x^4

in case of remainder 0
Raise ④

Ex. ① $(5127)^{2139}$

$\frac{2139}{4} = 3$ (Remainder)

$7^3 = 343 \Rightarrow 3$ Ans

② $(2163)^{11111}$

$\frac{11111}{4}$ → 3

3^3

27

7 Ans

③ $(7722)^{4488}$

$\frac{4488}{4} = 0$

2^4

16

$\Rightarrow 6$

④ $(1238)^{9814}$

$\frac{9814}{4} \Rightarrow 2$

8^2

$= 64$

$\Rightarrow 4$

Q1 $(2163)^{111111} \times (7722)^{4488} + (1238)^{7814}$

$7 \times 6 + 4$ BODMAS

$42 + 4$

$4\underline{6}$

$\hookrightarrow 6$

Q1 $211^{\frac{870}{4}} + 146^{\frac{127}{4}} \times 3^{\frac{424}{4}}$

$1^2 + 6^3 \times 3^4$

$1 + 6 \times 1$

$1 + 6$

7

Q1 $(2171)^{\frac{7}{4}}_{(3)} + (2172)^{\frac{9}{4}}_{(1)} + (2173)^{\frac{11}{4}}_{(3)} + (2174)^{\frac{13}{4}}_{(1)}$

$1^3 \quad 2^1 \quad 3^3 \quad 4^1$

$1 + 2 + 7 + 4$

$1\underline{4} \Rightarrow 4$

10^{th} digit

$(12)^3 \Rightarrow (10+2)^3 = {}^3C_0\ 10^3\ 2^0$
$+ {}^3C_1\ 10^2\ 2^1$
$+ {}^3C_2\ 10^1\ 2^2$
$+ {}^3C_3\ 10^0\ 2^3$

$\boxed{\begin{array}{r}120\\ +\ 8\end{array}}$

$\Rightarrow 1\,28$

FACTORS

Type (1) Total no. of factors

Factorial form $N = a^{p} \times b^{q}$

Total no. of factors $= \boxed{(p+1)(q+1)}$

How many factors 72 has?

$(3+1)\times(2+1) = 12$

$72 = 2^3 \times 3^2$

$(2^0+2^1+2^2+2^3)(3^0+3^1+3^2)$

$(1+2+4+8)(1+3+9) = 195$ Sum of factors of 72

15×13

$8=2^3$	$9=3^2$
$1=2^0$	$1=3^0$
$2=2^1$	$3=3^1$
$4=2^2$	$9=3^2$
$8=2^3$	
4 factors	3 factors

$1+2+4+8$
$3+6+12+24$
$9+18+36+72$

12 factors

Total factors

$$360 = 2^{3} \times 3^{2} \times 5^{1}$$

$(3+1)(2+1)(1+1)$

360
$= 36 \times 10$
$= 9 \times 4 \times 2 \times 5$
$= 3^2 \times 2^2 \times 2 \times 5$
$= 2^3 \times 3^2 \times 5^1$

$\Rightarrow (2^0+2^1+2^2+2^3)(3^0+3^1+3^2)(5^0+5^1)$

$\Rightarrow (1+2+4+8)(1+3+9)(1+5) = 1170$ (sum of factors of 360)

$\Rightarrow (1+2+4+8)(1+3+9+5+15+45)$

$\Rightarrow 1+3+9+5+15+45$
$2+6+18+10+30+90$
$4+12+36+20+60+180$
$8+24+72+40+120+360$

$(3+1)(2+1)(1+1) \Rightarrow 24$

$10800 \Rightarrow 108 \times 100 \Rightarrow 12 \times 9 \times 25 \times 4$
$\Rightarrow 2^2 \times 3 \times 3^2 \times 5^2 \times 2^2$
$\Rightarrow 2^4 \times 3^3 \times 5^2$
$\Rightarrow (4+1)(3+1)(2+1)$
$\Rightarrow 60$

Type ② Sum of the Factors

factorized form $\boxed{N = a^p \times b^q}$

$$\Rightarrow \underbrace{(a^0+a^1+a^2+\dots a^p)}_{(p+1)\text{ term}} \underbrace{(b^0+b^1+b^2+\dots b^q)}_{(q+1)\text{ term}}$$

$$\Rightarrow (1+a+a^2+\dots a^p)(1+b+b^2+\dots b^q)$$

$$\Rightarrow 1\frac{(a^{p+1}-1)}{(a-1)} \times 1\frac{(b^{q+1}-1)}{(b-1)}$$

$$\Rightarrow \boxed{\left(\frac{a^{p+1}-1}{a-1}\right) \times \left(\frac{b^{q+1}-1}{b-1}\right)}$$

Type ③ Prime Factors — Count Prime no in factorial form.

Type ④ Composite Factors

$\Downarrow$ $\boxed{\text{Total factor} - \left(\text{No. of prime factors} + 1\right)}$

Q1 How many factors of 72 are prime?

① $72 = \underline{2}^3 \times \underline{3}^2$

$= (2^0+2^1+2^2+2^3)(3^0+3^1+3^2)$

$= (1+2+4+8)(1+3+9)$

$= 1+2+4+8$

$3+6+12+24$

$9+18+36+72$

2 prime factors

② $360 = \underline{2}^3 \times \underline{3}^2 \times \underline{5}^1$ — 3 prime factors

$2^1, 3^1, 5^1$

③ $10800 = \underline{2}^4 \times \underline{3}^4 \times \underline{5}^2$ — 3 prime factors

$= 2, 3, 5$

④ $N = (60)^2 \times (42)^3$

$\Rightarrow (2^2 \times 3 \times 5)^2 (2 \times 3 \times 7)^3$

$\Rightarrow \underline{2}^7 \times \underline{3}^5 \times \underline{5}^2 \times \underline{7}^3$

4 prime factors

Q. How many factors of 72 are Composite number.

$\Rightarrow \quad 72 = 2^3 \times 3^2$

Total factors = $(3+1)(2+1) \Rightarrow 12$

prime factors = 2 (2, 3)

Composite factors = $12 - (2+1) = 9$

(2) $360 = 2^3 \times 3^2 \times 5^1$

Composite = Total factor $\Rightarrow (3+1)(2+1)(1+1) = 24$

$-$(prime + 1) $\Rightarrow (3+1) = -4$

Composite 20

Type 5 Even factors $\Rightarrow$ $\boxed{\text{Total factors} - \text{odd factors}}$

Type 6 Odd factors

$\downarrow$

$\boxed{\text{Factorial Form}} \rightarrow$ Delete 2^n

{ get total factors of the rest }

Ex. $360 = \cancel{2^3} \times 3^2 \times 5^1$

$(2+1)(1+1) = 6$ odd factors

Ex. $10800 = \cancel{2^4} \times \boxed{3^3 \times 5^2}$

$(3+1)(2+1)$

$4 \times 3 \Rightarrow 12$ factors (odd)

Even = Total $-$ odd

$(4+1)(3+1)(2+1) - 12$

$60 - 12 = 48$

Type (7) Perfect Square $N = a^p \times b^q$

$\frac{p}{2}$ $\frac{q}{2}$

$(Q_1+1)(Q_2+1)$

Perfect Square

Type (8) Perfect Cube

$\frac{p}{3}$ $\frac{q}{3}$

$(Q_1+1)(Q_2+1)$

Q1 How many factors of 72 are perfect Square no's?

$\frac{3}{2} = (1+1)(1+1) = 4$

$72 \Rightarrow 2^3 \times 3^2$

$\Rightarrow (2^0+2^1+2^2+2^3)(3^0+3^1+3^2)$

$\Rightarrow (1+2+4+8)(1+3+9)$

$\Rightarrow 1+2+4+8$

$3+6+12+24$

$9+18+36+72$

Q. $N = 2^7 \times 3^5$

$\frac{7}{2} \Rightarrow (3+1)$ $\frac{5}{2} \Rightarrow (2+1)$ $\Rightarrow (3+1)\times(2+1)$

$\Rightarrow 12$

$(2^0+2^2+2^4+2^6)(3^0+3^2+3^4)$

4×3

12 factors {perfect Square}

Q. $N = 2^{10} \times 3^{11}$

P Square	P Cube
$\frac{10}{2}$ $\frac{11}{2}$	$\frac{10}{3}$ $\frac{11}{3}$
$(5+1)(5+1)$	$(3+1)(3+1)$
6×6	4×4
36	16

Type (9)

a, b are 2 diff factors of 72
Such that

$\boxed{a \times b = 72}$

How many such pairs (a, b) exists?

Total pairs

$= \frac{\text{Total factor}}{2}$

$= \frac{12}{2} = 6$ pairs

$a \times b = 72$
$1 \times 72 = 72$
$36 \times 2 = 72$
$18 \times 4 = 72$
$9 \times 8 = 72$
$24 \times 3 = 72$
$12 \times 6 = 72$

Perfect sq no

Even Even

☆ $36 = 6^{2} = 2^{2} \times 3^{2}$

Total factors $= (2+1)(2+1) = 9$ odd

$(1+2+4)(1+3+9)$

$1+2+4$
$3+6+12$
$9+18+36$

NOTE

☆ If a number is a perfect square no then its total number of factors are always odd in number and vice-versa.

Q 1000 Door { each student go to the multiples of his roll no.
- If door is found close → Pls open
- open → Pls close

	1	2	3	4	5	6	-------	1000
1	✓	✓	✓	✓	✓	✓	-------	✓
2		×		×		×		×
3			×			✓		
4				✓				
5					×			
6						×		
⋮								
1000								

After the entire exercise

(1) how many doors are found opened? → 31

* Perfect square open rahega (odd no. of factors)
* Baki close (even factors)

$1^2, 2^2, 3^2, 4^2, 5^2 \text{ ------ } 31^2$

$1 \quad 4 \quad 9 \quad 16 \quad 25 \qquad 961$

(2) highest gate no. open? 961

$31^2 = 961$

FACTORS

$7200 = 72 \times 100 = 12 \times 6 \times 100 = 2^5 \times 3^2 \times 5^2$

① Number of factors.
$\Rightarrow 6 \times 3 \times 3 \Rightarrow 54$

② Sum of all factors.
$\Rightarrow (2^0+2^1+2^2+2^3+2^4+2^5)(3^0+3^1+3^2)(5^0+5^1+5^2)$

③ Number of even factors
$\Rightarrow$ Total factors − odd factors
$\Rightarrow 54 - 9 = 45$
or just $5(2+1)(2+1) = 45$

④ Sum of even factors (Delete 2^0)
$\Rightarrow (2^1+2^2+2^3+2^4+2^5)(3^0+3^1+3^2)(5^0+5^1+5^2)$

⑤ Number of odd factors (Delete 2^n)
$\cancel{2} \times 3^2 \times 5^2$
$\Rightarrow (2+1)(2+1) = 9$

⑥ Sum of odd factors {Delete all ② ki power}
$\Rightarrow (2^0)(3^0+3^1+3^2)(5^0+5^1+5^2)$

⑦ Number of factors divisible by 25
$\hookrightarrow 5^2$
$\Rightarrow 5^2 (2^5 \times 3^2)$
$(5+1)(2+1)$
$\Rightarrow 18$

⑧ Sum of factors divisible by 25
$\hookrightarrow (2^0+2^1+2^2+2^3+2^4+2^5)(3^0+3^1+3^2)(\cancel{5^0}+\cancel{5^1}+5^2)$

bcoz for divisibility we need atleast 5^2

or
$(2^0+2^1+2^2+\ldots 2^5)(3^0+\ldots 3^2)(5^2)$
$^6C_1 \cdot {}^3C_1 \cdot 1 \Rightarrow 18$
Jese karna hai humko

In type ke question mei hum pehle jo no. pucha hai usko prime factor mei tod ke alag kar lete hai jo bhi given no. ka prime factorization hai. Uske baad solve karte hai

Ex1 Find the number of and sum of factors of 700 which are divisible by ~~700~~ 35

$700 = 2^2 \times 5^2 \times 7^1$
$= 5 \times 7 (2^2 \times 5^1)$
$= 35 (2^2 \times 5^1)$

No. of factors which are divisible by 35
$\Rightarrow \cancel{(2+1)(1+1)} \Rightarrow 6$
$\Rightarrow 3 \times 2 \times 1 \Rightarrow 6$
$3 \times 2 \times 1$

Sum of factors $\Rightarrow (2^0+2^1+2^2)(\cancel{5^0}+5^1+5^2)(7^1)$ → atleast one 5 hona chahiye
$\Rightarrow \cancel{42}$ 1470

Ex-8 Find the number of factors of the value
$2^3 \times 3^6 \times 5^2 \times 7^1$
which are divisible by 50 but not divisible by 100.

$\Rightarrow$ $2^3 \times 3^6 \times 5^2 \times 7^1$

$50 = 2^1 \times 5^2$

$2^1 \times 5^2 \, (\cancel{2^2} \times 3^6 \times 7^1)$

50

isko isliye hataya kyunki agar ek bhi (2) aaya toh woh 100 se divisible ho jayega.

→ .ko alag hum atleast condition ke liye bhi use kar sakte hai matlab pata karne ke liye

Factors div by 50 but not by 100 $\} = 7 \times 2 = 14$

(9) Number of factors divisible by 40

I method

$4 \times 10 \Rightarrow 2^3 \times 5^1$

$2^3 \times 5^1 \, (2^2 \times 3^2)$

~~(3+1)(2+1)(1+1) → (12)(11)(24)~~

$(2+1)(2+1)(1+1) \Rightarrow 18$

atleast

$(\cancel{2^0} + \cancel{2^1} + \cancel{2^2} + 2^3 + 2^4 + 2^5)(3^0 + 3^1 + 3^2)(\cancel{5^0} + 5^1 + 5^2)$

atleast

II method

${}^3C_1 \times {}^3C_1 \times {}^2C_1$

$3 \times 3 \times 2 \rightarrow 18$

(10) Sum of factors divisible by 40.

we need atleast three 2 and one 5. Thus if any term will have $(2^4 \cdot 5^2)$ then this term will obviously be divisible by 40 because it already of three 2's and one 5.

(11) Number of factors which are divisible by 150.

1st method

$3 \times 5 \times 2 \times 5 = 2^1 \times 3^1 \times 5^2$

$2^1 \times 3^1 \times 5^2 \, (2^4 \times 3^1)$ } Or II method

$(4+1)(1+1)(1)$

$5 \times 2 \times 1 = 10$

(12) Sum of the factors which are divisible by 150

$2^1 \times 3^1 \times 5^2$

must contain atleast one (2), one (3), two (5).

$7200 = 2^5 \times 3^2 \times 5^2$

$(2^1 + 2^2 + 2^3 + 2^4 + 2^5)(3^1 + 3^2)(5^2)$

${}^5C_1 \times {}^2C_1 \times {}^1C_1$

$5 \times 2 \times 1$

$\rightarrow 10$

Q. Find the ~~number~~ sum of divisors of 544 which are perfect squares.

$\Rightarrow$ $544 \Rightarrow 2^5 \times 17^1$

Sum $\Rightarrow (2^0+2^2+2^4)(17^0) \Rightarrow$ 21 Ans

Q. How many factors of $\underbrace{4^5 \times 5^4 \times 6^3}_{\Rightarrow 2^{13} \times 3^3 \times 5^4}$ are perfect squares but not perfect cubes?

$\Rightarrow$ Sum $\Rightarrow \underbrace{(2^0+2^2+2^4+\dots 2^{12})}_{7} \underbrace{(3^0+3^2)}_{2} \underbrace{(5^0+5^2+5^4)}_{3}$

$7 \times 2 \times 3 \Rightarrow$ 42 terms

$\hookrightarrow$ factors (perfect squares)

out of these, three factors viz: $2^0 \times 3^0 \times 5^0$, $2^6 \times 3^0 \times 5^0$ and $2^{12} \times 3^0 \times 5^0$ are perfect cubes too.

Req factors $\Rightarrow$ 42 $-$ 3

$\Rightarrow$ 39

Type (10) Product of Factors

$\Rightarrow (N)^{\frac{\text{Total Factors}}{2}}$

Q Product of factors of (72) ?

$72 = 2^3 \times 3^2 \Rightarrow (3+1)(2+1) \Rightarrow 12$

$(1+2+4+8)\ (1+3+9)$

$(1\times72)\times(2\times36)\times(24\times3)\times(18\times4)\times(12\times6)\times(9\times8)$

$(72)^6$

Q $36 = 6^2 = 2^2 \times 3^2 \longrightarrow (36)^{9/2} = (6^2)^{9/2}$

$= 6^{2\times 9/2}$

$= 6^9$

Remainder Concept

- general
- Binomial
  - $x+1$
  - $x-1$
  - $x+y$

$\frac{\cancel{10}}{\cancel{6}}\ \frac{5}{3}$

$\frac{2139 \times 10^{40}}{5 \times 10^{40}}$

$4 = 2 \times 2$

4×10^{40}

(Jis no. se cancel kiya hai woh add karo ans ke)

$\Downarrow$

Right ans.

$$\frac{N^P}{x}$$

$\frac{(x+1)^P}{x}$ $\frac{(x-1)^P}{x}$ $\frac{(x+y)^P}{x}$

① $\frac{7^{40}}{6} = \frac{(6+1)^{40}}{6}$ ② $\frac{5^{40}}{6} = \frac{(6-1)^{40}}{6}$ ③ $\frac{10^{40}}{6} = \frac{(6+4)^{40}}{6}$

$$\boxed{(x+1)^{\overset{\text{integer}}{P}} = xQ+1}$$

① $\frac{7^{60}}{342} = \frac{(7^3)^{20}}{342} = (343)^{20} = (342+1)^{20} = \frac{342Q+1}{342}$
$= 1$

② $\frac{7^{100}}{342} = 7^1(7^{99}) = 7^1(7^3)^{33} \Rightarrow 7(343)^{33}$
$\Rightarrow 7(342+1)^{33}$
$\Rightarrow 7(342Q+1)$
$\Rightarrow \frac{\cancel{7\times342Q}+7}{\cancel{342}} \Rightarrow 7$ Ans

③ $\frac{13\times7^{25}}{48} = 13\times7^1(7)^{24} = 91(7^2)^{12}$
$= 91(49)^{12}$
$= 91(48+1)^{12}$
$= 91(48Q+1)$
$\Rightarrow \frac{\cancel{91\times48Q}+91}{\cancel{48}}$
$\Rightarrow \frac{91}{48} = 43$ Ans

④ $\frac{(15)^{60}}{7} = (14+1)^{60}$
$= 14Q+1$
$= 7(2Q)+1$
$= 1$

$(x-1)^{P}$

$(x-1)^{\text{even}}$ — $\boxed{xQ+1}$

$(x-1)^{\text{odd}}$ — $\boxed{xQ-1}$

Rem	Ans
-1	$x-1$
$-y$	$x-y$

Ex:- (1) $\frac{4^{10}}{5} \Rightarrow \frac{(5-1)^{10}}{5} = \frac{5Q+1}{5}$ (1)

(2) $\frac{4^{3}}{5} \Rightarrow \frac{(5-1)^{3}}{5} = 5^{3} - 3\cdot 5^{2} + 3\cdot 5 - 1$

$= 5(5^{2} - 3\cdot 5 + 3) - 1$

$= 5(13) - 1$

$= 5(12+1) - 1$

$= \frac{\cancel{5(12)} + 5 - 1}{\cancel{5}}$

$= 5 - 1$

$= 4$

$$\frac{(x-1)^{odd}}{x} = xQ-1$$

$$= x[(Q-1)+1]-y$$

$$= \frac{\cancel{x(Q-1)}+\boxed{x-y}}{\cancel{x}}$$

$$\frac{6^{100}}{7} = (7-1)^{100} = \frac{7Q+1}{7} = 1$$

even

$$\frac{6^{25}}{7} = (7-1)^{25} = \frac{7Q-1}{7} \Rightarrow -1 \Rightarrow 7-1$$

$$\Rightarrow 6$$

odd

pahle (–) ke time dekhni hai

$$\frac{7^{50}}{50} = (7^2)^{25} = (49)^{25} = (50-1)^{25} = \frac{50Q-1}{50}$$

odd

$$\Rightarrow -1$$

$$\Rightarrow 50-1 \Rightarrow 49$$

H.W $\frac{7^{61}}{344} \Rightarrow \frac{7\times7^{60}}{344} = \frac{7\times(7^3)^{20}}{344} = \frac{7\times(344-1)^{20}}{344}$ 20 → even

$$\Rightarrow \frac{7(344Q+1)}{344} \Rightarrow \frac{\cancel{7\times344Q}+7}{\cancel{344}} \Rightarrow 7 \text{ Ans}$$

24/02/

Q1 $\frac{7^{100}}{344} \Rightarrow \frac{7^{1}(7^3)^{33}}{344} = \frac{7(343)^{33}}{344} = \frac{7(344-1)^{33}}{344}$ odd

$\Rightarrow \frac{7(344Q-1)}{344} \Rightarrow \frac{7\times 344Q-7}{344} \Rightarrow -7$

$\Rightarrow 344-7 \Rightarrow 337$

Q. $\frac{(13)^{60}}{7} \Rightarrow \frac{(14-1)^{60}}{7} = \frac{14Q+1}{7} = 1$ (60 even)

$\boxed{(x+y)^p} \rightarrow xQ+y^p$

$(x-y)^p$

$[x+(-y)]^p \rightarrow xQ+(-y)^p$

Ex1 $\frac{9^{60}}{7} = (7+2)^{60} = \frac{\cancel{7Q}+2^{60}}{\cancel{7}}$

$\frac{2^{60}}{7} = (2^3)^{20} = (8)^{20}$

$= (7+1)^{20}$

$= \frac{7Q+1}{7}$

$\Rightarrow 1$

Ex1 $\frac{(10)^{100}}{7} = \frac{(7+3)^{100}}{7} = \frac{\cancel{7Q}+3^{100}}{\cancel{7}}$

$\Rightarrow \frac{3^{100}}{7} = \frac{(3^2)^{50}}{7} = \frac{(9)^{50}}{7} \Rightarrow \frac{(7+2)^{50}}{7} \Rightarrow \frac{\cancel{7Q}+2^{50}}{\cancel{7}}$

$\Rightarrow \frac{2^{50}}{7} = \frac{2^2(2^3)^{16}}{7} = \frac{2^2(8)^{16}}{7} \Rightarrow \frac{2^2(7+1)^{16}}{7} = 2^2\frac{(7Q+1)}{7}$

$\Rightarrow \frac{\cancel{2^2 7Q}}{\cancel{7}} + 2^2$

$\Rightarrow \frac{4}{7} \Rightarrow 4$

Ex: $\frac{3^{100}}{7} = 3^1(3^3)^{33} = 3(27)^{33}$

$= 3(28-1)^{33}$

$= 3[28Q + (-1)^{33}]$

$= 3[28Q - 1]$

$= \frac{3 \times \cancel{28Q} - 3}{\cancel{7}}$

$= -3$

$= 7-3 \Rightarrow 4$

Other Approach

Ex: $\frac{10^{100}}{7} \Rightarrow 10^1(10^3)^{33}$

$\Rightarrow 10(1000)^{33}$

$\Rightarrow 10(1001-1)^{33}$

$\Rightarrow 10(1001Q-1)$

$\Rightarrow \frac{10 \times 1001Q - 10}{7}$

$\Rightarrow -3$

$\Rightarrow 7-3$

$\Rightarrow 4$

For ouen purpose

Remainder theorem

$\frac{a \times b \times c}{n} = \frac{a_n \times b_n \times c_n}{n}$

Ex: Find the rem of $15 \times 17 \times 19$ when divided by 7.

$\Rightarrow \frac{15 \times 17 \times 19}{7}$

$\Rightarrow \frac{1 \times 3 \times 5}{7} = \frac{15}{7} = \frac{1}{7} \Rightarrow 1$

Q Prime no. $P>1000$

$$\frac{(P)^{3456}}{6} \rightarrow \text{rem} = ?$$

$$\frac{(6K+1)^{3456}}{6} \qquad \frac{(6K-1)^{3456}}{6}$$

$$\cancel{\frac{6QK+1}{6}} \qquad \cancel{\frac{6QK+1}{6}}$$

Ans

1 — 1 — 1

Q Unit place

only this matters

this also for checking even/odd

$$\frac{(123)^{4567^{6789^{12^{34^{56^{78^{90}}}}}}}}{4}$$

3^R (Ans)

$$\frac{(4567)^{6789^{12^{34^{56^{78^{90}}}}}}}{4}$$ odd

$$(4568-1)^{odd}$$

$$\Rightarrow \frac{4568Q-1}{4}$$

$$\Rightarrow -1 \Rightarrow 4-1 \Rightarrow 3$$

$$3^3 \Rightarrow 7 \text{ Ans}$$

Q:

$7^{7^{7^{7^{7^{7^{7^{7^{7^{7 \cdots \infty}}}}}}}}}$ / 4

7^R

7^3

343

3 Ans

$\frac{(7)^{7^{7^{7 \cdots \infty}}}}{4} = \frac{(8-1)^{7^{7^{7}}}}{4}$ → odd

$= (8-1)^{odd}$

$= \frac{8Q-1}{4}$

$= -1$

$\Rightarrow 4-1 \Rightarrow 3$

Q: $\frac{14!}{17}$

$\frac{16!}{17} = \frac{16 \times 15 \times 14!}{17}$ (−1) (−2)

$16 = 2 \times \frac{14!}{17}$

$\frac{14!}{17} = 8$

(Jo aata hai uske LHS mai likho)

(Jo puchha hai woh dikhna chahiye) (RHS)

Proof

$(p-1)! \bmod p = (p-1)$

$[(p-1)(p-2)!] \bmod p = (p-1)$

$[(p-1) \bmod p] * [(p-2)! \bmod p] = (p-1)$

$(p-1) * [(p-2)! \bmod p] = (p-1)$

$\boxed{(p-2)! \bmod p = 1}$

Q:

$\frac{N}{85}$, $R = 39$

$\frac{N}{17}$, $R = ?$

$85 \overline{)N(} Q_1$

39

$\frac{N}{17} = \frac{85 Q_1 + 39}{17}$

$\frac{39}{17} = 5$ Ans

Yeh khud se kiya hai

★ Fermats Theorem

$\frac{a^{p-1}}{p} \Rightarrow$ remainder "1"

p is a prime number,
a, p are co-prime each.

Ex: ① $\frac{26^{57}}{29} \Rightarrow \frac{26^{28} \times 26^{28} \times 26^1}{29} \Rightarrow \frac{1 \times 1 \times 26}{29} \Rightarrow \frac{26}{29} = 26$

② $\frac{5^{119}}{59} \Rightarrow \frac{5^{58} \cdot 5^{58} \times 5^3}{59} \Rightarrow \frac{1 \cdot 1 \cdot 125}{59} \Rightarrow \frac{125}{59}$
$\Rightarrow 7$

★ Wilsons Theorem

$\frac{(p-1)!}{p} \Rightarrow$ remainder "−1" or $(p-1)$

p is a prime number.

① $\frac{21!}{22} = 22-1 = 21$

$\frac{(p-2)!}{p} \Rightarrow$ remainder "1"

① $\frac{5!}{7} \Rightarrow 1$

BASE CONCEPTS

	Base (N)	digits allowed to use (upto $N-1$)
Decimal	10	0,1,2,3,4,5,6,7,8,9
Binary	2	0,1
	3	0,1,2
	4	0,1,2,3
	⋮	
	9	0,1,2,3,4,5,6,7,8
	11	0,1,2,3,4,5,6,7,8,9,~~10~~ A
	⋮	
	16	0,1,2,3,4,5,6,7,8,9,~~10~~ A,~~11~~ B,~~12~~ C,~~13~~ D,~~14~~ E,~~15~~ F,~~16~~

Ex: $(45\cancel{8})_7$ number invalid

$(\quad)_{any\ Base} \longrightarrow (\quad)_{10}$

Ex→ ① $\overset{8^2}{(1}\overset{8^1}{2}\overset{8^0}{3})_8 \longrightarrow (83)_{10}$

$1\times 8^2 + 2\times 8^1 + 3\times 8^0$

$64 + 16 + 3$

83

② $(\overset{2^5}{1}\overset{2^4}{0}\overset{2^3}{1}\overset{2^2}{1}\overset{2^1}{0}\overset{2^0}{1})_2 \longrightarrow (45)_{10}$

$1\times 2^5 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0$

$2^5 + 2^3 + 2^2 + 2^0$

$32 + 8 + 4 + 1$

45

③ $(\overset{7^2}{3}\overset{7^1}{4}\overset{7^0}{5})_7 \longrightarrow (180)_{10}$

$3\times 7^2 + 4\times 7^1 + 5\times 7^0$

$147 + 28 + 5$

180

④ $(\overset{16^2}{A}\overset{16^1}{1}\overset{16^0}{E})_{16} \rightarrow (2591)_{10}$

$16^2\times A + 16^1\times 1 + 16^0\times E$

$16^2\times 10 + 16^1\times 1 + 16^0\times 15$

$2560 + 16 + 15$

2591

$(\quad)_{10} \longrightarrow (\quad)_{\text{any Base}}$

Ex 1 ① $(83)_{10} \longrightarrow (123)_8$

8	83	
8	Q_1	R_1
8	Q_2	R_2
	Q_3	R_3

Bottom to up

8	83	
8	10	3
	1	2

Ex ② $(35)_{10} \longrightarrow (100011)_2$

2	35	
2	17	1
2	8	1
2	4	0
2	2	0
	1	0

Ex ③ $(141)_{10} \longrightarrow (261)_7$

7	141	
7	20	1
	2	6

Rukna Jab hai Tab yeh 7 se chota nahi aa jaye.

Ex ④ $(1311)_{10} \longrightarrow (51F)_{16}$

16	1311	
16	81	~~15~~ F
	5	1

Addition

$$
\begin{array}{r}
\begin{pmatrix} 3 & 4 & 5 \\ 2 & 3 & 1 \end{pmatrix}_{10} \\
+ \quad \\
\hline
5 \;\; 7 \;\; 6 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
\begin{pmatrix} 3 & 4 & 5 \\ 2 & 3 & 1 \end{pmatrix}_{8} \\
+ \quad \\
\hline
(5 \;\; 7 \;\; 6)_8
\end{array}
$$

$$
\begin{array}{r}
1 \quad\;\; \\
\begin{pmatrix} 3 & 4 & 5 \\ 2 & 3 & 7 \end{pmatrix}_{10} \\
+ \quad \\
\hline
5 \;\; 8 \;\; 2 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
1 \qquad\quad 1 \quad\;\; \\
\begin{pmatrix} 1 & 3 & 4 & 5 \\ 3 & 5 & 1 & 6 \end{pmatrix}_{8} \\
+ \quad \\
\hline
5 \;\; 0 \;\; 6 \;\; 3 \\
\hline
\end{array}
$$

$$
10 \,\overline{)\,12\,}\,(1 \qquad\qquad 8 \,\overline{)\,8\,}\,(1 \qquad\qquad 8 \,\overline{)\,11\,}\,(1
$$

$$
\begin{array}{r} 10 \\ \hline 2 \end{array} \qquad\qquad\qquad \begin{array}{r} 8 \\ \hline 0 \end{array} \qquad\qquad\qquad \begin{array}{r} 8 \\ \hline 3 \end{array}
$$

Subtraction

$$
\begin{array}{r}
-\left(\begin{array}{ccc} 4 & 5 & 6 \\ 2 & 1 & 3 \end{array}\right)_{10} \\
\hline
2 \quad 4 \quad 3 \\
\hline
\end{array}
\qquad
\begin{array}{r}
-\left(\begin{array}{ccc} 4 & 5 & 6 \\ 2 & 1 & 3 \end{array}\right)_{8} \\
\hline
(2 \quad 4 \quad 3)_8 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
\quad 1=10 \qquad 1=10 \\
-\left(\begin{array}{ccc} \overset{5}{\cancel{6}} & \underset{3}{\cancel{4}} & 3 \\ 2 & 6 & 7 \end{array}\right)_{10} \\
\hline
3 \quad 7 \quad 6 \\
\hline
\end{array}
$$

$10+3=13$
$13-7=6$

$1=8$ {1 carry li aur woh base ke barabar hoti hai}

$$
\begin{array}{r}
1=8 \qquad\qquad 1=8 \\
-\left(\begin{array}{cccc} \overset{5}{\cancel{6}} & \underset{12}{4} & \overset{4}{\cancel{5}} & 2 \\ 1 & 5 & 2 & 7 \end{array}\right)_{8} \\
\hline
4 \quad 7 \quad 2 \quad 3 \\
\hline
\end{array}
$$

Q1 $(7526)_8 - (Y)_8 = (4364)_8$

Find Y

$(7526)_8 - (4364)_8 = (Y)_8$

$1 = 8$ $\quad (8+2)-6$ $\quad 10-6=4$

$$
\begin{array}{r}
\left(\begin{array}{cccc} 7 & \cancel{5}_4 & 2 & 6 \\ 4 & 3 & 6 & 4 \end{array}\right)_8 \\
- \quad \\
\hline
(3 \quad 1 \quad 4 \quad 2)_8
\end{array}
$$

$$\boxed{Y = 3142}$$

Q2

$$
\begin{array}{r}
{}^{1}\,{}^{1} \quad \\
\left(\begin{array}{ccc} 1 & 3 & 7 \\ 2 & 7 & 6 \end{array}\right)_8 \\
+ \quad \\
\hline
4 \quad 3 \quad 5
\end{array}
\qquad
\begin{array}{r}
{}^{1} \quad \\
1\left(\begin{array}{ccc} 7 & 3 & 1 \\ 6 & 7 & 2 \end{array}\right)_8 \\
+ \quad \\
\hline
1 \quad 6 \quad 2 \quad 3
\end{array}
$$

$\frac{11}{8}$ $Q=1$ $R=3$

$8\overline{)13}(1$

$\underline{8}$

5

$\frac{14}{8}$ $Q=1$ $R=6$

$\frac{10}{8}$ $Q=1$ $R=2$

LCM / HCF

Highest Power

LCM → Least Common Multiple

$2^3 \times 3^2$
$= 72$ LCM of 24, 36

GCD Greatest common divisor

HCF Highest Common Factor

Lowest power

$2^2 \times 3^1$
$= 12$ HCF of 24, 36

24 36

$2^3 \times 3^1$ $2^2 \times 3^2$

Workbook (8)

10th	15th
10	15
20	30 sec
30	45
40	60
50	
60	

Total $\Rightarrow \dfrac{60\text{ min} \times 60\text{ sec} \times 24}{30}$

$\Rightarrow$ 120 times × 24 hrs

$\Rightarrow$ 2880 Ans

NOTE
* Common values find karni ho toh LCM ka use hoga.

LCM

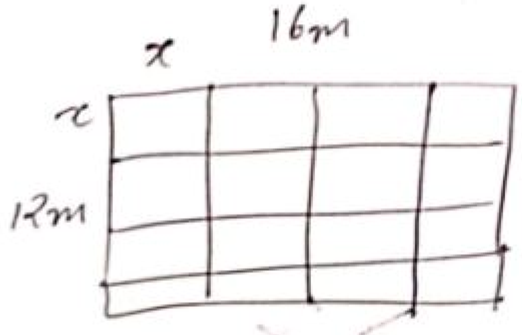
H.C.F — Biggest things are divided into smaller parts

Ex1

	Coke	Pepsi
	$\boxed{24 \text{ litre}}$	$\boxed{36 \text{ litre}}$
	$\boxed{1}$ $\boxed{1}$ $\boxed{1}$ ---- 24	$\boxed{1}$ $\boxed{1}$ $\boxed{1}$ ---- 36 = 60
	$\boxed{2}$ $\boxed{2}$ $\boxed{2}$ ---- 12	$\boxed{2}$ $\boxed{2}$ $\boxed{2}$ .. 18 = 30
highest no.	$\boxed{x}$ $\boxed{x}$ $\boxed{x}$ ----	$\boxed{x}$ $\boxed{x}$ $\boxed{x}$ ..
	$\Downarrow$ $\Downarrow$ $\Downarrow$	$\Downarrow$ $\Downarrow$ $\Downarrow$
	$\boxed{12}$ $\boxed{12}$ $\boxed{12}$ ---- 2	$\boxed{12}$ $\boxed{12}$ $\boxed{12}$ .. 3 = 5

5 glasses of 12 litres

Ex2



All tiles are of same size. Total tiles in given area of what size.

$\frac{16}{x}$ $\frac{12}{x}$ $\Rightarrow$ 4 (HCF)

total = 12

size = 4m

LCM {Applications}

$LCM(12, 15) = 60$

$2^2 \times 3^1 \quad 3 \times 5$

$2^2 \times 3 \times 5$

60

Type ①

(Remainder not given)

$\frac{N}{12} \qquad \frac{N}{15}$

60 → a) Least number

120

180 → b) III Smallest number

240 → c) greatest ③ digit number

300 d) Least ④ digit number

⋮

900

960

1020

1080

⋮

Greatest ③ digit no.

↓

actual ③ digit.

$$\begin{array}{r|l} LCM\ 60 & 999 \ (16 \\ & \underline{60} \\ & 399 \\ & \underline{360} \\ & 39 \end{array}$$

999 − 39

= 960

Least 4 digit no.

= 1 digit less

= $\boxed{\text{greatest ③ digit no.} + \text{LCM}}$

= 960 + 60

= 1020

Type (2)

Remainder given and it is same

$\frac{N}{12}$ $\frac{N}{15}$

Rem 7 7

Solve as Type (1) and add 7 (remainder) in the answers.

(a) Least no. ⇒ 60 + 7 ⇒ 67

(b) II smallest no. ⇒ 180 + 7 ⇒ 187

(c) Greatest (3) digit no. ⇒ 960 + 7 ⇒ 967

(d) Least (4) digit no. ⇒ 1020 + 7 ⇒ 1027

Type (3)

diff Remainder are given but diff is same

diff $\frac{N}{12}$ Rem 4 → 8 diff $\frac{N}{15}$ 7 → 8

diff same

Solve as Type (1) and ~~add~~ subtract difference in the answers.

(a) 60 − 8 = 52

(b) 180 − 8 = 172

(c) 960 − 8 = 952

(d) 1020 − 8 = 1012

(N) coins

$\frac{N}{2}$ $\frac{N}{3}$ $\frac{N}{4}$ $\frac{N}{5}$ → $60-1 = 59$

$\frac{1}{1}$ $\frac{2}{1}$ $\frac{3}{1}$ $\frac{4}{1}$

Type (4)

when Rem are given and they are not same and also the diff is not same.

	$\frac{N}{12}$	$\frac{N}{15}$
Rem	5	11

$12\times1+5 = \cancel{17} \rightarrow \frac{17}{15} = 2$

$12\times2+5 = \cancel{29} \rightarrow \frac{29}{15} = 14$

$12\times3+5 = 41 \rightarrow \frac{41}{15} = 11$ ✓

$12\times4+5 = 53$

(a) Smallest no. ← 41

41 + 60

(b) 3rd smallest no. ← 41 + 120 = 161

41 + 180

(c) greatest (3) digit no. ← 941

(d) Least (4) digit no. ← 1001

41 + 900 = 941

41 + 960 = 1001

41 + 1020 = 1061

* main part is to find out 1st no. by hit and trial method.

2 numbers (a, b)

$$a \times b = \underset{(a,b)}{LCM} \times \underset{(a,b)}{HCF}$$

$$LCM\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{LCM(a, c)}{HCF(b, d)}$$

$$HCF\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{HCF(a, c)}{LCM(b, d)}$$

Q1 $LCM\left(\frac{10}{3}, \frac{35}{6}\right)$

$10 = 2 \times 5$
$35 = 5 \times 7$
$3 = 3 \times 1$
$6 = 2 \times 3$

$$\frac{LCM(10, 35)}{HCF(3, 6)} \Rightarrow \frac{70}{3}$$

Q1 $HCF\left(\frac{10}{3}, \frac{35}{6}\right)$

$$\frac{HCF(10, 35)}{LCM(3, 6)} \Rightarrow \frac{5}{6}$$

HCF of 2 numbers = 12
(a, b)

a b

12 x 12 y

HCF 12 × HCF 1

24x

24y

Q1 HCF of 2 number = 24. Find the numbers?
their sum = 144

24x 24y
(x, y)
HCF = 1

$$24x + 24y = 144$$

$$24(x+y) = 144$$

$$\boxed{x+y=6}$$

HCF (x, y)	$x+y=6$
1	1, 5
~~2~~	2, 4
~~3~~	3, 3

Numbers

24x	24y
24 × 1	24 × 5
24	120

For own purpose

* $n_1 \times n_2 = HCF \times LCM$

Q1 $HCF = 11$ Find out how many pair of such
$LCM = 242$ number exists.

$n_1 \times n_2 = HCF \times LCM$

$11x \times 11y = 11 \times 242$

$x \times y = 22$

$x \quad y$

1×22

2×11

Koi number hai jiska HCF 4 hai Toh hum keh sakte hai ki woh 2 number 4x, 4y format ke honge. aur x, y both co-prime honge kyunki hum HCF toh nikal chuke hai

$11x, 11y$ | $11x, 11y$
$11 \times 1, 11 \times 22$ | $11 \times 2, 11 \times 11$
$(11, 242)$ | $(22, 121)$
1st pair | 2nd pair

* LCM and HCF of fractions :

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}$$

* $$\text{HCF of fraction} = \frac{\text{HCF of numerator terms}}{\text{LCM of denominator terms}}$$

* $$\text{LCM of fraction} = \frac{\text{LCM of numerator terms}}{\text{HCF of denominator terms}}$$

Q: Find HCF and LCM of $\frac{1}{2}, \frac{2}{3}, \frac{3}{7}$

$$HCF = \frac{HCF(1,2,3)}{LCM(2,3,7)} = \frac{1}{42} \qquad LCM = \frac{LCM(1,2,3)}{HCF(2,3,7)} = \frac{6}{1} = 6$$

Q) Find the greatest number which will divide 215, 167 and 135 so as to leave the same remainder in each case.

diff b/w 135 and 167 = 32, while 167 and 215 is 48

Req Ans ⇒ HCF(32, 48) ⇒ 16 Ans

Q) What will be the least possible number of the planks, if three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length?

⇒ The least possible number of planks would occur when we divide each plank into a length equal to the HCF of 42, 49 and 63. The HCF of these numbers is clearly 7 and this should be the size of each plank.

No. of planks in this case ⇒ $42/7 + 49/7 + 63/7 = 6 + 7 + 9$

⇒ 22 planks

Q) Find HCF of $(3^{125}-1)$ and $(3^{35}-1)$

★ HCF of (a^m-1) and (a^n-1) is $\boxed{(a^{HCF\ of\ m,n}-1)}$

↳ $3^{HCF(125,35)}-1 \Rightarrow (3^5-1)$

Q) The least perfect square number which is divisible by 3, 4, 6, 8, 10 and 11.

$3,\ 2^2,\ 2^1\times3^1,\ 2^3,\ 2^1\times5^1,\ 11^1$ → The answer should have atleast one 3, three 2's, one 5 and one 11 for it to be divisible by 3, 4, 6, 8, 10 & 11

Further, each of the prime factors should be having an even power in order to be a perfect square. Thus, the correct answer will be, $\underbrace{3\times3\times2\times2\times2\times2\times5\times5\times11\times11}_{Ans}$

LCM/HCF

Q→ what is the smallest number which when increased by 6 is divisible by 36, 63 and 108?

LCM(36, 63, 108) ⇒ 756

Req ans ⇒ 756 − 6 ⇒ 750 **Ans**

Applications of HCF

↳ ① To split things into smaller sections.

② To equally distribute any number of sets of items into their largest grouping.

③ To figure out how many people we can invite.

④ To arrange something into rows or groups.

LCM

↳ ① About an event that is or will be repeating over and over.

② To purchase or get multiple items in order to have enough.

③ To analyze when something will happen again at the same time.

Q→ The greatest no. which will divide 4003, 4136, 4249

↳ HCF(4249, 4003, 4136) ⇒ Ans

Q8 A forester ~~wants to~~ plant 44 apple trees, 66 ~~banana trees~~ and 110 mango-trees in equal rows (in terms of number of trees). Also, he wants ~~to~~ make distinct rows of trees (i.e., only one type of tree in one row). The no. of rows (min) that are req are.

~~44~~ HCF(44, 66, 110) ⇒ 22

(Ans→ $\frac{44}{22} + \frac{66}{22} + \frac{110}{22}$

agar ques tiles wala hota.
Toh size of tile
↳ LCM.
aur no. of tiles = $\frac{\text{[illegible]}}{\text{LCM}}$.

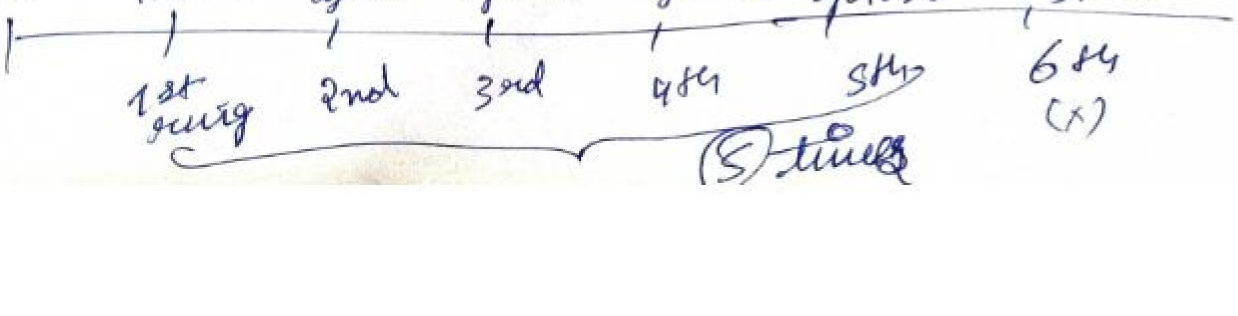
Q7 Four bells toll together at 9:00 a.m. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?

⇒ LCM(7, 8, 11, 12) = 1848 ~~seconds~~ ⇒ 30 min 48 sec.
Hence, the 4 bells would toll together every 30 min 48 sec.

No. of times they toll together in the next 3 hours

$\Rightarrow \frac{3 \times 60 \times 60}{1848} \Rightarrow 5$ tim[es]

or



Q1 Find the unit digit of

$$1!+2!+3!+4!+5!+\cdots\cdots+999!$$

$$\begin{array}{rr}
1! & 1 \\
+2! & 2 \\
+3! & 6 \\
+4! & 24 \\
\hline
+5! & 120 \\
\vdots & 720 \\
\vdots & 0 \\
\vdots & 0 \\
\vdots & 0 \\
999! & \vdots \\
\hline
 & 3
\end{array}$$

3 Ans

Q1

$$\frac{\overset{1+2+6+24}{1!+2!+3!+4!}+\cancel{5!}+\cdots\cancel{999!}}{\cancel{5}}$$

$$\frac{33}{5} \Rightarrow 3$$ Ans

*NOTE { for own purpose }

(1) **Squares of Numbers**

$1^2 = 1$	$11^2 = 121$
$2^2 = 4$	$12^2 = 144$
$3^2 = 9$	$13^2 = 169$
$4^2 = 16$	$14^2 = 196$
$5^2 = 25$	$15^2 = 225$
$6^2 = 36$	$16^2 = 256$
$7^2 = 49$	$17^2 = 289$
$8^2 = 64$	$18^2 = 324$
$9^2 = 81$	$19^2 = 361$
$10^2 = 100$	$20^2 = 400$

☆ Square of a number always ends with 0, 1, 4, 5, 6 and 9 as unit digit.

Square of a number can never ends with 2, 3, 7 and 8

(2) 21 to 29 { Pattern formed }

21 = 4 | 41
22 = 4 | 84
23 = 5 | 29
24 = 5 | 76
25 = 6 | $\boxed{25}$
26 = 6 | 76
27 = 7 | 29
28 = 7 | 84
29 = 8 | 41

reverse

Class Date → 24/02/23

Q4 $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots\cdots + \frac{1}{\sqrt{80}+\sqrt{81}}$

$\Rightarrow \frac{1}{\sqrt{2}+\sqrt{1}} * \frac{(\sqrt{2}-\sqrt{1})}{(\sqrt{2}-\sqrt{1})} + \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})} + \cdots\cdots + \frac{1}{\sqrt{81}+\sqrt{80}} \times \frac{(\sqrt{81}-\sqrt{80})}{(\sqrt{81}-\sqrt{80})}$

$\Rightarrow \frac{\sqrt{2}-\sqrt{1}}{(\sqrt{2})^2-(\sqrt{1})^2} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} + \cdots\cdots + \frac{(\sqrt{81}-\sqrt{80})}{(\sqrt{81})^2-(\sqrt{80})^2}$

$\Rightarrow \cancel{\sqrt{2}} - \sqrt{1} + \cancel{\sqrt{3}} - \cancel{\sqrt{2}} + \cancel{\sqrt{4}} - \cancel{\sqrt{3}} + \cancel{\cdots} \cdots + \sqrt{81} - \cancel{\sqrt{80}}$

$\Rightarrow \sqrt{81} - \sqrt{1}$

$\Rightarrow 9 - 1 \Rightarrow 8$ Ans

---

workbook (24) for own purpose

In a consecutive series,

$$\frac{\text{sum of numbers}}{\text{total numbers}} = \text{middle number of the series}$$

As $\frac{425}{5} = 85$ is the middle of 5 numbers i.e., 3rd number

∴ series is 81, 83, 85, 87, ------ 101, 103

$95 + 97 + 99 + 101 + 103 = 495$

★ **Prime No.**

If $P > 5$ → ① $\dfrac{\overbrace{888888}^{(7-1)\text{ times repeat karna}}}{7}$ { remainder '0' }

prime no.

② $\dfrac{3333333333 \,(10\text{ times})}{11}$ (11-1) times → remainder '0'

11 → prime no.

③ $\dfrac{8888 \text{----} (18 \text{ times})}{19}$

remainder → '0'.

④ $\dfrac{8888 \text{-----} (19 \text{ times})}{19}$ Rem → ?

$\dfrac{888 \text{-----} 18 \text{ times}}{19} \to 0 \Rightarrow \dfrac{8}{19}$

rem → 8

⑤ $\dfrac{888 \text{----} (20 \text{ times})}{19}$

$\dfrac{\overbrace{8888 \text{------} (18 \text{ times})}\;.\;88}{19}$

$\dfrac{88}{19} \Rightarrow$ Rem → 12

⑥ $\dfrac{8888 \text{-----} 37 \text{ times}}{19}$

$\dfrac{\underbrace{888 \text{-----} 8}_{18 \text{ times}} \to 0 \;\; \underbrace{888 \text{----} 8}_{18 \text{ times}} \to 0 \;\; 8}{19} = \dfrac{8}{19} \Rightarrow 8$

Trick to find cube root

~~1 – 1~~
2 – 8
3 – 7 (27)
~~4 – 4~~ (64)
~~5 – 5~~ (125)
~~6 – 6~~ (216)
7 – 3 (343)
8 – 2 (512)
~~9 – 9~~ (729)

1. 328509

$\sqrt[3]{328\cancel{509}}$

69

~~first~~ we choose this bcoz (343) is larger.

① Take last digit and change accordingly as $3 \to 7$ and $2 \to 8$ or vice versa

② Now delete last 3 no's

③ Now search the number and choose accordingly.

2. 50653

$\sqrt[3]{50\cancel{653}}$

37

3. 970299

$\sqrt[3]{970\cancel{299}}$

99

4. 1442897

$\sqrt[3]{1442\cancel{897}}$

113

$$\boxed{abcd = 1000a + 100b + 10c + d}$$

$$\frac{1000a+100b+10c+d}{2} = \frac{d}{2}$$

$$\frac{1000a+100b+10c+d}{4} = \frac{cd}{4}$$

$$\frac{1000a+100b+10c+d}{8} = \frac{bcd}{8}$$

÷5 $\frac{1000d+100c+10b+a}{5}$

5 → 1 digit

25 → $\frac{1000d+100c+10b+a}{25}$ → 2 digit

125 → abcdef

625 → abcdef

① ÷2	2^1	1 digit dekhte hai	
② ÷4	2^2	2	"
③ ÷8	2^3	3	"
④ ÷16	2^4	4	"
⑤ ÷32	2^5	5	"

÷3 abcd → $\frac{1000a+100b+10c+d}{3}$ = $\frac{[a+b+c+d]}{3}$ (1×a + 1×b + 1×c + d)

÷7 abcd → $\frac{1000a+100b+10c+d}{3} = \frac{700a+300a+70b+30b+7c+3c+7d-6d}{7}$

⇓

$\boxed{\frac{abc-2d}{7}} \Leftarrow \frac{3(100a+10b+c-2d)}{7} \Leftarrow \frac{(300a+30b+3c-6d)}{7}$

÷13 abcd → $\frac{1000a+100b+10c+d}{13} = \frac{1300a-300a+130b-30b+13c-3c+13d-12d}{13}$

⇓

$\boxed{\frac{abc+4d}{13}} \Leftarrow -3\frac{(abc+4d)}{13} \Leftarrow \frac{-3(100a+10b+c+4d)}{13}$

÷17 $\boxed{abc-5d}$ ÷23 $\boxed{abc+7d}$

$\div 11$

$$\frac{10000e+1000d+100c+10b+a}{11} = \frac{9999e+e+1001d-d+99c+c+11b-b+a}{11}$$

$$\Downarrow$$

$$\frac{\boxed{e-d+c-b+a}}{11}$$

Fedcba ⑥

$$\frac{100000F+10000e+1000d+100c+10b+a}{1001}$$

$$\frac{100100F-100F+10010e-10e+1001d-d+100c+10b+a}{1001}$$

$$\frac{-100F-10e-d+100c+10b+a}{1001}$$

$$\frac{-Fed+cba}{1001}$$

edcba ⑤

dcba ④

{Peeche se 3 Ka Pair Banao)

aur dekho woh pair - karne par 0 hai ya 1001 divisiblity ke liye

Ex ① 34678942

942
34
976
678
298

② 34672462345

Ex 36478 / 1001

478
36
442 (x)

$1001 = \underbrace{7 \times 11 \times 13}$

Toh hum inki divisiblity check karne ke liye ③-③ ka Pair Bana sakte hai

$9 \rightarrow abcd \rightarrow a+b+c+d$ Ex 32454 → 3+2+4+5+4 = 18/9 → 0

$99 \rightarrow abcd \rightarrow \frac{ab+cd}{99}$

(2,2 ka pair banao aur phir add kardo)

Ex ① 32454 → 3+24+54 → 81/99

② 212454 → 21+24+54 → 99/99 → 0

$999 \rightarrow abcdef \rightarrow abc+def$ Ex 32454 → 32+454 = 486/999

* → 10^n-1 / $9 \rightarrow 10^1-1$ $999 \rightarrow 10^3-1$ → 3 ke pair

$99 \rightarrow 10^2-1$ → 2 ke pair

$101 \Rightarrow$ abcde $\rightarrow (a+de)-(bc)$ Exp 32445 $\rightarrow 45-32 = 13$

$1001 \Rightarrow$ abcdef $\rightarrow$ (def - abc)

$10^n+1 \rightarrow$

$11 \rightarrow 10^{1}+1$

$101 \rightarrow 10^{2}+1$ — (2) ka pair banane hai

$1001 \rightarrow 10^{3}+1$

$10001 \rightarrow 10^{4}+1$

Vedic Mathematics

+

$9 \xrightarrow{+1} 10$

$19 \xrightarrow{+1} 20$ $\times 2(+)$ last digit $\times 2$ + remaining digit

$29 \xrightarrow{+1} 30$ $\times 3(+)$

$39 \xrightarrow{+1} 40$ $\times 4(+)$

$49 \xrightarrow{+1} 50$ $\times 5(+)$

$59 \xrightarrow{+1} 60$ $\times 6(+)$

$69 \xrightarrow{+1} 70$ $\times 7(+)$

$79 \xrightarrow{+1} 80$ $\times 8(+)$

$89 \xrightarrow{+1} 90$ $\times 9(+)$

$99 \xrightarrow{+1} 100$ $\times 10(+)$

Exp (1) $\frac{361}{19} \rightarrow$ 36 + 2 = 38; $\frac{38}{19} \rightarrow 0$

(2) $\frac{9309}{29} \rightarrow$ 930 + 27 = 957; 95 + 21 = 116; 11 + 18 = 29 $\Rightarrow 0$

(3) $\frac{25529}{49}$ → 2552 + 45 = 2597; 259 + 35 = 294; 29 + 20 = 49; $\frac{49}{49} \Rightarrow 0$

−

$11 \xrightarrow{-1} 10$

$21 \xrightarrow{-1} 20$ $(\times 2)(-)$

$31 \xrightarrow{-1} 30$ $\times 3(-)$

$41 \xrightarrow{-1} 40$ $\times 4(-)$

$51 \xrightarrow{-1} 50$ $\times 5(-)$

$61 \xrightarrow{-1} 60$ $\times 6(-)$

$71 \xrightarrow{-1} 70$ $\times 7(-)$

$81 \xrightarrow{-1} 80$ $\times 8(-)$

$91 \xrightarrow{-1} 90$ $\times 9(-)$

remaining digit − last digit $\times 2$

for (−ve) $\quad 11 - 1 = 10$

Ex: (1) $\frac{253}{11}$

$$\begin{array}{r} 253 \\ -3 \\ \hline 22 \end{array}$$

(2) $\frac{462}{21}$ $\quad 21 \xrightarrow{-1} 20$

$$\begin{array}{r} 46\,2^{\times 2} \\ -4 \\ \hline \frac{42}{21} \end{array} \Rightarrow 0$$

(3) $\frac{73315}{31}$

$$\begin{array}{r} 7331\,5^{\times 3} \\ 15 \\ \hline 731\,6^{\times 3} \\ 18 \\ \hline 71\,3 \\ -9 \\ \hline \frac{62}{31} \end{array} \Rightarrow 0$$

$17 \longrightarrow 51 \longrightarrow \times 5(-)$ { Jese humne 17 ka rule nikala tha toh humm 51 ka dekhenge toh woh 17 se toh hoga hi }

* Prime no's ke rule kese nikale

↳ Prime no. never ends with

2, 4, 6, 8, 0, 5 (×)

It ends with

1 ✓

3 → ~~2/9~~ 3/7

7 → 3/7

9 ✓

Ex: $47 \xrightarrow{\times 3} 141$ $(\times 14\,(-))$ → last digit ko 14 se × karke remaining se (−) karna hai.

$23 \xrightarrow{\times 3} 69$ $(\times 7\,(+))$

17: $\times 3 \to 51 \to \times 6(-)$; $\times 7 \to 119 \to \times 12(+)$

6 → $\boxed{2 | 3}$

99 → $\boxed{9 | 11}$

II

★ abab → ab(100+1) → ab × 101 → always divisible by 101.

Ex1 ① 3737

3700+37

37(100+1)

37 × 101

② 6262

62×101

★ abcabc → abc000+abc → always divisible by 1001 -

abc(1000+1)

abc×1001

7, 11, 13

★ ababab → ab × 10101

91 × 111

7×13×3×37

★ aaaaaa → 3/7/11/13/37

★ abab → 101

★ abcabc → 7/11/13

★ ababab → 3/7/13/37

★ aaaaaa → 3/7/11/13/37

Ex1 ① $\frac{737373}{37}$ 6

② $\frac{7373734}{37}$ 4

③ $\frac{73737343}{37}$ $\frac{43}{37}$ → 6

④ $\frac{7373737373737373}{37}$

$\frac{73}{37}$ → 36

☆ Jese humo ques diye hote hai jisme x and y ki value find karni hoti hai aur usme 8 ki divisibility dekhni hai toh hum last ke teen no. check karte hai woh karne ki zarurat nahi hai

TRICK

Ex 1 $1230x558y2$
even mei last ke 2 no. dekhte hai

even

y2

0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

$y \to 3/7$

Ex 1 $74x29y6$
odd mei last ke 2 no. + y dekhte hai

odd

66 + 4 = 70
76 + 4 = 80
86 + 4 = 90
96 + 4 = 100

06 + 4 = 10
16 + 4 = 20
26 + 4 = 30
36 + 4 = 40
46 + 4 = 50
56 + 4 = 60

$y \to 3/8$

☆ $7\,8\,5\,x\,3\,6\,7\,8\,1$ → agar yeh no. 9 se divisible hai aur humko x ki value find karni hai.
9

(hum pair banate hai jinka sum 9 ho ya 9 ka multiple aur cancel kar dete hai)
↳ Problem bahut choti ho jati hai

$\frac{x+8}{9}$

$x \to 1$

2 I 1 2 3 4 5 6, 7

II 1 2 3 4 5 6 7 8 9, → 9 digits

III 1 2 3 4 5 6 7 8 9 1 0 → 11 digits

IV $\underbrace{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9}_{9\ digit}\ \underbrace{10\ 11\ 12\ 13\ 14}_{10\ digit\ (2\times5)}$ → 19 digits.

1 to 99
→ 99
−9
90

1 digit → 1 to 9 ⟶ 9 No. × 1 ⟶ 9 digits

2 digit → 10 to 99 ⟶ 90 No. × 2 ⟶ 180 digits

↳ Har ek no. ki digit

3 digit → 100 to 999 ⟶ 900 × 3 ⟶ 2700 digits

1 digit no. → 9×10^0 ⟶ 9 × 1 ⟶ 9

2 digit no. → 9×10^1 ⟶ 90 × 2 ⟶ 180

3 digit no. → 9×10^2 ⟶ 900 × 3 ⟶ 2700

4 digit no. → 9×10^3 ⟶ 9000 × 4 ⟶ 36000

} digits hote hai Total

1 to 9 ⟶ 9

1 to 99 ⟶ 189 (9+180)

1 to 999 → 2889 (9+180+2700)

1 to 9999 → 38889

} matlab digit kitne hote hai Total

Q 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 → Total no. of digits?

$\underbrace{1\,2\,3\,4\,5\,6\,7\,8\,9}_{9\times1} + \underbrace{10\,11\,12\,13\,14\,15\,16\,17}_{8\times2} \Rightarrow$ 25 digits.

Q: 12345 ------- (Total 14 digits) Then find last number?

→ $\underbrace{1\,2\,3\,4\,5\,6\,7\,8\,9\,10\,11}$ 1 ~~X~~ {2 nahi lenge}

Q: 12345 ------- { Total 43 digits }

12345 ------- { hum poore no. todna llPKhenge Bhosda ho Jayega dimag ka }

$43-9 = \frac{34 \text{ digits}}{2}$

$= 17$ no.

1 to 9 ---- 25 26 (9+17)

★ If total digit

9 ↑ → then it contains $\boxed{1+2}$

189 ↑ → $\boxed{1+2+3}$

2889 ↑ → $\boxed{1+2+3+4}$

Q: 123456 ---- (13 digits)

↳ $13-9 = \frac{4 \text{ digits}}{2} =$ 2 no.

↳ Har no. 2 digit ka hoga

1 to 9 10 11 ✓

Q: 12345 ------- 52 digits.

↳ $52-9 = \frac{43 \text{ digits}}{2} \Rightarrow 21\frac{1}{2}$

↳ 1 to 9 ------ 303 (9+21)

Q: 12345 ----- (14 digits)

↳ $14-9 = \frac{5 \text{ digits}}{2} = 2\frac{1}{2}$

1 to 9 10 11 1

2 no. complete aur last no. ka 1 digit lena hai

Q1 123456 ------ (192 digits)

$$\begin{array}{r} 192 \\ -189 \\ \hline 3 \\ \hline 3 \end{array} = 1$$

1 to 99 100

Q1 123456 ----- (Total 193 digits)

1 to 99 100 1

$$\begin{array}{r} 193 \\ -189 \\ \hline \end{array}$$
$\frac{4}{3} = 1\frac{1}{3}$

Q1 123456 ----- (Total 194 digits)

1 to 99 100 10

$$\begin{array}{r} 194 \\ -189 \\ \hline \end{array}$$
$\frac{5}{3} = 1\frac{2}{3}$

Q1 Find last 3 digits

123456 ------ (279 digits total)

1 to 99 ------- 129 Ans

$$\begin{array}{r} 279 \\ 189 \\ \hline 90 \end{array}$$
$\frac{90}{3} = 30$

$$\begin{array}{r} 99 \\ 30 \\ \hline 129 \end{array}$$

Q1 123456 ------ (2989 digits)

1 to 999 ----- 1024

(999+25)

$$\begin{array}{r} 2989 \\ 2889 \\ \hline 100 \end{array}$$
$\frac{100}{4} = 25$

Q1 1 to 10000 {Find total digits}

[$\boxed{1234567 ------ 9999}$ 10000]

38889+5 → 38894

* 1 to 9 → 9
* 1 to 99 → 189
* 1 to 999 → 2889
* 1 to 9999 → 38889
*

Q1 1 to 100015 → 1 to 9999 + ~~to~~ 10000 to 10015

↳ 38889 + 16×5

★ 1 to 9 Times

1 → 1
2 → 1
3 → 1
4 → 1
5 → 1
6 → 1
7 → 1
8 → 1
9 → 1

Q: 1 to 508

1 → Times ?

1	10	20	30	40	50	60	70	80	90
1	11	21	31	41	51	61	71	81	91
2	12	22							
3	13	23							
4	14	24							
5	15	25							
6	16	26							
7	17	27							
8	18	28							
9	19	29							

1 to 99
1 → Times?

	Unit digit	Tens digit	1 to 99
1 →	10	10	20 Ans
2 →	10	10	20
3 →	10	20	20
⋮			
9 →	10	10	20

100	110	120	130	140
101	111	121		
102	112	122		
103	113	123		
104	114	124		
105	115	125		
106	116	126		
107	117	127		
108	118	128		
109	119	129		

1 to 149

UNIT/TENS → 20 + 100 (hundred)

→ 120

Q: 1 to 299
2 times → ? → 1 to 99 | 100 to 199 | 200 to 299

20 | 100+20 | 20

→ 160

U T	U T H	U T H
10+10	10 10 100	10 100

2 times → 20/20/120 → 160

3 → 20/20/20 → 60

4 → 20/20/20 → 60

5 →

6 →

7 →

8 →

9 → → 60

1 to 499

	1 to 99	100-199	200-299	300-399	400-499	
3 times?	20	20	20	120	20	→ 200
5 times?	20	20	20	20	20	→ 100 times

Q1 1 to 508

500	505
501	506
502	507
503	508
504	

5 times?

1 to 99 / 100 to 199 / 200 to 299 / 300 to 399 / 400 to 499 / 500 to 508

20 | 20 | 20 | 20 | 20 | 10

→ 110

H T O

9 + 0 + 1

→ 10

Face Value / Place Value

324578 → 300000 + 20000 + 4000 + 500 + 70 + 8

	Face value	Place value
8	8	8
7	7	70
5	5	500
4	4	4000
2	2	20000
3	3	300000

* $ab \rightarrow 10a + b$
* $abc \rightarrow 100a + 10b + c$

---

① (ab, ba)

diff: agar ko no. hai ab aur usse palat de tou diff kiska multiple hoga -

$(10a + b) - (10b + a)$

$9(a - b)$

Sum → $(10a + b) + (10b + a)$

$11(a + b)$

② (abc, cba)

diff: $(100a + 10b + c) - (100c + 10b + a)$

$99(a - c)$

Sum → ~~$101a + 20b + 101c$~~

No rule

★ Prime No.

If $P > 5$ $\longrightarrow$ ① $\dfrac{\overbrace{888888}^{(7-1)\text{ times repeat karna}}}{7}$ {remainder '0'}

prime no.

② $\dfrac{3333333333 \text{ (10 times)}}{11}$ $\rightarrow$ remainder '0'

(11-1) times

$\rightarrow$ prime no.

③ $\dfrac{8888\text{----(18 times)}}{19}$

remainder $\rightarrow$ '0'.

④ $\dfrac{8888\text{----(19 times)}}{19}$ Rem $\rightarrow$?

$\dfrac{888\text{-----18 times}}{19} \rightarrow 0 \Rightarrow \dfrac{8}{19}$

rem $\rightarrow$ 8

⑤ $\dfrac{888\text{----(20 times)}}{19}$

$\dfrac{\overbrace{8888\text{-------}8888 \text{ (18 times)}}\ .88}{19}$

$\dfrac{88}{19} \Rightarrow$ Rem $\rightarrow$ 12

⑥ $\dfrac{8888\text{------37 times}}{19}$

$\dfrac{\underbrace{888\text{......}8}_{18\text{ times}}^{\rightarrow 0}\ \underbrace{888\text{.....}8}_{18\text{ times}}^{\rightarrow 0}\ 8}{19} = \dfrac{8}{19} \Rightarrow 8$

Trick to find cube root

1 – 1
2 – 8
3 – 7 (27)
4 – 4 (64)
5 – 5 (125)
6 – 6 (216)
7 – 3 (343)
8 – 2 (512)
9 – 9 (729)

1. 328509

$\sqrt[3]{328\cancel{509}}$

~~first~~ we choose this bcoz (343) is larger.

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① Take last digit and change accordingly as 3 → 7 and 2 → 8 or vice versa

② Now delete last 3 no's

③ Now search the number and choose accordingly

2. 50653

$\sqrt[3]{50\cancel{653}}$

37

3. 970299

$\sqrt[3]{970\cancel{299}}$

99

4. 1442897

$\sqrt[3]{1442\cancel{897}}$

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