Assignment-6. PSA

1) Height of B-tree is $h \leq \log_t \frac{(n+1)}{2}$. where t is the ~~degree~~ minimum degree of B-tree. ~~Here~~ and $t \geq 2$. ~~and~~, $n \geq 1$.

The root contains at least one key. All other nodes contain at least $t-1$ keys. At depth 1, we can find at least 2 nodes, At depth 2, we can find at least $2t$. nodes. Similiarly at depth i, we will find $2t^{i-1}$ nodes at depth i and $2t^{h-1}$ nodes at depth h.

$$\text{Thus, } n \geq 1 + (t-1)\sum_{i=1}^{h} 2t^{i-1}$$

$$= 1 + 2\cancel{(t-1)}\left(\frac{t^h - 1}{\cancel{t-1}}\right)$$

$$= 2t^h - 1.$$

so, $t^h \leq (n+1)/2$ as required.

$$h \lg t \leq \lg\left(\frac{n+1}{2}\right).$$

$$h \leq \frac{\lg\left(\frac{n+1}{2}\right)}{\lg t} = \lg_t\left(\frac{n+1}{2}\right).$$

we know that t is the degree of the B-tree. This means an internal node can have at most $2t$ children, which is given m in question.

So, $2t = m \Rightarrow t = m/2$.

Thus, $h \leq \lg_{m/2}\left(\frac{n+1}{2}\right)$.

2). In this, we will try to find the ~~optimal~~ minimum number of egg droppings in closed-~~form~~ form and then prove that it's $O(\sqrt{n})$. The no. of eggs given to us is 2. and let the no. of floors in building be n.

Let the number of egg dropping be x. Then the first floor that we will try is floor x. Suppose in worst case, if egg breaks, then we will try ~~for~~ every floor from 1 to $x-1$ with the remaining egg. So, total # of trials $= 1 + (x-1)$.

other possibility is that the egg does not break in ~~the~~ first attempt. Then the next floor, we will try is $x+(x-1)$ because our optimal answer is x and if egg breaks at $x+(x-1)$, then we will have to try linearly from floor $x+1$ to $x+(x-2)$.

Now, let's generalise this -

If first egg has not broken so far, then the i^{th} trial has to be floor number $x+(x-1)+\cdots+(x-i-1)$. Thus, we can observe that we can cover $x+(x-1)+(x-2)+\cdots+2+1$ floors with x trials. The closed form value is $x(x+1)/2$.

So, the optimal x for a given building with n floors.

$$\frac{x(x+1)}{2} \geq n$$

$$x^2+x-2n \geq 0$$

The optimal value of x i.e min # of trials can be written as

$$\frac{-1+\sqrt{1+8n}}{2} \qquad (\because \sqrt{1+8n} \geq 1).$$

Proof that $\frac{-1+\sqrt{1+8n}}{2} = \Theta(\sqrt{n})$. -

We will that $f(n) = \frac{-1+\sqrt{1+8n}}{2}$, $f(n) = O(\sqrt{n})$ and $f(n) = \Omega(\sqrt{n})$.

So, for $f(n) = O(\sqrt{n})$

$$\frac{-1+\sqrt{1+8n}}{2} \leq \frac{0+\sqrt{n+8n}}{2}$$

$$\frac{-1+\sqrt{1+8n}}{2} = \frac{3}{2}\sqrt{n} \qquad \text{for } n \geq 100.$$

So, take $n_0 = 10$ and $c = 3/2$ to get $f(n) = O(\sqrt{n})$.

for $f(n) = \Omega(\sqrt{n})$

$$\frac{\sqrt{1+8n}-1}{2} \geq \sqrt{n}$$

$$1+8n \geq 1+2n$$

$$\sqrt{1+8n} \geq \sqrt{1+2n} \qquad (as\ n \geq 0)$$

$$4n + 4n + 1 \geq 4n + 4\sqrt{n} + 1. \quad (\text{as } n \geq 0)$$

we know

$$8n + 1 \geq (2\sqrt{n}+1)^2$$

$$\sqrt{8n+1} \geq 2\sqrt{n} + 1.$$ (rooting both sides as LHS & RHS ≥ 0)

$$\sqrt{8n+1} - 1 \geq 2\sqrt{n}$$

$$\frac{\sqrt{8n+1} - 1}{2} \geq \sqrt{n}$$ (for $n \geq 10$).

Take $n_0 = 10$ and $c = 1$, to get $f(n) = \Omega(\sqrt{n})$.

Thus, $f(n) = \Theta(\sqrt{n})$.

4) It is given that there is no negative weight cycles in the graph G and all negative weight edges are connected to the source vertex s.

Let $\delta(u, v)$ be the shortest path weight from vertex u to v. For any graph $G(V, E)$, we maintain an attribute v.d, which is an upper bound on the weight of a shortest path from s to v, for each vertex v, 's' is for source.

In the proof of correctness for Djikstra algorithm, we had taken into consideration that $\delta(s, y) \leq \delta(s, u)$ where y is the first vertex along shortest path p such that $y \in V - S$. S is a set of vertices whose final shortest path weights from the source s have already been determined. This is true because there are no negative edge weights. Actually, this always holds if y occurs on a shortest path from s to u and $y \notin S$ because all edges on the path from y to u have non-negative weight. If any edge had a negative weight, it would simply mean that we had stepped back to an edge incident with s, which implies that a cycle is involved in path, but it is given that there are no-negative weight cycles in the graph.

1 which would only be the case if it were a negative cycle;

5) Take a three-vertices acyclic graph $G(V,E)$ (drawn below).

a — b — c.

So, for finding the shortest path from a (source) to c (destination), we have to find the shortest path from a to b, then from b to c. This means there exists a graph in which for finding shortest path between points, we have to find for all vertices.

3) We will use kadane's algorithm here.

Initialize:

~~max~~ currentmax = 0

finalmax = 0

Loop for each element of the array -

a) finalmax = finalmax + a[i] (where a[i] is an element of array a)

b) if (~~fi~~ currentmax < finalmax)

currentmax = finalmax.

c) if (finalmax < 0)

finalmax = 0.

return ~~finalm~~ currentmax.

Kadane's algorithm looks for all contiguous segments of the array. In the pseudocode above, we have use finalmax, we keep track of maximum sum contiguous segment among all positive segments (currentmax)

When final max is positive and greater than current max, we update current max and finally return/print it.

Since we are traversing the array only once, the time complexity will be $\theta(n)$.