Communication Theory Lab Report

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Communication Theory Lab
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Lab Assignment # 9

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Signal Detection at the Receiver End

Abstract

In this experiment, we have tried to understand the concept of detecting the symbols, that were actually transmitted over the channel, from the received signal. Here, we have analyzed only the ML (maximum likelihood) detector and considered the prior probabilities of sending signal symbols as same across all symbols. We analyzed the theoretical and practical waveforms of the SNR (sound to noise ratio), by plotting their graphs in MATLAB.

1 Theory

In detection theory, we are concerned with determining the signal that has been sent in presence of noise. So we have -

$$r(k) = s_i(k) + n(k)$$

where n(k) follows a gaussian distribution with 0 mean and σ^2 variance. Suppose a signal consists of M symbols, then $s_i(k)$ can be any one of the M symbols. Hence we will have M hypotheses that will be defined as -

H_i : $s_i(t)$ is sent with probability $\Pr(H_i)$.

In this report, we have taken M = 2. So the signal is having only 2 symbols that are +1 and -1. We also assume that the prior probabilities of receiving $s_m(k)$ is equal. So,

$$Pr(s_m(k) = 1) = Pr(s_m(k) = -1) = 0.5$$

Hence in our case, we will be implementing the maximum likelihood (ML) detector. The detector can also make error in predicting the symbols and we characterize it using error of probability (P_e). Error happens when the predicted signal is not equal to actual signal sent. It is given as the ratio of number of errors and number of symbols sent. So,

$Pr_e(k) = \frac{\#of Errors}{K}$
Theoretically, probability of error is given by,

$$P_e = Q\sqrt{SNR}$$

where SNR is sound to noise ratio. SNR is given by the ratio of signal power and noise power. To convert it to dB, we take the log of it i.e.

$$SNR = 10log(\frac{P_{signal}}{P_{noise}})$$

Power of noise is given by its variance. Power of signal is taken to be 1. And the boundary is detection is defined as follows :

$$s_p(k) = -1 \text{ if y(k)} < 0$$

$$s_p(k) = 1 \text{ if y(k)} > 0$$

where $s_p(k)$ is the predicted signal.

1.1 Block Diagram

The block diagram is for detection of signal. The signal received is fed into the decision maker. If the received signal is more than 0, then the sent signal is predicted to be +1, and vice-versa.

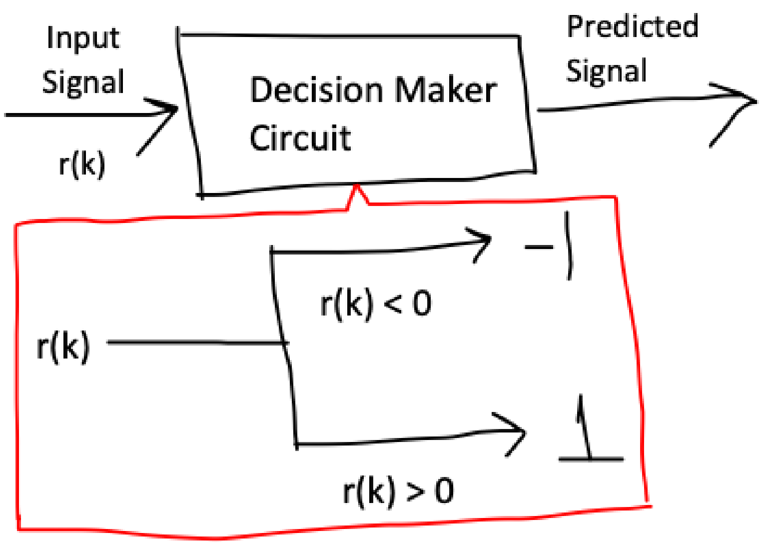

Figure 1: Quantization block diagram

1.2 Expected Outcome

The figures below are not drawn to scale.

1.2.1 Answer 1

This question was taken directly from what was taught in the lectures. The practical and the theoretical SNR should be similar.

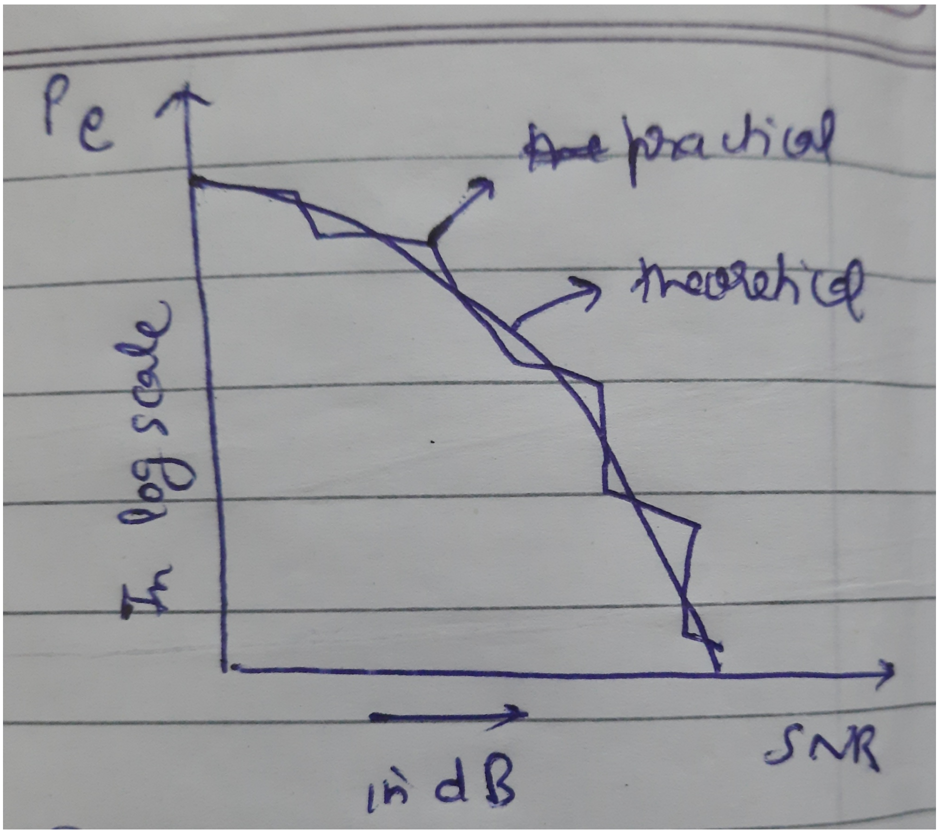

Figure 2: Figure for part A

1.3 Applications

The broad concept of reconstruction is used in various fields such as :

1. `Hearing tests:` When doctors emit different pitches of sound (the signal) to check a patient's hearing capabilities. The patient is expected to identify its presence, and must raise a hand whenever they hear the noise (the response).

2. `Sensor Detection at PNNL` : It is used in sensor technologies which is aimed at detecting threats to national and personal security in Pacific Northwest National Laboratory (PNNL) in Washington State.

3. `Compressed sensing` : It is used to recover high dimensional but with low complexity entities from only a few measurements.

4. `Image and file decompression:` Detection is used to decompress JPEG compressed images or compressed files/folders. Without proper detection, the image/files decompressed will be incorrect.

2 Results and Inferences

2.1 Answer 1

The plot in red depicts the theoretical curve, while the practical plot (which I have simulated in the lab) has been represented using blue colour. It is given that the system is having a Additive White Gaussian Noise. Its Power Spectral Density is constant over all frequencies. Noise follows a Gaussian distribution with mean 0 and standard deviation σ_n.

Inferences are :

1. In the plot, we can see that the probability of error is inversely proportional to SNR. That is at lower SNR values, error probability is high, and vice versa. It is inline with the analytical and theoretical plots.

2. In the plot, we can also see that our practical plot and the theoretical plot are very close to each other. This is because the number of bits we have taken i.e. 10^4 is very large. As a result, the practical SNR and theoretical SNR are almost same.

3. In real communication, we need a high SNR for small probability error. Getting a high SNR is practically hard, so we need to have a tradeoff between the two.

4. Since the probability of getting a symbol is same for all symbols, it is a maximum likelihood detector.

From the experiment, we can conclude that to have a small probability of error, a high SNR value is required.

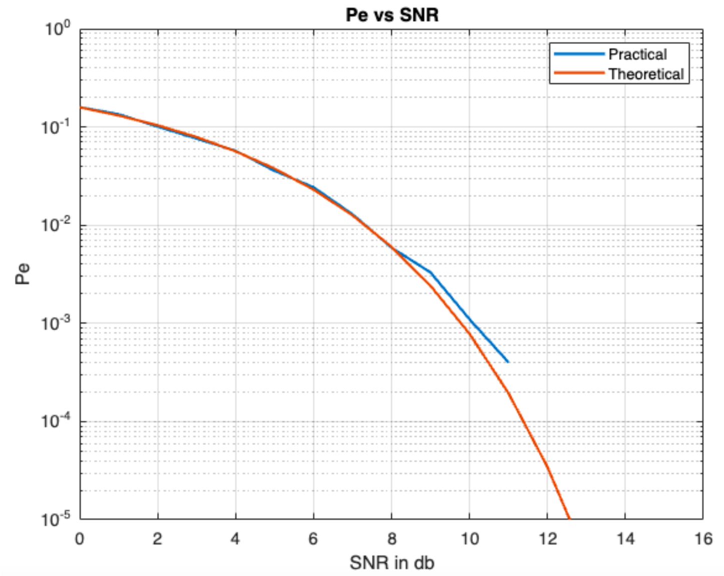

Figure 3: Figure 1 for part A

Appendix

A Matlab Commands

Table 1: Matlab commands used in this lab.

Matlab Command	**Function**
`plot(`x, y`)`	plots values of the simulation series y along the y-axis, with values of the simulation series x along the x-axis.
`figure()`	creates a new figure in MATLAB.
`title(x)`	adds a title x to the plot
`xlabel(x)`	adds a horizontal label x (along x axis) to the plot
`ylabel(x)`	adds a vertical label x (along y axis) to the plot
`grid on`	adds a grid to the plot.
`clc`	clears everything from the matlab command line window.
`linspace(x1,x2,p)`	generates p equally distant points between x1 and x2.
`subplot(abc)`	generates a subplot of size a x b, and the current image is of index c
`ones(length)`	creates an array of 1s.
`size`	gives length of an array
`xticks`	marks the ticks in x axis
`yticks`	marks the ticks in y axis
`randi`	generate random integral numbers
`qfunc`	finds the value of gaussian error function
`semilogy`	log base 10 scale for y axis

B Matlab Code

Matlab codes for each part.

B.1 Q1(a)

```
clc;
close all;

% number of bits
N=10^4;

% two bits
m=[-1,1];

%step 1
x=[];
for i=1:N
    x(i)=m(randi(length(m)));
end

%SNR ratio in db
%will go from 0 to 14
snr_db=0:1:14;

% snr
snr=10.^(snr_db/10);

%variance
var=(1./snr);

% Probability of error
Pe=[];
for k=1:length(var)

    % step 2
    %noise
    % cholesky factorization for GD of specific
    % standard deviation. chol of a scalar is under root
    % of the scalar. eg - chol(4) = 2. See
    % docs "https://www.mathworks.com/help/matlab/ref/randn.html"
    % for more info.
    % 0 mean added
    n=sqrt(var(k))*randn(1,N);
```

```
    % step 3
    % recieved signal
    y=x+n;

    % step 4
    %decoding
    r=[];
    for l=1:N
        if y(l)<0
            r(l)= -1;
        else
            r(l)= 1;
        end
    end

    % step 5
    %error calculation
    e=0;
    for j=1:N
        if r(j)~=x(j)
            e=e+1;
        end
    end

    % step 6
    % probability of error
    Pe(k)=e/N;
end

%theoretical Pe
tPe=qfunc(sqrt(snr));

figure(1);
semilogy(snr_db,Pe,'linewidth',1.5);
hold on;
semilogy(snr_db,tPe,'linewidth',1.5);
xlabel('SNR in db');
ylabel('Pe');
title('Pe vs SNR');
xlim([0 16]);
```

```
ylim([10^-5 1]);
grid on;
legend('Practical','Theoretical');
```

References

[1] IIT Mandi lectures on EE304 offered by Dr Adarsh https://cloud.iitmandi.ac.in/d/4bb3a5f304334160ab67/

[2] Tutorialspoint lectures on detection https://www.tutorialspoint.com/digital_communication/digital_communication_detection.htm

[3] Wikipedia notes on signal detection for signal processing https://en.wikipedia.org/wiki/Detection_theory

[4] Detection of signals on Youtube https://youtu.be/cUdnGkymAWU

[5] Signal Detection lectures https://ptolemy.berkeley.edu/projects/chess/eecs124/reading/LeeAndVaraiya11.pdf