

# Communication Theory Lab Report

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Date: April 1, 2022  
Communication Theory Lab  
(EE304P, Spring-2022)

Lab Assignment # 5



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# Line Coding techniques

## Abstract

In this experiment, we have tried to understand the concept of line coding techniques used in data transmission of a digital signal. We analysed them by plotting their graphs in MATLAB. There are many line coding techniques, but we primarily focused on polar, unipolar and bipolar types of line coding. We have also looked into their sub types that are RZ (return to zero) and NRZ (not return to zero).

## 1 Theory

We use line coding techniques to efficiently transmit digital signal over transmission lines. This is done primarily to avoid any kind of interference which leads to distortion of the message. There are three major types of line coding techniques, but we have implemented two that are:

1. Polar
2. Unipolar

All these have their own subtypes that are RZ (return to zero) and NRZ (not return to zero).

**Unipolar NRZ:** In this, all the higher signal levels take the same polarity. It can be either positive or negative. We usually work with the positive polarity for high signal. For the low signal, it is always zero. Also note that the polarity that the high signal takes is for the whole bit time.

**Unipolar RZ:** It is similar to Unipolar NRZ. But the polarity that the high signal takes is not for the whole bit time. It returns to zero halfway in the bit time.

**Polar NRZ:** Here the high signal value takes the positive pulse while the low signal values takes the negative pulse. Note that the polarity that the high/low signal takes is for the whole bit time.

**Polar RZ:** It is similar to Polar NRZ, but the polarity that the high/low signal takes is for the whole bit time. It is for half bit time. The remaining half bit time is zero level.

## 1.1 Block Diagram

The figure is not drawn to scale. The block diagram shows the classification of the different line coding techniques mentioned in the previous section.

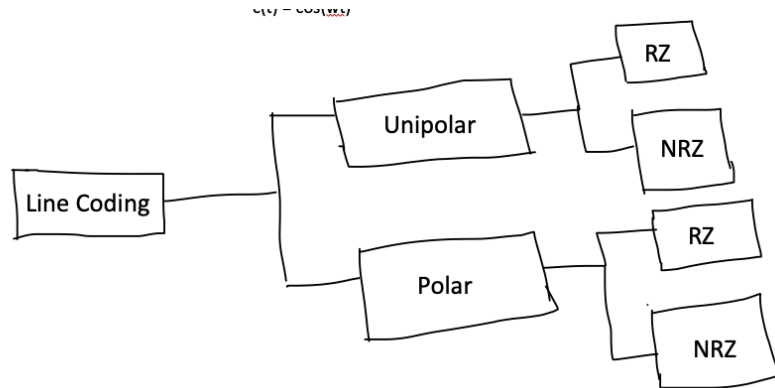


Figure 1: PM block diagram

## 1.2 Expected Outcome

The figures below are not drawn to scale.

### 1.2.1 Answer 1

This question was taken directly from what was taught in the lectures.

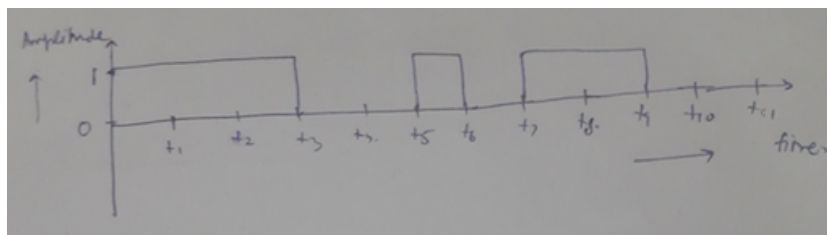


Figure 2: Answer 1

**1.2.2 Answer 2**

This is directly taken from the lecture.

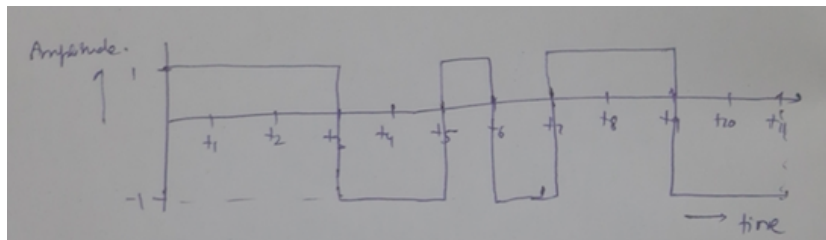


Figure 3: Answer 2

**1.2.3 Answer 3**

This question was taking inference from what was taught in the lectures.

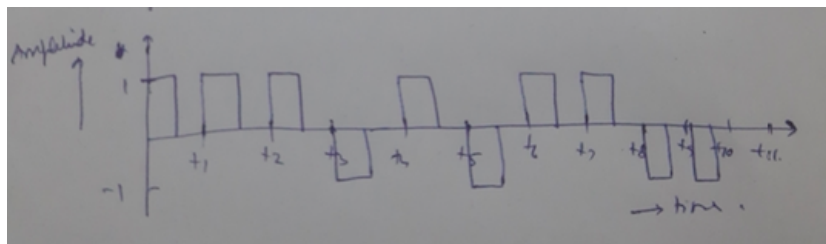


Figure 4: Answer 3

**1.2.4 Answer 4**

This question was taking inference from what was taught in the lectures.

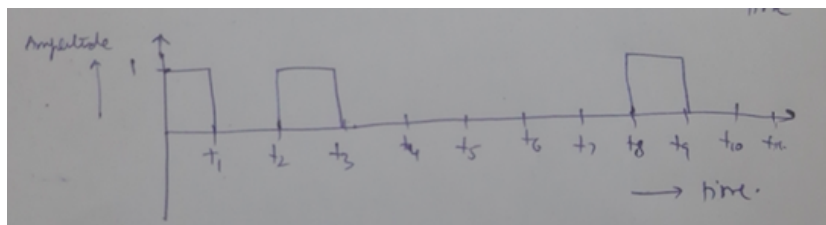


Figure 5: Answer 4

### 1.3 Application

Applications of line coding:

1. **Local Area Network (LAN)**: Line coding techniques such as Manchester and differential Manchester are widely used in local area networks.
2. **Wide Area Network (WAN)**: Bipolar line coding is used for long distance transmissions in United States, Canada, European nations and Japan.

## 2 Results and Inferences

### 2.1 Answer 1

We have  $ck = 11100101100$

This is **Unipolar NRZ**. Its waveform is similar to that described in the theory section.

Inferences are:

1. It is given in the question that 1 of  $ck$  is mapped to 1 for  $ak$ , and 0 of  $ck$  is mapped to 0 of  $ak$ . So  $ak$  and  $ck$  will be exactly same. Also we can see that the message signal  $m(t)$  exactly follows the shape (digital signal) and size (amplitude) of the  $ak$  signal.
2. We also observed that the message signal has only 2 levels, i.e 0 and 1.

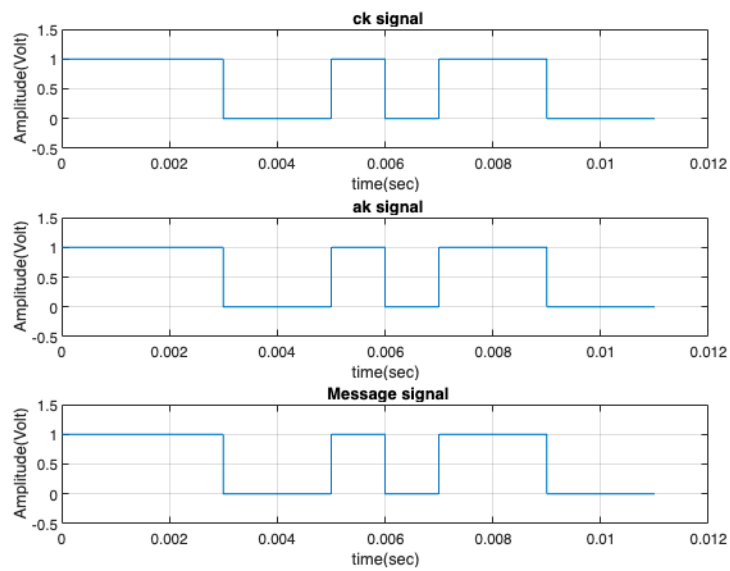


Figure 6: Answer 1

## 2.2 Answer 2

We have  $ck = 11100101100$

This is **Polar NRZ**. Its waveform is similar to that described in the theory section.

Inferences are:

1. It is given in the question that 1 of  $ck$  is mapped to 1 for  $ak$ , and 0 of  $ck$  is mapped to -1 of  $ak$ . So  $ak$  and  $ck$  will not be exactly same. Instead of 0 in  $ck$ ,  $ak$  will have -1. It is also evident from the plots that message signal follows  $ck$  signal whenever  $ck = 1$ . And for the interval when  $ak = 0$ , the message signal has the magnitude of -1.
2. We can also note that the message signal has only 2 levels, i.e 1 and -1.

$m(t)$  can be written as,  $m(t) = 1 \ 1 \ 1 \ -1 \ -1 \ 1 \ -1 \ 1 \ 1 \ -1 \ -1$ . Here  $T_b = .001$  sec as per the question.

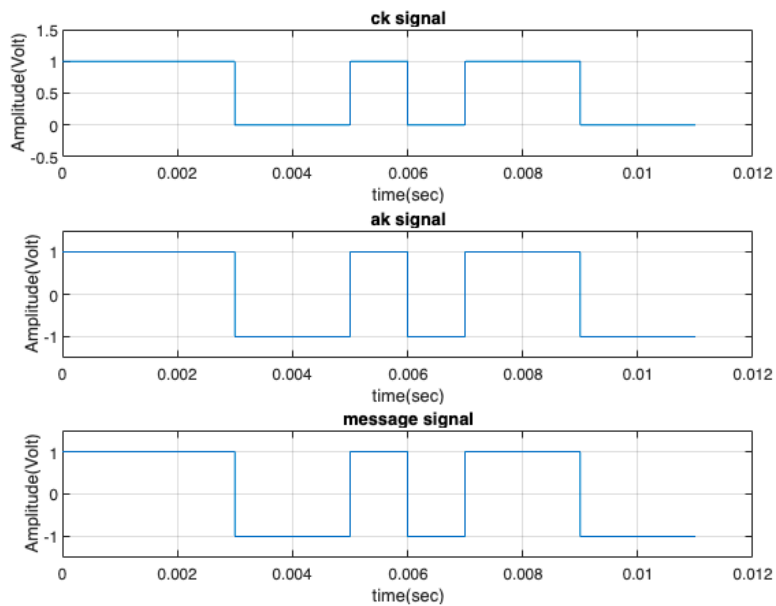


Figure 7: Answer 2

## 2.3 Answer 3

Part 1a:

Here  $ck = 11100101100$

It is mentioned in the question that  $T_b$  has got divided into two equal halves and the amplitude of message signal in second half is always zero. However the first half will vary as question 1a, that is 1 mapped to 1 and 0 to 0.

This is **Unipolar RZ**. Its waveform is similar to that described in the theory section.

Inferences are :

1. We have seen the message signal in part a that only two digital levels are present for the message signal that are 1 (when  $ck = 1$ ) and 0 (when  $ck = 0$ , as well as second half amplitude for interval  $T_b/2$ , which is always zero). In this case, for the interval  $nT_b/2$ , where  $n = \text{odd integer}$ , the message signal is either 1 or 0 as per  $ck$ . However, for interval  $nT_b/2$ , where  $n = \text{even integer}$ ,  $m(t)$  will always be zero. This is also evident from the plots.

**Part 1b:**

Here  $ck = 11100101100$

It is mentioned in the question that  $T_b$  has got divided into two equal halves and the amplitude of message signal in second half is always zero. However the first half will vary as question 1b, that is 1 mapped to 1 and 0 to -1.

This is **Polar RZ**. Its waveform is similar to that described in the theory section.

Inferences are :

1. We have seen the message signal in part b that there are 3 digital levels present (1, 0, -1). The level 1/-1 is being witnessed in the first half of  $T_b$  interval whenever  $ck$  is 1/0. Please note that here +1 for  $ck$  is mapped to +1 and 0 for  $ck$  is mapped -1. However, 0 level is being observed for all second halves of interval  $T_b$ . This is also evident from the plot.



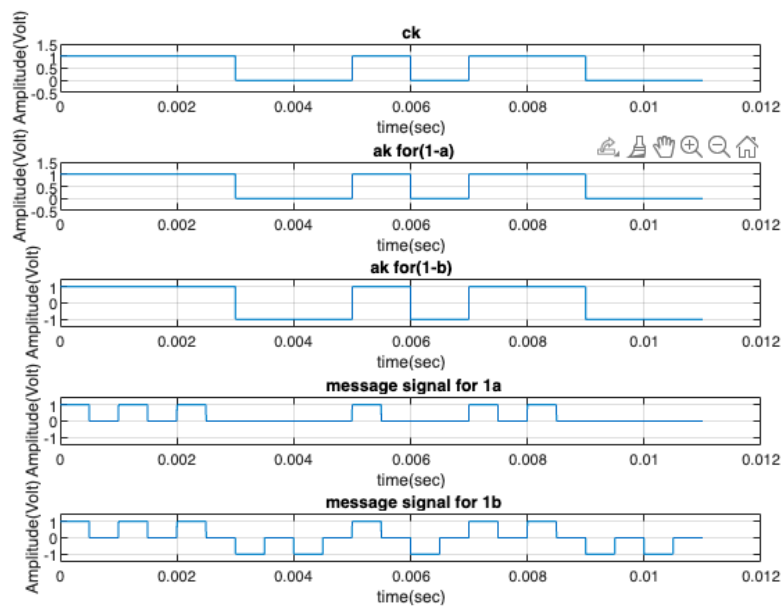


Figure 8: Answer 3

## 2.4 Answer 4

Here  $ck = 11100101100$

In this case,  $T_b$  interval was divided into two equal halves and the message signal is always zero in the second half. Also it is zero when  $ck$  is zero. In all other cases, it will be  $+1/0$  depending on the odd/even number of 1s in  $ck$ . This is **Unipolar RZ**. Its waveform is similar to that described in the theory section.

Inferences are :

1. In the message graph plot, we can see that there are 2 digital levels present (1 and 0). This is because  $A1/A0$  is taking values according to what given in question 1a. The level 0 in the first half of  $T_b$  is obtained when  $ck=0$ . 0 level is also obtained for all second halves of interval  $T_b$ .

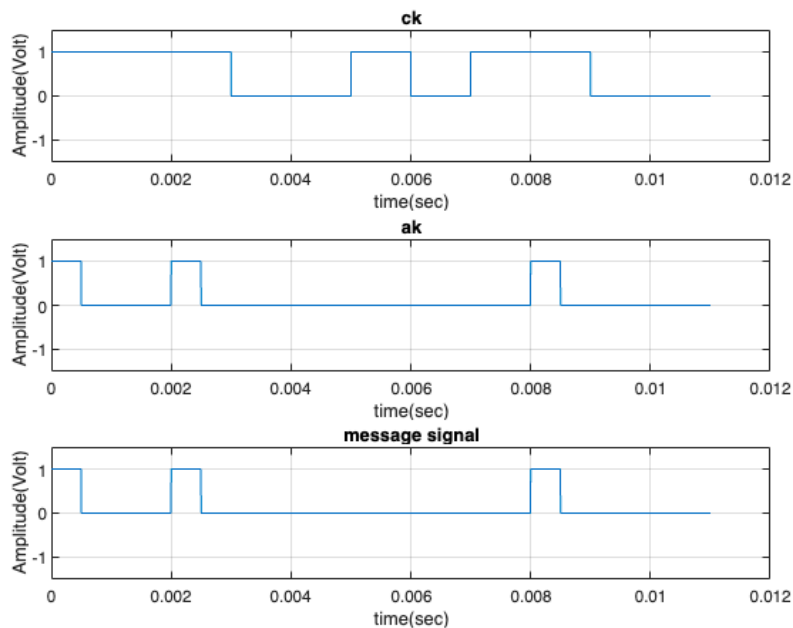


Figure 9: Answer 4

## Appendix

### A Matlab Commands

Table 1: Matlab commands used in this lab.

Matlab Command	Function
<code>plot(x, y)</code>	plots values of the simulation series $y$ along the $y$ -axis, with values of the simulation series $x$ along the $x$ -axis.
<code>figure()</code>	creates a new figure in MATLAB.
<code>title(x)</code>	adds a title $x$ to the plot
<code>xlabel(x)</code>	adds a horizontal label $x$ (along $x$ axis) to the plot
<code>ylabel(x)</code>	adds a vertical label $x$ (along $y$ axis) to the plot
<code>grid on</code>	adds a grid to the plot.
<code>clc</code>	clears everything from the matlab command line window.
<code>linspace(x1,x2,p)</code>	generates $p$ equally distant points between $x1$ and $x2$ .
<code>subplot(abc)</code>	generates a subplot of size $a \times b$ , and the current image is of index $c$
<code>ones(length)</code>	generates an array of 1s of size $length$
<code>size()</code>	gives size of array

### B Matlab Code

Matlab codes for each part.

#### B.1 Q1(a)

```
% question1(a)
clc;
close all;
```

```
Tb = .001;

% defined ak
t1 = 0:Tb/999:Tb;
t2 = Tb:Tb/999:2*Tb;
t3 = 2*Tb:Tb/999:3*Tb;
t4 = 3*Tb:Tb/999:4*Tb;
t5 = 4*Tb:Tb/999:5*Tb;
t6 = 5*Tb:Tb/999:6*Tb;
t7 = 6*Tb:Tb/999:7*Tb;
t8 = 7*Tb:Tb/999:8*Tb;
t9 = 8*Tb:Tb/999:9*Tb;
t10 = 9*Tb:Tb/999:10*Tb;
t11 = 10*Tb:Tb/999:11*Tb;

%defining time
t = [t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11];

%initialising ck
y1 = 1*ones(size(t1));
y2 = 1*ones(size(t2));
y3 = 1*ones(size(t3));
y4 = 0*ones(size(t4));
y5 = 0*ones(size(t5));
y6 = 1*ones(size(t6));
y7 = 0*ones(size(t7));
y8 = 1*ones(size(t8));
y9 = 1*ones(size(t9));
y10 = 0*ones(size(t10));
y11 = 0*ones(size(t11));

%defining ck
yck = [y1 y2 y3 y4 y5 y6 y7 y8 y9 y10 y11];

%plot ck
figure(1);
subplot(3,1,1);
plot(t,yck);
ylim([-0.5 1.5]);
title("ck signal");
```

```
xlabel(" time(sec)");
ylabel(" Amplitude(Volt)");
grid on;

% 1 is mapped to 1 and 0 to 0
% both ak and ck are same
yak = yck;

%plot ak
subplot(3,1,2);
plot(t ,yak);
ylim([-0.5 1.5]);
title(" ak signal");
xlabel(" time(sec)");
ylabel(" Amplitude(Volt)");
grid on;

%Given that 1 is mapped to 1, while 0 to 0
%mt and ak are same
ym = yck;

%plot mt
subplot(3,1,3);
plot(t ,ym);
ylim([-0.5 1.5]);
title(" Message signal");
xlabel(" time(sec)");
ylabel(" Amplitude(Volt)");
grid on;
```

## B.2 Q1(b)

```
% question1(b)
clc;
close all;

Tb = .001;

%defining ck
t1 = 0:Tb/999:Tb;
```

```
t2 = Tb:Tb/999:2*Tb;
t3 = 2*Tb:Tb/999:3*Tb;
t4 = 3*Tb:Tb/999:4*Tb;
t5 = 4*Tb:Tb/999:5*Tb;
t6 = 5*Tb:Tb/999:6*Tb;
t7 = 6*Tb:Tb/999:7*Tb;
t8 = 7*Tb:Tb/999:8*Tb;
t9 = 8*Tb:Tb/999:9*Tb;
t10 = 9*Tb:Tb/999:10*Tb;
t11 = 10*Tb:Tb/999:11*Tb;

%defining time
t = [t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11];

%initialising ck
y1 = 1*ones(size(t1));
y2 = 1*ones(size(t2));
y3 = 1*ones(size(t3));
y4 = 0*ones(size(t4));
y5 = 0*ones(size(t5));
y6 = 1*ones(size(t6));
y7 = 0*ones(size(t7));
y8 = 1*ones(size(t8));
y9 = 1*ones(size(t9));
y10 = 0*ones(size(t10));
y11 = 0*ones(size(t11));

%defining ck
yck = [y1 y2 y3 y4 y5 y6 y7 y8 y9 y10 y11];

%plot ck
figure(1);
subplot(3,1,1);
plot(t,yck);
ylim([-0.5 1.5]);
title("ck signal");
xlabel("time(sec)");
ylabel("Amplitude(Volt)");
grid on;

%new mapping for ak.
```

```
% 0 matches to 1, 1 matches to -1
y1 = 1*ones(size(t1));
y2 = 1*ones(size(t2));
y3 = 1*ones(size(t3));
y4 = -1*ones(size(t4));
y5 = -1*ones(size(t5));
y6 = 1*ones(size(t6));
y7 = -1*ones(size(t7));
y8 = 1*ones(size(t8));
y9 = 1*ones(size(t9));
y10 = -1*ones(size(t10));
y11 = -1*ones(size(t11));

yak = [y1 y2 y3 y4 y5 y6 y7 y8 y9 y10 y11];

%plotting ak
subplot(3,1,2);
plot(t,yak);
ylim([-1.5 1.5]);
title("ak signal");
xlabel("time(sec)");
ylabel("Amplitude(Volt)");
grid on;

%since mt and ak are same
ym = yak;

%plotting mt
subplot(3,1,3);
plot(t,ym);
ylim([-1.5 1.5]);
title("message signal");
xlabel("time(sec)");
ylabel("Amplitude(Volt)");
grid on;
```

### B.3 Q1(c)

```
% question1(c)
clc;
close all;

Tb = .001;

%defining ck
t1 = 0:Tb/999:Tb;
t2 = Tb:Tb/999:2*Tb;
t3 = 2*Tb:Tb/999:3*Tb;
t4 = 3*Tb:Tb/999:4*Tb;
t5 = 4*Tb:Tb/999:5*Tb;
t6 = 5*Tb:Tb/999:6*Tb;
t7 = 6*Tb:Tb/999:7*Tb;
t8 = 7*Tb:Tb/999:8*Tb;
t9 = 8*Tb:Tb/999:9*Tb;
t10 = 9*Tb:Tb/999:10*Tb;
t11 = 10*Tb:Tb/999:11*Tb;

%defining time
t_1 = [t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11];

%initialising ck
y1 = 1*ones(size(t1));
y2 = 1*ones(size(t2));
y3 = 1*ones(size(t3));
y4 = 0*ones(size(t4));
y5 = 0*ones(size(t5));
y6 = 1*ones(size(t6));
y7 = 0*ones(size(t7));
y8 = 1*ones(size(t8));
y9 = 1*ones(size(t9));
y10 = 0*ones(size(t10));
y11 = 0*ones(size(t11));

%defining ck
yck = [y1 y2 y3 y4 y5 y6 y7 y8 y9 y10 y11];

%plot ck
figure(1);
subplot(5,1,1);
```



```
plot(t_1 ,yck);
ylim([-0.5 1.5]);
title("ck");
xlabel("time(sec)");
ylabel("Amplitude(Volt)");
grid on;

% since ck = yak in part 1a.
yak = yck;

% plot ak for 1a
subplot(5,1,2);
plot(t_1 ,yak);
ylim([-0.5 1.5]);
title(" ak for(1-a)");
xlabel("time(sec)");
ylabel("Amplitude(Volt)");
grid on;

%defining ak for part 1b
y1 = 1*ones(size(t1));
y2 = 1*ones(size(t2));
y3 = 1*ones(size(t3));
y4 = -1*ones(size(t4));
y5 = -1*ones(size(t5));
y6 = 1*ones(size(t6));
y7 = -1*ones(size(t7));
y8 = 1*ones(size(t8));
y9 = 1*ones(size(t9));
y10 = -1*ones(size(t10));
y11 = -1*ones(size(t11));

%defining ak for 1b
yak1 = [y1 y2 y3 y4 y5 y6 y7 y8 y9 y10 y11];

%plotting of ak
figure(1);
subplot(5,1,3);
plot(t_1 ,yak1);
ylim([-1.5 1.5]);
title(" ak for(1-b)");
```

```
xlabel(" time(sec)");
ylabel(" Amplitude(Volt)");
grid on;

% message for 1a
Tb = .0005;

% redefining time
t1 = 0:Tb/999:Tb;
t010 = 1*Tb:Tb/999:2*Tb;
t2 = 2*Tb:Tb/999:3*Tb;
t20 = 3*Tb:Tb/999:4*Tb;
t3 = 4*Tb:Tb/999:5*Tb;
t30 = 5*Tb:Tb/999:6*Tb;
t4 = 6*Tb:Tb/999:7*Tb;
t40 = 7*Tb:Tb/999:8*Tb;
t5 = 8*Tb:Tb/999:9*Tb;
t50 = 9*Tb:Tb/999:10*Tb;
t6 = 10*Tb:Tb/999:11*Tb;
t60 = 11*Tb:Tb/999:12*Tb;
t7 = 12*Tb:Tb/999:13*Tb;
t70 = 13*Tb:Tb/999:14*Tb;
t8 = 14*Tb:Tb/999:15*Tb;
t80 = 15*Tb:Tb/999:16*Tb;
t9 = 16*Tb:Tb/999:17*Tb;
t90 = 17*Tb:Tb/999:18*Tb;
t10 = 18*Tb:Tb/999:19*Tb;
tA0 = 19*Tb:Tb/999:20*Tb;
t11 = 20*Tb:Tb/999:21*Tb;
tB0 = 21*Tb:Tb/999:22*Tb;

%defining time
t_1 = [t1 t010 t2 t20 t3 t30 t4 t40 t5 t50
t6 t60 t7 t70 t8 t80 t9 t90 t10 tA0 t11 tB0];

%defining mt for 1a
y1 = 1*ones(size(t1));
Y10 = zeros(size(t010));
y2 = 1*ones(size(t2));
Y20 = zeros(size(t20));
y3 = 1*ones(size(t3));
```

```
Y30 = zeros(size(t30));
y4 = 0*ones(size(t4));
Y40 = zeros(size(t40));
y5 = 0*ones(size(t5));
Y50 = zeros(size(t50));
y6 = 1*ones(size(t6));
Y60 = zeros(size(t60));
y7 = 0*ones(size(t7));
Y70 = zeros(size(t70));
y8 = 1*ones(size(t8));
Y80 = zeros(size(t80));
y9 = 1*ones(size(t9));
Y90 = zeros(size(t90));
y10 = 0*ones(size(t10));
YA0 = zeros(size(tA0));
y11 = 0*ones(size(t11));
YB0 = zeros(size(tB0));

%defining ak
ym = [y1 Y10 y2 Y20 y3 Y30 y4 Y40 y5 Y50 y6
Y60 y7 Y70 y8 Y80 y9 Y90 y10 YA0 y11 YB0];

%plotting of ak
subplot(5,1,4);
plot(t_1,ym);
ylim([-1.5 1.5]);
title(" message signal for 1a");
xlabel("time(sec)");
ylabel("Amplitude(Volt)");
grid on;

%defining mt for 1b
y1 = 1*ones(size(t1));
Y10 = zeros(size(t010));
y2 = 1*ones(size(t2));
Y20 = zeros(size(t20));
y3 = 1*ones(size(t3));
Y30 = zeros(size(t30));
y4 = -1*ones(size(t4));
Y40 = zeros(size(t40));
y5 = -1*ones(size(t5));
```

```
Y50 = zeros(size(t50));
y6 = 1*ones(size(t6));
Y60 = zeros(size(t60));
y7 = -1*ones(size(t7));
Y70 = zeros(size(t70));
y8 = 1*ones(size(t8));
Y80 = zeros(size(t80));
y9 = 1*ones(size(t9));
Y90 = zeros(size(t90));
y10 = -1*ones(size(t10));
YA0 = zeros(size(tA0));
y11 = -1*ones(size(t11));
YB0 = zeros(size(tB0));

%defining ak
ym2 = [y1 Y10 y2 Y20 y3 Y30 y4 Y40 y5 Y50 y6
Y60 y7 Y70 y8 Y80 y9 Y90 y10 YA0 y11 YB0];

%plot of ak
subplot(5,1,5);
plot(t_1,ym2);
ylim([-1.5 1.5]);
title(" message signal for 1b");
xlabel(" time(sec)");
ylabel(" Amplitude(Volt)");
grid on;

% question1(d)
% question1(d)
clc;
close all;

Tb = .001;

% defining ck
t1 = 0:Tb/999:Tb;
t2 = Tb:Tb/999:2*Tb;
t3 = 2*Tb:Tb/999:3*Tb;
t4 = 3*Tb:Tb/999:4*Tb;
t5 = 4*Tb:Tb/999:5*Tb;
t6 = 5*Tb:Tb/999:6*Tb;
```

```
t7 = 6*Tb:Tb/999:7*Tb;
t8 = 7*Tb:Tb/999:8*Tb;
t9 = 8*Tb:Tb/999:9*Tb;
t10 = 9*Tb:Tb/999:10*Tb;
t11 = 10*Tb:Tb/999:11*Tb;

%defining time
t_1 = [t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11];

%initialising ck
y1 = 1*ones(size(t1));
y2 = 1*ones(size(t2));
y3 = 1*ones(size(t3));
y4 = 0*ones(size(t4));
y5 = 0*ones(size(t5));
y6 = 1*ones(size(t6));
y7 = 0*ones(size(t7));
y8 = 1*ones(size(t8));
y9 = 1*ones(size(t9));
y10 = 0*ones(size(t10));
y11 = 0*ones(size(t11));

%defining ck
yck = [y1 y2 y3 y4 y5 y6 y7 y8 y9 y10 y11];

%plot of ck
figure(1);
subplot(3,1,1);
plot(t_1 ,yck);
ylim([-1.5 1.5]);
title("ck");
xlabel("time(sec)");
ylabel("Amplitude(Volt)");
grid on;

% defining ak
Tb = .0005;

% defining time
t1 = 0:Tb/999:Tb;
t010 = 1*Tb:Tb/999:2*Tb;
```

```

t2 = 2*Tb:Tb/1999:3*Tb;
t20 = 3*Tb:Tb/999:4*Tb;
t3 = 4*Tb:Tb/1999:5*Tb;
t30 = 5*Tb:Tb/999:6*Tb;
t4 = 6*Tb:Tb/1999:7*Tb;
t40 = 7*Tb:Tb/999:8*Tb;
t5 = 8*Tb:Tb/1999:9*Tb;
t50 = 9*Tb:Tb/999:10*Tb;
t6 = 10*Tb:Tb/1999:11*Tb;
t60 = 11*Tb:Tb/999:12*Tb;
t7 = 12*Tb:Tb/1999:13*Tb;
t70 = 13*Tb:Tb/999:14*Tb;
t8 = 14*Tb:Tb/1999:15*Tb;
t80 = 15*Tb:Tb/999:16*Tb;
t9 = 16*Tb:Tb/1999:17*Tb;
t90 = 17*Tb:Tb/999:18*Tb;
t10 = 18*Tb:Tb/999:19*Tb;
tA0 = 19*Tb:Tb/1999:20*Tb;
t11 = 20*Tb:Tb/1999:21*Tb;
tB0 = 21*Tb:Tb/999:22*Tb;

```

```
%defining time
```

```

t_2 = [t1 t010 t2 t20 t3 t30 t4 t40 t5 t50 t6 t60 t7
t70 t8 t80 t9 t90 t10 tA0 t11 tB0];

```

```
%defining ak
```

```

y1 = 1*ones(size(t1));
Y10 = zeros(size(t010));
y2 = 0*ones(size(t2));
Y20 = zeros(size(t20));
y3 = 1*ones(size(t3));
Y30 = zeros(size(t30));
y4 = 0*ones(size(t4));
Y40 = zeros(size(t40));
y5 = 0*ones(size(t5));
Y50 = zeros(size(t50));
y6 = 0*ones(size(t6));
Y60 = zeros(size(t60));
y7 = 0*ones(size(t7));
Y70 = zeros(size(t70));
y8 = 0*ones(size(t8));

```

```
Y80 = zeros(size(t80));
y9 = 1*ones(size(t9));
Y90 = zeros(size(t90));
y10 = 0*ones(size(t10));
YA0 = zeros(size(tA0));
y11 = 0*ones(size(t11));
YB0 = zeros(size(tB0));

%defining ak
yak = [y1 Y10 y2 Y20 y3 Y30 y4 Y40 y5 Y50 y6 Y60
y7 Y70 y8 Y80 y9 Y90 y10 YA0 y11 YB0];

%plotting of ak
subplot(3,1,2);
plot(t_2 ,yak);
ylim([-1.5 1.5]);
title("ak");
xlabel("time(sec)");
ylabel("Amplitude(Volt)");
grid on;

% since ak = ym
ym = yak;
subplot(3,1,3);
plot(t_2 ,ym);
ylim([-1.5 1.5]);
title("message signal");
xlabel("time(sec)");
ylabel("Amplitude(Volt)");
grid on;
```

## References

- [1] IIT Mandi lectures on EE304 offered by Dr Adarsh <https://cloud.iitmandi.ac.in/d/4bb3a5f304334160ab67/>
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- [4] Line Coding by All About Circuits <https://www.allaboutcircuits.com/textbook/radio-frequency-analysis-design/radio-frequency-modulation/phase-modulation-theory-time-domain-frequency-domain/>