Time-Varying Betas in Foreign Exchange Returns: An IPCA Approach*

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Abstract

Building a prediction model for foreign exchange market prices is remarkably challenging. One possible reason is that, for the purpose of identification, the beta coefficients are typically assumed to be constant over time. Following Kelly et al. (2019), we utilize the Instrumented Principal Component Analysis (IPCA), a flexible factor model, to reduce the dimensionality of diverse information sets encompassing FX data and various risk factors from G10 countries, all while maintaining model traceability and accommodating time-varying betas. Our results demonstrate IPCA's superior out-of-sample predictability compared to the random-walk model and the conventional PCA in terms of both statistic and economic significance. Furthermore, we find the medium-term interest rate differential, stock market momentum, and idiosyncratic volatility as crucial components for predicted returns.

Keywords: Exchange Rates, Instrumented Principal Component Analysis, Cross Section of Returns, Conditional Beta, Machine Learning
JEL Classification: F31, F37, G15, I12.

1 Introduction

It is surprisingly difficult to build a prediction model for the prices in foreign exchange (FX) market. The reason could be that FX returns actually follow a random walk process (Meese and Rogoff, 1983), or that the set of useful predictors that comove with FX returns varies with time (Engel et al., 2015; Rossi, 2013). At the same time, the proliferation of currency investment strategies continues to boost the trading volume in the currency markets.[1] Therefore, there is firmly growing interests not only for the academics but also for the practitioners and investors to search for a model with good predictive ability at least for a sufficiently long period that is relevant, if not for all subperiods where the FX data histories are available.

The predictability in equity market has been well explored and demonstrated in regression models (Rapach et al., 2010, 2013) in the short horizon. However, such predictability concerning the FX returns remains an open debate (Kilian and Taylor, 2003; Filippou et al., 2023). This paper aims to better understand the cross-sectional risk premium in the FX markets by applying a dimension reduction method to extract the relevant information from potential predictors, which include macroeconomic fundamentals and risk premium proxies derived from the stock markets. In addition, we want to evaluate to which extent that the predictors are useful

Inspired by the two-factor model proposed by Lustig et al. (2011), this paper applies a methodology of dimension reduction in order to introduce different sources of information set and at the same time to maintain a tractable model. Particularly, we follow Kelly et al. (2019) and use Instrumented Principal Component Analysis (IPCA) as a factor model to explain the cross section of FX market. A distinctive aspect of the IPCA approach is its allowance for time-varying beta coefficients, offering not only greater econometric flexibility but also theoretical alignment with the Euler equation for asset pricing in investment returns (Ross, 1976; Hansen and Richard, 1987). Furthermore, there is a large body of literature on the forward premium puzzle in which the empirical findings concerning the Uncovered Interest Rate Parity (UIP) deviation are large and the reasons remain as an open debate. The time-varying beta could also be a plausible explanation that introduces the bias in a commonly used linear regression model.

The IPCA method allows the beta to vary according to the macroeconomic condition. Notably, the IPCA method specifies beta coefficients as functions of country's characteristics, enabling them

[1] According to the latest *BIS Triennial Central Bank Survey 2022*, the FX transaction has reached 7.5 trillion US dollar per day in April 2022. Compared with the same survey published in 2019, the market has grown 14% larger.

to vary according to the macroeconomic conditions of each country in our application. This specification serves a dual purpose. First, it establishes a statistical link between the macroeconomic conditions of different countries and the expected currency returns. Second, it allows us to incorporate additional data as instruments, which enhances the efficiency of the estimation of the return factor model.

We collect the FX data of G10 country from January of 2008 to December of 2020 at monthly frequency. The risk factors considered in our paper are consistent with Filippou et al. (2023), including market volatility and macroeconomic variables. The preview of our findings can be summarized in three fold. First, PCA generally explains the cross-sectional returns better than IPCA based on the total R-squared using contemporaneous factors. On the contrary, the predictive R-squared which is using only the past information indicates better performance in IPCA than PCA. Intuitively, it implies that PCA suffers from an overfitting problem. Second, although it is not a surprise that the standard PCA outperforms the random walk model, we show that the IPCA-based in-sample forecast performs better than the above-mentioned two benchmark models. Third, in terms of out-of-sample forecast, PCA's performance is systematically worse than the Random Walk model, regardless of the number of latent factors. On the other hand, the IPCA presents consistent results better than the Random Walk model when choosing a moderate number of latent factors.

One of the very interesting features of the IPCA framework is its compatibility with statistical inference. Not only we can test whether the intercept of the asset pricing model is different from zero, but also we will examine the variable importance contributed to the latent factors. The result of the former test cannot reject the null of zero intercept, implying the latent factors overall provide good explanations on the cross-sectional FX returns. Hence, throughout our paper, we focus only on the conditional version of specification which constrains the intercept to be zero. According to the latter test, the most important factor is the interest rate differential in the medium term and the idiosyncratic volatility. Consistent with the literature, we did not find significance in the widely-used equity market factors, such as dividend yield and price earning ratio.

The contribution of this paper is threefold. First, we employed a dimension-reduction method to predict the cross section of FX returns. We successfully demonstrate that the in-sample explanatory power of risk factors in the post-2007 sample. Second, we show that the out-of-sample predictability clearly outperforms the two different benchmark models, i.e. the random-walk model and the conventional PCA. Third, our approach allows us to formally test the importance of the risk factors. We find that the interest rate differential in medium term and the FX idiosyncratic volatility contain important information in the latent factor that is used to predict the FX returns.

Our work is closely related to three streams of literature. First, FX predictability through global risk factor. Lustig et al. (2011, 2014); Verdelhan (2018) identified a *slope* factor to account for the currency returns, and demonstrated its close correlation to the global market volatility, which is fairly far from being time-invariant. In similar spirits, our work incorporates the information of financial market and macroeconomic conditions in identifying the beta coefficient in the FX determination model. Similarly, Opie and Riddiough (2020) exploited the predictable global risk factor to construct a currency portfolio which outperforms the popular alternatives such as carry trade, among others.

Second, *exchange rate reconnect* is a growing literature that aims to resolve the exchange-rate disconnect puzzle laid out by Obstfeld and Rogoff (2000). Deviating from conventional regression-based models, Wang and Wu (2015) employed an independent component based approach to improve the usefulness of information stored in economic fundamentals for the FX forecast. Our paper also add values to the economic fundamentals by using an instrumented PCA method. Furthermore, Rossi (2013); Lilley et al. (2022) found that the economic fundamentals as global risk proxies recovered significantly their in-sample explanatory power after the year 2007. Engel et al. (2015) showed better predictive power in a sample of early 2000s but not before. Featuring a non-linear method, our work also reflects similar findings in modeling the comovement between macroeconomic fundamentals and exchange rates.

Third, latent factor. Our work is closely related to Nucera et al. (2023) who studied cross-sectionally FX markets using also a PCA-based three-pass method by Giglio and Xiu (2021) and Lettau and Pelger (2020). Their work examined a large number of risk factors and evaluated the corresponding price of risk. Our paper differentiates from searching for more risk factors and focuses instead on the economic interpretations of the latent factor as well as on the model's forecast performance. Ponomareva et al. (2019) showed that PCA can be useful in forecast some bilateral exchange rates in Sterling pound and Australian dollar, but their study was on the time series while our focus is on the cross section of FX markets.

This paper proceeds as follows. Section 2 provides a review of IPCA method and explain how we implement to the currency returns. Section 3 presents the data sources and summary statistics. Next, we report and discuss the empirical findings in Section 4 and analyze the variables' contributions in Section 5. Section 6 summarizes the robustness tests. Finally, Section 7 concludes.

2 Methodology: asset pricing tests

In this section, we provide a review of the Instrumented Principal Component Analysis (IPCA, Kelly et al., 2019) model. Consider the factor model for the currency return $r_{i,t+1}$ of N assets over T periods, incorporating a $K \times 1$ vector of latent factors f_{t+1}:

$$r_{i,t+1} = \alpha_{i,t} + \beta_{i,t} f_{t+1} + \epsilon_{i,t+1}. \tag{1}$$

Following the Euler equation for investment returns (Ross, 1976; Hansen and Richard, 1987), the factor framework (1) is the setting for most empirical analyses of expected returns across assets. While applying this framework, the factor loadings $\beta_{i,t}$ and the intercept $\alpha_{i,t}$ are typically constrained to be static. Many studies either treat the factors $\{f_{t+1}\}$ as completely observable, estimating betas and alphas through regression (e.g., Fama and French, 1993), or regard the factors $\{f_{t+1}\}$ as entirely latent, simultaneously estimating betas, alphas, and the factors using factor techniques like the principal component analysis (PCA, e.g., Chamberlain and Rothschild, 1983; Connor and Korajczyk, 1988).

The IPCA model allows for the dynamic factor loadings $\beta_{i,t}$ and the intercept $\alpha_{i,t}$ to depend on observable $L \times 1$ country characteristics $z_{i,t}$:

$$\alpha_{i,t} = z_{i,t}^{\top} \Gamma_{\alpha} + \nu_{\alpha,i,t}, \text{ and } \beta_{i,t} = z_{i,t}^{\top} \Gamma_{\beta} + \nu_{\beta,i,t}. \tag{2}$$

The specification of $\beta_{i,t}$ and $\alpha_{i,t}$ serves as a statistical linkage between country's characteristics and expected currency returns. Furthermore, it becomes possible to incorporate additional data to estimate the factor model for returns more efficiently by instrumenting the latent factor loadings with observable characteristics $z_{i,t}$. By aggregating L characteristics into K linear combinations, the IPCA model also effectively averages out the characteristics' noise and reduces the characteristic space's dimensionality. For the estimation of the model, combining equations (1) and (2) yields:

$$r_{i,t+1} = z_{i,t}^{\top} \Gamma_{\alpha} + z_{i,t}^{\top} \Gamma_{\beta} f_{t+1} + \epsilon_{i,t+1}^{*}, \tag{3}$$

where $\epsilon_{i,t+1}^{*} = \epsilon_{i,t+1} + \nu_{\alpha,i,t} + \nu_{\beta,i,t} f_{t+1}$ denotes the composite error. Equation (3) can be further written in a vector form:

$$r_{t+1} = Z_t \Gamma_{\alpha} + Z_t \Gamma_{\beta} f_{t+1} + \epsilon_{t+1}^{*}, \tag{4}$$

where r_{t+1} is an $N \times 1$ vector that stacks the individual currency returns $r_{i,t+1}$, Z_t is an $N \times L$ matrix stacking the country's characteristics $z_{i,t}$, and ϵ^*_{t+1} is an $N \times 1$ vector of individual composite error $\epsilon^*_{i,t+1}$. Γ_β, Γ_α and $F = \{f_{t+1}\}_{t=1}^{T-1}$ can be estimated via the alternating least squares:

$$\min_{\Gamma_\beta,\Gamma_\alpha,F} \sum_{t=1}^{T-1} \left(r_{t+1} - Z_t\Gamma_\alpha - Z_t\Gamma_\beta f_{t+1}\right)^\top \left(r_{t+1} - Z_t\Gamma_\alpha - Z_t\Gamma_\beta f_{t+1}\right), \tag{5}$$

subject to the constraints $\Gamma_\beta^\top \Gamma_\beta = \mathbf{I}_K$ and $\Gamma_\beta^\top \Gamma_\alpha = \mathbf{0}_{K\times 1}$.

To ensure the convergence of our algorithm, we then standardize[2] all the time series of the above-mentioned country characteristics $z_{i,t}$ by using the corresponding mean $\overline{z}_i = \frac{1}{T}\sum_{t=1}^T z_{i,t}$ and the standard deviation $\hat{\sigma}_i = \sqrt{\frac{1}{T-1}\sum_{t=1}^T (z_{i,t} - \overline{z}_i)}$,

$$\tilde{z}_{i,t} = \frac{z_{i,t} - \overline{z}_i}{\hat{\sigma}_i}. \tag{6}$$

2.1 Bootstrap Procedure

To evaluate the significance of the alpha and each individual characteristic, Kelly et al. (2019) proposes a bootstrap procedure similar to the wild bootstrap (Wu, 1986; Liu, 1988). Let

$$\Gamma_\beta = \left[\gamma_{\beta,1}, \gamma_{\beta,2}, \ldots, \gamma_{\beta,L}\right]^\top,$$

where $\gamma_{\beta,\ell}$ for $\ell = 1, 2, \ldots, L$ is a $K \times 1$ vector that maps characteristic ℓ to the loadings on the K factors. Let

$$W_\alpha = \hat{\Gamma}_\alpha^\top \hat{\Gamma}_\alpha \text{ and } W_{\beta,\ell} = \hat{\gamma}_{\beta,\ell}^\top \hat{\gamma}_{\beta,\ell} \tag{7}$$

be the Wald-type test statistics for Γ_α and $\gamma_{\beta,\ell}$. Let $x_{t+1} = Z_t^\top r_{t+1}$. Then

$$x_{t+1} = Z_t^\top r_{t+1} = Z_t^\top Z_t \hat{\Gamma}_\alpha + Z_t^\top Z_t \hat{\Gamma}_\beta f_{t+1} + \hat{d}_{t+1}.. \tag{8}$$

where $\hat{d}_{t+1} = Z_t^\top \hat{\epsilon}^*_{t+1}$. For $b = 1, 2, \ldots, B$, the b^{th} bootstrap sample is generated as

$$\tilde{x}^b_{t+1} = Z_t^\top Z_t \hat{\Gamma}_\alpha + Z_t^\top Z_t \hat{\Gamma}_\beta f_{t+1} + \tilde{d}^b_{t+1}, \text{ in which } \tilde{d}^b_{t+1} = q^b_{1,t+1} \hat{d}_{q^b_{2,t+1}}, \tag{9}$$

[2] Kelly et al. (2019) rescale each of the input variable to the interval of $(-1, 1)$. Note that they examined firm-level data which can have extreme values that the analysis would still be largely disturbed with a simple Z-score transformation.

where $q^b_{2,t+1}$ is a uniformly drawn random time index, and $q^b_{1,t+1}$ is a standardized Student's t random variable with unit variance and five degrees of freedom. The inference is made according to the empirical null distribution of estimated test statistics using the bootstrap sample.

3 Data and preliminary findings

3.1 Sources and variable definition

We collect data from the following sources for our analysis. First, we collect the foreign exchange spot and forward rates at monthly frequency from *Datastream* from January 2008 to December 2020. The collected exchange rates are US dollar denominated, where the price currency is always the US dollar while the base currency comes from the non-U.S. country. Thus, the signs of exchange rate returns reflect the strength of the non-U.S. currency: a positive value implies an appreciation (depreciation) in the foreign currency (the U.S. dollar). The collected dataset contains currency of G10 members , including Australia, Canada, Switzerland, Denmark, the euro area, the United Kingdom, Japan, Norway, New Zealand, and Sweden.

As in Filippou et al. (2023), we consider 10 country characteristics computed using macroeconomic and financial data from multiple sources, which will be specified in each of the following variable.

Inflation differential ($INF_{i,t}$) Difference in inflation rates calculated from consumer price index for country i and the United States. The inflation data is collected from the International Financial Statistics (IFS) database of International Monetary Fund (IMF), except for New Zealand and the euro zone where the data is sourced from Bloomberg and Organization for Economic Co-operation and Development (OECD), respectively.

Unemployment rate gap differential ($UN_{i,t}$) Difference in unemployment rate gaps for country i and the United States, where the unemployment rate gap is the cyclical component extracted from the unemployment data[3] using the band-pass filter by Christiano and Fitzgerald (2003).

Bill yield differential ($BILL_{i,t}$) Difference in three-month government bill yields for country i and the United States. The bill data, sourced from OECD, represent the short-term interest rate differential.

[3]The unemployment data is collected from the same sources as inflation differential.

Note yield differential ($NOTE_{i,t}$) and Bond yield differential ($BOND_{i,t}$) Different in five- and ten-year government debt yields for country i and the United States. The note and bond data, sourced from Bloomberg, represent the medium- and long-term interest rate differential.

Dividend yield differential ($DP_{i,t}$) Difference in dividend yields for country i and the United States. The dividend yield data is mainly sourced from Datastream except for Canada and Norway where we collect the information from Bloomberg to ensure the data availability of the whole sample period.

Price-earnings differential ($PE_{i,t}$) Difference in price-earning ratios for country i and the United States. Similar to dividend yield differential, the price-earning data is mainly sourced from Datastream except for Canada, New Zealand, and Norway where the information from Bloomberg is used to complete the data set.

Stock market momentum differential ($SRET12_{i,t}$) Difference in cumulative 12-month returns of stock indices, collecting from MSCI, for country i and the United States.

Idiosyncratic volatility ($IV_{i,t}$) and Idiosyncratic skewness ($IS_{i,t}$) Integrated volatility and skewness computed using the fitted residuals for the two-factor model by Lustig et al. (2011) which is estimated using daily data of currency returns for month t in country i.

3.2 Summary statistics

Table 1 reports the summary statistics of currency returns from the G10 member country. The equally-weighted average returns across the 10 currencies is 1.2% per annum, and the standard deviation is 10.9% per annum. Both are consistent with the literature. We found that the skewness are mostly positive, implying that there are generally more incidents of negative than positive returns in the FX markets. In addition, the kurtosis are all positive, implying that the return distribution do not suffer from tail risk.

The correlation structures of input variables are reported in Table 2. As our analysis is based on panel data, there exist two possible ways of calculating the correlation matrices. We present both in Panel A and B. First, one might want to calculate the correlation across country's characteristics in order to examine the possibility of colinear concern. For each country, we first obtain a correlation matrix among its characteristics and keep only the upper triangle as the matrix is symmetric. Then, we take average across 10 countries to obtain the result in Panel A.

It is easy to spot high correlation among three bond-based variables. In Table 2, the average correlation between bill yield (BILL) and bond yield (BOND) is 80.5%, between BILL and NOTE is 84.3%, and between BOND and note yield (NOTE) is as high as 96.8%. Next, the stock market momentum (SRET12) has moderate correlation of −39.6% with Dividend yield (DP). Additionally, we note that idiosyncratic volatility (IV) is moderately correlated with BOND at 22.2% and with inflation (INF) at 23.2%. Interestingly, the correlation between BOND and INF is found to be weaker at 12.9%, implying that the differentials used in the international context are more likely to capture the risk premium dynamics. Lastly, it is not surprising to find negative and moderate correlation of −21.7% between DP and price-earning ratio (PE) as the opposing behavior seems to be driven by the price component. As a few characteristics are likely to capture the same source of risks, it becomes important to refocus only on the informative variables by reducing the dimension of the input dataset. Therefore, it justifies the choice of IPCA models to be applied in this study.

In Panel B of Table 2, we compute the correlation matrix in a different way. For each country's characteristics, we calculate a correlation matrix across countries, which are denoted by their own currency code. Then, we take average across 10 country's characteristics. It is evident that the correlations are positive and strong, which are generally above 50% but it can get as high as 67.8%. This finding corresponds to Lustig et al. (2011) who argued that there exists common factors in the global currency markets. The only one pair that falls below 20% is between Japan and New Zealand at 16.5%. This is consistent with the fact that Japanese yen and capital market has been used as the funding currency in the carry trade or a tool of diversifying risks thanks to its weak correlation to the rest of countries.

3.3 Beta-sorting portfolios

The time-varying risk premium is documented to be an important feature in the foreign exchange markets (Kaminsky and Peruga, 1990; Tai, 1999; Calomiris and Mamaysky, 2019). This may raise concerns of analyzing the foreign exchange rate returns by a conventional linear regression model where the beta coefficients are constant.

To examine whether our data has the time-varying beta, we divide our in-sample data evenly into two subsamples. Thus, each contains five years of observations. The results are presented in Table 3. We first estimate an Ordinary Least Square (OLS) regression without intercept in each subsample. Then, we rank the currencies according to its beta for each characteristic in the earlier subsample using the data from 2008 to 2012. As we only have ten currencies, we assign them into three portfolios. For the portfolios of high- and low-beta, each of them is assigned with three currencies. That leaves four currencies for the middle portfolios.

We show the average beta of each portfolio in Table 3. During Period 1, all the average betas increase monotonically because that is when the sorting takes place. In Period 2, we observe for almost all characteristics, the betas from the high group decrease and those from the low group increase. Out of 30 portfolios, there are only two exceptions which are marked by a star sign. This finding confirms the time-varying feature of beta coefficient in a linear regression model and reveals the possibility of misspecification there.

IPCA model allows the beta to be time-varying in a linear framework. Furthermore, the variation at the time dimension has a macroeconomic interpretation. Indeed, it is intuitive to model the time-varying sensitivity with respect to market performance based on the real business cycle. The difference unlocked by IPCA model is that the country's characteristics are now used as an instrument, rather as explanatory factors, in the asset pricing model to explain the exchange rate returns.

4 Empirical findings

4.1 Measures of model's performance

We estimate the K-factor IPCA model for various choice of K and consider restricted specification $\Gamma_\alpha = \mathbf{0}$. Two R^2 statistics can help us to evaluate the model's performance. The first is called "Total R^2" which we define as

$$\text{Total } R^2 = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}\hat{\Gamma}_\beta \hat{f}_{t+1}\right)^2}{\sum_{i,t} r^2_{i,t+1}}. \tag{10}$$

It represents the fraction of return variance explained by both the dynamic behavior of conditional loadings as well as by the contemporaneous factor realizations $\hat{f}_{t+1}$, aggregated over all currencies i and all time periods t.

The second measure, named as "Predictive R^2" is slightly different and defined as

$$\text{Predictive } R^2 = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z_{i,t}^{'}\hat{\Gamma}_{\beta}\hat{\lambda}\right)^2}{\sum_{i,t} r_{i,t+1}^2}, \tag{11}$$

where $\hat{\lambda} = \frac{1}{t}\sum_{j\leq t}\hat{f}_j$ is the estimated risk premium at time t, using all past available information to calculate the historical mean of the latent factors. It represents the fraction of realized return variation explained by the model's description of conditional expected returns, which are often estimated by the historical mean. In the constrained specification, the predictive R^2 summarizes the model's ability to describe risk compensation solely through exposure to systematic risk.

4.2 Model's performance: In-sample vs. out-of-sample tests

We split our sample into two parts where the first ten years belong to the in-sample period and the rest of three years are considered to be the out-of-sample period. First, let us begin examining the model's performance in the in-sample period in Table 4.

A common baseline case in the FX literature is the no-change benchmark involved a random walk model consistent with Meese and Rogoff (1983). The random walk model implied the absence of predictability. As a result, both Total and Predictive R^2 will yield a value of zero as there does not exist any informative predictor. This lays out a clean threshold at zero to determine whether the foreign exchange returns are predictable or not.

In Table 4, we evaluate the performance of IPCA models by comparing the two R^2 measures. Naturally, PCA is our alternative model in this analysis because we can estimate different numbers of latent factors in both framework. In this study, we consider up to five latent factors, $K = 1, \cdots, 5$. We first observe in Panel A that the Total R^2 of PCA are systemically higher than those of IPCA, indicating a better in-sample fit of the PCA than the IPCA. Nevertheless, the Total R^2 increases rapidly with the number of latent factors. When there is only one latent factor $K = 1$, the Total R^2 seem quite different at 68.21% and 51.11% between the PCA and IPCA models. This gap become smaller gradually when there are more latent factors. For instance, when $K = 5$, the model performance of IPCA are catching up with the PCA model, since the Total R^2 are 92.18% and 83.81%, respectively.

Next, we examine the Predictive R^2 in Panel B of Table 4. Immediately, we spot a sharp difference in terms of scale that the Predictive R^2 are much smaller than the Total R^2 for both models. This is aligned with the random walk hypothesis (Meese and Rogoff, 1983; Obstfeld and Rogoff, 2000) that the information extracted from the historical returns is not useful for the prediction. Especially for the PCA models, the Predictive R^2 are systematically negative regardless the number of latent factors. In consequence, we find that the performance of trading strategy based on the PCA models cannot beat the no-change benchmark. At this point, our finding agrees with Meese and Rogoff (1983) that macroeconomic fundamentals do not seem to be very informative regarding the foreign exchange returns.

On the contrary, there is hope in the IPCA models, which seem to be able to outperform the benchmark of random walk with a careful selection on the number of latent factors. From the in-sample period, we find one positive Predictive R^2 when $K = 2$ at 1.93%. Furthermore, in the out-of-sample period, we also find two positive Predictive R^2 when $K = 1$ and 3. It is not surprising that the magnitude is smaller in the out-of-sample period (0.03% and 0.28% for the models of $K = 1$ and 3, respectively) than the in-sample period. Overall, it is a common pattern that the in-sample R^2 is higher than out-of-sample R^2 as all models have certain level of over-fitting, which does not seem a serious concern in our analysis as the R^2 across subsamples are fairly similar in their quality and quantity. In short, an appropriate selection on the number of latent factors in the IPCA framework can add value to the historical information for the return prediction.

Finally, given the input dataset, we also test whether the intercept is sufficiently close to zero in order to validate the choice of the constrained IPCA model. The null hypothesis is $\Gamma_\alpha = \mathbf{0}$. Since the standard t test does not apply to IPCA framework, we have to bootstrap the corresponding p-values for our inference. The results are presented in both panels of Table 4. We find non-rejections for all choices of K with the obtained p-values well above 10%, which implies the currency returns could be sufficiently well-explained by the systematic risks approximated by the macroeconomic and financial factors. This finding is close to the dollar-factor narrative of Lustig et al. (2011, 2014); Verdelhan (2018) that there exists common factors for explaining the cross section of currency risk premium. To keep our analysis neat and tractable, we executive our empirical exercises only with the constrained model .

Regarding the comparison between in-sample and out-of-sample periods, we find that generally the former shows a higher Total R^2 than the latter in Panel A of Table 4. The difference is found to be much larger for the IPCA than the PCA models. Taking $K = 3$ for example, the out-of-sample R^2 is 81.48% by the PCA model and hence is 2.17% higher than the in-sample R^2. For the IPCA model, the out-of-sample R^2 is 62.43%, implying a 11.39% drop from the in-sample R^2. Essentially, the

R^2 difference between two subsamples in the IPCA is 5.25 ($\approx 11.39\%/2.17\%$) times larger than the PCA models, indicating the potential overfitting problem in the PCA models. That also provides a hint to explain why the IPCA might have a better chance to outperform the PCA's predictability.

Alternatively, if we examine the Predictive R^2 in Panel B of Table 4, we found similar results in the PCA models where the out-of-sample R^2 are less negative than the in-sample ones. Note that all the R^2 remain negative regardless the sub-samples. Thus, the PCA models still underperform the benchmark of Random walk model. On the other hand, several IPCA models manage to flip the negative in-sample R^2 to deliver a positive out-of-sample R^2, particularly when we select moderate number of latent factors such as $K = 1$, and 3. Similar to the findings with Total R^2, we observe once again much larger difference across the in-sample and out-of-sample Predictive R^2 in the IPCA than the PCA models. In short, we demonstrate the better out-of-sample predictability of the IPCA model than the conventional PCA and the random walk model, which is known to be difficult to defeat.

Focusing on the magnitude of Predictive R^2, the model with three latent factor ($K = 3$) has a higher value of 0.28% than that with one latent factor ($K = 1$) which implies 0.03%. It is a valid concern on the proximity of Predictive R^2 to zero in the model of one latent factor. As a result, we believe that it is a better choice of $K = 3$ as our baseline model in the rest analyses of the paper.

4.3 Economic significance based on the IPCA predictions

To further illustrate the economic significance of the IPCA models, we run a horserace of trading strategies by exploiting the predicted returns by the PCA and IPCA models at currency level. The trading strategies are constructed based on the sign of predicted returns by the PCA or IPCA models. Specifically, if the predicted return of certain foreign currency is positive, the strategy buys the foreign currency or sell the US dollar and hence maintains a long position on foreign currency for that period. Alternatively, the negative predicted return implies a short position on foreign currency. To ensure the comparability, we focus only on the models with three latent factors ($K = 3$) for both models.

Table 5 presents the full-sample and out-of-sample results at currency level. Based on a portfolio of equally-weighted returns across ten currencies, we find that the IPCA model outperforms the PCA in both whole-sample and out-of-sample periods. Noted that the reported returns are annualized to allow the comparison across two periods. We divided the cumulative returns by 13 in the whole sample and by three in the out-of-sample period. Next, we also observe that the out-of-sample returns are higher than the full sample, implying the poor performance in the ten-year in-sample period. Interestingly, the PCA models usually provide good model's fit in the in-sample data with high R^2 measures. In this exercise, we uncover very limited economic significance delivered by the PCA model. Hence, it remains challenging to transform the statistic significance into the economic significance.

Nevertheless, the PCA model reports loss only in one currency (-1.72% by Swiss Franc) while the IPCA leads to losses in two currencies (-1.02% and -0.17% by New Zealand dollar and Swedish Krona, respectively). Thus, one could argue that the IPCA model seems to outperform the PCA only marginally. We further examine the difference between the cumulative returns of two models. Figure 1 plot the difference defined as the IPCA minus the PCA returns. Therefore, positive values imply better performance found in the IPCA model. As in the tabulated results, we show the bar plot using full sample data in the upper subplot, and out-of-sample predictions in the lower subplot. In both periods, the IPCA model outperforms the PCA model in three out of ten currencies. It is clear that IPCA deliver better performance than PCA in most currencies.

By comparing both samples in Figure 1, we have two further remarks on the economic significance that the IPCA model can materialize. First, the difference of cumulative returns seems to be amplified in the out-of-sample period, especially in Swiss Franc and Australian dollar. This finding echoes our main results that the PCA models generally suffer the overfitting issue during the in-sample period. Second, the scale of performance difference is asymmetric. We observe that the negative difference in returns is relatively small compared with positive differences. That is to say, in the case of underperformance, the IPCA strategies fall behind of the PCA model but only slightly. In brief, this result only strengthens the previous findings of the dominating performance of IPCA models over the PCA models.

5 Interpreting IPCA factors

5.1 Which country characteristic matters?

As advocated in the Cochrane (2011) presidential address, it would be pertinent to any empirical asset pricing paper to investigate the incremental contribution of the explanatory variables. In this paper, we propose to test the statistical significance of a country's characteristic by simultaneously holding all other characteristics constant. Since each character enters the beta specification only through the Γ_β matrix, we can set the l^{th} row to be all zeros and then recalculate the Total R^2. If the Total R^2 drops dramatically, that would imply the l^{th} characteristic to be very important for the model's fit.

The results concerning R^2 reduction can be found in Table 6. We report the analysis of the models of one and three latent factors in Panel A and B, respectively. Since it might be interesting to compare the results across both models, we then follow Gu et al. (2020, 2021) to rescale the sum of R^2 reduction on all ten characteristics to one. Among ten country's characteristics, we find Note yield, Idiosyncratic volatility, and Stock momentum as the top three characteristics that drive the predicted returns in the model with only one latent factor. Furthermore, we also find the same characteristics in the top three list of the model with additional latent factors. The findings are fairly consistent in the ranking of feature importance that both models agree on Note yield to be the most important characteristics which cause R^2 reduction as high as 43.62% and 40.64% in the model with one and three latent factors, respectively. This finding reaffirms that the useful information of interest rates can be found in the medium term instead of short or long term. Specifically, it is consistent with better predictability performance in the medium-term exchange rates (Engel et al., 2015; Yesin, 2016; Eichenbaum et al., 2021).

One piece of information that the R^2 reduction did not provide is the direction of driving force with respect to each of the country characteristic. The IPCA models allow us to estimate the Γ_β coefficient whose sign reveals a bit more information on the directional contribution. It is clear in Panel A of Table 6 that the correlation between country's characteristics and exchange rate returns is not uniformly positive or negative. This is aligned with the findings in the regression-based analysis. However, one should be more careful in interpreting these coefficients because the exposure of country's characteristics is actually the inner product of two components, i.e. the Γ_β coefficient and the latent factor, f. In the model with three latent factors in Panel B, it becomes less obvious if one wants to find a clean interpretation on the directional influence of the explanatory variables. Since there are now three different factors, there is a chance that they do not agree with each other on the coefficient signs. Take Note yield for instance, the Γ_β coefficients are negative for Γ_{β_1} and Γ_{β_2}, but it flips to positive for Γ_{β_3}. Similar phenomenon is also spotted in the important variables, such as Idiosyncratic volatility and Stock momentum, as well as other less important variables. Although, the additional latent factors can enrich the IPCA model and strengthen its predictability, the interpretability on the Γ_β coefficient seems to degenerate rapidly in the number of latent factors.

Therefore, a formal test on whether $\Gamma_\beta = 0$ can perhaps provide better insight on the statistical significance of country's characteristics. The bootstrapped p-values in the last column of Table 6 can illustrate the relevance of country's characteristics. Under the 10% significance level, the majority of characteristics in the model of $K = 1$ passes the relevance test, except for Bond yield and Price-earning ratio with p-values at 32.9% and 62.0%, respectively. With the same significance level, the model of $K = 3$ is left with only Price-earning ratio, whose p-value is still as high as 30.3% in Panel B and thus could not pass the relevance test. Note that this finding is consistent across the number of latent factors. In short, the IPCA models are able to make the best use of macro-finance indicators as only one out of ten pre-determined characteristics is found to be redundant in the analysis.

If we choose a stricter significance level at 5% instead, one would find impossible to reject the null hypothesis in Panel B for the characteristic of Idiosyncratic volatility. Based on the amount of R^2 reduction, which is indeed a measure of economic significance, the Idiosyncratic volatility is highly important for the IPCA model of $K = 3$. Nevertheless, the statistical significance seems to be marginal only with the p-value of 6.6% in Table 6. With the increasing number of latent factors, we could witness mild difference in the ranking of characteristic contribution, depending on the economic or statistical significance. However, the conclusion that one can draw is generally consistent across the number of latent factors in IPCA models. Overall, the design of instrumented variables seems useful to reinforce the informativeness of the macroeconomic fundamentals, which were found to have weak signal in traditional regression-based models. Additionally, one cannot rule out that both macroeconomic and finance variables are important to understand the dynamics of foreign exchange rate return.

5.2 Comparison with existing models

We now compare IPCA with alternative models such as the OLS and the fixed effects regressions since they are widely used in the international finance literature. Table 7 reports the findings of which the model's performance expressed by Total and Predictive R^2 as defined in equations (10) and (11) can be found in Panel A, and the feature importance demonstrated by the significance of coefficients is shown in Panel B.

Regression (1) is the simple OLS with constant term. From Panel A, we can tell immediately that the OLS regression has a very poor performance. Among the four R^2 measures, the in-sample Total R^2 is the only one that stays positive at 11.09%. While it seems to outperform the random-walk model, it performs considerably worse than any of the dimension-reduction method, including PCA and IPCA. Furthermore, the constant α (≈ -0.016 with standard error at 0.004) is found to be positive and significant, which implies an unexplained risk premium equivalent to -1.6% per month. Therefore, the OLS regression does not seem a good candidate to explain foreign exchange rate risk premium.

Regressions (2) and (3) take various fixed effects into account in order to improve the explanatory power of the regression models. The country and the calendar month are common and obvious options when it comes to choose the fixed effects in an international context. Surprising, in Table 7 we found that the Total R^2 does not increase with the inclusion of additional fixed effects in the in-sample period. Instead, it drops slightly from 7.45% to 7.17%. This is at odds with the common intuition in panel regression studies where a large fraction of variations in financial returns can be explained away with the careful selection of fixed effects. We want to stress that, in order to keep our results comparable with the baseline IPCA models, the Total R^2 we are using here is based on Equation (10), which has a different definition from the adjusted R^2 computed from regressions. The principal discrepancy between them lies in the denominator where the Total R^2 in Equation (10) is not net of mean but simply the gross variation in foreign exchange returns. Normally, the useful fixed effects are capable of absorbing the unexplained amount found in the constant term, which essentially contribute to the global mean of foreign exchange returns. Without addressing this part, the Total R^2 would carry a larger denominator that eventually dilutes and and thus underestimates the model's performance. Hence, that explains why the Total R^2 of regression (3) is lower than regression (2).

Given the poor results of Total R^2 from the fixed-effects regression, we expect to find negative Predictive R^2 at least in the in-sample period. Panel A of Table 7 confirms our intuition. However, we find in both in-sample and out-of-sample periods that Predictive R^2 are slightly less negative in regression (3) than (2). It is worth noting that the improvements on Predictive R^2 after considering the calendar month fixed effects seem to agree with the usefulness of fixed effects in β-invariant models. That justifies somehow the evolution of using multiple fixed effects in studying risk premium of exchange rates.

Despite the fast-growing literature on the non-linear prediction models or even the machine learning models, one important motivation to adopt the regression-based approach is that people are interested in the drivers of the dependent variable. In Panel B, we find that the coefficients generally have consistent sign and significance level across different regression models. Nevertheless, the number of significant characteristics decreases if we control for additional fixed effects. Again, this finding certifies partially the usefulness of fixed effects. Clustered at both time and cross-sectional dimensions, regression (3) seems to be able to present a sparse set of important explanatory variables. The choice of top three characteristics by the fixed-effects regression is mostly different from the IPCA selection. The two approaches agree only on the importance of Stock market momentum and disagree on most of country characteristics. Alternatively, the characteristics that are statistically significant across three regressions in Table 7, including Bill yield and Idiosyncratic skewness, are not so significant in the IPCA models. The misalignment here points out the risk of misinterpreting the important characteristics for currency returns if the time-varying feature of coefficients is not properly addressed. To summarize, we stress the necessity of instrumenting the country characteristics in order to upgrade both the model's fitness and predictability from the traditional regression-based models.

5.3 What does each latent factor represent?

To better understand the latent factors of IPCA and the information contained in them, we must dig further into the IPCA models. In particular, we can extract the product of Γ_β and f from Equation (3) where you can see that this product captures the association of country's characteristics with exchange rate returns. Note that Γ_β alone, as reported in Table 6, is only half of the story when one intends to interpret the exposure of country's characteristics because the values of latent factors f can be either positive or negative.

We show in this paper the results from the baseline model with three latent factors ($K = 3$). Figure 2 presents two time-series plots of the product of $\Gamma_\beta \times f$ aggregated across three factors. The upper plot 2.A shows three most important country's characteristics according to the R^2 reduction, while the lower plot 2.B illustrates the rest of seven characteristics in our dataset. From the plot A, we notice that Note yield in solid line is generally very important even among the top three characteristics as it has higher value in $\Gamma_\beta \times f$ over time. This is aligned with the carry-trade profitability in the international currency markets that the interest rate differential reflected also by Note yield can explain well the foreign exchange rates dynamics and thus can be exploited by investors to earn profit.

However, there are a few occasions in which Idiosyncratic volatility show higher exposures than Note yield in Figure 2.A. The first occasion was the 2008 Financial Crisis, which is not at all surprising as the high risk premium due to the enormous uncertainty dominated not only in currency markets but also in stock markets and other asset classes. Next, the second occasion was in the year 2012 where the market was pessimistic at first due to the European debt crisis but the realized performance in the economic and hence the financial markets was better than expected. As the inflation was still rising since after the 2008 Financial Crisis, the interest-rate-based measures, including Note yield, had to remain high following the normal monetary policy response. Overall, these two incidents reveal clearly that the factors based on interest rate differential are not the only determinants in the foreign exchange markets. By applying the IPCA models, we are able to separate additional characteristics such as Idiosyncratic volatility that are essential for explaining the currency returns. More interestingly, we are capable of identifying the timing when the characteristics flip the signs or modify the magnitudes in their exposures. The fact that we do not have to restrict the sign of exposure throughout the estimation period makes the models more appealing in their performance in terms of the fitness and as well as the prediction.

Comparing with the values of top three characteristics, we find in the plot 2.B that the rest of seven characteristics have much weaker exposures as the range on y-axis is smaller. Among these seven characteristics, we recognize that Bill yield in dashed line and Bond yield in dotted line are shown to have high exposure to the foreign exchange rate returns. This results can be previewed by the high correlation among the three characteristics based on interest rates. Again, this is consistent with the carry-trade profitability most of the time. Additionally, it is worth noting that the Price-earning ratio seems to demonstrate similar pattern as the Idiosyncratic volatility at least in the 2008 Financial Crisis and around the year of 2012. Nevertheless, the magnitude was much smaller. Generally, the exposures are so weak that one cannot really distinguish a clear patterns of certain characteristics in the time series.

Next, we illustrate the product of $\Gamma_\beta \times f$ at the level of latent factors. Thus, there are three subplots in Figure 3. To keep our results tractable, we exclude the seven characteristics that are less important in our dataset in order to avoid the noise that comes with them. We find a clean pattern that each factor represents the information of one top-three characteristics. Specifically, the factor one is dominated by Note yield in solid line, the factor two is captured principally by Stock market momentum in dashed line, and the factor three is driven by Idiosyncratic volatility in dotted line. This finding supports the validity of IPCA models which seem to perform pretty well in preserving the important and relevant information in a small number of factors.

6 Robustness and discussions

6.1 Currency excess returns

Our baseline model was built to test the gross returns of foreign exchange markets. However, in the literature, there is considerable interests in understanding the foreign exchange risk premium. In order to extend the usefulness of IPCA model, we run robustness tests on the currency excess returns instead.

By the Covered Interest Rate Parity, we use the forward premium defined as $fp_{i,t} = \frac{F_{i,t} - S_{i,t}}{S_{i,t}} \approx i_{i,t} - i_{US,t}$, to approximate the interest rate differential between the G10 member countries i and the US. We then subtract the forward premium from the gross returns[4]

$$xr_{i,t+1} = \ln(S_{i,t+1}) - \ln(S_{i,t}) - fp_{i,t}. \tag{12}$$

The results are presented in Table 8. The patterns are similar to the baseline results in Table 4. First, all the predictive R^2 are negative in the PCA models. Second, Total R^2 increase in the number of latent factors and PCA models show higher Total R^2 than IPCA models in both subsamples. For PCA models alone, the in-sample Total R^2 is higher than the out of sample. Next, we find a few positive values in Predictive R^2 with the same two latent factors $K = 1, 3$ as in the main finding that IPCA shows the best predict ability with moderate number of latent factors. In short, we validate that both excess and gross returns of the currency markets deliver similar results regarding the IPCA performance.

6.2 Limitation of our studies

While the IPCA models successfully outperform the alternative models especially in out-of-sample period, there were at least two limitations affected our analysis. First, the selection on the number of latent factors is neither obvious nor intuitive. Our analysis of time-varying beta estimated by IPCA models delivers better performance than the no-change benchmark which was often found

[4] Alternatively, there is another definition for currency excess return as $xr_{i,t+1} = \frac{S_{i,t+1} - S_{i,t}}{S_{i,t}} - \frac{S_{i,t} - F_{i,t}}{S_{i,t}} = \frac{S_{i,t+1} - F_{i,t}}{S_{i,t}}$, where two $S_{i,t}$ in the numerator cancel out each other. Usually the results are very similar, but this presentation is closer to the common practice, for instance in Nucera et al. (2023).

to be challenging in the literature (Meese and Rogoff, 1983; Kilian and Taylor, 2003). Specifically, the good performance is found in the IPCA models with one and three latent factors, $K = 1$ and 3. Note that the performance of model $K = 1$ is weaker than $K = 3$, implying that the model's fit is not improving in the number of latent factors. In other words, there is a non-linear relationship between the number of K and the model's performance. While we are confident with the claim that a moderate number of latent factors is the best choice for the time-varying betas, the final number of K seems to be data-dependent.

Secondly, there are plenty of negative out-of-sample R^2 in our analysis, which may be due to the modest cross-section of foreign exchange returns. Alternatively, in the study with a large cross-section at firm level, we often find higher values in R^2, and thus it is more likely to observe positive Predictive R^2 as in Kelly et al. (2019). The total number of currencies is much smaller than the that of firms. Not to mention that many currencies are subject to the government's capital control and limited availability in the return history. Therefore, the such currency returns containing a lot of idiosyncratic risks would often result in discontinued or non-stationary time series which one would have no choice but to exclude them from a general analysis on risk premium. That is why we focus on the comparison with the alternative PCA models and other regression-based approaches, rather than the R^2 values across articles. It is worth noting that even when we obtain negative values on predictive R^2, the IPCA models always generate R^2 that are less negative than the PCA models and other existing, regression-based models.

7 Conclusion

Building a prediction model for foreign exchange market prices is remarkably challenging. Following Kelly et al. (2019), we utilize the Instrumented Principal Component Analysis (IPCA), a flexible factor model, to reduce the dimensionality of diverse information sets encompassing various macro-finance characteristics from G10 countries, all while maintaining model traceability and accommodating time-varying betas. The contribution of this paper can be summarized by three aspects. Firstly, it utilizes a dimension-reduction method for predicting the cross-section of foreign exchange returns, showcasing enhanced in-sample explanatory power in the post-2007 period. Secondly, the out-of-sample predictability surpasses two benchmark models, i.e. the random-walk model and the conventional PCA. Thirdly, the approach facilitates formal tests on the significance of risk factors, revealing that the interest rate differential in the medium term serves as crucial information in predicting foreign exchange returns.

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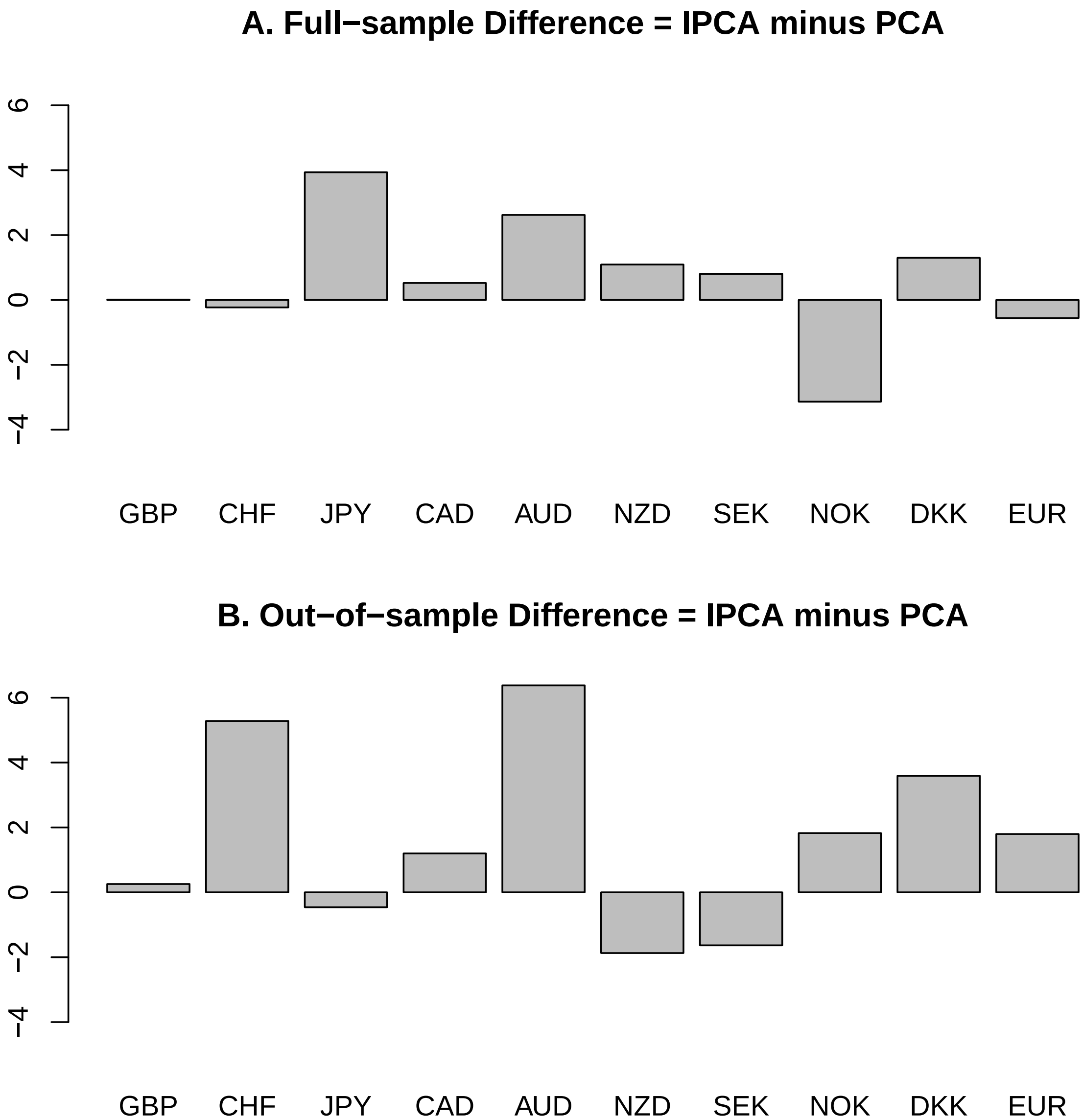

Figure 1. Performance difference of trading strategies: PCA vs. IPCA

This chart compares the performance of trading strategies based on the predictions of the PCA and IPCA models. The difference is obtained by subtracting the cumulative returns of IPCA from PCA models. Panel A uses the full-sample predictions while Panel B only processes the out-of-sample predictions. To ensure comparability, both models have three latent factors ($K = 3$) and the cumulative returns are annualized and expressed in percentage point.

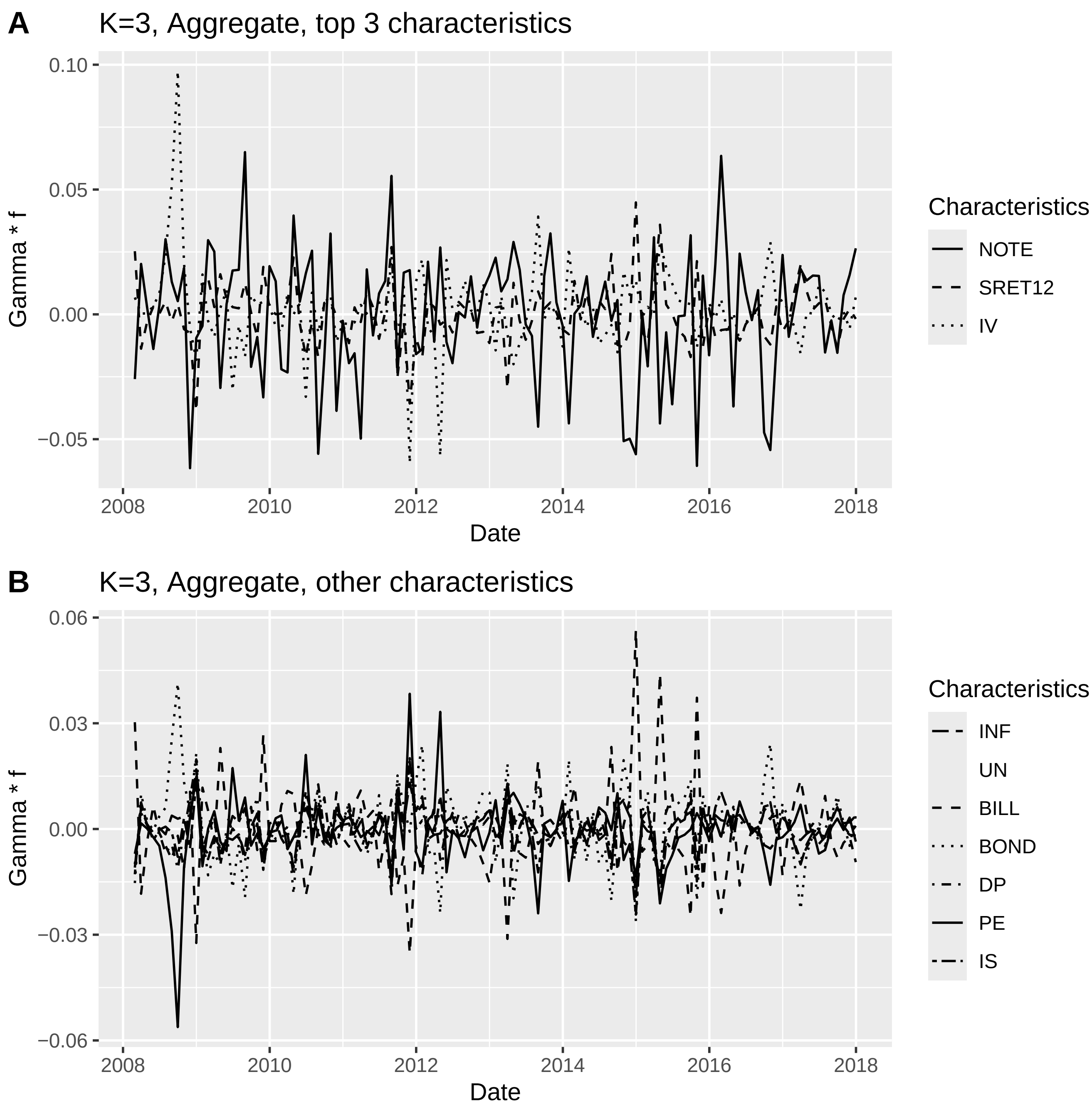

Figure 2. Time-varying beta by characteristic: Aggregate latent factors

This chart presents the time-series coefficients, i.e. the product of Γ_β and f in Equation (3), for each country characteristic based on the IPCA model with three latent factors ($K = 3$).

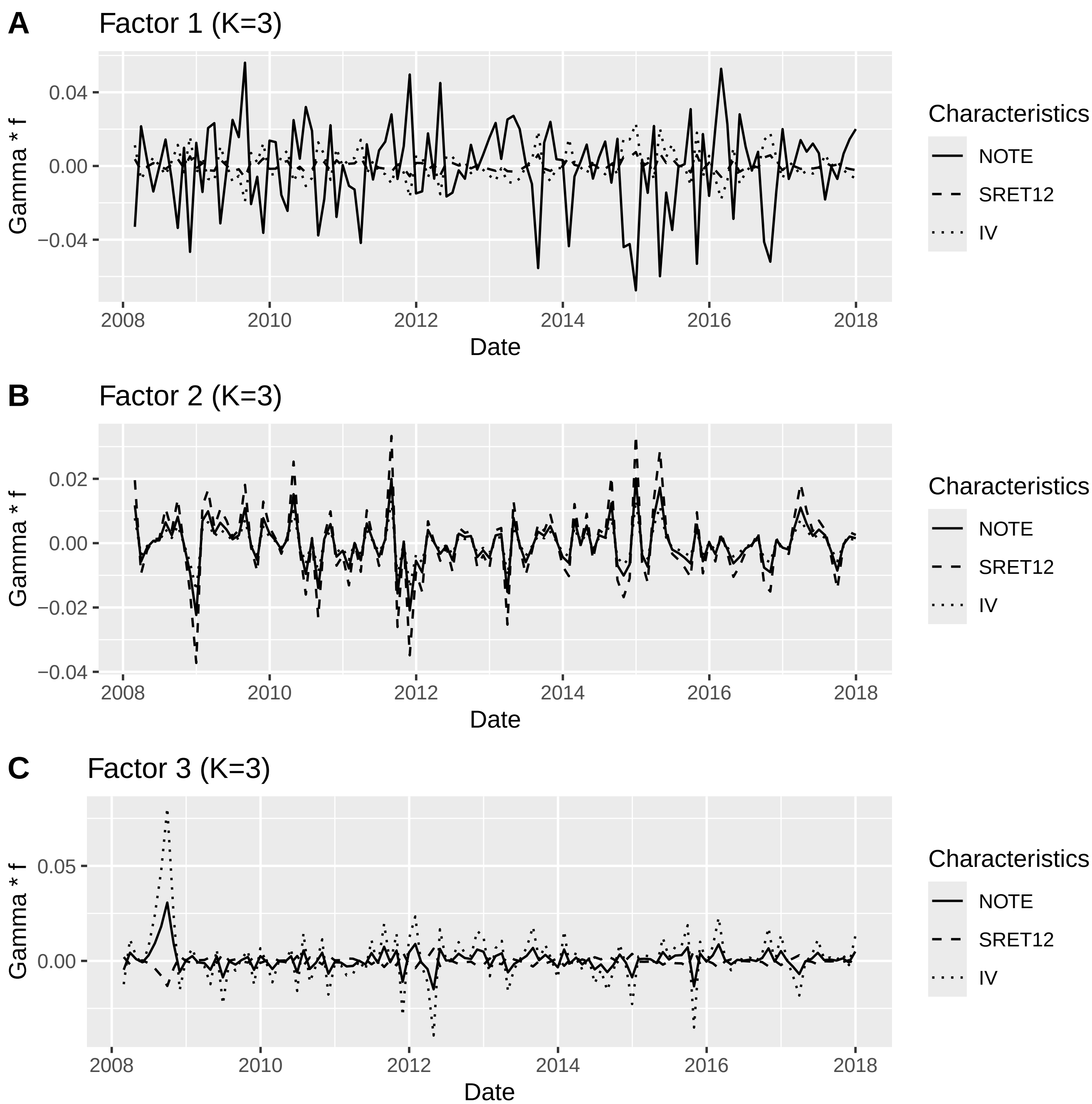

Figure 3. Time-varying beta by characteristic: individual latent factors

This chart presents the time-series coefficients, i.e. the product of f and Γ_{β_j} for $j = 1, 2, 3$ in Equation (3), for each country characteristic based on the IPCA model with three latent factors ($K = 3$).

Table 1. Summary Statistics

This table presents the summary statistics of the US dollar denominated foreign exchange spot rate returns from the G10 member country. The last row shows the equally-weighted returns across 10 currencies. The reported mean (Column 'Mean') and standard deviation (Column 'Std') are annualized. The characteristics of higher moments are also recorded. Skewness measures the degree of asymmetry. Particularly, negative skewness indicates a distribution with an asymmetric tail extending toward more negative values. Additionally, Kurtosis characterizes the relative peakedness of the currency return distribution compared with the normal distribution, where the positive kurtosis indicates the absence of fat tails on the distribution. The monthly returns are from January of 2008 to December 2020. In total, there are 165 observations for each currency. The data is collected from Datastream.

Currency	Code	Mean	Std	Skewness	Kurtosis
Australian Dollar	AUD	0.029	0.094	0.392	1.507
Canadian Dollar	CAD	−0.019	0.104	−0.099	3.073
Swiss Franc	CHF	−0.006	0.094	0.370	1.223
Danish Krone	DKK	0.019	0.095	0.505	2.995
Euro	EUR	0.010	0.133	0.626	2.168
British Pound	GBP	0.005	0.138	0.333	1.452
Japanese Yen	JPY	0.018	0.115	0.127	0.462
Norwegian Krone	NOK	0.035	0.122	0.335	0.711
New Zealand Dollar	NZD	0.013	0.098	0.311	1.667
Swedish Krona	SEK	0.014	0.099	0.346	1.621
Equally-Weighted Mean		0.012	0.109	0.325	1.688

Table 2. Correlation Structures

This table presents the correlation structures of the explanatory variables used in this paper. The correlation among the country's characteristics is reported in Panel A while the cross-currency correlation is shown in Panel B. All the correlation matrices are calculated at the currency or characteristics levels. Each correlation matrix is the average of 10 currencies or characteristics. The full variable names of the country's characteristics and currencies can be found in Tables 6 and 1, respectively. The monthly returns are from January of 2008 to December 2020. In total, there are 1650 currency-month observations. The data is sourced from multiple databases, including Bloomberg, Datastream, International Financial Statistics database of International Monetary Fund, OECD and MSCI stock returns.

Panel A: Correlation matrix of country characteristics

	BILL	BOND	DP	INF	IS	IV	NOTE	PE	SRET12	UN
BILL	1	0.805	−0.099	0.076	−0.007	0.140	0.843	0.110	0.067	0.077
BOND		1	−0.072	0.129	−0.016	0.222	0.968	0.051	0.133	−0.047
DP			1	−0.039	0.034	0.003	−0.096	−0.217	−0.396	−0.029
INF				1	−0.024	0.232	0.066	0.082	−0.145	−0.096
IS					1	−0.037	−0.009	0.010	0.029	−0.002
IV						1	0.190	0.004	−0.113	−0.155
NOTE							1	0.074	0.162	−0.016
PE								1	0.136	0.030
SRET12									1	−0.009
UN										1

Panel B: Correlation matrix of G10 currencies

	GBP	CHF	JPY	CAD	AUD	NZD	SEK	NOK	DKK	EUR
GBP	1	0.449	0.367	0.495	0.644	0.498	0.520	0.557	0.535	0.593
CHF		1	0.412	0.417	0.439	0.377	0.595	0.539	0.508	0.499
JPY			1	0.418	0.283	0.165	0.277	0.366	0.256	0.313
CAD				1	0.522	0.283	0.490	0.498	0.456	0.378
AUD					1	0.593	0.586	0.514	0.547	0.584
NZD						1	0.473	0.413	0.491	0.530
SEK							1	0.557	0.623	0.530
NOK								1	0.499	0.549
DKK									1	0.678
EUR										1

Table 3. Beta-sorting portfolios

This table demonstrates the time-varying nature of beta with the beta-sorting currency portfolios in two subsamples of equal lengths. Period 1 and 2 use the data from the first and last five years of the in-sample period, i.e. 2008:01–2017:12. Column 'Char.' represents the country's characteristics and column 'Port.' numerates the portfolios from 1 (Low-beta) to 3 (High-beta). In Period 1, an linear regression model without intercept is estimated at currency level and the currencies are ranked according to the beta of each country's characteristics. Specifically, Portfolios 1 and 3 contain three currencies and the middle portfolio contains four. * denotes the absence of beta-reversal in a specific portfolio. The monthly currency return data are collected from Datastream while the input variables are sourced from multiple databases, including Datastream, Bloomberg, IMF, OECD, and MSCI.

		Period 1	Period 2			Period 1	Period 2
Char.	Port.	2008–2012	2013–2017	Char.	Port.	2008–2012	2013–2017
Bill yield	1	−1.631	1.694	Idiosyncratic volatility	1	−2.111	1.504
	2	0.783	1.863		2	4.444	3.107
	3*	2.497	2.593		3	16.214	2.545
Bond yield	1	−6.199	0.706	Price-earning ratio	1	−0.060	0.018
	2	−1.117	2.334		2	−0.002	−0.001
	3	4.925	4.216		3	0.150	0.024
Dividend-price ratio	1	−2.599	−2.004	Stock momentum	1	−0.183	−0.109
	2	−0.004	0.002		2	−0.107	−0.037
	3*	0.022	0.032		3	−0.013	−0.022
Inflation	1	−1.843	−1.631	Unemployment gap	1	−0.027	−0.012
	2	−1.055	−0.419		2	−0.007	0.010
	3	−0.274	−0.714		3	0.014	−0.014
Idiosyncratic skewness	1	−0.016	−0.005	Note yield	1	−7.947	−4.816
	2	−0.005	−0.009		2	−1.337	−3.104
	3	0.007	−0.002		3	1.531	−1.535

Table 4. Model's performance

This table shows both the model's in-sample and out-of-sample performance across two models. First, PCA represents the Principal Component Analysis model, and second, IPCA represents the Instrumented Principal Component Analysis. We record Total R^2 and Predictive R^2 in Panel A and B, using the equations (10) and (11), respectively. The R^2 statistics of each model are presented in percentage and calculated up to five latent factors. In the second column, we also show the p-value corresponding to the constrained model where the null hypothesis is $\Gamma_\alpha = \mathbf{0}$. The monthly currency return data are collected from Datastream while the input variables are sourced from multiple databases, including Datastream, Bloomberg, IMF, OECD, and MSCI. We use the first ten years, i.e. from January of 2008 to December 2017, as our in-sample period and the rest of three years as our out of sample. In total, there are 120 and 36 observations for each currency during the in-sample and out-of-sample periods.

	In-sample 2008:01–2017:12			Out of Sample 2018:01–2020:12	
K	$H_0 : \Gamma_\alpha = 0$ p-value	PCA	IPCA	PCA	IPCA
Panel A: Total R^2					
1	0.42	68.40	51.11	66.58	43.87
2	0.83	77.36	66.07	73.59	56.22
3	0.98	83.73	73.82	80.84	62.43
4	0.15	88.72	79.89	85.44	74.62
5	0.23	92.22	83.81	89.41	75.64
Panel B: Predictive R^2					
1		−4.07	−0.14	−0.37	0.03
2		−4.92	1.93	−0.31	−2.31
3		−5.31	−1.62	−0.36	0.28
4		−5.55	−0.95	−0.36	−4.27
5		−5.70	−3.57	−0.42	−16.20

Table 5. Trading strategies based on PCA vs. IPCA models

This table compares the trading strategy performance between the PCA and IPCA models for each G10 currency. We compute the cumulative returns depending on the model's predictions. Specifically, the trading strategy buys the foreign currency (sells the US dollar) when the predicted returns are positive, and sells the foreign currency (buys the US dollar) when the predicted returns are negative. Both full sample and out-of-sample returns are annualized and expressed in percentage point. The last row shows the equally-weighted returns across 10 currencies. The monthly returns are from February of 2008 to December 2020 in the full sample, while the out-of-sample exercise began from February of 2018. The foreign exchange rate data is collected from Datastream.

		Full sample 2008:02–2018:01		Out-of-sample 2018:02–2020:12	
Currency	Code	PCA	IPCA	PCA	IPCA
Australian Dollar	AUD	−4.216	−1.595	1.532	7.912
Canadian Dollar	CAD	−0.662	−0.139	1.195	2.396
Swiss Franc	CHF	−0.633	−0.863	−1.718	3.565
Danish Krone	DKK	−0.509	0.789	0.562	4.155
Euro	EUR	−0.516	−1.075	0.567	2.363
British Pound	GBP	1.559	1.571	1.308	1.566
Japanese Yen	JPY	0.020	3.955	1.895	1.435
Norwegian Krone	NOK	2.915	−0.220	3.609	5.435
New Zealand Dollar	NZD	−0.248	0.843	0.852	−1.019
Swedish Krona	SEK	−0.630	0.175	1.466	−0.169
Equally-Weighted Mean	EWR	−0.292	0.344	1.127	2.764

Table 6. Characteristic contribution

This table reports the contribution of each country characteristic to overall model fit in the restricted IPCA specification when $K = 1$ and 3 in Panel A and B, respectively. The recorded R^2 reductions are presented in percentage when setting all Γ_β elements pertaining to that characteristic to zero. 'Std' standardize the R^2 reduction values to 100% while the original R^2 reduction is reported in the column 'Non-Std'. Columns 'Γ_β' report the estimated Γ coefficient(s) of latent factor(s). The shaded rows indicate the top three country characteristics according to the standardized R^2 reduction. We also present the coefficient and p-value of the test $H_0 : \Gamma_\beta = 0$.

Panel A: R^2 reduction (K = 1)				Gamma Coefficient			$H_0 : \Gamma_\beta = 0$
		Non-Std	Std	Γ_β			p-value
Inflation	INF	1.58	3.08	0.16			0.003
Unemployment gap	UN	2.91	5.69	−0.21			0.001
Bill yield	BILL	1.55	3.04	0.16			0.020
Note yield	NOTE	22.30	43.62	0.67			0.004
Bond yield	BOND	1.54	3.01	0.19			0.329
Dividend-price ratio	DP	0.70	1.38	−0.10			0.008
Price-earning ratio	PE	0.02	0.04	−0.02			0.620
Stock momentum	SRET12	5.60	10.96	0.33			0.001
Idiosyncratic volatility	IV	14.61	28.58	0.54			0
Idiosyncratic skewness	IS	0.31	0.61	−0.08			0.034
Panel B: R^2 reduction (K = 3)				**Gamma Coefficients (Γ_β)**			**$H_0 : \Gamma_\beta = 0$**
		Non-Std	Std	Γ_{β_1}	Γ_{β_2}	Γ_{β_3}	p-value
Inflation	INF	1.97	2.67	−0.02	0.25	0.09	0.006
Unemployment gap	UN	0.97	1.31	−0.01	−0.11	−0.16	0.007
Bill yield	BILL	7.22	9.78	0.32	−0.42	−0.22	0.009
Note yield	NOTE	30.00	40.64	−0.86	−0.34	0.27	0.021
Bond yield	BOND	6.50	8.81	0.09	0.34	0.41	0.051
Dividend-price ratio	DP	2.66	3.61	0.00	0.30	−0.03	0.005
Price-earning ratio	PE	4.37	5.91	−0.19	0.19	−0.39	0.303
Stock momentum	SRET12	8.89	12.04	0.10	−0.57	−0.12	0.001
Idiosyncratic volatility	IV	9.29	12.58	0.29	−0.22	0.71	0.066
Idiosyncratic skewness	IS	1.95	2.64	−0.17	0.16	0.03	0.009

Table 7. Time-invariant β in fixed effects regression

This table reports the results of the fixed effects regression models. Panel A shows the model's performance measures of in-sample (Panel A.1) and out-of-sample (Panel A.2) periods. Next, Panel B presents the coefficient results of the fixed effects regressions. Note that the Total and Predictive R^2 follow the definition in Equation (10) and (11), respectively. The shaded rows highlight the country characters with significance according to the R^2-reduction results in the feature importance exercise of Table 6. Standard errors (in parentheses) are clustered by country and time (calendar month) dimension. The superscripts *, **, and *** indicate statistical significance at 10%, 5%, and 1% respectively.

	(1)	(2)	(3)
Panel A.1: In-sample $\mathbf{R^2}$			
Total R^2	11.093	7.453	7.170
Predictive R^2	−0.597	−4.732	−1.907
Panel A.2: Out-of-sample $\mathbf{R^2}$			
Total R^2	−0.686	−0.286	4.329
Predictive R^2	−1.346	−7.974	−3.388
Panel B: In-sample Regressions			
Bill yield	0.828***	0.984***	0.703***
	(0.196)	(0.184)	(0.205)
Bond yield	0.352	−1.065*	−0.842
	(0.483)	(0.574)	(0.464)
Dividend-price ratio	0.000	0.000	−0.001
	(0.001)	(0.003)	(0.001)
Inflation	−0.458***	−0.517**	0.010
	(0.101)	(0.181)	(0.087)
Idiosyncratic skewness	−0.006***	−0.006*	−0.006**
	(0.001)	(0.003)	(0.002)
Idiosyncratic volatility	2.887***	4.206**	1.521
	(0.999)	(1.648)	(1.230)
Note yield	−1.008**	−0.249	−0.205
	(0.455)	(0.387)	(0.413)
Price-earning ratio	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
Stock momentum	−0.048***	−0.049***	−0.034***
	(0.008)	(0.008)	(0.008)
Unemployment gap	0.000	0.000	0.003
	(0.004)	(0.005)	(0.003)
α	−0.016***		
	(0.004)		
Calendar Month Fixed Effects			✓
Country Fixed Effects		✓	✓

Table 8. Robustness test: currency excess returns

This table shows both the model's in-sample and out-of-sample performance across two models using currency excess returns as dependent variable. First, PCA represents the Principal Component Analysis model, and second, IPCA represents the Instrumented Principal Component Analysis. We record Total R^2 and Predictive R^2 in Panel A and B, using the equations (10) and (11), respectively. The R^2 statistics of each model are presented in percentage and calculated up to five latent factors. In the second column, we also show the p-value corresponding to the constrained model where the null hypothesis is $\Gamma_\alpha = \mathbf{0}$. The monthly currency return data are collected from Datastream while the input variables are sourced from multiple databases, including Datastream, Bloomberg, IMF, OECD, and MSCI. We use the first ten years, i.e. from January of 2008 to December 2017, as our in-sample period and the rest of three years as our out of sample. In total, there are 120 and 36 observations for each currency during the in-sample and out-of-sample periods.

	In-sample 2008:01–2017:12			Out of Sample 2018:01–2020:12	
K	$H_0 : \Gamma_\alpha = 0$ p-value	PCA	IPCA	PCA	IPCA
		Panel A: Total R^2			
1	0.43	68.55	51.13	66.46	43.69
2	0.88	77.34	66.17	73.29	56.33
3	0.80	83.81	73.98	80.61	59.05
4	0.06	88.78	80.02	85.26	73.77
5	0.62	92.25	84.33	89.05	77.02
		Panel B: Predictive R^2			
1		−4.21	−0.23	0.24	0.68
2		−5.05	2.18	0.20	−2.91
3		−5.32	−2.05	0.27	0.38
4		−5.59	−2.13	0.22	−9.54
5		−5.73	−1.03	0.14	−13.66