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Q.1 if $R \in SO(3)$ & To prove $R(v \times w) = R(v) \times R(w)$

$\therefore$ lets consider dot product $\langle R(v\times w), Rv\rangle = [R(v\times w)]^T Rv$

$$= (v\times w)^T R^T R v$$

$(\because R^T R = I)$

$$= (v \times w)^T v$$

$\therefore (v\times w)$ is $\perp$ar to v thus $\langle R(v\times w), Rv\rangle = 0$

means $R(v\times w) \perp Rv$ —(1)

Similarly, $\langle R(v\times w), Rw\rangle = [R(v\times w)]^T Rw = (v\times w)^T R^T R w$

$$= 0$$

$\Rightarrow R(v\times w) \perp Rw$ —(2)

Hence $R(v\times w) = a\cdot[Rv \times Rw]$ from (1) & (2)

where a is some const.

$\therefore$ taking L2 norm on LHS & RHS

$\therefore$ LHS $= \|R(v\times w)\|_2$

$= \|v\times w\|_2$ (since SO(3) does not change magnitude)

$= \|v\|\,\|w\| \sin\theta$ (let θ be angle b/w v & w)

$\therefore \cos\theta = v^T w / \|v\|\,\|w\|$

$\therefore$ RHS $= \|a[Rv \times Rw]\|_2$

$= |a|\,\|Rv \times Rw\|_2$

$= |a|\,\|Rv\|\cdot\|Rw\|\cdot\sin(\theta_R)$ (Let θ be angle b/w Rv & Rw)

$= |a|\,\|v\|\cdot\|w\|\cdot\sin\theta_R$

Now $\cos\theta_R = (Rv)^T(Rw) = v^T R^T R w = v^T w$

$$\frac{\quad}{\|Rv\| \; \|Rw\|} = \frac{v \quad R \quad R\,w}{\|v\| \; \|w\|} = \frac{\quad}{\|v\| \; \|w\|}$$

$$\therefore \cos\theta_R = \cos\theta \Rightarrow \theta_R = \theta$$

$$\therefore \|v\| \, \|w\| \sin\theta = |a| \, \|v\| \, \|w\| \sin\theta$$

$$|a| = 1$$

$$\therefore \quad R(v \times w) = |a| \; R(v) \times R(w) \quad \text{where } |a| = 1 \qquad \text{—— (3)}$$

$\therefore$ Now R is made up of finite numbers which are evaluated values of polynomials. Hence R has to be continuous & cannot yield 2 different answers

$\therefore$ If we find a for $R = I$, then its valid $\forall \, R \in SO(3)$

$$\therefore \quad I(v \times w) = Iv \times Iw = v \times w \quad \text{Hence } a = 1$$

$$\therefore \quad R(v \times w) = Rv \times Rw$$

$\therefore$ Now to show $Rv \times w = (R\hat{v}R^T)\,w$

$\therefore$ Lets take any vector $z \in \mathbb{R}^3$ such that $Rz = w$

$$\begin{aligned} \therefore Rv \times Rz &= R(v \times z) \\ &= R(\hat{v} z) \\ &= R(\hat{v} I z) \end{aligned}$$

Now we know if $R \in SO(3)$ then $R^T R = I$

$$= R(\hat{v} R^T R z)$$

$$\therefore \quad Rv \times w = R\hat{v}R^T w$$

Q.2

$e^{\hat{\omega}t} \in SO(3)$ where $\hat{\omega}$ is skew symmetric

For this to be $SO(3)$ (1) $e^{\hat{\omega}t}$ must satisfy $(e^{\hat{\omega}t})^T (e^{\hat{\omega}t}) = I$

(2) $\det(e^{\hat{\omega}t}) = 1$

for (1)

$$\therefore \left(e^{\hat{\omega} t}\right)^T = \left[I + \hat{\omega} t + \frac{(\hat{\omega} t)^2}{2!} + \cdots \right]^T$$

$$= \left[I^T + \hat{\omega}^T t + \frac{(\hat{\omega}^2)^T t^2}{2!} + \cdots \right]$$

$$= \left[I - \hat{\omega} t + \frac{(\hat{\omega} t)^2}{2!} - \cdots \right] \qquad (\because \hat{\omega}^T = -\hat{\omega})$$

$$\left(e^{\hat{\omega} t}\right)^T = e^{-\hat{\omega} t}$$

Thus we know if some matrix $R = e^{\hat{\omega} t}$ then $R^T = R^{-1}$

$\therefore$ $R^T R = I$ Hence proved

& thus $e^{\hat{\omega} t}$ is also orthogonal

for ②

$\det(e^{\hat{\omega} t})$ is either ± 1 $\because$ its orthogonal

$\therefore$ since the elements of $e^{\hat{\omega} t} = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}$

are obtained through a convergent series of polynomials.

Hence the elements must be continuous

$\therefore$ If the elements are continuous then the determinent must also be continuous as its combinations of elements.

Thus if $\det(e^{0t}) = \det\left(I + 0t + \frac{0^2 t^2}{2!} + \cdots\right)$

$$= \det(I) = 1$$

it must be true $\forall$ $\hat{\omega} t$

$$\therefore \det(e^{\hat{\omega}\theta}) = 1$$

Q.3

$$F : so(3) \longrightarrow SO(3)$$

we want to show that for $\forall\, R \in SO(3)$

there exists a $F(\hat{\omega}\theta) = R = e^{\hat{\omega}\theta}$ where $\hat{\omega} \in so(3)$

$\theta \in \mathbb{R}$

$\|\omega\| = 1$

Let

$$\therefore R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

& Using Rodrigues formula

$$e^{\hat{\omega}\theta} = I + \hat{\omega}\sin\theta + \hat{\omega}^2(1-\cos\theta)$$

.....(proved in Q.4)

$\therefore$ Let $S_\theta = \sin\theta$, $C_\theta = \cos\theta$, $u_\theta = 1-\cos\theta$ & $\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$ where $\|\omega\| = 1$

$$\therefore e^{\hat{\omega}\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 S_\theta & \omega_2 S_\theta \\ \omega_3 S_\theta & 0 & -\omega_1 S_\theta \\ -\omega_2 S_\theta & \omega_1 S_\theta & 0 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}}_{\begin{bmatrix} -(\omega_2^2+\omega_3^2) & \omega_1\omega_2 & \omega_1\omega_3 \\ \omega_1\omega_2 & -(\omega_1^2+\omega_3^2) & \omega_2\omega_3 \\ \omega_1\omega_3 & \omega_2\omega_3 & -(\omega_1^2+\omega_2^2) \end{bmatrix}} u_\theta$$

$$= \begin{bmatrix} 1-(\omega_2^2+\omega_3^2)u_\theta & -\omega_3 S_\theta + \omega_1\omega_2 u_\theta & \omega_1\omega_3 u_\theta + \omega_2 S_\theta \\ \omega_3 S_\theta + \omega_1\omega_2 u_\theta & 1-(\omega_1^2+\omega_3^2)u_\theta & \omega_2\omega_3 u_\theta - \omega_1 S_\theta \\ -\omega_2 S_\theta + \omega_1\omega_3 u_\theta & \omega_2\omega_3 u_\theta + \omega_1 S_\theta & 1-(\omega_1^2+\omega_3^2)u_\theta \end{bmatrix}$$

we can eliminate some things if we take trace

$$\therefore \text{Trace of } R = r_{11} + r_{22} + r_{33} = 3 - 2\left[\omega_1^2 + \omega_2^2 + \omega_3^2\right]u_\theta$$

$$= 3 - 2u_\theta \quad \cdots (\because \|\omega\| = 1)$$

$$= 3 - 2(1 - C_\theta)$$

$$= 1 + 2C_\theta$$

$$\therefore R^T R = R R^T = I$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}\begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}\begin{bmatrix} \quad \end{bmatrix} = I$$

$$\therefore \begin{bmatrix} r_{11}^2 + r_{12}^2 + r_{13}^2 & \cdots \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} r_{11}^2+r_{12}^2+r_{13}^2 & - & - \\ - & r_{21}^2+r_{22}^2+r_{23}^2 & - \\ - & - & r_{31}^2+r_{32}^2+r_{33}^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore r_{11}^2 + r_{12}^2 + r_{13}^2 = 1$$

$$\therefore r_{21}^2 + r_{22}^2 + r_{23}^2 = 1$$

$$\therefore r_{31}^2 + r_{32}^2 + r_{33}^2 = 1$$

& $\det R = 1 \Rightarrow$ $\quad R^T = R^{-1} \Rightarrow$

Q.4 To prove : $e^{\hat{w}\theta} = I + \hat{w}\sin\theta + \hat{w}^2(1-\cos\theta)$

$$\therefore \sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots\cdots$$

$$\therefore \cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots\cdots$$

$$\therefore 1-\cos\theta = \frac{\theta^2}{2!} - \frac{\theta^4}{4!} + \cdots\cdots$$

$$\therefore \hat{w}\sin\theta = \hat{w}\theta + \frac{\hat{w}\theta^3}{3!} + \frac{\hat{w}\theta^5}{5!} + \cdots\cdots \quad \text{— (1)}$$

$$\therefore \hat{w}^2(1-\cos\theta) = \frac{\hat{w}^2\theta^2}{2!} - \frac{\hat{w}^2\theta^4}{4!} + \cdots\cdots \quad \text{— (2)}$$

$$\therefore \hat{w} = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix} \qquad \therefore \hat{w}^2 = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -(w_3^2+w_2^2) & w_1w_2 & w_1w_3 \\ w_1w_2 & -(w_3^2+w_1^2) & w_2w_3 \\ w_1w_3 & w_2w_3 & -(w_1^2+w_2^2) \end{bmatrix}$$

$\therefore \hat{w}^3 = \hat{w}^2\hat{w}$ ...

$$\therefore \hat{w}^3 = \hat{w}\,\hat{w}^2 = \begin{bmatrix} 0 & w_3(w_3^2+w_2^2+w_1^2) & -w_2(w_3^2+w_2^2+w_1^2) \\ -(\text{—}\|\text{—}) & 0 & w_1(w_3^2+w_2^2+w_1^2) \\ (\text{—}\|\text{—}) & -(\text{—}\|\text{—}) & 0 \end{bmatrix}$$

$\therefore \hat{w}^3 = -\|w\|^2\,\hat{w}$ $\qquad\qquad$ $\therefore \hat{w}^5 = \|w\|^2\,\hat{w}^2\cdot\hat{w} = \|w\|^4\,\hat{w}$

$\therefore \hat{w}^4 = -\|w\|^2\,\hat{w}^2$ $\qquad\qquad$ $\therefore \hat{w}^6 = \|w\|^4\,\hat{w}^2$ $\quad$ ..... & so on

$\therefore$ Eq 1 & 2 can be written as,

$$\hat{w}\sin\theta = \hat{w}\theta + \frac{(\hat{w}^3)\theta^3}{\|w\|^2\,3!} + \cdots\cdots \qquad (3)$$

$$\hat{w}^2(1-\cos\theta) = \frac{\hat{w}^2\theta^2}{2!} + \frac{\hat{w}^4\theta^4}{\|w\|^2\,4!} + \cdots\cdots \qquad (4)$$

$\therefore$ Adding 3 & 4 & I

$$\Rightarrow \left. I + \hat{w}\theta + \frac{(\hat{w}\theta)^2}{2!} + \frac{(\hat{w}\theta)^3}{\|w\|^2\,3!} + \frac{(\hat{w}\theta)^4}{\|w\|^2\,4!} + \cdots \right\} \Rightarrow e^{\hat{w}\theta} \text{ if } \|w\|=1$$

$\therefore$ if $\|w\|=1$ then,

$$e^{\hat{w}\theta} = I + \hat{w}\sin\theta + \hat{w}^2(1-\cos\theta)$$

else if $\|w\| \neq 1$ then,

$$e^{\hat{w}\theta} = I + \frac{\hat{w}}{\|w\|}\cdot\sin(\|w\|\theta) + \frac{\hat{w}^2}{\|w\|^2}\big(1-\cos(\|w\|\theta)\big)$$

Q.5

$$\hat{w} = \begin{bmatrix} 0 & -w \\ w & 0 \end{bmatrix}$$

$\therefore$ it can be shown that $\hat{w} \in so(2)$

$\because\ \hat{w} + \hat{w}^T = 0$

$$\therefore \hat{w}^2 = \begin{bmatrix} 0 & -w \\ w & 0 \end{bmatrix}\begin{bmatrix} 0 & -w \\ w & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -w^2 & 0 \\ 0 & -w^2 \end{bmatrix} = -w^2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -w^2 I$$

$\therefore \hat{w}^3 = \hat{w}^2\,\hat{w} = -w^2\,\hat{w}$

& $\hat{w}^6 = \hat{w}^5\,\hat{w} = w^4\,\hat{w}^2 = -w^6 I$

$\therefore \hat{\omega}^4 = \hat{\omega}^3 \hat{\omega} = -\omega^2 \hat{\omega}^2 = \omega^4 I$

$\therefore \hat{\omega}^5 = \omega^4 \hat{\omega}$

$$\therefore e^{\hat{\omega}\theta} = I + \hat{\omega}\theta + \frac{\hat{\omega}^2\theta^2}{2!} + \frac{\hat{\omega}^3\theta^3}{3!} + \frac{\hat{\omega}^4\theta^4}{4!} + \frac{\hat{\omega}^5\theta^5}{5!} + \frac{\hat{\omega}^6\theta^6}{6!} + \cdots$$

$$= I + \hat{\omega}\theta + \frac{(-\omega^2 I)\theta^2}{2!} + \frac{(-\omega^2\hat{\omega})\theta^3}{3!} + \cdots$$

Transforms into Rodrigues Formula proved in Q. 4

$\therefore \|\omega\| = \omega$

$$\therefore e^{\hat{\omega}\theta} = I + \frac{\hat{\omega}}{\omega}\sin(\omega\theta) + \frac{\hat{\omega}^2}{\omega^2}(1-\cos(\omega\theta))$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\sin(\omega\theta) + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}(1-\cos(\omega\theta))$$

$$= \begin{bmatrix} 1-(1-\cos\omega\theta) & -\sin\omega\theta \\ \sin\omega\theta & 1-(1-\cos\omega\theta) \end{bmatrix} = \begin{bmatrix} \cos\omega\theta & -\sin\omega\theta \\ \sin\omega\theta & \cos\omega\theta \end{bmatrix}$$

$\therefore$ we know for it to be surjective, $\forall R \in SO(2)$ there should be some $\hat{\omega}$ & θ in $e^{\hat{\omega}\theta}$

$\therefore \quad R = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}$ $\quad \therefore \det(R) = 1 = r_{11}\cdot r_{22} - r_{12}\cdot r_{21}$

$$\& \underbrace{\begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} r_{11} & r_{21} \\ r_{12} & r_{22} \end{bmatrix}}_{R^T} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\& \quad r_{11}^2 + r_{12}^2 = 1$

$\& \quad r_{21}\cdot r_{11} + r_{12}\cdot r_{22} = 0 \Rightarrow r_{11} r_{21} = -r_{12}\cdot r_{22}$

$\& \quad r_{12}\cdot r_{11} + r_{21}\cdot r_{22} = 0 \Rightarrow r_{11}\cdot r_{12} = -r_{21}\cdot r_{22}$

$\& \quad r_{22}^2 + r_{21}^2 = 1$

$\Rightarrow \underline{r_{12}^2 = r_{12}^2}$

Let

$\therefore r_{21}^2 = r_{12}^2 = b^2$

then $r_{11}^2 = 1-b^2$

$\& \; r_{22}^2 = 1-b^2$ $\Rightarrow \because$ elements of R are real

then $-1 \le r_{11} \le 1$

$\& \; -1 \le r_{22} \le 1$

thus r_{11} is a valid number for $\cos\omega\theta$

$\Rightarrow r_{21}$ & r_{11} are also $[-1, 1]$

thus $r_{12}^2 = 1 - r_{11}^2 = \sin^2\omega\theta = \pm\sin\omega\theta$

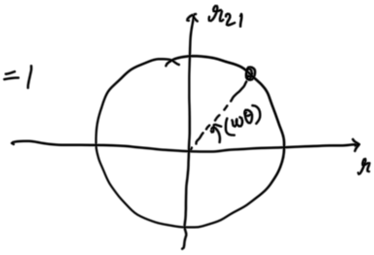
$\therefore \; r_{21} = -r_{12} = \mp\sin\omega\theta$

$\therefore$ for every $r_{11}, r_{12}, r_{21}, r_{22}$ there will be a valid ω, θ. Thus it is surjective

∴ for injective

∴ $\left.\begin{array}{l} \omega\theta = \cos^{-1}(r_{11}) \\ \& \ \omega\theta = \sin^{-1}(r_{21}) \end{array}\right\}$ only one pair of (ω, θ) should satisfy this

∴ $r_{11}^2 + r_{21}^2 = 1$



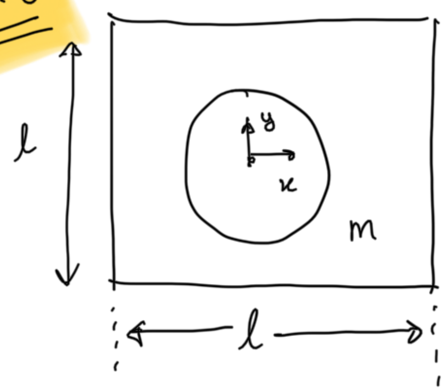
∴ There can be only one $(\omega\theta)$ that yield only one pair of r_{11}, r_{21}

But this means there can be multiple ω & θ individual variables that yied the product $\omega\theta$

eg: $\frac{\omega}{2}$ & 2θ

∴ Hence not injective !

Q.6



$l = 250$ mm $\quad m = \frac{1}{2}$ kg

$r = 50$ mm

The hole can be viewed as -ve mass & since Inertia is scalar we can directly add it for plate & hole

∴ Let m be the mass of plate with hole & assuming uniform mass density

∴ Area of plate with hole $= l^2 - \pi r^2$

∴ Area of plate without hole $= l^2$ & Area of hole $= \pi r^2$

∴ m_p = Mass of plate without hole $= m\left(\frac{l^2}{l^2 - \pi r^2}\right)$

∴ m_c = Mass of hole $= m\left(\frac{\pi r^2}{l^2 - \pi r^2}\right)$

a) $I_p = I_{square} - I_{circle}$

$= \underline{m_p (l^2 + l^2)} - \underline{m_c r^2}$ (obtained from class notes)

$12 \qquad 2$ (as derivation is trivial)

$$= \frac{ml^2}{l^2-\pi r^2} \cdot \frac{2l^2}{12} - m\left(\frac{\pi r^2}{l^2-\pi r^2}\right) \cdot \frac{r^2}{2}$$

$$= \frac{m}{(l^2-\pi r^2)}\left(\frac{l^4}{6} - \frac{\pi r^4}{2}\right) = \frac{m}{(l^2-\pi r^2)}\left(\frac{l^4 - 3\pi r^4}{6}\right)$$

$$= \frac{0.5}{(0.25^2 - \pi \cdot 0.05^2)} \times \frac{(0.25^4 - 3\pi\, 0.05^4)}{6}$$

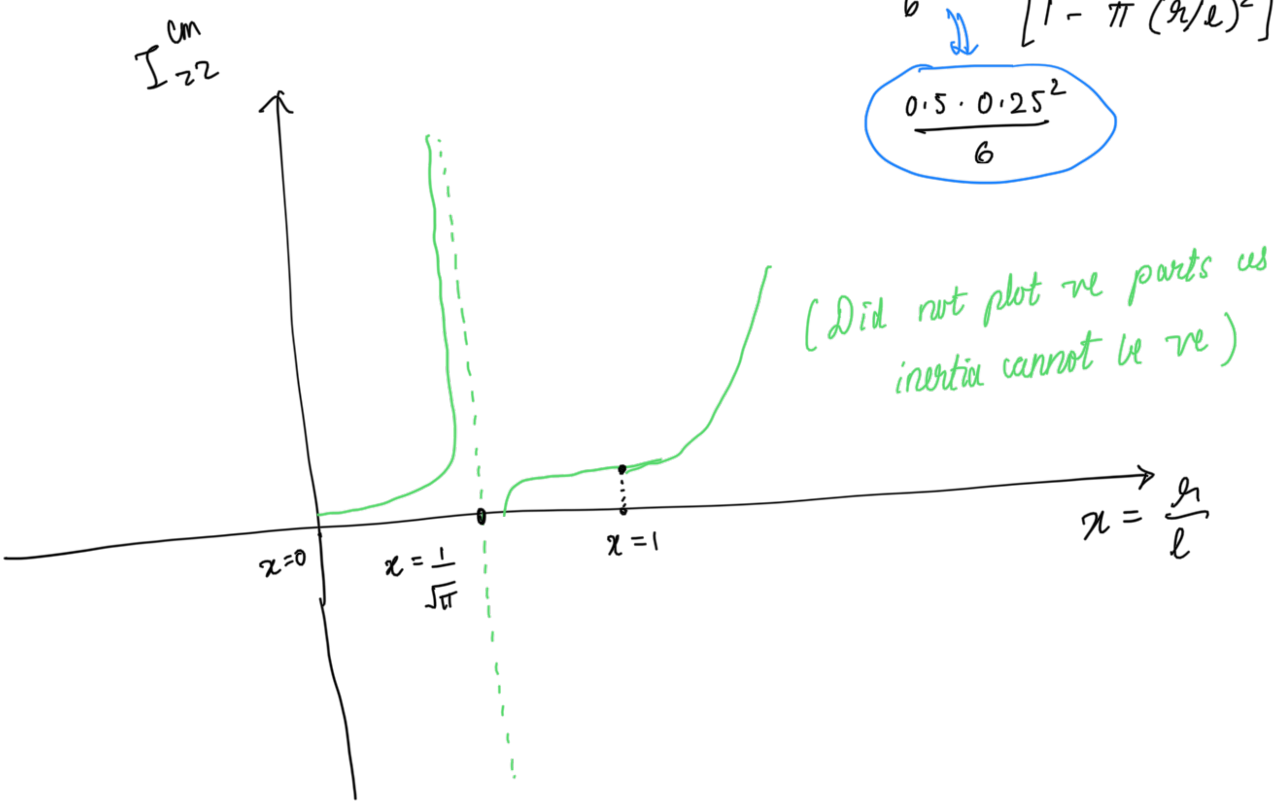
$$= \frac{0.5}{0.05464} \times 0.003847 = 0.0352 \ kg\, m^2$$

b) $$I_{zz}^{cm} = I_p = \frac{m}{(l^2-\pi r^2)}\left(\frac{l^4-3\pi r^4}{6}\right) = \frac{ml^4 \cdot \left[1 - 3\pi (r/l)^4\right]}{l^2\left[1-\pi\left(\frac{r}{l}\right)^2\right] 6}$$

$$= \frac{ml^2}{6} \frac{\left[1 - 8\pi\,(r/l)^4\right]}{\left[1-\pi\,(r/l)^2\right]}$$

$$\Downarrow$$

$$\frac{0.5 \cdot 0.25^2}{6}$$



c) put $x = 1/4 l$

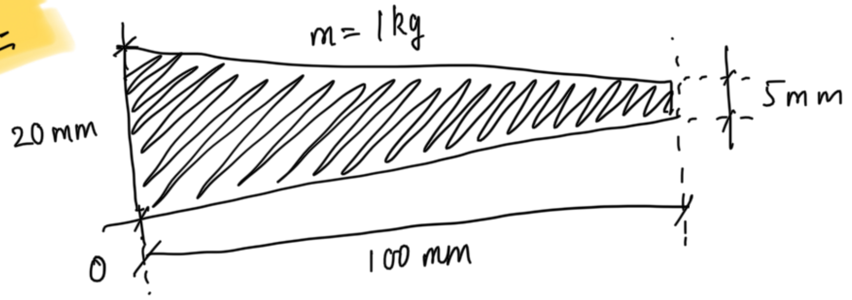
$$\therefore \lim_{x \to 0} I_{22}^{cm} = \lim_{x \to 0} \left\{ \frac{ml^2}{6} \cdot \frac{(1 - 3\pi x^4)}{(1 - \pi x^2)} \right\}$$

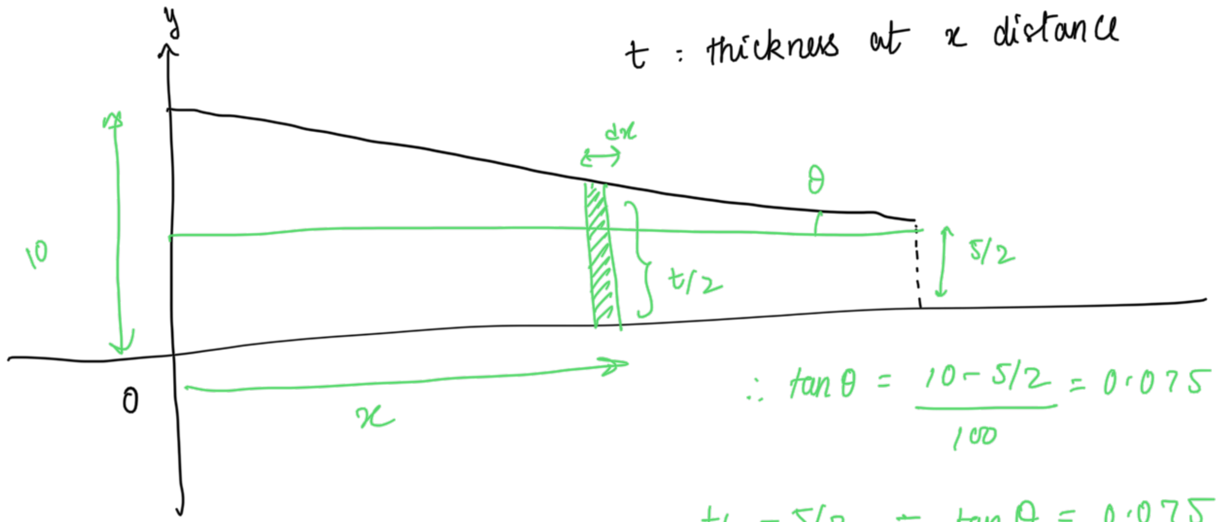
$$= \frac{ml^2}{6}$$

$$\therefore \lim_{x \to 1} I_{22}^{cm} = \lim_{x \to 1} \left\{ \frac{ml^2}{6} \frac{(1 - 3\pi x^4)}{(1 - \pi x^2)} \right\}$$

$$= \frac{ml^2}{6} \cdot \frac{(1 - 3\pi)}{(1 - \pi)} = \left(\frac{3\pi - 1}{\pi - 1} \right) \cdot \frac{ml^2}{6}$$

Q.7





$$\therefore \tan\theta = \frac{10 - 5/2}{100} = 0.075$$

$$\frac{t/2 - 5/2}{(100 - x)} = \tan\theta = 0.075$$

$$\frac{t}{2} = 10 - 0.075x$$

$$t = 20 - 0.15x$$

$\therefore$ Mass of element at distance x

$\Rightarrow dm = dA \cdot \rho$

assuming uniform mass density per unit area (ρ)

$$\therefore \rho = \frac{m}{\ldots} = \frac{1}{\ldots} = \frac{1}{1250} \text{ kg/mm}^2$$

Area of rod $\left(\frac{20+5}{2}\right) \times 100$

$$\therefore dm = dx \cdot t \cdot \rho$$

$$\therefore dm = (20 - 0.15x)\rho \, dx$$

$$\therefore (r_{com})_x = \frac{\int x \, dm}{\int dm} = \frac{\int_{x=0}^{x=100} x(20-0.15x) \cdot \rho \cdot dx}{1 \text{ kg}}$$

$$= \frac{1}{1250} \int_0^{100} 20x - 0.15x^2 \, dx$$

$$= \frac{1}{1250} \left[\frac{20x^2}{2} - \frac{0.15x^3}{3} \right]_0^{100}$$

$$= \frac{1}{1250} \left[10x^2 - 0.05x^3 \right]_0^{100} = 40 \text{ mm}$$

$$\therefore I_{zz}^{o} = \int x^2 dm = \int_{x=0}^{100} x^2 (20 - 0.15x) \rho \, dx$$

$$= \rho \int_{x=0}^{100} 20x^2 - 0.15x^3 \, dx = \rho \left[\frac{20x^3}{3} - \frac{0.15x^4}{4} \right]_0^{100}$$

$$= 2333.33 \text{ kg mm}^2$$

$\therefore$ Using $\parallel^{el}$ axis Theorem

$$I_{zz}^{o} = I_{zz}^{cm} + m r_{com}^2$$

$$I_{zz}^{cm} = I_{zz}^{o} - m r_{com}^2$$

$$= 2333.33 - 1 \times 40^2$$

$$= 733.33 \text{ kg mm}^2$$