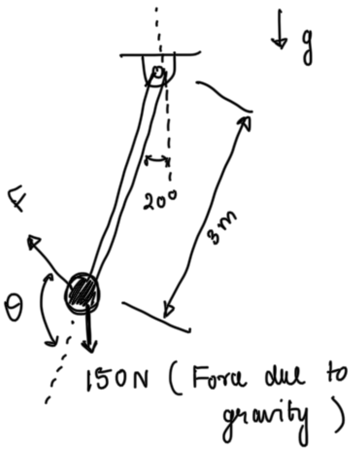
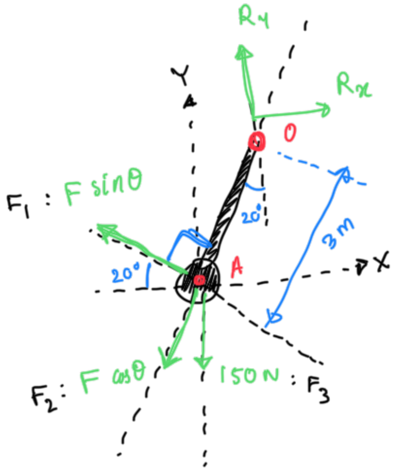
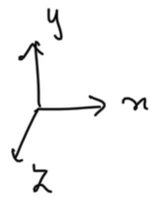
HW3 Aditya Shirwalkar 21232

Q. 5.1.18

a]







$$\therefore \sum \vec{M}_0 = 0$$

$$- F \sin\theta \times 3 + 150 \cos 70^\circ \times 3 = 0$$

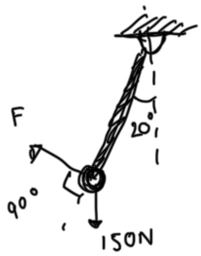
$$F = \frac{150 \cos 70^\circ}{\sin\theta} = \frac{150 \cos 70^\circ}{\sin\theta}$$

$$F(\theta) = \frac{51.3}{\sin\theta}$$

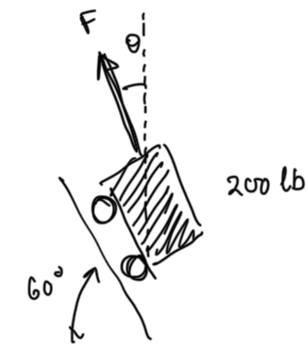
↳ $\therefore$ $F(\theta)$ is minimum when $\sin\theta$ is maximum

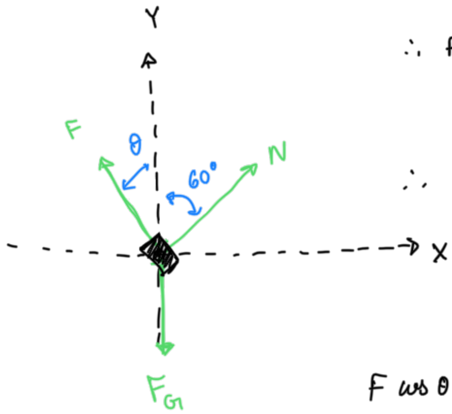
$\therefore \theta = 90^\circ$

& $F = 51.3\,N$



b]





$\sum F_Y = 0$

$\therefore F \cos\theta + N \cos 60^\circ = F_G$ — ①

$\sum F_X = 0$

$\therefore F \sin\theta = N \sin 60^\circ$ — ②

from ① & ②

$$F \cos\theta + F \sin\theta / \tan 60^\circ = F_G$$

$$F = \frac{F_G \tan 60^\circ}{\tan 60^\circ \cos\theta + \sin\theta} = \frac{F_G \sqrt{3}}{(\sqrt{3}\cos\theta + \sin\theta)}$$

$$\frac{d}{d\theta}F(\theta) = \sqrt{3}F_G \frac{d}{d\theta}\left(\frac{1}{(\sqrt{3}\cos\theta + \sin\theta)}\right)$$

$$= \sqrt{3}F_G \frac{-(-\sqrt{3}\sin\theta + \cos\theta)}{(3\cos^2\theta + \sin^2\theta + 2\sqrt{3}\cos\theta\sin\theta)}$$

$$= \sqrt{3}F_G \frac{(-\sqrt{3}\sin\theta + \cos\theta)}{(2\cos^2\theta + 2\sqrt{3}\cos\theta\sin\theta)}$$

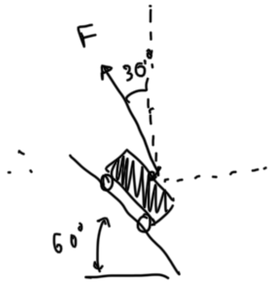
$$= \sqrt{3}F_a \frac{(\cos\theta - \sqrt{3}\sin\theta)}{2\cos\theta(\cos\theta + \sqrt{3}\sin\theta)} \Rightarrow \text{if } = 0$$

then $\cos\theta - \sqrt{3}\sin\theta = 0$

else no other possibility

$\therefore \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

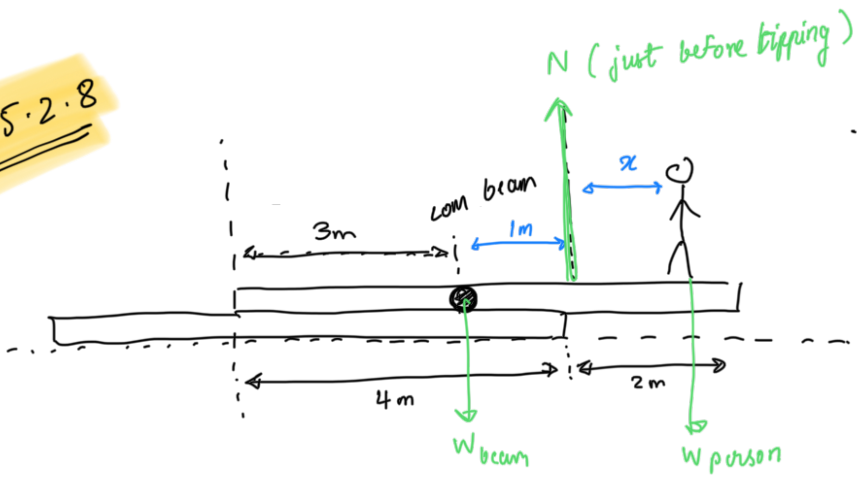
$= 30^\circ$



$\therefore$ & $F = \frac{\sqrt{3}}{2}F_G = 200 \times 0.454 \times 9.8 \times \frac{\sqrt{3}}{2}$

$F = 770.6\ N$

Q. 5.2.8



$\therefore W_{beam} \times 1 = x \times W_{person}$

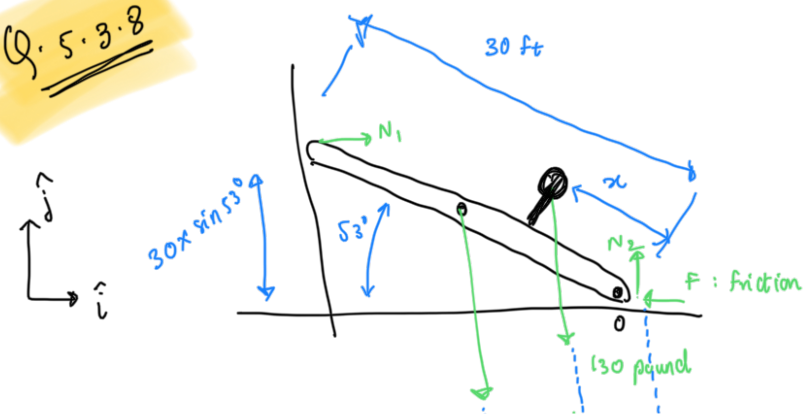
↳ just before tipping after some x^+ the beam will tip

$\therefore 1000 \times 1 = x \times 800$

$x = \frac{10}{8} = \frac{5}{4}\ m$

(from overhang pt.)

Q. 5.3.8



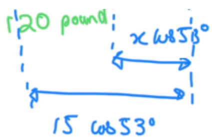
a] $x = 30/3 = 10$ ft.

what is N_2 & F?

$\therefore N_2 = (130 + 150) \times 9.8 \times 0.454$

$= 1245.77\ N$

$\therefore N_1 = F$ & $\Sigma M_o = 0$



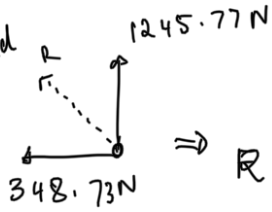
$$0.454 \times 9.8 \times \left(130 \times 10 \cos 53^\circ + 120 \times 15 \cos 53^\circ\right) = N_1 \cdot 30 \cdot \sin 53^\circ$$

$$N_1 = \left(\frac{3100 \cos 53^\circ}{30 \sin 53^\circ}\right) \times 0.454 \times 9.8$$

$$= 348.74\,N$$

$$\therefore F_1 = 348.73\,N$$

$\therefore$ Net force by ground



$$\Rightarrow \quad R = 1245.77\,N\,\hat{j} - 348.73\,\hat{i}$$

b] when $x = \frac{2}{3} \times 30 = 20$ ft

what is μ ?

$\therefore$ if bottom starts to slip, then $F = \mu N_2$ & $N_2 = 1245.77\,N$

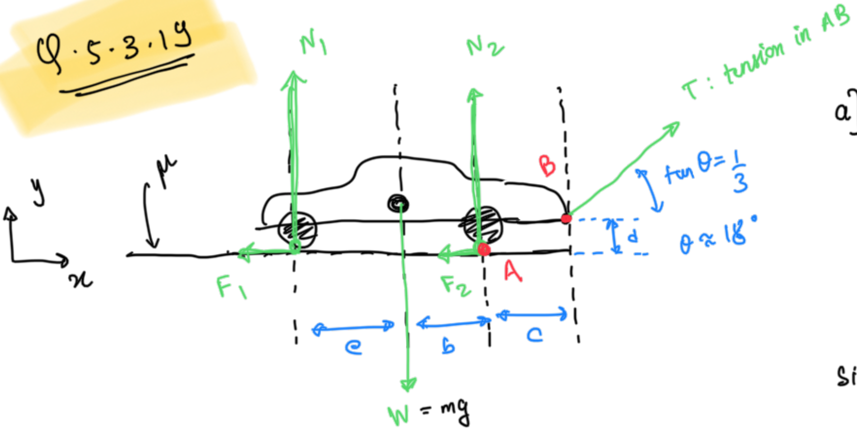
$\therefore N_1 = F$ & $\Sigma M_o = 0$

$$0.454 \times 9.8 \left(120 \times 15 \cos 53^\circ + 130 \times 20 \cos 53^\circ\right) = N_1 \cdot 30 \sin 53^\circ$$

$$\therefore N_1 = \left(\frac{4400 \cos 53^\circ}{30 \sin 53^\circ}\right) \times 0.454 \times 9.8 = 491.73\,N$$

$$\mu = F/N_2 = \frac{N_1}{N_2} = 0.39$$

Q.5.3.19



a] $\Sigma F_x = 0$

$$\therefore -F_1 - F_2 + T \cos\theta = 0 \quad \text{—(1)}$$

$\Sigma F_y = 0$

$$\therefore N_1 + N_2 + T \sin\theta - W = 0 \quad \text{—(2)}$$

since skidding, $\left.\begin{array}{l} F_2 = \mu N_2 \\ \& \; F_1 = \mu N_1 \end{array}\right\}$ —(3)

$\therefore$ put $(N_1 + N_2) = N$

$$\therefore \quad -\mu N + T \cos\theta = 0$$
$$\& \quad N + T \sin\theta - W = 0$$

$$\Rightarrow \quad T(\cos\theta + \mu \sin\theta) = \mu W$$

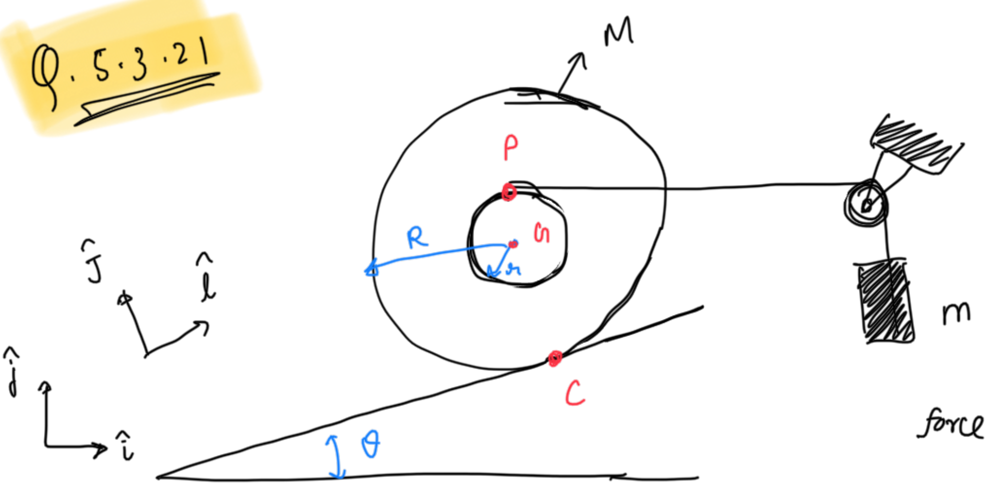
$$T = \frac{\mu W}{(\cos\theta + \mu \sin\theta)} = \frac{\mu W}{\frac{3}{\sqrt{10}} + \mu \cdot \frac{1}{\sqrt{10}}} = \frac{\sqrt{10}\,\mu W}{(3 + \mu)}$$

$$T = \frac{\sqrt{10}\,\mu m g}{(3 + \mu)}$$

b] For T to be minimum, $\frac{dT}{d\theta} = 0 \Rightarrow \mu W \cdot \underbrace{\frac{(-\sin\theta + \mu\cos\theta)}{(\cos^2\theta + \mu^2\sin^2\theta + 2\mu\sin\theta\cos\theta}}$

∴ for this to be zero, $\mu = \tan\theta$

Q.5.3.21



$r = \frac{R}{2}$

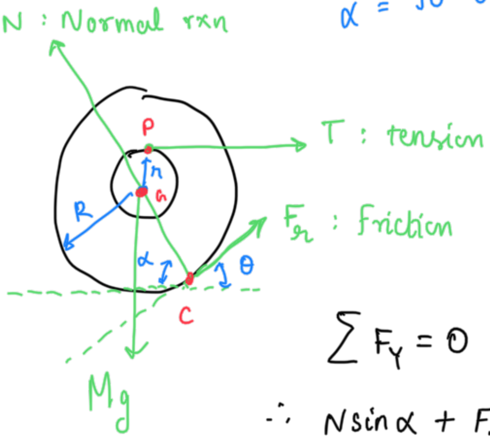
Since there is no slip b/w reel & slope, it means some friction force is acting

a]

$\sum F_y = 0$

$T = mg$ —(4)

$\alpha = 90 - \theta$



$\sum F_x = 0$

$T + F_r\cos\theta - N\cos\alpha = 0$ —(1)

$\sum F_y = 0$

∴ $N\sin\alpha + F_r\sin\theta - Mg = 0$ —(2)

$\sum M_G = 0$

$-T \times r + F_r \times R = 0$ —(3)

∴ From (3) & (4) $F_r = mg\frac{r}{R} = \frac{mg}{2}$

∴ from (1); $N = \frac{mg + \frac{mg}{2}\cos\theta}{\cos\alpha} = \frac{mg(2+\cos\theta)}{2\sin\theta}$

but from (2) ; $N = \dfrac{Mg - \frac{mg}{2}\sin\theta}{\sin\alpha} = \dfrac{mg\,(2M/m - \sin\theta)}{2\cos\theta}$

$$\therefore \quad \frac{\cancel{m}g\,(2+\cos\theta)}{\cancel{2}\sin\theta} = \frac{\cancel{m}g\,(2M/m - \sin\theta)}{\cancel{2}\cos\theta}$$

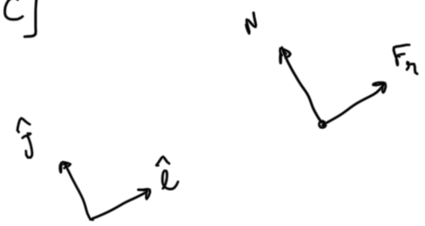
$$2\cos\theta + \cos^2\theta = \frac{2M}{m}\sin\theta - \sin^2\theta$$

$$\frac{2M}{m}\sin\theta = 2\cos\theta + 1$$

$$\underline{\frac{M}{m} = \frac{(1+2\cos\theta)}{2\sin\theta}} = \text{mass ratio}$$

b] $\therefore T = mg = \left[\dfrac{2M\sin\theta}{(1+2\cos\theta)}\right] g$

c]



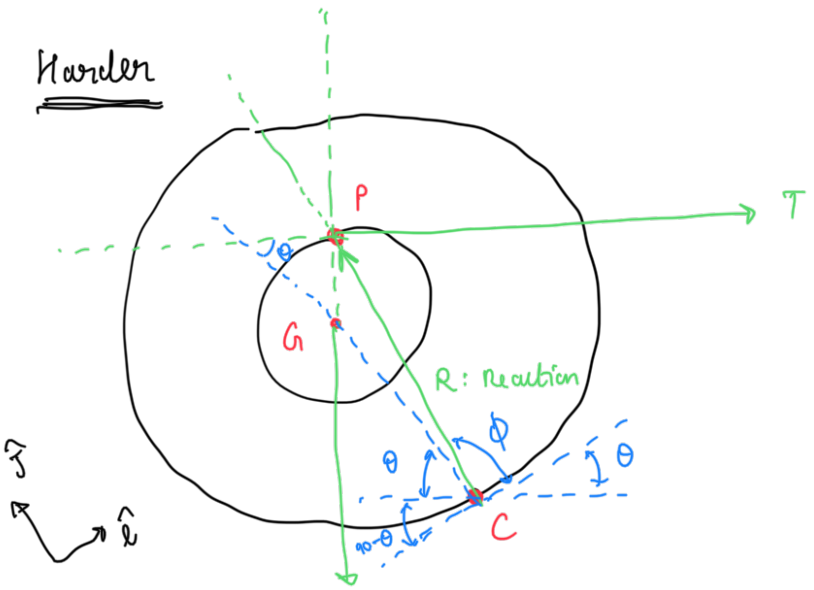
$$\therefore N = \frac{mg\,(2+\cos\theta)}{2\sin\theta} = \left[\frac{2M\sin\theta}{(1+2\cos\theta)}\right] g \,\frac{(2+\cos\theta)}{2\sin\theta}$$

$$= Mg\,\frac{(2+\cos\theta)}{(1+2\cos\theta)}$$

$$\therefore F_r = \frac{mg}{2} = \left(\frac{2M\sin\theta}{1+2\cos\theta}\right)\frac{g}{2}$$

$\therefore$ Force on the reel at pt. C $= \left(\dfrac{\sin\theta}{1+2\cos\theta}\right) Mg\,\hat{i} + \left(\dfrac{2+\cos\theta}{1+2\cos\theta}\right) Mg\,\hat{j}$

Harder



$\sum F_{\hat{j}} = 0$

$\therefore R\sin\phi - T/\cos\theta - Mg/\cos(90-\theta) = 0$

$\therefore R\sin\phi = T/\cos\theta + Mg/\sin\theta$

& $\sum F_{\hat{i}} = 0$

$\therefore R\cos\phi + T/\sin\theta - Mg/\sin(90-\theta) = 0$

$\therefore R\cos\phi - Mg/\cos\theta \quad T/\sin\theta$

$$\ldots = Mg/\cos\theta - T/\sin\theta$$

Mg

$$\therefore \tan\phi = \frac{T/\cos\theta + Mg/\sin\theta}{Mg/\cos\theta - T/\sin\theta}$$

$$= \frac{mg/\cos\theta + Mg/\sin\theta}{Mg/\cos\theta - mg/\sin\theta}$$

$$= \frac{\frac{1}{\cos\theta} + \left(\frac{1+2\cos\theta}{2\sin\theta}\right)\cdot\frac{1}{\sin\theta}}{\left(\frac{1+2\cos\theta}{2\sin\theta}\right)\frac{1}{\cos\theta} - \frac{1}{\sin\theta}}$$

$$= \frac{(2\sin^2\theta + \cos\theta + 2\cos^2\theta)}{2\sin^2\cos\theta} \cdot \frac{2\sin\theta\cos\theta\cdot\sin\theta}{\sin\theta + 2\cos\theta\sin\theta - 2\sin\theta\cos\theta} = \frac{(2+\cos\theta)}{\sin\theta}$$

matches the observation from c]. check d]

d] $\therefore \tan\phi = N/F_n = \dfrac{(2+\cos\theta)}{(1+2\cos\theta)} \Big/ \dfrac{\sin\theta}{(1+2\cos\theta)} = \dfrac{2+\cos\theta}{\sin\theta}$

$$\therefore \frac{M}{m} = \text{mass ratio} = \frac{(1+2\cos\theta)}{2\sin\theta} = \frac{(1+2\cos\theta)}{2(2+\cos\theta)}\cdot\frac{(2+\cos\theta)}{\sin\theta}$$

$$= \frac{(1+2\cos\theta)}{2(2+\cos\theta)}\cdot\tan\phi$$

e] $\therefore \mu < \tan\phi$ for it to not slip

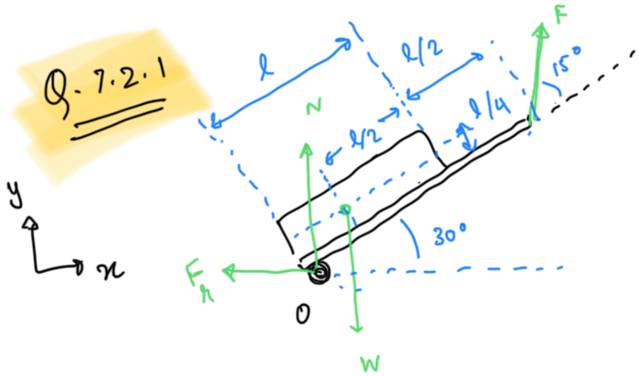
$$\therefore \mu < \frac{2+\cos\theta}{\sin\theta}$$

for $\theta = 0$; $\dfrac{m}{M} = 0$ & $\vec{F_c} = \left(\dfrac{2+1}{1+2\cdot 1}\right) Mg\,\hat{j} = Mg\,\hat{j}$

for $\theta = \dfrac{\pi}{2}$; $\dfrac{m}{M} = \dfrac{2\cdot 1}{1+2\cdot 0} = 2$ & $\vec{F_c} = \dfrac{1}{1+2\cdot 0} Mg\,\hat{i} + \dfrac{2+0}{1+2\cdot 0} Mg\,\hat{j}$

$$= Mg\,\hat{i} + 2Mg\,\hat{j}$$

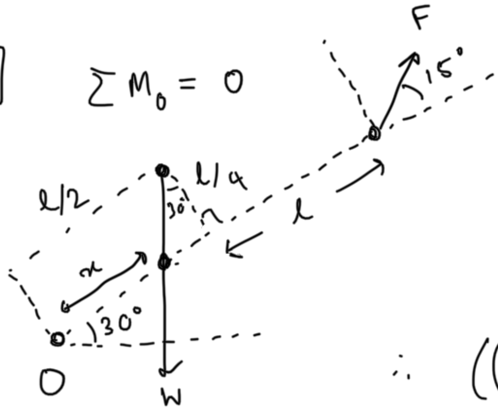
Q. 7.2.1



$\therefore \Sigma F_x = 0$ & $\Sigma F_y = 0$ & $\Sigma M_O = 0$

$\hookrightarrow \because$ suitcase is pulled steadily !

a] $\Sigma M_O = 0$



$-(x \cos 30°) W + F \sin 15° \cdot 3l/2 = 0$

& $\frac{l/2 - x)}{l/4} = \tan 30°$ $\Rightarrow x = \frac{l}{2} - \frac{l}{4} \tan 30°$

$\therefore \left(\left(\frac{l}{2} - \frac{l}{4} \tan 30°\right) \cos 30°\right) W = F \sin 15° \cdot 3l/2$

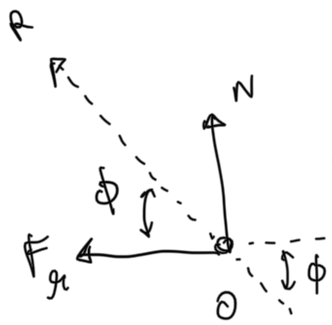
$$W = \frac{3F \sin 15°}{\cos 30° (1 - \frac{\tan 30°}{2})}$$

$W = 1.26 F = 126 N$

b] $\therefore F_R = F \cos(30 + 15°)$ & $N = W - F \sin(30 + 15)$

$= 100 \cos 45°$ $\qquad = 55.3 N$

$= 70.7 N$



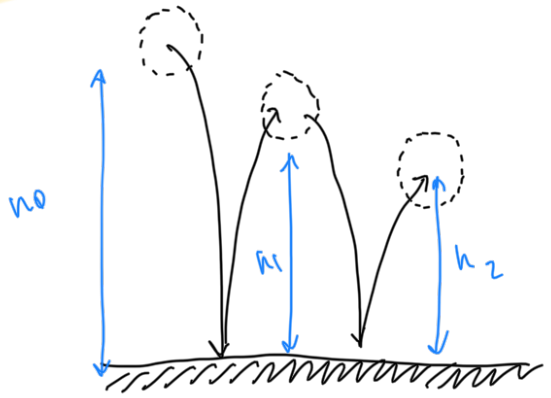
$\therefore$ Net force $= (-70.7 \hat{i} + 55.3 \hat{j}) N$

$\therefore \|R\|_2 = 89.75 N$

& $\phi = \tan^{-1}\left(\frac{55.3}{-70.7}\right) = -38°$

c] $\therefore$ if F is the input then $\frac{W}{F} = 1.26$

Q. 10.5.2



∴ what is e ? & h_2 ?

∴ using equations of motion,

$$v^2 = u^2 + 2as$$

∴ when dropped from h_0 let v_0^- be the velocity before impact
& v_0^+ ——||—— after impact

∴ If consider the entire scene as one system then there is no external non conservative force acting

∴ Conservation of Angular Momentum is applicable

$$\therefore |v_0^+| = e|v_0^-| \quad \text{—} ①$$

$$\therefore {v_0^-}^2 = 0^2 + 2 \times (-g) \times (-h_0)$$
$$= 2gh_0 \quad \text{—} ②$$

∴ when the h_1 ; velocity is zero ; $\therefore {v_0^+}^2 + 2(-g) \times (h_1) = 0$

$${v_0^+}^2 = 2gh_1 \quad \text{—} ③$$

$$\therefore \frac{{v_0^+}^2}{{v_0^-}^2} = \frac{h_1}{h_0} = e^2$$

$$\therefore e = \sqrt{\frac{6.4}{10}} = \frac{8}{10} = 0.8$$

∴ let v_1^- be the velocity just before the balls hits the ground after coming from height h_1

v_1^+ ... after impact $\Rightarrow |v_1^+| = e|v_1^-|$

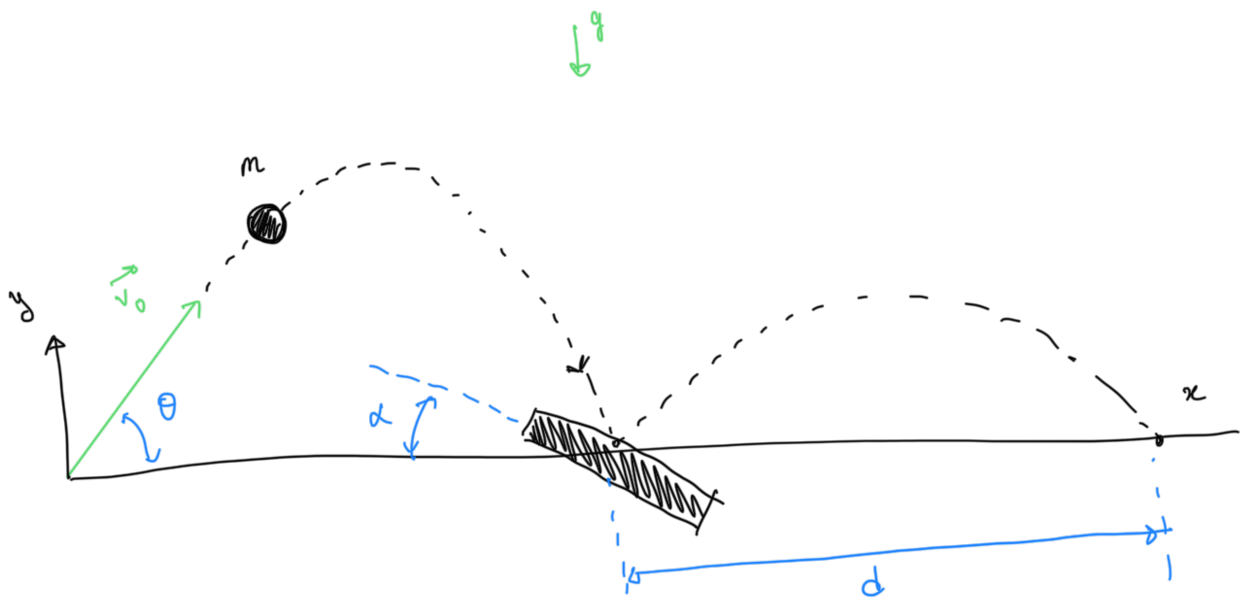
$\therefore$ v_1 is after impact ...

$$v_1^{+2} = e^2 \, v_1^{-2}$$
$$= e^2 \left(0^2 + 2 \times (-g) \times (-h_1) \right)$$
$$v_1^{+2} = e^2 (2 g h_1) \Rightarrow v_1^{+} = 8.96$$

$\therefore$ when ball reaches h_2 ; velocity $= 0$

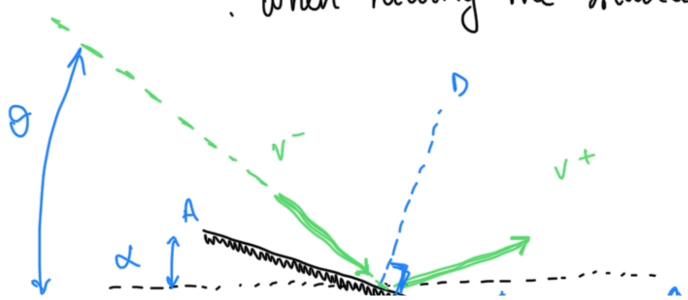
$$\therefore \quad 0 = v_1^{+2} + 2(-g)\, h_2$$
$$\therefore \; h_2 = 4.096 \text{ m}$$

Q.13.2.9 $\quad m = 0.5$ kg $\quad v_0 = 50$ m/s $\quad \theta = 60°$ $\quad e = 1$



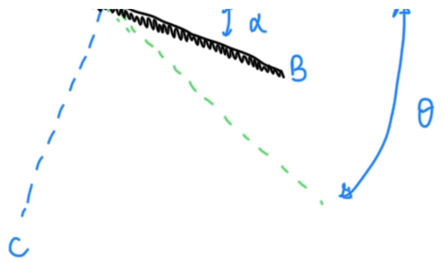
a] using eq^n of motion ; $v^2 = u^2 + 2aS$

$\because$ when hitting the slated floor , $s = 0$; $v^{-2} = v_0^2$ (but in downward)



$\therefore$ The v^- component along the line AB remains unchanged

where v^- component along line CD ...

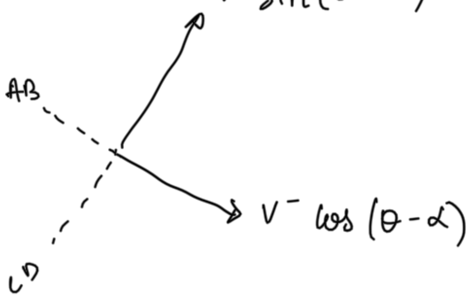


gets changed with $e=1$

$$v_{CD}^{+} = -v^{-}\sin(\theta-\alpha)$$

& $$v_{AB}^{+} = v^{-}\cos(\theta-\alpha)$$

$\therefore\ v^{+} = v^{-}\sin(\theta-\alpha)$



$\therefore$ Impulse of collision

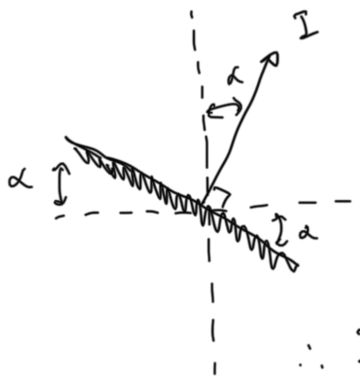
$$= F\Delta t = \frac{\Delta p}{\Delta t}\cdot\Delta t$$

$$= \Delta p$$

$$= m(\Delta v) = m\begin{bmatrix}\Delta v_{AB}\\ \Delta v_{CD}\end{bmatrix}$$

not vector in x-y but in AB-CD

$$I = m\begin{bmatrix}0\\ 2v^{-}\sin(\theta-\alpha)\end{bmatrix}$$



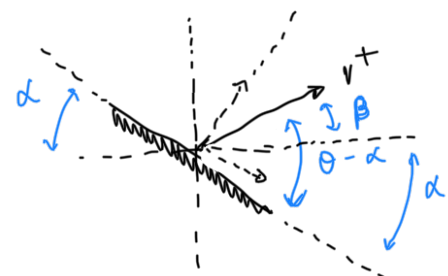
$\therefore$ Impluse $= I\sin\alpha\,\hat{i} + I\cos\alpha\,\hat{j}$

$$= 2mv^{-}\sin(\theta-\alpha)\sin\alpha\,\hat{i} + 2mv^{-}\sin(\theta-\alpha)\cos\alpha\,\hat{j}$$

$$= 2\cdot 0.5\cdot 50\cdot\sin(60^\circ-15^\circ)\left[\sin 15^\circ\,\hat{i} + \cos 15^\circ\,\hat{j}\right]$$

$$= 9.15\,\hat{i} + 34.15\,\hat{j}$$

b]



$$v^{+2} = v^{-2}\sin^2(\theta-\alpha) + v^{-2}\cos^2(\theta-\alpha)$$

$$\Rightarrow\ v^{+} = 50\,m/s$$

$\therefore\ \theta-\alpha = \alpha+\beta$

$\beta = \theta - 2\alpha$

$\therefore x = v^{+} \cos\beta \times t$

& $t = \dfrac{-v^{+}\sin\beta - v^{+}\sin\beta}{-g}$

$$\Rightarrow x = \frac{v^{+2}\, 2\sin\beta\cos\beta}{g} = \frac{v^{+2}\sin 2\beta}{g} = \frac{v^{+2}\sin(2\theta - 4\alpha)}{g}$$

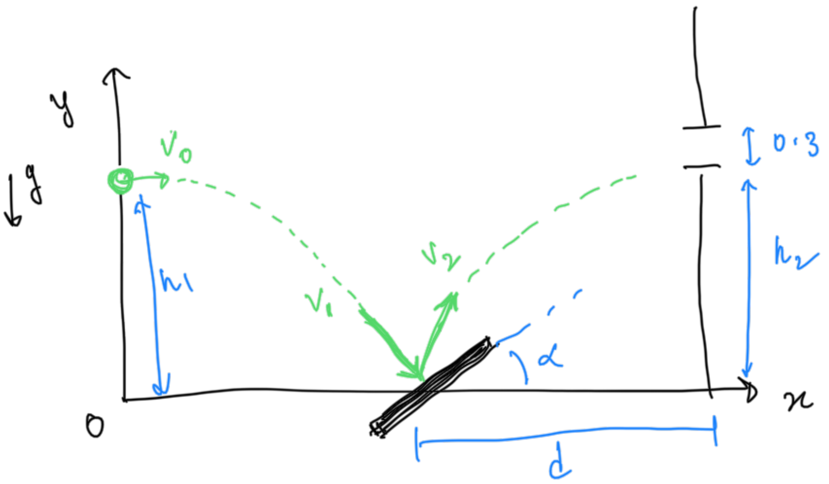
$$= \frac{50^2 \times \sin(120-60)}{9.8}$$

$$= 220.92 \text{ m}$$

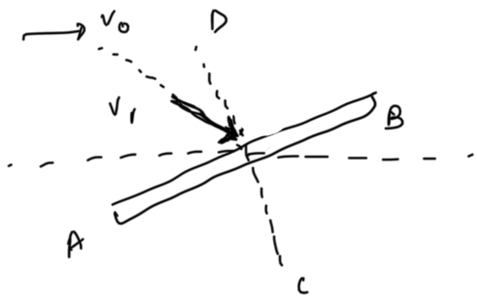
Q. 13.2.14

$h_1 = h_{ball} = 3$ m & $d = 2$ & $h_2 = h = 2$ m & width of the slot 0.3 m & $e = 0.9$ & $v = v_0 = 10$ m/s

α ?



$\therefore$

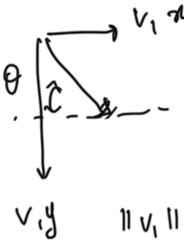


$v_{1x} = v_0$

& $v_{1y}^2 = 2(-g)(-h_1)$

$v_{1y} = -\sqrt{2gh_1}$

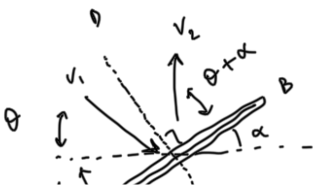
$\therefore$ The angle at which the ball hits the pedal



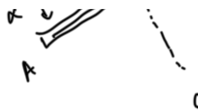
$$\tan\theta = \frac{|v_{1y}|}{|v_{1x}|} = \frac{\sqrt{2gh_1}}{v_0} = \frac{\sqrt{2\times 9.8\times 3}}{10}$$

$\|v_1\|_2 = \sqrt{2gh_1 + v_0^2}$

$\theta = \tan^{-1}(\cdot) \approx 37.45°$



$\therefore$ component along CD is changed, whereas component along AB remains unchanged!



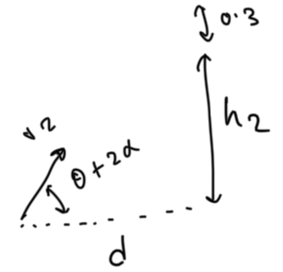
$\therefore (v_1)_{CD} = -v_1 \cdot \sin(\theta+\alpha)$

$\therefore (v_1)_{AB} = v_1 \cos(\theta+\alpha)$

$|(v_2)_{CD}| = e\,|(v_1)_{CD}|$

$(v_2)_{CD} = 0.9 \times v_1 \sin(\theta+\alpha)$ & $(v_2)_{AB} = v_1 \cos(\theta+\alpha)$

$\|v_2\|_2 = \sqrt{0.9^2\, v_1^2 \sin^2(\theta+\alpha) + v_1^2 \cos^2(\theta+\alpha)}$



$$d = \frac{v_2^2 \sin(2(\theta+2\alpha_1))}{g}$$ } Range of projectile / 2

$$h_2 = \frac{v_2^2 \sin^2(\theta+2\alpha_1)}{2g}$$ } Max height of projectile

$$1 = \frac{\sin(2(\theta+2\alpha_1))}{\sin^2(\theta+2\alpha_1)/2} \Rightarrow \frac{\sin(\theta+2\alpha_1)}{2} = 2\cos(\theta+2\alpha_1)$$

$$\therefore \tan(\theta+2\alpha_1) = 4$$

$$\alpha_1 = \frac{\tan^{-1}(4)-\theta}{2} \quad \text{.... (if } h_2)$$

$$\approx 19.2^\circ$$

if

$$h_2 + 0.3 = \frac{v_2^2 \sin^2(\theta+2\alpha_2)}{2g}$$

then

$$\frac{2}{2.3} = \frac{v_2^2 \sin(2(\theta+2\alpha_2))}{g} \times \frac{1}{\frac{v_2^2 \sin^2(\theta+2\alpha_2)}{2g}}$$

$$\sin(\theta+2\alpha_2) = 2.3 \times 2 \times \cos(\theta+2\alpha_2)$$

$$\Rightarrow \tan(\theta+2\alpha_2) = 4.6$$

$$\alpha_2 = \frac{\tan^{-1}(4.6)-\theta}{2} \quad \text{--- (if } h_2+0.3)$$

$\approx$

$\approx 20.14°$

$\therefore \alpha \in [19.2°, 20.14°]$