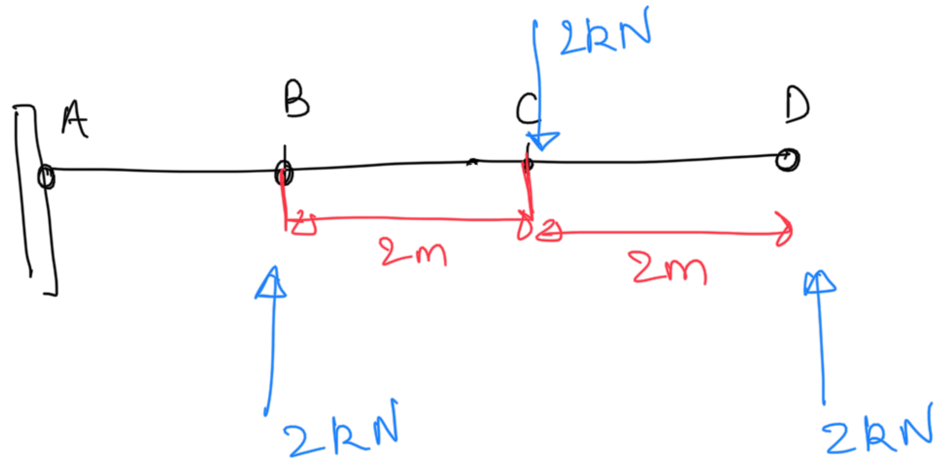
Aditya Shirwatkar

Homework 2

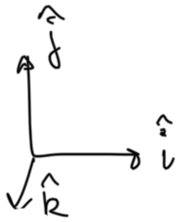
21232

Q. 2.1.11



Equivalent force couple at B & D ?

$\Longrightarrow$



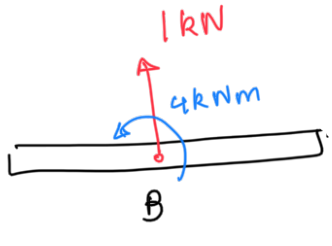
Since nothing is mentioned about weather the body is in equillibrium or not, ignoring the reaction forces & considering entire figure as FBD

a] $\therefore \sum$ Forces at B $= +1kN + 2kN - 2kN = 1kN\,\hat{j}$

$\therefore \sum$ Moment at B $= -(2kN \times 2m)\,\hat{k} + (2kN \times 4m)\,\hat{k}$

$= 4\,kNm\,\hat{k}$

$\therefore$ Equivalent Force-couple system

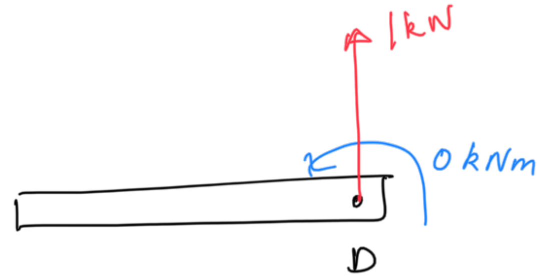


b] $\therefore \sum$ Forces at D $= +1kN + 2kN - 2kN = 1kN$

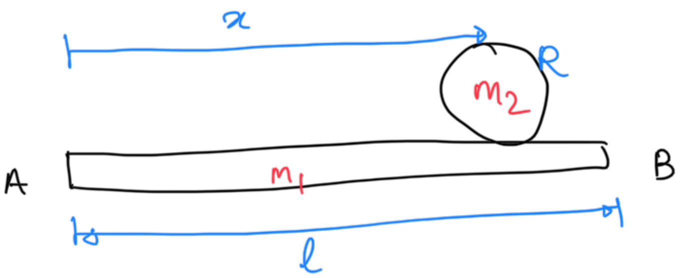
$\therefore \sum$ Moment at D $= (1kN \times -4m)\,\hat{k} + (-2kN \times -2m)\,\hat{k}$

$= 0 \text{ kNm}$

$\therefore$ Equivalent force-couple system



Q. 2.2.4



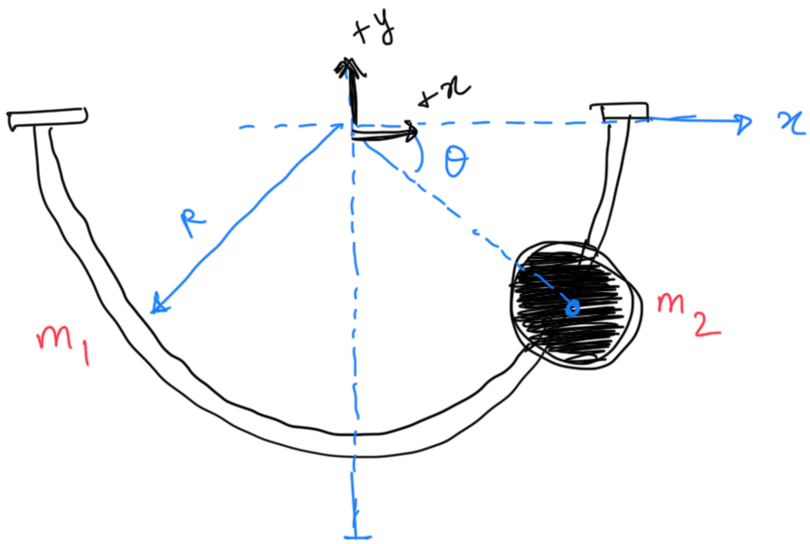
& $M = \dfrac{m_1}{m_2}$

$\therefore \; r_{com}^{m_1} = \dfrac{l}{2}$ & $r_{com}^{m_2} = x$ [assuming x is measured from A to centre of cylinder]

$$\therefore \; r_{com}^{total} = \frac{m_1 r_{com}^{m_1} + m_2 r_{com}^{m_2}}{m_1 + m_2}$$

$$r_{com}^{total} = \frac{m_1 (l/2) + m_2 (x)}{m_1 + m_2} = \underline{\underline{\frac{M\, l/2 + x}{1 + M}}}$$

Q. 2.2.9



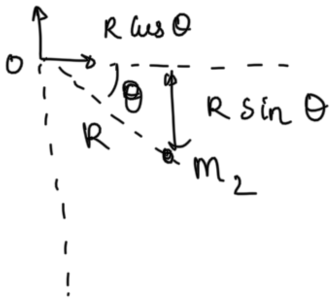
$R = 1\text{m}$

y

$m_1 = 0.1\,kg$

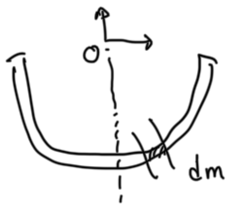
$m_2 = 0.25\,kg$

$\therefore r_{com} = ?$ @ $\theta = 30^\circ$



$$\therefore (r_{com})_{m_2} = \begin{bmatrix} x_{m_2} \\ y_{m_2} \end{bmatrix} = \begin{bmatrix} R\cos\theta \\ -R\sin\theta \end{bmatrix}$$

$\therefore$ for m_1



$dm = \left(\frac{m_1}{\pi R}\right) R\, d\theta$

due to symmetry

$$\therefore (r_{com})_{m_1} = \begin{bmatrix} x_{m_2} \\ y_{m_2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-\int_0^{\pi} R\sin\theta\, dm}{m_1} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -\frac{\int_0^{\pi} R\sin\theta \left(\frac{\cancel{m_1}}{\pi \cancel{R}}\right) \cancel{R}\, d\theta}{\cancel{m_1}} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{2R}{\pi} \end{bmatrix}$$

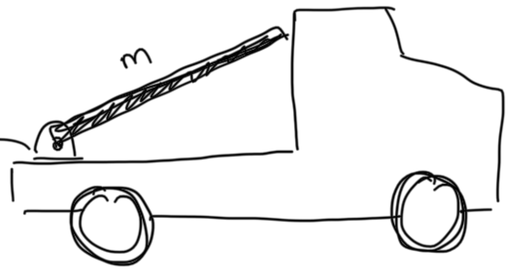
$$\therefore r_{total} = \frac{m_1 (r_{com})_{m_1} + m_2 (r_{com})_{m_2}}{m_1 + m_2} = \begin{bmatrix} \frac{m_2 R\cos\theta + 0}{m_1 + m_2} \\ \frac{-m_2 R\sin\theta + m_1\left(-\frac{2R}{\pi}\right)}{m_1 + m_2} \end{bmatrix}$$

$$\therefore r_{total} = \begin{bmatrix} R m_2 \cos\theta / m_1 + m_2 \\ -\frac{R(\pi m_2 \sin\theta + 2m_1)}{\pi (m_1 + m_2)} \end{bmatrix}$$

$$\therefore r_{total} \ @ \ \theta = 30^\circ = \begin{bmatrix} 1 \times 0.25 \cos 30^\circ / 0.1 + 0.25 \\ \\ -\dfrac{1 \times (\pi \cdot 0.25 \cdot \sin 30^\circ + 2 \times 0.1)}{\pi \ (0.1 + 0.25)} \end{bmatrix}$$

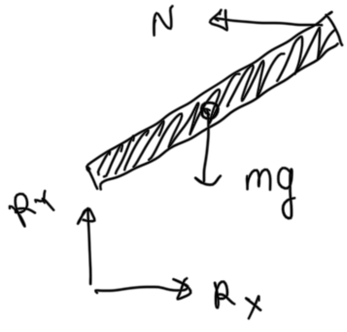
$$= \begin{bmatrix} 0.61 \text{ m} \\ \\ -0.53 \text{ m} \end{bmatrix}$$

Q. 2.3.12



assuming this is a pin joint with no friction

$\therefore$ FBD is



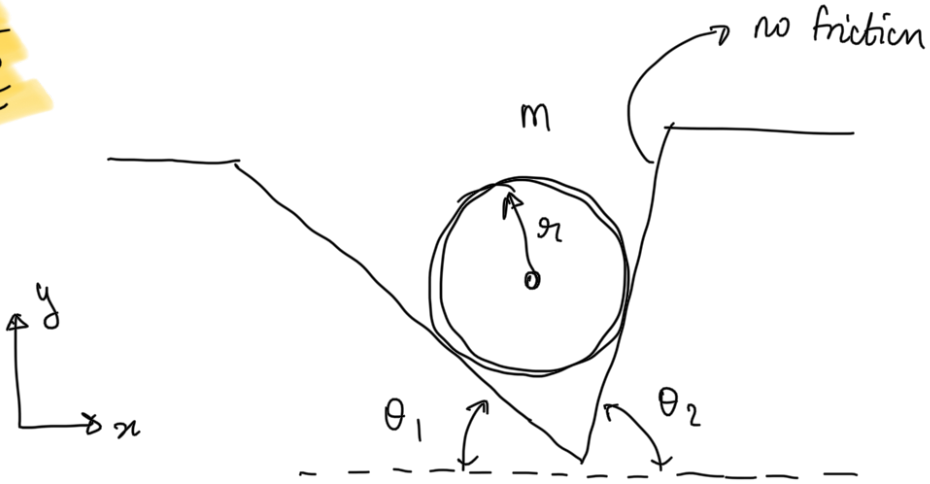
N : normal rxn from the flatbed truck

mg : weight due to gravity

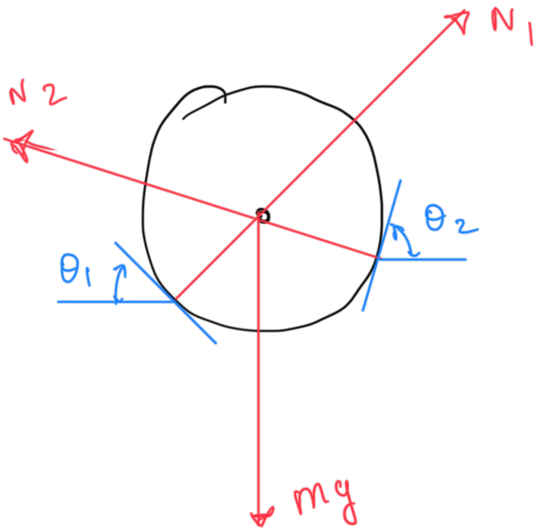
R_Y : Vertical rxn due to the joint

R_X : Horizontal rxn due to the joint

Q. 2.3.15



a]



b]

$\therefore \sum F_y = 0$ (since at rest)

$$\therefore -mg + N_1 \cos\theta_1 + N_2 \cos\theta_2 = 0 \quad \text{——— (1)}$$

Similarly $\sum F_x = 0$; $N_1 \sin\theta_1 - N_2 \sin\theta_2 = 0$ ——— (2)

$\therefore$ using (1) & (2) ;

$$mg = \left(\frac{N_2 \sin\theta_2}{\sin\theta_1}\right)\cos\theta_1 + N_2\cos\theta$$

$$\therefore N_2 = mg \sin\theta_1$$

$$\overline{\sin\theta_2 \cos\theta_1 + \cos\theta_2 \sin\theta_1}$$

$$= \frac{mg \sin\theta_1}{\sin(\theta_1+\theta_2)}$$

$$\therefore N_1 = \frac{mg \sin\theta_2}{\sin(\theta_1+\theta_2)}$$

$$\therefore \vec{N_1} = N_1 \sin\theta_1 \hat{i} + N_1 \cos\theta_1 \hat{j}$$

$$= \frac{mg \sin\theta_1 \sin\theta_2}{\sin(\theta_1+\theta_2)} \hat{i} + \frac{mg \cos\theta_1 \sin\theta_2}{\sin(\theta_1+\theta_2)} \hat{j}$$

& $\vec{N_2} = -N_2 \sin\theta_2 \hat{i} + N_2 \cos\theta_2 \hat{j}$

$$= -\frac{mg \sin\theta_1 \sin\theta_2}{\sin(\theta_1+\theta_2)} \hat{i} + \frac{mg \sin\theta_1 \cos\theta_2}{\sin(\theta_1+\theta_2)} \hat{j}$$

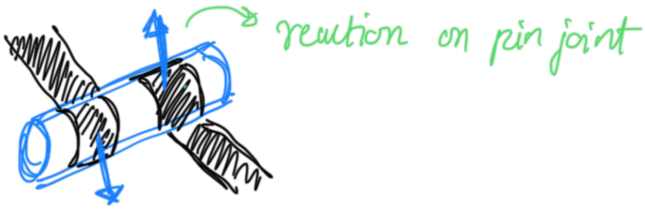
c] $\vec{N_1} + \vec{N_2} + m\vec{g} = 0$ (one can easily verify using results from b])

d] Net moment $= M_{com} = 0$ since there is point contact at the surface & all normals to a circle pass through its centre, so moment due to normal is zero. Similarly, weight due to gravity is through centre so moment $= 0$

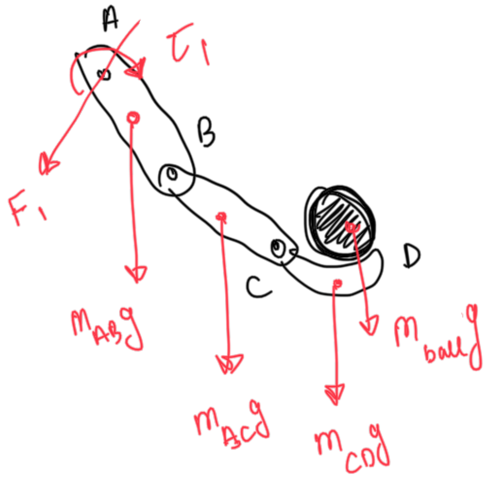
Q. 2.3.16

Assuming the muscles apply only torques

& also assuming the pin joint only apply reaction forces



a] arm + ball

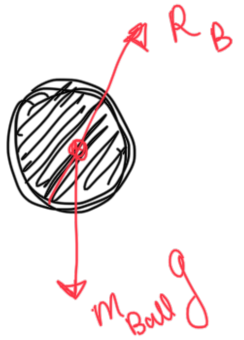


τ_1 : Torque applied by the muscles of shoulder on link AB

F_1 : rxn forces due to pin joint between link AB & shoulder

$m_{AB}g$, $m_{BC}g$, $m_{CD}g$ & $m_{ball}g$ are force due to gravity

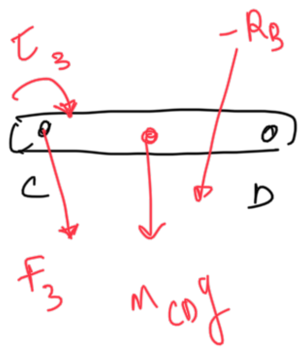
b] Ball



R : reaction on the ball due to link CD

$m_{Ball}g$: weight

c] Hand



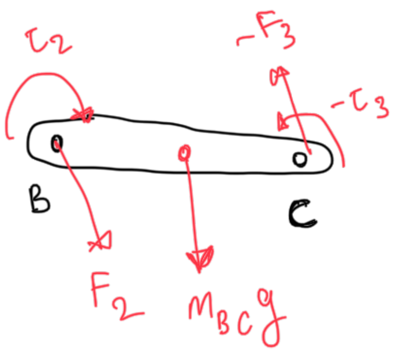
F_3 : rxn force due to pin joint on link CD

τ_3 : torque applied on CD due to muscles of link BC

$M_{CD}g$: force due to gravity

$-R_B$: rxn force due to ball on

d] Fore arm

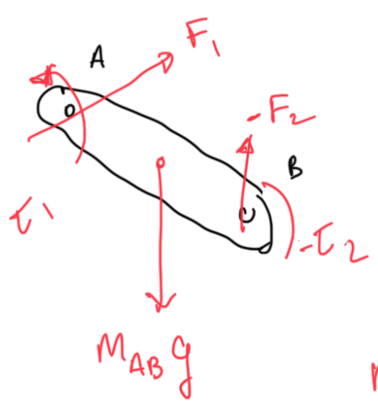


F_2 : rxn force on BC due to pin joint b/w BC & AB

τ_2 : torque applied on BC due to muscles of link AB

$M_{BC}g$: force due to gravity

e] upper arm



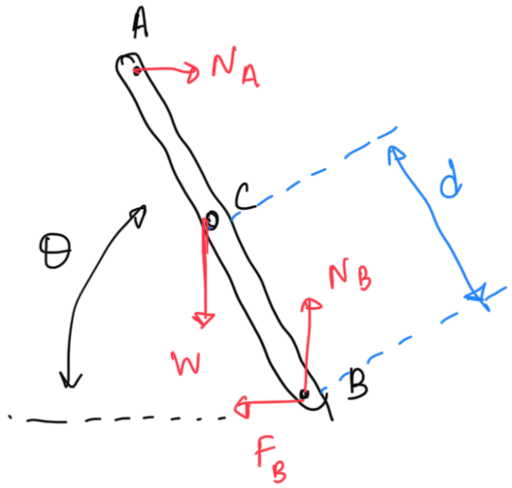
F_1 : rxn force on AB due to pin joint

τ_1 : torque applied on AB due to muscles of shoulder

$M_{AB}g$: force due to gravity

Q. 2.4.4

a]



b] If the ladder is about to slip then ; $F_B = \mu N_B$

$\therefore \sum F_Y = 0$ & $\sum F_x = 0$

$\therefore W = N_B$ $\therefore N_A = F_B$

& $\sum M_B = 0$

$\therefore (W \cancel{d} \cos\theta) - (2\cancel{d} \sin\theta N_A) = 0$

$$N_A = \frac{W}{2\tan\theta}$$

$$\therefore F_B = \frac{W}{2\tan\theta}$$

$$\therefore N_B = \frac{W}{2\mu\tan\theta}$$

$\therefore$ Total force

$$= \begin{bmatrix} -W/2\tan\theta \\ W/2\mu\tan\theta \end{bmatrix}$$

$\therefore$ The direction of total force at B is given by

$$\hat{u} = \frac{\frac{-W}{2\tan\theta}\hat{i} + \frac{W}{2\mu\tan\theta}\hat{j}}{\sqrt{\frac{W^2}{2^2\tan^2\theta} + \frac{W^2}{2^2\mu^2\tan^2\theta}}} = \frac{-1\hat{i} + 1/\mu\,\hat{j}}{\sqrt{1+1/\mu^2}}$$

$$= \frac{-\mu\hat{i} + 1\hat{j}}{\sqrt{}}$$

$\sqrt{\mu^2 + 1}$

Now relative positions of A & B will alter θ, but there is no θ present in the final direction vector $\hat{u}$.

$\therefore$ No effect on $\hat{u}$!