HW 5 Aditya Shirwalkar 21232

Pg 10 :

Q.3

$u + v + w + z = 6$ what is the intersection of 3 planes in $\mathbb{R}^4$?

$u + w + z = 4$

$u + w = 2$

$\Rightarrow z = 2, v = 2 \Rightarrow u + w = 4$ $\therefore$ ∞ points in $\mathbb{R}^2$

$\therefore$ line

$\therefore$ if $u = -1$

then $-1 + v + w + z = 6$

$-1 + 0 + w + z = 4$ $\Rightarrow$ single point

$-1 + 0 + w = 2 \Rightarrow w = 3$

if we take a 4^{th} plane as $u + w = b$ where $b \neq 4$

then we have no sol^n

Pg 18 :

Q.12

$2x + 5y = 0$

$4x + dy + z = 2$

$y - z = 3$

$$\left[\begin{array}{ccc|c} 2 & 5 & 0 & 0 \\ 4 & d & 1 & 2 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 2 & 5 & 0 & 0 \\ 0 & d-10 & 1 & 2 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

$\therefore$ if $d = 10$, will need to exchange rows

$\therefore$ if $d=10$: $\left[\begin{array}{ccc|c} 2 & 5 & 0 & 0 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$ $\Big\}$ Triangular (non singular)

$\therefore$ if $d \neq 10$: $\left[\begin{array}{ccc|c} 2 & 5 & 0 & 0 \\ 0 & (d-10) & 1 & 3 \\ 0 & 0 & -1-\left(\frac{1}{d-10}\right) & 2-\frac{1}{d-10} \end{array}\right]$

if this system is singular,

$$-1-\left(\frac{1}{d-10}\right)=0$$

$$(d-10) = -1$$

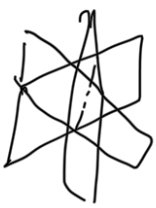
$$d = 9$$

Pg 19 :

Q. 18

A system of linear eq's can have only 1, ∞ many or no solution otherwise the linearity of the equations won't be satisfied

a] An entire line passing through (x, y, z) & (X, Y, Z)

b]  they meet on the entire line passing on those 2 points

Pg 20 :

Q. 24

$$\left[\begin{array}{cccc|c} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \end{array}\right.$$

$$\left[\begin{array}{cccc|c} 0 & 0 & -1 & 2 & 5 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 2 & -1 & 0 & 0 & 0 \\ 0 & 3/2 & -1 & 0 & 0 \\ \text{---} & \text{''} & \text{---} & & \\ \text{---} & \text{''} & \text{---} & & \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 2 & -1 & 0 & 0 & 0 \\ 0 & 3/2 & -1 & 0 & 0 \\ 0 & 0 & 4/3 & -1 & 0 \\ 0 & 0 & -1 & 2 & 5 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 2 & -1 & 0 & 0 & 0 \\ 0 & 3/2 & -1 & 0 & 0 \\ 0 & 0 & 4/3 & -1 & 0 \\ 0 & 0 & 0 & 5/4 & 5 \end{array}\right] \begin{array}{l} u = 1 \\ v = 2 \\ w = 3 \\ z = 4 \end{array}$$

Pg 29 :

Q.4

$A \Rightarrow m \times n$

$x \Rightarrow n$

$$A = \overbrace{x_1 \begin{bmatrix} a_1 \end{bmatrix}_m + x_2 \begin{bmatrix} a_2 \end{bmatrix}_m + \cdots\cdots}^{n}$$

$$\# = mn$$

if $B \Rightarrow n \times p$

$$\# = mnp$$

$$\begin{bmatrix} a_1 & \cdots\cdots & a_n \end{bmatrix}_{m \times n} \begin{bmatrix} b_1 & b_2 \cdots & b_p \end{bmatrix}_{n \times p} =$$

Pg 30 :

Q.10

a) $b_1 = b_3$ then what about AB

$$AB = A \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} = \begin{bmatrix} Ab_1 & Ab_2 & Ab_3 \end{bmatrix}$$

Yes True $\because Ab_1 = Ab_3$

b] If $b_1 = b_3$ for rows

$$AB = A\begin{bmatrix} b_1 \longrightarrow \\ b_2 \longrightarrow \\ b_3 \longrightarrow \end{bmatrix} = \begin{bmatrix} a_1 \longrightarrow \\ a_2 \longrightarrow \\ a_3 \longrightarrow \end{bmatrix}$$

c] If $a_1 = a_3$ for rows

then each row of AB = (row i of A) $\times$ B

$\therefore a_1 B = a_3 B$

$\therefore$ True

d] $(AB)^2 = A^2B^2$

$$\begin{array}{cc} (AB)^2 & A^2B^2 \\ \downarrow & \downarrow \\ ABAB & AABB \end{array}$$

$\because$ matrix mul is not commutative

$\therefore$ False

Pg 31

Q.21

$$A = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad C = AB = \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & -0.5 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} = A$$

$$\therefore A^3 = A^2 A$$

$$= A^2 = A$$

$$\therefore A^k = A$$

$$\therefore B^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore B^3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\therefore B^3 = B$$

$$\therefore B^4 = B^3 B$$

$$= B^2$$

$$\therefore C^2 = \begin{bmatrix} 0.5 & -0.5 \\ 0.5 & -0.5 \end{bmatrix}\begin{bmatrix} 0.5 & -0.5 \\ 0.5 & -0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\therefore C^k = 0_{2\times 2}$$

if $k > 1$

$$\therefore B^k = \begin{cases} B & \text{if } k \text{ is odd} \\ B^2 & \text{if } k \text{ is even} \end{cases}$$

Q.22

a] E_{21} subtracts 5 times row 1 from row 2

$$\begin{bmatrix} \leftarrow a_1 \rightarrow \\ \leftarrow a_2 \rightarrow \\ \leftarrow a_3 \rightarrow \end{bmatrix} \Rightarrow \begin{bmatrix} \leftarrow a_1 \rightarrow \\ \leftarrow a_2 - 5a_1 \rightarrow \\ \leftarrow a_3 \rightarrow \end{bmatrix}$$

$$\begin{bmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 - 5a_1 & a_5 - 5a_2 & a_6 - 5a_3 \\ a_7 & a_8 & a_9 \end{bmatrix}$$

$$\Downarrow \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) E_{32} subtracts -7 times row 2 from row 3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -7 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

c) P exchanges rows 1 & 2 then rows 2 & 3

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Pg 33 :

Q.38

$AB = I$ & $BC = I$

associative Law : $(AB)C = A(BC)$

$\therefore$ if $AB = I$ & $BC = I$ then $A = C$

Q.42

a) True $(m \times n) \times (m \times n)$ is possible if $m = n$

b) False $(m \times n) \times (p \times q) \Rightarrow n = p$

$(p \times q) \times (m \times n) \Rightarrow q = m$

$A \Rightarrow (m \times n)$

$B \Rightarrow (n \times m)$

c) True : using b) $AB \Rightarrow (m \times q) \Rightarrow (m \times m)$

$BA \Rightarrow (p \times n) \Rightarrow (n \times n)$

d) $AB = B$

$\hookrightarrow$ BA should also be B then only I

$\therefore BA \Leftrightarrow AB$?

$\Downarrow$

$ABA \Leftrightarrow AAB$

$BA \Leftrightarrow AAB$

$BAB \Leftrightarrow AABB$

$B \Leftrightarrow AAB^2$

$B \Leftrightarrow AB\,B$

✓ Thus True

Pg 45

Q.5

$$Ax = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix} = b$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$\begin{bmatrix} & & \\ -3 & 0 & 1 \end{bmatrix}$ & $E_1 b = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$

$\therefore E_1 A = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix} = U$

$\therefore \quad A = LU$

$\therefore E_1^{-1} = L$

$E_1^{-1} U = A$

$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

$\therefore \ A = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix}}_{U}$

& $\underbrace{\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}}_{c}$

Pg 46

Q. 11

$LUx = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}}_{b}$

we know $Ux = c$

$\therefore Lc = b$

$\therefore C = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$ (using substitution)

$\therefore x = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$ (using back substitution)

$\therefore Ax = b$

$\therefore LUx = b$

Q.14

$P_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad P_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\quad P_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$P_I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad P_{23,12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$P_{13,23} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\quad P_{12,23} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$\therefore P_{12}^{-1} = P_{12}$ ($\because$ repeated permutation results in the same matrix)

Similarly $P_{13}^{-1} = P_{13}$ & $P_{23}^{-1} = P_{23}$

$\therefore P_{23,12} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = P_{13,23} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\therefore P_{12,23} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$$\therefore\ P_{23,12}\cdot P_{23,12} = \begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix} = \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix} = P_{12,23}$$

$$\therefore\ P_{12,23}\cdot P_{12,23} = \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix} = \begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix} = P_{23,12}$$

Pg 48

Q. 29

$$A = \begin{bmatrix}a&a&a&a\\a&b&b&b\\a&b&c&c\\a&b&c&d\end{bmatrix}$$

$$A_1 = \begin{bmatrix}a&a&a&a\\0&b-a&b-a&b-a\\0&b-a&c-a&c-a\\0&b-a&c-a&d-a\end{bmatrix} \Rightarrow A_2 = \begin{bmatrix}a&a&a&a\\0&b-a&b-a&b-a\\0&0&c-b&c-b\\0&0&c-b&d-b\end{bmatrix}$$

$$\Updownarrow$$

$$E_1 = \begin{bmatrix}1&0&0&0\\-1&1&0&0\\-1&0&1&0\\-1&0&0&1\end{bmatrix} \qquad E_2 = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&-1&1&0\\0&-1&0&1\end{bmatrix}$$

$$\therefore\ A_3 = U = \begin{bmatrix}a&a&a&a\\0&b-a&b-a&b-a\\0&0&c-b&c-b\\0&0&0&d-c\end{bmatrix} \Leftrightarrow E_3 = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&-1&1\end{bmatrix}$$

$$\therefore\ E_3E_2E_1A = U$$

$$A = E_1^{-1}E_2^{-1}E_3^{-1}U$$

$$L = \begin{bmatrix}1&0&0&0\\1&1&0&0\\ \cdots\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\ \cdots\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\ \cdots\end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

conditions are $a>0$, $b \neq a$, $c \neq b$, $d \neq c$

Pg 60

Q.18

$$A = \begin{bmatrix} a & b & c \\ d & e & 0 \\ f & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & e \end{bmatrix}$$

$\therefore \det(A) = -cef$ $\qquad \therefore \det(B) = ade - bce$

$\therefore cef \neq 0$

& $ade \neq bce$

$\therefore c \cdot e \cdot f \neq 0$

$(ad - bc) \cdot e \neq 0$

$\Downarrow$

$ad - bc \neq 0$

& $\because c \neq 0 \Rightarrow ad \neq 0 \Rightarrow a \neq 0,\ d \neq 0$

& $b \neq 0$

$\therefore a \neq 0,\ b \neq 0,\ c \neq 0,\ d \neq 0,\ e \neq 0,\ f \neq 0$

& $ad \neq bc$

Pg 61

Q.28 $M = ABC$ if M^{-1} exists then $A^{-1}\ B^{-1}\ C^{-1}$ also exist

$\therefore M^{-1} = C^{-1}B^{-1}A^{-1}$

$\therefore A^{-1}M = BC$

$\therefore A^{-1}MC^{-1} = B$

$\Rightarrow$ $A^{-1}MC^{-1}B^{-1} = BB^{-1}$

$A^{-1}M\ C^{-1}B^{-1} = I$

$MC^{-1}B^{-1} = A$

$C^{-1}B^{-1} = M^{-1}A$

$B^{-1} = CM^{-1}A$

Pg 62

Q.38 (a)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \therefore \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -3 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

A^{-1}

(b)

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \end{array}\right.$$

$$\left[\begin{array}{ccc:ccc} 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc:c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ 0 & -1 & 0 \end{matrix}}_{A^{-1}} \\ 0 & 0 & 1 & \end{array}\right]$$

Pg 64

Q.54

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\therefore M = \begin{bmatrix} a_{11}\ a_{12} \cdots a_{1n} & b_{11}\ b_{12} \cdots b_{1q} \\ a_{21} & b_{21} \\ \vdots & \end{bmatrix}$$

$$\begin{bmatrix} \vdots & & \vdots & \\ a_{m1} & & b_{m1} & \\ c_{11}\ c_{12} \cdots c_{1n} & & d_{11}\ b_{12} \cdots b_{1q} & \\ \vdots & & \vdots & \\ c_{p1} & & d_{p1} & \end{bmatrix}$$

$$\therefore M^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} & c_{11} & c_{21} & \cdots & c_{p1} \\ a_{12} & \ddots & & \vdots & c_{12} & \ddots & & \vdots \\ \vdots & & & & \vdots & & & \\ a_{1n} & \cdots & & & c_{1n} & \cdots & & \\ & — & 11 & — & & — & 11 & — \end{bmatrix}$$

$$M^T = \begin{bmatrix} A^T & C^T \\ B^T & D^T \end{bmatrix}$$

For M to be symmetric $\Rightarrow$ $M^T = M$

$\therefore$ if $A \Rightarrow m \times n$ $\quad C \Rightarrow p \times n$

$B \Rightarrow m \times q$ $\quad D \Rightarrow p \times q$

$\therefore$ $A^T = A$ & $D^T = D$

& $B^T = C$ & $C^T = B$

Q.56

If $A = A^T$, $B = B^T$

a) $A^2 - B^2$

$= AA^T - BB^T$ & $(AA^T - BB^T)^T = AA^T - BB^T$

True

b) $(A+B)(A-B)$

$\Rightarrow \therefore ((A+B)(A-B))^T = (A-B)^T (A+B)^T$

$= (A^T - B^T)(A^T + B^T)$

$= \underbrace{(A-B)(A+B)}_{\text{may or may not be}}$

c) ABA

$$\Rightarrow (ABA)^T = A^T B^T A^T = ABA$$

True

d) $ABAB$

$$\Rightarrow (ABAB)^T = B^T A^T B^T A^T = \underbrace{BABA}_{\text{may or may not be true}}$$

Pg 72

Q. 7

$$H = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix}$$

a] $\therefore$ since not mentioned to use Gauss Jordan, using MATLAB

$$\therefore H^{-1} = \begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix}$$

b] Approximating to 3 figures

$$H = \begin{bmatrix} 1 & 0.500 & 0.333 \\ 0.500 & 0.333 & 0.250 \\ 0.333 & 0.250 & 0.200 \end{bmatrix}$$

$$\therefore H^{-1} = \begin{bmatrix} 9.670 & -39.502 & 33.283 \\ -39.508 & 210.185 & -196.951 \\ 33.283 & -196.951 & 195.771 \end{bmatrix}$$

Answer differs a lot

Q.10

$$A = \begin{bmatrix} 0.001 & 0 \\ 1 & 1000 \end{bmatrix}$$

$$\therefore \left[\begin{array}{cc|cc} 0.001 & 0 & 1 & 0 \\ 1 & 1000 & 0 & 1 \end{array}\right]$$

a] Direct elim

$$\Rightarrow \left[\begin{array}{cc|cc} 0.001 & 0 & 1 & 0 \\ 0 & 1000 & -1000 & 1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1000 & 0 \\ 0 & 1 & -1 & 0.001 \end{array}\right]$$

b] Partial Pivoting

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 1000 & 0 & 1 \\ 0.001 & 0 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 1000 & 0 & 1 \\ 0 & -1 & 1 & -0.001 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -1000 & 0 \\ 0 & 1 & -1 & 0.001 \end{array}\right]$$