E1 222 Stochastic Models and Applications
Problem Sheet 1.1
(No need to submit answers)

1. A fair coin is tossed n times. What is the probability that the difference between the number of heads and the number of tails is equal to $n-3$?

2. Suppose we want to find mutually exclusive events, $A_1, \cdots, A_n$ such that $P(A_i) \geq 0.1$, $\forall i$. What is the maximum possible value for n.

3. State whether the following sets are finite, countably infinite or uncountably infinite.

   (i). The sample space for the situation when each of a million people vote for one of ten candidates.

   (ii). All rational numbers between 0 and 1.

   (iii). All irrational numbers between 0 and 1.

   (iv). All real numbers between 0 and 1.

4. A standard deck of playing cards is made into four piles. Let E_i denote the event that the i^{th} pile contains exactly one ace. Find $P(E_1E_2E_3E_4)$.

5. A chord is drawn at random in the unit circle. What is the probability that the length of the chord is greater than the side of the inscribed equilateral triangle. (The side of an equilateral triangle inscribed inside a unit circle is $\sqrt{3}$).
   (The solution to the problem depends on how we define what is meant by a random chord. One possibility is as follows. We can think of choosing a random chord to be same as that of choosing a point inside the circle. This is because any given point inside the circle can be uniquely corresponded to a chord: join the point to the center of the circle and then draw a line through the point perpendicular to the line joining the point to the center. The resulting chord is what we can uniquely associate with the point. Using this idea, calculate the above probability.)

6. Consider a student answering a multiple-choice question. The student knows the answer with probability p, $0 < p < 1$. If the student knows the answer, the student marks the correct answer with probability 0.99. When the student does not know the answer, the student guesses and hence the probability of marking the correct answer is $\frac{1}{k}$, where k is the number of choices. Calculate the probability that the student knows the answer given that the student marked the correct answer.

7. State whether the following sequences of sets are monotone.

   (i). $A_k = [0,\ 1 + \frac{(-1)^k}{k}],\ k = 1, 2, \cdots$

(ii). $A_k = [1/k,\ 1],\ k = 1, 2, \cdots$

8. Let $A_k = (-1/k,\ 1],\ k = 1, 2, \cdots$. Let $B = \cap_{k=1}^{\infty} A_k$. For any $x < 0$, show that there is a K such that $x \notin A_K$. For any x, such that $0 < x < 1$, show that $x \in A_k,\ \forall k$. Now determine what B is.

9. Let $A_k = [1/k,\ 1],\ k = 1, 2, \cdots$. Let $B = \cup_{k=1}^{\infty} A_k$. For any $0 < x < 1$, show that there is a K such that $x \in A_K$. Now determine what B is.