# E1 222 Stochastic Models and Applications

**Problem Sheet 3.2**

(You need not submit solutions)

1. Let $(X, Y)$ have joint density
$$f_{XY}(x, y) = x + y, \ 0 < x < 1, \ 0 < y < 1.$$
Find the marginal densities and the conditional densities. Find $P[X > 2Y]$. Are $X, Y$ independent? Calculate $P[X > 0.5 \mid Y = 0.5]$ and $P[X > 0.5]$.

2. Let $X, Y$ be discrete random variables taking values in $\{1, 2, \cdots, N\}$ with joint probability mass function
$$f_{XY}(x, y) = \frac{1}{Nx}, \ 1 \leq y \leq x \leq N$$
Find the marginal mass function of $X$, and the conditional mass function $f_{Y|X}$.

3. Let $X$ be uniform from 0 to 1, let $Y$ be uniform from 0 to $X$ and let $Z$ be uniform from 0 to $Y$. What is the joint density of $X, Y, Z$? Find the marginal densities, $f_X, f_Y, f_Z$ and the joint density of $Y, Z$.

4. Let $X, Y$ be independent discrete random variables each being uniform over $\{0, 1, \cdots, N\}$. Find $P[X > Y]$, $P[X < Y]$ and $P[X = Y]$.

5. Let $X, Y$ be iid random variables which are uniform over $(0, 1)$. Find $P[X > Y]$ and $P[Y > X]$.

6. Let $A, B$ be two events. Let $I_A$ and $I_B$ denote the indicator random variables of these events. Show that $I_A$ and $I_B$ are independent iff $A$ and $B$ are independent.

7. Let $X, Y \in \{0, 1, \cdots, N\}$ be two discrete random variables with joint mass function given by $f_{XY}(i, j) = 1/(N + 1)^2, \ 0 \leq i, j \leq N$. Are $X, Y$ independent? Find the pmf of $Z$ when (i). $Z = |X - Y|$, and (ii). $Z = X + Y$.

8. Let $X$ and $Y$ be independent random variables each having an exponential distribution with the same value of parameter $\lambda$. Show that $Z = \min(X, Y)$ is exponential with parameter $2\lambda$.