E1 222 Stochastic Models and Applications

Problem Sheet 2.1

(You need not submit solutions)

1. Let $(\Omega,\ \mathcal{F},\ P)$ be a probability space and let $A_1, A_2 \in \mathcal{F}$. Consider the following random variable:

$$\begin{aligned} X(\omega) &= -1 \quad \text{if} \quad \omega \in A_1 \\ &= +1 \quad \text{if} \quad \omega \in A_1^c A_2 \\ &= \ \ 0 \quad \text{if} \quad \omega \in A_1^c A_2^c \end{aligned}$$

   What is the event $[X < 0.5]$? Find the distribution function of X.

2. Two fair dice are rolled and X is the maximum of the two numbers. Specify a reasonable probability space and then define this random variable as a function on the appropriate Ω. Derive its probability mass function.

3. Consider the probability space with $\Omega = [0, 1]$ and the usual probability assignment (where probability of an interval is the length of the interval). Define X by $X(\omega) = 2\omega$ if $0 \leq \omega \leq 0.5$, and $X(\omega) = 2\omega - 0.5$ if $0.5 < \omega \leq 1$. What is the event $[X \in (0.5,\ 0.75)]$? Find the distribution function of X.

4. Let X be a random variable with $P[X = a] = 0$. Express $P[|X| \geq a]$ in terms of the distribution function of X.

5. Let X be geometric. Calculate probabilities of the events (i). $[X \leq 10]$, (ii). $[X = 3 \ \text{ or } \ 5 \leq X \leq 7]$.

6. Let X be exponential random variable. Calculate probabilities of (i). $[|X| \leq 3]$, (ii). $[X \leq 4 \ \text{ or } \ X \geq 10]$.

7. Let X be a rv with density function

$$f(x) = cx^3, \quad \text{if} \quad 0 \leq x \leq 1.$$

   ($f(x)$ is zero for all other values of x). Find the value of c and the distribution function of X. Find $P[X > 0.5]$.