

E1 222 Stochastic Models and Applications

Problem Sheet 2.2

(You need not submit the solutions)

1. A fair dice is rolled repeatedly till the sum of all numbers obtained exceeds 6. Let X denote the number of rolls needed. Find the values of $F_X(1)$, $F_X(7)$ and $F_X(2)$.
2. Let X be a discrete random variable having a uniform distribution over the set $\{1, 2, \dots, 20\}$. Find the mass function of the random variables: (i). $U = \frac{1}{X}$, (ii). $U = \max(X, 10)$.
3. Let X be continuous random variable with uniform density over $(-1, 1)$. Find the density (or mass function) of the random variables: (a). $U = e^X$, (b). $U = \frac{X}{1+X}$, (c). $U = g(X)$ where $g(x) = -1$ if $x < 0$, $g(x) = 0$ if $x = 0$, and $g(x) = 1$ if $x > 0$.
4. Let X be a continuous random variable having uniform density over $[0, 3]$. Let $Y = (X - 1)^2$. Find the density of Y .
5. Let X be a random variable, g be some density function and ϕ a differentiable strictly increasing function on $(-\infty, \infty)$. Suppose that

$$P[X \leq x] = \int_{-\infty}^{\phi(x)} g(z) dz$$

Show that the density of $Y = \phi(X)$ is $g(y)$.

6. We have a coin with probability p of coming up heads, $0 < p < 1$. Now consider the following procedure that determines value of a random variable, X .
 1. Flip the coin and let the result (heads or tails) be denoted by O_1 .
 2. Flip the coin again and let the result be O_2 .
 3. If $O_1 = O_2$ go to step 1; else go to 4.
 4. If O_2 is heads set $X = 0$; otherwise set $X = 1$.

Find the mass function of X .

7. Let X be a random variable uniformly distributed over $\{0, 1, \dots, N\}$. Find $E[X]$.

8. A darts board consists of concentric circles with radius $\frac{k}{n}$, $k = 1, 2, \dots, n$. Thus there are n annular regions. A dart is thrown randomly. If it hits the k^{th} annular region we get $1/(2k - 1)$ rupees. What is the expected amount one gets if a dart is thrown randomly.

9. Let X be a rv with density function

$$f(x) = cx(1 - x), \quad \text{if } 0 \leq x \leq 1.$$

($f(x)$ is zero for all other values of x). Find the value of c and $P[X > 1]$.
Let $Y = 2X^3 - 3X^2 + 3X + 5$. Find $E[Y]$.

10. Let X be an exponential random variable. Find EX^3 .