Lab 4 - Modelling and Simulation - Market dynamics

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In this assignment, we explore bifurcations in one-dimensional (1D) nonlinear systems, a fundamental area of study within the field of dynamical systems theory used in a real-life example.

I. INTRODUCTION

Here, we will look into market competition dynamics between 2 companies, A and B, in a local regional market. Then, later on, we also analyzed the growth/decay of A's market share in the global market, which is influenced by both local and international market factors.

II. MODEL

A. Local Market

The equation for growth/decay of A's market share is given as:

$$\frac{dx}{dt} = ax(1-x)(x-y) \tag{1}$$

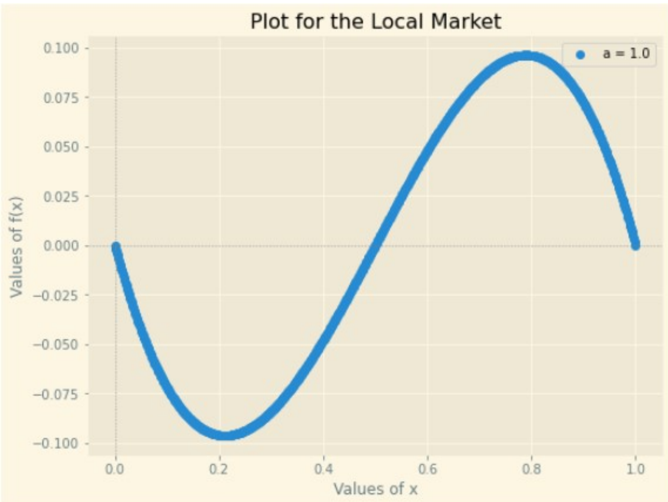

FIG. 1: The Local Market at a = 1

In Eq 1, x represents the market share of A, (1-x) is the size of the available potential customer base, and (x-y) represents the relative competitive edge of A over B in the local market, and $\frac{dx}{dt}$ represents the rate of growth/decay of A's market share.

We know that there are only two companies in the local market, therefore $x + y = 1$ Substitute $y = 1 - x$ in the Equation 1, then the equation is given as below:

$$\frac{dx}{dt} = ax(1-x)(2x-1) \tag{2}$$

1. Fixed Points and their Stabilities

To analyze Eq 2, we need to calculate the equilibrium, stable, and unstable points for the given Equation. To find the equilibrium points, we equate the right-hand side of Eq 2 to zero, and then we will get the fixed issues. A fixed point is said to be stable when a slight disturbance away from it causes it to return to its original position. Otherwise, the fixed point is stated to be unstable.

$$ax(1-x)(2x-1) = 0$$

We get the fixed points as $x = 0$, $x = 1$ and $x = 1/2$. We will sketch the Eq 2, to determine the stabilities. we check the derivative of a function at fixed points; if the value turns out to be negative, then it is a stable point. If the deal turns out to be positive, then it is an unstable point, and if it turns out to be zero, we need to perform the second derivative test.

$$\frac{d(a(x)(1-x)(2x-1))}{dx} = -a(6x^2 - 6x + 1) \tag{3}$$

In Eq 3, if we calculate it at point $x = 0$, then the value comes out to be $-a$, assuming $a > 0$, so the value is negative therefore, $x = 0$ is a stable point. Similarly, $x = 1$ becomes a stable point, and $x = 1/2$ will be unstable.

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B. Global Market

Assuming the regional market is connected and influenced by the global market, the Equation 1 will have an additional term of $r(p-x)$ where p is the market share of company A in the global market, and r denoted the strength of influence from the global call to local demand.

$$\frac{dx}{dt} = ax(1-x)(x-y) + r(p-x) \tag{4}$$

Eq 4 is being sketched and shown in figure 3.

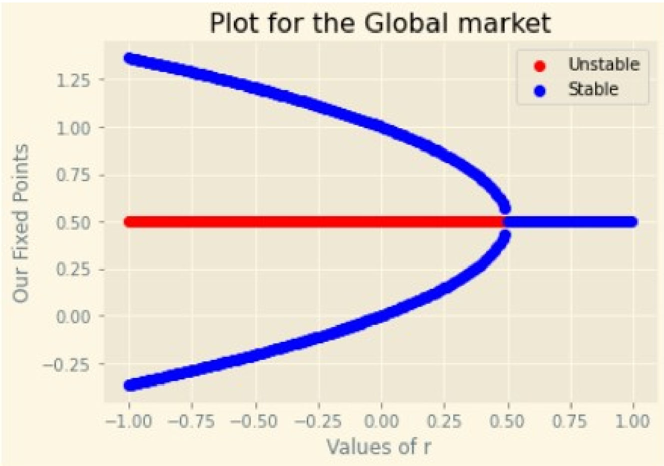

FIG. 2: The Global Market at a=1 and p=0.5

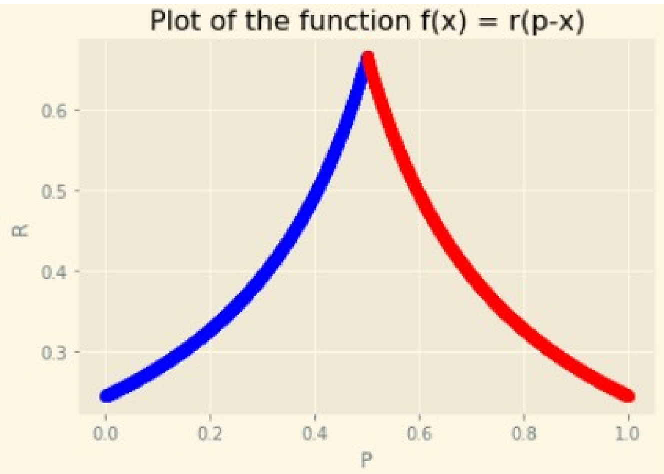

FIG. 3: Plot of r and p given a = 1

To get the critical condition for bifurcation, we equate Equate 3 to zero, assuming a = 1 and then get the two extremes. The extremas we get are $x1 = (3+\sqrt{3})/6$ and $x2 = (3-\sqrt{3})/6$.We sketched the figure to obtain r and p values such that bifurcation occurs.

Therefore, we get the relation of r and p as $r(p-x1) = f(x1)$ where $f(x) = x(1-x)(2x-1)$ and on plotting r vs p we observe that initially, the x is less than p and also the function value will be negative so the value of r will increase till it reaches the maximum value and then when the r value will be dropped as p will increase because the $f(x)$ will be positive and also the denominator term will decrease. We observe in the plot of r vs. p that A's market will increase as the influence of A's market from the global to the local market increases till the market share of A becomes 50 percent of the total global market. Then, the value of the strength of influence from the international to the local market will decrease as p increases.

III. RESULTS

For our given question about which market strategy to apply, we need to maximize the derivative (i.e., dx/dt) when x is our company's market share. So, if I were to be the director of the marketing company A and my market share was x, so to flip the market, according to our bifurcation, I would have to essentially have a higher value of 'a', which would eventually maximize our dx/dt and our market share (x) would increase.

IV. CONCLUSION

We were able to investigate bifurcations in one-dimensional nonlinear systems in this facility. These exercises improved our knowledge of bifurcation phenomena and their importance in complex systems.

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