

### (Unit: 3)

Grammar:  $G = (V_n, \Sigma, P, S)$

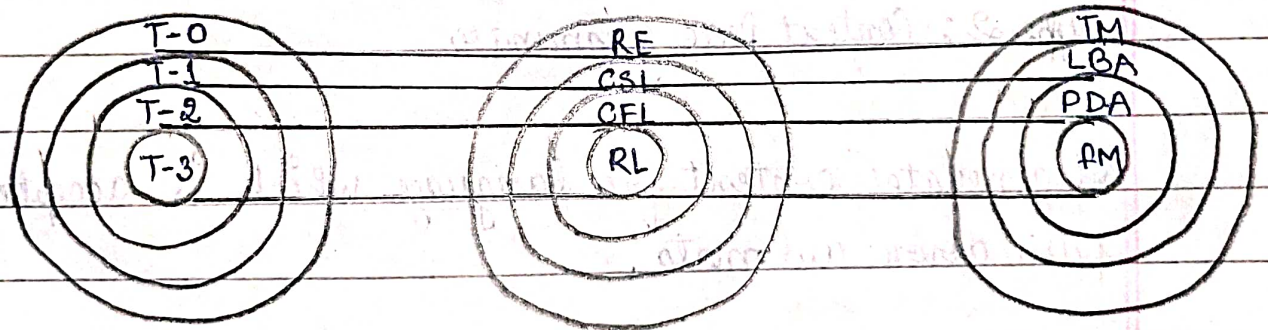
$V_n$  = Set of non-terminals.

$\Sigma$  = Set of alphabets (Terminals).

$P$  = Production rule.

$S$  = Starting Symbol.

Grammar: The set of all strings that can be derived from the grammar is said to be the language generated from the grammar.



### (GRAMMER)

FM: Finite Machine

LBA: Linear Bounded Automata

RE: Recurring Enumerable

TM: Turing Machine

Type 0: Recursive enumerable / unrestricted grammar /  
Phased Structure Grammar.

$$\alpha \rightarrow \beta; \alpha \in (\Sigma \cup V_n)^*, V_n (\Sigma \cup V_n)^*, \beta \in (\Sigma \cup V_n)^*$$

Type 1 'Grammar': Length increasing grammar (Non-  
contracting language). It generates context sensitive language  
which is accepted by linear automata.

$$\alpha \rightarrow \beta$$

$$\alpha \in (\Sigma \cup V_n) V_n (\Sigma \cup V_n)^*$$

$$\beta \in (\Sigma \cup V_n)^*$$

$$|\alpha| \leq |\beta|$$

$V_n$  = Set of non-terminal.

Type 2: Context free grammar.

It generates context free language which is accepted by  
push down automata.

$$\alpha \rightarrow \beta$$

$$\alpha \in (\Sigma \cup V_n) V_n (\Sigma \cup V_n)^*$$

$$\beta \in (\Sigma \cup V_n)^*$$

$$|\alpha| \leq |\beta|$$

$$\alpha \rightarrow \beta$$

$$\alpha \in V_n \quad |\alpha| = 1$$

$$\beta \in (\Sigma \cup V_n)^*$$



Type 3 Grammar: Regular Grammar.

It generates regular language which is accepted by finite automata.

Left linear grammar

$$A \rightarrow a/Ba.$$

$$A, B \in V_n$$

$$|A| = |B| = 1$$

Right linear grammar

$$A \rightarrow a|aB.$$

$$A, B \in V_n$$

$$|A| = |B| = 1.$$

Ques 1)  $G = \{S, \{0, 1\}, \{S \rightarrow OS1, S \rightarrow \Lambda\}, S\}$  Find  $L(G)$ .

Soln

$$S \rightarrow OS1$$

$$\rightarrow OOS11$$

$$\rightarrow OOS111$$

$$\rightarrow \dots$$

$$\rightarrow 0^n 1^n$$

$$L(G) = \{0^n 1^n \mid n \geq 0\}.$$

Ques 2)  $G = \{S, \{a\}, \{S \rightarrow SS\}, S\}$  Find language generated by the Grammar

$$S \rightarrow SS.$$

$$L(G) = \phi$$

Ques 3)  $G = \{(S, c), (a, b), P, S\}$   $S \rightarrow aca.$

$$c \rightarrow aca/b$$

Find  $L(G)$ .

Sol<sup>n</sup>

$$S \rightarrow aca$$

$$S \rightarrow aba \in L$$

$$S \rightarrow aca$$

$$S \rightarrow aacaa$$

$$S \rightarrow aaacaaa$$

$$\rightarrow a^n c a^n$$

$$L \rightarrow a^n b a^n$$

$$\text{So, } L(G) = \{a^n b a^n \mid n \geq 1\}$$



Find  $L(G)$ .

Sol

- $S \rightarrow aca$
- $S \rightarrow aba \in L$
- $S \rightarrow aca$
- $S \rightarrow aacaa$
- $S \rightarrow aaacaaa$
- $\rightarrow a^n c a^n$
- $\rightarrow a^n b a^n$

$$S_0, L(G) = \{a^n b a^n \mid n \geq 1\}$$

Derivation tree: A derivation tree or parse tree is a rooted, nested tree, that typically represent the semantic information of strings derived from a context free grammar.

Example: For the grammar

$$G = (V, T, P, S) \text{ where}$$

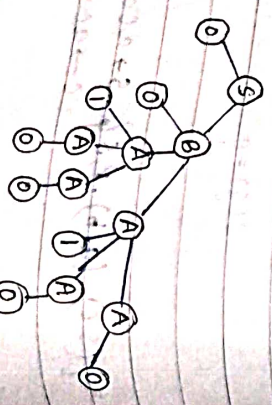
$$S \rightarrow aB, B \rightarrow bAA, A \rightarrow aAA, A \rightarrow a$$

$V$  = Set of variables

$T$  = Set of terminals

$P$  = Production Rule

$S$  = Start



Root vertex: Root vertex must be labelled by start symbol.

Vertex: Vertex is labelled by non-terminal symbol.

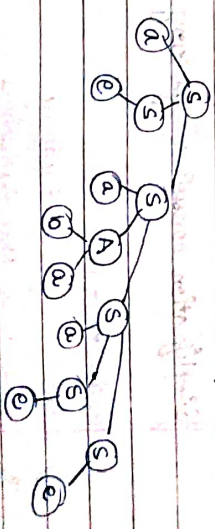
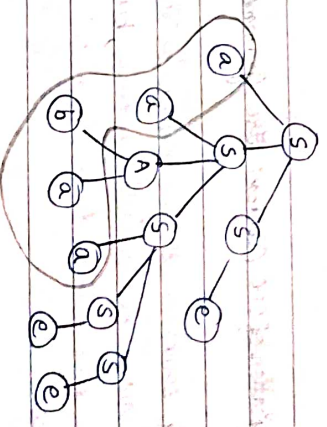
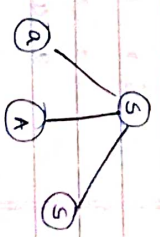
Leaves: leaves are labelled by terminal symbol etc.

Left Derivation tree: A left derivation tree is obtained by a prime production of the left most variable in each step.

For example: For generating the starting symbols (aabaab) from the grammar.

$$S \rightarrow aAS \mid aSS \mid \epsilon$$

$$A \rightarrow bAA \mid ba$$



**Ambiguous:** A grammar is said to be ambiguous if there exist two or more derivation tree for a string  $w$  that means two or more left derivation tree.

ques)  $G = \{S\}, \{a+b, +, \times\}, P, S\}$

$P$  consist of;

$$S \rightarrow S+S \mid S \times S \mid a \mid b$$

Sol<sup>n</sup>

$$w = a + a \times b$$

$$S \rightarrow S + S$$

$$S \rightarrow S \times S$$

$$S + S \times S$$

$$S \rightarrow S + S \times S$$

$$a + a \times b$$

$$a + a \times b$$

**Simplification of CFG (Context Free Grammar):**

In CFG, sometimes all the production rules and symbols are not needed for the derivation of strings, besides this, there may also be some null productions and unit productions and symbols.

Simplification consists of following steps:

- 1) Reduction of CFG
- 2) Removal of Unit Productions
- 3) Removal of Null Productions



- 1) Reduction of CFG (Phase 1): Derivation of an equivalent grammar  $G'$  from the CFG ( $G$ ) such that each symbol appears in sentential form.

Derivation Procedure: \* Include the start symbol  $\gamma_1$  and initialize  $i=1$

- \* Include all symbols  $\gamma_{i+1}^0$  that can be derived from  $\gamma_i^0$  and include all production rules that have been applied.
- \* Increment  $i$ , and repeat Step 2, until  $\gamma_{i+1}^0 = \gamma_i^0$
- example: Find the reduced grammar equivalent to the grammar  $G$  having production rules.

$P: S \rightarrow AC | B, A \rightarrow a, E \rightarrow c | Bc, E \rightarrow aA | e$

Phase 1:  $T = \{a, c, e\}$

$$w_1 = \{A, C, E\}$$

$$w_2 = \{A, C, E, S\}$$

$$w_3 = \{A, C, E, S\}$$

$$G' = \{A, C, E, S\}, \{a, c, e\}, \{S\}$$

$$P: A \rightarrow a, C \rightarrow c, E \rightarrow e, S \rightarrow AC, E \rightarrow aA$$

Phase 2:  $\gamma_1 = \{S\}$

$$\gamma_2 = \{S, A, C\}$$

$$\gamma_3 = \{S, A, C, a, e\}$$

$$\gamma_4 = \{S, A, C, a, e\}$$

$$q'' = \{(A, C, S), (a, C), P, \{S\}\}$$

$$S \rightarrow AC, A \rightarrow a, C \rightarrow e$$

Chomsky Normal Form: In Chomsky Normal Form, we have a restriction on the length of RHS {Right Hand Side} which is elements in RHS, should either be 2 variables or a terminal.

$$A \rightarrow a$$

$$A \rightarrow BC$$

Steps to convert in CNF: ★ If the start symbol 'S' occurs on right side, create a new start symbol 'S'', and a new product ( $S' \rightarrow S$ )

★ Remove Null productions.

★ Remove Unit production  $\{S' \rightarrow S\}$  [eg.  $A \rightarrow B$ ]

★ Replace each production (eg:  $A \rightarrow B_1, B_2, \dots, B_n$ ) where,  $n > 2$  with  $A \rightarrow B_1 C$ , where  $C \rightarrow B_2, \dots, B_n$



- ★ Repeat this step for all productions, having 2 or more symbols, on the right side. If the right side of all production is in the form of  $A \rightarrow aB$ , then production is replaced by  $A \rightarrow XB$  and  $X \rightarrow a$ .
- ★ Repeat this step, for every production which is of the form  $A \rightarrow aB$ .