28.11.2023

Ques- Solve $x^4 - 5x^3 + 20x^2 - 40x + 60 = 0$ by Newton Raphson method, given that all the roots of given equation are complex number.

Here, $f(x) = x^4 - 5x^3 + 20x^2 - 40x + 60$

$$f'(x) = 4x^3 - 15x^2 + 40x - 40$$

We know that,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{n+1} = \frac{3x_n^4 - 10x_n^3 + 20x_n^2 - 60}{4x_n^3 - 15x_n^2 + 40x_n - 40}$$

Let $x_0 = 2(1+i)$

$$\Rightarrow x_1 = \frac{3[2(1+i)]^4 - 10[2(1+i)]^3 + 20[2(1+i)] - 60}{4(2(1+i))^3 - 15(2(1+i))^2 + 40(2(1+i)) - 40}$$

$$x_1 = 1.92(1+i)$$

$$\Rightarrow x_2 = \frac{3[1.92(1+i)]^4 - 10[1.92(1+i)]^3 + 20[1.92(1+i)] - 60}{4[1.92(1+i)]^3 - 15[1.92(1+i)]^2 + 40[1.92(1+i)] - 40}$$

$\alpha + i\beta = x_2 = 1.915 + 1.908\,i$ (1st Root)

As roots are occuring in parts.

$\alpha - i\beta = 1.915 - 1.908\,i$ (2nd Root)

Let $\alpha \pm i\beta$ are the 2nd pair of parts of equation.

Since, We know for a quadratic polynomial

$$ax^2 + bx + c = 0.$$

$$\Rightarrow \alpha + \beta = \frac{-b}{a}$$

and $\alpha\beta = \frac{c}{a}$

$$\Rightarrow \alpha_1 + i\beta_1 + \alpha_1 + i\beta_1 + \alpha_2 + i\beta_2 + \alpha_2 - i\beta_2 = 05$$

$$\Rightarrow 2\alpha_1 + 2\alpha_2 = 5$$

$$2[1.915] + 2\alpha_2 = 5$$

$$2\alpha_2 = 1.17.$$

$$\Rightarrow \alpha_2 = 0.585$$

and

$$(\alpha_1 + i\beta_1)(\alpha_1 - i\beta_1) \cdot (\alpha_2 + i\beta_2)(\alpha_2 - i\beta_2) = 60$$

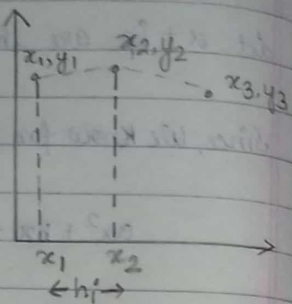
$$\Rightarrow (\alpha_1^2 + \beta_1^2)(\alpha_2^2 + \beta_2^2) = 60$$

$$\beta_2 = 2.805$$

Hence, four roots are :- $(1.915 \pm 1.908\,i)$ and $(0.585 \pm 2.805i)$

Muller's Method :-

If $y = f(x) = 0$; first 3 points
x_1, x_2, x_3
and y_1, y_2, y_3



★ It is an iterative method that requires 3 starting points here $y = f(x)$ is approximated by a second degree parabola passing through 3 points
(x_{i-2}, y_{i-2})
(x_{i-1}, y_{i-1})
(x_i, y_i)

Let these 3 distinct points on the curve $y = f(x)$ are approximation to a root of $f(x) = 0$.

NOTE: A Second degree curve passes through these three points by Langrange's Interpolation formula is

$$y = A(x - x_i)^2 + B(x - x_i) + y_i$$

$$y = \frac{(x - x_{i-1})(x - x_i)}{(x_{i-2} - x_{i-1})(x_{i-2} - x_i)} \times y_{i-2} + \frac{(x - x_{i-2})(x - x_i)}{(x_{i-1} - x_{i-2})(x_{i-1} - x_i)} \times y_{i-1} + \frac{(x - x_{i-2})(x - x_{i-1})}{(x_i - x_{i-2})(x_i - x_{i-1})} \times y_i$$

★ h_i = Stepsize = $x_i - x_{i-1}$

★ ∇_i = Backward difference operator.

$\nabla_i = y_{i-1} - y_i$

★ Δ_i = Forward difference operator.

$\Delta_i = y_i - y_{i-1}$

★ $$A = \frac{1}{h_{i-1} + h_i}\left[\frac{\Delta_i}{h_i} - \frac{\Delta_{i-1}}{h_{i-1}}\right]$$

$$B = \frac{\Delta_i}{h_i} + A h_i$$

for better root.

★ $$x_{i+1} = x_i + \left[\frac{-B \pm \sqrt{B^2 - 4AB}}{2A}\right]$$

for second iteration