Solution of Algebraic and Transcendental Equation:

Method 1:

Bisection Method.

Ques 1) Find the root of a Equation $x^3-4x-9=0$ using Bisection Method and correct it to three decimal places.

Solution: Here, $f(x) = x^3-4x-9=0$

For $x=0$; $f(0)=-9$
$x=1$; $f(1)=-12$
$x=2$; $f(2)=-9$ (-ve)
$x=3$; $f(3)=6$ (+ve)

Now, $x_0 = 2$ and $x_1 = 3$

Root must lie in the interval [2,3]

$\Rightarrow x_2 = \dfrac{x_0+x_1}{2} = 2.5$

$f(2.5) = (2.5)^3 - 4(2.5) - 9 = -3.375$

Replacing 2 by 2.5 and root will lie in the interval [2.5, 3]

$$x_3 = \frac{2.5+3}{2} = 2.75$$

$$f(2.75) = (2.75)^3 - 4(2.75) - 9$$

$$= 0.7968$$

Now replacing 3 by 2.75 and root will lie in the interval [2.5, 2.75]

$$x_4 = \frac{2.5+2.75}{2} = 2.625$$

$$f(2.625) = (2.625)^3 - 4(2.625) - 9$$

$$= -1.4121$$

Now replacing 2.5 by 2.625 and root will lie in the interval [2.625, 2.75]

$$x_5 = \frac{2.625+2.75}{2} = 2.6875$$

$$f(2.6875) = (2.6875)^3 - 4(2.6875) - 9$$

$$= -0.3391$$

Now replacing 2.625 by 2.6875 and root will lie in the interval [2.6875, 2.75]

$$x_6 = \frac{2.6875+2.75}{2} = 2.71875$$

$$f(2.71875) = (2.71875)^3 - 4(2.71875) - 9$$

$$= 0.2209$$

Now replacing 2.75 by 2.71875 and root will lie in the interval [2.6875, 2.71875]

$$x_7 = \frac{2.6875+2.71875}{2} = 2.703125$$

$$f(2.703125) = (2.703125)^3 - 4(2.703125) - 9$$

$$= -0.0610$$

Now replacing 2.6875 by 2.703125 and root will lie in the interval [2.703125, 2.71875]

$$x_8 = \frac{2.703125+2.71875}{2} = 2.7109375$$

$$f(2.7109375) = (2.7109375)^3 - 4(2.7109375) - 9$$

$$= 0.0794$$

Now replacing 2.71875 by 2.7109375 and root will lie in the interval [2.703125, 2.7109375]

$$x_9 = \frac{2.703125+2.7109375}{2} = 2.70703125$$

$$f(2.70703125) = (2.70703125)^3 - 4(2.70703125) - 9$$

$$= 9.049 \times 10^{-3}$$

Now replacing 2.7109375 by 2.70703125 and root will lie in the interval [2.703125, 2.70703125]

$$x_{10} = \frac{2.703125 + 2.70703125}{2}$$

$$= 2.7050708125$$

$$f(2.7050708125) = \frac{(2.7050708125)^3 - 4(2.70507812)}{-9}$$

$$= -0.0260$$

Now replacing 2.703125 by 2.7050708125 and root will lie in the interval [2.7050708125, 2.70703125]

$$x_{11} = \frac{2.7050708125 + 2.70703125}{2}$$

$$= 2.706054688$$

$$f(2.706054688) = \frac{(2.706054688)^3 - 4(2.70605468)}{-9}$$

$$= -8.5055 \times 10^{-3}$$

Now replacing 2.7050708125 by 2.706054688 and root will lie in the interval [2.706054688, 2.70703125].

$$x_{12} = \frac{2.706054688 + 2.70703125}{2}$$

$$x_{12} = 2.706542969$$

Hence, the root which is correct upto three decimal places for the given algebraic equation is $x_{12} = 2.70654 \approx 2.706$

Ques 2) Find the root of the equation, $\cos x = xe^x$ by bisection method and correct it upto four decimal places.

solution is wrong

Sol^n $f(x) = \cos x - xe^x = 0$

$f(0) = \cos 0 - 0e^0 = 1$ (+ve Value)

For $x = 1$ = $\cos 1 - 1e^1 = -1.7184$ (-ve Value)

Root must lie in the interval [0,1]

where $x_0 = 0$ and $x_1 = 1$

$\Rightarrow x_2 = \frac{0+1}{2} = 0.5$

$f(0.5) = \cos(0.5) - (0.5)e^{0.5} = 0.1756$

Now replacing 0 by 0.5 and root will lie in the interval [0.5, 1]

$\Rightarrow x_3 = \frac{0.5+1}{2} = 0.75$

$f(0.75) = \cos(0.75) - (0.75)e^{0.75} = -0.58783$

Now replacing 1 by 0.75 and root will lie in the interval $[0.5, 0.75]$

$\Rightarrow x_4 = \frac{0.5+0.75}{2} = 0.625$

$f(0.625) = \cos(0.625) - (0.625)e^{0.625}$

$= -0.16771$

Now replacing 0.75 by 0.625 and root will lie in the interval $[0.5, 0.625]$

$\Rightarrow x_5 = \frac{0.5+0.625}{2} = 0.5625$

$f(0.5625) = \cos(0.5625) - (0.5625)e^{0.5625}$

$= 0.01273$

Now replacing 0.5 by 0.5625 and root will lie in the interval $[0.5625, 0.625]$

$\Rightarrow x_6 = \frac{0.5625+0.625}{2} = 0.59375$

$f(0.59375) = \cos(0.59375) - (0.59375)e^{0.59375}$

$= -0.07519$

Now replacing 0.625 by 0.59375 and root will lie in the interval $[0.5625, 0.59375]$

$\Rightarrow x_7 = \frac{0.5625+0.59375}{2} = 0.578125$

$f(0.578125) = \cos(0.578125) - (0.578125)e^{0.578125}$

$= -0.03067$

Now replacing 0.59375 by 0.578125 and root will lie in the interval $[0.5625, 0.578125]$

$\Rightarrow x_8 = \frac{0.5625+0.578125}{2} = 0.5703125$

$f(0.5703125) = \cos(0.5703125) - (0.5703125)e^{0.5703125}$

$= -8.8295 \times 10^{-3}$

Now replacing 0.578125 by 0.5703125 and root will lie in the interval $[0.5625, 0.5703125]$

$f(0.5703125) = -ve.$ solution is wrong

$\Rightarrow x_9 = \frac{0.5625+0.5703125}{2} = 0.56640625$

At $x_7 = 0.5156$ must be the answer.

upto 4 decimal places which is correct.

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Ques 3) Find the real roots of the equation $x^3 - 2x - 5 = 0$ by the method of false position, and correct it upto three decimal places.

According to Regula-False Position Method.

$(a, f(a))$ and $(b, f(b))$ are 2 points

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

When $y = 0$,

$$\Rightarrow x_2 = \frac{(x_0 - x_1) f(x_0)}{f(x_1) - f(x_0)} + x_0$$

So, $$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}.$$

where, $a = x_0$ (-ve Value),

and $b = x_1$ (+ve Value)

According to question,

$f(x_0) = x_0^3 - 2x_0 - 5$

ie $f(x) = x^3 - 2x - 5$

$\Rightarrow f(0) = -5$ (-ve)

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$\Rightarrow f(1) = (1)^3 - 2(1) - 5 = -6$ (-ve)

$\Rightarrow f(2) = (2)^3 - 2(2) - 5 = -1$ (-ve)

$\Rightarrow f(3) = (3)^3 - 2(3) - 5 = 16$ (+ve)

$\therefore$, $x_0 = 2$ and $x_1 = 3$

now, $$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{2 \times 16 - 3 \times (-1)}{16 - (-1)} = \frac{32 + 3}{17}$$

So, $x_2 = 2.0588$

Now, $f(x_2) = ?$

$f(2.0588) = (2.0588)^3 - 2(2.0588) - 5$
$= -0.391$ (-ve)

$\therefore$, Now, $x_0 = 2.0588$ and $x_1 = 3$

now, $$x_3 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{2.0588 \times 16 - 3 \times (-0.391)}{16 - (-0.391)}$$

$$= \frac{34.1138}{16.391} = 2.0812$$

So, $f(x_3) = ?$

$f(20812) = (2.0812)^3 - 2(2.0812) - 5$
$= -0.147 \quad (-ve)$

$\therefore$, Now, $x_0 = 2.0812$ and $x_1 = 3$

now, $x_4 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{(20812) \times 16 - 3 \times (-0.147)}{16 - (-0.147)}$

$= \dfrac{33.7402}{16.147} = 2.0895$

So, $f(x_4) = ?$

$f(2.0895) = (2.0895)^3 - 2(2.0895) - 5$
$= -0.056 \quad (-ve)$

$\therefore$, Now, $x_0 = 2.0895$ and $x_1 = 3$.

now, $x_5 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{(2.0895) \times 16 - 3 \times (-0.056)}{16 - (-0.056)}$

$= \dfrac{33.6}{16.056} = 2.0926$

So, $f(x_5) = ?$

$\Rightarrow f(2.0926) = (2.0926)^3 - 2(2.0926) - 5$
$= -0.021 \quad (-ve)$

$\therefore$, Now, $x_0 = 2.0926$ and $x_1 = 3$

$\Rightarrow x_6 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{(2.0926) \times 16 - 3 \times (-0.021)}{16 - (-0.021)}$

$= \dfrac{33.5446}{16.021} = 2.0937$

$\Rightarrow f(2.0937) = (2.0937)^3 - 2(2.0937) - 5$
$= -9.499 \times 10^{-3} \quad (-ve)$.

$\therefore$, Now, $x_0 = 2.0937$ and $x_1 = 3$.

$\Rightarrow x_7 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$= \dfrac{(2.0937) \times 16 - 3 \times (-9.499 \times 10^{-3})}{16 - (-9.499 \times 10^{-3})}$

$= \dfrac{33.5276}{16.009} = 2.09429$.

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$\Rightarrow f(x_7) = ?$

$\Rightarrow f(2.09429) = (2.09429)^3 - 2(2.09429) - 5$

$= -2.92 \times 10^{-3}$ (-ve)

Now, $x_0 = 2.09429$ and $x_1 = 3$.

$$\Rightarrow x_8 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{(2.09429) \times 16 - 3 \times (-2.92 \times 10^{-3})}{16 - (-2.92 \times 10^{-3})}$$

$$= \frac{33.5174}{16.00292} = 2.0944.$$

Hence, $x_8 = 2.0944$ must be the correct answer which is correct upto 3 decimal places.

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Newton Raphson Rule:

$\Rightarrow f(x) = 0, x = ??$

$x_0 + h \Rightarrow f(x_0 + h) = 0.$

So, $f(x_0) + hf'(x_0) + \frac{h^2}{2!} f''(x_0) + \text{------}$

$\Rightarrow f(x_0) + hf'(x_0) = 0$

Now, $h = -\frac{f(x_0)}{f'(x_0)}$

Now, $x = x_0 + h.$

$\Rightarrow x = x_0 - \frac{f(x_0)}{f'(x_0)}$

So, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Ques - Find the +ve root of $x^4 - x = 10$, correct it to 3 decimal places using Newton Raphson Method.

$f(x) = x^4 - x - 10 = 0. \quad \Rightarrow f'(x) = 4x^3 - 1$

$f(1) = 1^4 - 1 - 10 = -10$ (-ve)

$f(2) = 2^4 - 2 - 10 = 4$ (+ve)

Hence, $x_0 = 2$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$$x_1 = 2 - \frac{f(2)}{f'(2)}.$$

$$x_1 = 2 - \frac{4}{31} = 1.87096$$

Now, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$= 1.87096 - \frac{0.38247}{25.19711}$$

$$x_2 = 1.85578$$

Now, $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$$= 1.85578 - \frac{4.8008 \times 10^{-3}}{24.56462}$$

$$x_3 = 1.85558$$

and this is correct upto 3 decimal places.

Ques- Evaluate the following. Correct it upto 4 decimal places by Newton Raphson Method.

i) $\frac{1}{31}$

Let $x = \frac{1}{31}$

$$\Rightarrow x - \frac{1}{31} = 0.$$

So, $f(x) = x - \frac{1}{31}$ and $f'(x) = 1$.

at $x = 1 \Rightarrow f(1) = 1 - \frac{1}{31} = 0.96774$ (+ve)

Hence, $x_0 = 1$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$$x_1 = 1 - \frac{f(1)}{f'(1)} = \frac{1 - 0.96774}{1} = 0.03226$$

Now, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$= 0.03226 - 1.93548 \times 10^{-6}$$

$$x_2 = 0.032258$$

which is correct upto 4 decimal places.

ii) $\sqrt{5}$

Let $x = \sqrt{5}$.

$\Rightarrow x^2 = 5$.

$\Rightarrow x^2 - 5 = 0$.

So, $f(x) = x^2 - 5$ and $f'(x) = 2x$.

$f(2) = 2^2 - 5 = -1$ (-ve).

$f(3) = 3^2 - 5 = 4$ (+ve).

So, $x_0 = 3$.

$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 - \frac{4}{6} = 2.33333$

Now, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.33333 - \frac{0.44442}{4.66666}$

$= 2.23809$

$\Rightarrow x_2 = 2.23809$

$\Rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.23809 - \frac{9.04684 \times 10^{-3}}{4.47618}$

$x_3 = 2.236068$

Now, $x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$.

$= 2.23606 - \frac{(-3.56764 \times 10^{-5})}{4.47212}$

$= 2.236067$

which is correct upto four decimal places.

iii) $\frac{1}{\sqrt{14}}$

Let $x = \frac{1}{\sqrt{14}}$ So, $f(x) = 14x^2 - 1$.

$\Rightarrow x^2 = \frac{1}{(\sqrt{14})^2}$ and $f'(x) = 28x$.

$\Rightarrow x^2 - \frac{1}{14} = 0$.

$\Rightarrow 14x^2 - 1 = 0$

At $x=1$.

$f(1) = 14-1 = 13$ (+ve).

$f(0.5) = 14(0.5)^2 - 1 = 2.5$ (+ve)

$f(0.3) = 14.(0.3)^2 - 1 = 0.26$ (+ve).

So, $x_0 = 0.3$

$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 0.3 - \frac{0.26}{8.4}$

$x_1 = 0.26904$

Now, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.26904 - \frac{0.01335}{7.53312}$

$x_2 = 0.2672678$

Now, $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 0.26726 - \frac{(-9.2936 \times 10^{-6})}{7.48328}$

$x_3 = 0.2672612$

which is correct upto 4 decimal places.

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iv) $\sqrt[3]{24}$

Let $x = \sqrt[3]{24}$.

$\Rightarrow x - \sqrt[3]{24} = 0$.

$\Rightarrow f(x) = x - \sqrt[3]{24}$.

and $f'(x) = 1$.

At $x = 2$. $\Rightarrow f(2) = 2 - \sqrt[3]{24} = -0.88449$ (-ve)

At $x = 3$ $\Rightarrow f(3) = 3 - \sqrt[3]{24} = 0.11550$ (+ve)

So, $x_0 = 3$.

$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 3 - \frac{0.11550}{1}$

$x_1 = 2.8845$

Now, $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.8845 - \frac{8.59385 \times 10^{-7}}{1}$

$x_2 = 2.884499141$

Now, $x_3 = x_2 - \frac{f(x_1)}{f'(x_1)}$

$= 2.88449 - (-9.14061 \times 10^{-6})$

$= 2.884499141$

which is correct upto 4 decimal places.

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v) $(30)^{-1/5}$

Let $x = (30)^{-1/5}$

$\Rightarrow x - (30)^{-1/5} = 0.$

$\Rightarrow f(x) = x - (30)^{-1/5}.$

$\Rightarrow f'(x) = 1.$

At $x = 0$, $f(0) = -6.66667 \times 10^{-3}$

At $x = 1$, $f(1) = 0.99333$

At $x = 0.03 \Rightarrow f(0.03) = 0.023333$ (+ve)

$x_0 = 0.03.$

$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 0.03 - \frac{0.023333}{1}$

$x_1 = 6.667 \times 10^{-3} = 0.006667$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}.$

$= 0.006667 - \frac{3.33 \times 10^{-7}}{1}$

$x_2 = 0.006666667$

which is correct upto 4 decimal places.