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Date

→ Soft Computing

Fuzzy logic : developed by Lotfi A Zadeh (1965)
Not clear

- It is mathematical language (Advance version)
- It is an approach of computing based on degree of truth rather than usual true or false boolean logic on which modern computing is based.
- It is an attempt to use more human like real things, by applying degree of truth.
- It seems closer to way our ~~mow~~ brain works.
- The term fuzzy refers to the thing which are not clear.
- It deals with Fuzzy set & Fuzzy algebra.

→ Operation on Fuzzy set.

1) Union (AUB) = Max(A, B).

Fuzzy set
small = {(1,1), (2,1), (3,0.9), (5,0.4), (6,0.3)}.
Medium = [(1,0), (2,0), (3,0), (5,0.5), (6,0.8)).

Fuzzy Union = [Small U Medium].
= [(1,1), (2,1), (3,0.9), (5,0.5), (6,0.8)}

2) Fuzzy Intersection : Min [A, B].

(small ∩ medium) = [(1,0), (2,0), (3,0), (5,0.4), (6,0.3)]

3) Difference - Fuzzy difference - $(A-B)x$

$= \min(A(x), \{1-B(x)\})$

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→ Properties Related to Fuzzy Union, Intersection & Difference

- Identity - $A \cup \phi = A$, $A \cap \phi = \phi$.
- Idempotance - $A \cup A = A$, $A \cap A = A$.
- Commutative - $A \cup B = B \cup A$, $A \cap B = B \cap A$.
- Associativity - $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$.

Ques What is soft Computing & Hard Computing.
Ques What is hybrid Computing.
Ques - Difference b/w fuzzy set & Crisp set.
Ques - What is membership Function in fuzzy logic.
Ques - Define Fuzzy logic.

- Distributivity - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

- Transitivity - If $(A \underset{\substack{\Downarrow \\ \text{subset}}}{\subseteq} B) \cap (B \subset C)$ then $A \subset C$

- Involution - $\overline{\overline{A}} = A$.

- De-Morgan's Law - $\overline{A \cap B} = \overline{A} \cup \overline{B}$.
  $\overline{A \cup B} = \overline{A} \cap \overline{B}$

→ Operation on Fuzzy set.

$X = \{a, b, c, d, e, f\}$

let $A = \{(a, 0.2), (b, 0.4), (c, 0.7), (d, 0.9)\}$.

$B = \{(a, 0.4), (b, 0.1), (c, 0.9), (d, 0.2)\}$.

→ Union (OR).

$A \cup B = \text{Max}\{\mu_A(x), \mu_B(x)\}, \quad x \in U$.

$A \cup B = \{(a, 0.4), (b, 0.4), (c, 0.9), (d, 0.9)\}$.

→ Intersection (AND).

$A \cap B = \text{Min}\{\mu_A(x), \mu_B(x)\}, \quad x \in U$

$A \cap B = \{(a, 0.2), (b, 0.1), (c, 0.7), (d, 0.2)\}$.

→ Compliment (Not).

$\overline{A} = \overline{\mu_A(x)} = 1 - \mu_A(x)$ where $x \in U$.

$\overline{A} = \{(a, 0.8), (b, 0.6), (c, 0.3), (d, 0.1)\}$

→ Difference (A − B).

$A - B = \text{Min}\{\mu_A(x), 1 - \mu_B(x)\}$.

$A - B = \{(a, 0.2), (b, 0.4), (c, 0.1), (d, 0.8)\}$.

→ Algebric sum

$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$.

$A = \{(a, 0.2), (b, 0.3), (c, 0.4), (d, 0.5)\}$.

A ~~$\mu_{A+B}(x) =$~~

$B = (\{a, 0.1), (b, 0.2), (c, 0.2), (d, 1)\}$.

$\mu_{A+B}(x) = \{(a, 0.28), (b, 0.44), (c, 0.52), (d, 1.0)\}$

→ Algebric Product

$$\boxed{\mu_{A\cdot B}(x) = \mu_A(x) \cdot \mu_B(x)}$$

$A = \{0.1/2 + 0.3/4 + 0.2/5 + 0.6/8\}.$
$B = \{0.2/2 + 0.4/4 + 0.1/5 + 0.3/8\}.$

→ $A = \{(2, 0.1), (4, 0.3), (5, 0.2), (8, 0.6)\}$
→ $B = \{(2, 0.2), (4, 0.4), (5, 0.1), (8, 0.3)\}.$

$\mu_{A\cdot B}(x) = \{(2, 0.02), (4, 0.12), (5, 0.02), (8, 0$

$\cancel{\mu_{A\cdot B}(x) = \{0.02/0.02 + 4/0.12}$
$\mu_{A\cdot B}(x) = \{0.02/2 + 0.12/4 + 0.02/5 + 0.18/8\}.$

→ Bounded Sum.

$\mu_{BS}(x) = \min\{1, \mu_A(x) + \mu_B(x)\}.$

$A = \{(Amit, 0.4), (Rahul, 0.3), (Ram, 0.5), (Ajay, 0.8$
$B = \{(Amit, 0.2), (Rahul, 0.4), (Ram, 0.7), (Ajay, 0.2$

$\mu_{BS}(x) = \{(Amit, 0.6), (Rahul, 0.7), (Ram, 1), (Ajay, 1$

→ Bounded Difference.

$\mu_{BD} = \max\{0, \mu_A(x) - \mu_B(x)\}.$

$\mu_{BD}(x) = \{(Amit, 0.2), (Rahul, 0.0), (Ram, 0.0), (Ajay,$

→ Bounded Product.

$\mu_{BP} = \max\{0, \mu_A(x) + \mu_B(x) - 1\}.$

→ Drastic Product = $\begin{cases} \mu_A(x), & \text{if } \mu_B(x) = 1 \\ \mu_B(x), & \text{if } \mu_A(x) = 1 \\ 0, & \text{if } \mu_A(x), \mu_B(x) < 1 \end{cases}$

→ Fuzzy relations & operation.

→ Fuzzy relation it relate element of one univase X to those of another universe Y through the cartesian product of two universe.

$A \in X$, $B \in Y$ $\Rightarrow$ $\boxed{R = A \times B \subset X \times Y}$

$A = \{(a, 0.2), (b, 0.7), (c, 0.4)\}$

$B = \{(a, 0.5), (b, 0.6)\}$

$\boxed{R = A \times B.}$

$\mu_R(x, y) = \mu_R(A, B).$

$= \min(\mu_A(x), \mu_B(Y))$

$$= \begin{array}{c|cc} & a & b \\ \hline a & 0.2 & 0.2 \\ b & 0.5 & 0.6 \\ c & 0.4 & 0.4 \end{array}$$

This matrix is also Called fuzzy Matrix. and represent fuzzy relation.

→ ~~Fuzzy~~ Operation of fuzzy relation.

- $\boxed{\text{Union (OR)} - \mu_{R \cup S}(X, Y) = \max\{\mu_R(X, Y), \mu_S(X, Y)\}}$

$$\mu_R(X, Y) = \begin{array}{c|cc} & a & b \\ \hline a & 0.2 & 0.2 \\ b & 0.5 & 0.6 \\ c & 0.4 & 0.4 \end{array} \qquad \begin{array}{c|cc} & a & b \\ \hline a & 0.3 & 0.5 \\ b & 0.1 & 0.4 \\ c & 0.3 & 0.6 \end{array}$$

$$\mu_{R \cup S}(X, Y) = \begin{array}{c|cc} & a & b \\ \hline a & 0.3 & 0.5 \\ b & 0.5 & 0.6 \\ c & 0.4 & 0.6 \end{array}$$

$\mu_{R \cap S}(X, Y) = \min\{\mu_R(x, y), \mu_S(x, y)\}$

$$\mu_{R\cap S}(X,Y) = \begin{array}{c|cc} & a & b \\ \hline a & 0.2 & 0.2 \\ b & 0.1 & 0.4 \\ c & 0.3 & 0.4 \end{array}$$

→ Compliment

$$\boxed{\overline{\mu_R(X, Y)} = 1 - \mu_R(X, Y)}$$

$$\overline{\mu_R(X,Y)} = \begin{array}{c|cc} & a & b \\ \hline a & 0.8 & 0.8 \\ b & 0.5 & 0.4 \\ c & 0.6 & 0.6 \end{array}$$

→ Composition. – The operation executed on two compatable fuzzy relation to get a single fuzzy relation

$$\boxed{\begin{array}{l} R = X * y \\ S = y * z \end{array}}$$

1. Fuzzy max-min composition
2. Fuzzy max-product composition.

→ Fuzzy max-min composition

refer to phone once

$\mu_{R\circ S} = \max[\min[\mu_R(x_1, y_1), \mu_S(y_1, z_1)], \min[\mu_R(x_1, y_2), \mu_S(y_2, z_1)]]$

$\mu_{R\circ S} = \max(\min(\mu_R(X, Y), \mu_S(Y, Z)))$

$$R_{X,Y} = \begin{array}{c|cc} & Y_1 & Y_2 \\ \hline X_1 & 0.6 & 0.3 \\ X_2 & 0.2 & 0.9 \end{array}$$

$$S_{Y,Z} = \begin{array}{c|ccc} & Z_1 & Z_2 & Z_3 \\ \hline Y_1 & 1 & 0.5 & 0.3 \\ Y_2 & 0.8 & 0.4 & 0.7 \end{array}$$

$$\mu_{R \circ S} = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

→ Fuzzy max-product Composition.

$$\mu_{R \cdot S} = \max(\mu_R(x,y) \cdot \mu_S(y,z)).$$

$$(x_1, y_1) = \max[(x_1,y_1)\cdot(y_1,z_1), (x_1,y_2)\cdot(y_2,z_1)]$$

$$= \max[0.6, 0.3 \times 0.8)$$
$$= \max[0.6, 0.24)$$
$$= 0.6$$

$$\cdot T. = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.30 & 0.21 \\ 0.72 & 0.36 & 0.63 \end{bmatrix} \end{array}$$

→ Fuzzy Implication. OR Fuzzy If-then Rule OR Fuzzy Rule OR Fuzzy Conditional statement.

(1) If X is A then Y is B.
(2) If X is A then Y is B else Y is C.

ex:- If Mango is Yellow then Mango is sweet
If Mango is Yellow then mango is sweet else sour.

If X is A then Y is B.

Its implication relation R can be written as

$$\boxed{R = (A \times B) \cup (\bar{A} \times Y)}$$

(2) If X is A then Y is B else Y is C.

$$\boxed{R = (A \times B) \cup (\bar{A} \times C)}$$

Ques - $X = \{a, b, c, d\}$, $Y = \{1, 2, 3, 4\}$

$A' = \{(a, 0), (b, 0.8), (c, 0.6), (d, 1)\}$.

$B' = \{(1, 0.2), (2, 1), (3, 0.8), (4, 0)\}$.

$C' = \{(1, 0.8), (2, 0.4), (3, 1), (4, 0.8)\}$.

Determine the implication relation.

a) If X is A' then Y is B' $\Rightarrow (A' \times B') \cup (\bar{A'} \times Y)$

b) If X is A' then Y is B' else C'.

$\rightarrow$

$$A' \times B' = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

$\bar{A'} = 1 - \mu_{A'}(x)$

$= \{(a, 1), (b, 0.2), (c, 0.4), (d, 0)$.

$$\bar{A'} \times Y = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$(A' \times B') \cup (\bar{A'} \times Y) = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ a \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.4 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

b) $R = (A' \times B') \cup (\bar{A'} \times C')$.

$$\bar{A'} \times C' = \begin{array}{c} \\ a \\ b \\ c \\ a \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0.8 & 0.8 \\ 0 & 0.4 & 0.6 & 0.8 \\ 0 & 0.4 & 1 & 0.8 \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0.8 \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$$\bar{A'} \times C' = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 1 & 0.8 \\ 0 & 0.2 & 0.2 & 0.2 \\ 0 & 0.4 & 0.7 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

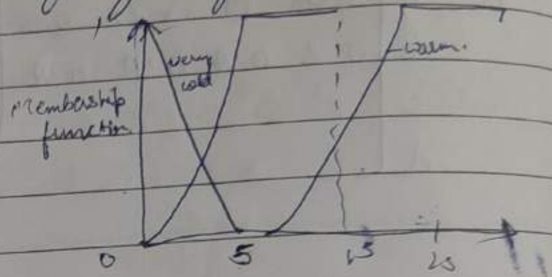
→ Algebric Product ⇒ Same as fuzzy set.
Bounded Product, Drastic Product.

Ques $A = \{(x_1, 0.2), (x_2, 0.3), (x_3, 1), (x_4, 0.5), (x_5, 0.1)\}$
$B = \{(x_1, 0), (x_2, 0.3), (x_3, 0.6), (x_4, 0.7), (x_5, 0.9)\}$

i) $\mu_{AP} = \{(x_1, 0), (x_2, 0.09), (x_3, 0.6), (x_4, 0.35), (x_5, 0.0$
ii) $\mu_{BP} = \{(x_1, 0), (x_2, 0), (x_3, 0.6), (x_4, 0.2), (x_5, 0)$
iii) $\mu_{DP} = \{(x_1, 0), (x_2, 0), (x_3, 0.6), (x_4, 0), (x_5, 0)\}$

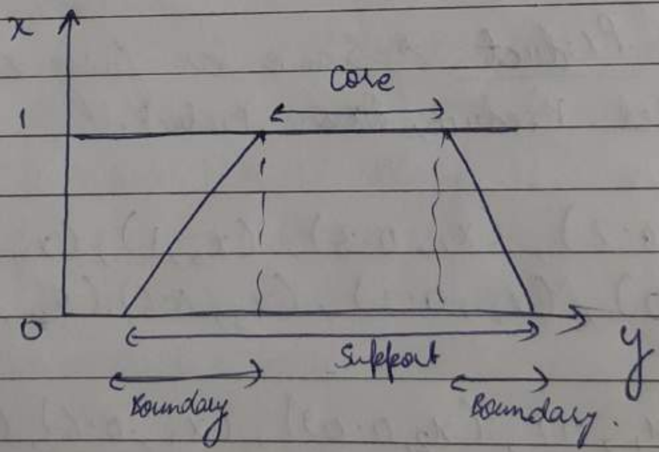
→ **Membership function in fuzzy logic** – It represents the degree of membership and its values lie between 0 ~~to~~ and 1, including 0 and 1. It can be defined as a technique to solve practical problem by experience rather than Knowledge. Membership function are used in fuzzification and de-fuzzification of a fuzzy system.

if $\mu_A(x) = 1$
$\mu_B(x) = 0$
$\mu_A(x) < 1$



→ **Features of Membership**

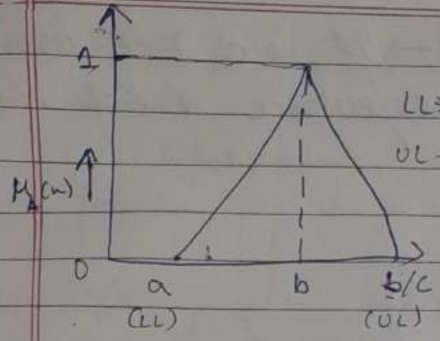
① Core $\mu_x(A) = 1$ ② Support $\mu_A(x) > 0$ ③ Boundary $0 < \mu_A(x) < 1$



→ Fuzzy logic system.

Type of Membership function.

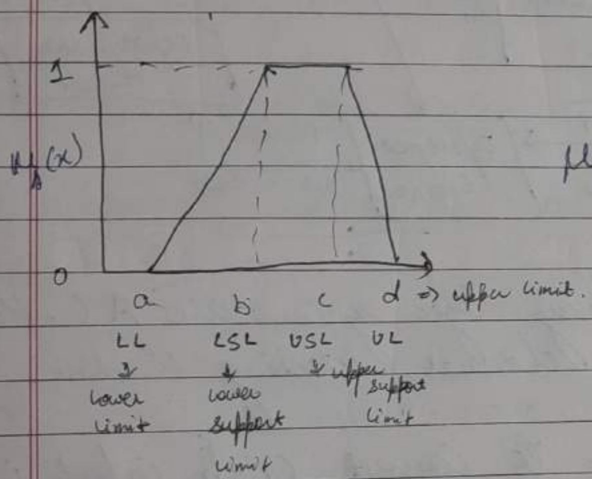
- Triangular Membership function.



LL → Lower limit
UL → upper limit

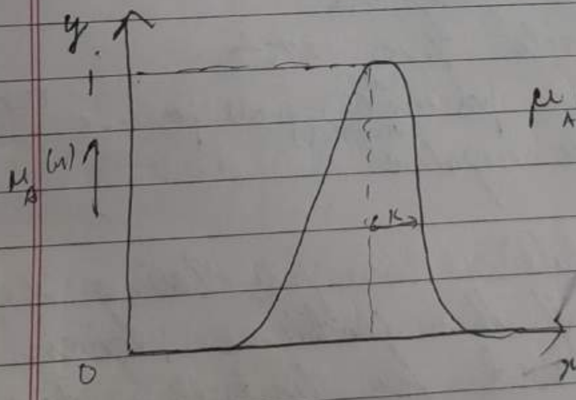
$$\mu_A(x) \begin{cases} 0 & \text{if } x<a \\ \frac{x-a}{x-b} & \text{if } a\le x\le b \\ \frac{c-x}{c-b} & \text{if } b\le x\le c \\ 0 & \text{if } x>c \end{cases}$$

- Trapezoidal Membership function.



$$\mu_{Ax} \begin{cases} 0 & , \text{If } (x<a) \text{ or } (x>d) \\ \frac{x-a}{b-a} & , \text{if } b\ge x\ge a. \\ 1 & , c\ge x\ge b \\ \frac{d-x}{d-c} & c\le x\le d. \end{cases}$$

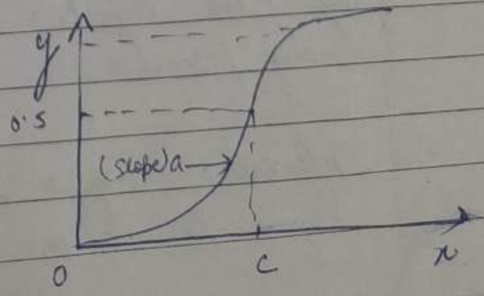
- Gaussian Membership function.



$$\mu_A(x) = e^{-\frac{(x-b)^2}{2k^2}}$$

where K ⇒ standard deviation > 0.

- Sigmoid Membership function.



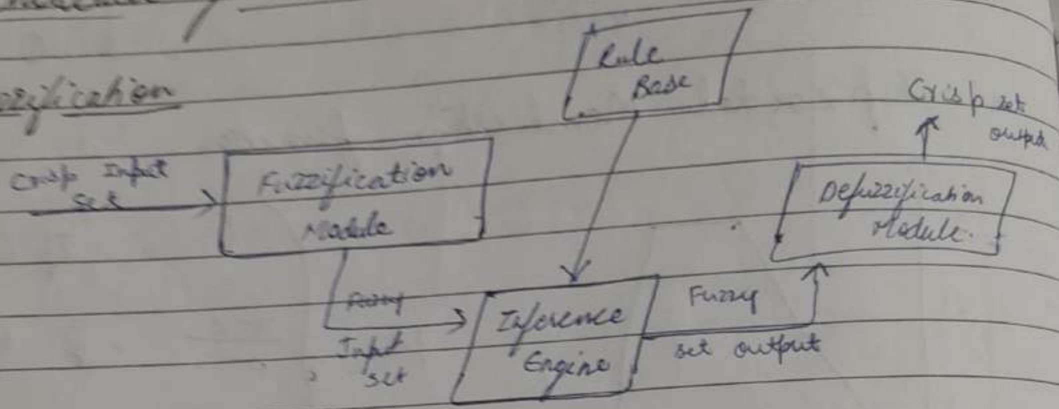
$$\mu_A(x) = \frac{1}{1+e^{-a(x-c)}}$$

$\boxed{\text{At } x = C, \ \mu = 0.5}$

→ Fuzzy logic system (FLS) — It is a rule based system that relies on experience rather than knowledge.

- Architecture of FLS

- Fuzzification



The application of fuzzy logic system are AC, Heater, microwave, Vacuum cleaner, washing Machine, Pattern recognition.
The Module is use to convert crisp input into fuzzy input using membership function this process of conversion is called fuzzification.
It divide the crisp set ~~system~~ into five state
① Large positive (very cold) ② Medium positive (cold) ③ Small positive (Warm)
④ Medium negative (Hot) ⑤ Large negative (very Hot)

- Rule Based. — This Module is use for storing the set of rules & the if then conditions given by the experts are used for controlling decision making process.

- Inference Engine — Processing is done in Inference Engine. It allow users to find the matching degree between fuzzy inputs & rule after matching degree the system determine which rule is to be added according to the given input. When all rules are fired then they are combined for developing the control action.

- Defuzzification - takes fuzzy output generated by the Inference Engine as ~~output~~ input then transforms into crisp output set. This is the last step of FLS. It is the output for automation system.

→ **Defuzzification system** -

① **Lambda-cut Method OR Alpha-cut Method.**

$$R = \begin{bmatrix} 1 & 0.8 & 0 \\ 0.8 & 1 & 0.4 \\ 0 & 0.4 & 1 \end{bmatrix}, \lambda = 1.$$

$\lambda = 0.2.5$

$$R_{\lambda=1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_{.2.5} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\lambda = 0 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

If $A = \left\{ \frac{1}{a} + \frac{0.9}{b} + \frac{0.6}{c} + \frac{0.3}{d} + \frac{0.01}{e} + \frac{0}{f} \right\}$

find crisp set for different λ, for $\lambda = 1$.

$A_{\lambda=1} = \{a\}$

$$A_{\lambda=1} = \left\{ \frac{1}{a} + \frac{0}{b} + \frac{0}{c} + \frac{0}{d} + \frac{0}{e} + \frac{0}{f} \right\}.$$

- <u>Properties of λ - cut Method</u>

If A and B are two fuzzy set, then.

(1) $(A \cup B)_\lambda = A_\lambda \cup B_\lambda$

(2) $(A \cap B)_\lambda = A_\lambda \cap B_\lambda$

(3) $(\overline{A}_\lambda) = \overline{A}_\lambda$ (except for $\lambda = 0.5$).

→ If R and S are two fuzzy Relation.

(1) $(R \cup S)_\lambda = R_\lambda \cup S_\lambda$.

(2) $(R \cap S)_\lambda = R_\lambda \cap S_\lambda$

(3) $(\overline{R_\lambda}) \neq \overline{R_\lambda}$.

Ques Two fuzzy set P and Q are defined on X as following

$\mu(x)$	x_1	x_2	x_3	x_4	x_5
P	0.1	0.2	0.7	0.5	0.4
Q	0.9	0.6	0.3	0.2	0.8

Find (I) $(P \cap Q)_{0.4}$ (II) $(P \cup Q)_{0.5}$

$(P \cap Q)_{0.4} = P_{0.4} \cap Q_{0.4} \Rightarrow \{X_5\}$

$P_{0.4} = \{X_3, X_4, X_5\}$.

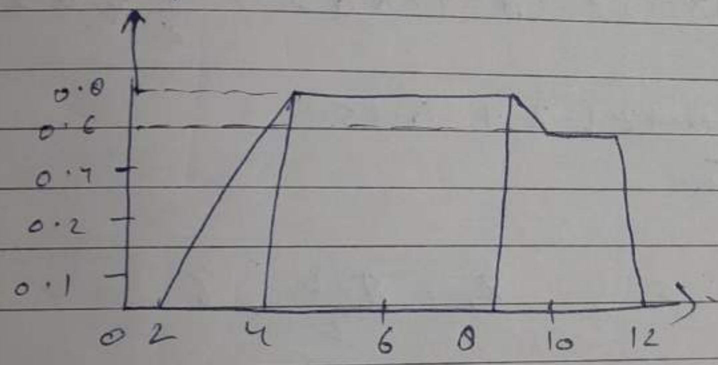
$Q_{0.4} = \{X_1, X_2, X_5\}$.

$(P \cup Q)_{0.5} = \{x_1, x_2, x_3, x_4, x_5\}$.

$P_{0.5} = \{x_3, x_4, x_5\}$
$Q_{0.5} = \{x_1, x_2, x_5\}$.

② Maxima method — This method deals with the maximum membership value to convert fuzzy value to crisp value.
In this method there are three type of Category.

a) First of Maxima method (FOM)
b) Last of Maxima method (LOM)
c) Mean of Maxima method (MOM)

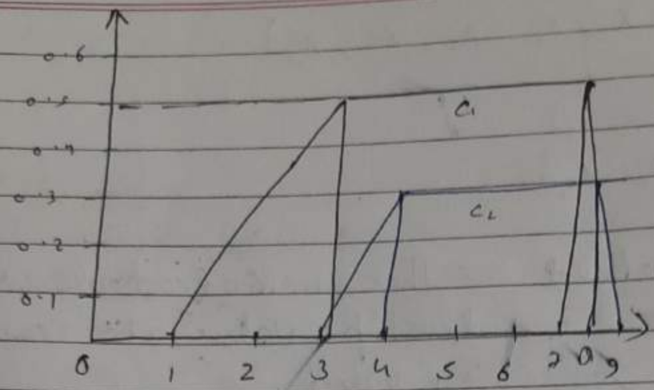


$X_{FOM} = 4$
$X_{LOM} = 8$
$X_{MOM} = \frac{4+6+8}{3} = 6.$

③ <u>Centroid</u> <u>Method</u> OR Centre of sum (COS).
In this method we calculate the sum of area of each individual fuzzy set then find the centre of this area.

$C_1 = \{(1,0), (3, 0.5), (7, 0.5), (8, 0)\}$.
$C_1 = \{(1,0), (3, 0.5), (7, 0.5), (8, 0.5), (9, 0)\}$.
$C_2 = \{(3,0), (4, 0.3), (8, 0.3), (9, 0)\}$

crisp value = $\frac{A_1 X_1 + A_2 X_2}{A_1 + A_2}$



$$A_1 = \frac{1}{2}(3-1)(0.5) + (7-3)\times 0.5 + \frac{1}{2}\times 0.5\times 1$$

$$= 0.5 + 2 + 0.25 \Rightarrow 2.75$$

$$A_2 = \frac{1}{2}\times(4-3)\times 0.3 + (8-4)\times 0.3 + \frac{1}{2}\times 0.3\times 1$$

$$\Rightarrow 0.15 + \cancel{0.12}\, 1.2 + 0.15$$

$$= 1.5$$

$$X_1 = \frac{7+3}{2} = 5, \quad X_2 = \frac{8+4}{2} = 6$$

$$\text{Crisp value} = \frac{A_1X_1 + A_2X_2}{A_1 + A_2} = \frac{2.75\times 5 + 1.5\times 6}{2.75 + 1.5}$$

$$= 5.35$$

→ weighted Average Method — It is also one of the defuzzification method to convert fuzzy set to crisp set.

$A = \{(0,0), (1,0.3), (4,0.3), (5,0)\}$

$B = \{(3,0), (4,0.5), (6,0.5), (7,0)\}$

$C = \{(5,0), (6,1), (7,1), (8,0)\}$

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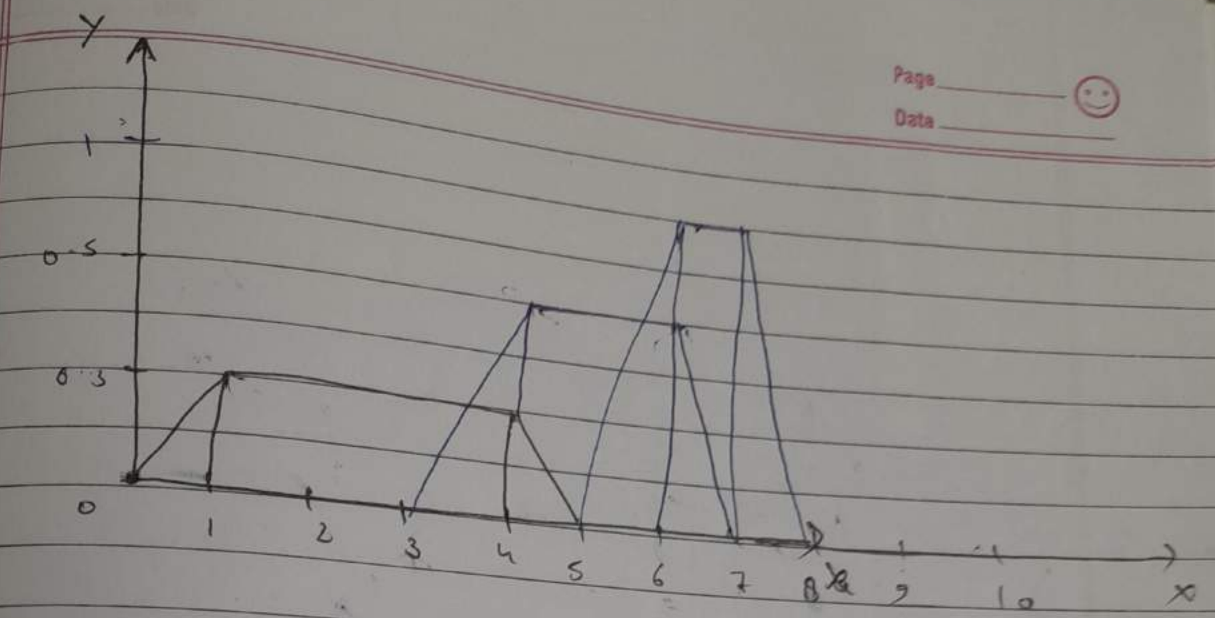

Crisp value $= \left(\frac{1+4}{2}\right) \times 0.3 + \left(\frac{4+6}{2}\right) \times 0.5 + \left(\frac{6+7}{2}\right) \times 1$

$\Rightarrow 2.5 \times 0.3 + 5 \times 0.5 + 6.5 \times 1$

$\Rightarrow .75 + 2.5 + 6.5 \Rightarrow 9.75.$

$= \frac{9.75}{1.8} = 5.416.$