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## Unit: 2

## Regular Expressions.

**Regular Expression:** ★ Regular Expression are used for representing certain set of strings in an algebraic function.

★ Any terminal Symbol belongs to Σ Including NULL & ϕ are also a Regular Expression.

★ The union of 2 Regular Expressions R1 and R2 written as R1+R2 is also a Regular Expression.

★ The Concatenation of a 2 Regular Expressions R1&R2 is also a Regular Expression written as R1R2.

★ The iteration (closure) of a Regular Expression written as $R^*$ is also a Regular Expression.

Ⓐ R1 & R2 (Union) → R1+R2
Ⓐ R1 & R2 (Concatenation) → R1R2
Ⓐ $(R^*) \rightarrow R^*$
Ⓐ $R \rightarrow (R)$

★ If R is a Regular Expression, then (R) is also a Regular Expression.

Eg: Δ = {0, 1, 2}
RE = 0+1+2

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## Identity of Regular Expression:

★ $\phi + R = R$ --- I1
★ $\phi R + R\phi = \phi$ --- I2
★ $\epsilon R = R\epsilon = R$ --- I3
★ $\epsilon^* = \epsilon$ and $\phi^* = \epsilon$ --- I4
★ $R+R = R$ --- I5
★ $R^* R^* = R^*$ --- I6
★ $RR^* = R^* R$ --- I7
★ $(R^*)^* = R^*$ --- I8
★ $\epsilon + RR^* = \epsilon + R^*R = R^*$ --- I9
★ $(PQ)^* P = P(QP)^*$ --- I10
★ $(P+Q)^* = (P^*Q^*)^* = (P^* + Q^*)^*$ --- I11
★ $(P+Q)R = PR + QP$ and --- I12
$R(P+Q) = RP + RQ$

**Arden Theorem:** Let P and Q be two Regular Expressions over Σ If P does not contain "NULL", then the following equation in R = Q+RP, has a unique solution $[R = QP^*]$

$R = Q + QP^* P$
$R = Q(\wedge + P^* P)$ (Using I9) ($\wedge$ = NULL)
$R = Q(P^*) = QP^*$

To prove uniqueness by replacing R = Q+RP on the RHS.

$R = QP$ --- ①
$R = Q + (Q+RP)P$
$= Q + QP + RP^2$
$= Q + QP + (Q+RP)P^2$

$= Q + QP + QP^2 + RP^3$

$= Q + QP + QP^2 + (Q + RP)P^3$

$= Q + QP + QP^2 + QP^3 + RP^4$

$= Q(\wedge + P + P^2 + P^3 + \cdots\cdots + P^{n-1} + P^n)$

$= Q(P^*)$

$[R = QP^*]$

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Ques) Given a Regular Expression for representing a set of string in which every 0 is immediately followed by atleast two 1's.

1 or 011

$RE = (1 + 011)^*$

If W is the language then;

(A) W does not contain any 0

OR

(B) It contains a 0 preceeding '1' followed by 11.

W is either 1 or 011.

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Ques) Prove that the Regular Expression $R = \wedge + 1^*(0+1)^*(1+(01$ Also describe the string $(1+011)^*$

Let $P = 1^*(011)^*$

$R = \wedge + PP^*$

$R = P^*$ (By I9)

$R = (\underbrace{1^*}_{P}\ \underbrace{(011)^*}_{Q})^*$

$R = (1 + 011)^*$ (By $I_{11}$)

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$$(P+Q)^* = (P^* + Q^*)^*$$

$$= (P^* + Q^*)^* = (P^* Q^*)^*$$