CHITKARA UNIVERSITY

2022-2023

Sessional Test I– Nov, 2022

Semester I

ID No: ....

[Total No. of Pages: 02]

Time: 90 minutes

Title of the Course: Calculus and Statistical Analysis
Max. Marks: 40
Course Code: 22AS001

Instructions:

For Section A

- There is one question having five parts. Each part is having four distinct options out of which only one choice will be correc
  There is no negative marking for incorrect answers.

For Section B

- There are 5 Questions of 3 marks each. All are compulsory.

For Section C

- There are 2 Questions of 10 marks each. All are compulsory.

Section-A

(All Questions are Compulsory, Each question carries 01 mark)

Q1.If $z = x^2 + y^2$, where $x = r\cos\theta, y = r\sin\theta$. Then the derivative $\frac{\partial z}{\partial r}$ is

(A) r (B) 2r (C) 0 (D) -1

Q2.Which of the following system of equations represents the chemical balancing problem

$$x(C_3H_8) + y(O_2) \to z(CO_2) + w(H_2O)$$

(A) $x - z = 0,$
$x - 2w = 0,$
$2y + z + w = 0.$

(B) $3x - 2z = 0,$
$8x - w = 0,$
$y - 2z + w = 0.$

3 + 2y

(C) $3x - z = 0,$
$8x - 2w = 0,$
$2y - 2z - w = 0.$

(D) $3x - 2z = 0,$
$x - 2w = 0,$
$y + z + w = 0.$

Q3. The characteristic roots of the matrix A are 1, 0, -1. Then characteristic roots of A^{100} are

(A) 1, 1, 1 (B) 1, 0,-1 (C) 1, 0, 0 (D) 1, 0, 1

Q4. The order of the homogeneous function $\frac{\sqrt{x}+\sqrt{y}}{x^2+y^2}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) $-\frac{3}{2}$ (D) 2

Q.5 The eigen vectors corresponding to distinct eigen values are

(A) Linearly dependent (B) Linearly Independent
(C) May or may not Linearly Independent (D) May or may not Linearly dependent

Section-B

(Attempt all questions, each question carries 03 marks)

Q.6 Find the rank of given matrix

$$A=\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

19 24 25 25

Q.7 Test for the consistency of the following system of equations using elementary operations

$$x+y+z=9$$
$$2x+5y+7z=52$$
$$2x-y-z=-6$$

Q.8 Find the equation of the tangent plane for the surface $x^3+y^3+3xyz=3$ at point $(1,2,-1)$.

Q.9 Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u\,log\,(u)$, where $log\,(u)=\frac{x^3+y^3}{3x+4y}$.

Q.10 Check whether the function $f(x,y)=\begin{cases}\frac{x^3-y^3}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

continuous or not.

Section-C

(Attempt all questions, each question carries 10 marks)

Q.11

(A) Find the eigen values and eigen vectors of the following matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}.$$

(B) If $z=\frac{x^2+y^2}{x+y}$, show that

$$\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right).$$

Q.12 For the given matrix A

$$\begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

(A) Find the characteristic equation of the matrix A.
(B) Verify the Cayley Hamilton theorem.
(C) Find A^{-1}.

CHITKARA
UNIVERSITY

2022-2023

Sessional Test II– Dec, 2022

Semester I

D No: ....

[Total No. of Pages: 01]

Time: 90 minutes

itle of the Course: Calculus and Statistical Analysis
Max. Marks: 40
Course Code: 22AS001
nstructions:
or Section A

- There are 5 questions of 1 marks each. Each question is having four distinct options out of which only one choice will be correct. There is no negative marking for incorrect answers.

or Section B

- There are 5 Questions of 3 marks each. All are compulsory.

or Section C

- There are 2 Questions of 10 marks each. All are compulsory.

Section-A

(All Questions are Compulsory, Each question carries 01 mark)

1. The curve $x^3 + y^3 = 3axy$ is symmetric about
   (A) x-axis (B) y-axis (C) $y = x$ (D) Origin.
2. The Jacobian of transformation of cartesian coordinates to polar coordinates is
   (A) r^2 (B) $2r$ (C) $\frac{r}{2}$ (D) r
3. To find the stationary points of function $f(x, y) = x^2 + y^3$ the condition is
   (A) $2x = y$ (B) $2x + y = 0$ (C) $2x = 0, 3y^2 = 0$ (D) $2x = 0, 3y = 0$
4. The value of $\Gamma\left(\frac{1}{2}\right) =$
   (A) $\frac{1}{4}$ (B) $\sqrt{\frac{\pi}{4}}$ (C) $\sqrt{\pi}$ (D) 2
5. $\beta(p, q) =$
   (A) $\beta(p-1, q)$ (B) $\beta(q, p)$ (C) $\beta(p+1, q+1)$ (D) $\beta(p^2, q^2)$

Section-B

(Attempt all questions, each question carries 03 marks)

6. Calculate the Jacobian $J = \frac{\partial(u,v)}{\partial(x,y)}$, where $u(x, y) = x + \frac{y^2}{x}$ and $v(x, y) = \frac{y^2}{x}$.

7. Examine for the extreme values $f(x, y) = x^2 + y^2 + 6x + 12$.

8. Change the order of integration and evaluate the integral

$$\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dx\, dy.$$

9. Evaluate the integral

$$\int_0^1 \int_0^2 \int_0^1 xyz \, dx \, dy \, dz.$$

10. Evaluate

$$\int_0^\infty e^{-3x} x^2 dx .$$

Section-C

(Attempt all questions, each question carries 10 marks)

11. (A) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$.

(B) Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.

12. (A) Expand the function $\cos(x)\cos(y)$ in powers of x, y up to third degree terms.

(B) Find the volume of ellipsoid using change of variables

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1.$$

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Total No. of Pages:02

END TERM EXAMINATION

1ST SEMESTER, 2022-23

22AS001-CALCULUS AND STATISTICAL ANALYSIS

Time allowed: 03 Hours **Max. Marks: 60**

General Instructions:

- **Follow the instructions given in each section.**
- **Make sure that you attempt the questions in order.**

SECTION-A (10x2 marks=20 marks)

(All questions are compulsory, each question carries 02 marks)

Q1 Determine the rank of the matrix A, $A = \begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -1.5 \end{bmatrix}$

Q2 For the given matrix B, determine the eigen values of $3B^3 + 5B^2 - 6B + 2I$; $B = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

Q3 Find the equation of the normal plane to the surface $x^2 + 2y^2 + 3z^2 = 12$ at the point (1,2,-1)

Q4 A balloon is in the form of right circular cylinder of radius 1.5m and length 4 m, is surmounted by hemispherical ends. If the radius is increased by 0.01 m and the length by 0.05m, find the percentage change in the volume of the balloon.

Q5 Discuss the continuity of the function $f(x,y) = \frac{x}{\sqrt{x^2+y^2}}$, when $(x,y) \neq (0,0)$ and $f(x,y) = 2$ when $(x,y) =$ $(0,0)$

Q6 Determine the area lying inside the circle $r = a\, sin\theta$ and outside the cardiod $r = a(1 - cos\theta)$

Q7 Evaluate $\iiint_R e^{x+y+z} dz\, dy\, dx$, where R is the region defined by $0 < x < log2, 0 < y < x, 0 < z < x + y$

Q8 Find $\beta\left(\frac{5}{2}, \frac{3}{2}\right)$

Q9 The probability of a man hitting a target is 1/3. (a) If he fires 5 times, what is the probability of his hitting the target at least twice? (b) How many times he fire so that the probability of his hitting the target at least once is more than 90%?

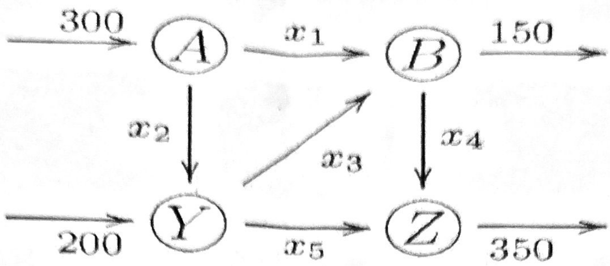
Q10 Find the correlation coefficient between x and y, when the lines of the regression are $2x - 9y + 6 = 0$, $x - 2y + 1 = 0$

SECTION-B (8x5 marks=40 marks)

(Attempt any Eight Questions, each question carries 05 marks)

Q11 The flow of traffic (in vehicle per hour) through a network of streets is shown in the following figure.

DO NOT WRITE ANYTHING ON QUESTION PAPER EXCEPT ROLL NO.



Formulate the above problem as a system of linear equations as $AX = B$, hence find normal form of A.

12 Determine the values of a and b for which the system of the following equations has (a) No solution (b) Unique number of solutions (c) Infinite number of solutions

$$x + 2y + 3z = 6$$
$$x + 3y + 5z = 9$$
$$2x + 5y + az = b$$

Mass $\int dxdy$

$\frac{1}{m}\iiint x f dxdy$

Change the order of integration and evaluate $\iint_R xy\ dy\ dx$, where R is the region defined by

$$x^2 < y < 2 - x;$$
$$0 < x < 1$$

The profit for some software company is given by $PR(x, y) = -100 + 80x - 0.1x^2 + 100y - 0.2y^2$, where x & y represents the levels of output of two products produced by the company. If the company knows its maximum combined feasible production to be 325. How can the company maximize its profit?

Find the centroid of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Find the points for which the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ attains maxima.

7 By transforming into cylindrical coordinates evaluate the integral $\iiint_R x^2 + y^2 + z^2\ dxdydz$, where R is the region bounded by $0 \le z \le x^2 + y^2 \le 1$

18 Mice with an average lifespan of 32 months will live upto 40 months when fed by a certain nutritious food. If 64 mice fed on this diet have an average lifespan of 38 months and $\sigma = 5.8$ months. Is there any reason to believe that average lifespan is less than 40 months? (Use 0.01 level of significance , $Z_{0.01} = 2.33$)

19 Determine the rank correlation coefficient for the following data which shows the marks/ranks obtained in two quizzes in Mathematics.

Marks in first quiz (X)	6	5	8	8	7	6	10	4	9	7
Marks in second quiz (Y)	8	7	7	10	5	8	10	6	8	6

Q20 Find the Maclaurin's series expansion of $f(x, y) = e^x \sin y$