Communication-Avoiding Machine Learning

Aditya Devarakonda

ASPIRE Summer Retreat

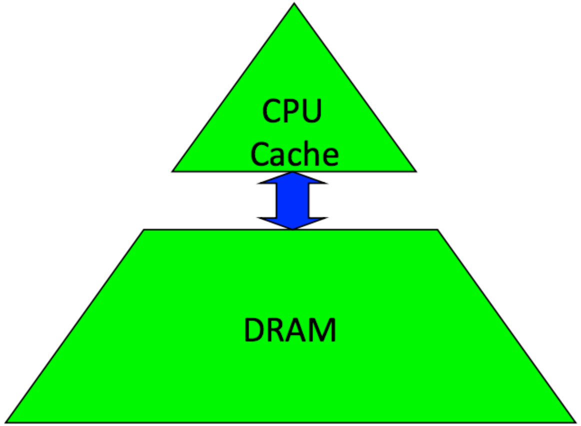
June 14th, 2017

Collaborators:
Jim Demmel (Adviser), Kimon Fountoulakis (Post-Doc), and Michael W. Mahoney (Co-adviser)

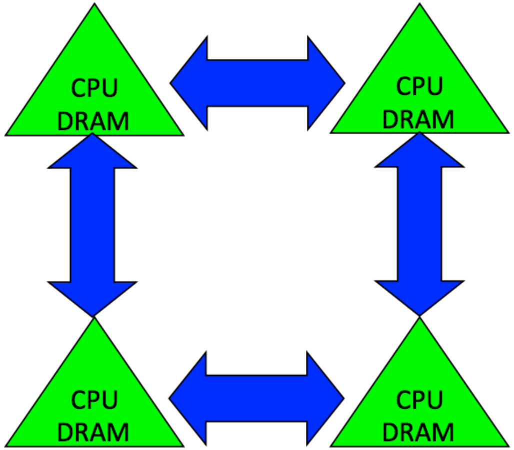
Definition

Communication is <u>data movement</u>.

Sequential



Parallel (Distributed-Memory)

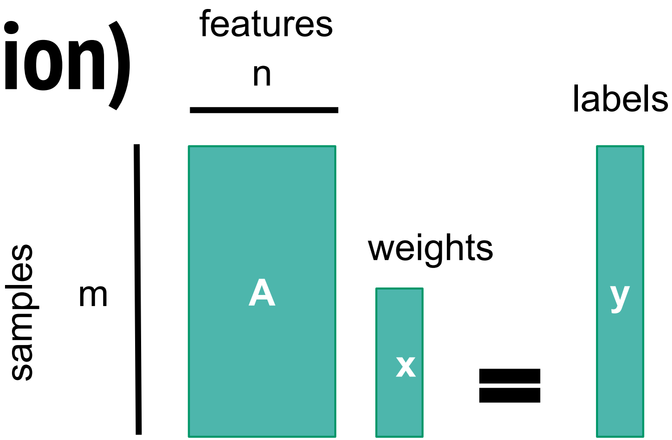


Courtesy:
Demmel

Least-Squares (Linear Regression)

Many ways to solve.

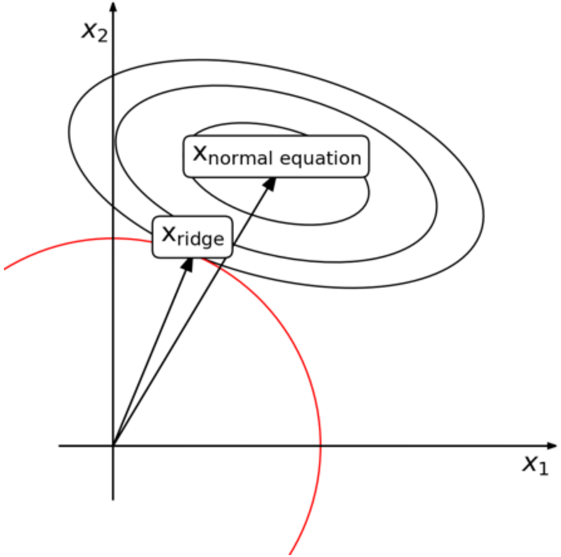
- Direct
  - Explicitly solve normal equation.
  - Implicitly through matrix factorizations.
- Iterative
  - Krylov methods (e.g. Conjugate Gradients).
  - **(Block) Coordinate Descent.**

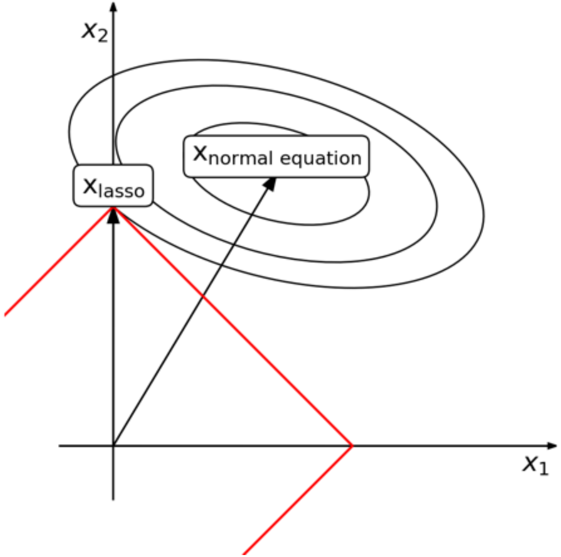


Least-Squares: $\mathrm{argmin}_{x \in \mathbb{R}^n} \frac{1}{2} ||Ax - y||_2^2$

Normal Equation: $x = (A^T A)^{-1} A^T y$

Regularized Least-Squares (Regression)



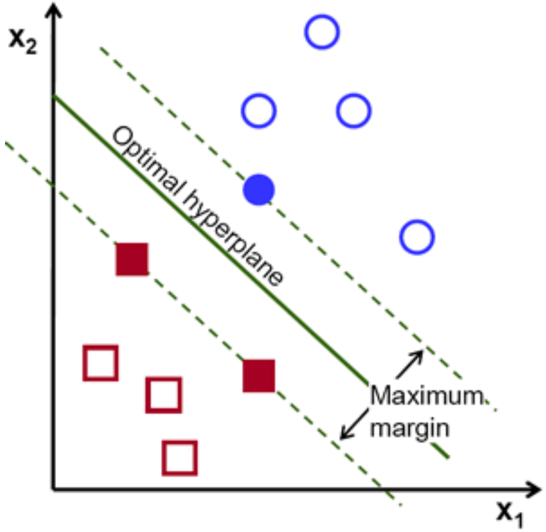


Ridge: $\operatorname{argmin}_{x\in\mathbb{R}^n} \underbrace{\frac{1}{2}\|Ax - y\|_2^2}_{\text{Loss function.}} + \underbrace{\frac{\lambda}{2}\|x\|_2^2}_{\text{Regularization function.}}$

LASSO: $\operatorname{argmin}_{x\in\mathbb{R}^n} \frac{1}{2}\|Ax - y\|_2^2 + \boxed{\lambda\|x\|_1}$

Binary Classification

Support Vector Machines



Ridge Regression with Block Coordinate Descent

Ridge solution:
(closed-form)

$$\underbrace{x}_{n} = \left(\underbrace{A^T}_{m} \underbrace{A}_{n} + \lambda I_n \right)^{-1} A^T y$$

λ = regularization parameter
I_n = n x n Identity matrix
y = labels

Similar to normal equation.

Ridge Regression with Block Coordinate Descent

Ridge solution:
(closed-form)

$$x = \left(A^T A + \lambda I_n \right)^{-1} A^T y$$

Block Coordinate Descent

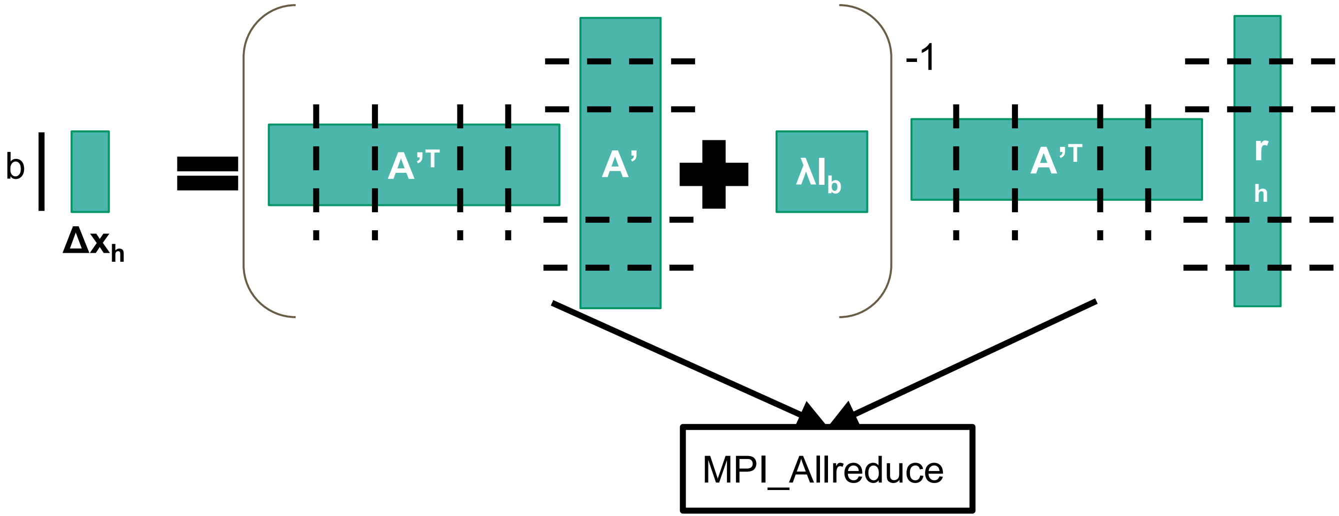
λ = regularization parameter
I_b = b x b Identity matrix
r = residual

A' = subsampled columns.

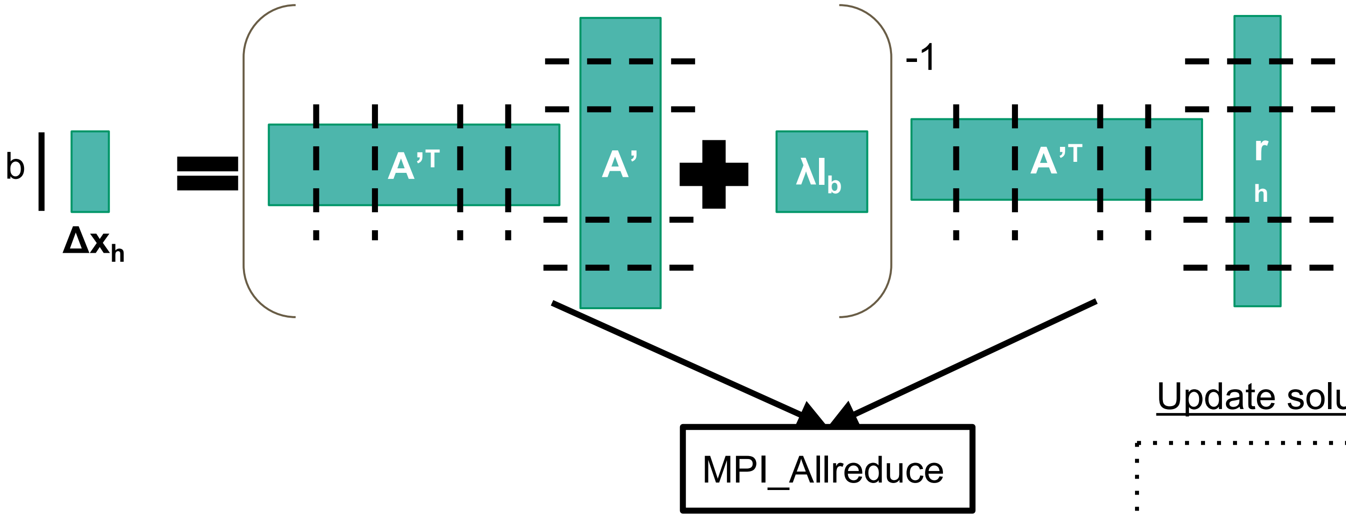
$$b \,\Big|\, \Delta x_h = \left(\overset{m}{\overline{A'^T}} \; \overset{b}{\overline{A'}} + \lambda I_b \right)^{-1} A'^T r$$

Solution to sub-problem.

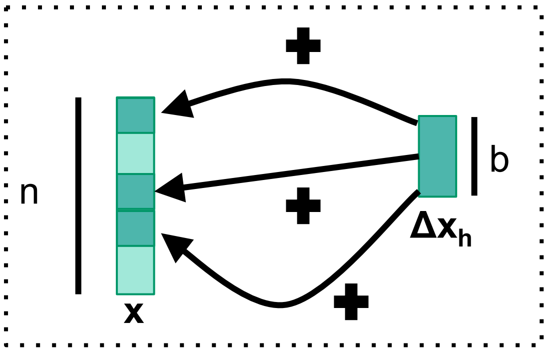
Block Coordinate Descent in Parallel



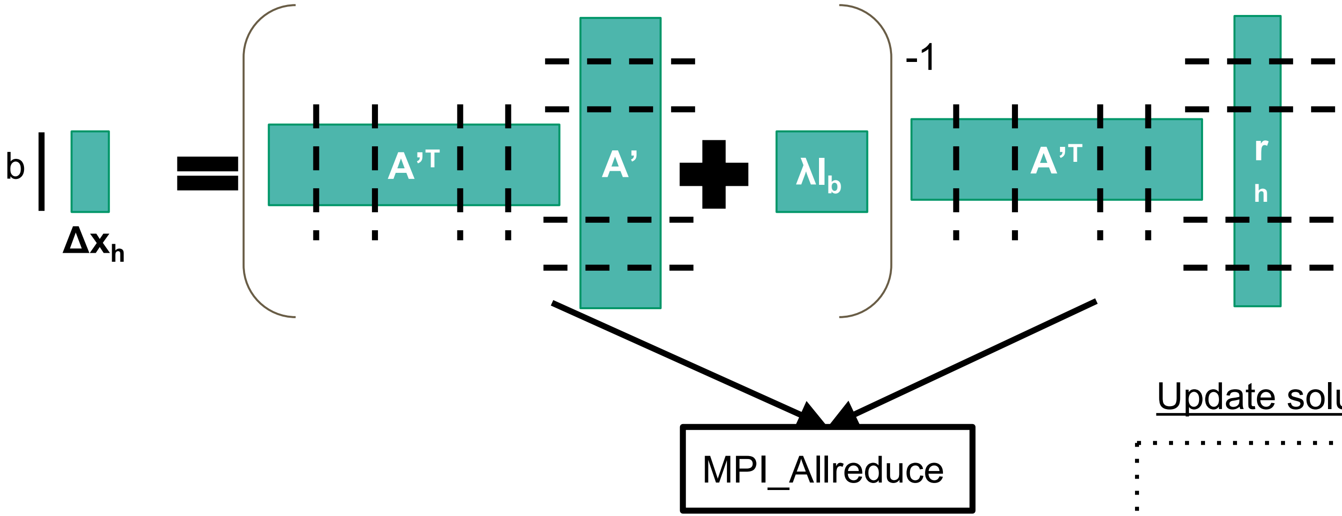
Block Coordinate Descent in Parallel

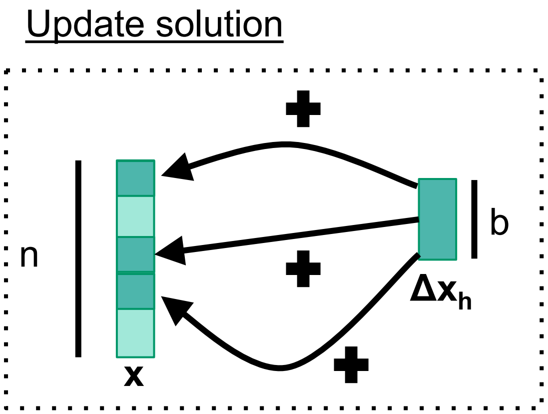


Update solution



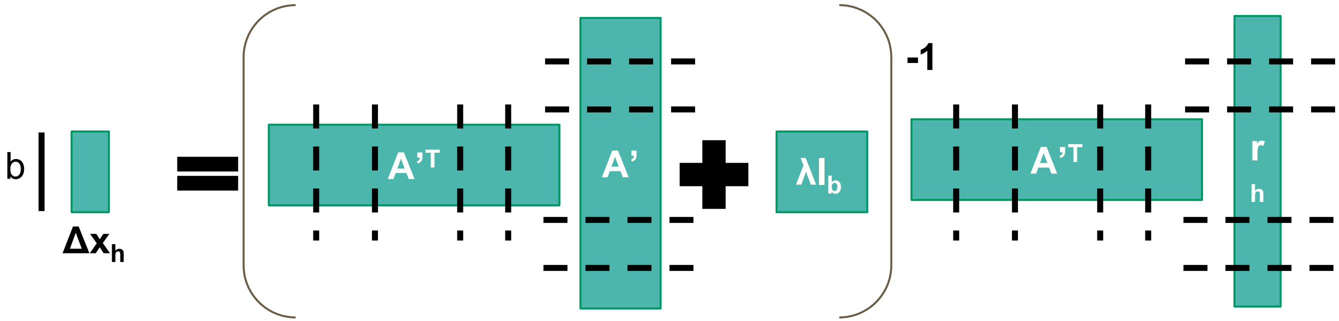
Block Coordinate Descent in Parallel





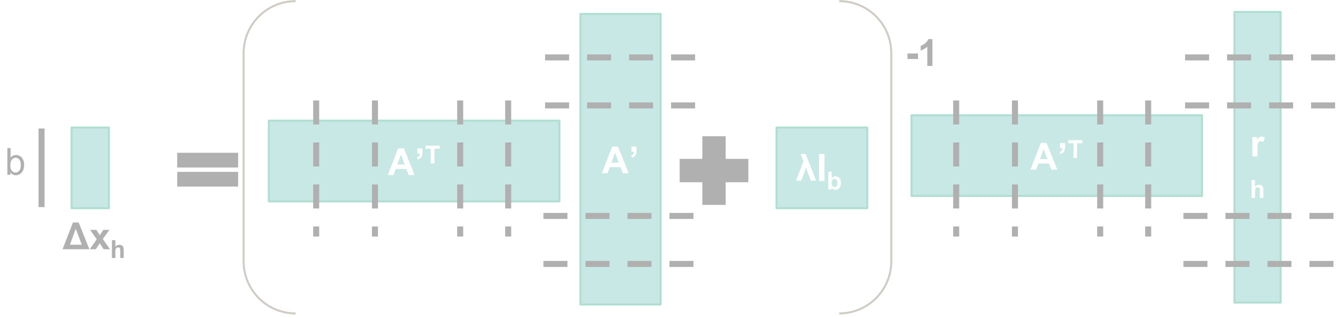
Select new coordinates.
Repeat until convergence.

Communication-Avoiding Block Coordinate Descent

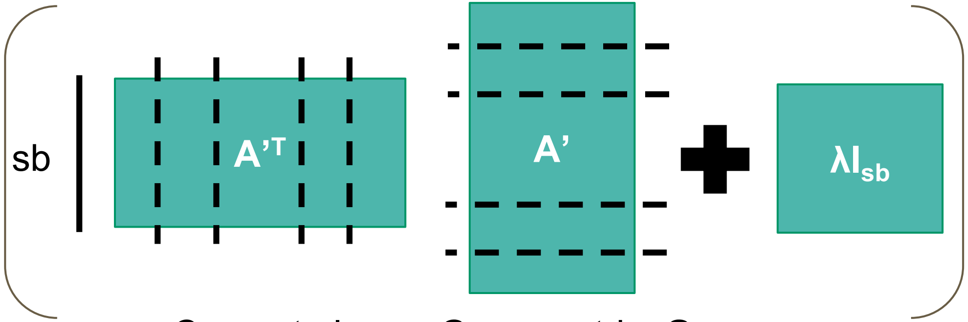


Block Coordinate Descent

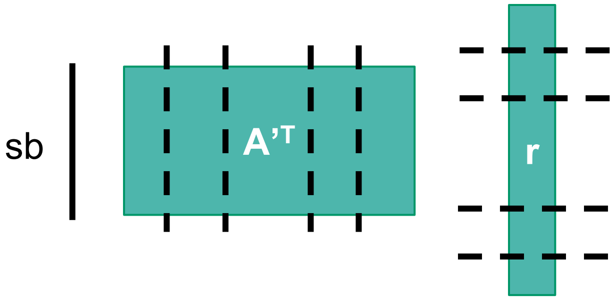
Communication-Avoiding Block Coordinate Descent



s = CA parameter
I_{sb} = sb x sb Identity
r = residual



Compute larger Gram matrix, **G**.
(Do not invert!)

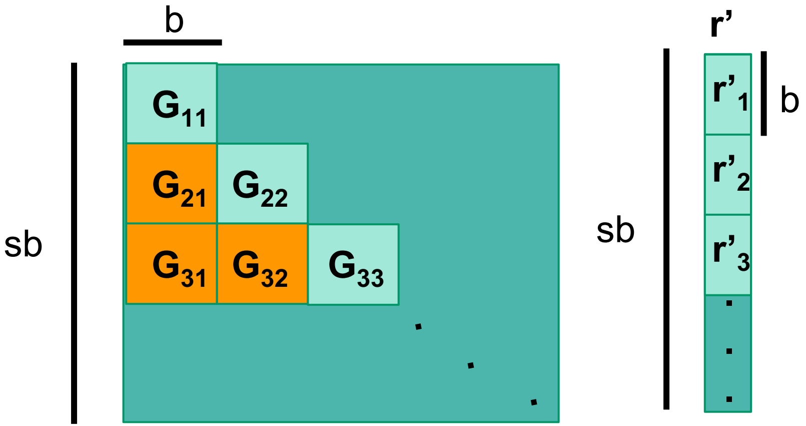


Larger residual contribution.
r is from this iteration

Redundantly on all processors.

Dimensions exaggerated for clarity.

Communication-Avoiding Block Coordinate Descent



Partition G and r'.

(**G** is symmetric)

Redundantly on all processors.

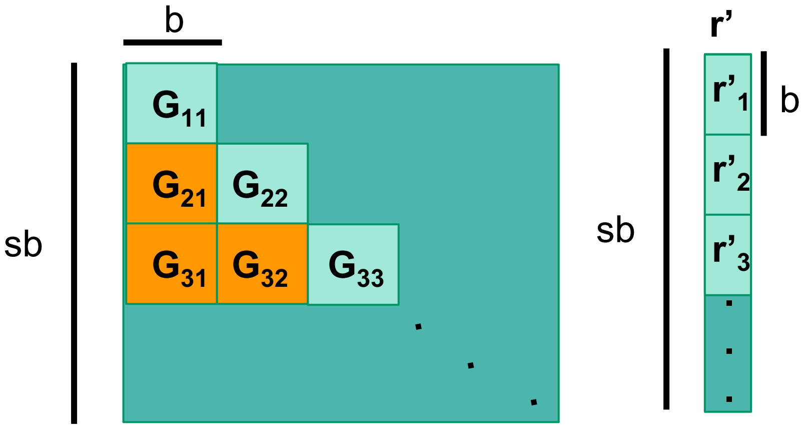
No communication for inner iterations.

Communication-Avoiding Block Coordinate Descent

Solution to 1st sub-problem.

$$\Delta x_1 = \left(G_{11} \right)^{-1} r'_1$$

Inner iteration 1



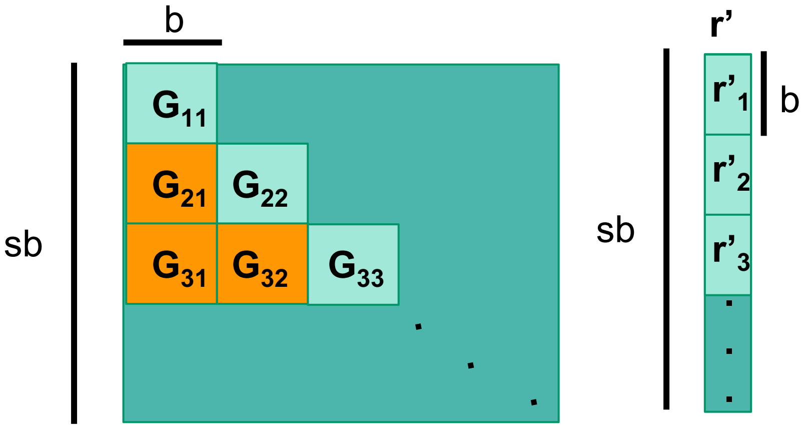
r_2', r_3',... become stale as soon as Δx_1 is computed.

Orange blocks are the updates

Redundantly on all processors.

No communication for inner iterations.

Communication-Avoiding Block Coordinate Descent



Inner iteration 1

$$\Delta x_1 = \left(G_{11}\right)^{-1} r'_1$$

Inner iteration 2

$$\Delta x_2 = \left(G_{22}\right)^{-1} r'_2 - G_{21}\,\Delta x_1$$

Solution to 2nd sub-problem.

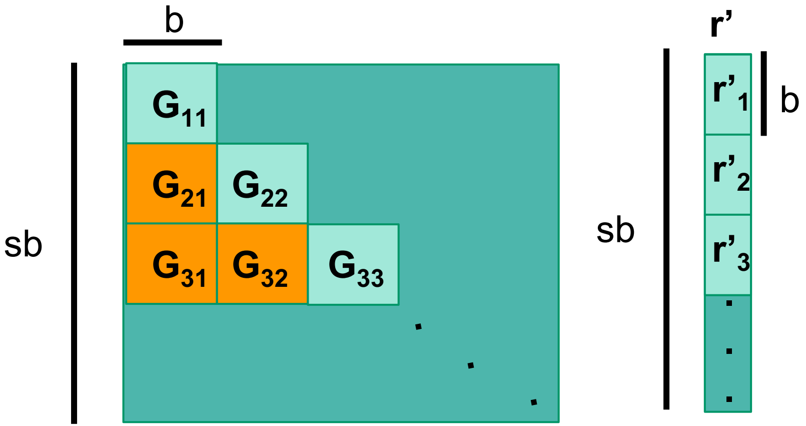
r_2', r_3',... become stale as soon as **Δx_1** is computed.

Orange blocks are the updates

Redundantly on all processors.

No communication for inner iterations.

Communication-Avoiding Block Coordinate Descent



Inner iteration 1

$$\Delta x_1 = \left(G_{11}\right)^{-1} r'_1$$

Inner iteration 2

$$\Delta x_2 = \left(G_{22}\right)^{-1} r'_2 - G_{21} \Delta x_1$$

Inner iteration 3

$$\Delta x_3 = \left(G_{33}\right)^{-1} r'_3 - \begin{bmatrix} G_{31} & G_{32} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

Solution to 3rd sub-problem.

Subtraction terms are needed for CA and non-CA solutions to agree.

Algorithm 1 Block Coordinate Descent (BCD) Algorithm

Input: $X \in \mathbb{R}^{d\times n}, y \in \mathbb{R}^n,\ H > 1,\ w_0 \in \mathbb{R}^d,\ b \in \mathbb{Z}_+$ s.t. $b \le d$
for $h = 1, 2, \cdots, H$ **do**
  choose $\{i_m \in [d] | m = 1, 2, \ldots, b\}$ uniformly at random without replacement
  $\mathbb{I}_h = [e_{i_1}, e_{i_2}, \cdots, e_{i_b}]$
  $\Gamma_h = \frac{1}{n}\mathbb{I}_h^T X X^T \mathbb{I}_h + \lambda \mathbb{I}_h^T \mathbb{I}_h$
  $\Delta w_h = \Gamma_h^{-1}\left(-\lambda \mathbb{I}_h^T w_{h-1} - \frac{1}{n}\mathbb{I}_h^T X z_{h-1} + \frac{1}{n}\mathbb{I}_h^T X y\right)$
  $w_h = w_{h-1} + \mathbb{I}_h \Delta w_h$
  $z_h = z_{h-1} + X^T \mathbb{I}_h \Delta w_h$
Output w_H

Communication (every iteration)

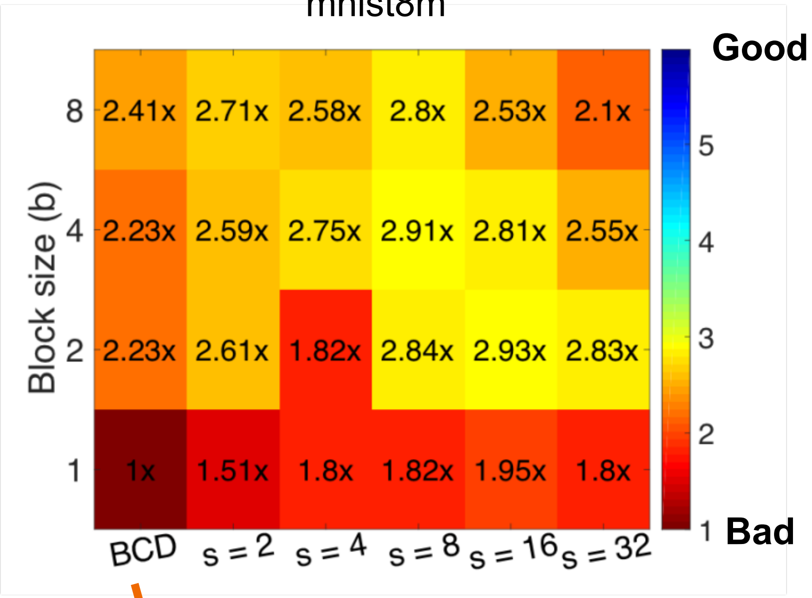
Algorithm 2 Communication-Avoiding Block Coordinate Descent (CA-BCD) Algorithm

Input: $X \in \mathbb{R}^{d\times n}, y \in \mathbb{R}^n,\ H > 1,\ w_0 \in \mathbb{R}^d,\ b \in \mathbb{Z}_+$ s.t. $b \le d$
for $k = 0, 1, \cdots, \frac{H}{s}$ **do**
  for $j = 1, 2, \cdots, s$ **do**
    choose $\{i_m \in [d] | m = 1, 2, \ldots, b\}$ uniformly at random without replacement
    $\mathbb{I}_{sk+j} = [e_{i_1}, e_{i_2}, \cdots, e_{i_b}]$
  let $Y = \left[\mathbb{I}_{sk+1}, \mathbb{I}_{sk+2}, \cdots, \mathbb{I}_{sk+s}\right]^T X$.
  compute the Gram matrix, $G = \frac{1}{n} Y Y^T + \lambda I$.
  for $j = 1, 2, \cdots, s$ **do**
    Γ_{sk+j} are the $b \times b$ diagonal blocks of G.
    $$\Delta w_{sk+j} = \Gamma_{sk+j}^{-1}\Big(-\lambda \mathbb{I}_{sk+j}^T w_{sk} - \lambda \sum_{t=1}^{j-1}\left(\mathbb{I}_{sk+j}^T \mathbb{I}_{sk+t} \Delta w_{sk+t}\right) - \frac{1}{n}\mathbb{I}_{sk+j}^T X z_{sk} - \frac{1}{n}\sum_{t=1}^{j-1}\left(\mathbb{I}_{sk+j}^T X X^T \mathbb{I}_{sk+t} \Delta w_{sk+t}\right) + \frac{1}{n}\mathbb{I}_{sk+j}^T X y\Big)$$
    $w_{sk+j} = w_{sk+j-1} + \mathbb{I}_{sk+j} \Delta w_{sk+j}$
    $z_{sk+j} = z_{sk+j-1} + X^T \mathbb{I}_{sk+j} \Delta w_{sk+j}$
Output w_H

Communication (every outer iteration)

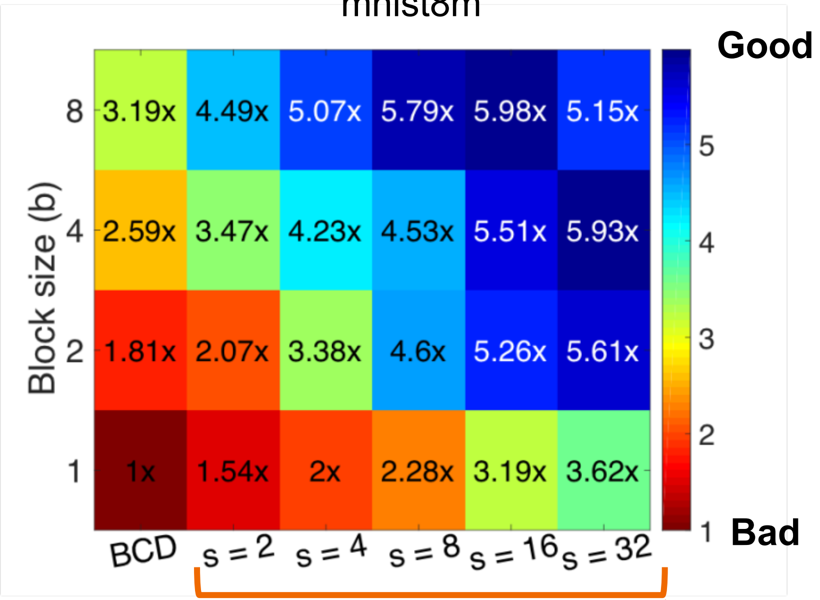
No communication

Speedups (Ridge Regression)



64 nodes of Edison

Standard Algorithm



1K nodes of Edison

Communication-Avoiding

Not Just Ridge Regression

Applies to **proximal methods** (e.g. LASSO, Group LASSO, elastic-net).

Recall that, Inner loop = no communication.

If nonlinearity is in inner loop, then **CA possible**.

Also applies to **Support Vector Machines** (current work).

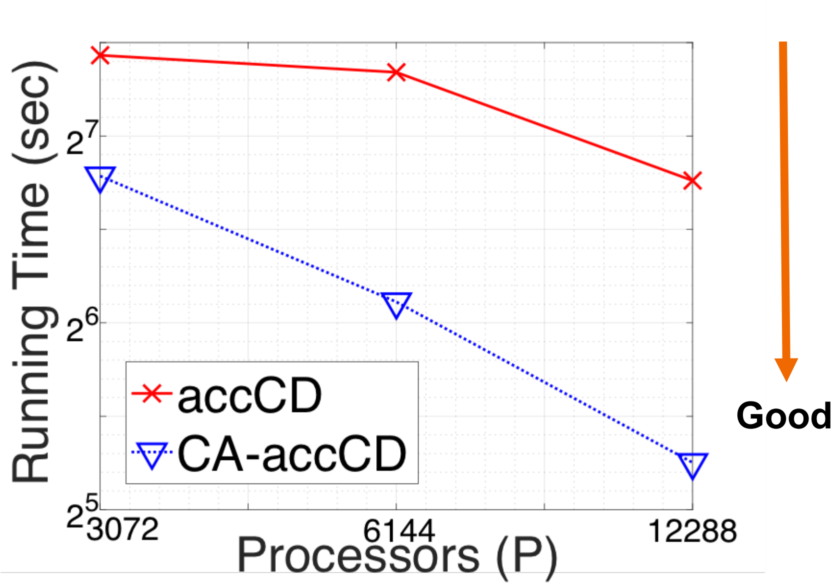
Nonlinearity is in inner loop.

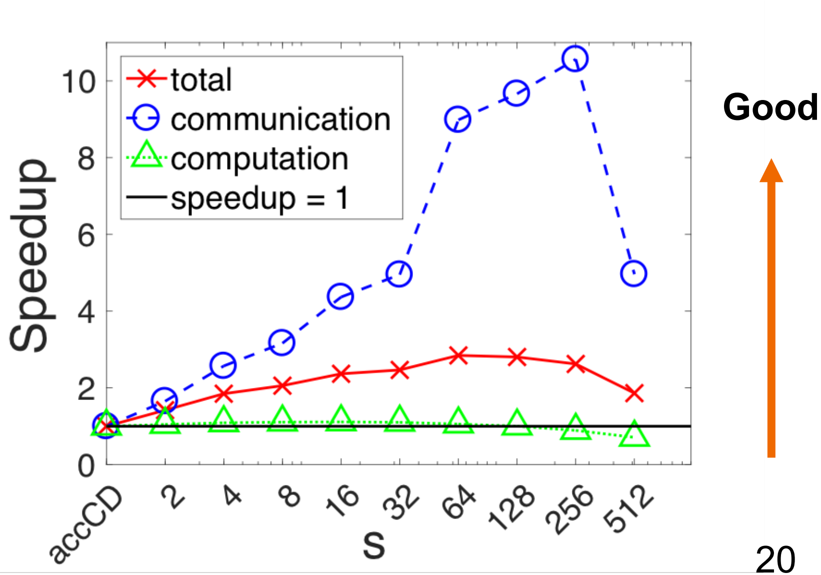
CALASSOS: Scalable Proximal Methods

accCD = accelerated Coordinate Descent

CA-technique applies to accelerated methods.

Strong scaling and speedups on **Url dataset (2M by 3M).**





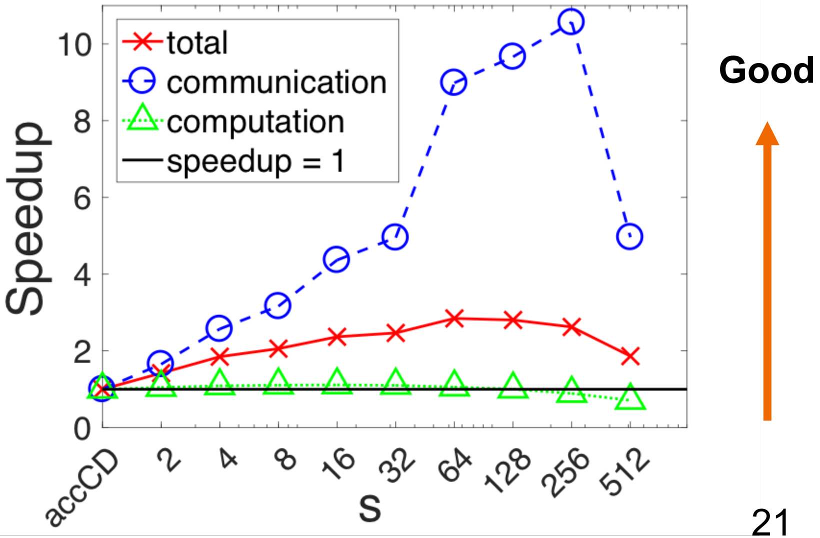
CALASSOS: Scalable Proximal Methods

CA-method perform **s^2 more flops.**

But still get **computation speedup.**

Due to BLAS-3 calls instead of BLAS-1 in non-CA.

BLAS-3 = Cache-efficient computation + higher flops rate.



CA-SVM: Preliminary Results

Based on Dual Coordinate Descent for Linear SVM (Hsieh, et. al.)

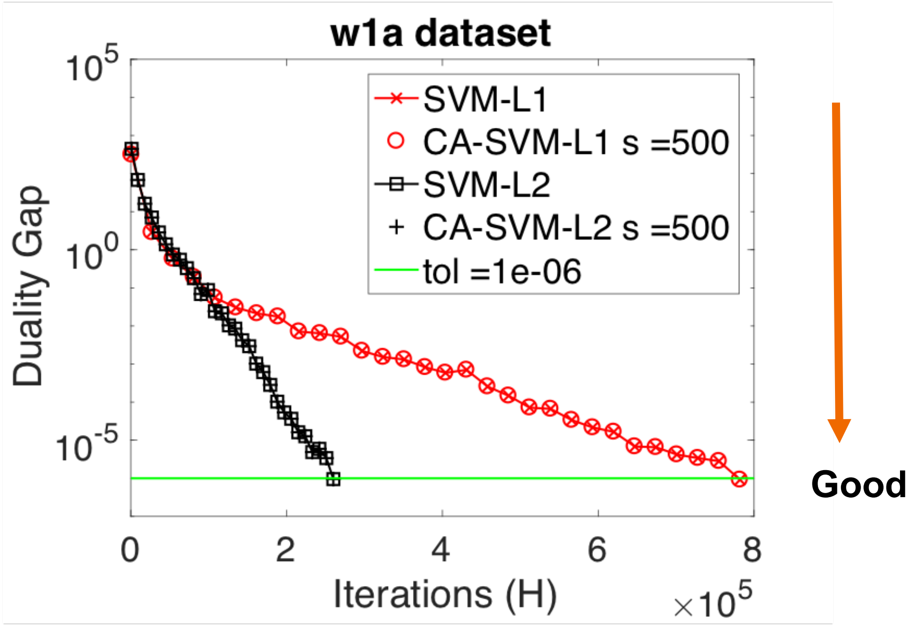
SVM-L1 = Hinge loss = $max(0, 1 - A^T_i x y_i)$

SVM-L2 = (Hinge loss)²

easier so, converges quickly.

Numerically stable **(unlike CA-Krylov)**!

Ditto for Ridge and Proximal.



Summary and Future Work

Large speedups when **latency dominates**.

Provably communication-avoiding.

CA-technique applies to **non-linear optimization**.

- How far can we go (e.g. Logistic regression)?

Speedups on **other platforms and frameworks**?

- Example: Cloud + Spark is latency dominated.
- Expect greater speedups!

Problem	MPI Speedup
Ridge Regression	Up to **6.1x**
Proximal Least-Squares	Up to **5.1x**
SVM	**Similar expected**

Questions?

Thanks to co-authors, collaborators, and sponsors!