Maths

Number Theory:

give a Number ~~x~~ n. check whether a i\ is prime or Not.

```
for ( int i=2; i<n-1; i++)
    if (x%i==0) return false
return true;
```

Time: $O(n)$
Space: $O(1)$.

⊙ $N = f_1 \times f_2$

$f_1 \le f_2$

$f_1 \Rightarrow \frac{N}{f_1} = f_2$

$f_1 \le \sqrt{N}$

$f_2 \ge \sqrt{N}$.

```
for (log int i=2; i*i<=N; i++)        N = 10^9
    if (N%i == 0) return false;
return true;
```

Time: $O(\sqrt{N})$
Space: $O(1)$.

sqrt(N) ⇒ log N

```
int sq = sqrt(N)
for (int i=2; i<=(int) sqrt(N); i++)
    if (N%i==0) return false;
return true;
```

sq

$O(\sqrt{n} + \log N)$;

Time: $O(\sqrt{n} \cdot \log N)$
Space: $O(1)$

Q. Given a Number N find its factors.

$12 \rightarrow \{1, 2, 3, 4, 6, 12\}$.

```
vector<int> ans;
for( int i=1; i<=N; i++)
        if( N%i==0)  ans.pb(i);
return ans;
```

$O(N)$.

```
for(int i=1;i*i<=N; i++)
        if( N%i == 0) {
            ans.pb(i);
            if( N/i != i ){
                ans.pb(N/i);
            }
        }
```

36

6   6

$O(\sqrt{N})$

Q. Given a Number N find all factors of 1 – N.

N = 6

$1 \rightarrow 1$

$2 \rightarrow 1, 2$

$3 \rightarrow 1, 3$

$4 \rightarrow 1, 2, 4$

$5 \rightarrow 1, 5$

$6 \rightarrow 1, 2, 3, 6$

$O(N^2)$

$O(N \cdot \sqrt{N})$

(i)

1 → 1,
2 → 1, 2
3 → 1, 3
4 → 1, 2, 4
5 → 1, 5
6 → 1, 2, 3, 6
7 → 1, 7
8 → 1, 2, 4, 8
9 → 1, 3, 9
10 → 1, 2, 5, 10.

```
for( i=1; i<=N; i++)
{  for(int j=i; j<=N; j+=i)
   {  fac[j].pb(i);
}
```

$i=1$ N
$i=2$ $\frac{N}{2}$
$i=3$ $N/3$

$$\left(\frac{N}{2} > \frac{N}{3}\right)$$

$$N + \frac{N}{2} + \frac{N}{3} + \frac{N}{4} + \frac{N}{5} + \frac{N}{6} + \frac{N}{7} + \frac{N}{8} + \dots \dots$$

$$N + \left(\frac{N}{2} + \frac{N}{2}\right) + \left(\frac{N}{4} + \frac{N}{4} + \frac{N}{4} + \frac{N}{4}\right) + \left(\frac{N}{8} + \frac{N}{8} + \dots \frac{N}{8}\right) + \dots$$

$$\underbrace{N + N + N + N + \dots N}_{\log N} \Rightarrow \underline{N \log N}$$

Time: $O(N \log N)$ $N\sqrt{N}$

$$\underset{2^0}{N} \quad \underset{2^1}{\left(\frac{N}{2}+\frac{N}{2}\right)} \quad \underset{2^2}{\left(\frac{N}{4} + \frac{N}{4} + \frac{N}{4} + \frac{N}{4}\right)} + \underset{2^4}{(8)} \dots 2^k$$

$$2^0 + 2^1 + 2^2 + \dots 2^k = N$$

$$\frac{a(r^k - 1)}{r-1}$$

$$\frac{1(2^k - 1)}{\cancel{2-1}} = N$$

$$2^k = N$$

$$k = (\log_2 N)$$

$$\underbrace{N \quad N \quad N \quad N \quad - - \quad N}_{k}$$

$$k \cdot N = (\log_2 N \cdot N)$$

Prime factorization

$$N \cdot (2^3 \cdot 3^3 \cdot 5^1)$$

$$\Downarrow$$

$$(2, 3, 5)$$

10^9+7

$$\sqrt{13} = 3 \quad 4$$

2, 3

N

$\sqrt{N}$ =) prime

$\sqrt{10} = 3$

5 2 3

2

$$20 = 2^2 \cdot 5$$

$$f_1 * f_2$$

$$f_1 \le N$$
$$f_2 \ge N$$

```
for (int i=2; i*i<=N; i++)
{
    if(N%i==0) {
        ans.pb(i);
        while(N%i==0) N/=i;
}
if(N>1) ans.pb(N);
```

$\{2, 5\}$

$\frac{\cancel{10}}{\cancel{2}} = 5$ 20

$3 * 3 \leq 5$

$20 = \cancel{2}^2 \; 5$

8 → 2

$N/2$

~~$i*i \leq N$~~

$N = p_1 * p_2$

$\boxed{i * i <= N}$ break $p_1 < p_2$

$\boxed{p_1 * p_1 <= N}$ $N = p_1^2$

① GCD & LCM

GCD (a, b) :

```
for (int i = 1; i <= min(a,b); i++) {
      if ( a%i == 0 && b%i == 0) {
            ans = i;
      }
}
            return ans;
```

O(min (a,b));

Euclid Algorithm.

$$\left\{ gcd(a,b) = gcd(a, a+b), \; gcd(a, a-b) \atop gcd(a+b, b) \quad gcd(a-b, b); \right\}$$

(10, 4) = (14, 4) (6, 4)

2 2 2

$$\gcd(a,b) \overset{a>b}{=} \gcd(a-b,\ b)$$

$$\Downarrow$$

$$\gcd(a-2b,\ b)$$

$$\Downarrow$$

$$\gcd(a-3b,\ b)$$

$$\vdots$$

$$\gcd(a-fb,\ b)$$

$$\gcd(a\%b,\ b)$$

$a - fb < b$

$$\boxed{\gcd(a,0) = a}$$

$$\gcd(a,b) = \gcd(b,a)$$

$$\gcd(a,b) = \gcd(b,\ a\%b)$$

$a>b$

```
gcd(a,b) {
  if (b==0) return a;
  return (b, a%b);
}
```

Time:

$O(\log(\max(a,b)))$

$\log_{\phi}(a,b)$

ϕ: golden ratio:

1.

$$\gcd(a,b) * lcm(a,b) = a*b.$$

$$= \frac{a*b}{\gcd(a,b)}$$

$(24,\ 30)$ 6

$\Downarrow$

$(24,\ 6)$ 6

$\Downarrow$

$(0,\ 6) \Rightarrow 6$

$$\boxed{\_\ \_\ \gcd(a,b)}$$

Prime Seive.

L - R

count of primes in L to R.

L, R

$\boxed{1 \le L \le R \le 10^6}$

$Q \le (10^5)$

2x3

3x2

3 x 3

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~	11	~~12~~
13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~	~~21~~	~~22~~	23	~~24~~
~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~	31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~
37	~~38~~	~~39~~	~~40~~	41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~

$sz = 10^6$

5x5

5x4

2x10

2x2x5

2x10

```
Primes[10^6+7] {True};

Primes[0] = Primes[1] = false;

for (int i = 2; i <= sz; i++) {
    [if (Prim[i] == false) continue;]
    for (int j = 2*i; j <= sz; j += i) {
        Prime[j] = false;
    }
}
```

i

L-1 → y

R → x

x - y

Time: $O(N \log\log N)$.

$\sqrt{50}$

```
for(i=2;i*i<=sz; i++){
    if(prime(i)==false) continue;
  for(int j=i*i; j<=sz; j+=i)
        Prime[j]=false;
  }
}
```

$(N \log \log N)$

Segmented Sieve:

$1 \le L \le R \le 10^{12}$

count primes in L to R

$1 \le R-L \le 10^6$

$10^{12} \to$ Array ✗

$1 - 10^{12}$

$(10^{12}-10^6, 10^{12})$

$sz \Rightarrow \sqrt{sz}$

$10^{12} \Rightarrow \sqrt{10^{12}} = 10^6 \to$ prime?

$\Downarrow$

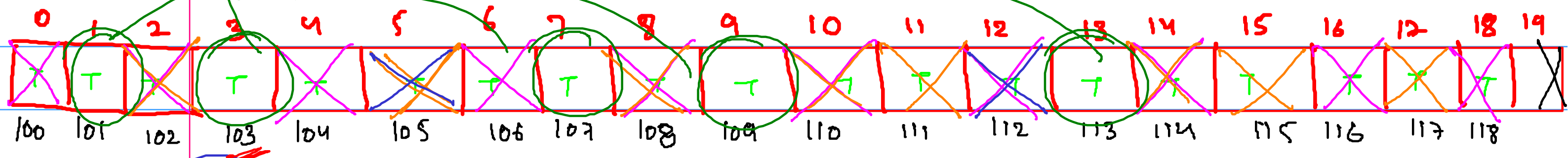
Normal Seive

10^6

$\sqrt{118} = 10$

5 . R-L+1

100 (L) 118 (R)



Range

$N \log N$

R-L+1

5

2

$\frac{102}{100}$ 2

100

$2 \to 4$, $2 \to 100$

Dummy 100 → 100-L

4 $\{2, 3, 5, 7\}$ 100 102 105 44 25

$2*2 = 4$

10

3

$p \to p*p < L$

$p \to \text{ceil}\left(\frac{L}{p}\right) \cdot p$

100

$\text{ceil}\left(\frac{100}{3}\right) * 3$

$(34) * 3$

$\to 2$ $\left(\frac{100}{2}\right) \times 2$

100

100

$(50+1) * 2 = 102$

$\frac{L}{p} =$

$\frac{\cancel{100}}{\cancel{2}} = \lfloor 50 \rfloor = 50+1$

$\frac{a}{b} = \left(\left(\frac{a+b-1}{b}\right) * b\right)$ $\lceil 50 \rceil =$

$L + L \% b$

$p \to p*p$ 4

$p \to \left(\frac{L+p-1}{p}\right) \cdot p$

121 max 100 110

11 121 110

1 $\sqrt{R}$ prime

2 Array[R-L+1]

3 Prime → p → $p*p$, p^2 ; $\left(\frac{L+P-1}{p}\right)*p$ } max

1 — 10^6

2 L — R (Arr(i) == true) ans++;

$\sqrt{sz}$ sz

b)

L — R

1 — $\sqrt{R}$

10^5

$\sqrt{R}$

$\boxed{L-R}$ $\sqrt{R}$

p → $p*p$ 1 — R

P

L —

$\boxed{ceil\left(\frac{L}{P}\right)*p}$ ⇒ $\left(\frac{L+P-1}{p}\right)*p$

121

P — 11 → 121, 110

110 Drum ordering

L - 100 R = 130

11 → p → $p*p$ 121

110 = 11×10

$L \le p$ $2\times5\times11$

$P_2 \cdot p$ p → $p*\frac{L}{p_1}$ p_2 $p_1 \cdot p_2 \cdot p$ 2×55 5×22

```cpp
vector<int> primes;
const int sz = 100005;
bool P[sz];
void pre(){
   for(int i = 2; i < sz ; i++)P[i] = true;
   for(int i = 2; i* i < sz ; i++){
      if(P[i] == false)continue;
      for(int j = i * i; j < sz ; j += i)
         P[j] = false;
   }
   for(int i = 2 ; i < sz ; i++){
      if(P[i])primes.push_back(i);
   }
}
```

```cpp
void solve(int T) {
   int L, R;
   cin >> L >> R;
   int N = R - L + 1;
   int Arr[N];
   for(int i = 0; i < N ; i++)Arr[i] = true;

   for(auto p : primes){
      if(p * p > R)break;
      int s1 = p * p;
      int s2 = ((L + p - 1) / p) * p;
      int start = max(s1, s2);
      for(int j = start - L; j < N; j += p){
         Arr[j] = false;
      }
   }

   for(int i = 0; i < N ; i++){
      int o = i + L;
      if(o != 1 and Arr[i]){
         cout << o <<endl;
      }
   }
   cout <<endl;
```

$$N = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{3} \cdot p_4^{\alpha_4} \cdots p_k^{\alpha_k}$$

$p_1, p_2, p_3 \ldots p_k$ are prime.

Number of factors: $\tau(n) = (\alpha_1+1)\cdot(\alpha_2+1)\cdot(\alpha_3+1)\cdots$

$$= \prod_{i=1}^{k} (\alpha_i+1)$$

$$36 = 2^2 \cdot 3^2 = (2+1)\cdot(2+1) = 9$$

(1, 2, 3, 4, 6, 9, 12, 18, 36)

1+ 37 18 55 48 8 56

Sum of factors: 3 6 91

$$\prod_{i=1}^{k} \left(\frac{p_i^{\alpha_i+1} - 1}{p_i - 1} \right)$$

$$\frac{2^3-1}{2-1} \cdot \frac{3^3-1}{3-1}$$

$$7 \cdot 13 = 91$$

Product of factor: $N^{\frac{\tau(n)}{2}} = 36^{\left(\frac{9}{2}\right)}$

$= \left(\sqrt{36}\right)^{9} = 6^{9}$

⊙ Conjectures:

⊙ Goldbach's Conjecture: each even integer $n > 2$ can be represented as $n = a + b$, so that both a & b are prime.

⊙ Twin prime Conjecture: There are infinite number of pairs such that $p, p+2$ both are prime.

$(3,5)$ $(5,7)$ $(11,13)$ $(17,19)$

⊙ Legendre Conjecture: There is always a prim number b/w n^2 & $(n+1)^2$, $n \in$ positive integer.

$$\left\{\begin{array}{l} 16 = 13 + 3 \\ 14 = 7 + 7 \\ 20 = 13 + 7 \end{array}\right\}$$

$P_1 = \alpha_1$ $\quad P_2 = \alpha_2$ $\quad P_3 = \alpha_3$ $\quad P_4 = \alpha_4$

P_1^0 $\quad P_2^0$ $\quad P_3^0$ $\quad P_4^0$

P_1^1 $\quad P_2^1$ $\quad P_4$

P_1^2 $\quad P_2^2$

P_1^3 $\quad P_2^3$

$\ldots P_1^{\alpha}$ $\quad \ldots$

101010

all even

(101010)

1 odd

(001010)
(111010)
(100010)
(101110)
(101000)
(101011)

① Modular Arithmetic :

MOD = $10^9 + 7$ (Prime).

Ans % (MOD)

$(a+b) \% MOD$

$(0 \le ans < MOD)$

Addition

$$(a+b)\%MOD = (a\%MOD + b\%MOD)\%MOD$$

Subtraction

$$(a-b)\%MOD = ((a\%MOD - b\%MOD)\%MOD + MOD)\%MOD$$

$$(5-9)\%7 = (-4\%7) = -4$$

$$(-4+7)\%7 = 3\%7 = 3$$

Multiplication:

$$(a*b)\%MOD = (a\%MOD * b\%MOD)\%MOD$$

Division

$$\left(\frac{a}{b}\right)\%MOD \neq \left(\frac{a\%MOD}{b\%MOD}\right)\%MOD \quad X.$$

$$= (a * modIn(b, MOD))\%MOD$$

Mod Inverse

$$5 \times \frac{1}{5}$$

$$\boxed{5 \times b = 1}$$

Multiplicative MOD Inverse

$$\boxed{(a \times b)\%MOD = 1}$$

$a = 17$ MOD = 5

$a = 17$ $\quad$ $MOD = 5$

$(a \times 1) \% MOD = (17 \times 1) \% 5 = 2$

$(17 \times 2) \% MOD = 34 \% 5 = 4$

$(17 \times 3) \% MOD = 51 \% 5 = 1$

17 modinverse = 3 $\quad$ $MOD = 5$

$\left(\frac{3}{17}\right) \% MOD = (3 * ModI(17,5)) \% 5$

$= (3 * 3) \% 5 = 9 \% 5 = 4$

Mod Inverse ?

① Fermats Little Theorem.

Binary Exponentiation

$(A^b) \% MOD$ $\quad$ A^b

$A^{b/2} \cdot A^{b/2}$

$A^{b/4} \cdot A^{b/4}$

b odd

A^b $\quad$ b odd

$A \cdot A^{b/2} \cdot A^{b/2}$

```
// fastExpo(a, b, mod)
{ if (b == 0) return 1;
  ll ans = fast(a, b/2, mod);
  ans = (ans * ans) % MOD;
  if ( b % 2 == 1)
      ans = (a * ans) % MOD;
  return ans;
}
```

$O(\log b)$ $\quad$ Space: $(\log b)$ stack

$\left(\frac{y \% MOD}{x \% MOD}\right)^{2^{64}} \% MOD.$ $\quad$ $2^{100} \rightarrow$

$2^{50} \cdot 2^{50} = x$

$(a * b) \% MOD$

$(a \% MOD \cdot b \% MOD) \% MOD$

LONG = 4e18 MOD = 10^9+7

$X = 10^9+6$
$Y = 10^9+6$
10^{18} {Store}

$10^{18} \% (10^9+7) < (10^9+7)$

- Fermats Little Theorem.

a, m find mod inverse of a given that m prime.

$$\boxed{\text{modIn}(a) = \{a^{m-2} \% m\}}$$

↳ binary Expon.

$a = 17$, $M = 5$

$= (17)^3 \% 5$
$= ((17\%5) \cdot (17\%5) \cdot (17\%5))\%5$
$(2 \cdot 2 \cdot 2)$ $8\%5 = 3$

$m \rightarrow$ prime

$$\text{modIn}(a) = a^{\phi(m)-1} \% M$$

$\phi(m)$: Euler Toetent function. $a^{m-1-1} \% \text{MOD}$
$(a^{m-2} \% \text{MOD})$

↳ No. of integer X $(1 \le X \le M)$ that are coprime with m.

$\phi(7) = m-1$ $\phi(\text{prime}) = \text{prime}-1$

10

$\phi(10)$

Ans = N

$10 - \frac{10}{2} = 5$

(1, 10) = 1
(2, 10) = 2
(3, 10) = 1
(4, 10) = 2
(5, 10) = 5
(6, 10) = 2
(7, 10) = 1
(8, 10) = 2
(9, 10) = 1
(10, 10) = 10

(N%i==0)

N/i

Ans i

```
for(i=2; i*i<=N; i++){
    if(N%i==0){
        while(N%i==0)
            N/=i;
        Ans-=(Ans)/i;
}
if(N>1)
    Ans -= (Ans/n)
```

5

Complexity $O(\sqrt{N})$

3 4 6

$\frac{10}{2}$

5

$5 -= (5/5) = 4$

6/5

10

4

$5 - 1 = 4$

1 —— 10

$\frac{10}{2}$

10^9+7

$\phi(p_1 \cdot p_2) = \phi(p_1) \cdot \phi(p_2)$

$\phi(N_1 \cdot N_2) = \phi(N_1) \cdot \phi(N_2) \cdot \frac{d_1}{\phi(d_1)}$

6

$d_1 = \gcd(N_1 \cdot N_2)$

(1, 5)

$6 - \frac{6}{2} = \frac{3}{3}$

$3 - 1 = 2$

```
phi[sz];          phi[i] = i;

for( int i=2; i<= sz; i++ ){
    if ( phi[i] == i )
    for( int j= i; j<sz; j+=i)
        phi[j] -= (phi[j]/i);
    }
}
```

$$\boxed{O(N \log \log N)}$$

Mod Inverse

Linear Diaphantine Equation.

$Ax + By = C$ $\quad$ $A, B, C \in I$

↳ Integral Solution

$$\phi(\text{prime}) = \text{prime} - 1$$

$$\phi(\underbrace{p_1}_{\text{prime}} \cdot \underbrace{p_2}_{\text{prime}}) = \phi(p_1) \cdot \phi(p_2)$$

$$\phi(N_1 \cdot N_2) = \phi(N_1) \cdot \phi(N_2) \cdot \frac{d}{\phi(d)}$$

$d = \gcd(N_1 \cdot N_2)$

4

$$\sum_{d|N} \phi(d) = N$$

$N = 10$ $\{1, 2, 5, 10\}$

$\phi(1) + \phi(2) + \phi(5) + \phi(10) = 10$

$\boxed{1 + 1 + 4 + 4} = \underline{10}$

- Linear Diophantine Equation:

$Ax + BY = C$

Integral Sol^n will exis iff gcd(A,B) divides C.

$3x + 6y = 12$ $(x=2, y=1)$

$\{Ax + BY = \cancel{G}\} \; \underline{gcd(A,B)}$ && (A,B) are co-prime.

$\Downarrow$

$\boxed{AX + BY = \cancel{B} \; \underline{GCD(A,B)}}$ $(gcd(A,B) = 1)$

$4x + 5Y = 1$ $\underline{\{x = -1, y = 1\}}$

$\hookrightarrow$ $AX + BY = gcd(A,B)$

$AX + BY = gcd(\overset{a'}{B}, \overset{b'}{A\%B}) = a'x + b'y$

$AX + BY = BX + (A\%B)Y$

~~gcd(A,B) = 1~~

$Ax + By = \text{GCD}(A, B)$

$Ax + By = \gcd(A, B)$ ——— (1)

$\gcd(a, b) = \gcd(b, a \% b)$ — (1) $5 \% 3 = 2$

$\gcd(a', b')$ $\quad 5 - \left(\frac{5}{3}\right) \cdot 3$

$\gcd(a', b') = a'x + b'y$ using Eq^n (1)

$\gcd(B, A \% B) = BX + (A \% B) Y$

$= BX + \left(A - \left\lfloor \frac{A}{B} \right\rfloor \cdot B\right) \cdot Y$

$= BX + AY - \left\lfloor \frac{A}{B} \right\rfloor \cdot B \cdot Y$

$\gcd(B, A \% B) = B\left(X - \left\lfloor \frac{A}{B} \right\rfloor Y\right) + AY$. $\{x_1, y_1\}$

$Ax + By = \left\{ A y_1 + B\left(x_1 - \left\lfloor \frac{A}{B} \right\rfloor \cdot y_1 \right) \right\}$

$$\left\{ \begin{array}{l} X = y_1 \\ Y = x_1 - \left\lfloor \frac{A}{B} \right\rfloor \cdot y_1 \end{array} \right\}$$

x_1, y_1

$\left(y_1, x_1 - \left\lfloor \frac{A}{B} \right\rfloor \cdot y_1\right)$.

(Extended Euclid Algorithm)

$ax + by = \text{Gcd}(a, b)$

$ax + 0 \cdot y = a\ \text{G}(a, 0)$

```
{gcd, x1, y1} f(int a, int b}
{       if(b==0) {
              {a, 1, -1};
   }
        {g1, x1, y1} = f(b, a%b);
```

a, b

return $\{g, x, y\} : \{g, y_1, x_1 - (\frac{A}{b}) y_1\}$;

};

⊙ Mod Inverse.

$$(A\cdot B)\ \%\ MOD = 1$$

$$(A\cdot B - 1)\ \%\ MOD = 0$$

$$A\cdot B - 1 = MOD\cdot q \qquad \{q \in I\}.$$

$$A\cdot B - MOD\cdot q = 1$$

$$A\cdot B + MOD\cdot Q = 1 \qquad \{q = -Q\}.$$

$$\boxed{A(X) + MOD(Y) = 1}$$

$$Ax + BY = 1 \qquad \{\ gcd(A, MOD) = 1)$$

$$\boxed{A(x) + MOD(y) = 1}$$

A & MOD should be Co-prime.

X, y, gcd

(MOD inverse of A);

10 + 0

9 − 4

$A_i + A_i\ \%\ MOD$

$+ A_j - A_j\ \%\ MOD$

$A_j + (A_i - A_j)\ \%\ MOD$

$(A_j) + (A_i - A_j + MOD)\ \%\ MOD.$

$A_i + A_j + [(A_i)\ \%\ MOD - (A_j)\ MOD] + MOD]\ \%\ MOD$

⊙ Sample Input 1

```
4
2 18
12 1
3 5
4 5 6
5 4
79 29 80 58 80
3 20
33 46 56
```

0   10   10   MOD = 17

$0 + 10$ $10 + 10 + 0 = 20$

$+ (0 - 10) \% 17$

17   7

3, 5, ~~0~~,

(4, 5, 6)

$6 \times 2 = 12$

$5 + 6 + (0 - 1 + 7)$

15

10   $(2 \times MAX,$ MAX_1 $MAX_2)$

$A_i + A_j + (A_i - A_j) \% MOD$

MAX   MAX   MAX

SMAX   SMAX

$A_i + A_j + (A_i - A_j) \% MOD$

$(A-x)$ $(A-x)$

MOD =   MOD = 7

{6, 7}

$x = x = y =$

$(B + A + y)$

$(B + A + y)$

$B + A - x + (A_i - (A_j - x)) \% MOD$

$+ ((A_i - A_j) \% MO + y) \% MOD$

$B + A - x + (y + x) \% MOD$

$y + x$ $(B + A - y)$

$$A + B + \underbrace{(A-B)\%MOD}_{y} \Rightarrow (A+B+y)$$

$$A + (B-x) + (A-(B-x))\%MOD$$

$$A+B-x+((A-B)\%MOD + x)\%MOD)$$

$$A+B-x+\underbrace{(y+x)\%MOD}_{(y+x)}$$

$$A+B-\cancel{x}+y+\cancel{x} = A+B+y$$

⊙· L − R Gcd

| a_0 a_1 a_2 a_3 | ~~a_4 a_5 a_6 a_7 a_8~~ | a_9 - - - - a_{n-1} |

L R

left[i]: 0 — i gcd. L R

Right[i]- n-1 − i gcd.

gcd. (GCD(L-1) ; GCD(R+1))

$$B + MAX + \underbrace{(B-MAX)\%MOD}_{y} \rightarrow B+MAX+y.$$

$$B+(MAX-x)+(B-(MAX-x))\%MOD$$

$$B+(MAX-x)+((\underbrace{B-MAX)\%MOD}_{y}+x)\%MOD$$

$$(B+MAX-\cancel{x}+\cancel{(y+x)\%MOD}) \Rightarrow y+x \quad = (B+MAX+y)$$

Q.

1	2	3	4	5
4	2	7	8	8

Q1 = 1 4 = 8

Q2 : 2 4 = 4

Q3 : 2 3 = 1

gcd(2, 8) = 2

Left Gcd

4 2 1 1 1

1 1 1 8 8

Right

gcd(a, b, c)

gcd(gcd(a, b), c)

1. Segment Tree
2. Sparse Table
3. Sqrt Decomposition

4. BIT

1. A_0 A_1 A_2 A_3 A_4 A_5

Q : X

A. Row GCD

time limit per test: 2 seconds

memory limit per test: 512 megabytes

input: standard input

output: standard output

You are given two positive integer sequences $a_1, \dots, a_n$ and $b_1, \dots, b_m$. For each $j = 1, \dots, m$ find the greatest common divisor of $a_1 + b_j, \dots, a_n + b_j$.

Input

The first line contains two integers n and m ($1 \le n, m \le 2 \cdot 10^5$).

The second line contains n integers $a_1, \dots, a_n$ ($1 \le a_i \le 10^{18}$).

The third line contains m integers $b_1, \dots, b_m$ ($1 \le b_j \le 10^{18}$).

Output

Print m integers. The j-th of them should be equal to GCD$(a_1 + b_j, \dots, a_n + b_j)$.

Example

input

```
4 4
1 25 121 169
1 2 7 23
```

= 1 4

output

```
2 3 8 24
```

+ - - - - - $A_{n-1} + X$

1 25 121 169

q (1, 2, 7, 23)

(2, 3, 8, 24)

1+1 = 2

1+2 = 3

1+7 = 8

1+23 = 24

8

$1+4=5$ $\quad 25+4=29$ $\quad 121+4=125$

$169+4=173$ $\quad =1$

$\gcd(a+x, b+x, c+x)$ $\qquad \gcd(a,b)=\gcd(a,b-a)$

$\gcd(\gcd(a+x, b+x), \gcd(a+x, c+x))$

$\gcd(\gcd(a+x, b+x-(a+x)), \gcd(a+x, c+x-(a+x)))$

$=\gcd(\gcd(a+x, b-a), \gcd(a+x, c-a));$

$\Rightarrow \gcd(a+x, b-a, c-a);$

$\Rightarrow \gcd(a_0+x, \underbrace{a_1-a_0, a_2-a_0, a_3-a_0, a_4-a_0 \cdots, a_{n-1}-a_0}_{f})$

$=\gcd(a_0+x, f);$

$f=\gcd(a_1-a_0, a_2-a_0, a_3-a_0 \ldots)$

x

$\gcd(f, a_0+x);$

$O(N.\log A_i)$

$aaaa = a$

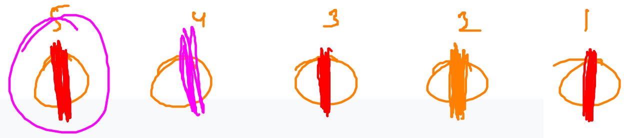
$l \qquad r$

$\Rightarrow$ A B B A C D A

op = 1 + help(l+1, r);

for(k = l+1; k <= r; k++)
if (s[k] == s[l]) min(help(l, k-1) + help(k+1, r).)

Example 1:




```
Input: arr = [3,5,1,2,4], m = 1
Output: 4
Explanation:
Step 1: "00100", groups: ["1"]
Step 2: "00101", groups: ["1", "1"]
Step 3: "10101", groups: ["1", "1", "1"]
Step 4: "11101", groups: ["111", "1"]
Step 5: "11111", groups: ["11111"]
The latest step at which there exists a group of size 1 is step 4.
```

ans = 1
ans = 2
ans = 3
$\boxed{ans = 4}$

Example 2:

```
Input: arr = [3,1,5,4,2], m = 2
Output: -1
Explanation:
Step 1: "00100", groups: ["1"]
Step 2: "10100", groups: ["1", "1"]
Step 3: "10101", groups: ["1", "1", "1"]
Step 4: "10111", groups: ["1", "111"]
Step 5: "11111", groups: ["11111"]
No group of size 2 exists during any step.
```

-1

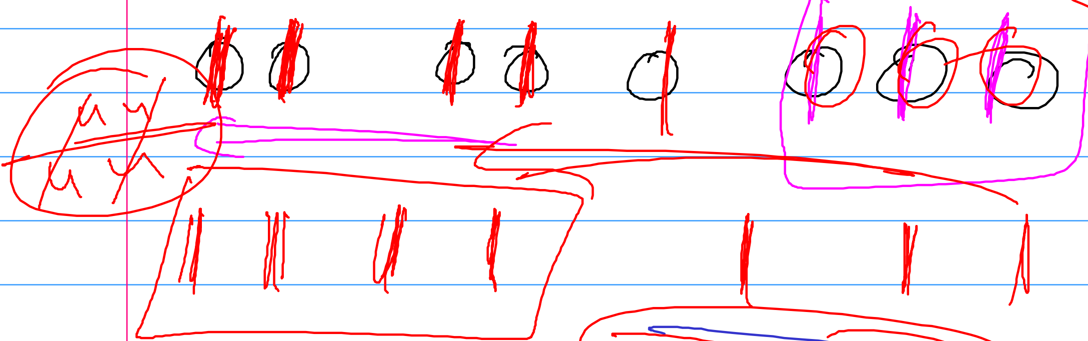
1 2 3 4 5
1 1 1 1 1

2
(3, 3) = 1
(1, 1) = 1
(5, 5) = 1

[1, 2, 3, 4, 5]
[3, 1, 5, 4, 2]

1 2 3 4 5

m = 2
(3, 4, 5) = 3
(1, 2, 5) = 5
M = 3
(2, 3, 5)



M = 4

k = 2 p = 2

4 4 4 = 3

4 4
4 4 4