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Experiment No-06

Topic- HOTELLING'S T^2 DISTRIBUTION

Problem- Suppose we are given the data matrix for the random sample of size n=3 from a BVND as $\underset{\sim}{X}=\begin{bmatrix} 6 & 9 \\ 10 & 6 \\ 8 & 3 \end{bmatrix}$

Evaluate the value of the Hotelling's T^2 statistics for $\underset{\sim}{\mu}=\begin{pmatrix} 9 \\ 5 \end{pmatrix}$ and hence test the hypothesis

$H_0:\underset{\sim}{\mu}=\begin{pmatrix} 9 \\ 5 \end{pmatrix}$

Theory-

Let, $\underset{\sim}{X}_1$, $\underset{\sim}{X}_2$,............ $\underset{\sim}{X}_n$ be a random sample from a $N_p(\underset{\sim}{\mu}, \Sigma)$ population.

Then, $\underset{\sim}{\bar{X}} = \begin{pmatrix} \overline{X_1} \\ \overline{X_2} \\ \vdots \\ \overline{X_p} \end{pmatrix}$ is the sample mean vector

Where, $\bar{X_i} = \frac{1}{n}\sum_{i=1}^{n} X_{ij}$, the sample mean of X_i , i=1,2,……p and

$S_{ij} = \frac{1}{n-1}\sum_{i=1}^{n}(X_{ik} - \bar{X_i})(X_{jk} - \bar{X_j})$

The sample mean square of X_i ,X_j i.j=1,2,p

$S=(S_{ij})_{pxp}$

The Hotelling's T^2-statistic is given by

$T^2 = \sqrt{n}\,(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu})' \sqrt{n}\,(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu})$

The size of the critical region is given by (α) for testing $H_0{:}\underset{\sim}{\mu} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$ is given bys

$\alpha = P[T^2/\mu=\mu_0 > \frac{(n-1)p}{n-p} F_{p,n-p(\alpha)}]$

$= P[\sqrt{n}\,(\underset{\sim}{X} - \underset{\sim}{\mu}_0)' S^{-1} \sqrt{n}\,(\underset{\sim}{X} - \underset{\sim}{\mu}_0) > \frac{(n-1)p}{n-p} F_{p,n-p(\alpha)}$

Finally the calculated value is compared with the tabulated value and conclusions are drawn accordingly.

Calculation-

The R-programming for obtaining a solution of the given problem is as follows -

```
x=array(c(6,10,8,9,6,3),dim=c(3,2))

x

x1=x[,1]

x1

x2=x[,2]

x2
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x1_bar=mean(x1)

x1_bar

x2_bar=mean(x2)

x2_bar

x_bar=array(c(x1_bar,x2_bar),dim=c(2,1))

x_bar

n=3

s11=var(x1)*(n/(n-1))

s11

s12=cov(x1,x2)*(n/(n-1))

s12

s21=s12

s21

s22=var(x2)*(n/(n-1))

s22

s=array(c(s11,s12,s21,s22),dim=c(2,2))

s

mu=array(c(9,5),dim=c(2,1))

mu

d=x_bar-mu

d

t2_obs=n*t(d)%*%solve(s)%*%d

t2_obs

p=2

t2_tab=(((n-1)*p)/(n-p))*qf(0.95,2,1)
```

t2_tab

Conclusion-

(1) The value of Hotelling T^2 statistic for $\mu=\begin{pmatrix}9\\5\end{pmatrix}$ is 0.5185185

(2) The calculated value of the Hotelling's T^2 statistic is 0.5185185 and the tabulated value is 798.

Since, the calculated value of T^2(i.e. 0.5185185) is less than the tabulated value of F (i.e. 798) at 5% level of significance ; Hence we accept our null hypothesis and conclude that $\mu=\begin{pmatrix}9\\5\end{pmatrix}$.