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Experiment No-07

Topic- PAIRED HOTELLING'S T^2 DISTRIBUTION GENERALISATION OF STUDENTS t-TEST)

Problem- A group of 12 hypertension patients were administered 5 mg of the drug 'Metoprolol' . The drug is used to test high blood pressure and fast heart rate . The data on systolic blood pressure measured in mm/mg(X_1) and heart rate is measured in 'beats per minute' (X_2) of the patients before and 1 hour after administering the drug is given below .

Serial No.	**Before administering the drug**			
	BP ($^1X_1$)	**Heart rate ($^1X_1$)**	**BP ($^2X_1$)**	**Heart rate ($^2X_2$)**
1	160	88	155	83
2	180	99	162	85
3	130	80	125	70
4	175	92	165	89
5	140	83	137	80
6	110	77	103	75
7	155	85	150	81
8	145	83	132	80
9	135	81	130	73
10	117	78	112	71
11	169	90	160	85
12	150	82	147	78

Has the drug been able to significantly reduce the BP and heart rate on average ? Also find the 95% simultaneous confidence interval for the individual mean difference .

Theory-

Suppose $(X_1, X_2, \ldots\ldots\ldots\ldots\ldots X_p)' \sim N_p(\underset{\sim}{\mu}, \Sigma)$ are measured at two stages as represented below –

1st stage-

X_1	X_2	.....	X_p
${}^1X_{11}$	${}^1X_{21}$	.....	${}^1X_{p1}$
${}^1X_{12}$	${}^1X_{22}$	.....	${}^1X_{p2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
${}^1X_{1n}$	${}^1X_{2n}$	.....	${}^1X_{pn}$

2st stage-

X_1	X_2	.....	X_p
${}^2X_{11}$	${}^2X_{21}$	.....	${}^2X_{p1}$
${}^2X_{12}$	${}^2X_{22}$	.....	${}^2X_{p2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
${}^2X_{1n}$	${}^2X_{2n}$	.....	${}^2X_{pn}$

The sample of differences is obtained as follows-

X_1	X_2	.....	X_p
${}^1X_{11}-{}^2X_{11}=d_{11}$	${}^1X_{21}-{}^2X_{21}=d_{21}$	.....	${}^1X_{p1}-{}^2X_{p1}=d_{p1}$
${}^1X_{12}$	${}^1X_{22}-{}^2X_{22}=d_{22}$	.....	${}^1X_{p2}-{}^2X_{p2}=d_{p2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
${}^1X_{1n}-{}^2X_{1n}=d_{1n}$	${}^1X_{2n}-{}^2X_{2n}=d_{2n}$	.....	${}^1X_{pn}-{}^2X_{pn}=d_{pn}{}^2X_{pn}$

Here the observed differences are

$\underset{\sim}{d}_j = (d_{j1}, d_{j2}, \ldots\ldots\ldots\ldots d_{jn})'$; j=1,2,.......p

For a test of the hypothesis $H_0: \underset{\sim}{D} = \underset{\sim}{0}$ (D is the population difference) against $H_1: D \neq \underset{\sim}{0}$ for a $N_p(\underset{\sim}{\mu}, \Sigma)$ population, H_0 is rejected at the level of significance α

if observed T^2 > tabulated T^2 .

Or, $\quad n\overline{\underset{\sim}{d}}' S_d^{-1} \overline{\underset{\sim}{d}} > \frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)$

Where $F_{p,n-p}(\alpha)$ is the upper $(100\alpha)^{th}$ percentile of the F distribution with p and (n-p) degrees of freedom

Here,

$$\overline{\underset{\sim}{d}}' = \begin{pmatrix} \frac{1}{n}\sum_{j=1}^{n} d_{1i} \\ \frac{1}{n}\sum_{j=1}^{n} d_{2i} \\ \vdots \\ \frac{1}{n}\sum_{j=1}^{n} d_{pi} \end{pmatrix} \text{ and } S_d = (Sij)_{p\times p} \text{ where, } S_{ij} = \frac{1}{n-1}\sum_{k=1}^{n}\left(d_{ik} - \overline{d}_i\right)\left(d_{jk} - \overline{d}_j\right)$$

Also, an 100(1-α)% simultaneous confidence interval for the individual mean difference D_i are given by

$$d_i \pm \sqrt{\frac{(n-1)p}{n-p} F_{p,n-p}(\alpha)} \times \sqrt{\frac{s_{di}^2}{n}}$$

Where d_i is the i^{th} element of $\overline{\underset{\sim}{d}}$ and s_{di}^2 is the i^{th} diagonal element of S_d.

Calculation-

Here, the first stage of observation represents the values recorded before administering the drug. The second stage of observation represents the values recorded after administering the drug. The R-program for obtaining a solution of the given problem is as follows –

```
x11=c(160,180,130,175,140,110,155,145,135,117,169,150)

x12=c(88,99,80,92,83,77,85,83,81,78,90,82)

x21=c(155,162,125,165,137,103,150,132,130,112,160,147)

x22=c(83,85,70,89,80,75,81,80,73,71,85,78)

D1=x11-x21
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D1

D2=x12-x22

D2

D1_bar=mean(D1);D2_bar=mean(D2)

D1_bar

D_bar=array(c(D1_bar,D2_bar),dim=c(2,1))

D_bar

n=12

s11=var(D1)*(n/(n-1))

s11

s12=cov(D1,D2)*(n/(n-1))

s12

s21=s12

s21

s22=var(D2)*(n/(n-1))

s22

Sd=array(c(s11,s12,s21,s22),dim=c(2,2))

Sd

p=2

t2_cal=n*t(D_bar)%*%solve(Sd)%*%D_bar

t2_cal

t2_tab=(((n-1)*p)/(n-p))*qf(0.95,p,n-p,0)

t2_tab

LCL1=D_bar[1]-(sqrt(t2_tab)*sqrt(Sd[1,1]/n))

LCL1
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UCL1=D_bar[1]+(sqrt(t2_tab)*sqrt(Sd[1,1]/n))

UCL1

LCL2=D_bar[2]-(sqrt(t2_tab)*sqrt(Sd[2,2]/n))

LCL2

UCL2=D_bar[2]+(sqrt(t2_tab)*sqrt(Sd[2,2]/n))

UCL2

Conclusion-

Since the calculated value of HOTELLING'S T^2-statistic (i.e. 40.89267) is greater than the tabulated value vat 5% level of significance (i.e. 9.026206) , so we accept the null hypothesis and conclude that the drug has been able to significantly reduce the BP and Heart Rate on average

Also the 95% simultaneous confidence intervals for the individual mean differences are

Mean difference D_i	**95% confidence interval**
D_1=7.333333	(3.276116,11.39055)
D_2=5.666667	(2.473723,8.85961)