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Experiment No-01

Topic- DRAWING OF A RANDOM SAMPLE AND ESTIMATION OF PARAMETERS OF A BIVARIATE NORMAL DISTRIBUTION

Problem- Draw a random sample of size 10 from a bivariate normal population having the mean vector and covariance matrix as

$$\underset{\sim}{\mu} = \begin{pmatrix} 12 \\ 15 \end{pmatrix} \qquad \Sigma = \begin{pmatrix} 64 & 40 \\ 40 & 25 \end{pmatrix}$$

Hence, estimate the parameters based on the random sample thus obtained (assuming that the parameters are unknown).

Theory-

The procedure of drawing a random sample from the bivariate normal distribution say $(X,Y) \sim N(\mu_x, \mu_y, \sigma_x, \sigma_y, \rho)$ is as follows:

Step 1- We write down the marginal distribution of X, which is $X \sim N(\mu_x, \sigma_x)$

Step 2- We draw a random number from uniform(0,1) distribution and we denote it by R

Step 3- We consider the CDF of X given by

$F(X) = P(X \le x)$ and we set

$$F(x) = R$$
$$x = F^{-1}(R)$$
$$x = \phi(R)$$

Where, $\phi(.)$ is the quantile function of X. The value of x thus obtained is a random value of X.

Step 4- We write down the conditional distribution of Y/X=x which is given by

$$Y / X = x \sim N(\mu_{Y/X}, \sigma_{Y/X})$$

Where,
$$\mu_{Y/X} = \mu_Y + \rho \frac{\sigma_Y}{\sigma_X}(x - \mu_X)$$
$$\sigma_{Y/X} = \sqrt{\sigma_Y^2 (1 - \rho^2)}$$

And x is the value of X obtained in step 3.

Step 5- Since the conditional distribution of Y/X is a given univariate normal, we repeat the step 2 and 3 in order to obtain a random value of Y/X=x.

$$F_1\left(Y/X\right) = R_1 \Rightarrow Y = F_1^{-1}(R_1) = \phi_1(R_1)$$

Step 6- Step 1-5 are repeated n times in case a random sample of size n from (X,Y) is desired. The 'n' pairs of values $(x_1, y_1), (x_2, y_2), \ldots\ldots, (x_n, y_n)$ thus obtained constitute random sample of size n from the bivariate normal population X,Y . The maximum likelihood estimates of the parameters $\mu_X, \mu_Y, \sigma_X, \sigma_Y, \rho$ are calculated from the sample as shown below-

$$\hat{\mu}_X = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\hat{\mu}_Y = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

$$\hat{\sigma}_X = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$$

$$\hat{\sigma}_Y = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}$$

$$\hat{\rho} = r = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\hat{\sigma}_X\,\hat{\sigma}_Y}$$

<u>Calculation</u>-

The R-program for obtaining the random sample and estimating the parameters from the given BND is as follows-

```
mx=12; my=15; sdx=8; sdy=5

rho=40/(sdx*sdy)

rho

r<-runif(10,0,1)

x<-mat.or.vec(10,1)

for(i in 1:10){

x[i]<-qnorm(r[i],mx,sdx)}

x

my_x<-my+((rho*sdy/sdx)*(x-mx))

sdyx<-sqrt(sdy^2*(1-(rho^2)))

r1<-runif(10,0,1)

y<-mat.or.vec(10,1)

for(i in 1:10){
```

```
y[i]<-qnorm(r1[i],my_x[i],sdyx)}
y
mxhat<-mean(x)
mxhat
myhat<-mean(y)
myhat
sx_hat<-sqrt(var(x))
sx_hat
sy_hat<-sqrt(var(y))
sy_hat
rho_hat<-cov(x,y)/(sx_hat*sy_hat)
rho_hat
```

<u>Conclusion</u>-

The random sample of size 10 from the BND with given parameters $\underset{\sim}{\mu}$ and $\sum$ is given by-

(1.673798, 8.546124), (7.312850, 12.070531), (14.457159, 16.53572), (21.502800, 20.939250), (14.241883, 16.401177), (2.439168, 9.024480), (14.134589, 16.334118), (32.028724, 27.517953), (24.277234, 22.673272), (10.261240, 13.913275)

The value of the estimated parameters are-

$$\hat{\mu}_x = 14.23294$$

$$\hat{\mu}_y = 16.39559$$

$$\hat{\sigma}_x = 9.608884$$

$$\hat{\sigma}_y = 6.005552$$

$$\hat{\rho} = 1$$