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Experiment No-12

Topic- PERIODOGRAM ANALYSIS

Problem- The following are the temperature measurements taken every minute in a chemical reactor.

200,208,208,204,204,207,207,204,202,199,201,198,200,202,203,205,207,211,204,206,203,203,201,198,200,206,207,206,200,203,203,200,200,195,202,204

Construct the periodogram for the periods 36,18,12,9,36/5,6.Also draw up an analysis of variance table showing the mean square associated with these periods and the residual mean square.

Theory-

A way of analyzing a time series from which trend and seasonal variations have been eliminated is to assume that it is made up of sine and cosine waves of different frequencies such as given below.

$$y_t = a_0 + \sum_{i=1}^{n} a_i \cos\frac{2\pi t}{\lambda_i} + \sum_{i=1}^{n} b_i \sin\frac{2\pi t}{\lambda_i} + u_t$$

The objective is to estimate the periodicities λ_i (i.e. to find which are the best value of λ_i to select) and to evaluate the constants $a_1, a_2, a_3, ..., a_n$ and $b_1, b_2, b_3, ..., b_n$. This is known as periodogram analysis.

In periodogram analysis, we take a number of trial periods $\mu_1, \mu_2, \mu_3, ..., \mu_n$ and for each trial period μ_i, we compute the value of indicator function or intensity function. The intensity function is so choosen that when any of the trial period approaches any of true period, it attains a local maximum. The graph of the indictor function against the trial periods is known as the periodogram. From the periodogram, the trial period against which the maximum value of the indicator function occurs is taken to be estimate of the true periodicity.

Suppose the curve to be fitted to the observed time series is given by---

$$y_t = A_0 + A\cos\frac{2\pi t}{\mu} + B\sin\frac{2\pi t}{\mu} \quad , \quad t = 1,2,3,........, n$$

And n=kμ , where k is an integer and μ is a period. Then there will be k-complete rows of μ values of y_t when arranged serially and extra value are not considered in the computation. The n=kμ values can be arranged in a (k×μ) arrays as follows—

$$\begin{array}{cccccc}
y_1 & y_2 & y_3 & \cdots & \cdots & y_\mu \\
y_{\mu+1} & y_{\mu+2} & y_{\mu+3} & \cdots & \cdots & y_{2\mu} \\
y_{2\mu+1} & y_{2\mu+2} & y_{2\mu+3} & \cdots & \cdots & y_{3\mu} \\
\vdots & \vdots & \vdots & \cdots & \cdots & \vdots \\
\vdots & \vdots & \vdots & \cdots & \cdots & \vdots \\
y_{(k-1)\mu+1} & y_{(k-1)\mu+2} & y_{(k-1)\mu+3} & \cdots & \cdots & y_{k\mu}
\end{array}$$

This is known as Buys Ballot table. If n is not a multiple of μ (i.e. $n \neq k\mu$) we continue writing down its rows until there are fewer than μ terms left.

The constants A_0,A,B are obtained by minimizing the sum of squares of errors.

$$\sum_{t=1}^{n}\left(y_t - A_0 - A\cos\frac{2\pi t}{\mu} - B\sin\frac{2\pi t}{\mu}\right)^2$$

With respect to A_0,A and B

The least squares estimates of A_0,A and B are,

$$\widehat{A}_0 = \frac{1}{n}\sum_{t=1}^{n} y_t$$

$$\widehat{A} = \frac{2}{n}\sum_{t=1}^{n} y_t \cos\frac{2\pi\, t}{\mu}$$

$$\widehat{B} = \frac{2}{n}\sum_{t=1}^{n} y_t \sin\frac{2\pi\, t}{\mu}$$

The indicator function corresponding to the trial period μ is given by----

$$I(\mu) = \frac{n}{2}\left[\left\{\widehat{A}(\mu)\right\}^2 + \left\{\widehat{B}(\mu)\right\}^2\right]$$

Which is also known as the indicator function of the trial period μ. I(μ) reaches a local maximum when μ tends to λ (true period). Hence it's a plot of I(μ) versus μ , the value of μ which maximizes I(μ) say μ* is taken to be estimate of λ.

Finally we fit the the following Fourier series model to the given time series

$$y_t = \widehat{A}_0 + \widehat{A}\cos\frac{2\pi t}{\mu^*} + \widehat{B}\sin\frac{2\pi t}{\mu^*} \quad , \quad t = 1,2,3,........, n$$

The ANOVA table showing the mean squares associated with the periods 36, 18, 12, 9, 36/5, 6 is given by -

Trial period	Source of variation	df	SS	MS	Calculated F	Tabulated F
36	1/36	2	I(36)	MS1=I(36)/2	MS1/MSE	F(2,23)
18	1/18	2	I(18)	MS2=I(18)/2	MS2/MSE	F(2,23)
12	1/12	2	I(12)	MS3=I(12)/2	MS3/MSE	F(2,23)
9	1/9	2	I(9)	MS4=I(9)/2	MS4/MSE	F(2,23)
36/5	5/36	2	I(36/5)	MS5=I(36/5)/2	MS5/MSE	F(2,23)
6	1/6	2	I(6)	MS6=I(16)/2	MS6/MSE	F(2,23)
	Error	36-1-12=23	ESS	MSE		
	Total	36-1=35				

Where, TSS= $\sum_{t=1}^{n}(y_t - \bar{y})^2$

Calculation-

For calculating $I(\mu)$ and evaluating the ANOVA, we use the following R-program:

```
y=c(200,208,208,204,204,207,207,204,202,199,201,198,200,202,203,205,207,211,204,206,203,
203,201,198,200,206,207,206,200,203,203,200,200,195,202,204)

a0=mean(y)

a0

m=mat.or.vec(36,1)

n=mat.or.vec(36,1)

#36#

for(i in 1:36){

m[i]=y[i]*cos((2*pi*i)/36)

n[i]=y[i]*sin((2*pi*i)/36)}

a36=(2/36)*sum(m)

a36

b36=(2/36)*sum(n)

b36

i_36=(36/2)*((a36^2)+(b36^2))

i_36

#18#

for(i in 1:36){

m[i]=y[i]*cos((2*pi*i)/18)

n[i]=y[i]*sin((2*pi*i)/18)}

a18=(2/36)*sum(m)

a18

b18=(2/36)*sum(n)
```

```
b18
i_18=(36/2)*((a18^2)+(b18^2))
i_18
#12#
for(i in 1:36){
m[i]=y[i]*cos((2*pi*i)/12)
n[i]=y[i]*sin((2*pi*i)/12)}
a12=(2/36)*sum(m)
a12
b12=(2/36)*sum(n)
b12
i_12=(36/2)*((a12^2)+(b12^2))
i_12
#9#
for(i in 1:36){
m[i]=y[i]*cos((2*pi*i)/9)
n[i]=y[i]*sin((2*pi*i)/9)}
a9=(2/36)*sum(m)
a9
b9=(2/36)*sum(n)
b9
i_9=(36/2)*((a9^2)+(b9^2))
i_9
#36/5#
for(i in 1:36){
```

```
m[i]=y[i]*cos((2*pi*i)/(36/5))
n[i]=y[i]*sin((2*pi*i)/(36/5))}
a365=(2/36)*sum(m)
a365
b365=(2/36)*sum(n)
b365
i_36_5=(36/2)*((a365^2)+(b365^2))
i_36_5
#6#
for(i in 1:36){
m[i]=y[i]*cos((2*pi*i)/6)
n[i]=y[i]*sin((2*pi*i)/6)}
a6=(2/36)*sum(m)
a6
b6=(2/36)*sum(n)
b6
i_6=(36/2)*((a6^2)+(b6^2))
i_6
mu=c(36,18,12,9,36/5,6)
i_mu=c(i_36,i_18,i_12,i_9,i_36_5,i_6)
i_mu
plot(mu,i_mu,type="o",
    col="blue",
    xlab="Trial  period ----- >",
```

```
    ylab="Indicator  function ----- >",
    main="Periodogram",
    col.main="red",
    lwd=2)
grid()
tss=sum((y-a0)^2)
tss
ms=i_mu/2
ms
ess=tss-sum(i_mu)
ess
mse=ess/23
mse
obs_f=ms/mse
obs_f
tab_f=qf(1-0.05,2,23,0)
tab_f
```

The value of $I(\hat{\mu})$ corresponding to the trial values of the period $\hat{\mu}$ = 36, 18, 12, 9, 36/5, 6

Trial period (μ)	$I(\hat{\mu})$
36	7.1255719
18	59.7049174
12	164.3877996
9	32.1076067
36/5	0.3472193
6	9.3888889

ANOVA:

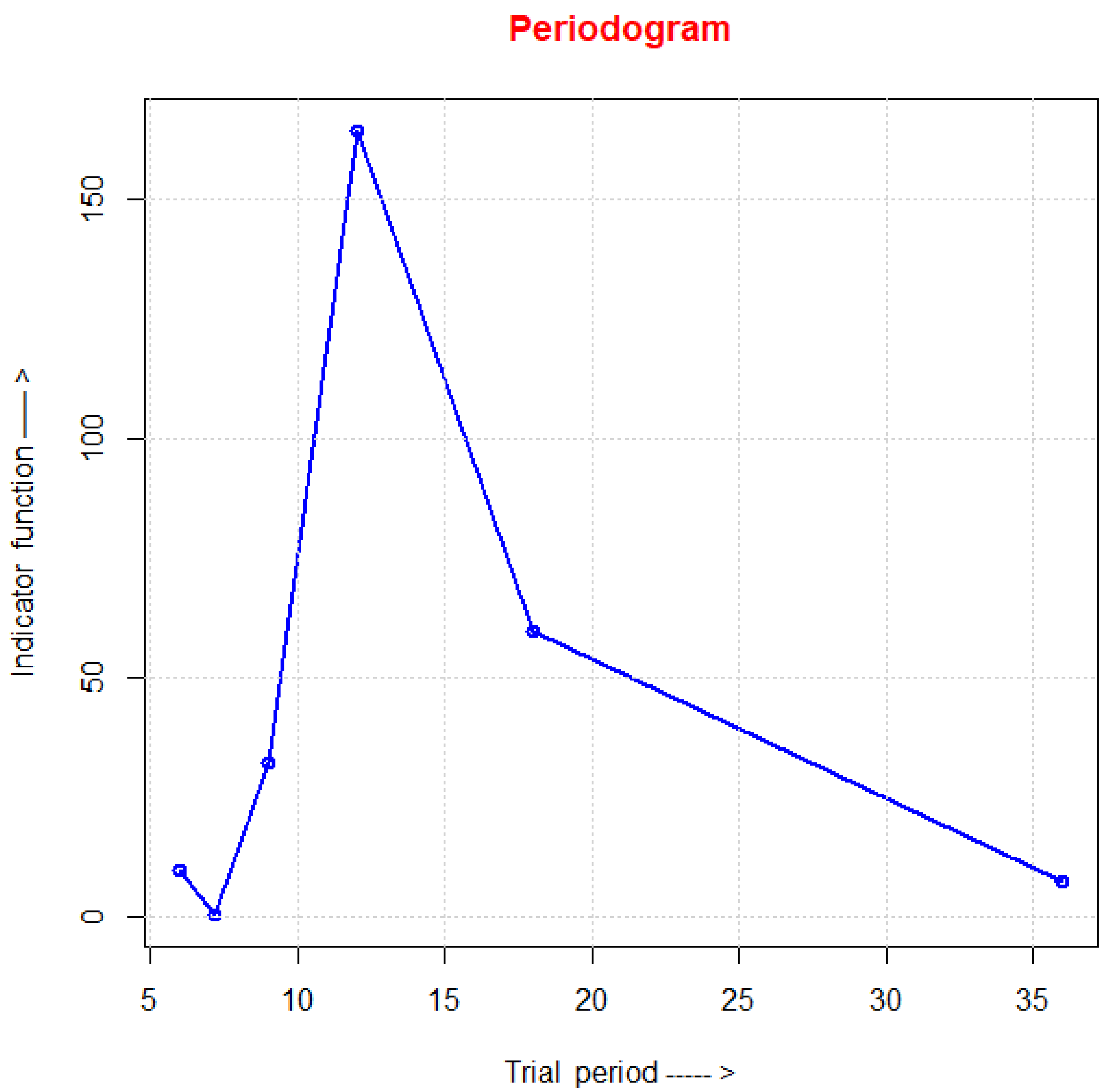
Trial period	Source of variation	df	SS	MS	Calculated F	Tabulated F
36	1/36	2	7.1255719	3.5627859	0.60391544	3.422132
18	1/18	2	59.7049174	29.8524587	5.06018638	3.422132
12	1/12	2	164.3877996	82.1938998	13.93240189	3.422132
9	1/9	2	32.1076067	16.0538034	2.72122434	3.422132
36/5	5/36	2	0.3472193	0.1736096	0.02942797	3.422132
6	1/6	2	9.3888889	4.6944444	0.79573894	3.422132
	Error	36-1-12=23	135.688	5.899478		
	Total	36-1=35	408.75			

Conclusion-

From the above ANOVA table we observe that F value is significant for the periods 18 and 12. Hence, we conclude that the data is a mixture of waves corresponding to the periods 18 and 12. The fitted Fourier series is

$$y_t = 203.0833 + 0.7161405\frac{2\pi t}{18} + 1.67454\sin\frac{2\pi t}{18} - 2.510363\cos\frac{2\pi t}{12} + 1.682478\sin\frac{2\pi t}{12}$$

Peridogram:



Periodogram
Indicator function ——— >
Trial period ----- >
0
50
100
150
5
10
15
20
25
30
35