Roll No-12

M.sc. 3rd semester

Date of Assignment-02/12/2020

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Experiment No -09

Topic- Tracing the power curve for Normal distribution with unknown mean and variance

Problem – Consider the following sample from N(μ,σ^2) Where μ and σ^2 both are unknown. The sample values are 5.2,10.8,7.1,16.4,12.5,12,10.3,10.0,12.7,9.7,10.5

Construct the UMP test for testing h_0:σ^2=4 against

(i) h_1:σ^2>4

(ii) h_1:σ^2<4

(iii) h_1:$\sigma^2 \neq 4$

Also, draw the power curve for each of the cases considering the level of significance as α=0.05

Theory and Calculation-

(i)From the theory of similar region we know that the CR for testing h_0:σ^2=4 against h_1:σ^2>4is given by

$$W_1 = \left\{\underset{\sim}{X}: \sum_{i=1}^{n}(x_i - \bar{x})^2 > k_1\right\}$$

where k_1 is a constant to be determined in such a way that α=0.05

∴ $\quad$ $\mathrm{P}[x \in W_1|H_0] = 0.05$

$$\Rightarrow \mathrm{P}\left[\sum_{i=1}^{n}(x_i - \bar{x})^2 > k_1|H_0\right] = 0.05$$

$$\Rightarrow \mathrm{P}\left[\sum_{i=1}^{n}\frac{(x_i - \bar{x})^2}{\sigma_0^2} > \frac{k_1}{\sigma_0^2}\right] = 0.05$$

$$\Rightarrow \mathrm{P}[\chi^2_{11-1} > a_1] = 0.05 \qquad where\ a_1 = \frac{k_1}{\sigma_0^2} = \frac{k_1}{4}$$

$$\Rightarrow P[\chi^2_{10} \leq a_1] = 0.95$$

To, find the value of a_1 we use the following R-command

a1=qchisq(0.95,10,0)

a1

∴a1=18.30704

∴k_1=4×a1=73.22815

We have, $\bar{x} = 10.654545$

∴ The S.R. is given by

$$W_1 = \left\{\underset{\sim}{X}: \sum_{i=1}^{n}(x_i - 10.654545)^2 > 73.22815\right\}$$

Power of the test is given by

Power=1-β=P[x∈W_1/H_1]

$$= \mathrm{P}\left[\sum_{i=1}^{n}(x_i - 10.654545)^2 > 73.22815/H_1\right]$$

$$= P\left[\chi_{10}^2 > \frac{73.22815}{\sigma_1^2}\right]$$

$$= 1 - P\left[\chi_{10}^2 < \frac{73.22815}{\sigma_1^2}\right]$$

Now, to trace the power curve we construct the following table considering different trial values of σ^2>4

TABLE 1

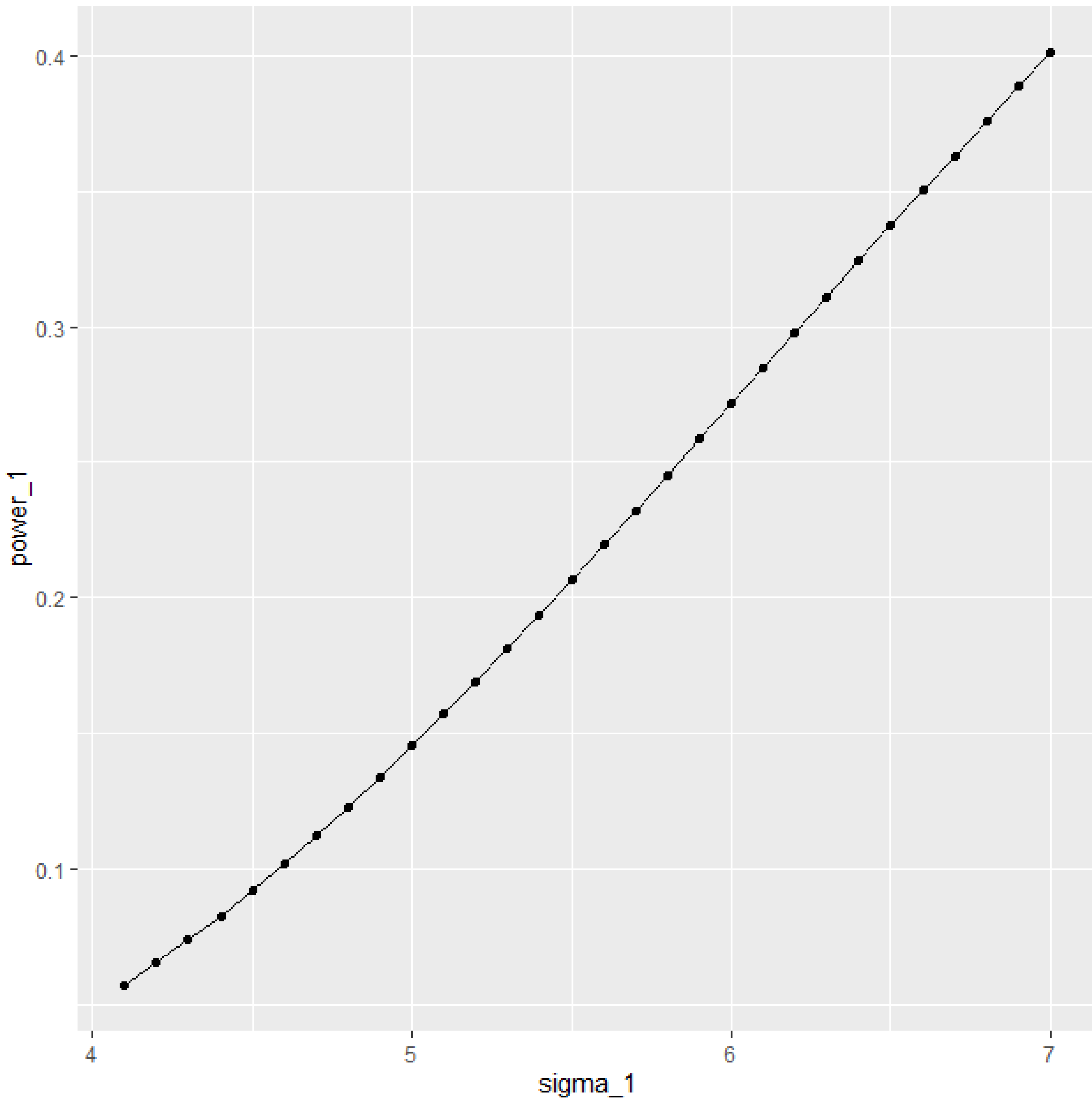
Sl no	sigma_1	power_1
1	4.1	0.057362
2	4.2	0.06527
3	4.3	0.073707
4	4.4	0.082652
5	4.5	0.092082
6	4.6	0.10197
7	4.7	0.112289
8	4.8	0.123008
9	4.9	0.134098
10	5	0.145526
11	5.1	0.15726
12	5.2	0.169269
13	5.3	0.181521
14	5.4	0.193985
15	5.5	0.206631
16	5.6	0.219428
17	5.7	0.23235
18	5.8	0.245368
19	5.9	0.258456
20	6	0.271591
21	6.1	0.284748
22	6.2	0.297905
23	6.3	0.311043
24	6.4	0.324141

25	6.5	0.337183
26	6.6	0.350151
27	6.7	0.363031
28	6.8	0.375808
29	6.9	0.388471
30	7	0.401007

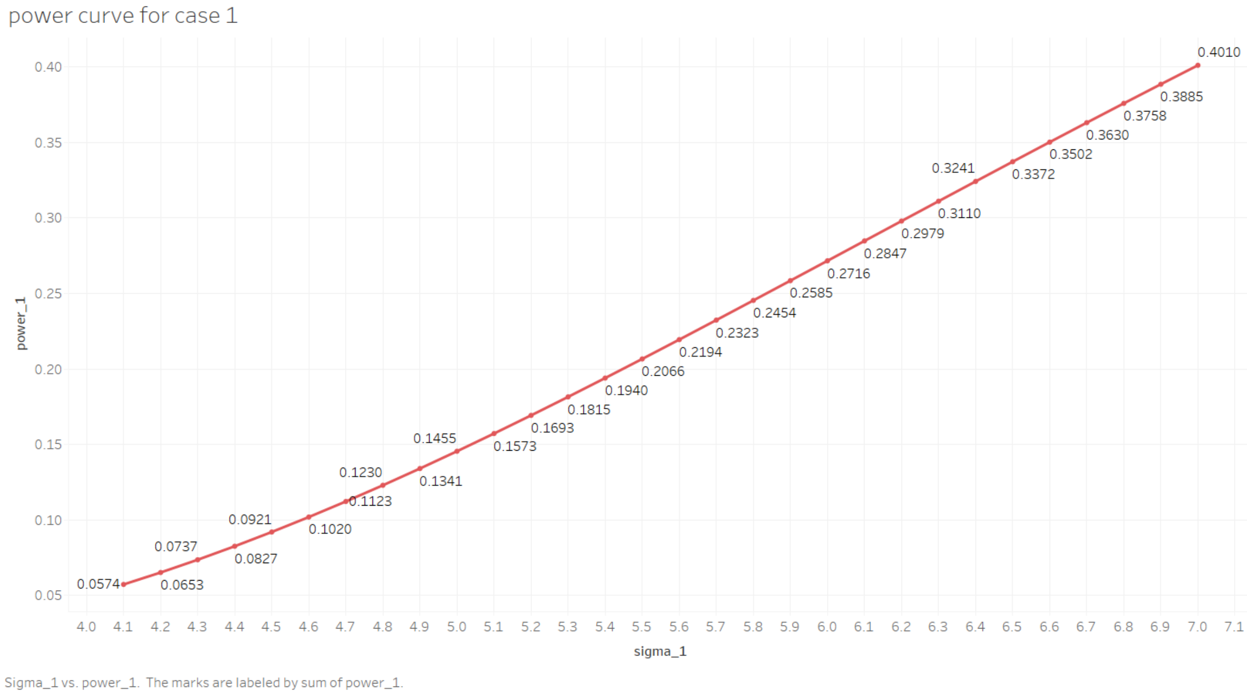
<u>Programming in R for case 1-</u>

```
library('ggplot2')

k_1 = 73.22815

sigma_1  =  seq(from=4.1, by=0.1, length.out=30)

sigma_1

sigma_11  =  k_1/sigma_1

sigma_11

power_1  =  mat.or.vec(30,1)

for(i in 1:30){

        power_1[i]  =  1-pchisq(sigma_11[i],10,0)

              }

power_1

Table  =  data.frame(sigma_1, power_1)

Table

View(Table)

ggplot(data  =  Table, mapping = aes(x = sigma_1,y = power_1))+geom_point()+geom_line()

data.frame(sigma_1)

data.frame(power_1)
```

Power curve by using ggplot 2



Power curve generated by using Tableau



ii) To test h₁:σ^2=4against h₁:σ^2<4, is given by

$$W_2 = \left\{ \underset{\sim}{X}: \sum_{i=1}^{n} (x_i - \bar{x})^2 < k_2 \right\}$$

where k_2 is a constant to be determined in such a way that α=0.05

$\therefore \qquad P[x \in W_2 | H_0] = 0.05$

$$\Rightarrow P\left[\sum_{i=1}^{n} (x_i - \bar{x})^2 < k_2 | H_0 \right] = 0.05$$

$$\Rightarrow P\left[\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{\sigma_0^2} < \frac{k_2}{\sigma_0^2} \right] = 0.05$$

$$\Rightarrow \mathrm{P}[\chi^2_{11-1} < a_2] = 0.05 \qquad , where\ a_2 = \frac{k_2}{\sigma_0^2} = \frac{k_2}{4}$$

$$\Rightarrow P[\chi^2_{10} < a_2] = 0.05$$

To, find the value of a_2 we use the following R-command,

a2=qchisq(0.05,10,0)

a2

∴a2=3.940299

So,k2=15.7612

∴The similar region is given by

$$W_2 = \left\{ \underset{\sim}{X} : \sum_{i=1}^{11} (x_i - 10.654545)^2 < 15.7612 \right\}$$

The power of the test is given by

$1-\beta = P(x \in W_2/H_1)$

$= P[\text{Reject } H_0/H_1 \text{ is true}]$

$$= \mathrm{P}\left[\sum_{i=1}^{n} (x_i - 10.654545)^2 < 15.7612/\mathrm{H}_1\right]$$

$$= P\left[\chi^2_{10} < \frac{15.7612}{\sigma_1^2}\right]$$

$$= 1 - P\left[\chi^2_{10} > \frac{73.22815}{\sigma_1^2}\right]$$

To draw the power curve we construct the following table considering different trial values of $\sigma < 4$

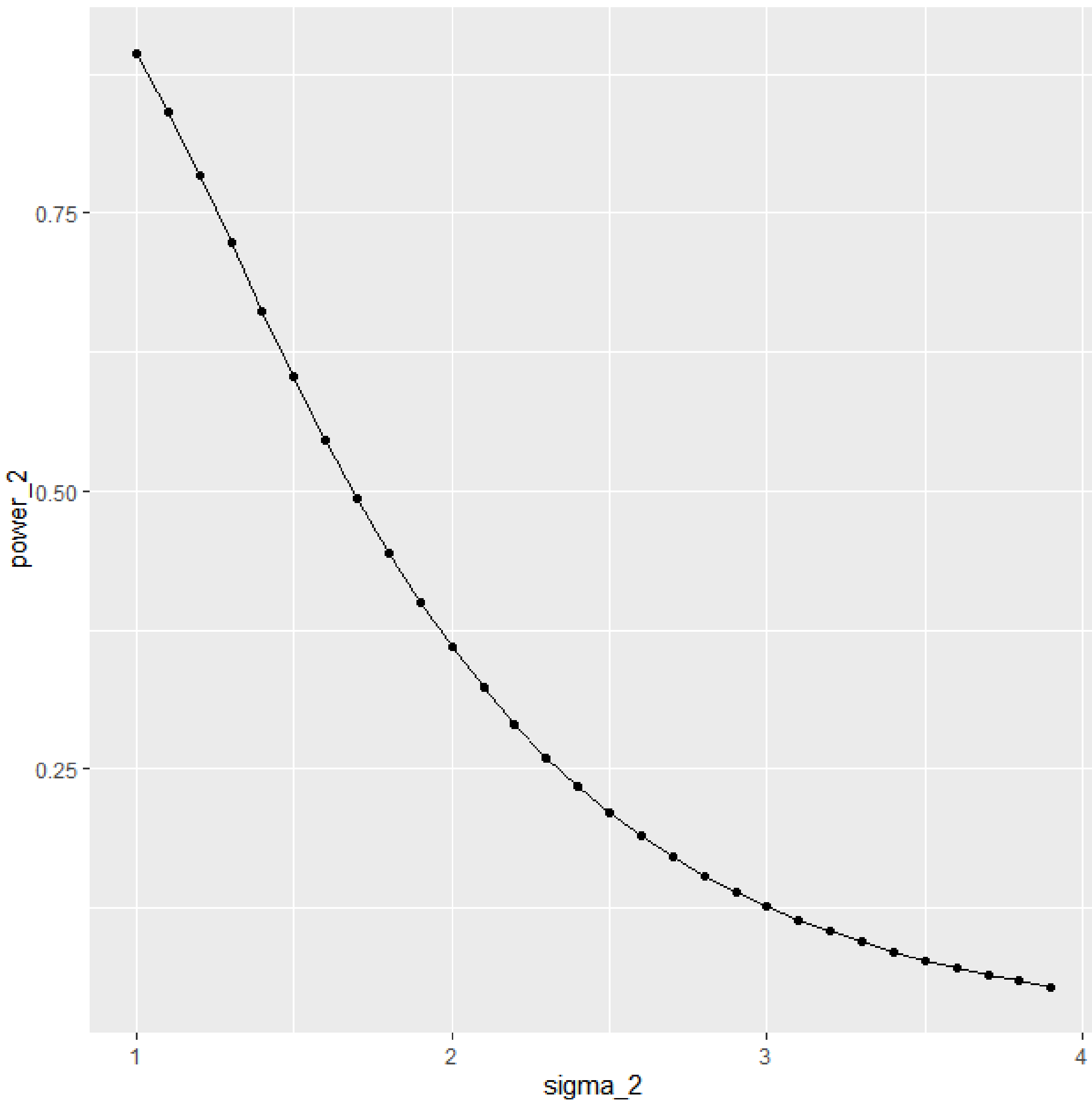
TABLE 2

sl no	sigma_2	power_2
1	1	0.893325
2	1.1	0.841466
3	1.2	0.783742
4	1.3	0.723157
5	1.4	0.662226
6	1.5	0.602846
7	1.6	0.546316
8	1.7	0.493435
9	1.8	0.444615
10	1.9	0.39999
11	2	0.359501
12	2.1	0.322969
13	2.2	0.29014
14	2.3	0.260726
15	2.4	0.234423
16	2.5	0.210934
17	2.6	0.189972
18	2.7	0.171273
19	2.8	0.154589
20	2.9	0.1397
21	3	0.126403
22	3.1	0.114521
23	3.2	0.103893
24	3.3	0.094377
25	3.4	0.085848
26	3.5	0.078194
27	3.6	0.071318
28	3.7	0.065132
29	3.8	0.059561
30	3.9	0.054537

Programming in R for case 2-

```
library('ggplot2')
k_2 = 15.7612
sigma_2 = seq(from=1.0, by=0.1, length.out=30)
sigma_2
sigma_22 = k_2/sigma_2
sigma_22
power_2 = mat.or.vec(30,1)
for(i in 1:30){
    power_2[i] = pchisq(sigma_22[i],10,0)
        }
power_2
Table = data.frame(sigma_2, power_2)
Table
View(Table)
ggplot(data = Table, mapping=aes(x = sigma_2, y = power_2))+geom_point()+geom_line()
data.frame(sigma_2)
data.frame(power_2)
```

<u>Power curve by using ggplot 2</u>



Power curve generated by using Tableau

Sigma_2 vs. power_2. The marks are labeled by sum of power_2.

(iii)To test H$_0$:σ^2=4against H$_1$:σ^2≠4 , the best similar region is given by

$$W_3 = \left\{\underset{\sim}{X}: \sum_{i=1}^{11}(x_i - \bar{x})^2 < k_3\right\} or \left\{\underset{\sim}{X}: \sum_{i=1}^{11}(x_i - \bar{x})^2 > k_4\right\}$$

Where, k_3 and k_4 are constants to be determined such that α=0.05

∴ $\quad$ $\mathrm{P}[x \in W_3|H_0] = 0.05$

$$\Rightarrow \mathrm{P}\left[\sum_{i=1}^{11}(x_i - \bar{x})^2 < k_3|H_0 \; or \sum_{i=1}^{11}(x_i - \bar{x})^2 > k_4|H_0\right] = 0.05$$

Assuming that the test is equitailed, we have

$$\Rightarrow \mathrm{P}\left[\sum_{i=1}^{11}\frac{(x_i - \bar{x})^2}{\sigma_0^2} < \frac{k_3}{\sigma_0^2}\right] = \frac{0.05}{2}$$

$$\&\mathrm{P}\left[\sum_{i=1}^{11}\frac{(x_i - \bar{x})^2}{\sigma_0^2} > \frac{k_4}{\sigma_0^2}\right] = \frac{0.05}{2}$$

$\Rightarrow P[\chi^2_{10} < a_3] = 0.025$

$\&\mathrm{P}[\chi^2_{10} > a_4] = 0.025$

Where, $a_3=\frac{k_3}{\sigma_0^2} = \frac{k_3}{4}$ and $a_4=\frac{k_4}{\sigma_0^2} = \frac{k_4}{4}$

$$\Rightarrow P[\chi^2_{10} < a_3] = 0.025$$

$$\&P[\chi^2_{10} > a_4] = 0.975$$

Now to determine the values of a_3 and a_4 we use the following R-command:

a3=qchisq(0.025,10,0)

a3

a4=qchisq(0.975,10,0)

a4

∴a_3=3.246973 and a_4=20.48318

$\therefore k_3$=12.98789 and k_4=81.93271

Also , $\bar{\boldsymbol{x}} = 10.654545$

$\therefore$The similar region is given by

$$W_3 = \left\{\underset{\sim}{X}: \sum_{i=1}^{11}(x_i - \bar{x})^2 < 12.98789\right\} \ or \ \left\{\underset{\sim}{X}: \sum_{i=1}^{11}(x_i - \bar{x})^2 > 81.93271\right\}$$

Now, Power of the test is given by

$1-\beta=P(x\in W_3/H_1)$

$=P[\text{Reject } H_0/H_1 \text{ is true}]$

$$= \mathrm{P}\left[\frac{\sum_{i=1}^{n}(x_i - 10.654545)^2}{\sigma_1^2} < \frac{12.98789}{\sigma_1^2} or \frac{\sum_{i=1}^{n}(x_i - 10.654545)^2}{\sigma_1^2} > \frac{81.93271}{\sigma_1^2}\right]$$

$$= \mathrm{P}\left[\chi_{10}^2 < \frac{12.98789}{\sigma_1^2}\right] + \mathrm{P}\left[\chi_{10}^2 > \frac{81.93271}{\sigma_1^2}\right]$$

$$= \mathrm{P}\left[\chi_{10}^2 < \frac{12.98789}{\sigma_1^2}\right] + 1 - \mathrm{P}\left[\chi_{10}^2 < \frac{81.93271}{\sigma_1^2}\right]$$

To trace the power curve we construct the following table considering different trial values of $\sigma^2 \neq 4$

TABLE 3

sl no	sigma_3	power_3
1	4.1	0.052213
2	4.2	0.055101
3	4.3	0.058642
4	4.4	0.062812
5	4.5	0.06759
6	4.6	0.07295
7	4.7	0.078868
8	4.8	0.085319
9	4.9	0.092276

10	5	0.099713
11	5.1	0.107602
12	5.2	0.115915
13	5.3	0.124625
14	5.4	0.133704
15	5.5	0.143123
16	5.6	0.152856
17	5.7	0.162874
18	5.8	0.173152
19	5.9	0.183662
20	6	0.19438
21	6.1	0.205281
22	6.2	0.21634
23	6.3	0.227536
24	6.4	0.238845
25	6.5	0.250246
26	6.6	0.26172
27	6.7	0.273248
28	6.8	0.284811
29	6.9	0.296392
30	7	0.307975
31	1	0.77565
32	1.1	0.701831
33	1.2	0.628546
34	1.3	0.558689
35	1.4	0.493973
36	1.5	0.435219
37	1.6	0.382633
38	1.7	0.33604
39	1.8	0.295045
40	1.9	0.259148
41	2	0.227813
42	2.1	0.200512
43	2.2	0.176754
44	2.3	0.156093
45	2.4	0.138133
46	2.5	0.122532
47	2.6	0.108994
48	2.7	0.097269
49	2.8	0.087149
50	2.9	0.078458
51	3	0.071053
52	3.1	0.064816

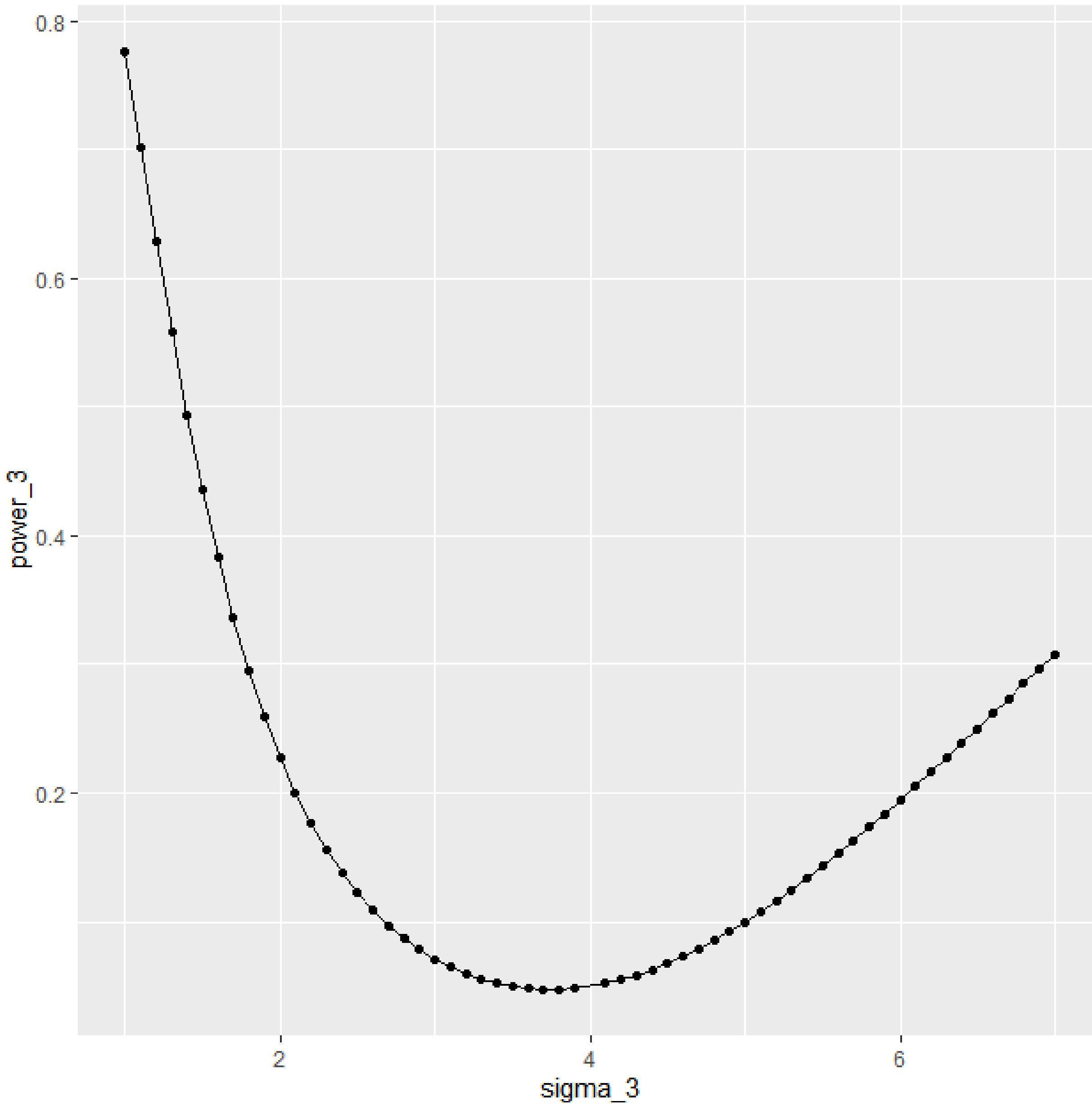
53	3.2	0.059652
54	3.3	0.055481
55	3.4	0.052242
56	3.5	0.049882
57	3.6	0.048359
58	3.7	0.047638
59	3.8	0.047689
60	3.9	0.048484

<u>Programming in R for case 3-</u>

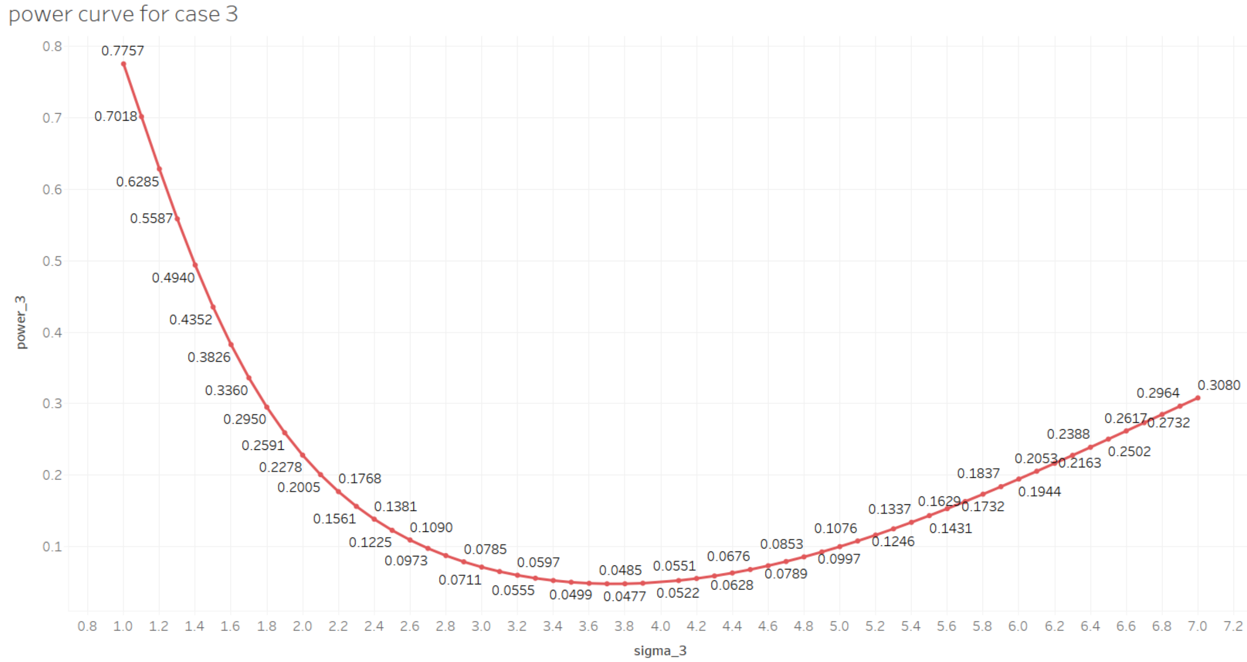
```
library('ggplot2')

k_3 = 12.98789

k_4 = 81.93271

sigma_1 = seq(from=4.1, by=0.1, length.out=30)

sigma_1

sigma_2 = seq(from=1.0, by=0.1, length.out=30)

sigma_2

sigma_3 = c(sigma_1,sigma_2)

sigma_3

sigma_31 = k_3/sigma_3

sigma_31

sigma_32 = k_4/sigma_3

sigma_32

power_3 = mat.or.vec(60,1)

for(i in 1:60){

    power_3[i] = pchisq(sigma_31[i],10,0)+1-pchisq(sigma_32[i],10,0)

        }

power_3
```

```
Table = data.frame(sigma_3, power_3)

Table

View(Table)

ggplot(data = Table, mapping = aes(x = sigma_3, y = power_3))+geom_point()+geom_line()

data.frame(sigma_3)

data.frame(power_3)
```

Power curve by using ggplot 2



Power curve generated by using Tableau



Sigma_3 vs. power_3. The marks are labeled by sum of power_3.