Roll No-12

M.sc. 3rd semester

Date of Assignment-02/12/2020

Date of Submission-08/12/2020

Experiment No -08

Topic- Tracing the power curve for Normal distribution with unknown mean and variance

Problem – Consider the following example of size 11 from $N(\mu, \sigma^2)$ where μ and σ^2 both are unknown. The sample values are 5.2, 10.8, 7.1, 16.4, 12.5, 12, 10.3, 10.0, 12.7, 9.7, 10.5

Obtain the most powerful similar regions for testing $h_0 : \mu = 11$ against

i) $h_1 : \mu > 11$

ii) $h_1 : \mu < 11$

iii) $h_1 : \mu \neq 11$

Also, draw the power curve for each of the cases considering the level of significance as $\alpha = 0.05$

Theory and Calculation-

i) The CR for testing $h_0 : \mu = 11$ against $h_1 : \mu > 11$ is given by

$$
\begin{aligned}
W_0 &= \left\{ \underset{\sim}{X} : x \in W \mid H_0 \right\} \\
&= \left\{ \underset{\sim}{X} : \frac{\sqrt{n}\left(\bar{x} - \mu_0\right)}{S} > t_{\alpha, n-1} \right\} \\
&= \left\{ \underset{\sim}{X} : \bar{x} > \frac{S\, t_{\alpha, n-1}}{\sqrt{n}} + \mu_0 \right\} \\
&= \left\{ \underset{\sim}{X} : \bar{x} > 11 + \frac{S}{\sqrt{n}} t_{\alpha, n-1} \right\}
\end{aligned}
$$

S can be computed from the sample values and to obtain the values of $t_{\alpha, n-1}$,we use the following R-command.

alfa=0.05

n = 11

t_tab_1=qt(1-alfa,n-1)

t_tab_1=qt(0.95,10)

t_tab_1

$\therefore$ t_tab1= 1.812461

To find out the value of S, we use the following R-program

sv = c(5.2,10.8,7.1,15.4,12.5,12,10.3,10,12.7,9.7,10.5)

variance_sv = var(sv)

variance_sv

sqs = sqrt(variance_sv)

sqs

$\therefore$ sqs=2.751826

To obtain the CR, we use the following R-command

sr_1 = 11+(sqs*t_tab_1)/(sqrt(n))

sr_1

$\therefore$ sr_1= 11.99752

$\therefore$ The CR is given by

$W_1 = \left\{ \underset{\sim}{X} : \bar{x} > 11.99752 \right\}$

Now, the power of the test is given by

$$Power = P\left\{ x \in W_1 \mid H_1 \right\}$$

$$= P\left\{ \frac{\sqrt{n}\left(\bar{x} - \mu_0\right)}{S} > t_{\alpha,n-1} \mid H_1 \right\}$$

$$= P\left\{ \frac{\sqrt{n}\left(\bar{x} - \mu_1 + \mu_1 - \mu_0\right)}{S} > t_{\alpha,n-1} \mid H_1 \right\}$$

$$= P\left\{ \left(\bar{x} - \mu_1\right) > \left(t_{\alpha,n-1} \, S \big/ \sqrt{n} \right) - \left(\mu_1 - \mu_0\right) \mid H_1 \right\}$$

$$= P\left\{ \frac{\sqrt{n}\left(\bar{x} - \mu_1\right)}{S} > t_{\alpha,n-1} - \frac{\left(\mu_1 - \mu_0\right)\sqrt{n}}{S} \mid H_1 \right\}$$

$$= P\left\{ t > t_{\alpha,n-1} - \frac{\left(\mu_1 - \mu_0\right)\sqrt{n}}{S} \mid H_1 \right\}$$

$$= 1 - P_{H_1}\left\{ t < t_{\alpha,n-1} - \frac{\left(\mu_1 - \mu_0\right)\sqrt{n}}{S} \right\}$$

Where $t = \left\{ \frac{\sqrt{n}\left(\bar{x} - \mu_1\right)}{S} \right\}$, $\mu_1 > \mu_0$ follows student's t-distribution with (n-1) df.

Now, to draw the power curve we construct the following table considering different values of $\mu > 11$

TABLE 1

	mu_1	power_1
1	11.5	0.1270818
2	11.6	0.1507847
3	11.7	0.1777487
4	11.8	0.2080664
5	11.9	0.2417317
6	12.0	0.2786215
7	12.1	0.3184837
8	12.2	0.3609342
9	12.3	0.4054635
10	12.4	0.4514549
11	12.5	0.4982127
12	12.6	0.5449989

	mu_1	power_1
13	12.7	0.5910740
14	12.8	0.6357376
15	12.9	0.6783660
16	13.0	0.7184407
17	13.1	0.7555676
18	13.2	0.7894846
19	13.3	0.8200591
20	13.4	0.8472765
21	13.5	0.8712228
22	13.6	0.8920630
23	13.7	0.9100195
24	13.8	0.9253507
25	13.9	0.9383330
26	14.0	0.9492454
27	14.1	0.9583579

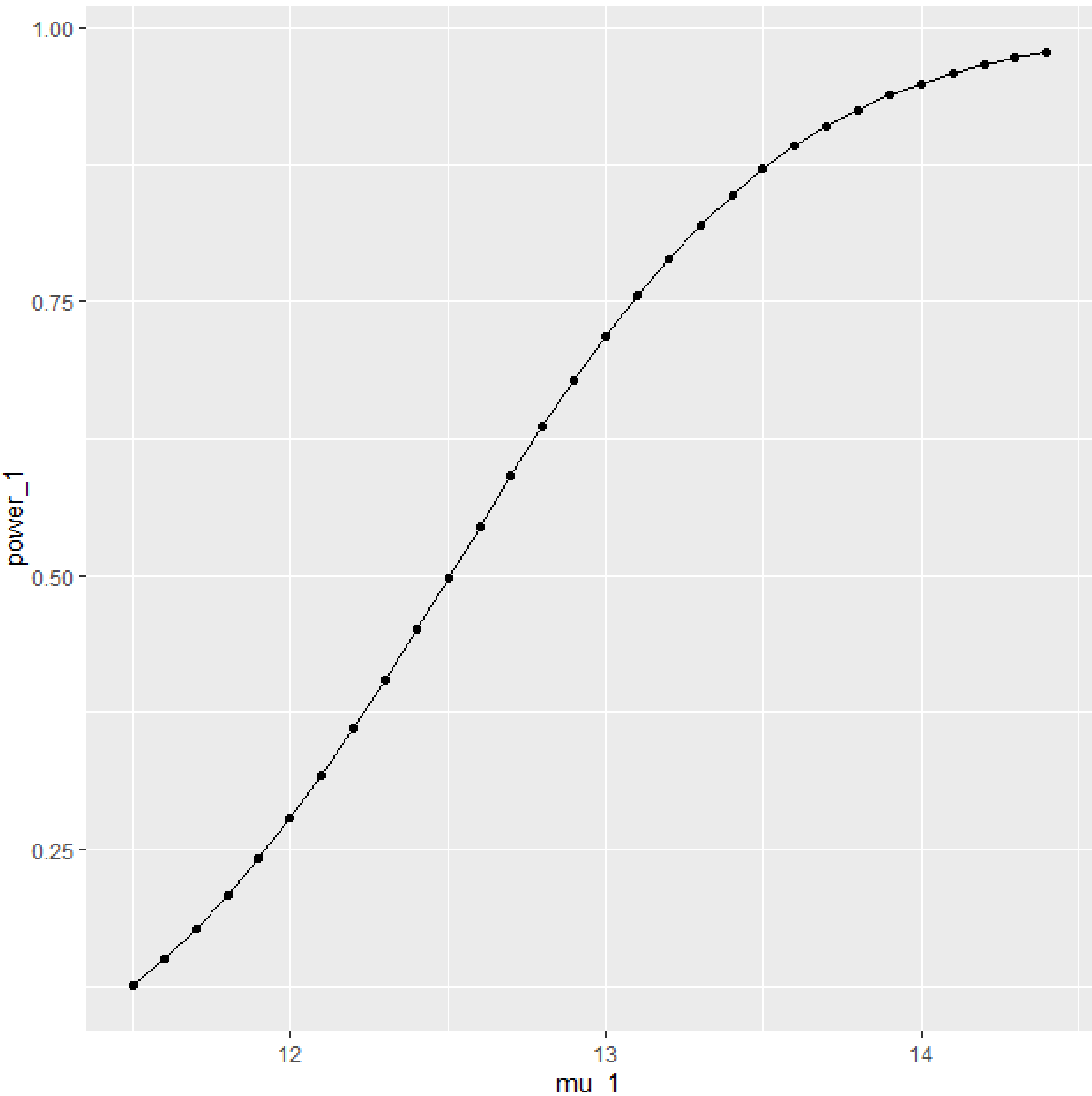
	mu_1	power_1
28	14.2	0.9659237
29	14.3	0.9721739
30	14.4	0.9773153

<u>Programming in R for case 1-</u>

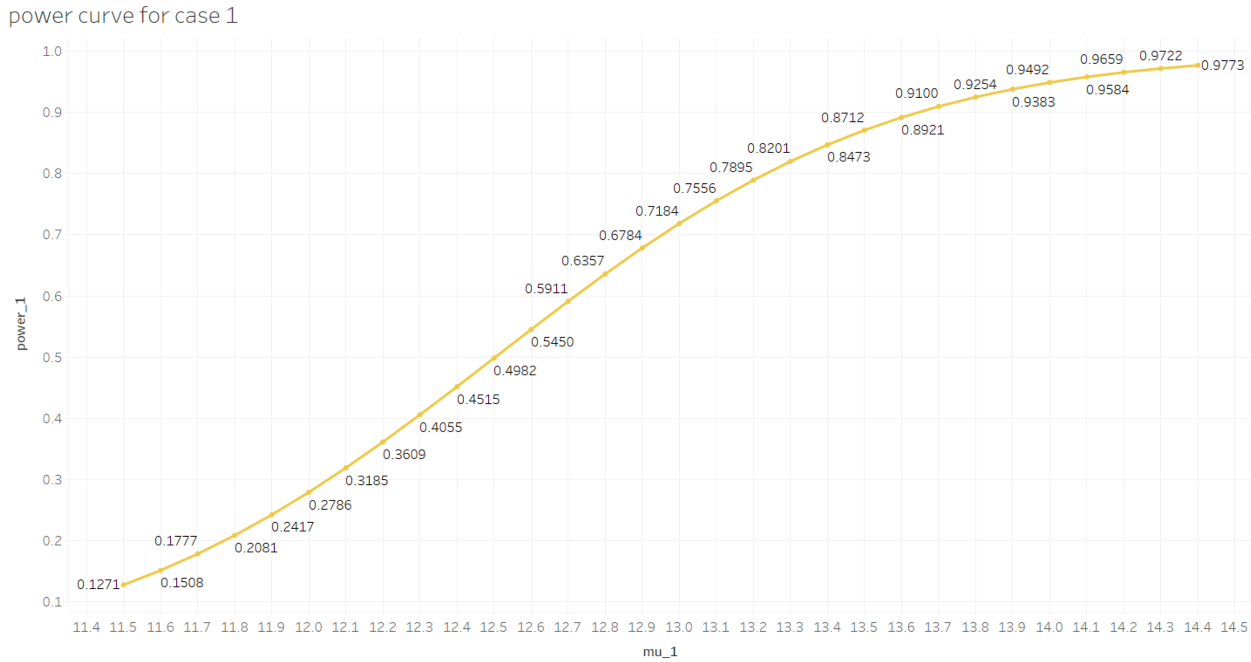
```
library('ggplot2')

alfa = 0.05

n = 11

t_tab_1 = qt(1-alfa, n-1)

t_tab_1

#To find out the value of similar region, we use the following R-Program

sv = c(5.2,10.8,7.1,15.4,12.5,12,10.3,10,12.7,9.7,10.5)

variance_sv = var(sv)

variance_sv

sqs = sqrt(variance_sv)

sqs

sr_1 = 11+(sqs*t_tab_1)/(sqrt(n))

sr_1

#To find the power curve

n_1 = 11

a = sqs

mu_1 = seq(from=11.5, by=0.1, length.out=30)

mu_1
```

```
power_1 = mat.or.vec(30,1)
for(i in 1:30){
    power_1[i] = 1-pt((t_tab_1-(((mu_1[i]-n_1)*sqrt(n))/a)), n-1)
        }
power_1
Table = data.frame(mu_1, power_1)
Table
View(Table)
ggplot(data=Table,mapping=aes(x=mu_1,y=power_1))+geom_point()+geom_line()
data.frame(mu_1)
data.frame(power_1)
```

Power curve by using ggplot 2



Power curve generated by using Tableau



Mu_1 vs. power_1. The marks are labeled by sum of power_1.

ii) The CR for testing $h_0 : \mu = 11$ against $h_1 : \mu < 11$ is given by

$$W_1 = \left\{ \underset{\sim}{X} : \frac{\sqrt{n}\left(\bar{x} - \mu_0\right)}{S} < -t_{\alpha,n-1} \right\}$$

$$= \left\{ \underset{\sim}{X} : \bar{x} > \mu_0 + \frac{S\left(-t_{\alpha,n-1}\right)}{\sqrt{n}} \right\}$$

$$= \left\{ \underset{\sim}{X} : \bar{x} > 11 + \left(-t_{\alpha,n-1}\right)\frac{S}{\sqrt{n}} \right\}$$

S can be computed from the sample values and to obtain the values of ($-t_{\alpha,n-1}$) ,we use the following R-command.

alfa = 0.05

n = 11

t_tab_2 = qt(alfa, n-1)

t_tab_2

$\therefore$ t_tab2= -1.812461

To find out the value of S, we use the following R-program

```
sv = c(5.2,10.8,7.1,15.4,12.5,12,10.3,10,12.7,9.7,10.5)

variance_sv = var(sv)

variance_sv

sqs = sqrt(variance_sv)

sqs
```

$\therefore$ sqs= 2.751826

To obtain the CR, we use the following R-command

```
sr_2 = 11+(sqs*t_tab_2)/(sqrt(n))

sr_2
```

$\therefore$ sr_2 = 9.496189

$\therefore$ The CR is given by

$$W_2 = \left\{ \underset{\sim}{X} : \bar{x} < 9.496189 \right\}$$

Now, the power of the test is given by

$$Power = P\{x \in W_2 \mid H_1\}$$

$$= P\left\{ \frac{\sqrt{n}(\bar{x} - \mu_0)}{S} < -t_{\alpha,n-1} \mid H_1 \right\}$$

$$= P\left\{ \frac{\sqrt{n}(\bar{x} - \mu_1 + \mu_1 - \mu_0)}{S} < -t_{\alpha,n-1} \mid H_1 \right\}$$

$$= P\left\{ (\bar{x} - \mu_1) < \frac{(-t_{\alpha,n-1})S}{\sqrt{n}} - (\mu_1 - \mu_0) \mid H_1 \right\}$$

$$= P\left\{ \frac{\sqrt{n}(\bar{x} - \mu_1)}{S} < -t_{\alpha,n-1} - \frac{(\mu_1 - \mu_0)\sqrt{n}}{S} \mid H_1 \right\}$$

$$= P\left\{ t < -t_{\alpha,n-1} - \frac{(\mu_1 - \mu_0)\sqrt{n}}{S} \mid H_1 \right\}$$

Where $t = \left\{ \frac{\sqrt{n}\left(\bar{x} - \mu_1\right)}{S} \right\}$, $\mu_1 < \mu_0$ follows student's t-distribution with (n-1) df.

Now, to draw the power curve we construct the following table considering different values of $\mu < 11$

TABLE 2

	mu_2	power_2
1	8.0	0.94924537
2	8.1	0.93833301
3	8.2	0.92535070
4	8.3	0.91001947
5	8.4	0.89206302
6	8.5	0.87122282
7	8.6	0.84727654
8	8.7	0.82005907
9	8.8	0.78948456
10	8.9	0.75556758
11	9.0	0.71844072

	mu_2	power_2
12	9.1	0.67836601
13	9.2	0.63573760
14	9.3	0.59107395
15	9.4	0.54499892
16	9.5	0.49821269
17	9.6	0.45145488
18	9.7	0.40546353
19	9.8	0.36093422
20	9.9	0.31848370
21	10.0	0.27862146
22	10.1	0.24173172
23	10.2	0.20806642
24	10.3	0.17774867
25	10.4	0.15078469
26	10.5	0.12708184
27	10.6	0.10646985

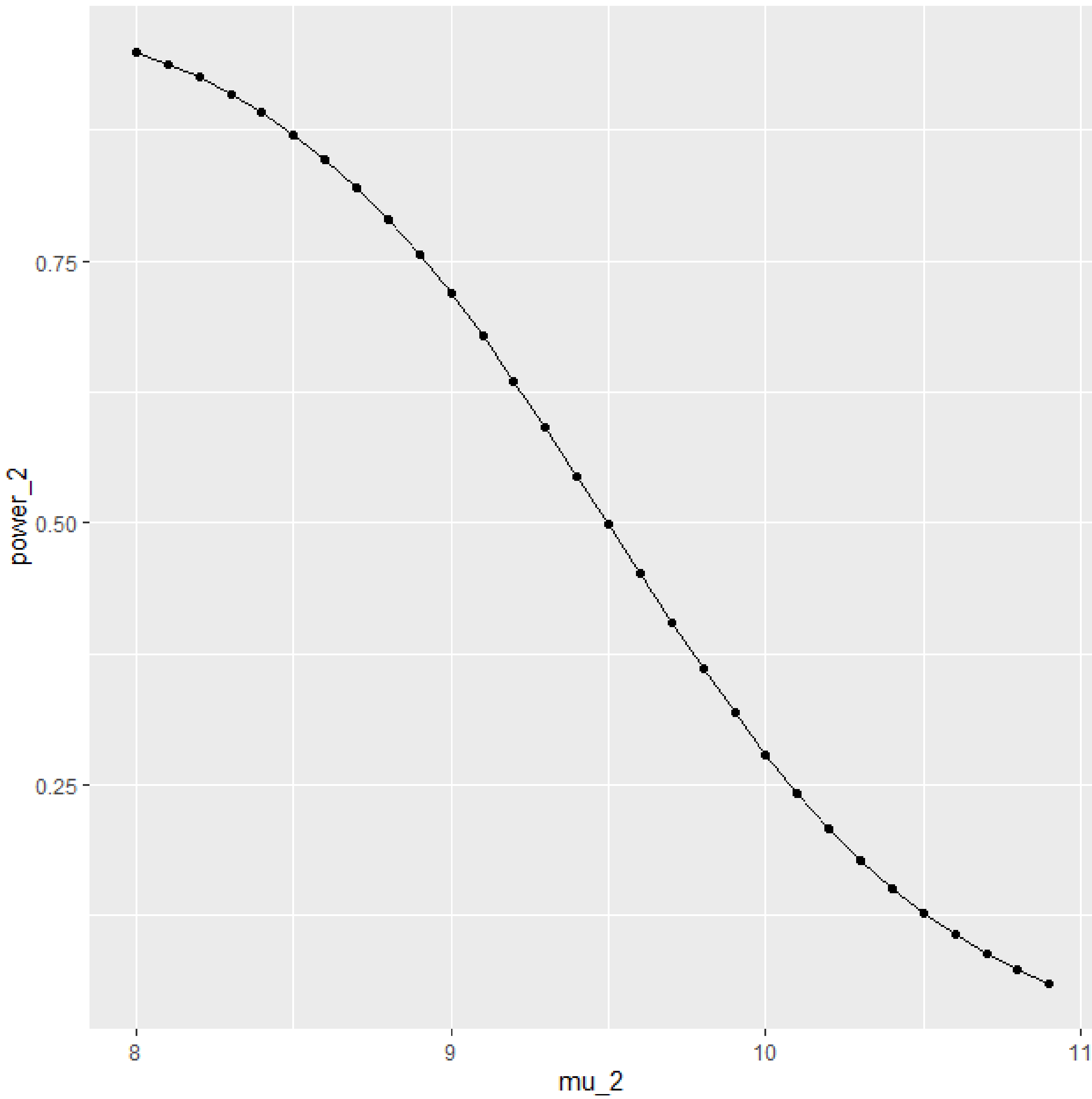
	mu_2	power_2
28	10.7	0.08872293
29	10.8	0.07358059
30	10.9	0.06076583

Programming in R for case 2-

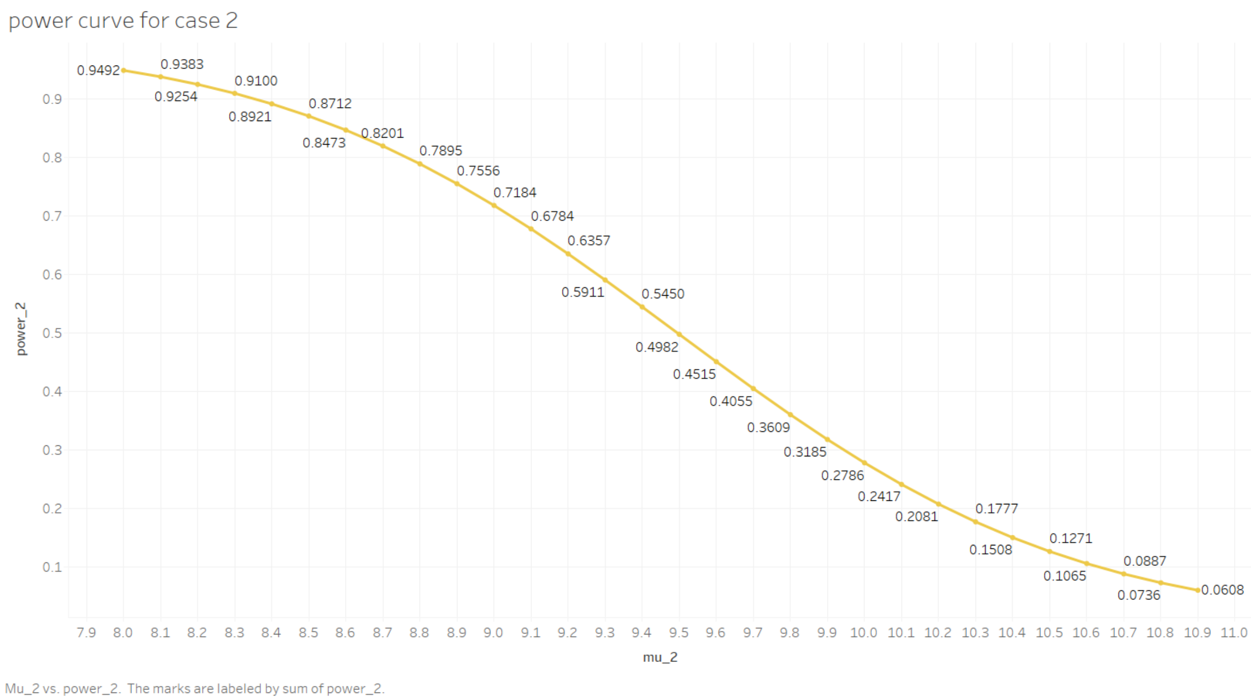
```
library('ggplot2')

alfa = 0.05

n = 11

t_tab_2 = qt(alfa, n-1)

t_tab_2

#To find out the value of similar region, we use the following R-Program

sv = c(5.2,10.8,7.1,15.4,12.5,12,10.3,10,12.7,9.7,10.5)

variance_sv = var(sv)

variance_sv

sqs = sqrt(variance_sv)

sqs

sr_2 = 11+(sqs*t_tab_2)/(sqrt(n))

sr_2

#To find the power curve

n_2 = 11

b = sqs

mu_2 = seq(from=8.0, by=0.1, length.out=30)

mu_2
```

```
power_2 = mat.or.vec(30,1)
for(i in 1:30){
    power_2[i] = pt((t_tab_2-(((mu_2[i]-n_2)*sqrt(n))/b)), n-1)
        }
power_2
Table = data.frame(mu_2, power_2)
Table
View(Table)
ggplot(data=Table,mapping=aes(x=mu_2,y=power_2))+geom_point()+geom_line()
data.frame(mu_2)
data.frame(power_2)
```

Power curve by using ggplot 2



Power curve generated by using Tableau



iii) The CR for testing $h_0 : \mu = 11$ against $h_1 : \mu \neq 11$ is given by

$$W_2 = \left\{ \underset{\sim}{X} : \frac{\sqrt{n}(\bar{x} - \mu_0)}{S} > t_{\alpha/2, n-1} \quad or \quad \frac{\sqrt{n}(\bar{x} - \mu_0)}{S} < -t_{\alpha/2, n-1} \right\}$$

$$= \left\{ \underset{\sim}{X} : \bar{x} > \mu_0 + \frac{S\, t_{\alpha/2, n-1}}{\sqrt{n}} \quad or \quad \bar{x} < \mu_0 + \frac{\left(-t_{\alpha/2, n-1}\right) S}{\sqrt{n}} \right\}$$

$$= \left\{ \underset{\sim}{X} : \bar{x} > 11 + \frac{S\, t_{\alpha/2, n-1}}{\sqrt{n}} \quad or \quad \bar{x} < 11 + \frac{\left(-t_{\alpha/2, n-1}\right) S}{\sqrt{n}} \right\}$$

To find out the value of $t_{\alpha/2,n-1}$ and the CR's , we use the following R-commands

```
t_tab_3=qt((1-(0.05/2)),11-1)

t_tab_3

t_tab_4=qt((0.05/2),11-1)

t_tab_4

sr_3=11+(sqs*t_tab_3)/(sqrt(n))

sr_3

sr_4=11+(sqs*t_tab_4)/(sqrt(n))

sr_4
```

$\therefore$ sr_3= 12.8487 and sr_4= 9.151298

$\therefore$ The CR is given by

$$W_3 = \{\underset{\sim}{X} : \bar{x} > 12.8487 \quad or \quad \bar{x} < 12.8487\}$$

And the power of the test is given by

$$Power = P\{x \in W_3 \mid H_1\}$$

$$= P\left\{\frac{\sqrt{n}(\bar{x}-\mu_0)}{S} > t_{\alpha/2,n-1} \quad or \quad \frac{\sqrt{n}(\bar{x}-\mu_0)}{S} < -t_{\alpha/2,n-1} \mid H_1\right\}$$

$$= P_{H_1}\left\{(\bar{x}-\mu_1) > \left(t_{\alpha/2,n-1} \, S/\sqrt{n}\right) - (\mu_1-\mu_0)\right\} + P_{H_1}\left\{(\bar{x}-\mu_1) < \frac{\left(-t_{\alpha/2,n-1}\right)S}{\sqrt{n}} - (\mu_1-\mu_0)\right\}$$

$$= P_{H_1}\left\{t > t_{\alpha/2,n-1} - \frac{(\mu_1-\mu_0)\sqrt{n}}{S}\right\} + P_{H_1}\left\{t < -t_{\alpha/2,n-1} - \frac{(\mu_1-\mu_0)\sqrt{n}}{S}\right\}$$

Now, to draw the power curve, we construct the following table considering different values of $\mu \neq 11$

TABLE 3

	mu_a	power_3
1	11.5	0.07647112
2	11.6	0.08887098
3	11.7	0.10406590
4	11.8	0.12226710
5	11.9	0.14367027
6	12.0	0.16843403
7	12.1	0.19665666
8	12.2	0.22835288
9	12.3	0.26343298
10	12.4	0.30168688
11	12.5	0.34277583
12	12.6	0.38623393
13	12.7	0.43148074
14	12.8	0.47784477

	mu_a	power_3
15	12.9	0.52459634
16	13.0	0.57098669
17	13.1	0.61628922
18	13.2	0.65983862
19	13.3	0.70106364
20	13.4	0.73951068
21	13.5	0.77485651
22	13.6	0.80691003
23	13.7	0.83560448
24	13.8	0.86098223
25	13.9	0.88317481
26	14.0	0.90238097
27	14.1	0.91884496
28	14.2	0.93283680
29	14.3	0.94463556
30	14.4	0.95451626

	mu_a	power_3
31	8.0	0.90238097
32	8.1	0.88317481
33	8.2	0.86098223
34	8.3	0.83560448
35	8.4	0.80691003
36	8.5	0.77485651
37	8.6	0.73951068
38	8.7	0.70106364
39	8.8	0.65983862
40	8.9	0.61628922
41	9.0	0.57098669
42	9.1	0.52459634
43	9.2	0.47784477
44	9.3	0.43148074
45	9.4	0.38623393
46	9.5	0.34277583

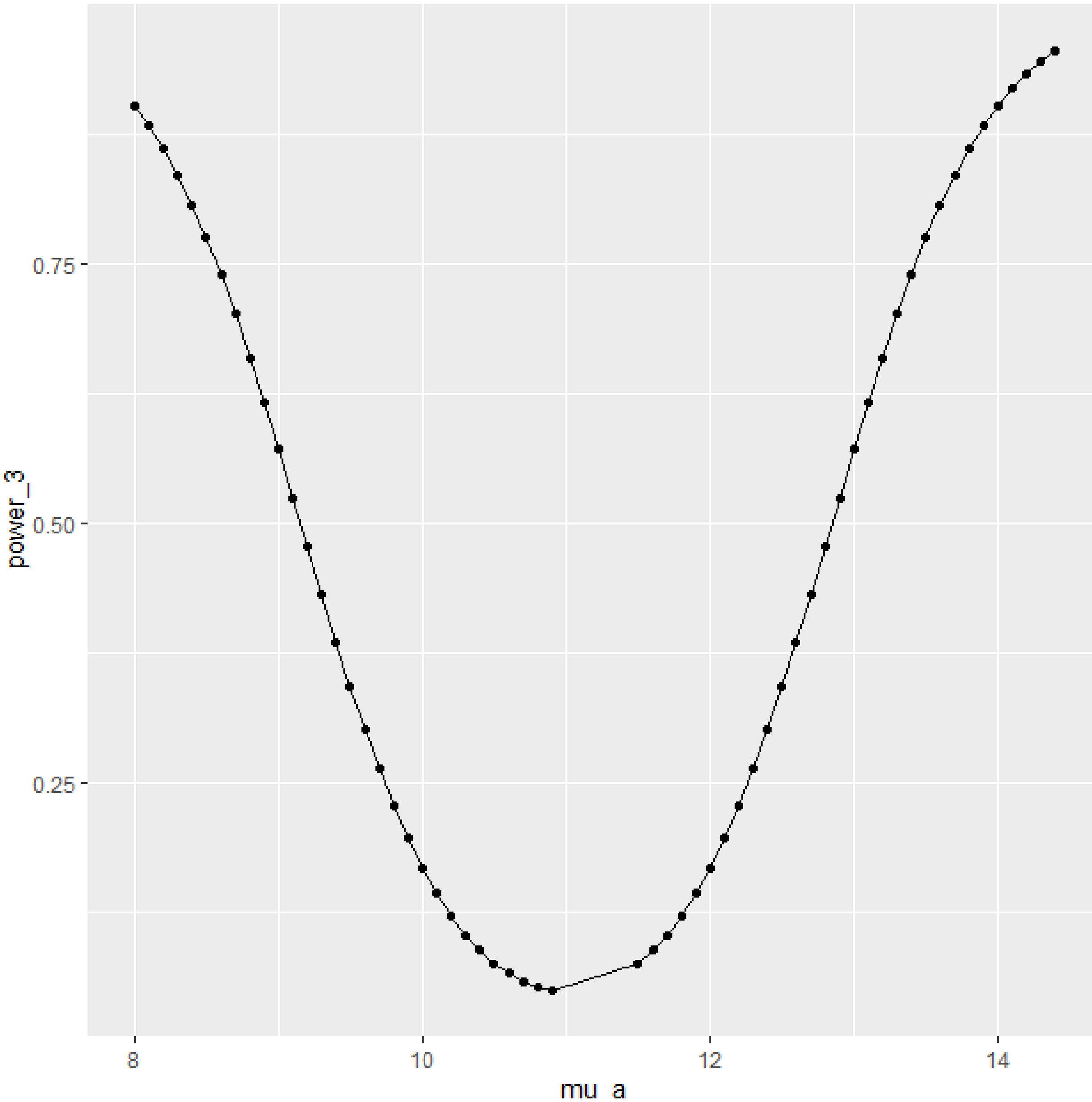
	mu_a	power_3
47	9.6	0.30168688
48	9.7	0.26343298
49	9.8	0.22835288
50	9.9	0.19665666
51	10.0	0.16843403
52	10.1	0.14367027
53	10.2	0.12226710
54	10.3	0.10406590
55	10.4	0.08887098
56	10.5	0.07647112
57	10.6	0.06665829
58	10.7	0.05924313
59	10.8	0.05406694
60	10.9	0.05101049

Programming in R for case 3-

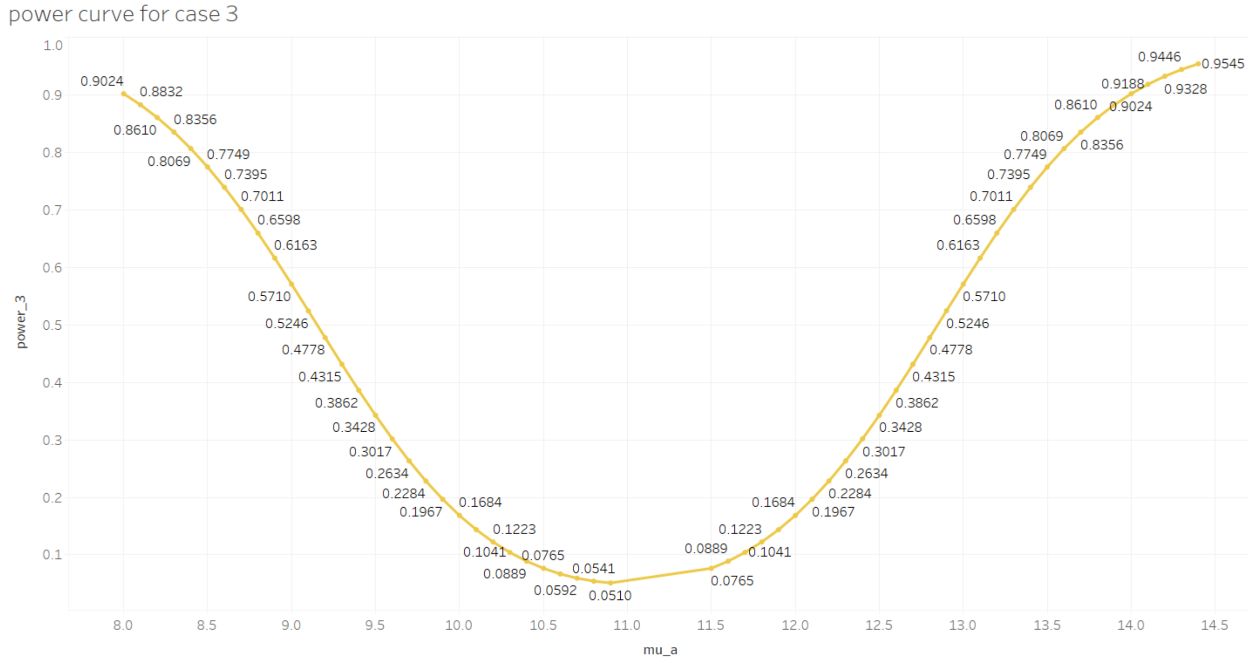
```
library('ggplot2')

alfa = 0.05

n = 11

t_tab_3=qt((1-(0.05/2)),11-1)

t_tab_3

t_tab_4=qt((0.05/2),11-1)

t_tab_4

#To find out the value of similar region, we use the following R-Program

sv = c(5.2,10.8,7.1,15.4,12.5,12,10.3,10,12.7,9.7,10.5)

variance_sv = var(sv)

variance_sv

sqs = sqrt(variance_sv)

sqs

sr_3=11+(sqs*t_tab_3)/(sqrt(n))

sr_3

sr_4=11+(sqs*t_tab_4)/(sqrt(n))

sr_4

#To find the power curve

n_3 = 11

c = sqs

n_4 = 11

d = sqs

mu_3 = seq(from=11.5, by=0.1, length.out=30)
```

```
mu_3

mu_4 = seq(from=8.0, by=0.1, length.out=30)

mu_4

mu_a = c(mu_3,mu_4)

mu_a

power_3 = mat.or.vec(60,1)

for(i in 1:60){

    power_3[i] =(1-pt((t_tab_3-(((mu_a[i]-n_3)*sqrt(n))/c)), n-1)) + pt((t_tab_4-(((mu_a[i]-
n_4)*sqrt(n))/d)), n-1)

        }

power_3

Table = data.frame(mu_a, power_3)

Table

View(Table)

ggplot(data=Table,mapping=aes(x=mu_a,y=power_3))+geom_point()+geom_line()

data.frame(mu_a)

data.frame(power_3)
```

Power curve by using ggplot 2



Power curve generated by using Tableau



Mu_a vs. power_3. The marks are labeled by sum of power_3.