Submitted by-Aditya Gautam

Roll No-12

M.sc. 3rd semester

Date of Assignment-26/10/2020

Date of Submission-01/10/2020

Experiment No -02

Topic- Tracing a power curve for testing the mean of Normal Distribution.

Problem –A random sample of size 100 is drawn from N$\sim(\mu, \sigma^2)$ where σ^2 = 25. Draw the power curve for testing $H_0 : \mu = 36.25$

Against

(1) $H_1 : \mu > 36.25$ (2) $H_1 : \mu < 36.25$ (3) $H_1 : \mu \neq 36.25$

Use the level of significance as $\alpha = 0.01$.

Theory and Calculation-

Using Neyman's pearson fundamental lemma, the critical region is given by-

W = $\{x : \frac{L(\tilde{x},\theta_1)}{L(\tilde{x},\theta_0)} \geq k\}$

Here, $f(x, \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$, $-\infty < x < \infty$, $-\infty < \mu < \infty$, $\quad \sigma > 0$

Therefore $\frac{L(\tilde{x},\mu_1)}{L(\tilde{x},\mu_0)} = \frac{\prod_{i=1}^{n} f(x_i,\mu_1,\sigma^2)}{\prod_{i=1}^{n} f(x_i,\mu_0,\sigma^2)} \geq k$

$$\Rightarrow \frac{(\frac{1}{\sigma\sqrt{2\pi}})^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu_1)^2}}{(\frac{1}{\sigma\sqrt{2\pi}})^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu_0)^2}} \geq k$$

$$\Rightarrow e^{-\frac{1}{2\sigma^2}[\sum_{i=1}^{n}(x_i-\mu_1)^2-\sum_{i=1}^{n}(x_i-\mu_0)^2]} \geq k$$

$$\Rightarrow e^{-\frac{1}{2\sigma^2}[n(\mu_1^2-\mu_0{}^2)-2\sum_{i=1}^{n}(\mu_1-\mu_0)]} \geq k$$

$$\Rightarrow e^{-\frac{1}{2\sigma^2}[n(\mu_1^2-\mu_0{}^2)-2n\bar{x}(\mu_1-\mu_0)]} \geq k$$

$$\Rightarrow -\frac{1}{2\sigma^2}[n(\mu_1^2-\mu_0{}^2) - 2n\bar{x}(\mu_1 - \mu_0)] \geq k', \ where\ k' = \text{logk}$$

$$\Rightarrow -\frac{n}{2\sigma^2}\left[n(\mu_1^2-\mu_0{}^2) + \frac{n\bar{x}}{\sigma^2}(\mu_1 - \mu_0)\right] \geq k'$$

$$\Rightarrow \frac{n\bar{x}}{\sigma^2}(\mu_1 - \mu_0) \geq k' + \frac{n}{2\sigma^2}((\mu_1^2-\mu_0{}^2)$$

$$\Rightarrow \bar{x}\left(\mu_1 - \mu_0\right) \geq \frac{\sigma^2}{n}[\,k' + \frac{n}{2\sigma^2}((\mu_1^2-\mu_0{}^2)]$$

$$\Rightarrow \bar{x}(\mu_1 - \mu_0) \geq k'', \quad \text{where } k'' = \frac{\sigma^2}{n}[k' + \frac{n}{2\sigma^2}((\mu_1^2-\mu_0{}^2)]$$

Case I: When $\mu_1 > \mu_0$, then the critical region is

$$W_1 = \{x: \bar{x} \geq k_1\}$$

Case II: When $\mu_1 < \mu_0$, then the critical region is

$$W_2 = \{x: \bar{x} \leq k_2\}$$

(I) We are to test $H_0: \mu = 36.25 \; against \; H_1: \mu > 36.25$. The critical region for testing this is given by

$$W_1 = \{x: \bar{x} \geq k_1\}$$

where k_1 is a constant to be determined such that

$$P(x \in W_1 | H_0) = \alpha$$

$$\Rightarrow P(\bar{x} \geq k_1 | H_0) = 0.01$$

To obtain the value of k_1, we need the following R command

k1=qnorm(0.99, 36.25,5/10)

k1

$\therefore k_1 = 37.41317$

Thus the critical region is $W_1 = \{x: \bar{x} \geq 37.41317\}$
Power of the test is given by=1-β= $P(x \in W_1 | H_1)$

$= P(\bar{x} \geq 37.41317 \; | \mu > 2)$
$=1- P(\bar{x} \leq 37.41317 \; | \mu > 2)$

Now to draw the power curve we construct the following table using the following R command-

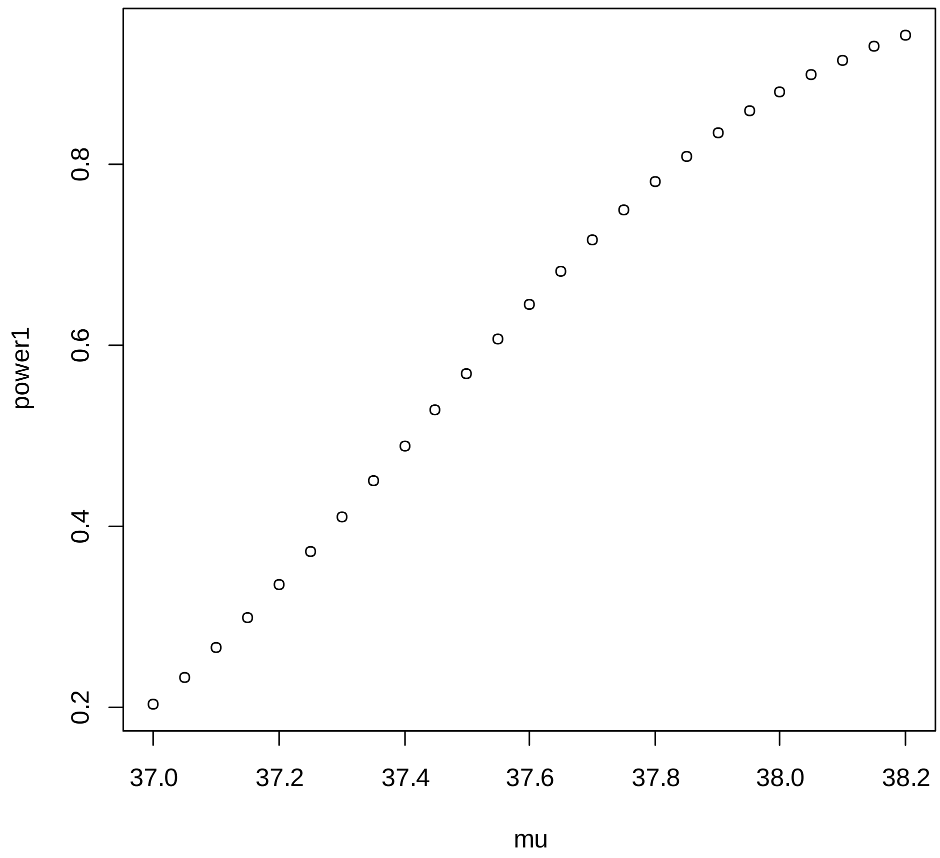
Trial values of μ(>36.25)	Power
37.00	0.2043034
37.05	0.2338128
37.10	0.2655434
37.15	0.2993233
37.20	0.3349272

37.25	0.3720806
37.30	0.4104654
37.35	0.4497283
37.40	0.4894899
37.45	0.5293564
37.50	0.5689306
37.55	0.6078240
37.60	0.6456684
37.65	0.6821260
37.70	0.7168984
37.75	0.7497337
37.80	0.7804317
37.85	0.8088461
37.90	0.8348854
37.95	0.8585107
38.00	0.8797328
38.05	0.8986066
38.10	0.9152251
38.15	0.9297124
38.20	0.9422161

Programming in R for case 1

```
sigma=5

sigma

n=100

n

sd=sigma/sqrt(n)

sd

k1=qnorm(0.99,36.25,sd)

k1

mu=c(37.00,37.05,37.10,37.15,37.20,37.25,37.30,37.35,37.40,37.45,37.50,37.55,37.60,37.65
,37.70,37.75,37.80,37.85,37.90,37.95,38.00,38.05,38.10,38.15,38.20)

mu

power=mat.or.vec(25,1)

power1=mat.or.vec(25,1)

for(i in 1:25){

power[i]=pnorm(k1,mu[i],sd)

power

power1[i]=1-power[i]}

power1

plot(mu,power1)
```

Power curve for case 1



ii) Here we are to test$H_0: \mu = 36.25\ against\ H_1: \mu < 36.25$. The critical region for testing this is given by

$$W_2 = \{x: \bar{x} \leq k_2\}$$

where k_2 is a constant to be determined such that

$$P(x \in W_2 | H_0) = \alpha$$

$$\Rightarrow P(\bar{x} \leq k_2 | H_0) = 0.01$$

To obtain the value of k_2, we need the following R command

k2=qnorm(0.01,36.25,5/10)

k2

$\therefore k_2 = 35.08683$

Thus the critical region is $W_2 = \{x: \bar{x} \geq 35.08683\}$

Power of the test is given by=1-β= $P(x \in W_2 | H_1)$

$$= P(\bar{x} \leq 35.08683\ |\mu < 2)$$

Now to draw the power curve we construct the following table considering different trail values ($\mu < 36.25$)

Trial values for $\mu < 36.25$	Power
30.50	1.0000000
30.70	1.0000000
30.90	1.0000000
31.10	1.0000000
31.30	1.0000000
31.50	1.0000000
31.70	1.0000000

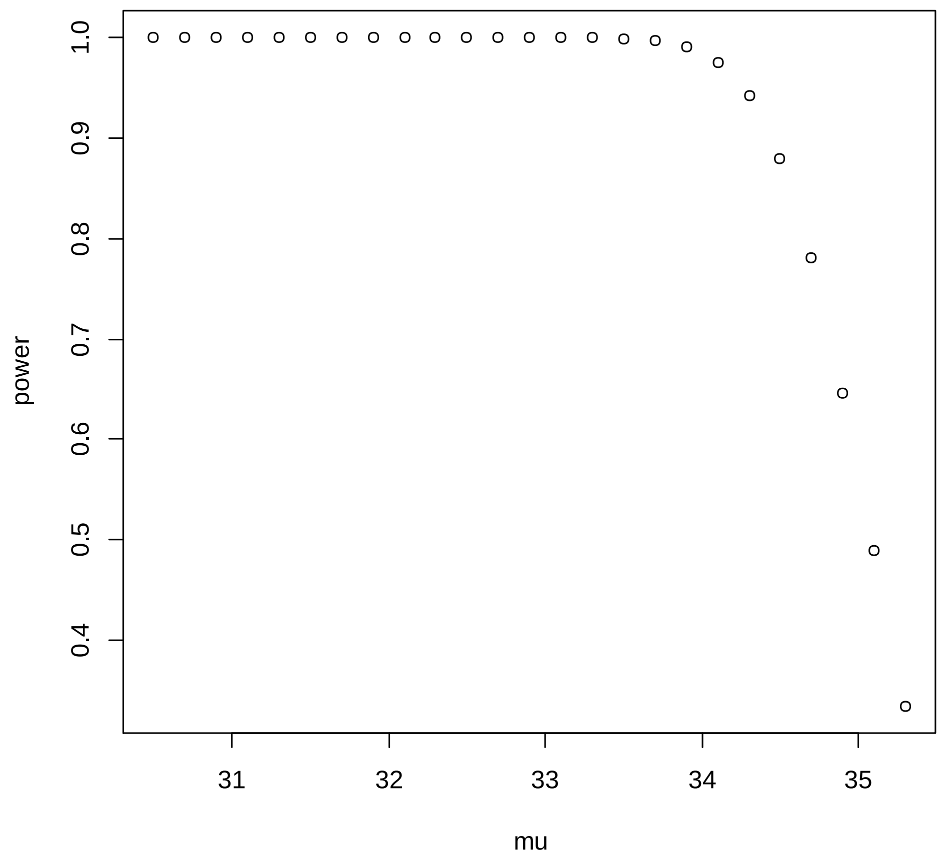
31.90	1.0000000
32.10	1.0000000
32.30	1.0000000
32.50	0.9999999
32.70	0.9999991
32.90	0.9999939
33.10	0.9999646
33.30	0.9998240
33.50	0.9992473
33.70	0.9972285
33.90	0.9911934
34.10	0.9757893
34.30	0.9422161
34.50	0.8797328
34.70	0.7804317
34.90	0.6456684
35.10	0.4894899
35.30	0.3349272

Programming in R for case 2

```
sigma=5

sigma
```

```
n=100

n

sd=sigma/sqrt(n)

sd

k2=qnorm(0.01,36.25,sd)

k2

mu=c(30.50,30.70,30.90,31.10,31.30,31.50,31.70,31.90,32.10,32.30,32.50,32.70,32.90,33.10,33.
30,33.50,33.70,33.90,34.10,34.30,34.50,34.70,34.90,35.10,35.30)

mu

power=mat.or.vec(25,1)

power

for(i in 1:25){

power[i]=pnorm(k2,mu[i],sd)}

power

plot(mu,power)
```

Power curve for case 2



iii) Here we are to test $H_0: \mu = 36.25$ $against$ $H_1: \mu \neq 36.25$. The critical region for testing this is given by

$$W_3 = \{x: \bar{x} \leq k_3 \ or\ \bar{x} \geq k_4 \}$$

where k_3 and $k_4$are some constants to be determined such that

$$P(x \in W_3 | H_0) = \alpha$$

$$\Rightarrow P(\bar{x} \leq k_3 \ or\ \bar{x} \geq k_4 | H_0) = 0.01$$

$$\Rightarrow P(\bar{x} \leq k_3) + P(\bar{x} \geq k_4) = 0.01$$

Assuming that the test is equitailed, we have

$P(\bar{x} \leq k_3) = \frac{0.01}{2}$=0.005

&$P(\bar{x} \geq k_4) = 0.005$

To obtain the values of k_3 and k_4, we use the following R command:

```
k3=qnorm(0.005,36.25,5/10)

k3

k4=qnorm(1-0.005,36.25,5/10)

k4
```

$$\therefore k_3 = 34.96209 \quad and\ k_4 = 37.53791$$

Thus, the critical region is

$$W_3 = \{x: \bar{x} \leq 34.96209 \ \ or\ \bar{x} \geq 37.53791\}$$

The power of the test is given by

1-β= $P(x \in W_3 | H_1)$

=P$(x: \bar{x} \leq 34.96209 \ or\ \bar{x} \geq 37.53791 | H_1)$

=$P_{\mu \neq 2}(x: \bar{x} \leq 34.96209) + P_{\mu \neq 2}(\bar{x} \geq 37.53791 | H_1)$

=$P_{\mu \neq 2}(x: \bar{x} \leq 34.96209) + [1 - P_{\mu \neq 2}(\bar{x} \leq 37.53791 | H_1)]$

To draw the power curve we construct the following table considering different trail values $of \mu \neq 36.25$.

Trial values for $\mu \neq 36.25$	Power
30.50	1.0000000
30.70	1.0000000
30.90	1.0000000
31.10	1.0000000
31.30	1.0000000
31.50	1.0000000
31.70	1.0000000
31.90	1.0000000
32.10	1.0000000
32.30	0.9999999
32.50	0.9999996
32.70	0.9999970
32.90	0.9999814
33.10	0.9999020
33.30	0.9995566
33.50	0.9982731
33.70	0.9942014
33.90	0.9831721
34.10	0.9576615

34.30	0.9072768
34.50	0.8223013
34.70	0.6999201
34.90	0.5494100
35.10	0.3913401
35.30	0.2495783

Trial values for $\mu \neq 36.25$	Power
37.00	0.1410247
37.05	0.1645894
37.10	0.1905709
37.15	0.2189309
37.20	0.2495783
37.25	0.2823677
37.30	0.3170995
37.35	0.3535228
37.40	0.3913401
37.45	0.4302143
37.50	0.4697776
37.55	0.5096419

37.60	0.5494100
37.65	0.5886878
37.70	0.6270956
37.75	0.6642793
37.80	0.6999201
37.85	0.7337423
37.90	0.7655195
37.95	0.7950787
38.00	0.8223013
38.05	0.8471227
38.10	0.8695297
38.15	0.8895561
38.20	0.9072768

<u>Programming in R for case 3</u>

```
sigma=5

sigma

n=100

n

sd=sigma/sqrt(n)

sd

k3=qnorm(0.005,36.25,sd)
```

```
k3

k4= qnorm(1-0.005,36.25,sd)

k4

mu=c(30.50,30.70,30.90,31.10,31.30,31.50,31.70,31.90,32.10,32.30,32.50,32.70,32.90,33.10,33.
30,33.50,33.70,33.90,34.10,34.30,34.50,34.70,34.90,35.10,35.30,37.00,37.05,37.10,37.15,37.20,
37.25,37.30,37.35,37.40,37.45,37.50,37.55,37.60,37.65,37.70,37.75,37.80,37.85,37.90,37.95,38.
00,38.05,38.10,38.15,38.20)

mu

power2=mat.or.vec(50,1)

for(i in 1:50){

power2[i]=pnorm(k3,mu[i],sd)+(1-pnorm( k4,mu[i],sd)) }

power2

plot(mu,power2)
```

power curve for case 3