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Experiment No -01

Topic- Tracing a power curve for testing the mean of Normal Distribution.

Problem –A random sample of size 25 is drawn from N$\sim(\mu, \sigma^2)$ where σ^2 = 4. Draw the power curve for testing $H_0 : \mu = 2$

Against

(1) $H_1 : \mu > 2$ (2) $H_1 : \mu < 2$ (3) $H_1 : \mu \neq 2$

Use the level of significance as $\alpha = 0.05$.

Theory and Calculation-

Using Neyman's pearson fundamental lemma, the critical region is given by-

W = $\{x : \frac{L(\tilde{x},\theta_1)}{L(\tilde{x},\theta_0)} \geq k\}$

Here, f(x, μ, σ^2) = $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$, $-\infty < x < \infty$, $-\infty < \mu < \infty$, $\sigma > 0$

Therefore $\frac{L(\tilde{x},\mu_1)}{L(\tilde{x},\mu_0)}$ = $\frac{\prod_{i=1}^{n} f(x_i,\mu_1,\sigma^2)}{\prod_{i=1}^{n} f(x_i,\mu_0,\sigma^2)} \geq k$

$$\Rightarrow \frac{(\frac{1}{\sigma\sqrt{2\pi}})^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu_1)^2}}{(\frac{1}{\sigma\sqrt{2\pi}})^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu_0)^2}} \geq k$$

$$\Rightarrow e^{-\frac{1}{2\sigma^2}[\sum_{i=1}^{n}(x_i-\mu_1)^2-\sum_{i=1}^{n}(x_i-\mu_0)^2]} \geq k$$

$$\Rightarrow e^{-\frac{1}{2\sigma^2}[n(\mu_1^2-\mu_0{}^2)-2\sum_{i=1}^{n}(\mu_1-\mu_0)]} \geq k$$

$$\Rightarrow e^{-\frac{1}{2\sigma^2}[n(\mu_1^2-\mu_0{}^2)-2n\bar{x}(\mu_1-\mu_0)]} \geq k$$

$$\Rightarrow -\frac{1}{2\sigma^2}[n(\mu_1^2-\mu_0{}^2) - 2n\bar{x}(\mu_1 - \mu_0)] \geq k', \ where\ k' = \text{logk}$$

$$\Rightarrow -\frac{n}{2\sigma^2}\left[n(\mu_1^2-\mu_0{}^2) + \frac{n\bar{x}}{\sigma^2}(\mu_1 - \mu_0)\right] \geq k'$$

$$\Rightarrow \frac{n\bar{x}}{\sigma^2}(\mu_1 - \mu_0) \geq k' + \frac{n}{2\sigma^2}((\mu_1^2-\mu_0{}^2)$$

$$\Rightarrow \bar{x}\left(\mu_1 - \mu_0\right) \geq \frac{\sigma^2}{n}[\, k' + \frac{n}{2\sigma^2}((\mu_1^2-\mu_0{}^2)]$$

$$\Rightarrow \bar{x}(\mu_1 - \mu_0) \geq k'', \ \text{where } k'' = \frac{\sigma^2}{n}[k' + \frac{n}{2\sigma^2}((\mu_1^2-\mu_0{}^2)]$$

Case I: When $\mu_1 > \mu_0$, then the critical region is

$$W_1 = \{x : \bar{x} \geq k_1\}$$

Case II: When $\mu_1 < \mu_0$, then the critical region is

$$W_2 = \{x: \bar{x} \leq k_2\}$$

(I) We are to test $H_0: \mu = 2$ $against$ $H_1: \mu > 2$. The critical region for testing this is given by

$$W_1 = \{x: \bar{x} \geq k_1\}$$

where k_1 is a constant to be determined such that

$$P(x \in W_1 | H_0) = \alpha$$

$$\Rightarrow P(\bar{x} \geq k_1 | H_0) = 0.05$$

To obtain the value of k_1, we need the following R command

```
k1=qnorm(0.95,2,2/5)

k1
```

$\therefore k_1 =$2.657941

Thus the critical region is $W_1 = \{x: \bar{x} \geq$2.657941$\}$

Power of the test is given by=1-β= $P(x \in W_1 | H_1)$

$$= P(\bar{x} \geq 2.657941 \; | \mu > 2)$$

$$=1- P(\bar{x} \leq 2.657941 \; | \mu > 2)$$

Now to draw the power curve we construct the following table using the following R command-

Trial values of μ(>2)	Power
3.01	0.8106100
3.02	0.8173061
3.03	0.8238523
3.04	0.8302482
3.05	0.8364931
3.06	0.8425868
3.07	0.8485294
3.08	0.8543208
3.09	0.8599615
3.10	0.8654519
3.11	0.8707927
3.12	0.8759848
3.13	0.8810290
3.14	0.8859266
3.15	0.8906789
3.16	0.8952872
3.17	0.8997532

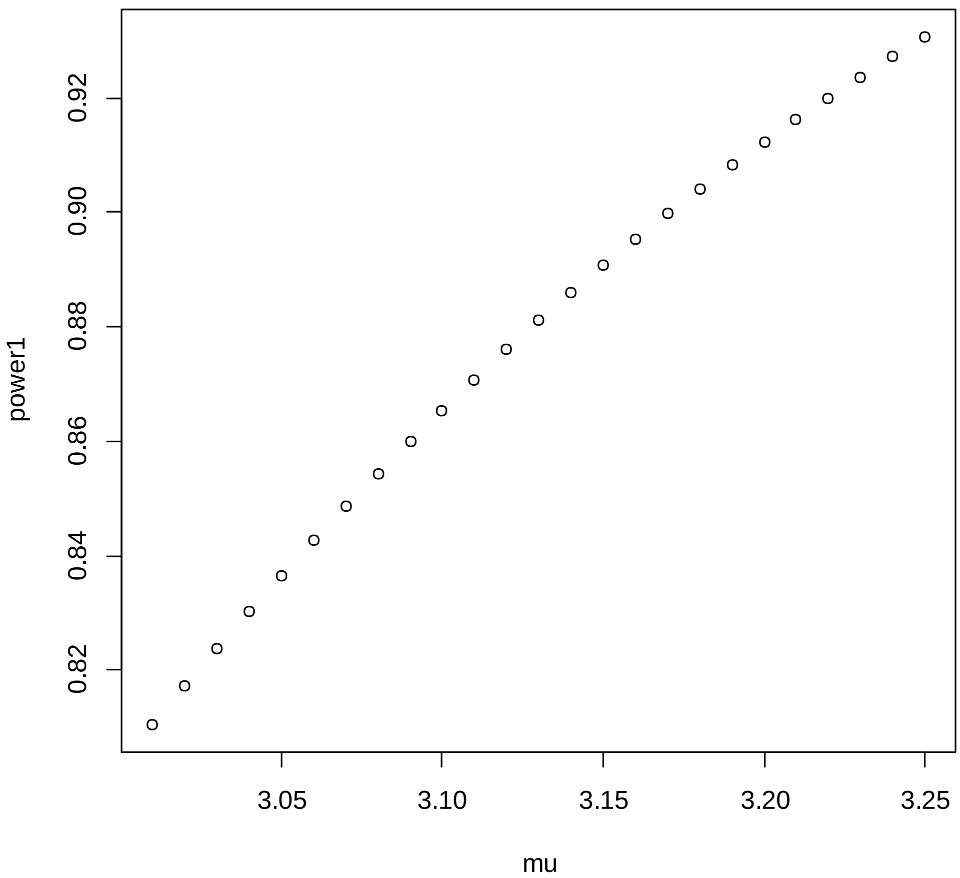
3.18	0.9040785
3.19	0.9082650
3.20	0.9123145
3.21	0.9162292
3.22	0.9200111
3.23	0.9236625
3.24	0.9271856
3.25	0.9305829

Programming in R for case 1

```
sigma=2

sigma

n=25

n

sd=sigma/sqrt(n)

sd

k1=qnorm(0.95,2,sd)

k1

mu=c(3.01,3.02,3.03,3.04,3.05,3.06,3.07,3.08,3.09,3.10,3.11,3.12,3.13,3.14,3.15,3.16,3.17,3.
18,3.19,3.20,3.21,3.22,3.23,3.24,3.25)

mu

power=mat.or.vec(25,1)

power
```

```
power1=mat.or.vec(25,1)

power1

for(i in 1:25){

power[i]=pnorm(k1,mu[i],sd)

power

power1[i]=1-power[i]}

power1

plot(mu,power1)
```

Power curve for case 1



ii) Here we are to test$H_0: \mu = 2\ against\ H_1: \mu < 2$. The critical region for testing this is given by

$$W_2 = \{x: \bar{x} \leq k_2\}$$

where k_2 is a constant to be determined such that

$$P(x \in W_2 | H_0) = \alpha$$

$$\Rightarrow P(\bar{x} \leq k_2 | H_0) = 0.05$$

To obtain the value of k_2, we need the following R command

```
k2=qnorm(0.05,2,2/5)

k2
```

$\therefore k_2$ =1.342059

Thus the critical region is $W_2 = \{x: \bar{x} \geq$1.342059$\}$

Power of the test is given by=1-β= $P(x \in W_2 | H_1)$

$$= P(\bar{x} \leq 1.342059\ |\mu < 2)$$

Now to draw the power curve we construct the following table considering different trail values ($\mu < 2$).

Trial values for $\mu < 2$	Power
1.60	0.2595110
1.61	0.2514756
1.62	0.2435735
1.63	0.2358076
1.64	0.2281801
1.65	0.2206934
1.66	0.2133493
1.67	0.2061498
1.68	0.1990963
1.69	0.1921902
1.70	0.1854327
1.71	0.1788246
1.72	0.1723668
1.73	0.1660597

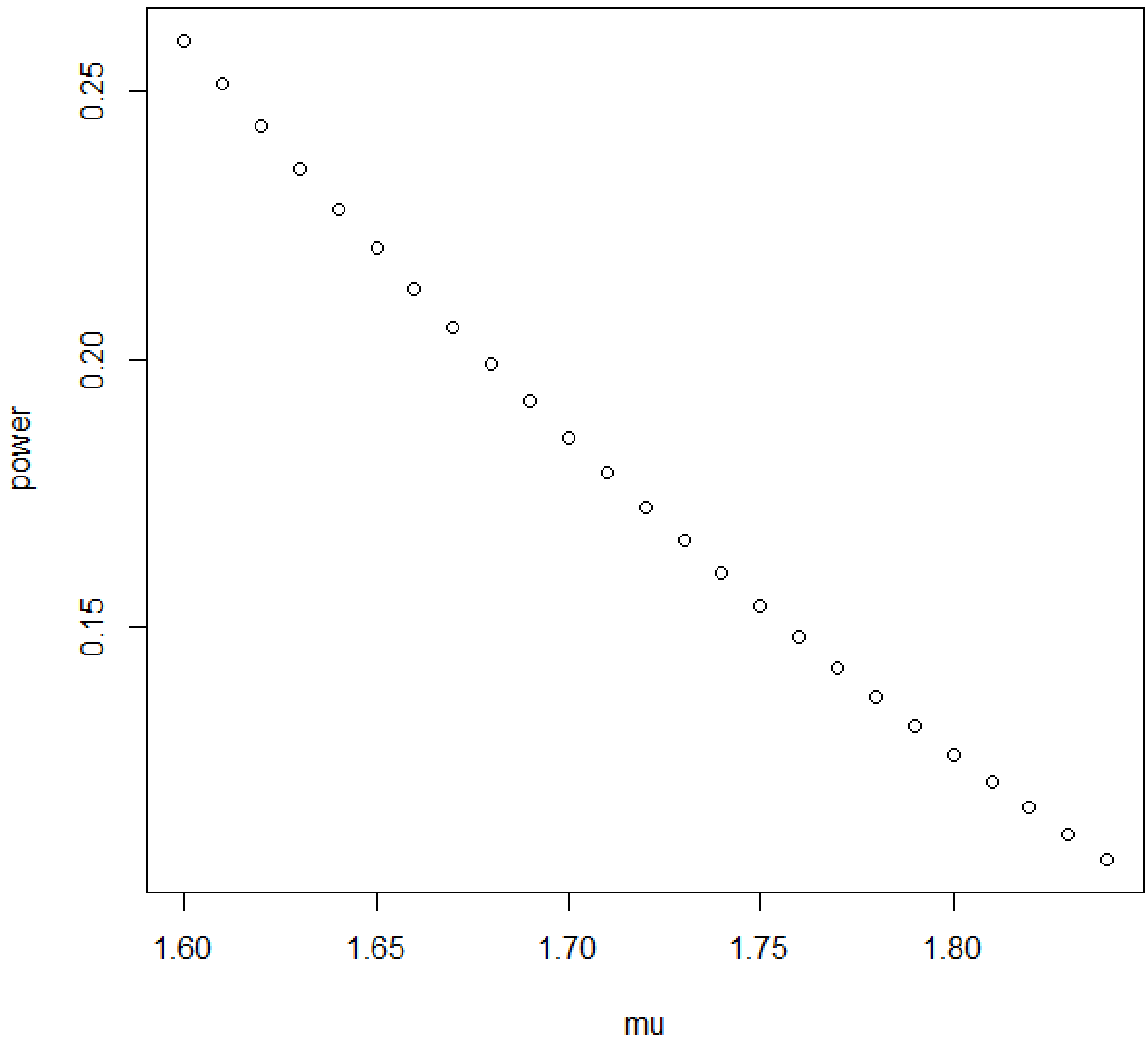
1.74	0.1599037
1.75	0.1538989
1.76	0.1480453
1.77	0.1423426
1.78	0.1367904
1.79	0.1313881
1.80	0.1261349
1.81	0.1210299
1.82	0.1160721
1.83	0.1112602
1.84	0.1065928

Programming in R for case 2

```
sigma=2

sigma

n=25

n

sd=sigma/sqrt(n)

sd

k2=qnorm(0.05,2,sd)

k2

mu=c(1.60,1.61,1.62,1.63,1.64,1.65,1.66,1.67,1.68,1.69,1.70,1.71,1.72,1.73,1.74,1.75,1.76,1.77,1.78,1.79,1.80,1.81,1.82,1.83,1.84)

mu

power=mat.or.vec(25,1)

power

for(i in 1:25){

power[i]=pnorm(k2,mu[i],sd)}

power
```

```
plot(mu,power)
```

Power curve for case 2



iii) Here we are to test $H_0: \mu = 2$ $against$ $H_1: \mu \neq 2$. The critical region for testing this is given by

$$W_3 = \{x: \bar{x} \leq k_3 \text{ } or \text{ } \bar{x} \geq k_4 \}$$

where k_3 and k_4 are some constants to be determined such that

$$P(x \in W_3 | H_0) = \alpha$$

$$\Rightarrow P(\bar{x} \leq k_3 \text{ } or \text{ } \bar{x} \geq k_4 | H_0) = 0.05$$

$$\Rightarrow P(\bar{x} \leq k_3) + P(\bar{x} \geq k_4) = 0.05$$

Assuming that the test is equitailed, we have

$P(\bar{x} \leq k_3) = \frac{0.05}{2}$=0.025

&$P(\bar{x} \geq k_4) = 0.025$

To obtain the values of k_3 and k_4, we use the following R command:

```
k3=qnorm(0.025,2,2/5)

k3

k4=qnorm(1-0.025,2,2/5)

k4
```

$$\therefore k_3 = 1.216014 \quad and \text{ } k_4 = 2.783986$$

Thus, the critical region is

$$W_3 = \{x: \bar{x} \leq 1.216014 \text{ } or \text{ } \bar{x} \geq 2.783986\}$$

The power of the test is given by

1-β= $P(x \in W_3 | H_1)$

=P$(x: \bar{x} \leq 1.216014 \text{ } or \text{ } \bar{x} \geq 2.783986 | H_1)$

=$P_{\mu \neq 2}(x: \bar{x} \leq 1.216014) + P_{\mu \neq 2}(\bar{x} \geq 2.783986 | H_1)$

=$P_{\mu \neq 2}(x: \bar{x} \leq 1.216014) + [1 - P_{\mu \neq 2}(\bar{x} \leq 2.783986 | H_1)]$

To draw the power curve we construct the following table considering different trail values $of \mu \neq 2$.

Trial values for $\mu \neq 2$	Power
1.60	0.17007505
1.61	0.16398881
1.62	0.15806362
1.63	0.15230016
1.64	0.14669894
1.65	0.14126035
1.66	0.13598463
1.67	0.13087189
1.68	0.12592212
1.69	0.12113520
1.70	0.11651088
1.71	0.11204884
1.72	0.10774863
1.73	0.10360975
1.74	0.09963160
1.75	0.09581353
1.76	0.09215481
1.77	0.08865468
1.78	0.08531233
1.79	0.08212691
1.80	0.07909753
1.81	0.07622332
1.82	0.07350335
1.83	0.07093673
1.84	0.06852255
3.01	0.71397901
3.02	0.72241999
3.03	0.73073741
3.04	0.73892797
3.05	0.74698854
3.06	0.75491624
3.07	0.76270839

3.08	0.77036251
3.09	0.77787635
3.10	0.78524787
3.11	0.79247525
3.12	0.79955687
3.13	0.80649134
3.14	0.81327748
3.15	0.81991430
3.16	0.82640104
3.17	0.83273713
3.18	0.83892220
3.19	0.84495607
3.20	0.85083877
3.21	0.85657049
3.22	0.86215163
3.23	0.86758274
3.24	0.87286456
3.25	0.87799798

Programming in R for case 3

sigma=2

sigma

n=25

```
n

sd=sigma/sqrt(n)

sd

k3=qnorm(0.025,2,sd)

k3

k4=qnorm(1-0.025,2,sd)

k4

mu=c(1.60,1.61,1.62,1.63,1.64,1.65,1.66,1.67,1.68,1.69,1.70,1.71,1.72,1.73,1.74,1.75,1.76,1.77,1.78,1.79,1.80,1.81,1.82,1.83,1.84,3.01,3.02,3.03,3.04,3.05,3.06,3.07,3.08,3.09,3.10,3.11,3.12,3.13,3.14,3.15,3.16,3.17,3.18,3.19,3.20,3.21,3.22,3.23,3.24,3.25)

mu

power2=mat.or.vec(50,1)

power2

for(i in 1:50){

power2[i]=pnorm(k3,mu[i],sd)+(1-pnorm(k4,mu[i],sd))}

power2

plot(mu,power2)
```

power curve for case 3