Roll No-12

M.sc. 3rd semester

Date of Assignment-22/11/2020

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Experiment No -06

Topic- Tracing the power curve of Poisson Distribution.

Problem – Draw the power curve for testing $H_0 : \lambda = 9.5$ against $H_0 : \lambda \rangle 9.5$

i) $H_1 : \lambda \rangle 9.5$ ii) $H_1 : \lambda \langle 9.5$

Where λ is the mean of the Poisson distribution and the significance level is $\alpha = 0.05$

Theory and Calculation-

Here we are testing $H_0 : \lambda = 9.5$ against $H_0 : \lambda \rangle 9.5$

Since $\bar{x}$ is a sufficient statistic for λ, therefore the critical region is given by-

$W_1 = \left\{ \underset{\sim}{x} : \bar{x} \rangle k_1 \right\}$, Where k_1 is a constant to be determined such that $\alpha = 0.05.$ Therefore-

$\alpha = P\left(X \in W_1 \mid H_0 \right)$

$\Rightarrow 0.05 = P\left(\bar{x} \rangle k_1 \mid H_0 \right) = P\left(Y \rangle k_1 \mid \lambda = 9.5 \right)$ Where $Y \sim P\left(\lambda = 9.5 \right)$

$\Rightarrow 1 - 0.05 = 1 - P\left(Y \rangle k_1 \mid \lambda = 9.5 \right)$

$\Rightarrow 0.95 = P\left(Y \langle k_1 \mid \lambda = 9.5 \right)$

To find out the value of k_1, we use the following R-command-

k1=qpois(0.95,9.5)

k1

Therefore, k1=15

The C.R. is given by-

$W_1 = \left\{ \underset{\sim}{x} : \bar{x} \rangle 15 \right\}$

Now the power of the test is given by-

Power $= 1 - \beta$

= [Reject $H_0 \mid H_1$ is true]

$= P\left(\bar{x} \rangle 15 \mid \lambda \rangle 9.5 \right)$

$= P\left(Y \rangle 15 \mid \lambda \rangle 9.5 \right)$ Where $Y \sim P\left(9.5 \right)$

$= 1 - P\left(Y \leq 15 \mid \lambda \rangle 9.5 \right)$

Now to draw the power curve we construct the following table considering different values of $\lambda\rangle 9.5$

<u>TABLE</u>

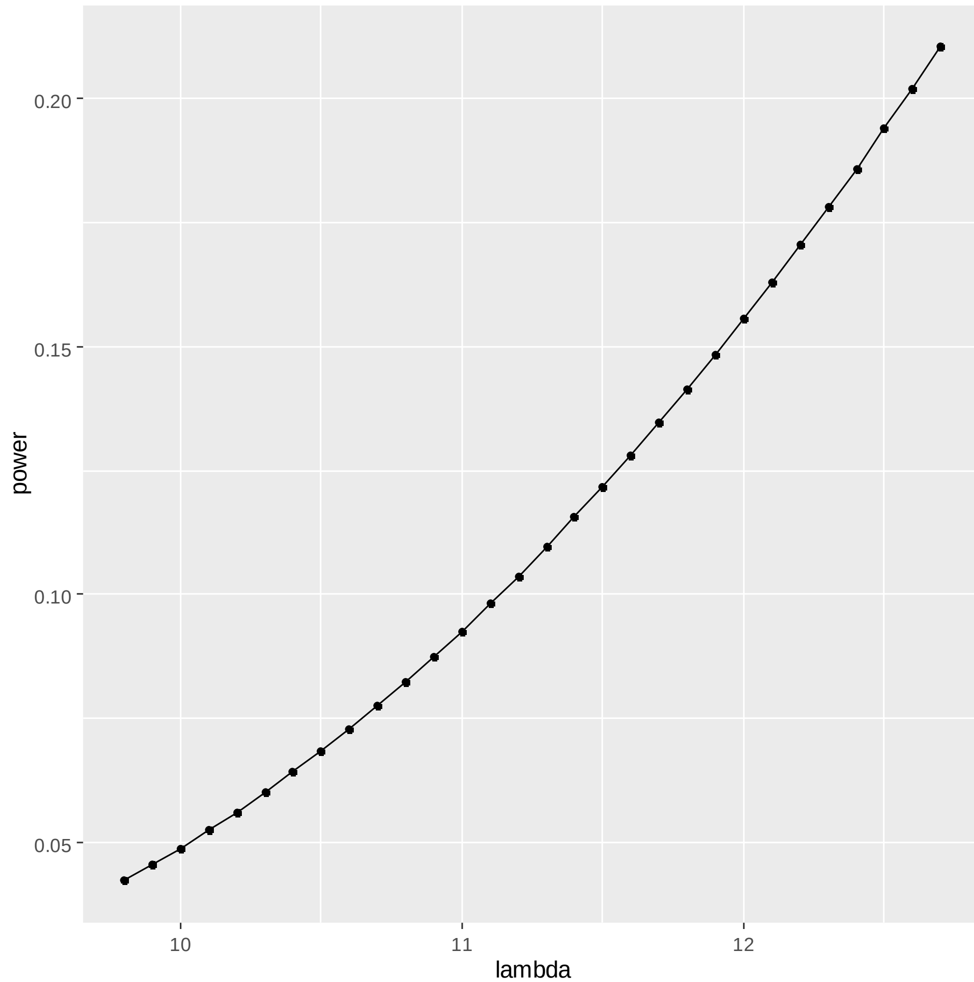
lambda	power	
1	9.8	0.04213918
2	9.9	0.04535480
3	10.0	0.04874040
4	10.1	0.05229957
5	10.2	0.05603566
6	10.3	0.05995175
7	10.4	0.06405067
8	10.5	0.06833494
9	10.6	0.07280680
10	10.7	0.07746818
11	10.8	0.08232069
12	10.9	0.08736562
13	11.0	0.09260391
14	11.1	0.09803618

	lambda	power
15	11.2	0.10366270
16	11.3	0.10948338
17	11.4	0.11549780
18	11.5	0.12170517
19	11.6	0.12810435
20	11.7	0.13469387
21	11.8	0.14147188
22	11.9	0.14843621
23	12.0	0.15558435
24	12.1	0.16291343
25	12.2	0.17042029
26	12.3	0.17810142
27	12.4	0.18595303
28	12.5	0.19397100
29	12.6	0.20215094
30	12.7	0.2104881

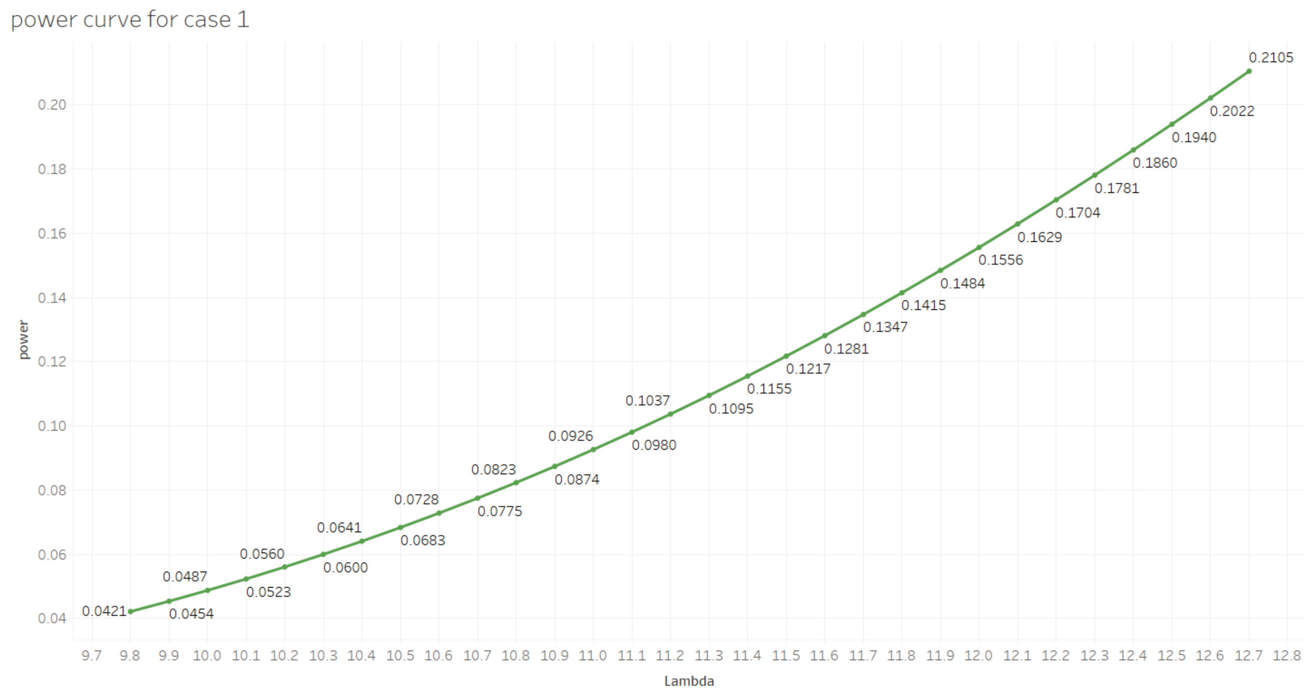
Programming in R for case 1-

```
library('ggplot2')

k1=qpois(0.95,9.5)

k1

lambda=seq(from=9.8, by=0.1, length.out=30)

power=mat.or.vec(30,1)

for(i in 1:30){

power[i]=1-ppois(15,lambda[i])}

power

Table=data.frame(lambda,power)

Table

View(Table)

ggplot(data=Table,mapping=aes(x=lambda,y=power))+geom_point()+geom_line()

data.frame(lambda)

data.frame(power)
```

Power curve by using ggplot 2



Power curve generated by using Tableau



Lambda vs. power. The marks are labeled by sum of power.

ii) For testing $H_0 : \lambda = 9.5$ against $H_1 : \lambda < 9.5$, the C.R. is given by-

$W_2 = \{\underset{\sim}{x} : \bar{x} < k_2\}$, Where k_2 is a constant to be determined such that $\alpha = 0.05$. Therefore,

$$\alpha = P(x \in W \mid H_0) = P(\bar{x} < k_2 \mid \lambda = 9.5)$$

$$= P(Y < k_2 \mid \lambda = 9.5)$$

$$\Rightarrow 0.05 = P(Y \le k_2 - 1 \mid \lambda = 9.5)$$

To find out the values of k_2, we use the following R-command-

k2_1=qpois(0.05,9.5)

k2_1

Therefore,   k2-1=5 => k2=6

The C.R. is given by- $W_2 = \{\underset{\sim}{x} : \bar{x} \langle 6\}$

Power of the test is given by-

Power= $1-\beta$ = [Reject $H_0 \mid H_1$ is true]

$$= P(\bar{x} \langle 6 \mid \lambda \langle 9.5)$$

$$= P(Y \langle 6 \mid \lambda \langle 9.5)$$

$$= P(Y \langle 5 \mid \lambda \langle 9.5)$$

Now to draw the power curve, we construct the following table considering different trial values of $\lambda \langle 9.5$

TABLE

	lambda	power
1	6.2	0.4141130
2	6.3	0.3987717
3	6.4	0.3837437
4	6.5	0.3690407
5	6.6	0.3546730

	lambda	power
6	6.7	0.3406494
7	6.8	0.3269771
8	6.9	0.3136619
9	7.0	0.3007083
10	7.1	0.2881194
11	7.2	0.2758975
12	7.3	0.2640432
13	7.4	0.2525566
14	7.5	0.2414365
15	7.6	0.2306808
16	7.7	0.2202869
17	7.8	0.2102511
18	7.9	0.2005691
19	8.0	0.1912361
20	8.1	0.1822465
21	8.2	0.1735944

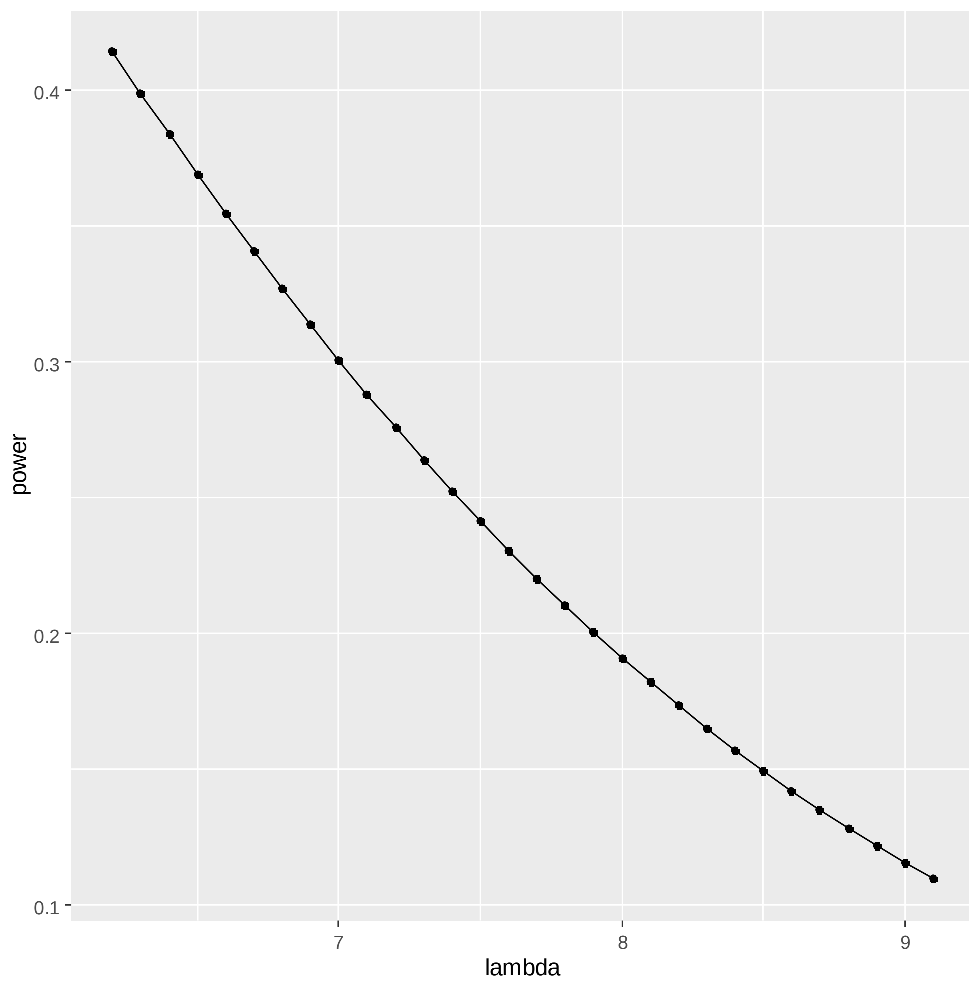
	lambda	power
22	8.3	0.1652734
23	8.4	0.1572768
24	8.5	0.1495973
25	8.6	0.1422276
26	8.7	0.1351600
27	8.8	0.1283866
28	8.9	0.1218995
29	9.0	0.1156905
30	9.1	0.1097514

Programming in R for case 2-

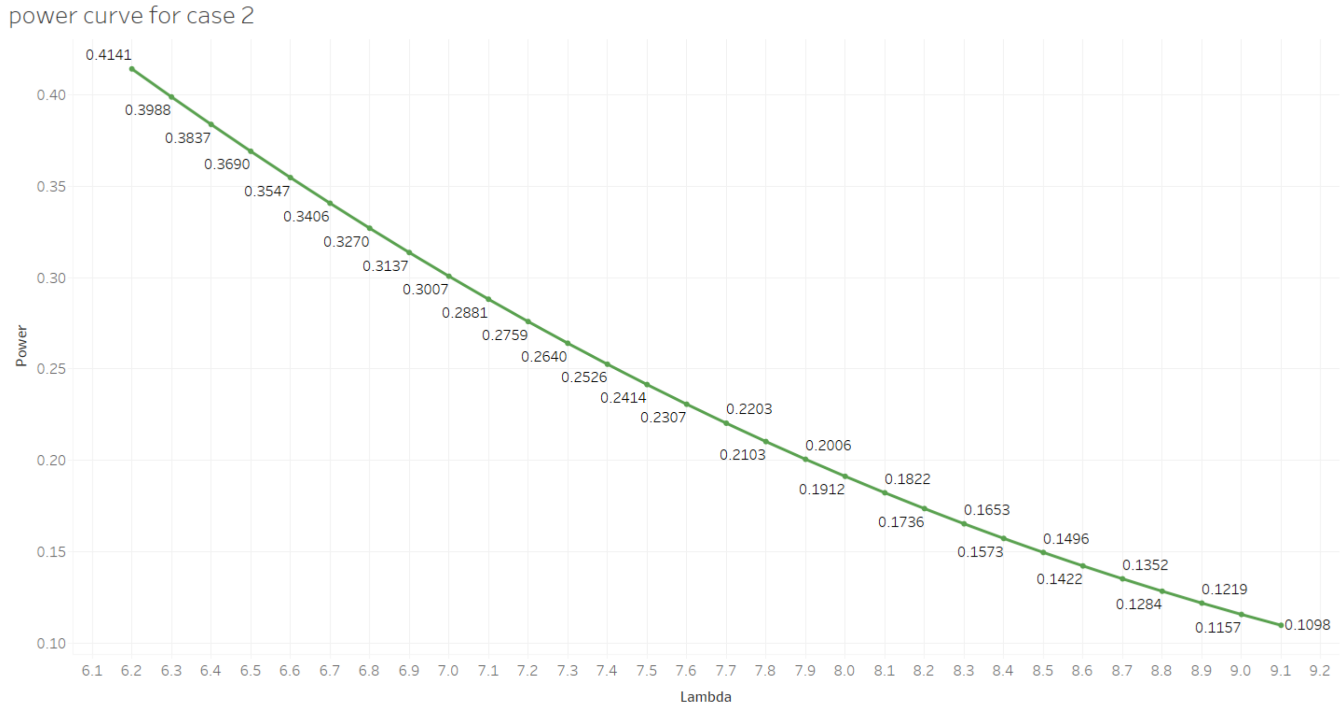
```
library('ggplot2')

k2=6

k2_1=5

lambda=seq(from=6.2, by=0.1, length.out=30)

power=mat.or.vec(30,1)

for(i in 1:30){

power[i]=ppois(k2_1,lambda[i])}

power

Table=data.frame(lambda,power)
```

```
Table
View(Table)
ggplot(data=Table,mapping=aes(x=lambda,y=power))+geom_point()+geom_line()
data.frame(lambda)
data.frame(power)
```

Power curve by using ggplot 2



Power curve generated by using Tableau



Lambda vs. Power. The marks are labeled by sum of Power.