Roll No-12

M.sc. 3^{rd} semester

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Experiment No -05

Topic- Tracing of power curve for Binomial Distribution.

Problem – A coin is tossed 20 times and we want to test whether the coin is unbiased. Construct the appropriate critical region and draw the power curve testing. The level of significance is- $\alpha = 0.527$

Theory and Calculation-

Let, $\theta =$ Probability of getting the heads.

To test whether the coin is unbiased we need to test the null hypothesis-

$$H_0 : \theta = \frac{1}{2} \quad against \quad H_1 : \theta \neq \frac{1}{2}$$

The critical region for testing the null hypothesis is given by-

$$W = \left\{ \underset{\sim}{x} : \sum_{i=1}^{20} x_i \rangle k_1 \quad or \quad \sum_{i=1}^{20} x_i \langle k_2 \right\}$$

Where k_1 and k_2 are the unknown constants to be determined such that $\alpha = 0.527$

Therefore, $\alpha = P(X \in W \mid H_0) = P\left[\sum_{i=1}^{20} x_i \rangle k_1 \quad or \quad \sum_{i=1}^{20} x_i \langle k_2 \mid H_0 \right]$

$$= P\left[X \rangle k_1 \quad or \quad X \langle k_2 \mid H_0 \right]$$

Where, $X = \sum_{i=1}^{20} x_i \sim Binom\left(20, \frac{1}{2}\right)$

Assuming that the test is equitailed we have-

$$P(X \rangle k_1) = \frac{0.527}{2}$$

$$\Rightarrow 1 - P(X \rangle k_1) = 1 - \frac{0.527}{2} = 1 - 0.2635$$

$$\Rightarrow P(X \leq k_1) = 0.7365$$

And $P(X \langle k_2) = \frac{0.527}{2} = 0.2635$

$$\Rightarrow P(X \leq k_2 - 1) = 0.2635$$

To find the value of k1 and k2, we use the following R-command-

K_one=qbinom(0.7365,20,1/2)

K_two=qbinom(0.2635,20,1/2)

K_one

K_two

Therefore, k_one=11 k_two=9 k_three=10

Therefore, the C.R. is given by-

$$W = \left\{ \underset{\sim}{x} : \sum_{i=1}^{20} x_i \rangle 11 \quad or \quad \sum_{i=1}^{20} x_i \langle 10 \right\}$$

Power of the test is given by-

Power=$1-\beta$ = [Reject $H_0 \mid H_1$ is true]

$$= P\left[\sum_{i=1}^{20} x_i \rangle 11 \quad or \quad \sum_{i=1}^{20} x_i \langle 10 \mid H_1 \right]$$

$$= P_{H_1}\left[\sum_{i=1}^{20} x_i \rangle 11 \right] + P_{H_1}\left[\sum_{i=1}^{20} x_i \langle 10 \right]$$

$$= P_{H_1}\left[X \le 9 \right] + \left[1 - P_{H_1}\left\{ X \le 11 \right\} \right]$$

Now to draw the power curve for testing $H_0 : \theta = \frac{1}{2} \quad against \quad H_1 : \theta \neq \frac{1}{2}$, we construct the

following table considering different trial values of $\theta \neq \frac{1}{2}$ and using the R- program given below-

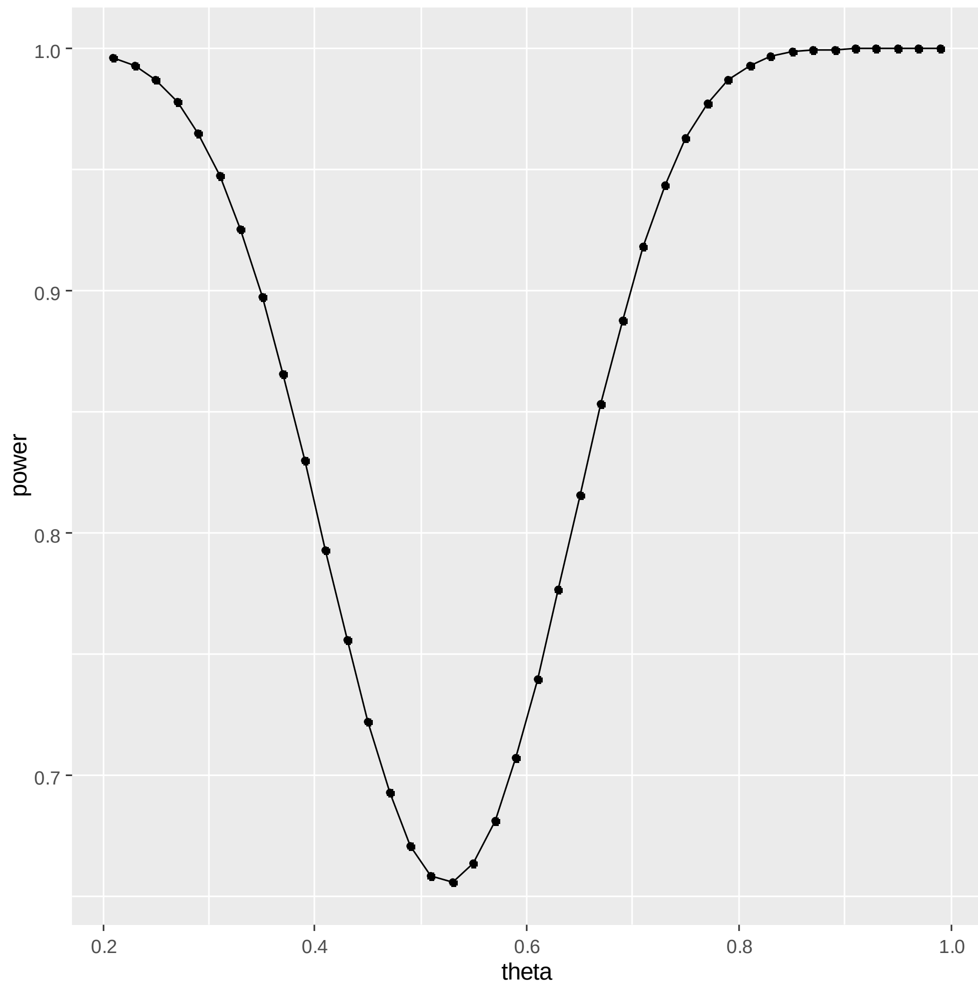
Programming in R

```
library('ggplot2')

k_one=qbinom(0.7365,20,1/2)

k_one

k_two=qbinom(0.2635,20,1/2)

k_two

k_three=k_two+1

k_three

theta=seq(from=0.21, by=0.02, length.out=40)

power=mat.or.vec(40,1)

for(i in 1:40){

power[i]=pbinom(9,20,theta[i])+1-pbinom(11,20,theta[i])}

power

Table=data.frame(theta,power)

Table

View(Table)

ggplot(data=Table,mapping=aes(x=theta,y=power))+geom_point()+geom_line()

data.frame(theta)

data.frame(power)
```

TABLE

Trial values of $\theta \neq \frac{1}{2}$	**Power**
	0.9963770
	0.9928696
	0.9870710
	0.9781552
	0.9653023
	0.9478274
	0.9253237
	0.8977988
	0.8657787
	0.8303570
	0.7931724
	0.7563092
	0.7221262
	0.6930302
	0.6712233
	0.6584551
	0.6558150
	0.6635956
	0.6812466
	0.7074289
	0.7401629
	0.7770509
	0.8155443
	0.8532157
	0.8880019
	0.9183819
	0.9434700
	0.9630170
	0.9773254
	0.9871034
	0.9932855
	0.9968570
	0.9987097
	0.9995518
	0.9998751
	0.9999744
	0.9999967
	0.9999998
	1.0000000
	1.0000000

Power curve by using ggplot 2



Power curve generated by using Tableau