Roll No-12

M.sc. 3rd semester

Date of Assignment-28/11/2020

Date of Submission-04/12/2020

Experiment No -07

Topic- Tracing the power curve of Exponential distribution.

Problem – A random sample of size 16 is drawn from a distribution with pdf

$$F(x,\theta)=\begin{cases}\theta e^{-\theta x}\ ; & x\rangle 0\\ 0\ , & otherwise\end{cases}$$

Draw the power curve for testing $H_0:\theta=3.6$ against

(i) $H_1:\theta\rangle 3.6$

(ii) $H_1:\theta\langle 3.6$

Assume, $\alpha=0.05$.

Theory and Calculation-

The BCR according to Neyman Pearson's fundamental lemma is given by -

$$W = \left\{ \underset{\sim}{x} : \frac{L(x,\theta_1)}{L(x,\theta_0)} \geq k \right\}$$

Now, $\dfrac{L(x,\theta_1)}{L(x,\theta_0)} = \dfrac{(\theta_1)^n e^{-\theta_1 \sum_{i=1}^{n} x_i}}{(\theta_0)^n e^{-\theta_0 \sum_{i=1}^{n} x_i}} = \left(\dfrac{\theta_1}{\theta_0} \right)^n e^{-(\theta_1 - \theta_0)\sum_{i=1}^{n} x_i} \geq k$

$\Rightarrow (\theta_0 - \theta_1)\sum_{i=1}^{n} x_i \geq k_1$ (Taking logarithm on both sides)

Case I: If $\theta_1 \rangle \theta_0$, then the C.R. is given by -

$$W_1 = \left\{ \underset{\sim}{x} : \sum_{i=1}^{n} x_i \langle k_2 \right\}$$

Case II: When $\theta_1 \langle \theta_0$, then the C.R. is given by -

$$W_2 = \left\{ \underset{\sim}{x} : \sum_{i=1}^{n} x_i \geq k_3 \right\}$$

(i) Here, we have to test $H_0 : \theta = 3.6$ against $H_1 : \theta \rangle 3.6$. The critical region for testing this is given by -

$$W_1 = \left\{ \underset{\sim}{x} : \sum_{i=1}^{n} x_i \langle k_2 \right\}$$

Where k_1 is a constant to be determined such that the size of the CR is 0.05.

$\therefore \quad \alpha = P(x \in W_1 / H_0)$

$\Rightarrow 0.05 = P\left[\sum_{i=1}^{n} x_i \leq k_2 \mid_{\theta=3.6} \right]$

$\Rightarrow 0.05 = P[X \leq k_2 \mid_{\theta=3.6}]$

Where, $X = \sum_{i=1}^{n} x_i \sim gamma(16,3.6)$

Now, to obtain the value of k_2 we use the following R-command-

k2=qgamma(0.05,16,3.6)

k2

$\therefore$ k2= 2.787766

$\therefore$ The CR is given by -

$$W_1 = \left\{ X : \sum_{i=1}^{n} x_i \le 2.787766 \right\}$$

The power of the test is given by -

$$1-\beta = P(x \in W / H_1) = P\left[\sum_{i=1}^{16} x_i \le 2.787766 \mid \theta \rangle 3.6 \right]$$

$$= P[X \le 2.787766 \mid \theta \rangle 3.6] \; ; \quad X = \sum_{i=1}^{n} x_i \sim gamma(16, \theta_1)$$

To draw the power curve, we construct the following table considering different trial values of $\theta \rangle 3.6$.

TABLE 1

	theta_1	power_1
1	3.7	0.36616699
2	3.8	0.34521723
3	3.9	0.32493218
4	4.0	0.30534186
5	4.1	0.28647044

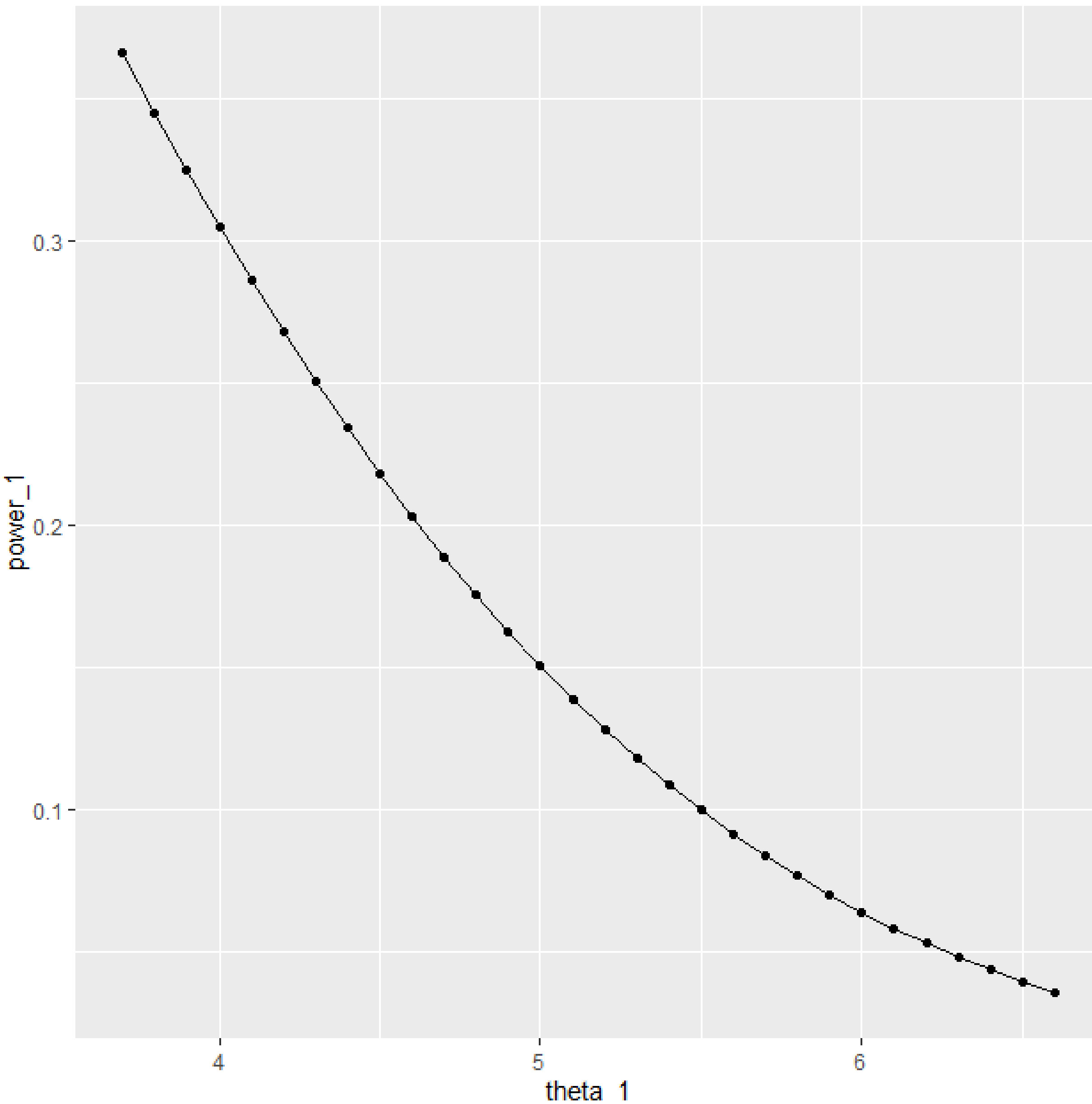
	theta_1	power_1
6	4.2	0.26833648
7	4.3	0.25095318
8	4.4	0.23432870
9	4.5	0.21846648
10	4.6	0.20336556
11	4.7	0.18902102
12	4.8	0.17542423
13	4.9	0.16256336
14	5.0	0.15042361
15	5.1	0.13898768
16	5.2	0.12823608
17	5.3	0.11814744
18	5.4	0.10869889
19	5.5	0.09986633
20	5.6	0.09162473
21	5.7	0.08394836

	theta_1	power_1
22	5.8	0.07681109
23	5.9	0.07018657
24	6.0	0.06404845
25	6.1	0.05837055
26	6.2	0.05312702
27	6.3	0.04829248
28	6.4	0.04384214
29	6.5	0.03975190
30	6.6	0.03599843

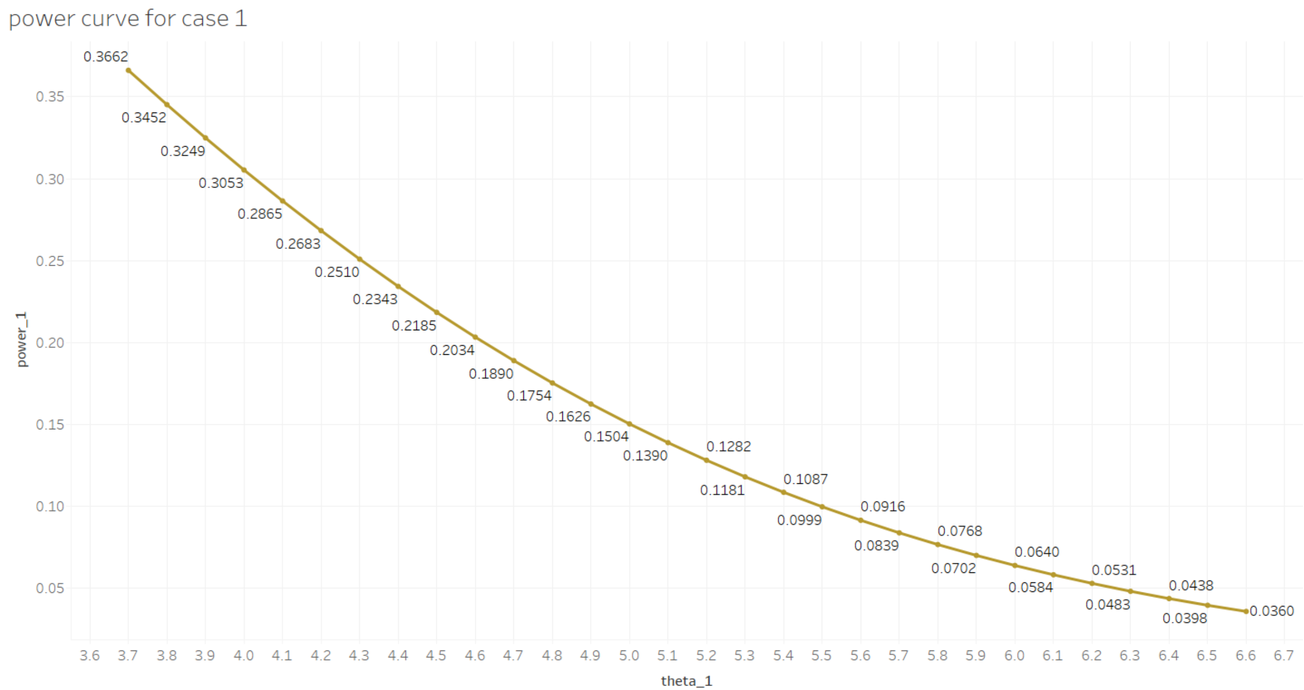
Programming in R for case 1-

```
library('ggplot2')

k2 = qgamma(0.05,16,3.6)

k2

n = 16

theta_1 = seq(from=3.7, by=0.1, length.out=30)

theta_1

power_1 = mat.or.vec(30,1)

for(i in 1:30){

power_1[i] = pgamma(k2, theta_1[i])}

power_1

Table = data.frame(theta_1,power_1)

Table

View(Table)

ggplot(data=Table,mapping=aes(x=theta_1,y=power_1))+geom_point()+geom_line()

data.frame(theta_1)

data.frame(power_1)
```

Power curve by using ggplot 2



Power curve generated by using Tableau



Theta_1 vs. power_1. The marks are labeled by sum of power_1.

(ii) Here we are testing $H_0 : \theta = 3.6$ against $H_1 : \theta < 3.6$. The CR for testing this is given by -

$$W_1 = \left\{ \underset{\sim}{x} : \sum_{i=1}^{16} x_i \geq k_3 \right\}$$

Where k_3 is a constant to be determined such that the size of the CR is 0.05.

$\therefore \alpha = P[P \in W_2 \mid H_0]$

$\Rightarrow 0.05 = P\left[\sum_{i=1}^{20} x_i \geq k_3 \mid \theta = 3.6 \right]$

$\Rightarrow 0.05 = P[X \geq k_3 \mid \theta = 3.6]$ where, X~gamma(16, 3.6)

$\Rightarrow 0.05 = 1 - P[X \leq k_3 \mid \theta = 3.6]$

$\Rightarrow 0.95 = P[X \leq k_3 \mid \theta = 3.6]$

Now, to obtain the value of k_3, we use the following R command-

k3=qgamma(0.95,16,3.6)

k3

$\therefore$ k3= 6.415869

$\therefore$ The CR is given by -

$$W_2 = \left\{ \underset{\sim}{X} : \sum_{i=1}^{n} x_i \leq 6.415869 \right\}$$

And the power of the test is given by -

$$1 - \beta = P[x \in W_2 \mid H_1]$$

$$= P\left[\sum_{i=1}^{16} x_i \geq 6.415869 \mid \theta < 3.6 \right]$$

$= P[X \geq 6.415869 \mid \theta < 3.6]$ Where, $X = \sum_{i=1}^{16} x_i$ ~ gamma(16, θ_1)

$$= 1 - P[X < 6.415869 \mid \theta < 3.6]$$

To draw the power curve, we construct the following table considering different trial values of $\theta < 3.6$.

TABLE 2

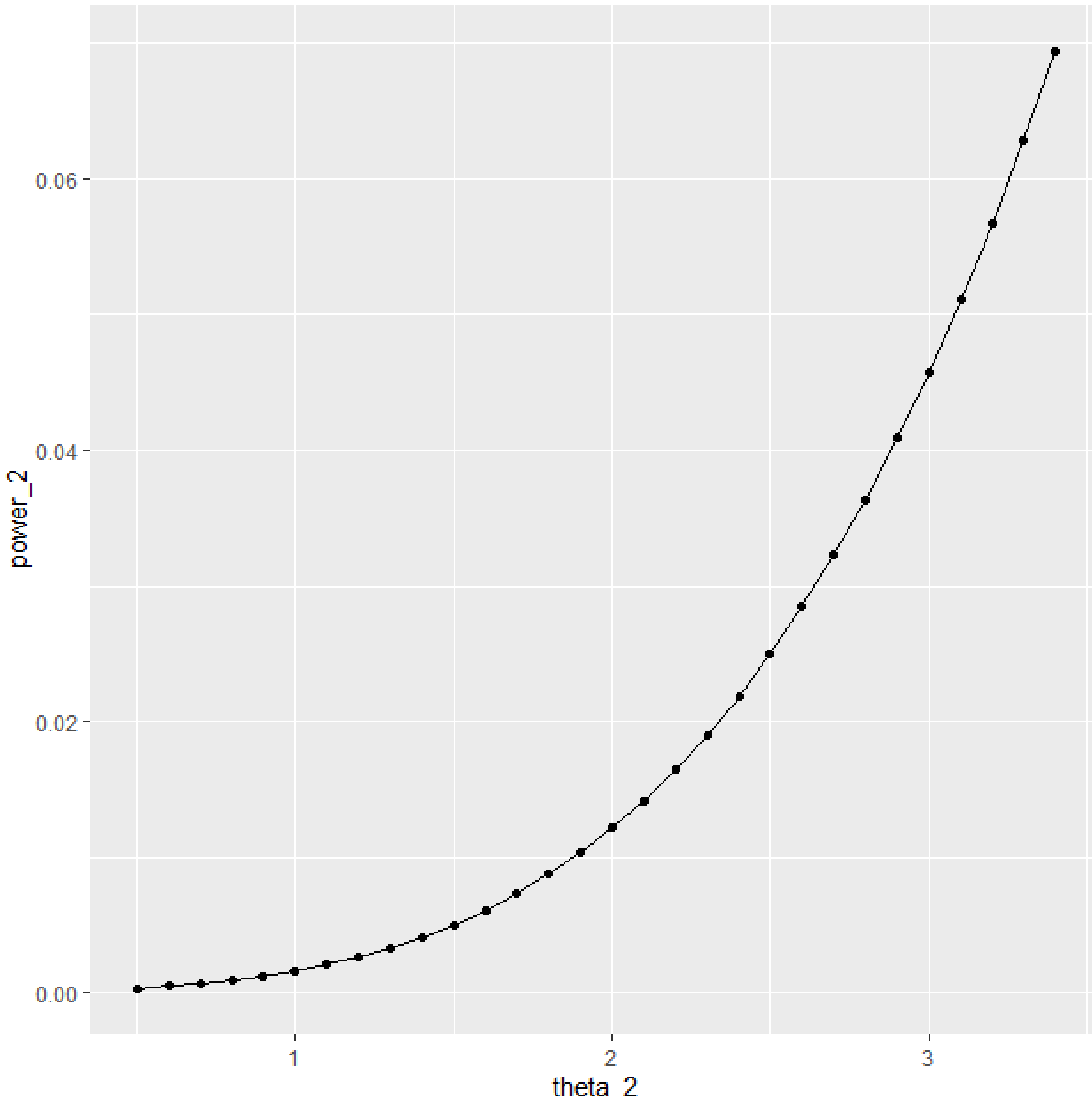
	theta_2	power_2
1	0.5	0.0003407889
2	0.6	0.0004948788
3	0.7	0.0006927973
4	0.8	0.0009426972
5	0.9	0.0012535981
6	1.0	0.0016353975
7	1.1	0.0020988716
8	1.2	0.0026556663
9	1.3	0.0033182768
10	1.4	0.0041000169
11	1.5	0.0050149766
12	1.6	0.0060779687
13	1.7	0.0073044639
14	1.8	0.0087105158

	theta_2	power_2
15	1.9	0.0103126751
16	2.0	0.0121278943
17	2.1	0.0141734240
18	2.2	0.0164667004
19	2.3	0.0190252271
20	2.4	0.0218664500
21	2.5	0.0250076285
22	2.6	0.0284657035
23	2.7	0.0322571636
24	2.8	0.0363979101
25	2.9	0.0409031249
26	3.0	0.0457871388
27	3.1	0.0510633052
28	3.2	0.0567438782
29	3.3	0.0628398974
30	3.4	0.0693610808

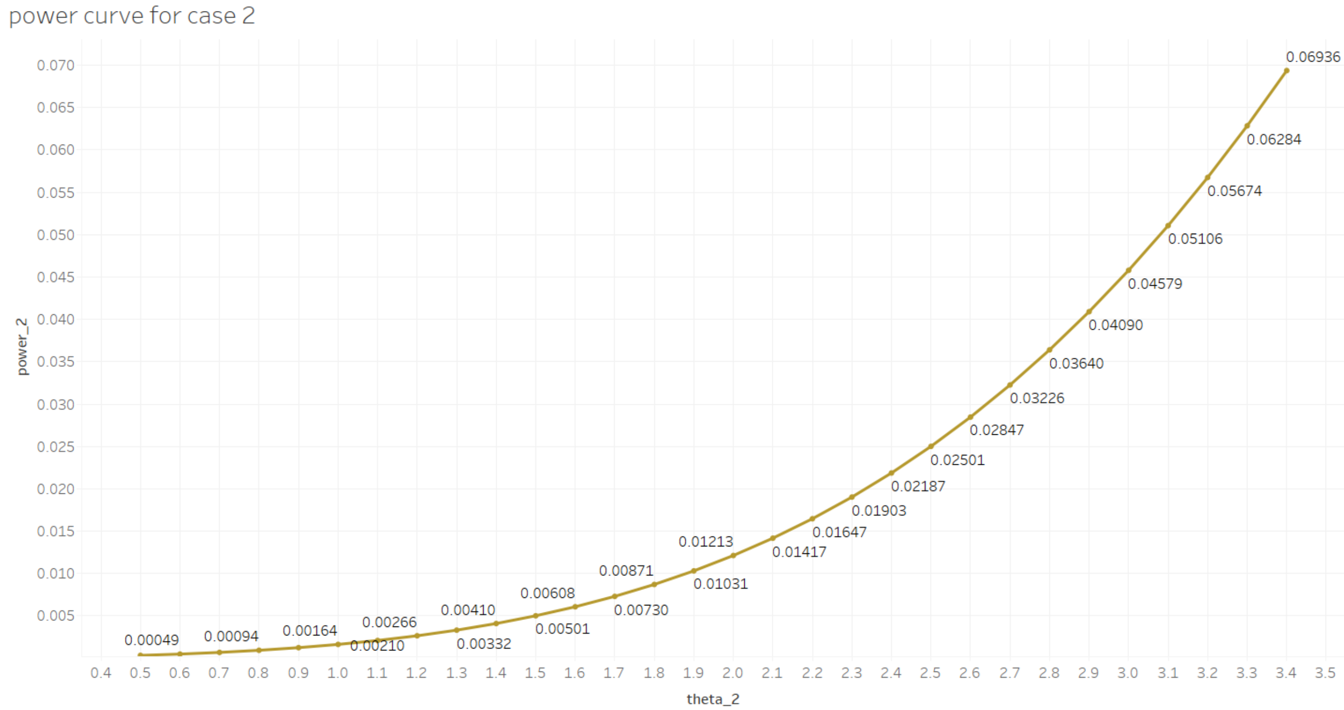
Programming in R for case 2-

```
library('ggplot2')

k3 = qgamma(0.95,16,3.6)

k3

n = 16

theta_2 = seq(from=0.5, by=0.1, length.out=30)

theta_2

power_2 = mat.or.vec(30,1)

for(i in 1:30){

power_2[i]=1-pgamma(k3, theta_2[i])}

power_2

Table_2 = data.frame(theta_2,power_2)

Table_2

View(Table_2)

ggplot(data=Table_2,mapping=aes(x=theta_2,y=power_2))+geom_point()+geom_line()

data.frame(theta_2)

data.frame(power_2)
```

Power curve by using ggplot 2



<u>Power curve generated by using Tableau</u>



Theta_2 vs. power_2. The marks are labeled by sum of power_2.