ECN-316 Image Processing

Course Project

Topic

Sampling, Aliasing and Reconstruction

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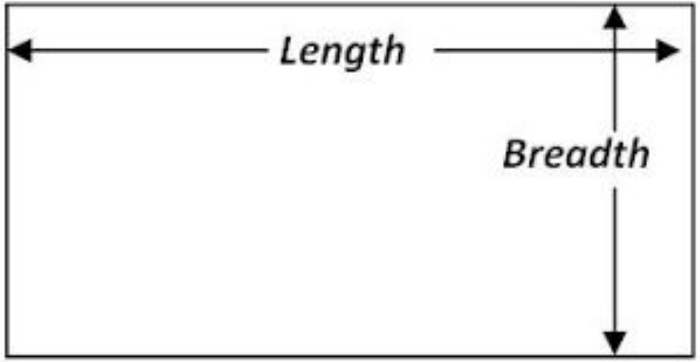
Sampling and Quantization

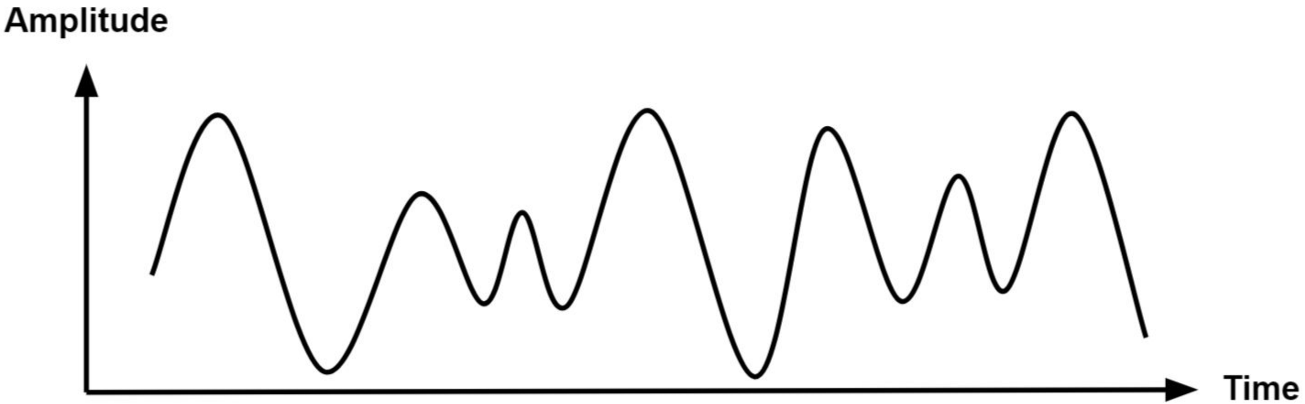
Why Digitization?



$0 <= x <= \text{Length}$

$0 <= y <= \text{Breadth}$

$f(x, y) = r(x, y) \times l(x, y)$



From theory of real numbers, there exists infinite number of points between any two points.
This implies an image should be represented by infinite number of points

Also, each such point may contain one of the infinite many possible intensity/color values.
That means the color values are also infinite.

Such a representation is not possible in any of the computer systems

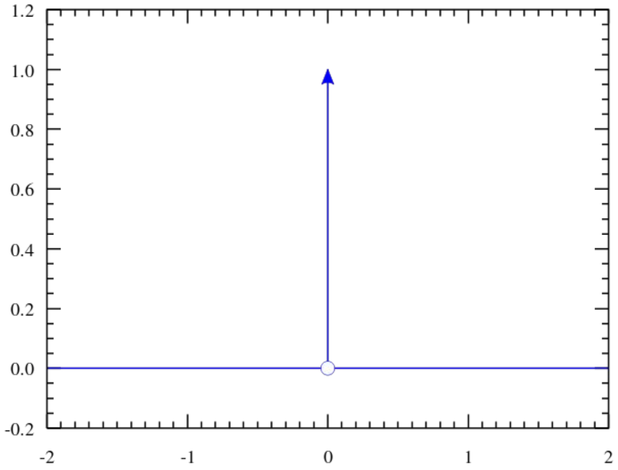
The process of digitizing the domain of a continuous signal is known as ***sampling***.

Sampling is basically recording the values at only some specific points.

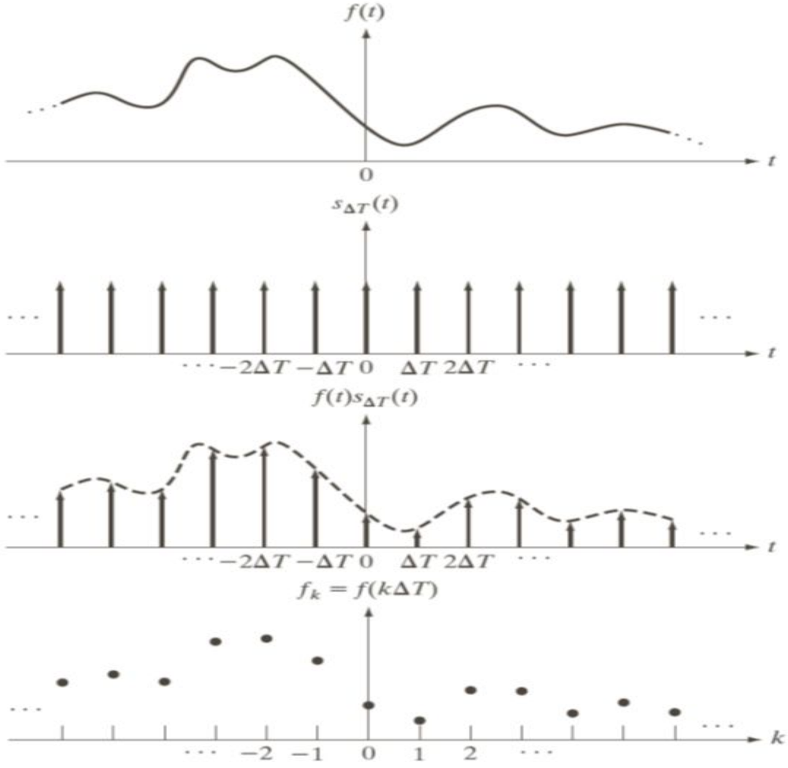
Sampling allows us to transmit specific data points, such that those data points could be used to retrieve the original signal.

The most important thing achieved by doing this is that now during the transmission of the signal, we don’t have to store infinite values

How Sampling is achieved?



Dirac delta function

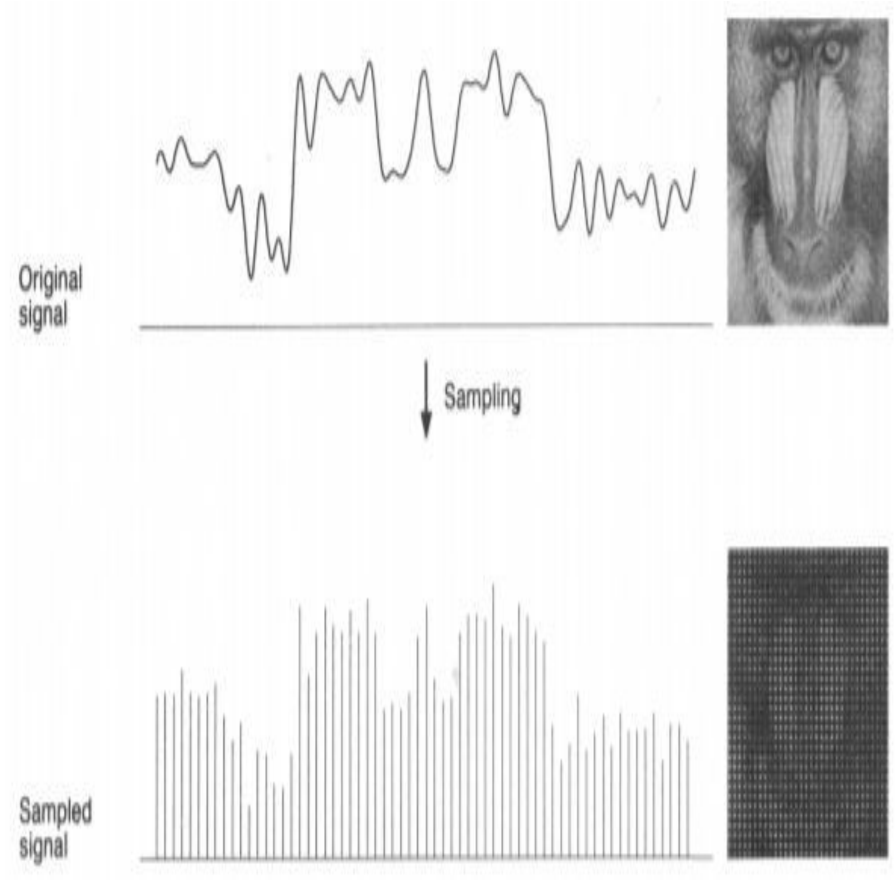


Achieved by multiplying continuous signal with impulse train (series of dirac delta function)

$$\bar{f}(t) = f(t)\, s_{\Delta T}(t)$$

$$= \Sigma_{n=-\infty}^{n=\infty} f(t)\, \delta(t - n\Delta T)$$

A special note regarding no. of samples to be taken

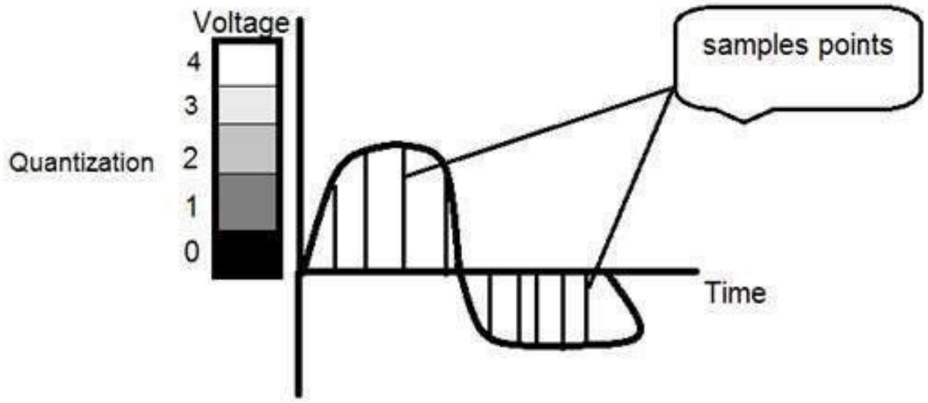


Quantization

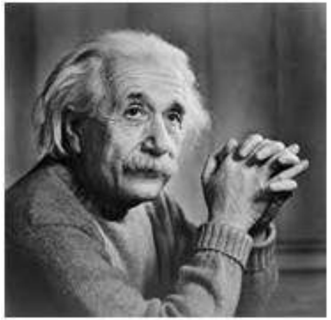
Quantization

Under quantization process the amplitude values of the image are digitized.

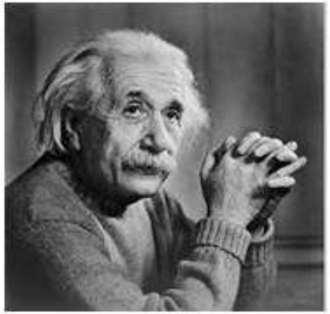
We assume the amplitude values to lie in a certain category of shades.



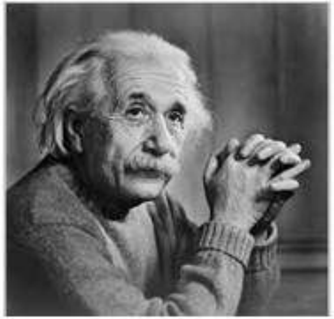
256 Gray Levels



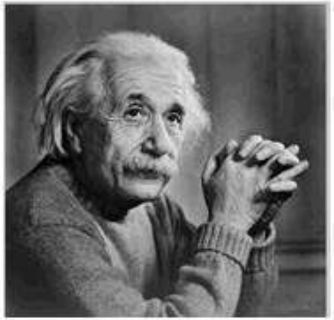
64 Gray Levels



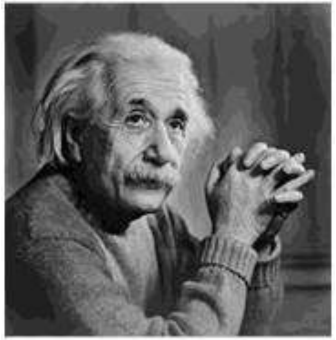
128 Gray Levels



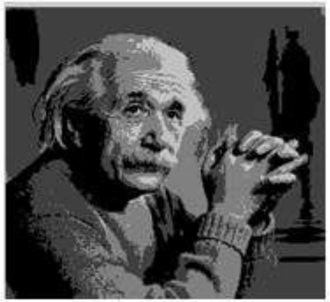
32 Gray Levels



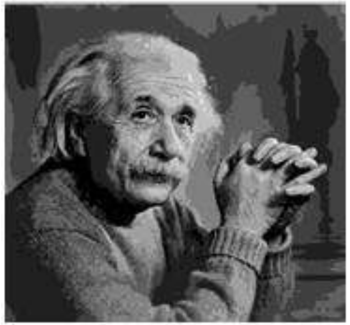
16 Gray Levels



4 Gray Levels



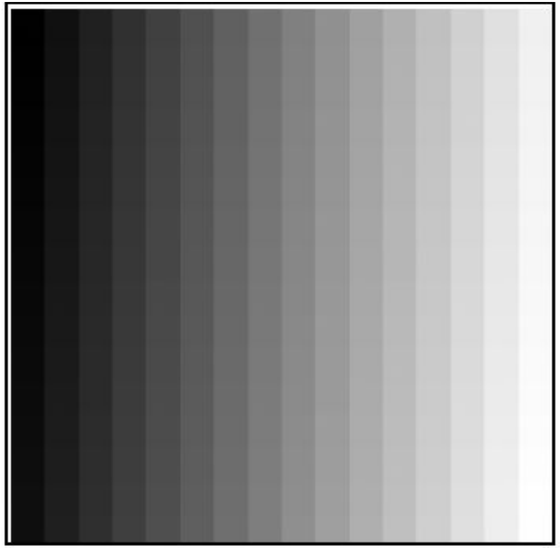
8 Gray Levels



2 Gray Levels

No. of quantization levels $= k = 2^b$

Image as matrix of numbers



0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240
1	17	33	49	65	81	97	113	129	145	161	177	193	209	225	241
2	18	34	50	66	82	98	114	130	146	162	178	194	210	226	242
3	19	35	51	67	83	99	115	131	147	163	179	195	211	227	243
4	20	36	52	68	84	100	116	132	148	164	180	196	212	228	244
5	21	37	53	69	85	101	117	133	149	165	181	197	213	229	245
6	22	38	54	70	86	102	118	134	150	166	182	198	214	230	246
7	23	39	55	71	87	103	119	135	151	167	183	199	215	231	247
8	24	40	56	72	88	104	120	136	152	168	184	200	216	232	248
9	25	41	57	73	89	105	121	137	153	169	185	201	217	233	249
10	26	42	58	74	90	106	122	138	154	170	186	202	218	234	250
11	27	43	59	75	91	107	123	139	155	171	187	203	219	235	251
12	28	44	60	76	92	108	124	140	156	172	188	204	220	236	252
13	29	45	61	77	93	109	125	141	157	173	189	205	221	237	253
14	30	46	62	78	94	110	126	142	158	174	190	206	222	238	254
15	31	47	63	79	95	111	127	143	159	175	191	207	223	239	255

Fourier Transform of a continuous function

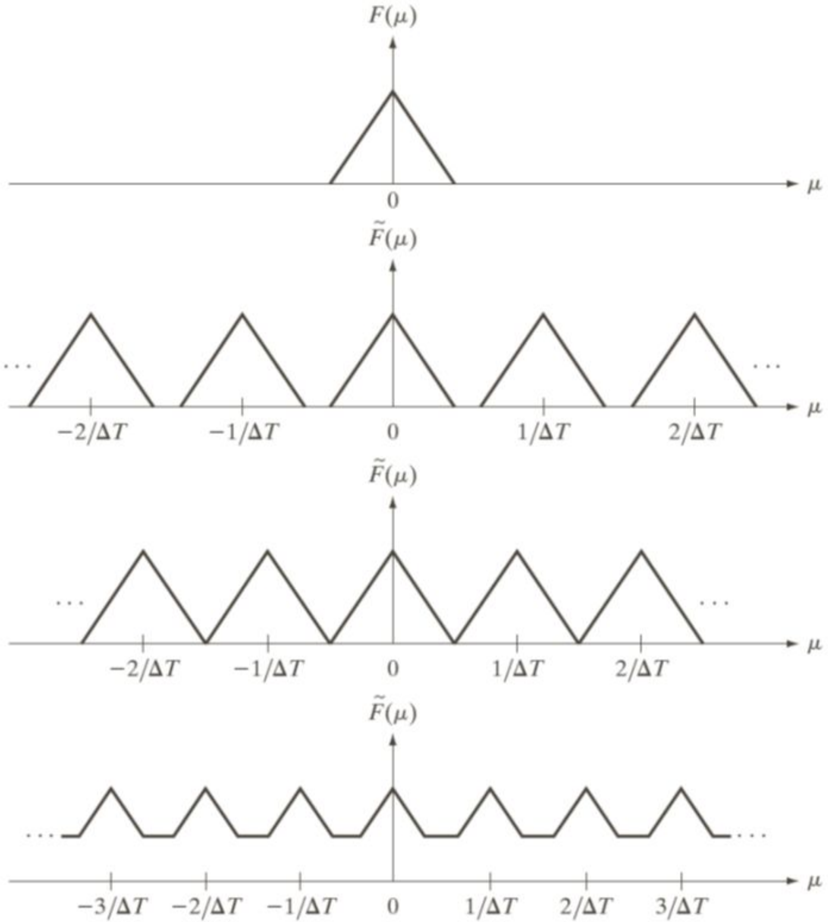
$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt \leftrightarrow f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu} d\mu$$

Our Sampled Signal: -

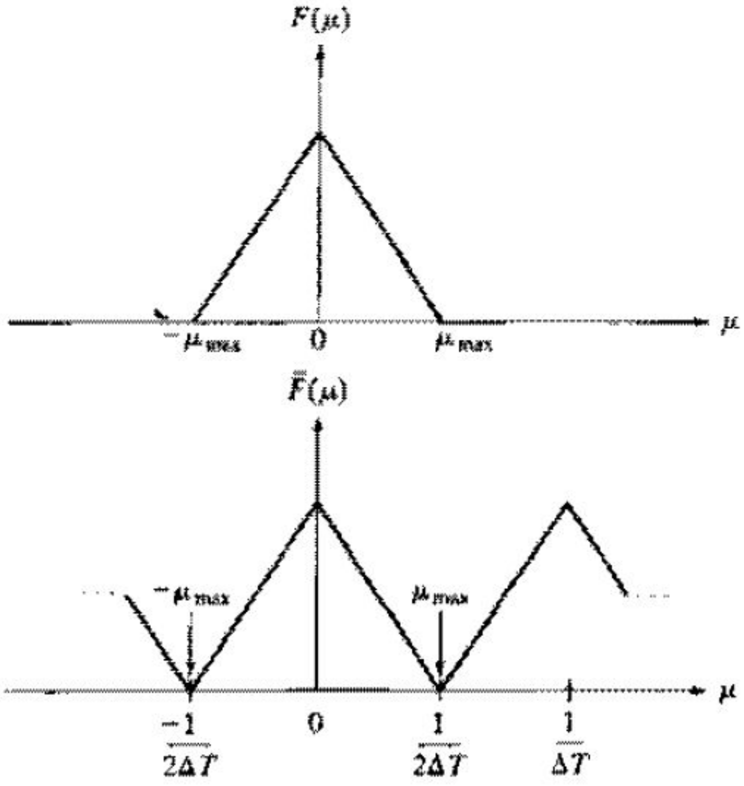
$$\begin{aligned} \bar{f}(t) &= f(t)\, s_{\Delta T}(t) \\ &= \Sigma_{n=-\infty}^{n=\infty} f(t)\, \delta(t - n\Delta T) \end{aligned}$$

- $\mathcal{F}(\bar{f}(t)) = \mathcal{F}(f(t)s_{\Delta T}(t)) = F(\mu) * \mathcal{F}(s_{\Delta T}(t)) = F(\mu) * S(\mu)$
  - $S(\mu) = \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty}\delta(\mu - \frac{n}{\Delta T})$
-

$$
\begin{aligned}
\mathcal{F}(\bar{f}(t)) = F(\mu) * S(\mu) &= \int_{-\infty}^{\infty} F(\tau)S(\mu-\tau)d\tau \\
&= \frac{1}{\Delta T}\int_{-\infty}^{\infty} F(\tau)\sum_{n=-\infty}^{\infty}\delta(\mu-\tau-\frac{n}{\Delta T})d\tau \\
&= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} F(\tau)\delta(\mu-\tau-\frac{n}{\Delta T})d\tau \\
&= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} F(\mu-\frac{n}{\Delta T})
\end{aligned}
$$



$F(\mu)$
μ
0
$\tilde{F}(\mu)$
μ
$-2/\Delta T$
$-1/\Delta T$
0
$1/\Delta T$
$2/\Delta T$
$\tilde{F}(\mu)$
μ
$-2/\Delta T$
$-1/\Delta T$
0
$1/\Delta T$
$2/\Delta T$
$\tilde{F}(\mu)$
μ
$-3/\Delta T$
$-2/\Delta T$
$-1/\Delta T$
0
$1/\Delta T$
$2/\Delta T$
$3/\Delta T$



$F(\mu)$
μ
$-\mu_{max}$
0
μ_{max}
$\bar{F}(\mu)$
$-\mu_{max}$
μ_{max}
μ
$\frac{-1}{2\Delta T}$
0
$\frac{1}{2\Delta T}$
$\frac{1}{\Delta T}$

Nyquist Theorem

Nyquist Theorem

A bandlimited continuous-time signal can be sampled and perfectly reconstructed from its samples if the waveform is sampled over twice as fast as it's highest frequency component.

Nyquist limit: the highest frequency component that can be accurately represented:

$$\frac{1}{2\Delta T} > \mu_{max}$$

Nyquist frequency: sampling rate required to accurately represent up to fmax:

$$\frac{1}{\Delta T} > 2\mu_{max}$$

Aliasing

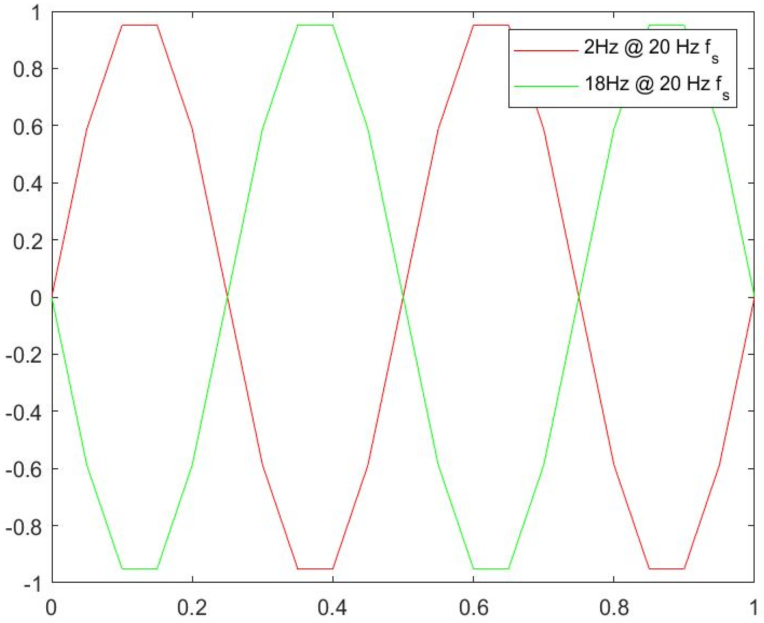
Aliasing

Aliasing occurs when a signal is sampled at a rate less than the Nyquist Rate.

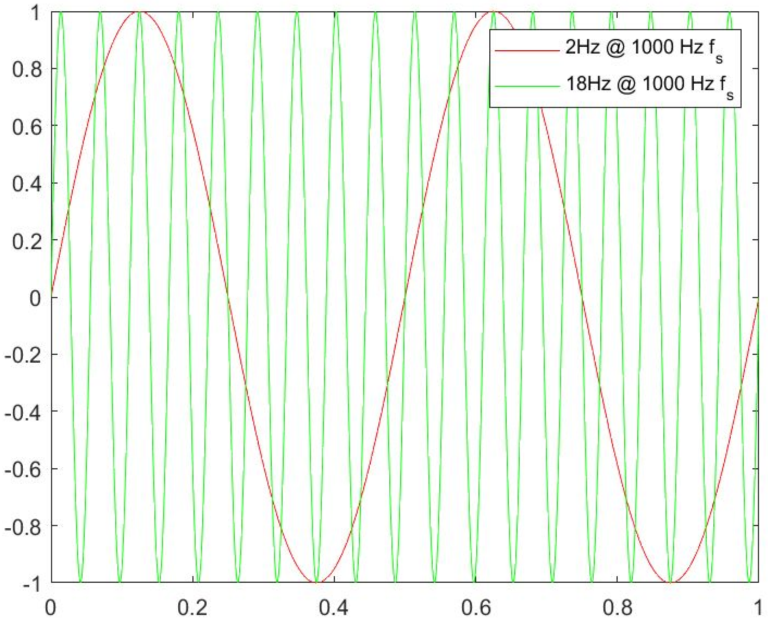
When the frequency of sampling function is less than the twice the maximum frequency component of original signal then the high frequency components of the original function interfere with each other and the original waveform is lost, making it difficult to retrieve the original signal.

These phenomenon is termed as Aliasing.

Anti Aliasing: Aliasing happens due to overlap of high frequency components, so if we are able to remove this high frequency components before sampling (without losing identity) then the original signal could still be recovered. This is achieved by first passing the original signal through a low pass filter, and this is known as anti-aliasing.

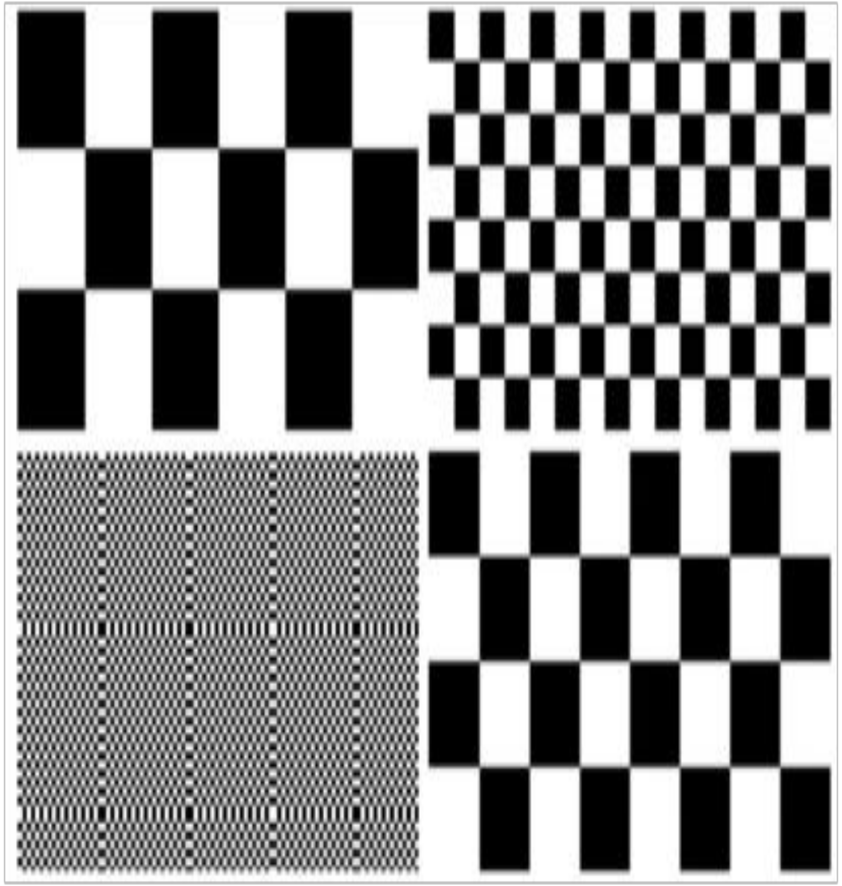


Sinusoid signals of 2Hz & 18 Hz sampled at 20Hz



Sinusoid signals of 2Hz & 18 Hz sampled at 1000Hz

Examples



Let us assume our camera has 96x96 imaging system. Each sensing unit can report only one intensity value.

If the size of the each chessboard square is more than the size of the each sensing area then we will be able to resolve the image correctly.

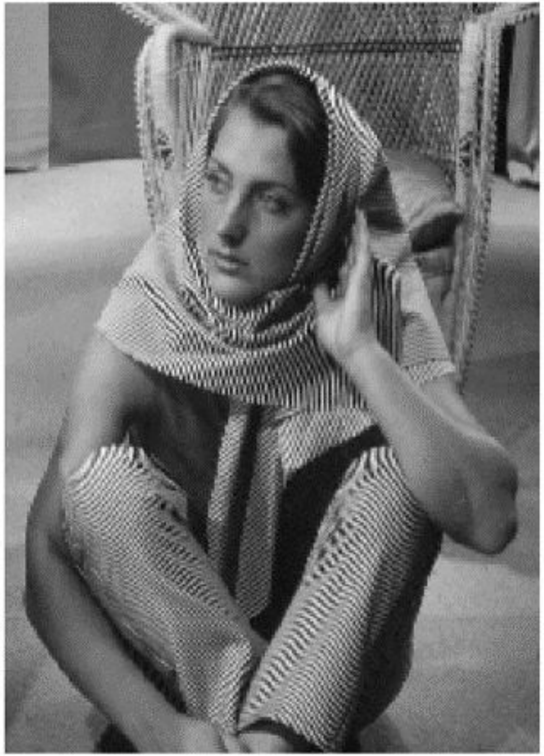
So this system is able to resolve patterns that are upto 96x96 squares, each having size 1x1 pixel.

Square lengths:

a. 16
b. 6
c. 0.9174
d. 0.4798 pixels

Last image look likes squares of length 12 pixels instead of 0.4798 pixels.

Original



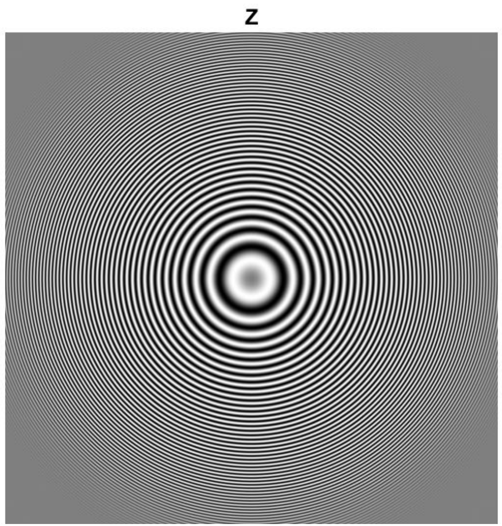
Aliasing (due to resizing)

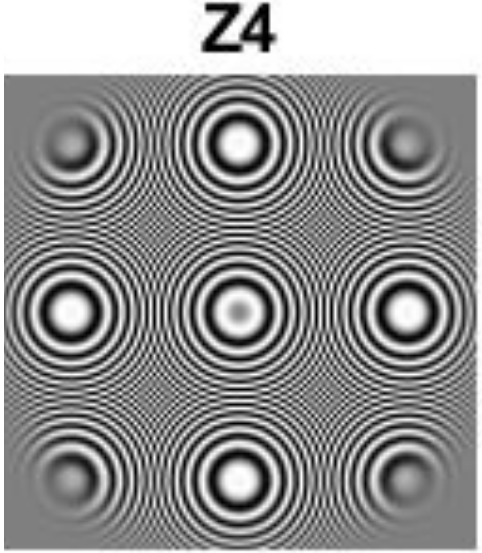
Anti-aliasing

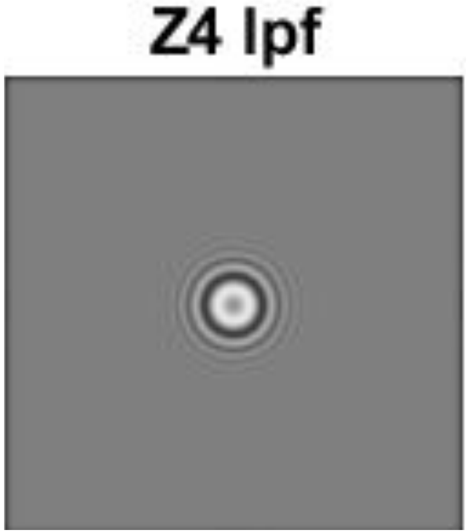
Original

Aliasing (due to resizing)

Anti-aliasing



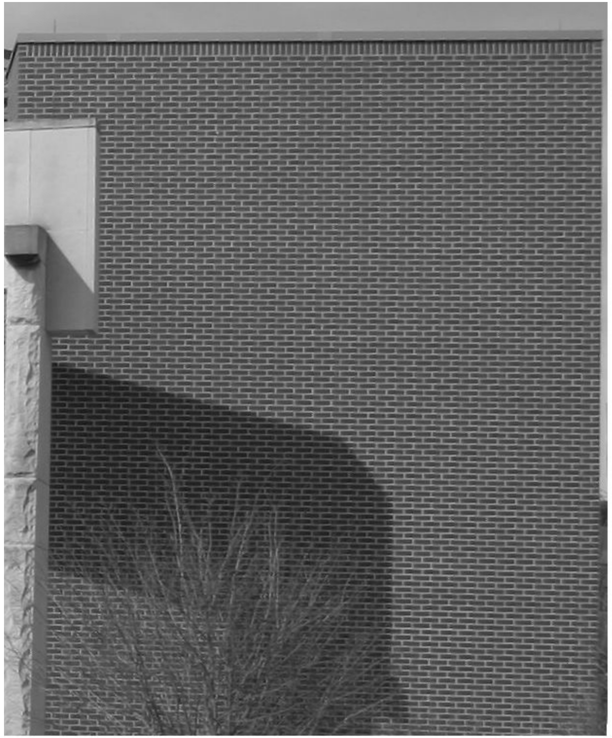




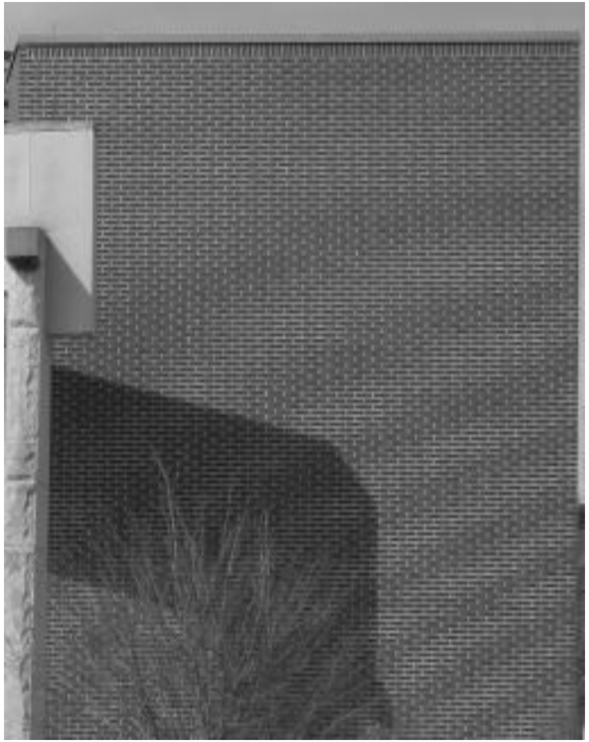
Original

Aliasing (due to resizing)

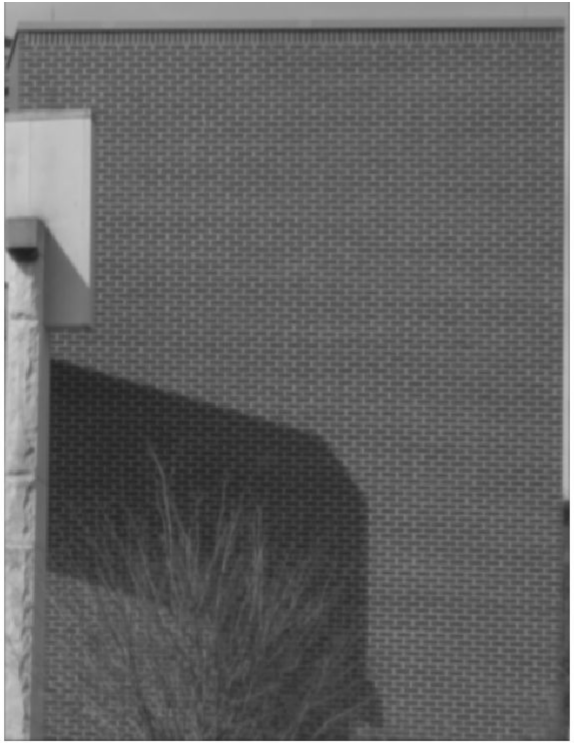
Anti-aliasing



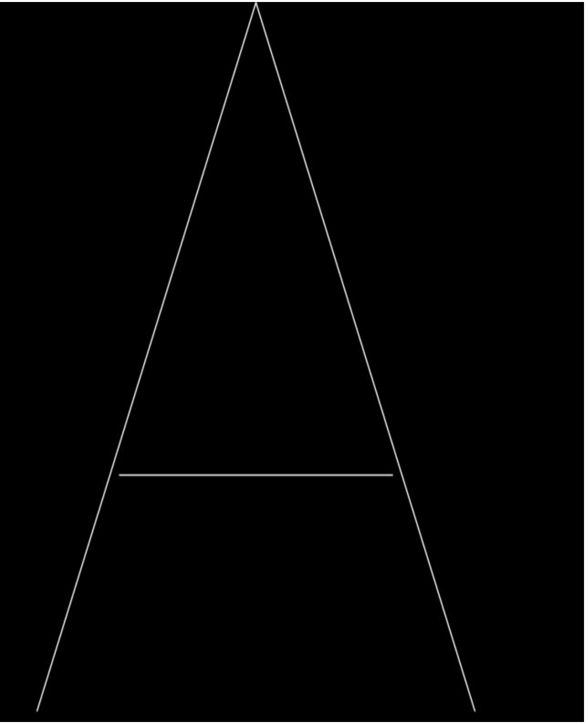
Original



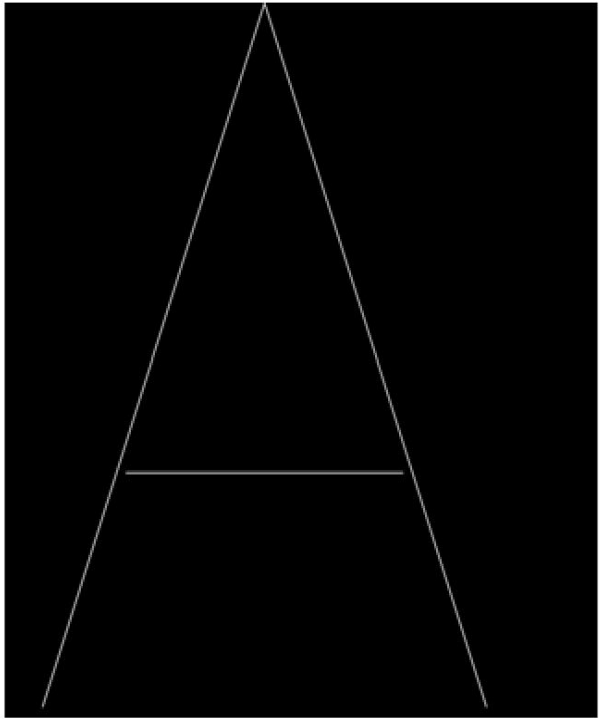
Aliasing (due to resizing)



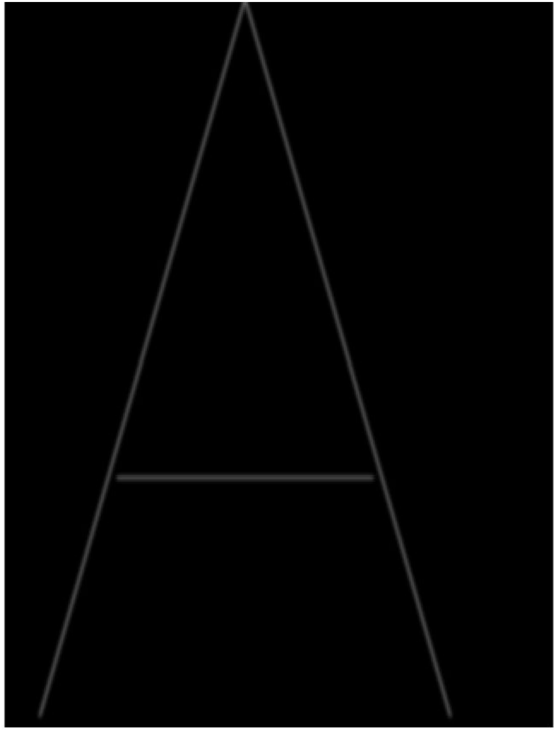
Anti-aliasing



Original



Aliasing (due to resizing)



Anti-aliasing

Original

Aliasing (due to resizing)

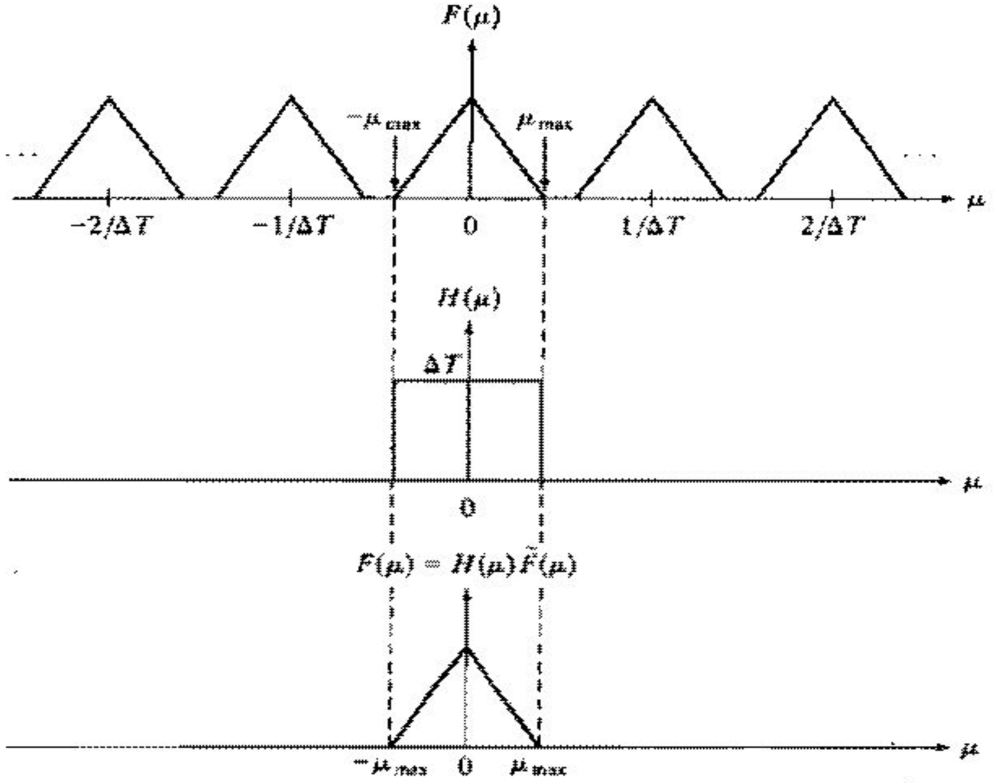
Anti-aliasing

Original Image

Aliasing

Image Reconstruction

Image Reconstruction

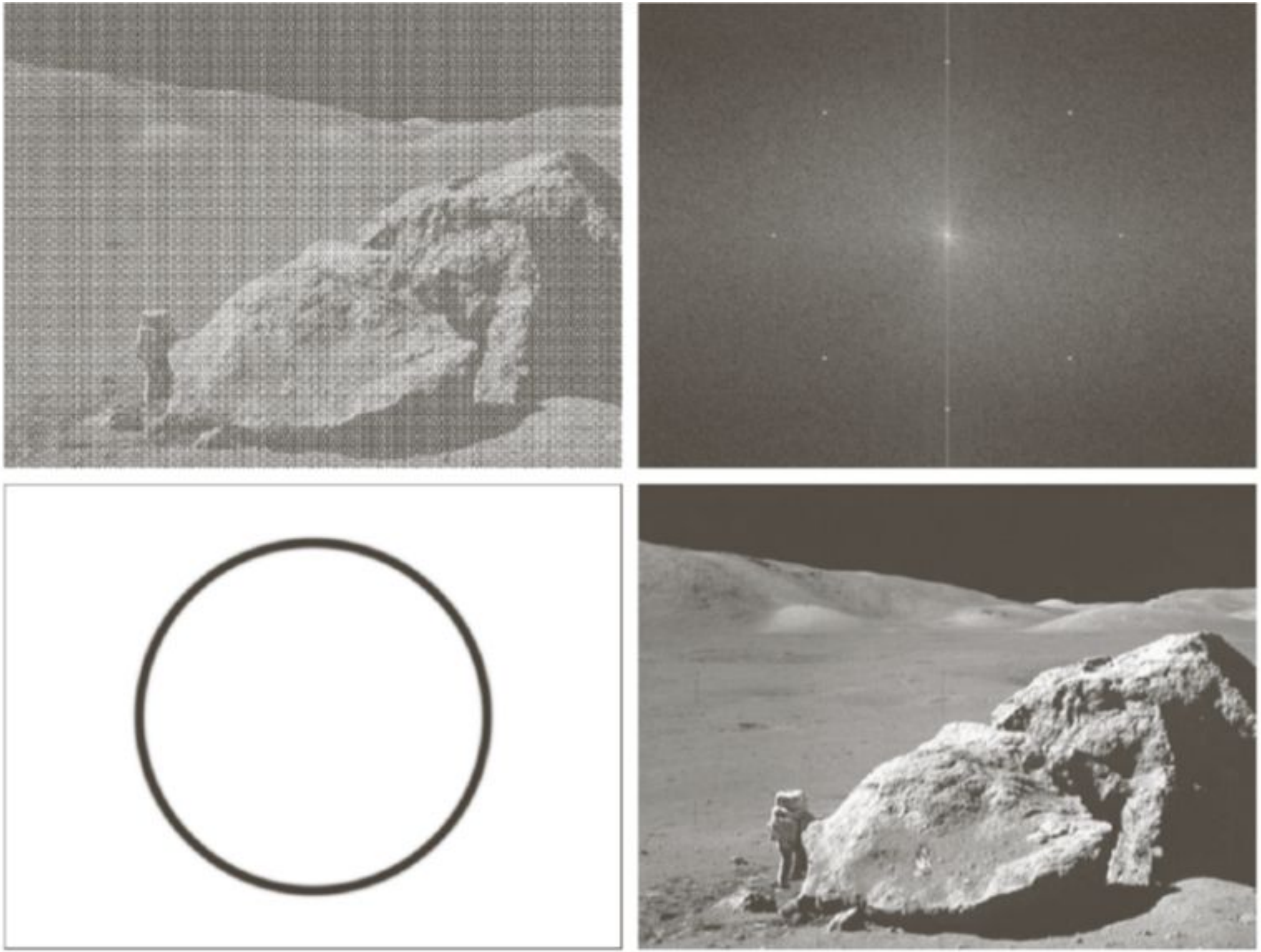


The process of retrieving the original signal from the sampled signal.

$$F(\mu) = H(\mu)\,\overline{F(\mu)}$$

$$f(t) = F^{-1}\{F(\mu)\}$$

$$f(t) = h(t) * \overline{f(t)}$$



a b
c d

(a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

References

https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=5278399

https://www.tutorialspoint.com/dip/index.htm

https://en.wikipedia.org/wiki/Aliasing

Digital Image Processing by Rafael C Gonzalez.

Study Material provided by Prof Dheeraj Kumar

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Thank You