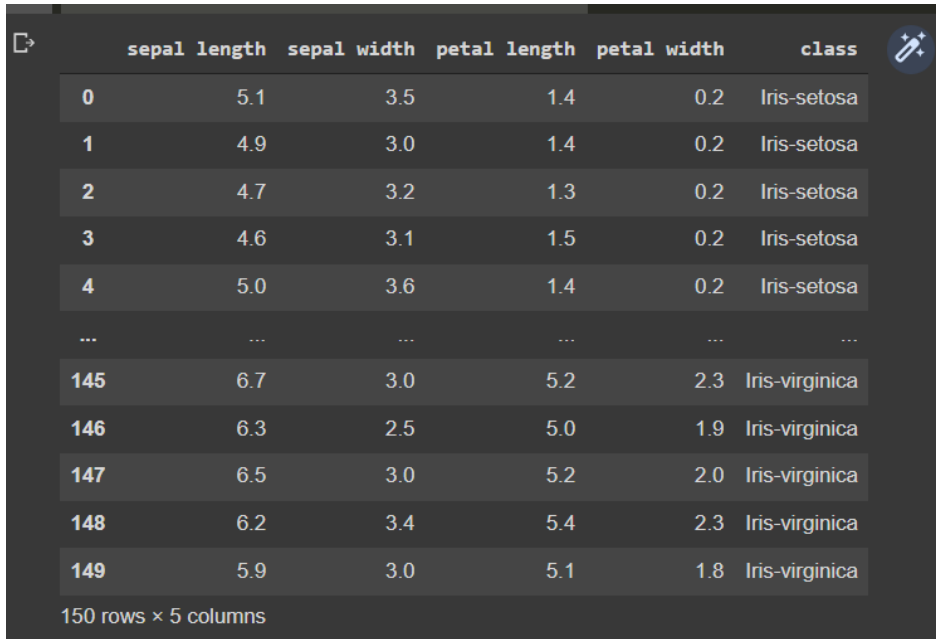


ML Assignment1 Report

Q1)

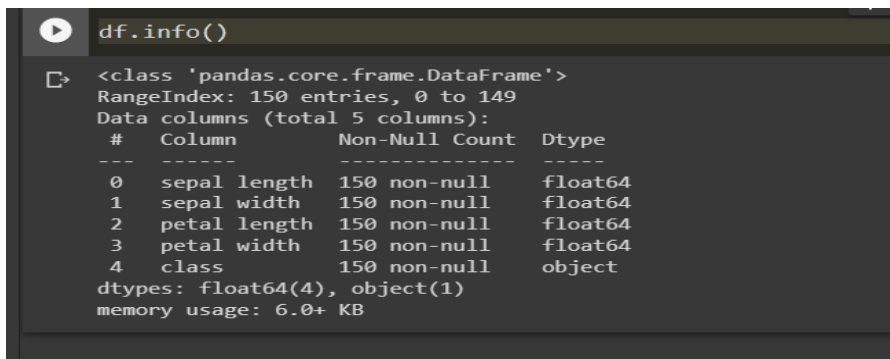
1) IRIS Dataset



| | sepal length | sepal width | petal length | petal width | class |
|-----|--------------|-------------|--------------|-------------|----------------|
| 0 | 5.1 | 3.5 | 1.4 | 0.2 | Iris-setosa |
| 1 | 4.9 | 3.0 | 1.4 | 0.2 | Iris-setosa |
| 2 | 4.7 | 3.2 | 1.3 | 0.2 | Iris-setosa |
| 3 | 4.6 | 3.1 | 1.5 | 0.2 | Iris-setosa |
| 4 | 5.0 | 3.6 | 1.4 | 0.2 | Iris-setosa |
| ... | ... | ... | ... | ... | ... |
| 145 | 6.7 | 3.0 | 5.2 | 2.3 | Iris-virginica |
| 146 | 6.3 | 2.5 | 5.0 | 1.9 | Iris-virginica |
| 147 | 6.5 | 3.0 | 5.2 | 2.0 | Iris-virginica |
| 148 | 6.2 | 3.4 | 5.4 | 2.3 | Iris-virginica |
| 149 | 5.9 | 3.0 | 5.1 | 1.8 | Iris-virginica |

150 rows × 5 columns

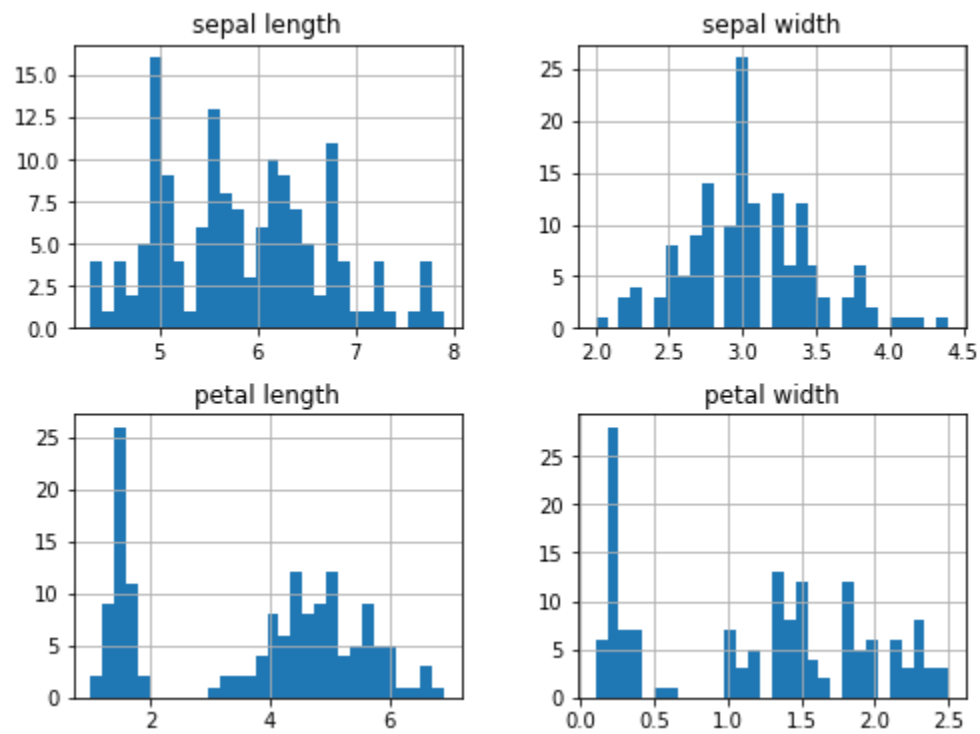
Column info-



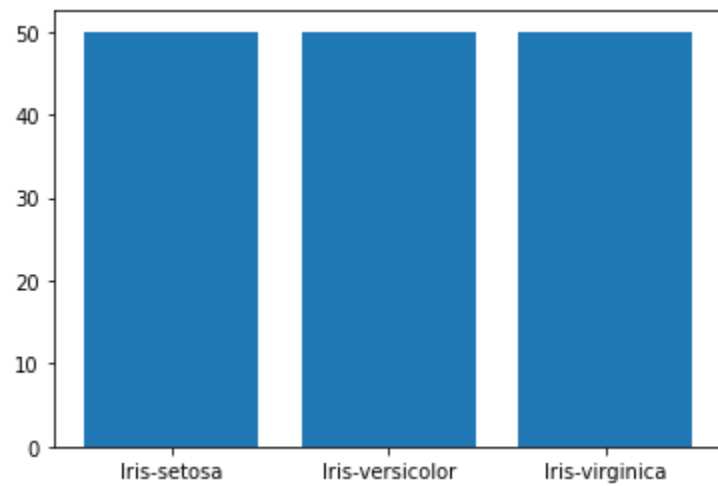
```
df.info()

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 150 entries, 0 to 149
Data columns (total 5 columns):
 #   Column        Non-Null Count  Dtype  
---  -
 0   sepal length  150 non-null   float64
 1   sepal width   150 non-null   float64
 2   petal length  150 non-null   float64
 3   petal width   150 non-null   float64
 4   class         150 non-null   object  
dtypes: float64(4), object(1)
memory usage: 6.0+ KB
```

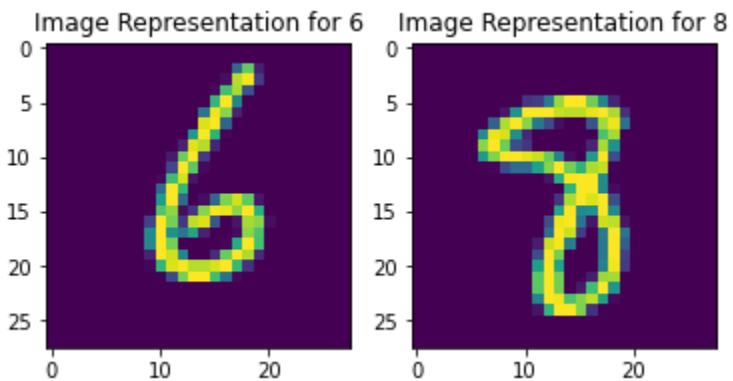
Histogram for attributes -



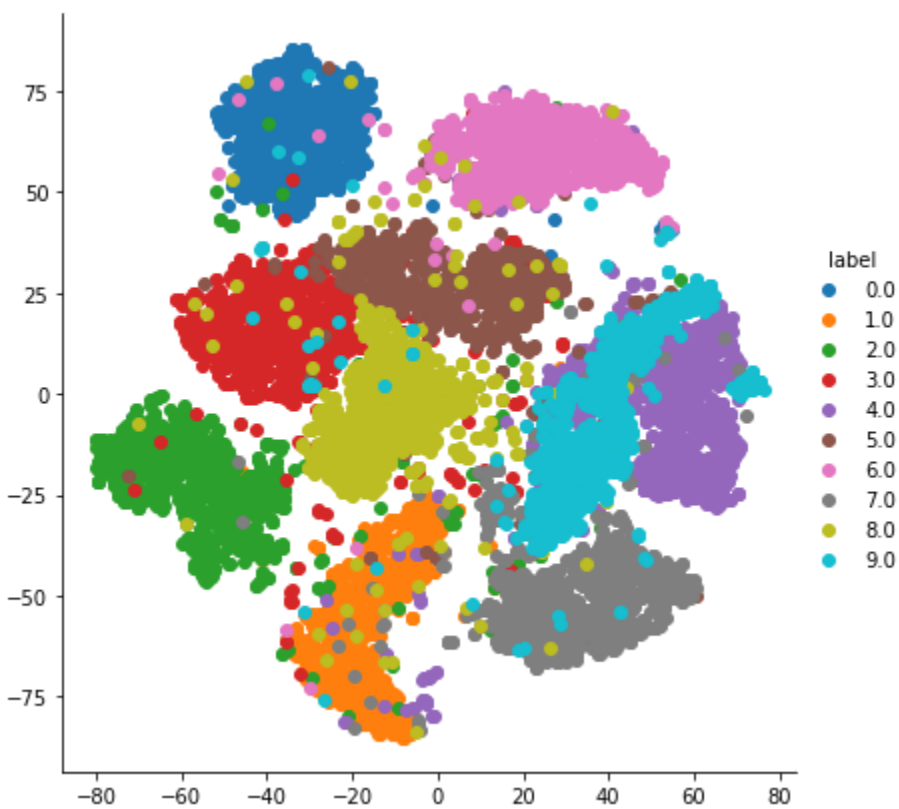
Bar Graph for class attribute -



2) Image visualization of MNIST dataset



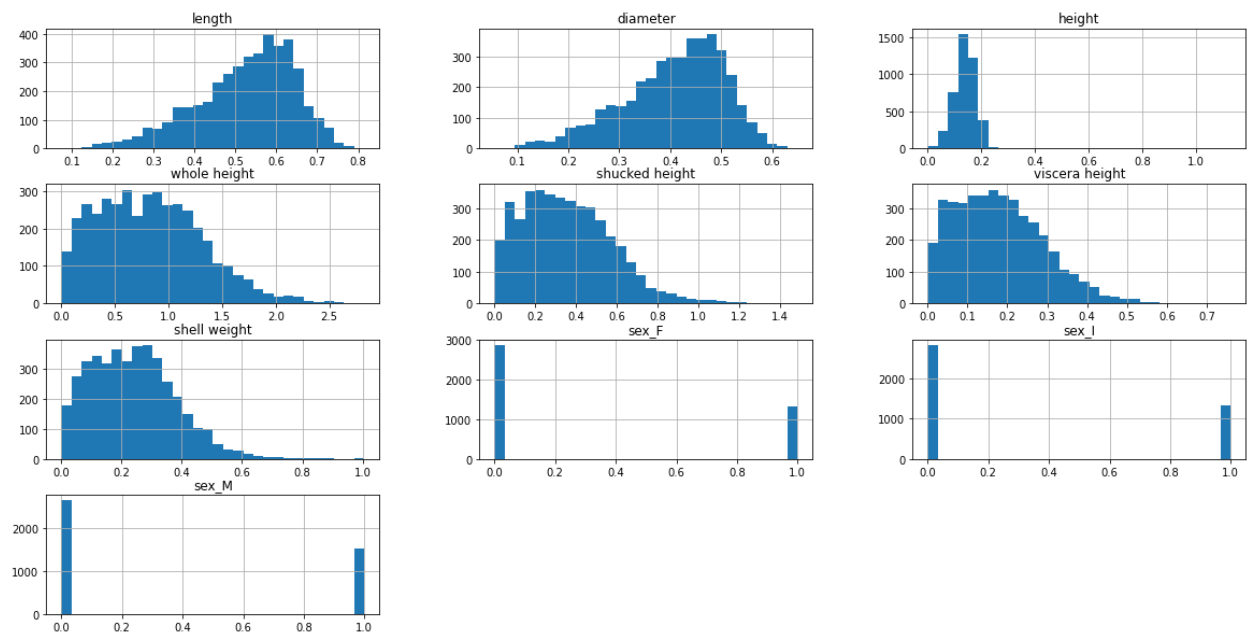
Plot after TSNE-



Here, the scatter plot shows different clusters of the data and we can see the labels(0-9 numbers) which look similar to each other have their clusters closer to each other in comparison to the labels that look distinct .

Q2)

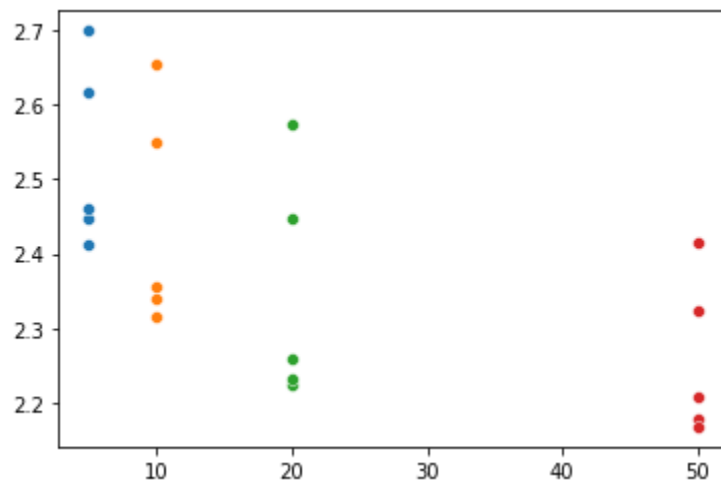
1) Feature visualization -



a)

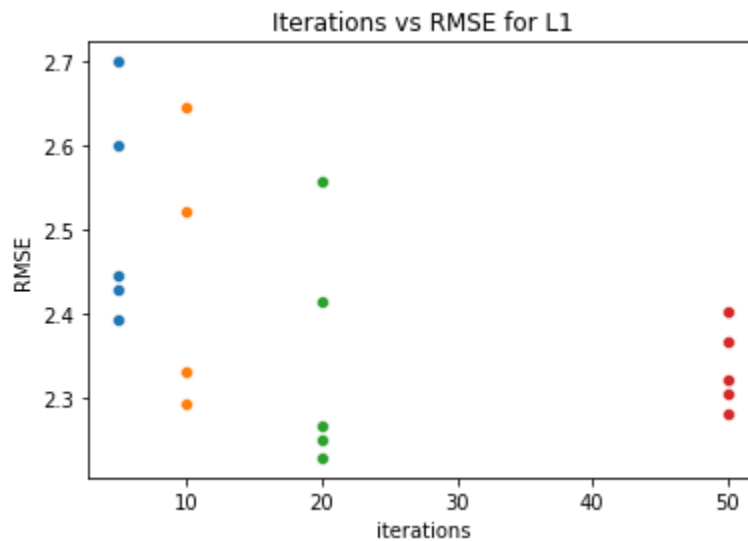
On checking for different learning rate value I have chosen it as 0.001

Iteration vs RMSE plot for Logistic Regression -

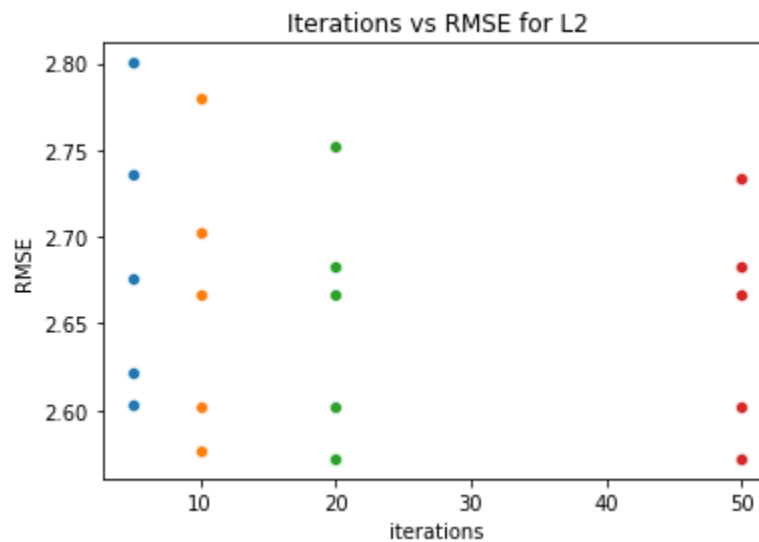


b)

Iteration vs RMSE plot for Logistic Regression with Lasso -
 $\lambda = 0.01$

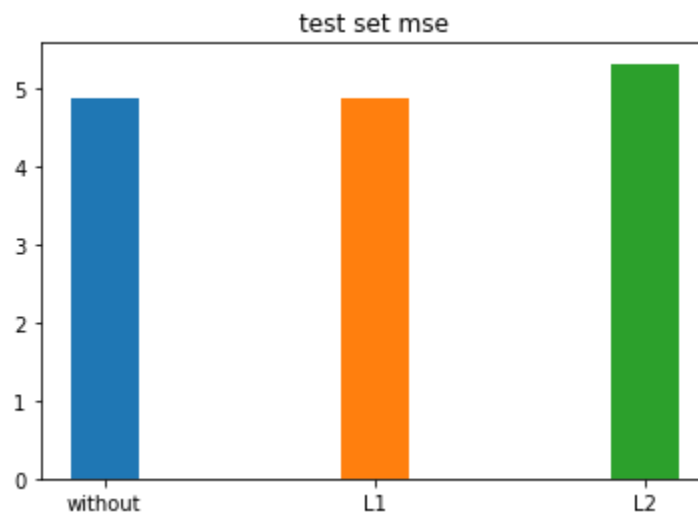


Iteration vs RMSE plot for Logistic Regression with L2 -
 $\lambda = 0.01$

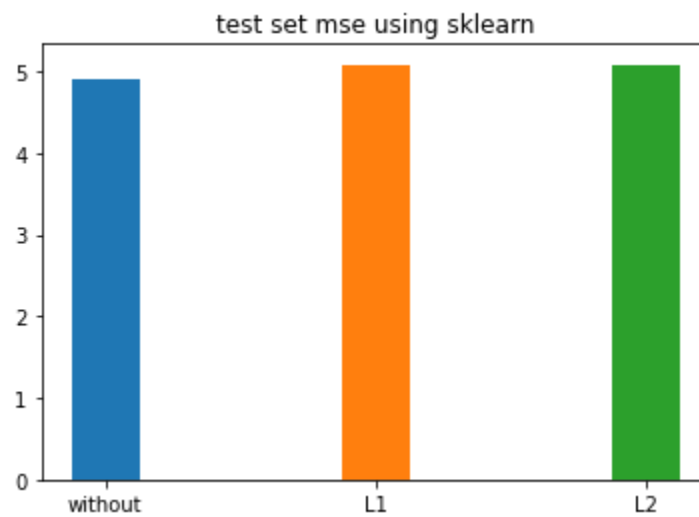


From all above graphs we can conclude that RMSE values decreases with increased iterations. And RMSE values on val set are greater with regularization

c) test set accuracies for all 3 above implemented models -



d) test set accuracies for all 3 models implemented with sklearn -



We can see here that sklearn implementation has given less test accuracies for all 3 models than my model

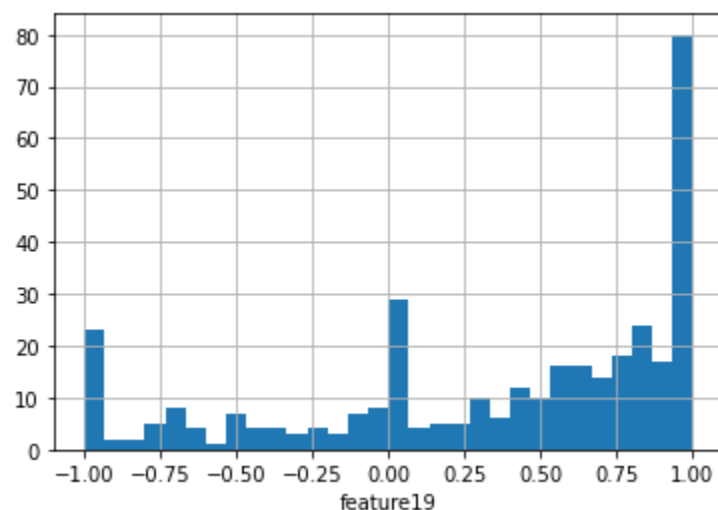
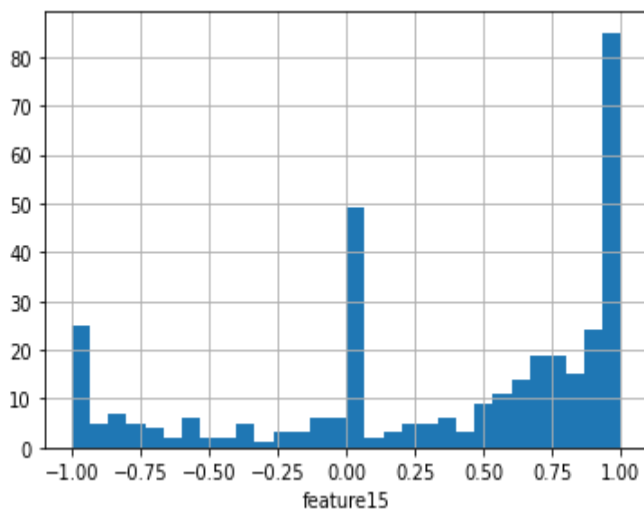
e) validation set accuracy for models with closed form of Linear Regression-

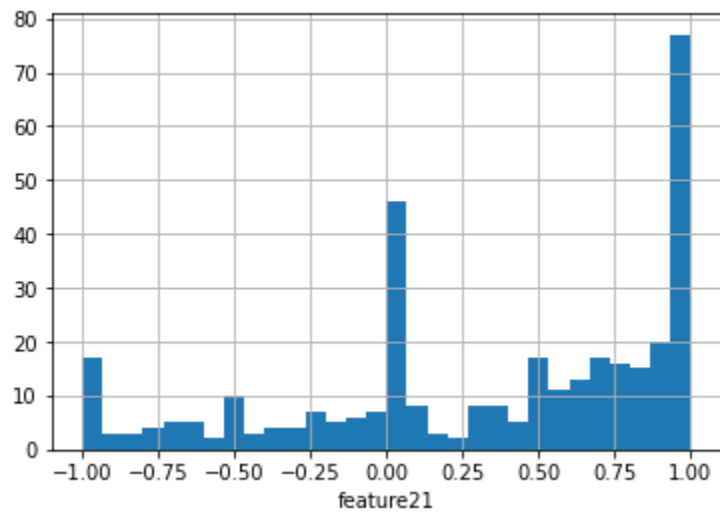
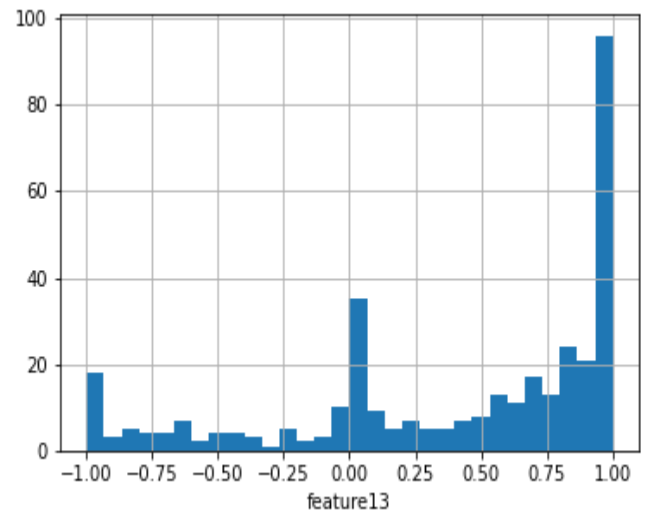
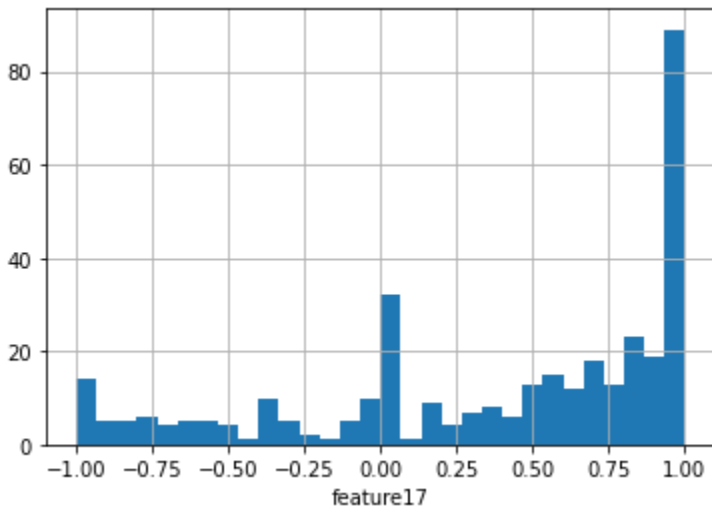
```
mse for model 1 = 5.0040500253146725  
mse for model 2 = 5.018084909369004  
mse for model 3 = 4.783313929136972  
mse for model 4 = 5.252756282087939  
mse for model 5 = 4.608845587491214
```

Here we can see that mean squared error values for model with closed form and gradient descent have similar values which can be due to small number of features and small training dataset size

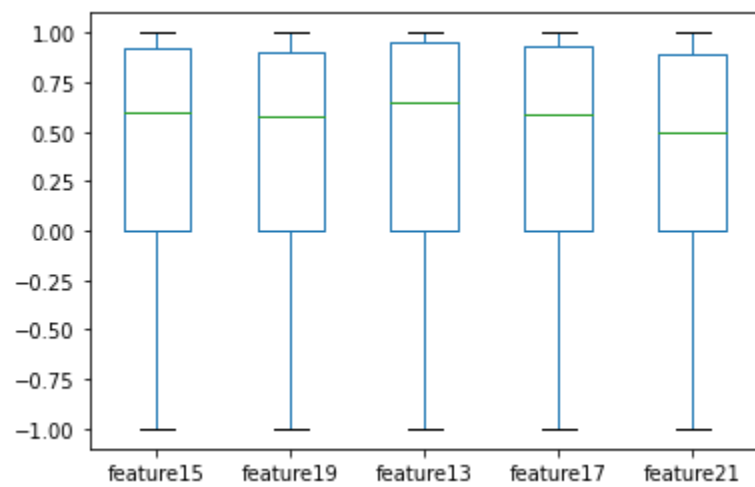
Q3)

1) Histogram for top 5 features with highest variances -

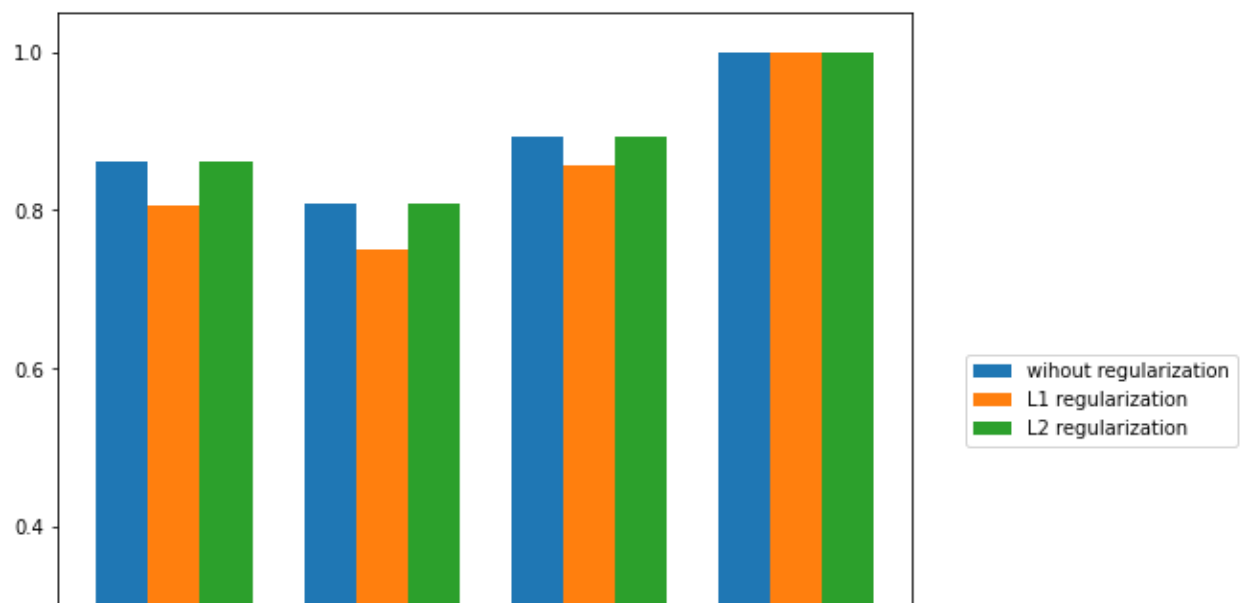




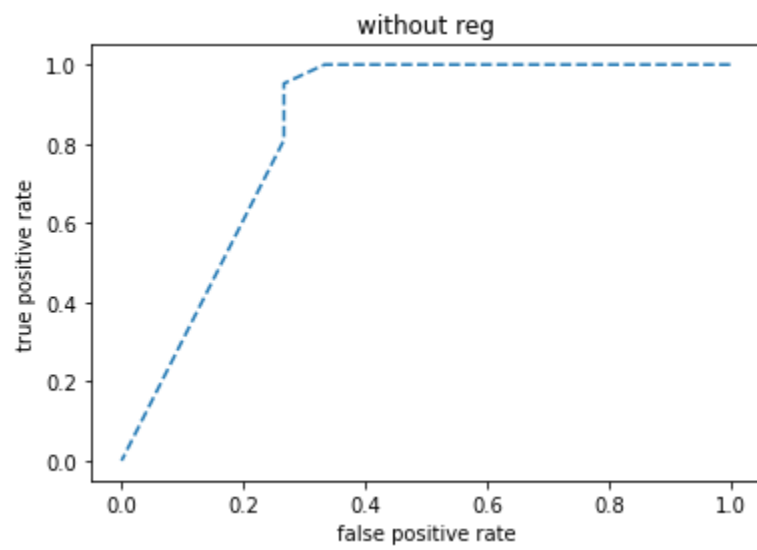
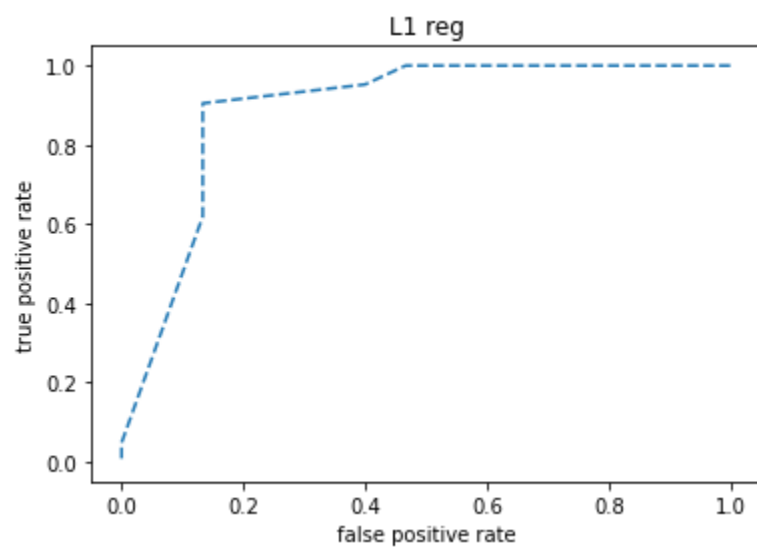
Boxplot for those same features -

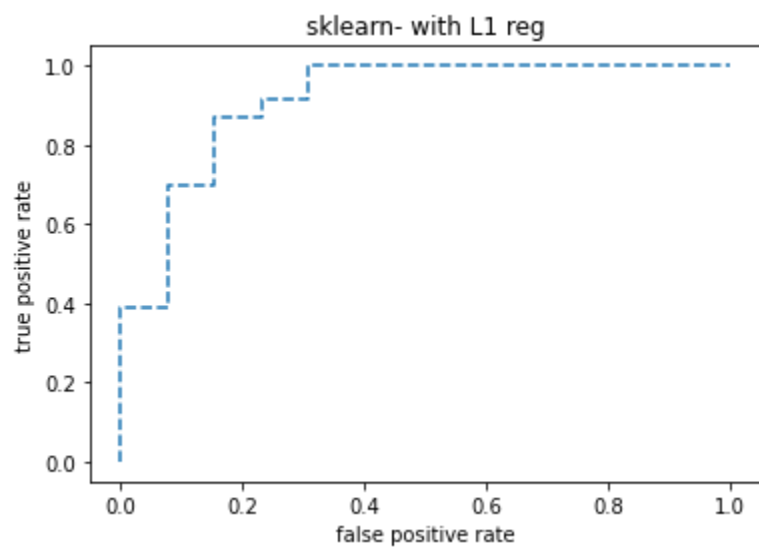
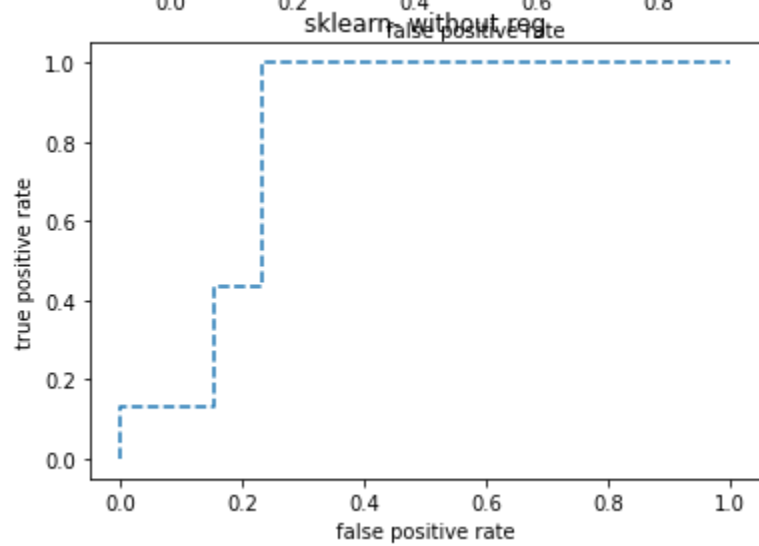
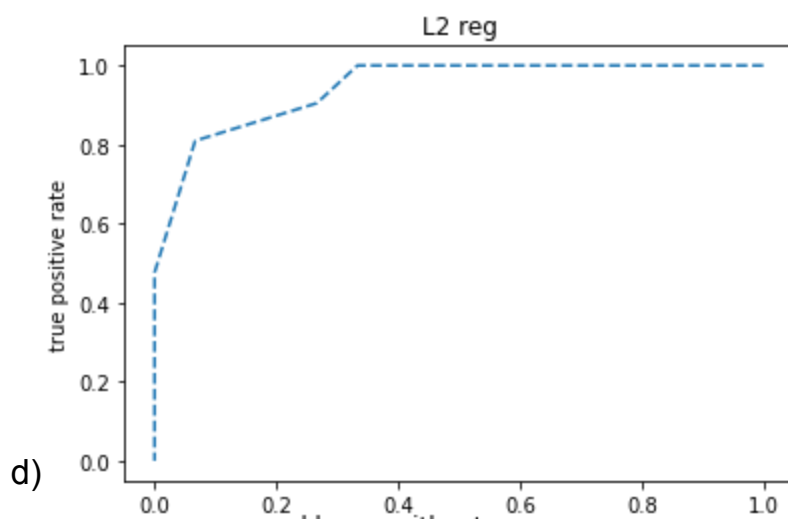


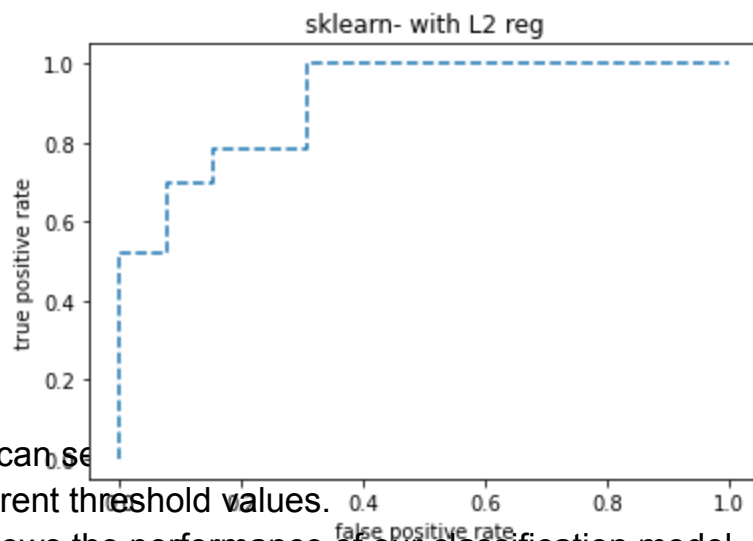
b)



c)







We can see the performance of our classification model with false positive rate for different threshold values. It shows the performance of our classification model

2)

a)

Metrics for OVO Logistic Regression without Regularization-

| OVO Logistic Regression without regularization | | | | |
|------------------------------------------------|-----------|--------|----------|---------|
| | precision | recall | f1-score | support |
| 0 | 0.9596 | 0.9684 | 0.9639 | 980 |
| 1 | 0.9763 | 0.9797 | 0.9780 | 1135 |
| 2 | 0.9175 | 0.9157 | 0.9166 | 1032 |
| 3 | 0.9127 | 0.9208 | 0.9167 | 1010 |
| 4 | 0.9255 | 0.9491 | 0.9372 | 982 |
| 5 | 0.8950 | 0.8700 | 0.8823 | 892 |
| 6 | 0.9483 | 0.9374 | 0.9428 | 958 |
| 7 | 0.9390 | 0.9280 | 0.9335 | 1028 |
| 8 | 0.8761 | 0.9076 | 0.8916 | 974 |
| 9 | 0.9304 | 0.9009 | 0.9154 | 1009 |
| accuracy | | | 0.9289 | 10000 |
| macro avg | 0.9280 | 0.9278 | 0.9278 | 10000 |
| weighted avg | 0.9290 | 0.9289 | 0.9289 | 10000 |

Metrics for OVO Logistic Regression with L2 Regularization-

| OVO Logistic Regression with L2 regularization | | | | | |
|------------------------------------------------|-----------|--------|----------|---------|--|
| | precision | recall | f1-score | support | |
| 0 | 0.9602 | 0.9837 | 0.9718 | 980 | |
| 1 | 0.9588 | 0.9833 | 0.9709 | 1135 | |
| 2 | 0.9405 | 0.9041 | 0.9219 | 1032 | |
| 3 | 0.9110 | 0.9218 | 0.9163 | 1010 | |
| 4 | 0.9225 | 0.9450 | 0.9336 | 982 | |
| 5 | 0.9097 | 0.8812 | 0.8952 | 892 | |
| 6 | 0.9360 | 0.9614 | 0.9485 | 958 | |
| 7 | 0.9393 | 0.9183 | 0.9287 | 1028 | |
| 8 | 0.9073 | 0.8943 | 0.9007 | 974 | |
| 9 | 0.9139 | 0.9049 | 0.9094 | 1009 | |
| accuracy | | | 0.9307 | 10000 | |
| macro avg | 0.9299 | 0.9298 | 0.9297 | 10000 | |
| weighted avg | 0.9305 | 0.9307 | 0.9305 | 10000 | |

b)
Metrics for OVR Logistic Regression without Regularization-

| OVO Logistic Regression without regularization | | | | | |
|------------------------------------------------|-----------|--------|----------|---------|--|
| | precision | recall | f1-score | support | |
| 0 | 0.9467 | 0.9786 | 0.9624 | 980 | |
| 1 | 0.9545 | 0.9797 | 0.9670 | 1135 | |
| 2 | 0.9316 | 0.8847 | 0.9076 | 1032 | |
| 3 | 0.8923 | 0.9109 | 0.9015 | 1010 | |
| 4 | 0.9190 | 0.9358 | 0.9273 | 982 | |
| 5 | 0.8910 | 0.8610 | 0.8757 | 892 | |
| 6 | 0.9380 | 0.9468 | 0.9423 | 958 | |
| 7 | 0.9253 | 0.9163 | 0.9208 | 1028 | |
| 8 | 0.8718 | 0.8727 | 0.8722 | 974 | |
| 9 | 0.9039 | 0.8860 | 0.8949 | 1009 | |
| accuracy | | | 0.9184 | 10000 | |
| macro avg | 0.9174 | 0.9173 | 0.9172 | 10000 | |
| weighted avg | 0.9182 | 0.9184 | 0.9181 | 10000 | |

Metrics for OVR Logistic Regression with L2 Regularization-

| OVO Logistic Regression with L2 regularization | | | | | |
|------------------------------------------------|-----------|--------|----------|---------|--|
| | precision | recall | f1-score | support | |
| 0 | 0.9375 | 0.9796 | 0.9581 | 980 | |
| 1 | 0.9470 | 0.9762 | 0.9614 | 1135 | |
| 2 | 0.9302 | 0.8779 | 0.9033 | 1032 | |
| 3 | 0.9021 | 0.9030 | 0.9025 | 1010 | |
| 4 | 0.8996 | 0.9308 | 0.9149 | 982 | |
| 5 | 0.8885 | 0.8487 | 0.8681 | 892 | |
| 6 | 0.9228 | 0.9478 | 0.9351 | 958 | |
| 7 | 0.9189 | 0.9144 | 0.9166 | 1028 | |
| 8 | 0.8698 | 0.8573 | 0.8635 | 974 | |
| 9 | 0.8966 | 0.8761 | 0.8862 | 1009 | |
| accuracy | | | 0.9124 | 10000 | |
| macro avg | 0.9113 | 0.9112 | 0.9110 | 10000 | |
| weighted avg | 0.9121 | 0.9124 | 0.9120 | 10000 | |

Theory Questions

Solⁿ 4) i) Let our hypothesis f^h be: $h_\theta(x) = \theta_0 + \theta_1 x_1 + \dots + \theta_n x_n$
where $\{\theta_1, \theta_2, \dots, \theta_n\} = \theta$ are parameters of our hypoth.
 $\hookrightarrow j = \text{no. of features}$

$$\text{Cost } f^h \rightarrow J(\theta) = \frac{1}{2n} \sum_{i=1}^n (h_\theta(x_i) - y_i)^2$$

size of dataset

~~Let X be~~

Writing $J(\theta)$ in terms of X we get,

$$J(\theta) = \frac{1}{2n} (X\theta - y)^T (X\theta - y) \quad \left\langle \because A^T A = |A|^2 \right\rangle$$

$$= \frac{1}{2n} ((X\theta)^T - y^T) (X\theta - y)$$

$$= \frac{1}{2n} [(X\theta)^T X\theta - (X\theta)^T y - y^T X\theta + y^T y]$$

$$\left\langle \because AB = B^T A^T \right\rangle$$

$$= \frac{1}{2n} [X\theta^T X^T X\theta - 2(X\theta)^T y + y^T y]$$

$$\left\langle \because \frac{\partial}{\partial \theta} (X\theta^T X^T X\theta) = 2X^T X\theta \right\rangle$$

On Taking Partial derivative,

$$\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{2n} [2X^T X\theta - 2X^T y] = 0$$

$$\Rightarrow X^T X\theta = X^T y$$

On multiplying $(X^T X)^{-1}$ on both sides we get

$$\theta = (X^T X)^{-1} X^T$$

$$\theta = (X^T X)^{-1} X^T y$$

↳ this is our closed form of Linear Regression

Sol 4) 2) It exists when X matrix is invertible

Sol 4) 3) ~~W~~ Closed form sol^{just} of Linear Reg. looks easy but it is computationally very expensive as it requires to invert $(X^T X)^{-1}$ and matrix which is very expensive and matrix multiplication for large data feature dataset is also expensive.

for We can use gradient descent when no. of features are small and no. of training sets are also small (i.e., < 20000)

Sol 4) 4) for Simple linear regression, line eqⁿ,

$$y = m \cdot x + b$$

$$\text{where } m = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} \quad b = \bar{y} - m \bar{x}$$

for Least Square estimate, for

$$\text{So, when } y = \bar{y} \Rightarrow b = \frac{\sum (x - \bar{x})(\bar{y} - \bar{y})}{\sum (x - \bar{x})^2}$$

Sol 5)

Sol 4) 5)

4) for simple linear regression, eqⁿ of line is:

$$y = mx + b$$

To for least square line, we have to minimize

$$J = \sum_{k=1}^n [y_k - (mx_k + b)]^2 \quad (n = \text{no. of dataset})$$

Here we have 2 parameters m & b

$$\therefore \text{for min}^m, \frac{\partial J}{\partial b} = 0 \quad \& \quad \frac{\partial J}{\partial m} = 0$$

$$\frac{\partial J}{\partial b} = -2 \sum_{k=1}^n [y_k - (mx_k + b)] = 0$$

$$\Rightarrow \sum_{k=1}^n y_k = m \sum_{k=1}^n x_k + \sum_{k=1}^n b$$

$$\Rightarrow \sum y_k = m \sum x_k + nb$$

$$\Rightarrow \frac{1}{n} \sum_{k=1}^n y_k = \frac{m}{n} \sum_{k=1}^n x_k + b$$

$$\Rightarrow \bar{y} = m\bar{x} + b$$

\therefore Since (\bar{x}, \bar{y}) satisfies the eqⁿ of for Linear Regression line

\Rightarrow for Least square line always passes through (\bar{x}, \bar{y})

Q4) ^{Yes,} We can use Linear Regression for classification by assigning a threshold. All the $h(x)$ below that threshold should go to class 1 and above the threshold should go to class 2. But ~~logit~~ for classification problems we should prefer logistic regression or other classification specific algos.