

Kernel (k)

$$K(x, y) = \langle \phi(x), \phi(y) \rangle$$

$$x = (x_1, x_2, x_3)$$

$$y = (y_1, y_2, y_3)$$

$$\phi(x) = \begin{bmatrix} x_1 x_1 \\ x_1 x_2 \\ x_1 x_3 \\ x_2 x_1 \\ x_2 x_2 \\ x_2 x_3 \\ x_3 x_1 \\ x_3 x_2 \\ x_3 x_3 \end{bmatrix}$$

$$x = [1, 2, 3]$$

$$y = [4, 5, 6]$$

$$\phi(y) = \begin{bmatrix} y_1 y_1 \\ y_1 y_2 \\ y_1 y_3 \\ y_2 y_1 \\ y_2 y_2 \\ y_2 y_3 \\ y_3 y_1 \\ y_3 y_2 \\ y_3 y_3 \end{bmatrix}$$

$$= [16, 20, 24, 20, 28, 30, 24, 30, 36]$$

$$\phi(x) = \begin{bmatrix} x_1 x_1 \\ x_1 x_2 \\ x_1 x_3 \\ x_2 x_1 \\ x_2 x_2 \\ x_2 x_3 \\ x_3 x_1 \\ x_3 x_2 \\ x_3 x_3 \end{bmatrix} \quad \begin{array}{l} x = [1, 2, 3] \\ y = [4, 5, 6] \end{array}$$

$$= [1, 2, 3, 2, 4, 6, 3, 6, 9]$$

$\langle \phi(x), \phi(y) \rangle \rightarrow$ dot product

$$[1, 2, 3, 2, 4, 6, 3, 6, 9] [16, 20, 24, 20, 28, 30, 24, 30, 36]$$

$$= 16 + 40 + 72 + 40 + 100 + 180 + 72 + 180 + 324$$

$$= 1024$$

Cons \rightarrow Computationally expensive, the calculation at higher dimensional space.

Kernel trick \rightarrow Work on lower dimensional space to compute the math at higher dimensional space

$$K(x, y) = (\langle x, y \rangle)^2$$

$$x = [1, 2, 3]$$

$$y = [4, 5, 6]$$

$$= (4 + 10 + 18)^2$$

$$= 14 + 18$$

$$= (32)^2$$

$$= 1024$$