13/03/25

Support vector machine

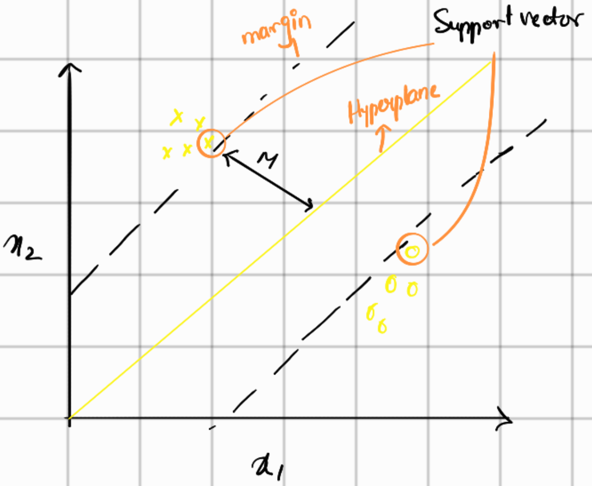
(ISLP book)

β is the vector of weight (coefficient for features)

intercept bias

$$d = \frac{|\beta_0 + \beta_1 x_1 + \beta_2 x_2|}{\sqrt{\beta_1^2 + \beta_2^2}}$$

discriminant or decisive function



$$p = \frac{a a^T}{a^T a} b \rightarrow \text{projection of } b \text{ onto } a$$

Lines. (Given)

a. $2x_1 + 3x_2 - 5 = 0$

b. $-x_1 + 4x_2 + 7 = 0$

c. $5x_1 - 12x_2 + 10 = 0$

x_1	x_2	y
3	4	+1
2	3	+1
1	-1	-1
-2	+1	-1

① Compute distance from each point
get the margin value

$d_A \Rightarrow$ A represents sample

a. $2x_1 + 3x_2 - 5 = 0$

$$d_1 = \frac{|\beta_0 + \beta_1 x_1 + \beta_2 x_2|}{\sqrt{\beta_1^2 + \beta_2^2}}$$

$4+9$

$$= \frac{|-5 + (2 \times 3) + (3 \times 4)|}{\sqrt{2^2 + 3^2}} = \frac{-5 + 6 + 12}{\sqrt{13}}$$

$$= \frac{13}{\sqrt{13}} = \sqrt{13}$$

$x_1 = 2$

$x_2 = 3$

$$d_2 = \frac{|-5 + (2 \times 2) + (3 \times 3)|}{\sqrt{2^2 + 3^2}} = \frac{-5 + 4 + 9}{\sqrt{13}} = \frac{8}{\sqrt{13}}$$

$$d_3 = \frac{|-5 + (2 \times 1) + (3 \times -1)|}{\sqrt{2^2 + 3^2}}$$

$x_1 = 1$

$x_2 = -1$

$$= \frac{-5 \quad +2 \; -3}{\sqrt{13}} = \frac{6}{\sqrt{13}}$$

$$d_4 = \frac{\left| -5 \quad +(2 \times -2) + (3 \times 1) \right|}{\sqrt{2^2 + 3^2}}$$

$x_1 = -2$
$x_2 = +1$

$$= \frac{-5 \quad -4 \; +3}{\sqrt{13}} = \frac{6}{\sqrt{13}}$$

$d_1^a \quad 13/\sqrt{13}$
$d_2^a \quad 8/\sqrt{13}$
$d_3^a \quad 6/\sqrt{13}$
$d_4^a \quad 6/\sqrt{13}$

x_1	3
x_2	4

b. $-x_1 + 4x_2 + 7 = 0$

$$d_1^b = \frac{\left| \beta_0 \; + \beta_1 x_1 \; + \beta_2 x_2 \right|}{\sqrt{\beta_1^2 + \beta_2^2}}$$

$$= \frac{\left| 7 + ((-1) \times 3) + (4 \times 4) \right|}{\sqrt{(-1)^2 + (4)^2}} = \frac{|7 - 3 + 16|}{\sqrt{17}} = \frac{20}{\sqrt{17}}$$

$$d_1^b = \frac{20}{\sqrt{17}}$$

$x_1 = 2 \;, \quad x_2 = 3$

$$d_2^b = \frac{\left| 7 + ((-1) \times 2) + (4 \times 3) \right|}{\sqrt{(-1)^2 + (4)^2}} = \frac{7 - 2 + 12}{\sqrt{17}}$$

$$= \frac{17}{\sqrt{17}}$$

$x_1 = 1 \qquad x_2 = -1$

$$d_3^b = \frac{|7 + ((-1) \times 1) + (4 \times -1)|}{\sqrt{(-1)^2 + (4)^2}} = \frac{7 - 1 - 4}{\sqrt{17}} = \frac{2}{\sqrt{17}}$$

$x_1 = -2 \qquad x_2 = +1$

$$d_4^b = \frac{|7 + ((-1) \times -2) + (4 \times 1)|}{\sqrt{(-1)^2 + (4)^2}}$$

$$= \frac{|7 - 3 + 4|}{\sqrt{17}} = \frac{8}{\sqrt{13}}$$

x_1	x_2	y
3	4	+1
2	3	+1
1	-1	-1
-2	+1	-1

c. $5x_1 - 12x_2 + 10 = 0$

$$d^c = \frac{|\beta_0 + \beta_1 x_1 + \beta_2 x_2|}{\sqrt{\beta_1^2 + \beta_2^2}}$$

$x_1 = 3 \qquad x_2 = 4$

$$d_1^c = \frac{10 + (5 \times 3) + (-12 \times 4)}{\sqrt{5^2 + 12^2}} = \frac{10 + 15 - 48}{13}$$

$$= \frac{23}{13}$$

$$d_2^c = \frac{10 + (5 \times 2) + (-12 \times 3)}{\sqrt{5^2 + 12^2}} = \frac{10 + 10 - 36}{13}$$

$$= \frac{16}{13}$$

$$d_3^c = \frac{10 + (5 \times 1) + (-12 \times -1)}{\sqrt{5^2 + 12^2}} = \frac{10 + 5 + 13}{13} = \frac{28}{13}$$

$$d_4^c = \frac{10 + (5 \times -2) + (-12 \times 1)}{\sqrt{5^2 + 12^2}} = \frac{10 - 10 - 12}{13}$$

$$= \frac{12}{13}$$

Select the minimum distance of the data point from the hyplane equation

$m_1 = 1.664$

$m_2 = 0.485$

$m_3 = 0.92$

Select the margin with maximum value

Read - Linear algebra topics

1. Projection
2. Norm of a vector
3. Vector of addition

① Maximal marginal classifier → This is not SVM

Maximize M

$\beta_0, \beta_1, \beta_2, \beta_p$ M

Subject to $\sum_{j=1}^{p} \beta_j^2 = 1$

$$y^{(i)}\left(\beta_0 + \beta_1 x_1^{(i)} + \beta_2 x_2^{(i)} + \ldots \beta_p x_p^{(i)}\right) \geq M$$

Hard margin -

Langrangian multipliers