Observations	Actual label	Predicted Score	Predicted label	0.7	0.5	0.4
1	1	0.85	1	1	1	1
2	0	0.60	1		1	1
3	1	0.70	1	1	1	1
4	1	0.40	0			1
5	0	0.55	1		1	1
6	1	0.50	1		1	1
7	0	0.65	1		1	1
8	0	0.35	0			
9	1	0.60	1		1	1
10	0	0.20	0			

ROC for $-[0.7, 0.5, 0.4, 0.2]$

Positives = 5
Negative = 5

All the values ≥ 0.7 predicted positive.

at predicted score

TP	FN
2	3
FP	TN
0	5

$$TPR = \frac{TP}{TP+FN}$$

$$= 1$$

$$FN = 5-2$$

$$TN = N - FP$$

$$= 5-0$$

$$FPR = \frac{FN}{TN+FN} = \frac{3}{8} = 3.75$$

$$\text{Precision} = \frac{TP}{TP+FN}$$

$$F_1 = \frac{2 \cdot \text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}} = \frac{2 \times 0.4 \times 1}{1.4} = \frac{2}{5} = 0.4$$

$$= \frac{0.8}{1.4} = \frac{4}{7} = 0.57$$

ROC at 0.5

TP	FN
4	1
FP	TN
3	2

$FN = P - TP$

$= 5 - 4$

$TN = N - FP = 5 - 3 = 2$

$TPR = \frac{4}{7} = 5.71$

$FPR = \frac{1}{3} = 0.33$

$Precision = \frac{4}{4+1} = \frac{4}{5} = 0.8$ $F_1 = \frac{2 \times 0.8 \times 5.7}{8 + 5.71} = 0.67$

ROC at 0.4

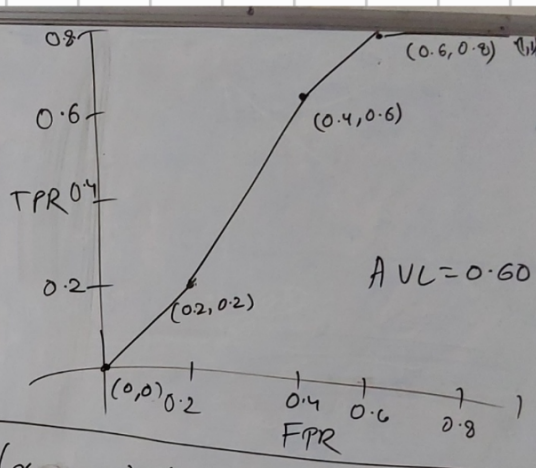
TP	FN
5	0
FP	TN
3	2

$TPR = \frac{5}{8} = 6.25$

$FPR = 0$

$Precision = \frac{5}{5+0} = 1$ $F_1 = \frac{2 \times 1 \times 6.25}{7.625} = 1.75$

Ob	True val	Pred. val	At 0.5 Predic	0.8	0.6	0.4
1	1	0.95		1	1	1
2	0	0.85		1	1	1
3	1	0.78		0	1	1
4	0	0.66		0	1	1
5	1	0.60		0	1	1
6	0	0.55		0	0	1
7	1	0.40		0	0	1
8	0	0.30		0	0	0
9	1	0.20		0	0	0
10	0	0.10		0	0	0



AUC = 0.60

@ 0.8

TP 1	FP 1
FN 4	TN 4

Sen = $\frac{1}{5}$ = 0.2

Sp = $\frac{4}{5}$ = 0.8 ; FPR : 0.2

@ 0.6

TP 3	FP 2
FN 2	TN 3

Sen = $\frac{3}{5}$ = 0.6

Spe = $\frac{3}{5}$ = 0.6 ; FPR = 0.4

@ 0.4

TP 4	FP 3
FN 1	TN 2

Sen = $\frac{4}{5}$ = 0.8

Spe = $\frac{2}{5}$ = 0.4 ; FPR = 0.6

$$AUC = \frac{\sum (x_{i+1} - x_i) \times (y_i + y_{i+1})}{2}$$

$$= \frac{(0.2-0)\times(0+0.2) + (0.4-0.2)\times(0.6+0.2) + (0.6-0.4)\times(0.8+0.6) + (1-0.6)\times(1+0.8)}{2}$$

$$= \frac{(0.2\times0.2) + (0.2\times0.8) + (0.2\times1.4) + (0.4\times1.8)}{2}$$

$$= \frac{0.04 + 0.16 + 0.28 + 0.72}{2} = \frac{1.20}{2} = \boxed{0.60}$$