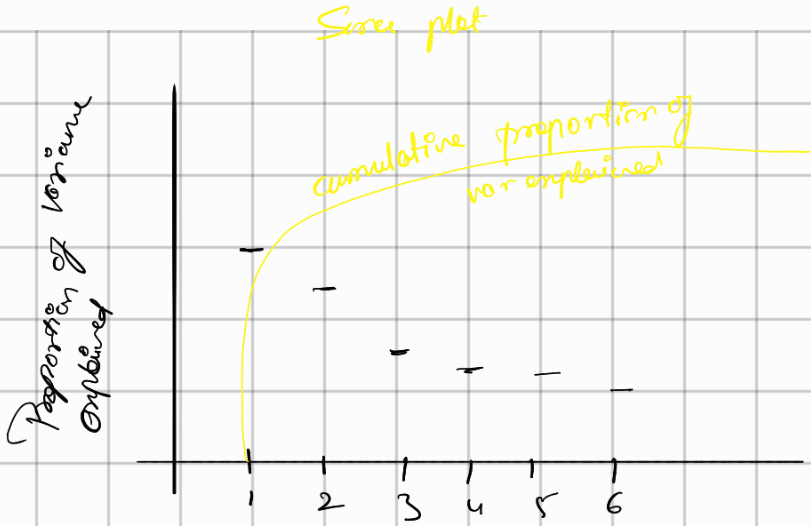

no. of component

Q – $X = \begin{bmatrix} 2 & 100 \\ 4 & 200 \\ 6 & 300 \end{bmatrix}_{3\times 2} \xrightarrow[\text{PCA}]{\text{apply}} Z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}_{3\times 1}$

$\mathbb{R}^2 \qquad \mathbb{R}^1$

① Standardization

Col 1 $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \rightarrow \mu_1 = \frac{2+4+6}{3} = 4$

$$\sigma_1 = \sqrt{\frac{(x_i - \mu)^2}{n}}$$

$$\sigma_1 = \sqrt{\frac{(2-4)^2}{2} + \frac{(4-4)^2}{2} + \frac{(6-4)^2}{2}}$$

$$= \sqrt{\frac{4}{2} + 0 + \frac{4}{2}}$$

$$\sigma_1 = \sqrt{\frac{8}{2}} = 2$$

$$\sigma_1 = 2$$

$$\mu_2 = \frac{600}{3} = 200$$

$$\sigma_2 = \sqrt{\frac{(100)^2 + 0 + (100)^2}{2}}$$

$$= \frac{2\times 10^4}{2}$$

$$\boxed{\sigma_2 = 100}$$

Col 1

$$x_1 = \frac{x_1 - \mu}{\sigma} = \frac{2-4}{2} = -1$$

$$x_2 = \frac{4-4}{2} = 0$$

Col 2

$$x_1 = \frac{100-200}{100} = -1$$

$$x_2 = 0$$

$x_3 = \frac{6-4}{2} = 1$ $\qquad\qquad$ $x_3 = \frac{100}{100} = 1$

② Covariance mat

$\cancel{\frac{1}{n} X^T X}$ $\quad \frac{1}{n-1} X . X^T$

$$X_{std} = \begin{array}{cc} x^1 & x^2 \end{array} \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\underset{2\times 3}{\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}}$$

$$= \frac{1}{2} \begin{bmatrix} 1+0+1 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

③ Compute eigen vector and values of the covariance mat

④ Decide the needed % of variance explained and choose 'K' principal component

⑤ Transform X into new space of principal component

⑥ Use this z to train a model.

28/03/25

Continuing

Eigen values

$\lambda_1 = 0$ $\lambda_2 = 2$

$\frac{1}{n} = \sqrt{(1)^2 + (1)^2}$

eigen vectors

$V_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

wat?in

for Eigen value solve

Satisfy this equation

$\det(P - \lambda I) = 0$

$\det \begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix} = 0$

$(1-\lambda)^2 - (1 \times 1) = 0$

$\cancel{1} + \lambda^2 - 2\lambda \cancel{-1} = 0$

$\boxed{\lambda_2 = 2}$

$\boxed{\lambda_1 = 0}$

For Eigen vector

$P - 0I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$x_1 + x_2 = 0$$
$$x_1 = -x_2$$

For $\lambda_1 = 0$ $\quad v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

For $\lambda_2 = 2$

$$P - 2I = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x_1 + x_2 = 0$$
$$\boxed{x_1 = x_2}$$

$$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\boxed{v_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

$\Rightarrow$ normalization $v_2 = \dfrac{v_2}{\|v_2\|}$

$$z_{pca} = X_{std} \times v_2$$

$$z_{pca} = \begin{bmatrix} -1 & -1 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} -1 \times 1/\sqrt{2} + -1 \times 1/\sqrt{2} \\ 0 + 0 \\ 1/\sqrt{2} + 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} -2/\sqrt{2} \\ 0 \\ -2/\sqrt{2} \end{bmatrix}$$