Tutorial -1 solutions

Q1

3. Determine the angle by which v_1 *leads* i_1 if $v_1 = 10\cos(10t - 45°)$ and i_1 is equal to (*a*) $5\cos 10t$; (*b*) $5\cos(10t - 80°)$; (*c*) $5\cos(10t - 40°)$; (*d*) $5\cos(10t + 40°)$; (*e*) $5\sin(10t - 19°)$.

Sol^n :

$v_1 = 10\cos(10t - 45°)$

converting v_1 into phasor Domain

$V_1 = 10 \angle -45°$

(Taking $\cos(10t)$ as reference)

* For any signal : $x = A\cos(\omega t + \phi)$

its phasor : $X = A\angle\phi$

(a) $i_1 = 5\cos(10t) \xrightarrow{\text{phasor}} I_1 = 5\angle 0°$

For angle between v_1 and i_1 : $\dfrac{V_1}{I_1} = \dfrac{\cancel{10}^{\,2} \angle -45°}{\cancel{5}\angle 0°}$

$= 2\angle -45°$

$\therefore$ v_1 leads i_1 by angle of $-45°$

(b) $\quad i_1 = 5\cos(10t - 80^\circ) \xrightarrow[\text{Phasor}]{} I_1 = 5\angle -80^\circ$

For angle between v_1 and i_1 :

$$\frac{V_1}{I_1} = \frac{\cancel{10}\angle -45^\circ}{\cancel{5}\angle -80^\circ}$$

$$= 2\angle(-45^\circ - (-80^\circ))$$

$$= 2\angle(-45^\circ + 80^\circ)$$

$$= 2\angle 35^\circ$$

$\therefore$ v_1 leads i_2 by angle : 35°

(c) $\quad i_1 = 5\cos(10t - 40^\circ) \xrightarrow[\text{Phasor}]{} I_1 = 5\angle -40^\circ$

For angle between v_1 and i_1 :

$$\frac{V_1}{I_1} = \frac{\cancel{10}\angle -45^\circ}{\cancel{5}\angle -40^\circ}$$

$$= 2\angle(-45^\circ - (-40^\circ))$$

$$= 2\angle(-45^\circ + 40^\circ)$$

$$= 2\angle -5^\circ$$

$\therefore$ v_1 leads i_2 by angle : -5°

(d) $i_1 = 5\cos(10t + 40^\circ) \xrightarrow{\text{Phasor}} I_1 = 5\angle +40^\circ$

For angle between v_1 and i_1 : $\dfrac{V_1}{I_1} = \dfrac{10\angle -45^\circ}{5\angle +40^\circ}$

$= 2\angle(-45^\circ - (+40^\circ))$

$= 2\angle(-45^\circ - 40^\circ)$

$= 2\angle -85^\circ$

$\therefore$ v_1 leads i_1 by angle : -85°

(e) $i_1 = 5\sin(10t - 19^\circ) \xrightarrow{\text{Phasor}} I_1 = 5\angle -109^\circ$

$= 5\cos(10t - 19^\circ - 90^\circ)$

For angle between v_1 and i_1 : $\dfrac{V_1}{I_1} = \dfrac{10\angle -45^\circ}{5\angle -109^\circ}$

$= 2\angle(-45^\circ - (-109^\circ))$

$= 2\angle(-45^\circ + 109^\circ)$

$= 2\angle 64^\circ$

$\therefore$ v_1 leads i_1 by angle : 64°

Q2

5. Determine which waveform in each of the following pairs is lagging: (*a*) cos $4t$, sin $4t$; (*b*) cos $(4t - 80°)$, cos $(4t)$; (*c*) cos $(4t + 80°)$, cos $4t$; (*d*) $-\sin 5t$, cos $(5t + 2°)$; (*e*) $\sin 5t + \cos 5t$, $\cos(5t - 45°)$.

Sol^n :

(a) $\underbrace{\cos 4t \,,\ \sin 4t = \cos(4t - 90°)}_{\text{phasor}}$

$1\angle 0°$, $1\angle -90°$

From the above $\sin 4t$ is lagging wrt $\cos 4t$ by $90°$

(b) $\underbrace{\cos(4t - 80°) \,,\ \cos(4t)}_{\text{phasor}}$

$1\angle -80°$, $1\angle 0°$

From the above $\cos(4t - 80°)$ is lagging wrt $\cos 4t$ by $80°$

(c) $\underbrace{\cos(4t + 80°) \,,\ \cos 4t}_{\text{phasor}}$

$1\angle 80°$, $1\angle 0°$

From the above $\cos 4t$ is lagging wrt $\cos(4t + 80°)$ by $80°$

(d) $\underbrace{-\sin 5t = \cos(5t + 90°) \,,\ \cos(5t + 2°)}_{\text{phasor}}$

$1\angle 90°$, $1\angle 2°$

$*\quad \dfrac{1\angle 90^\circ}{1\angle 2^\circ} = 1\angle 90^\circ - 2^\circ = 1\angle 88^\circ$

$\therefore$ $\cos(5t+2^\circ)$ is lagging wrt $-\sin 4t$ by 88°

(e) $\quad \underbrace{\sin 5t + \cos 5t \quad , \quad \cos(5t-45^\circ)}_{\text{Phasor}}$

$$\underbrace{1\angle -90^\circ + 1\angle 0^\circ}\ , \quad 1\angle -45^\circ$$

$$1(-j) + 1 = 1 - j = \sqrt{1^2+1^2}\ \angle\phi = \sqrt{2}\ \angle -45^\circ$$

$$\therefore\ \phi = -\tan^{-1}\left(\frac{1}{1}\right) = -45^\circ$$

$*$ For any complex Number : $z = x + jy$

Phasor form : $\sqrt{x^2+y^2}\ \angle\phi$

let $\theta = \tan^{-1}\left(\dfrac{|y|}{|x|}\right)$

$$\begin{array}{c|c} 180^\circ - \theta & \theta \\ \hline 180^\circ + \theta & -\theta \end{array}$$

$\therefore\quad 1\angle -45^\circ\ , \quad 1\angle -45^\circ$

$\therefore$ No pairs are leading / lagging

Q3

20. Perform the indicated operations, and express the answer in both rectangular and polar forms:

(a) $\dfrac{2+j3}{1+8\angle 90^\circ} - 4$; (b) $\left(\dfrac{10\angle 25^\circ}{5\angle -10^\circ} + \dfrac{3\angle 15^\circ}{3-j5}\right) j2$;

(a) $\dfrac{2+j3}{1+8\angle 90^\circ} = \dfrac{2+j3}{1+8j}$

$\boxed{\begin{aligned} j &= 1\angle 90^\circ \\ -j &= 1\angle -90^\circ \end{aligned}}$

- $2+3j = \sqrt{2^2+3^2} \angle \tan^{-1}\left(\frac{3}{2}\right)$

$= \sqrt{13} \angle 56.30^\circ$

$2+3j \simeq 3.60 \angle 56.30^\circ$

- $1+8j = \sqrt{1^2+8^2}\ \tan^{-1}\left(\frac{8}{1}\right) = \sqrt{65} \angle 82.87^\circ$

$= 8.06 \angle 82.87^\circ$

$\Rightarrow \dfrac{2+3j}{1+8j} - 4 = \dfrac{2+3j-4-32j}{1+8j} = \dfrac{-2-29j}{1+8j}$

$= 3.6 \angle 183.18^\circ$

In rectangular form: $3.6(\cos(183.18^\circ) + j\sin(183.18^\circ))$

$= 3.6(-0.998 - j0.055)$

$= -3.6 - j0.2$

(b) $\left(\dfrac{1\cancel{0}^{2}\angle 25^\circ}{\cancel{5}\angle -10^\circ} + \dfrac{3\angle 15^\circ}{3-j5} \right) j2$

$= \left(1.638 + j1.147 + \dfrac{3\angle 15^\circ}{5.83\angle -59.03^\circ} \right) j2$

$= (1.638 + j1.147 + 0.514\angle 74.03^\circ) j2$

$= (1.638 + j1.147 + 0.141 + j0.494) j2$

$= (1.779 + j1.641) j2$

$= -3.282 + j3.558$

$= \boxed{4.32\angle 111.56^\circ}$

In rectangular form: $4.32 \left(\cos(111.56^\circ) + j \sin(111.56^\circ) \right)$

$= \boxed{-1.58 + j4.02}$

$\underline{\underline{Q4}}$

26. Transform each of the following into phasor form: (*a*) 11 sin 100*t*; (*b*) 11 cos 100*t*; (*c*) 11 cos(100*t* – 90°); (*d*) 3 cos 100*t* – 3 sin 100*t*.

Sol^n :

(a) $11 \sin 100t \rightsquigarrow \boxed{11\angle -90^\circ}$

$= 11\cos(100t - 90^\circ)$

(b) $11 \cos 100t \rightsquigarrow \boxed{11\angle 0^\circ}$

(c) $\quad 11\cos(100t - 90^\circ) \rightsquigarrow \boxed{11\angle -90^\circ}$

(d) $\quad 3\cos 100t - \underbrace{3\sin 100t}_{\cos(100t-90^\circ)} \rightsquigarrow 3\angle 0^\circ - 3\angle -90^\circ$

$$= 3 - (3(-j))$$

$$= 3 + 3j$$

$$= 3\sqrt{2}\angle \tan^{-1}\left(\frac{3}{3}\right)$$

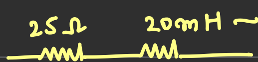
$$= \boxed{3\sqrt{2}\angle 45^\circ}$$

Q5

39. Determine the equivalent admittance of the following, assuming an operating frequency of 1000 rad/s: (*a*) 25 Ω in series with 20 mH; (*b*) 25 Ω in parallel with

20 mH

Sol^n: (a)



$\rightarrow \quad j\omega L = j(1000)(20\times 10^{-3})$

$$= 20j$$

$$Z_{eq.} = 25 + 20j$$

admittance $\quad Y_{eq.} = \dfrac{1}{Z_{eq}} = \dfrac{1}{25+20j} = \boxed{0.031\angle -38.65^\circ \text{ Seimens}}$

$$= \boxed{(0.024 - 0.019j) \text{ Seimens}}$$

(b)

20mH $\rightarrow$ $20j$

$$\frac{1}{Z_{eq.}} = \frac{1}{25} + \frac{1}{20j} = \frac{20j + 25}{500j} = \frac{4j+5}{100j}, \quad Y_{eq} = \frac{4j+5}{100j} = \boxed{64\angle -51.3 \text{ mS}}$$

Q6

125 μF in series, but $\omega = 4$ krad/s.

43. Calculate the equivalent impedance seen at the open terminals of the network shown in Fig. 10.56 if f is equal to (*a*) 1 Hz; (*b*) 1 kHz; (*c*) 1 MHz; (*d*) 1 GHz; (*e*) 1 THz.

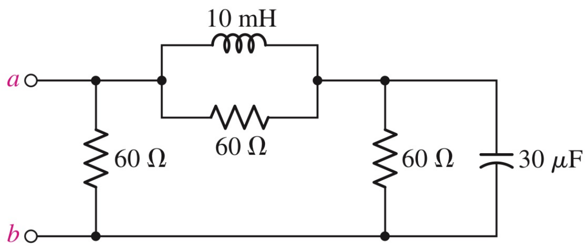
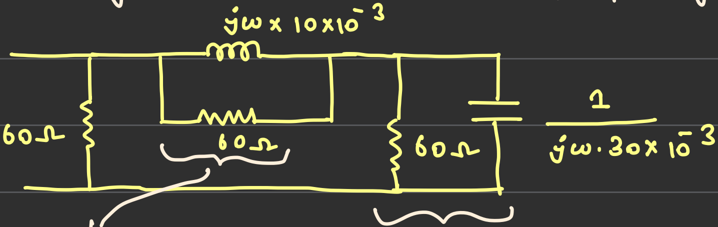


FIGURE 10.56

Sol^n :

converting the circuit into frequency domain



$$Z_2 = \frac{60 \times j\omega\, 10^{-2}}{60 + j\omega\, 10^{-2}}$$

$$Z_1 = \frac{60 \times \dfrac{1}{j\omega \cdot 30\times 10^{-3}}}{60 + \dfrac{1}{j\omega \cdot 30\times 10^{-3}}}$$

(a) $f = 1\,Hz \rightsquigarrow \omega = 2\pi f = 2\pi$ rad/sec

$$\Rightarrow \quad Z_1 = \frac{\overset{30\ 10}{\cancel{60}} \times \dfrac{1}{\cancel{2}\pi j \times \cancel{3}\times 10^{-2}}}{60 + \dfrac{1}{2\pi j \times 3\times 10^{-2}}} = \frac{\dfrac{-j \times 10^4}{\pi}}{60 + \dfrac{\cancel{100}\,50}{\cancel{2}\pi j \times 3}}$$

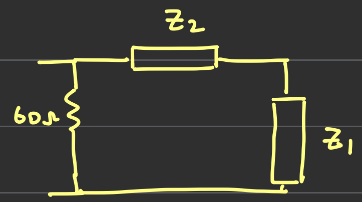
$$Z_1 = \frac{\frac{-j \times 10^4}{\pi}}{60 + \frac{\cancel{100}50}{2\pi j \times 3}} = \frac{-0.31j \times 10^4}{60 - 5.30j}$$

$$\therefore Z_1 = \frac{-3100j}{60 - 5.30j} = \boxed{51.46 \angle -84.95^\circ}$$

• $$Z_2 = \frac{60 \times j\omega\, 10^{-2}}{60 + j\omega\, 10^{-2}} = \frac{60 \times 10^{-2} j}{60 + j\, 10^{-2}} = \frac{0.6j}{60 + 0.01j}$$

$$= \boxed{0.01 \angle 89.99^\circ}$$

•



$$\therefore Z_{eq.} = 60 \parallel (Z_1 + Z_2)$$

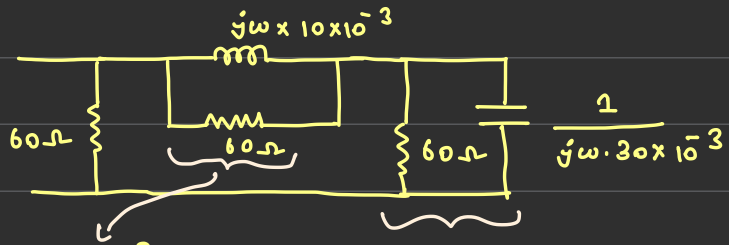
$$= 60 \parallel (51.45 \angle -84.94^\circ)$$

$$= \frac{60 \times 51.45 \angle -84.94^\circ}{60 + 51.45 \angle -84.94^\circ}$$

$$= \frac{3087 \angle -84.94^\circ}{82.41 \angle -38.45^\circ}$$

$$\boxed{Z_{eq} = 37.45 \angle -46.48^\circ}$$

(b) $f = 1\,KHz$



$$Z_2 = \frac{60 \times j\omega\, 10^{-2}}{60 + j\omega\, 10^{-2}}$$

$$Z_1 = \frac{60 \times \frac{1}{j\omega \cdot 30\times 10^{-3}}}{60 + \frac{1}{j\omega \cdot 30 \times 10^{-3}}}$$

- $$Z_1 = \frac{60 \times \frac{1}{30j}}{60 + \frac{1}{30j}} = \frac{-2j}{60 - 0.03j} = 0.033 \angle -89.97^\circ$$

- $$Z_2 = \frac{6j}{60 + 0.1j} = 0.09 \angle 89.90^\circ$$

$$\therefore \quad Z_{eq.} = 60 \parallel (0.057 \angle 89.82^\circ)$$

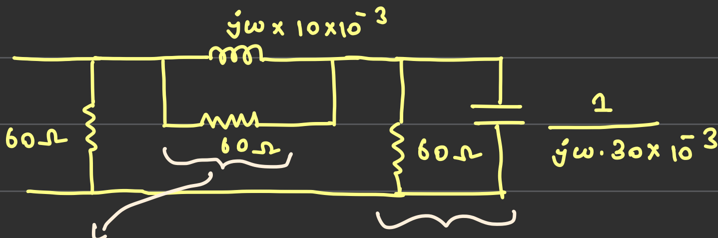
$$= \frac{60 \times 0.057 \angle 89.82^\circ}{60 + 0.057 \angle 89.82^\circ}$$

$$= \frac{3.42 \angle 89.82^\circ}{60 \angle 0.05^\circ}$$

$$Z_{eq} = 0.057 \angle 89.77^\circ$$

(c) $f = 1\,MHz$

$= 10^6\,Hz$



$$Z_2 = \frac{60 \times j\omega\, 10^{-2}}{60 + j\omega\, 10^{-2}}$$

$$Z_1 = \frac{60 \times \frac{1}{j\omega \cdot 30\times 10^{-3}}}{60 + \frac{1}{j\omega\cdot 30 \times 10^{-3}}}$$

• $$Z_1 = \frac{\frac{\cancel{6}0^2}{j\times 10^6 \times 10^{-3} \times \cancel{3}\cancel{0}}}{60 + \frac{1}{j\times 10^6\times 10^{-3}\times 30}} = \frac{-2\times 10^3 j}{60 - j\frac{100}{3}} = 29.13\angle -60.94^\circ$$

• $$Z_2 = \frac{60 \times j \times 10^6 \times 10^{-2}}{60 + j\times 10^6 \times 10^{-2}} = \frac{6\times 10^5 j}{60 + j\times 10^4} = 59.9\angle 0.34^\circ$$

$\therefore$ $$Z_{eq.} = 60 \parallel (78.18\angle -18.73^\circ)$$

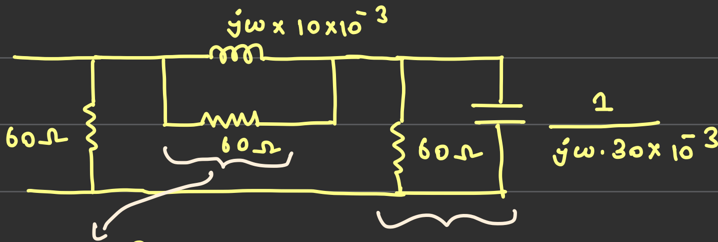
$$= \frac{60 \times 78.18\angle -18.73^\circ}{60 + 78.18\angle -18.73^\circ}$$

$$= \frac{4690\angle -18.73^\circ}{136.37\angle -10.60^\circ}$$

$$Z_{eq} = 4825.03\angle -18.5^\circ\ \Omega$$

(d) $f = 1\,GHz$

$= 10^9\,Hz$



$$Z_2 = \frac{60 \times j\omega\, 10^{-2}}{60 + j\omega\, 10^{-2}}$$

$$Z_1 = \frac{60 \times \frac{1}{j\omega \cdot 30\times10^{-3}}}{60 + \frac{1}{j\omega\cdot 30\times 10^{-3}}}$$

- $$Z_1 = \frac{\frac{\cancel{60}\;2}{j\times10^9\times\cancel{30}\times10^{-3}}}{60 + \frac{1}{j\times10^9\cdot30\times10^{-3}}} = \frac{-2\times10^{-6} j}{60 - \frac{10^{-7}}{3} j} = \boxed{\frac{1}{3}\times10^{-7}\angle -89.9^\circ\,\Omega}$$

- $$Z_2 = \frac{6\times10^8 j}{60 + 10^7 j} = \boxed{60\times10^{-4}\angle 3.43^\circ}$$

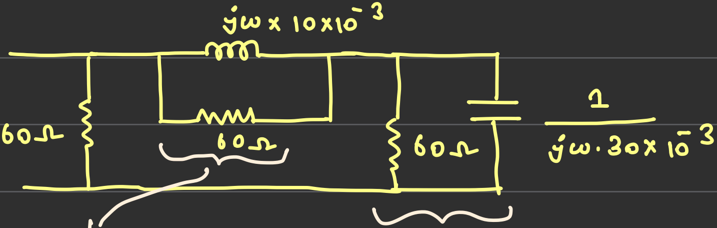
$$\therefore\quad Z_{eq.} = 60 \,||\, (5.99\times10^{-3}\angle 3.43^\circ)$$

$$= \frac{60\times 5.99\times10^{-3}\angle 3.43^\circ}{60 + 5.99\times10^{-3}\angle 3.43^\circ}$$

$$\boxed{Z_{eq} = 5.98\times10^{-3}\angle 3.42\,\Omega}$$

(e) $f = 1\,\mathrm{THz}$

$= 10^{12}\,\mathrm{Hz}$



$$Z_2 = \frac{60 \times j\omega\, 10^{-2}}{60 + j\omega\, 10^{-2}}$$

$$Z_1 = \frac{60 \times \frac{1}{j\omega \cdot 30\times 10^{-3}}}{60 + \frac{1}{j\omega \cdot 30 \times 10^{-3}}}$$

$$\cdot\ Z_1 = \frac{\dfrac{60^2}{j\times 10^{12}\times 30\times 10^{-3}}}{60 + \dfrac{1}{j\,10^{12}\cdot 30\times 10^{-3}}} = \frac{-2\times 10^{-9} j}{60 - \frac{10^{-10}}{3} j} \approx \frac{-2\times 10^{-9}}{60} j$$

$$\approx -\frac{1}{3}\times 10^{-10} \angle 90^\circ$$

$$\approx \frac{10^{-10}}{3} \angle -90^\circ \approx 3.33\times 10^{-11} \angle -90^\circ$$

$$\cdot\ Z_2 = \frac{6\times 10^{11} j}{60 + 10^{10} j} \approx 60\times 10^{-7} \angle 3.43^\circ\ \Omega$$

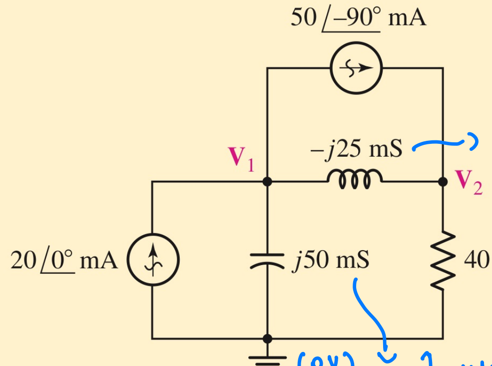
$$Z_{eq.} = 60 \parallel (5.99\times 10^{-6} \angle 3.42^\circ)$$

$$= \frac{60 \times 5.99\times 10^{-6} \angle 3.42^\circ}{60 + 5.99\times 10^{-6} \angle 3.42^\circ}$$

$$Z_{eq} \approx 5.98 \times 10^{-6} \angle 3.41^\circ\ \Omega$$

Q7

10.12 Use nodal analysis on the circuit of Fig. 10.23 to find $\mathbf{V}_1$ and $\mathbf{V}_2$.



$-j25\text{ mS} \rightarrow \frac{1}{-j25}\,\Omega = \frac{j}{25} \times 1000\,\Omega = 40j\,\Omega$

$40\text{ mS} \rightarrow \frac{1}{40} \times 10^{3}\,\Omega = 25\,\Omega$

$j50\text{ mS} \rightarrow \frac{1}{j50} \times 1000\,\Omega = -20j\,\Omega$

(0V)

■ FIGURE 10.23

Sol^n :

nodal at V_1 :

$$-20\angle 0^\circ \times 10^{-3} + \frac{V_1}{-20j} + \frac{V_1 - V_2}{40j} + 50\angle -90^\circ \times 10^{-3} = 0$$

$$\Rightarrow \quad \frac{V_1}{20j} - \frac{V_2}{40j} = 20 \times 10^{-3} + 50j \times 10^{-3}$$

$$\Rightarrow \quad -20 \times 10^{-3} + V_1\left[\frac{-1}{20j} + \frac{1}{40j}\right] - \frac{V_2}{40j} + 50(-j) \times 10^{-3} = 0$$

$$\Rightarrow \quad -20 \times 10^{-3} - 50j \times 10^{-3} = -\left[V_1\left[\frac{-1}{20j} + \frac{1}{40j}\right] - \frac{V_2}{40j}\right]$$

$$\Rightarrow \quad -40j \times 10^{-3}[20 + 50j] = V_1 + V_2 \qquad -(1)$$

nodal at V_2

$$V_2\left(\frac{-j}{40} + \frac{1}{25}\right) - \frac{V_1}{40j} = -50 \times 10^{-3} j$$

after solving,

$$-5V_1 + (5+8j)V_2 = 10 \qquad -(2)$$

from eq^n (1) & (2)

$$\boxed{\begin{aligned} V_1 &= 0.9756 + 0.4195j \\ V_2 &= 1.0244 - 1.2195j \end{aligned}}$$

In polar form

$$\boxed{\begin{aligned} V_1 &= 1.062 \angle 23.27^\circ V \\ V_2 &= 1.593 \angle -50^\circ V \end{aligned}}$$

Q8

10.13 Use mesh analysis on the circuit of Fig. 10.25 to find $\mathbf{I}_1$ and $\mathbf{I}_2$.

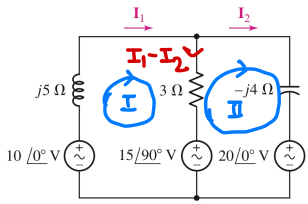


FIGURE 10.25

Solⁿ : KVL I : $-10\angle 0^\circ + I_1(5\angle 90^\circ) + 3(I_1 - I_2) + 15\angle 90^\circ = 0$

$\Rightarrow \quad I_1(3 + 5j) - 3I_2 = 10 - 15j \quad -(1)$

$$I_1(3+5j) - 3I_2 = 10-15j$$

KVL II : $-3(I_1 - I_2) \quad -4j\, I_2 + 20 - 15j = 0$

$\Rightarrow \quad -3I_1 + (3-4j)I_2 = -20+15j \quad -(2)$

By cramer's rule

$$\Delta = \begin{vmatrix} 3+5j & -3 \\ -3 & 3-4j \end{vmatrix}$$

$$= (3+5j)(3-4j) - 9$$

$$\Delta = 20+3j$$

$$\Delta_1 = \begin{vmatrix} 10-15j & -3 \\ -20+15j & 3-4j \end{vmatrix} = (10-15j)(3-4j) + 3(-20+15j)$$

$$= -30-85j - 60 + 45j$$

$$\Delta_1 = -90 - 40j$$

$$\Delta_2 = \begin{vmatrix} 3+5j & 10-15j \\ -3 & -20+15j \end{vmatrix} = -135-55j + 3(10-15j)$$

$$= -105 - 100j$$

$$\therefore \quad I_1 = \frac{\Delta_1}{\Delta} = \frac{-90-40j}{20+3j} = -4.69-1.29j$$

$$\therefore \quad I_1 = 4.87 \angle -164.6° \ A$$

$$\therefore \quad I_2 = \frac{\Delta_2}{\Delta} = \frac{-105-100j}{20+3j} = 7.17 \angle -144.9°$$

$$\therefore \quad I_2 = 7.17 \angle -144.9° \ A$$