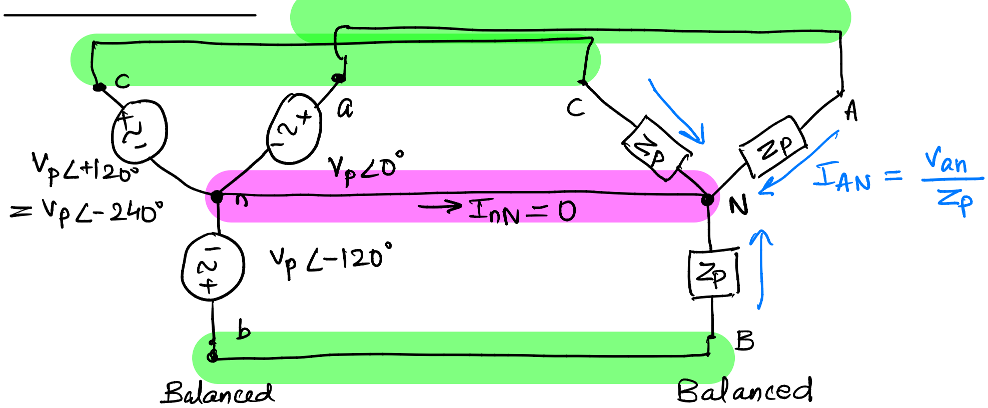
Three phase systems

Y - Y connection

"star - star" connection



Phase Voltages : $V_{AN} = V_{an}$, $V_{BN} = V_{bn}$, $V_{CN} = V_{cn}$

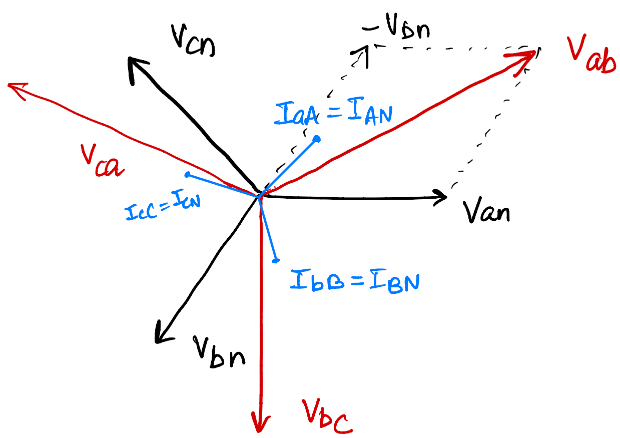
Line Voltages : V_{ab} , V_{bc} , V_{ca}

Line Currents : I_{aA} , I_{bB} , I_{cC}

Phase currents : $I_{AN} = I_{aA}$, $I_{BN} = I_{bB}$, $I_{CN} = I_{cC}$

$$\underbrace{\begin{aligned} V_{an} &= V_p\angle 0^\circ \\ V_{bn} &= V_p\angle{-120^\circ} \\ V_{cn} &= V_p\angle{-240^\circ} \end{aligned}}_{\text{phase voltages}}$$

$$\underbrace{\begin{aligned} V_{ab} &= V_{an} - V_{bn} = \sqrt{3}\,V_p\angle 30^\circ \\ V_{bc} &= V_{bn} - V_{cn} = \sqrt{3}\,V_p\angle{-90^\circ} \\ V_{ca} &= V_{cn} - V_{an} = \sqrt{3}\,V_p\angle{-210} \end{aligned}}_{\text{line voltages}}$$



V_p = magnitude of phase voltages $= |V_{AN}| = |V_{BN}| = |V_{CN}| = V_p$

V_L = magnitude of line voltages $= |V_{ab}| = |V_{bc}| = |V_{ca}| = \sqrt{3}V_p$

$$\boxed{V_L = \sqrt{3}V_p}$$

Line currents = phase currents.

$$I_{aA} = I_{AN} = \frac{V_{an}}{Z_p} = \frac{V_{AN}}{Z_p}$$

$$I_{bB} = I_{BN} = \frac{V_{BN}}{Z_p}$$

$$I_{cC} = I_{CN} = \frac{V_{CN}}{Z_p}$$

$$\boxed{\begin{aligned} Z_p &= \frac{V_{AN}}{I_{AN}} \\ &= \frac{V_p \angle \theta^\circ}{I_p \angle \phi^\circ} \\ &= \frac{V_p}{I_p} \angle \theta - \phi^\circ \end{aligned}}$$

I_p = magnitude of phase currents $= |I_{AN}| = |I_{BN}| = |I_{CN}|$

I_L = magnitude of line currents $= |I_{aA}| = |I_{bB}| = |I_{cC}|$

$$\boxed{I_p = I_L}$$

P_{avg} for any phase $= \frac{1}{2}\text{Re}\{V I^*\}$

$V_{AN} = V_p \angle 0^\circ$

$I_{AN} = I_p \angle \phi^\circ$

$$\begin{aligned} P_{avg, A} &= \frac{1}{2}\text{Re}\{V_{AN} I_{AN}^*\} \\ &= \frac{1}{2}\text{Re}\{V_p \angle \theta^\circ (I_p \angle \phi^\circ)^*\} \\ &= \frac{1}{2} V_p I_p \cos(\theta - \phi) \\ &= \frac{1}{2}\frac{1}{\sqrt{3}} V_L I_L \cos(\theta - \phi) \end{aligned}$$

P_{avg} per phase $= P_{avg}$ A phase $= P_{avg}$ B phase $= P_{avg}$ C phase $= \frac{1}{2} V_p I_p \cos(\theta - \phi')$

$$\boxed{P_{avg} \text{ per phase} = \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi) \text{, if } V_L, I_L \text{ are rms values.}}$$

$$\begin{aligned} P_{avg} \text{ total} &= \frac{3}{2} V_p I_p \cos(\theta - \phi) \\ &= 3 V_p I_p \cos(\theta - \phi) \text{, if } V_p \,\&\, I_p \text{ are rms values.} \end{aligned}$$

$$\boxed{P_{avg} \text{ total} = \sqrt{3} V_L I_L \cos(\theta - \phi) \text{, if } V_L \,\&\, I_L \text{ are rms values.}}$$

Total instantaneous power

$$p_{total}(t) = p_A(t) + p_B(t) + p_C(t)$$

$$p_A(t) = v_{AN}(t)\, i_{AN}(t)$$

$$v_{AN}(t) = V_p \cos(\omega t + \theta)$$

$$i_{AN}(t) = I_p \cos(\omega t + \phi)$$

$$\begin{aligned} p_A(t) &= V_p I_p \cos(\omega t + \phi) \cos(\omega t + \theta) \\ &= \frac{V_p I_p}{2} \Big[\cos\underbrace{(\theta - \phi)}_{\beta} + \cos\underbrace{(2\omega t + \theta + \phi)}_{\alpha} \Big] \\ &= \frac{V_p I_p}{2} \left[\cos\alpha + \cos\beta \right] \end{aligned}$$

$$p_B(t) = v_{BN}(t)\, i_{BN}(t) = \frac{V_p I_p}{2} \left[\cos\beta + \cos(\alpha - 240^\circ) \right]$$

$$p_C(t) = \frac{V_p I_p}{2} \left[\cos\beta + \cos(\alpha + 240^\circ) \right]$$

$$p_{total}(t) = \frac{V_p I_p}{2} \Big[3\cos\beta + \cos\alpha + \underbrace{\cos(\alpha - 240^\circ) + \cos(\alpha + 240^\circ)}_{-\cos\alpha} \Big]$$

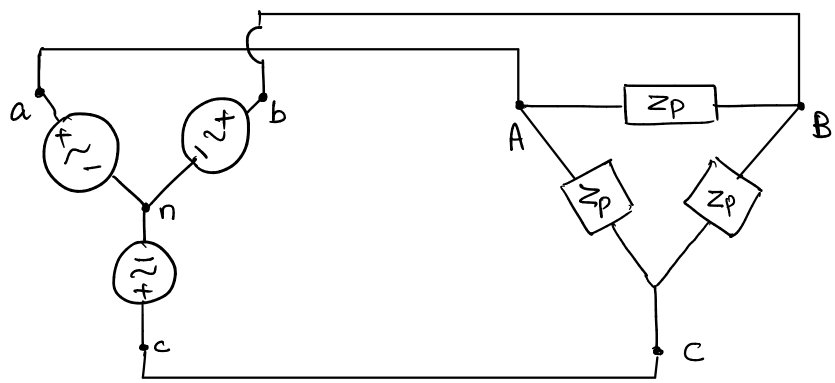
$$p_{total}(t) = \frac{3 V_p I_p}{2} \cos\beta = \frac{3 V_p I_p}{2} \cos(\theta - \phi)$$

$$P_{avg} = \frac{1}{T} \int_0^T p(t)\, dt$$

Y – Δ Connections

Y- connected source

Δ- connected load (no neutral node)



Phase voltages: $V_{AB} = V_{ab}$, $V_{BC} = V_{bc}$, $V_{CA} = V_{ca}$

Line voltages: V_{ab} , V_{bc}, V_{ca} } ← same as phase voltages

$$V_{AB} = V_{ab} = V_{an} - V_{bn} = \sqrt{3}\, V_{an} \angle 30^\circ$$
$$V_{BC} = V_{bc} = \sqrt{3}\, V_{an} \angle -90^\circ$$
$$V_{CA} = V_{ca} = \sqrt{3}\, V_{an} \angle -210^\circ$$

} Line voltages = phase voltages

V_P = magnitude of phase voltage $= |V_{AB}| = |V_{BC}| = |V_{CA}| = \sqrt{3}\,|V_{an}|$

V_L = magnitude of line voltage $= |V_{ab}| = |V_{bc}| = |V_{ca}| = \sqrt{3}\,|V_{an}|$

$$\boxed{V_P = V_L}$$

Line currents: $I_{aA} = I_{AB} - I_{CA} = \sqrt{3}\, I_{AB} \angle -30^\circ$

$$I_{bB} = \sqrt{3}\, I_{BC} \angle -30^\circ$$
$$I_{cC} = \sqrt{3}\, I_{CA} \angle -30^\circ$$

Phase currents: $I_{AB} = V_{AB} / Z_P$

$$I_{BC} = V_{BC} / Z_P$$
$$I_{CA} = V_{CA} / Z_P$$

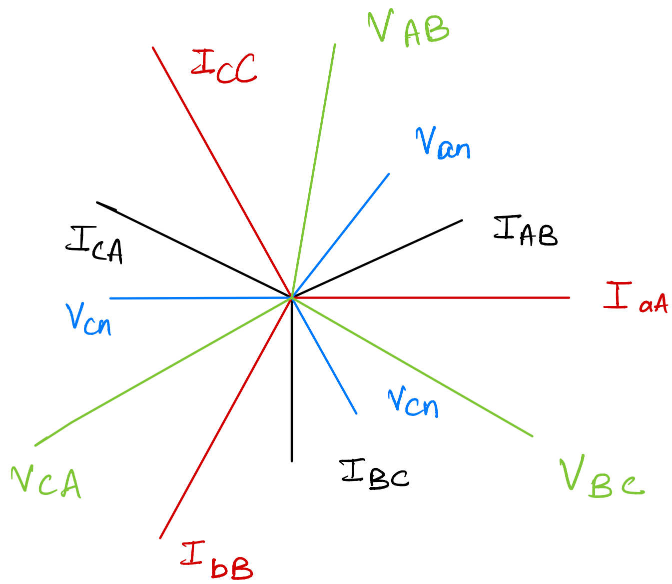
$$Z_P = \frac{V_{AB}}{I_{AB}} = \frac{V_P \angle \theta^\circ}{I_P \angle \phi^\circ} = \frac{V_P}{I_P} \angle \theta - \phi^\circ$$

I_P = magnitude of phase currents $= |I_{AB}| = |I_{BC}| = |I_{CA}|$

I_L = magnitude of line currents $= |I_{aA}| = |I_{bB}| = |I_{cC}|$

$$\Rightarrow I_L = \sqrt{3}\, \underbrace{|I_{AB}|}_{I_P}$$

$$\Rightarrow \boxed{I_L = \sqrt{3}\, I_P}$$



Similar to Y-Y connection:

$$P_{avg} \text{ per phase} = \frac{1}{2} Re\{V_{AB} I_{AB}^*\} = \frac{1}{2} V_p I_p \cos(\theta - \phi)$$

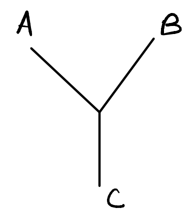
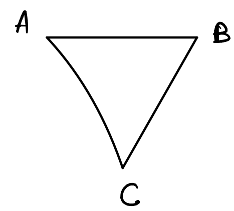
$$\boxed{P_{avg} \text{ per phase} = \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi) \text{, if } V_L, I_L \text{ are rms values.}}$$

$$P_{avg} \text{ total} = \frac{3}{2} V_p I_p \cos(\theta - \phi)$$

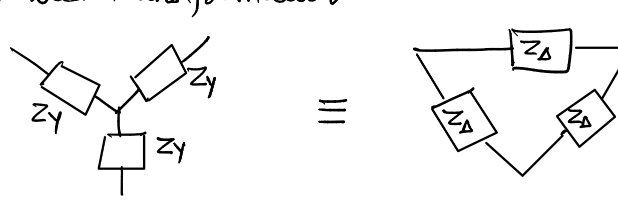
$$= 3 V_p I_p \cos(\theta - \phi) \text{, if } V_p \text{ \& } I_p \text{ are rms values.}$$

$$\boxed{P_{avg} \text{ total} = \sqrt{3} V_L I_L \cos(\theta - \phi) \text{, if } V_L \text{ \& } I_L \text{ are rms values.}}$$

Y-Y connection versus Y-Δ connection

		
Phase voltages Line voltages	V_{AN} , V_{BN} , V_{CN} V_{ab} , V_{bc} , V_{ca}	V_{AB} , V_{BC} , V_{CA} V_{ab} , V_{bc} , V_{ca} same
Phase currents Line currents	I_{AN}, I_{BN}, I_{CN} I_{aA}, I_{bB}, I_{cC} same	I_{AB}, I_{BC}, I_{CA} I_{aA}, I_{bB}, I_{cC}
	$V_L = \sqrt{3}\, V_p$ $I_L = I_p$	$V_L = V_p$ $I_L = \sqrt{3}\, I_p$
Z_p	$Z_p = \frac{V_p}{I_p} \angle \theta - \phi$	$Z_p = \frac{V_p}{I_p} \angle \theta - \phi$
P_{avg} per phase	$\frac{1}{2} V_p I_p \cos(\theta - \phi)$ $= \frac{1}{2} \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi)$	$\frac{1}{2} V_p I_p \cos(\theta - \phi)$ $= \frac{1}{2} \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi)$
P_{avg} total	$\frac{3}{2} V_p I_p \cos(\theta - \phi)$ $= \frac{1}{2} \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi)$	$\frac{3}{2} V_p I_p \cos(\theta - \phi)$ $= \frac{1}{2} \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi)$

Star to delta load transformation :-



$$\boxed{Z_Y = \frac{Z_\Delta}{3}}$$