14/08/24 # Signals And Systems 11:00AM

Evaluation : (relative grading) yoooooooooo :fire emoji:

Attendance	10%
Quiz	16%
Assignments	15%
mid sem	24%
end sem	35%

Book: Signals & System 2nd edition by Alan V. Oppenheimer

Lecture : 1 14/08/24 : 11:00AM

* Linear Algebra Revision

⇒ GROUP : consists of ....

- A set G
- A rule / binary operation "∗"
  - a. associative
    $$x * (y * z) = (x * y) * z \quad \forall x, y, z \in G$$
  - b. There exists an element "e" called the identity of group G such that
    $$e * x = x * e = x \quad \forall x \in G$$
  - c. $\forall x \in G, \exists x^{-1}$ such that
    $$x * x^{-1} = e = x^{-1} * x \quad \forall x \in G$$
  - d. if $x * y = y * x \quad \forall x, y \in G$, this group is called

Commutative / Abelian Group

Note: we only need (1), (2.a), (2.b), (2.c), for a set to be a group.

eg: A set of invertible $n \times n$ matrices with binary operation = matrix multiplicati
But it is not an Abelian Group

eg: set of continuous time periodic signals with time period T with "∗" = "+"

⇒ FIELD : consists of the following

- A set F
- Two binary operations "+" and "·" such that ....
  - $(F, +)$ is an abelian group
  - define $F^* = F - \{0\} \Rightarrow (F^*, \cdot)$ is an abelian group
  - multiplication operation distributes over addition
    - left distributive
      $$x \cdot (y + z) = xy + xz \quad \forall x, y, z \in F$$
    - Right distributive
      $$(x + y) \cdot z = xz + yz \quad \forall x, y, z \in F$$

eg: F = Real Numbers $\mathbb{R}$

* Vector Space : A set V with a map...

- '+' : $V \times V \to V$
  $(v_1, v_2) \to (v_1 + v_2)$ called vector addition
- '·' : $F \times V \to V$
  $(a, v_1) \to (a \times v_1)$ called scalar multiplication

... is called a F-vector space or vector space over the field F if the following are satisfied:

- $(V, +)$ is an abelian group
- $a \cdot (v_1 + v_2) = av_1 + av_2$
- $(a_1 + a_2) \cdot v = a_1 v + a_2 v$
- $(\alpha\beta) v = \alpha(\beta v) = \beta(\alpha v)$
- $1 \cdot v = v$
- $a \cdot 0 = 0$
- $0 \cdot v = 0$
- if $v \neq 0$, then $a \cdot v = 0$ implies $a = 0$
- if V is a vector space over field F, then any linear combination of vectors lying in V (with scalars from F) would again lie in V

* METRIC SPACE : metric is a map

$$d : X \times X \to R$$

satisfies the following: $(\forall x, y, z \in X)$

- $d(x, y) \geq 0$ and $d(x, y) = 0$ if $x = y$
- $d(x, y) = d(y, x)$
- $d(x, y) \leq d(x, z) + d(z, y)$

This map is called a metric and a set equipped with this map is called a metric space and is denoted by (X, d)

eg: Euclidean distance

Note: metric is merely a generalization of the notion of distance

* NORM : let V be a F-vector space.

A map $\|\cdot\| : V \to R$ is called a norm if it satisfies ...

- $\|\bar{v}\| \geq 0$ and $\|\bar{v}\| = 0$ if $\bar{v} = 0$
- $\|a\bar{v}\| = |a| \, \|v\|$
- $\|v_1 + v_2\| \leq \|v_1\| + \|v_2\|$

A vector space equipped with a norm is called a normed vector space

eg: let V be a F-vector space with a nor

prove that $d(v_1, v_2) = \|v_1 - v_2\|$ is a proper metric

Lecture: 2

16/08/24 : 9:30Am

* Inner Product:

let V be a F-vector space
A map,

$$\langle , \rangle : V \times V \to F$$

is called an inner product if ...

- $\langle v,v \rangle \geq 0$ and $\langle v,v \rangle = 0$ iff $v=0$
- $\langle v_1,v_2 \rangle = \overline{\langle v_2,v_1 \rangle}$ " $-$ " : complex conjugate operation
- it is linear in the first co-ordinate
  $$\langle a_1v_1 + a_2v_2, w \rangle = a_1\langle v_1,w \rangle + a_2\langle v_2,w \rangle$$
  $\forall\ v_1,v_2,w \in V$ and $a_1,a_2 \in F$
- it is conjugate linear in the second co-ordinate
  $$\langle v, a_1w_1 + a_2w_2 \rangle = \bar{a}_1\langle v,w_1 \rangle + \bar{a}_2\langle v,w_2 \rangle$$
  $\forall\ w_1,w_2,v \in V$ and $a_1,a_2 \in F$

eg: dot product → measures cosine similarity " norm(x)·norm(y)·$\cos\theta$

$$\langle x,y \rangle = x_1y_1 + x_2y_2 + \dots x_ny_n$$

complex inner Product

$$\langle x,y \rangle = \sum_{i=1}^{n} x_i \cdot \bar{y}_i$$ → complex conjugate

* Linear Independence of Vectors

A set of vectors $v_1, v_2, \dots v_n$ in $V_n(F)$ is called linearly independent (LI) if

$$\sum_{i=1}^{n} a_iv_i = 0 \quad \text{implies } a_i = 0\ \forall i$$

Note: In a 2d space, we represent all non-colinear vectors using 2 vectors

Definition: The number of maximal LI vectors in a vector space V is called the DIMENSION of the vector space and the maximal LI vectors is called a BASIS for V.

if $\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$ is a basis for $V_n(F)$, then $\forall\ v_i \in V$ & $a_i \in F$ → weighted linear combination of vector V

$$V = \sum_{i=1}^{n} a_ie_i$$

$V = a_1e_1 + a_2e_2 + \dots a_ne_n$

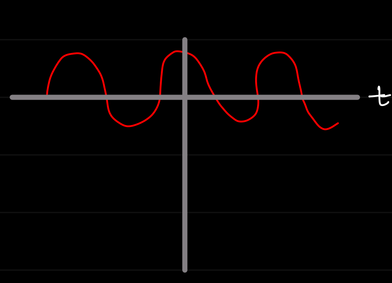
* ORTHOGONAL & ORTHONORMAL Basis

A set of basis vectors $(v_1, v_2 \dots v_n)$ spanning an inner product space V if:

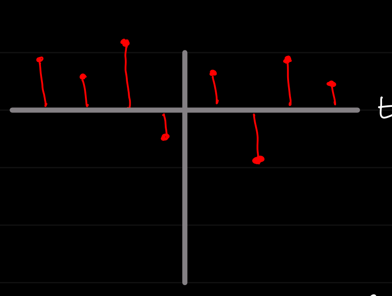
$$v_i \neq 0 \quad \forall\ i$$
$$\langle v_i,v_j \rangle \neq 0 \quad \forall\ i\ \&\ j$$

SIGNALS (CTS & DTS)

- Continuous time signal



- Discrete time signal



- Even Signal : symmetric about vertical axis
  $$x(t) = x(-t)\ \forall\ t$$
  eg: cosine function
- Odd Signal : non-symmetric about vertical axis
  $$x(-t) = -x(t)\ \forall\ t$$
  eg: sine function

signal notation: $x(t)$ continuous time signal
$x[n]$ discrete time signal

images are two dimensional signal
videos are 3d
videos with audio : 4d

* Even/odd component of a signal:

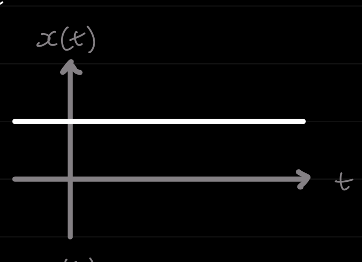
$$\text{even}\{x(t)\} = \frac{x(t)+x(-t)}{2}$$

$$\text{odd}\{x(t)\} = \frac{x(t)-x(-t)}{2}$$

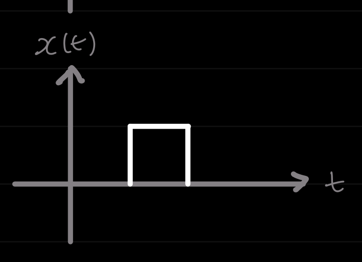
* PERIODIC: if $x(t) = x(t+T)\ \forall\ t$ then T is the time period

The smallest time period T defined for this property is called the fundamental time period

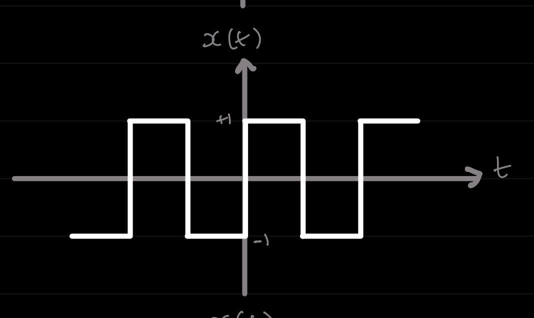
eg:



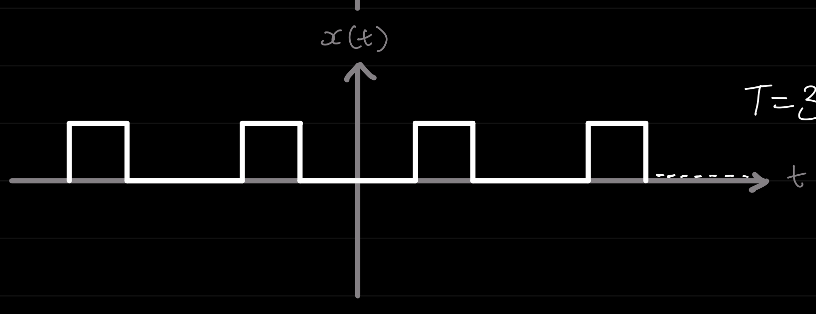
Not periodic



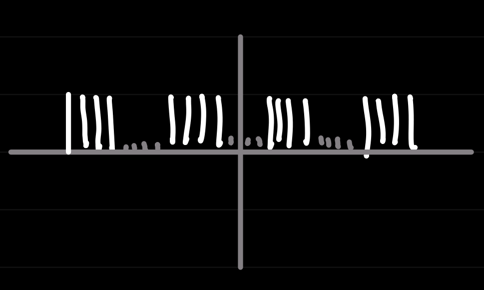
$T \to \infty$
Aperiodic



Periodic with $T=2$



$T=3$



$T=8$

classification { Probabilistic
Dataministic

* ENERGY OF SIGNAL

Continuous Time signal:

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

Discrete Time signal

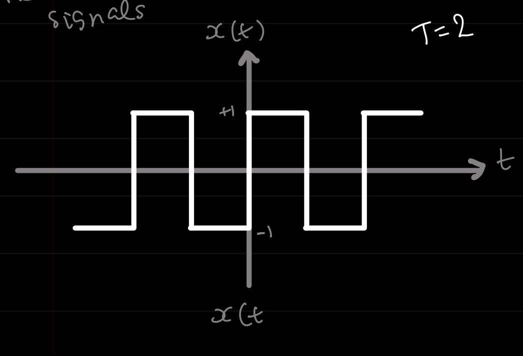
$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Note: Periodic signals are power signals ✓
Aperiodic signals are not power signals ✗

$T=\infty$ → so, power = not defined → hence, Aperiodic signals are not power signals

avg. energy in " a time duration

$$\text{Power} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\,dt$$



compute power?

$$P = \lim_{T\to\infty} \frac{1}{2T}\int$$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2$$

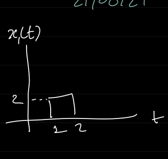
$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{x=-N}^{N} |x[n]|^2$$

* Lecture : 3 21/08/24

$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$= \int_{2}^{2} |2|^2 dt = 4\int_0^2 dt$

$= 4 < \infty$



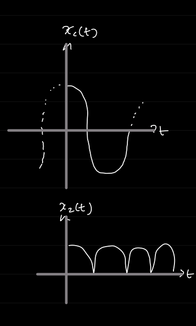
↑ energy signal

$x_1(t) \equiv$ energy signal, continuous signal, aperiodic, deterministic

$x_1(t) \neq x_1(t+T) \Rightarrow$ aperiodic signal

$x_2(t) =$ continuous signal, periodic signal, ← deterministic power signal

we know its value at every point ← not probabilistic



$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

after mod

$E = \lim_{T_0 \to \infty} \underbrace{\int_{-T_0}^{T_0} |x(t)|^2 dt}_{\infty}$

Since $E \to \infty$ and not $E < \infty$, this signal is not an energy signal

$P = \lim_{T_0 \to \infty} \frac{1}{2T_0} \int_{-T_0}^{T_0} |x(t)|^2 dt$

intuition $\left\{ \approx \frac{\text{infinite block}}{\text{infinit block}} = \text{finite} \right.$

$\boxed{0 < P < \infty} \Rightarrow$ Power Signal ✓

* TRANSFORMATION OF SIGNAL

① Translation

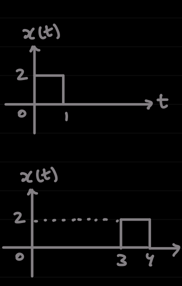
energy of signal doesn't change

(a) DELAY

$y(t) = x(t - t_0)$

let $t_0 = 3$

$y(t) = x(t-3)$



$y(0) = x(-3) = 0$
$y(3) = x(0) = 2$
$y(4) = x(1) = 2$
$y(5) = x(2) = 0$

} verification of transformation

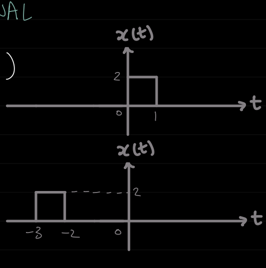
(b) ADVANCE A SIGNAL

$y(t) = x(t + t_0)$

let $t_0 = 3$

$y(-3) = x(0) = 2$
$y(-2) = x(1) = 2$
$y(-1) = x(2) = 0$

verified

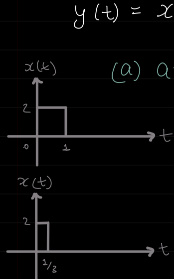


energy of scale changes in accordance with area under curve

② Scaling

$y(t) = x(at)$

(a) $a > 1$: compression of the signal on the t-axis

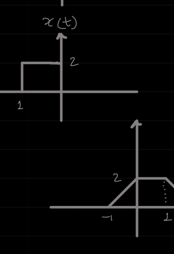


let $a = 3$

$y(t) = x(3t)$

$y(0) = x(0) = 2$
$y(\frac{1}{3}) = x(1) = 2$
$y(\frac{2}{3}) = x(2) = 0$

(b) $0 < a < 1$: Dilation on t-axis

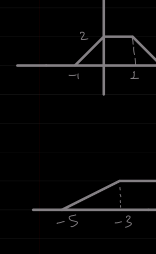


let $a = \frac{1}{3}$

$y(t) = x(at) = x(t/3)$

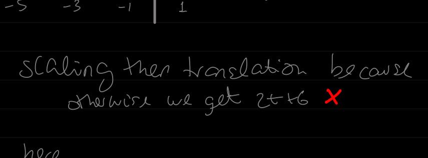
$y(0) = x(0) = 2$
$y(3) = x(1) = 2$
$y(6) = x(2) = 0$

(c) SPECIAL CASE: $a = -1$

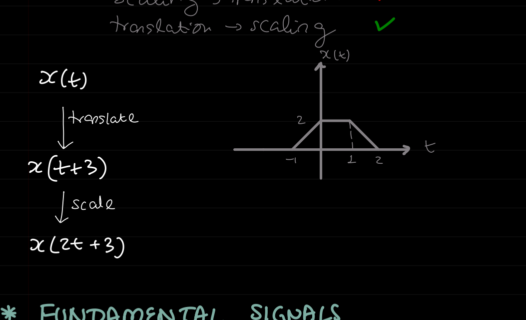


$y(t) = x(-t)$

flipped on the t-axis ≡ time reversal



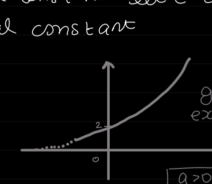
eg} $y(t) = x(2t+3)$ translation + scaling



scaling then translation because otherwise we get $2t+6$ ✗

here,
scaling → translation ✗
translation → scaling ✓

$x(t)$
↓ translate
$x(t+3)$
↓ scale
$x(2t+3)$



* FUNDAMENTAL SIGNALS

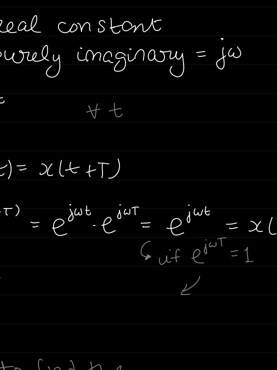
- Continuous Time Signals

$x(t) = ce^{at}$

- CASE: 1 c = real constant let $c = 2$
  a = real constant

$x(t) = 2e^{at}$

① let $a = 1$



growing exponential

$\boxed{a>0}$ $\boxed{a<0}$

② let $a = -1$

③ let $a = 0$

- CASE: 2 c ≡ real constant
  a ≡ purely imaginary $= j\omega$

$x(t) = e^{j\omega t}$ $\forall t$

periodic if $x(t) = x(t+T)$

$x(t+T) = e^{j\omega(t+T)} = e^{j\omega t} \cdot e^{j\omega T} = e^{j\omega t} = x(t)$
if $e^{j\omega T} = 1$

$e^{j\omega T} = 1$ if
$\omega T = 2\pi k$

$e^{j2\pi k}$

since we need to find the fundamental time period (smallest interval), let $K = 1$

So, $\omega T = 2\pi \Rightarrow \boxed{T = \frac{2\pi}{\omega}}$ fundamental time period

$E = \int_{-\infty}^{\infty} |e^{j\omega t}|^2 dt = \int_{-\infty}^{\infty} 1 \, dt = \infty$

- CASE: 3 C ≡ complex number
  a ≡ real number

$x(t) = |c| e^{j\theta} \cdot e^{at} = |c| e^{at + j\theta}$

e^{at} will define if it is growing / decaying exponential

we can also say, $|c|e^{j\theta} = c\cos(\theta) + c \cdot j\sin(\theta)$

$Re\{x(t)\} = |c| \cdot \cos\theta \cdot e^{at}$

$Im\{x(t)\} = |c| \cdot \sin\theta \cdot e^{at}$

- CASE: 4 C ≡ complex number $= |c|e^{j\theta}$
  a ≡ complex number $= r + j\omega$

$x(t) = ce^{at} = |c|e^{j\theta} \cdot e^{(r+j\omega)t}$

$= \underbrace{|c|e^{rt}}_{\text{envelope with either increasing or decaying exponential}} \cdot \underbrace{e^{j(\omega t+\theta)}}_{\cos(\omega t+\theta) + j\sin(\omega t+\theta)}$

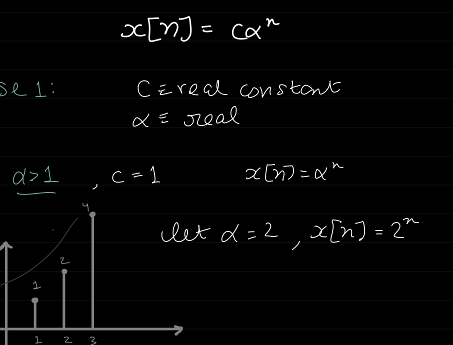
cosinosoidal signal, sinusoidal signal

$Re\{x(t)\} = |c|e^{rt} \cdot \cos(\omega t + \theta)$

$Im\{x(t)\} = |c|e^{rt} \cdot \sin(\omega t + \theta)$

let $r = 2$, $|c| = 1.5$

$Re\{x(t)\} = 1.5e^{2t} \cos(\omega t + \theta)$



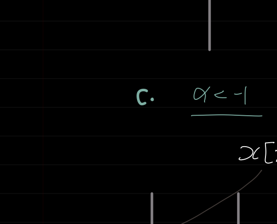
- Discrete Time Signals

$x[n] = c\alpha^n$

- Case 1: C ≡ real constant
  α ≡ real

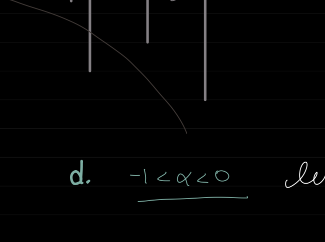
a. $\alpha > 1$, $c = 1$ $x[n] = \alpha^n$

let $\alpha = 2$, $x[n] = 2^n$



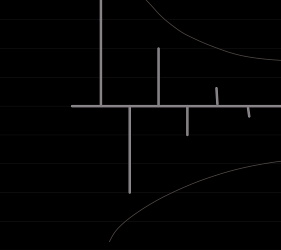
b. $\alpha < 1$ let $\alpha = 0.5$

$x[n] = (0.5)^n = (\frac{1}{2})^n$



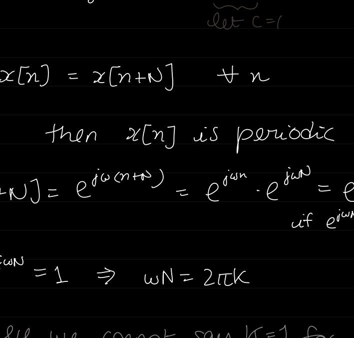
c. $\alpha < -1$ let $\alpha = -2$

$x[n] = (-2)^n$



d. $-1 < \alpha < 0$ let $\alpha = \frac{-1}{2}$

$x[n] = (\frac{-1}{2})^n$



- Case : 2 C ≡ real
  $\alpha = e^{j\omega}$

$x[n] = c\alpha^n = \underbrace{e^{j\omega n}}_{\text{let } C=1}$

if $x[n] = x[n+N]$ $\forall n$

then $x[n]$ is periodic

$x[n+N] = e^{j\omega(n+N)} = e^{j\omega n} \cdot e^{j\omega N} = e^{j\omega n}$
if $e^{j\omega N} = 1$

$e^{j\omega N} = 1 \Rightarrow \omega N = 2\pi K$

here we cannot say $K = 1$ for smallest interval necessarily because we want N to be an integer

$N = \frac{2\pi k}{\omega}$ ≡ has to be integer

$\frac{N}{K} = \frac{2\pi}{\omega}$ ≡ has to be rational number

Note: ω has to be a multiple of π

* Lecture: 4

28/08/24

- for continuous time signals
  → frequency is unique $(0 \to \infty)$

- for discrete time signal
  → frequency $\in [0, 2\pi]$ and then loops

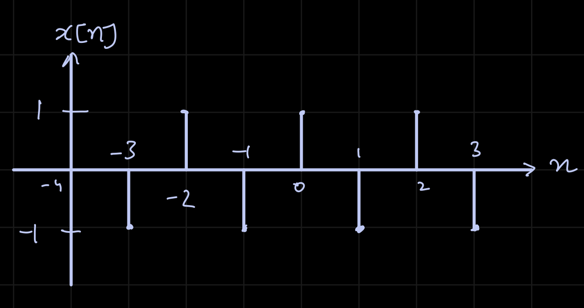
DTS

$$x[n] = e^{j\omega_0 n}$$

let $\omega_0 = 0 \Rightarrow e^{j0} = 1\angle 0 = \cos 0 + j\sin 0 = 1$

let $\omega_0 = 2\pi \Rightarrow e^{j\cdot 2\pi n} = \cos(2\pi n) + j\sin(2\pi n) = 1$

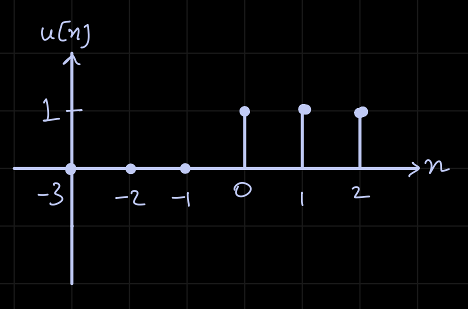
let $\omega_0 = \pi \Rightarrow e^{j\cdot \pi n} = \cos(\pi n) + j\sin(\pi n) = (-1)^n$



* Discrete Time Signals

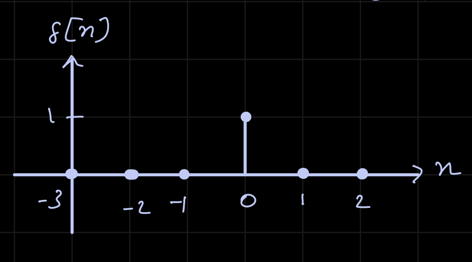
unit step signal $u[n]$

$$u[n] = \begin{cases} 1 : n \geq 0 \\ 0 : \text{else} \end{cases}$$



unit impulse function $\delta[n]$

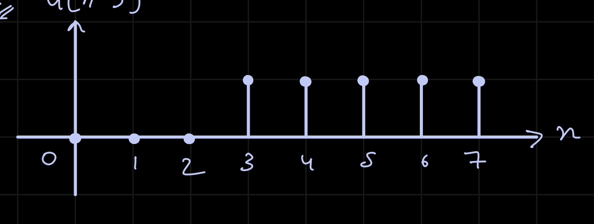
$$\delta[n] = \begin{cases} 1 : n = 0 \\ 0 : \text{else} \end{cases}$$



note:

$1 : n-3 \geq 0$
$n \geq 3$

$u[n-3]$



conversion btw u[] and δ[]

$$\boxed{\begin{aligned} \delta[n] &= u[n] - u[n-1] \\ u[n] &= \sum_{k=-\infty}^{n} \delta[k] \end{aligned}}$$

note: $u[n] = 0 \quad \forall\, n < 0$

so,
$$u[-2] = \sum_{k=-\infty}^{-2} \delta[k] = 0$$
$$u[-1] = \sum_{k=-\infty}^{-1} \delta[k] = 0$$
$$u[0] = \sum_{k=-\infty}^{0} \delta[k] = 1$$
$$u[1] = \sum_{k=-\infty}^{1} \delta[k] = 1 \quad \to 2?$$

- $$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

$\Rightarrow$ for $n = -2$, $\delta[\,]$ will be 1 only when $n - k = 0$ i.e. $k = -2$
but k is never negative since $k \in [0, \infty)$

hence $u[-2] = 0$

$\Rightarrow$ for $n = 0$, $n - k = 0$ to get a 1 from δ fn
$k = 0$ ✓
$k \in [0, \infty)$

$\Rightarrow$ for $n = 1$, $n - k = 0$ to get $\delta(0) = 1$
$k = 1$ ✓

$$u[1] = \sum_{k=0}^{\infty} \delta[1-k] = \underset{0}{\delta[1]} + \underset{1}{\delta[0]} + \underset{0}{\delta[-1]} + \dots$$
$$= \underline{1} \checkmark$$

- **SIFTING / SAMPLING**

$x[n] \cdot \delta[n] = ?$ $\quad x[n]\delta[n] = \begin{cases} x[0] & : n = 0 \\ 0 & : \text{otherwise} \end{cases}$

we cannot directly say $x[n]\delta[n] = x[0]$ since LHS: function and RHS: scalar

we can also say $x[n]\delta[n] = x[0]\delta[n]$

↳ gives only the value of $x[n]$ at $n = 0$
$\equiv x[0]$

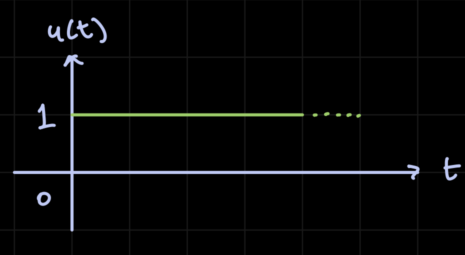
eg) $x[n] \cdot \delta[n - n_0] = ?$ $\quad x[n] \cdot \delta[n - n_0] = \begin{cases} x[n_0] & : n = n_0 \\ 0 & : \text{otherwise} \end{cases}$

↳ gives just the value of $x[n]$ at $n = n_0$
$\equiv x[n_0]$

* Continuous Time Signal

- unit step function

$$u(t) = \begin{cases} 1 & : t \geq 0 \\ 0 & : \text{else} \end{cases}$$



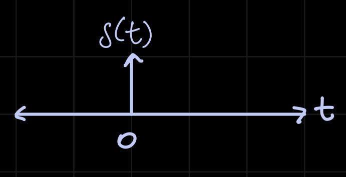
<u>OR</u>

$$u(t) = \begin{cases} 1 & : t > 0 \\ 1/2 & : t = 0 \\ 0 & : \text{else} \end{cases}$$

- Unit Impulse function $\equiv$ singularity function

$$\delta(t) = 0 \quad : t \neq 0$$

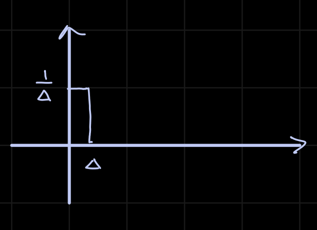
and $$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

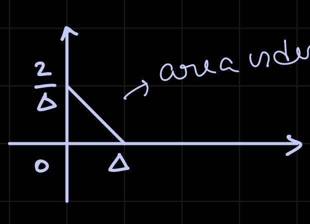


$\delta_\Delta(t)$:

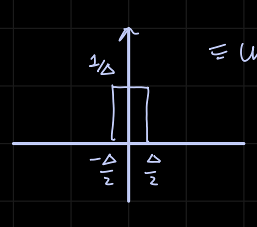
infinitesimally small

$$\delta_\Delta(t) = \begin{cases} 0 : t \geq \Delta \ \& \ t < 0 \\ \frac{1}{\Delta} : \text{otherwise} \end{cases}$$





area under the curve $= \frac{1}{2} \times \Delta \times \frac{2}{\Delta} = 1$
$\equiv$ unit imp use func ✓



$\equiv$ unit imp. ✓

$$\boxed{\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau = \int_{0}^{\infty} \delta(t - \tau)\, d\tau$$

$$\boxed{\delta_\Delta(t) = \frac{d}{dt} u_\Delta(t)}$$

* Tutorial : 1 30/08/24

(7) $\text{even}\{x[t]\} = \dfrac{x(t)+x(-t)}{2}$

$\text{odd}\{x[t]\} = \dfrac{x(t)-x(-t)}{2}$

8) (a) $x_1[n] = \sin(0.02\pi n)$

$\omega = 0.02\pi$

$T = \dfrac{2\pi}{\omega} =$

(3) $p(x,y) = a^2x_1y_1 + abx_1y_2 + b^2x_2y_2 + ab\,x_2y_1$

$$A = \begin{vmatrix} a^2 & ab \\ ab & b^2 \end{vmatrix}$$

$|A| = a^2b^2 - a^2b^2 = 0$

atleast one eigenvalue is zero

So, since not all eigenvalues are +ve

(d) for no values of a, b

eigenvalues are values of λ such that

$|A - \lambda I| = 0$

eigenvalues / eigenvectors

Q1) $B = \{V_1, V_2, \dots V_n\}$

B is span of V

$\bar{x} \in V$

$\bar{x} = K_1V_1 + K_2V_2 + \dots K_nV_n$

$- \quad \bar{x} = C_1V_1 + C_2V_2 + \dots C_nV_n$

$0 = (K_1 - C_1)V_1 + (K_2 - C_2)V_2 + \dots (K_n - C_n)V_n$

since $\bar{V_i} \neq 0$, $(K_i - C_i)$ need to be 0

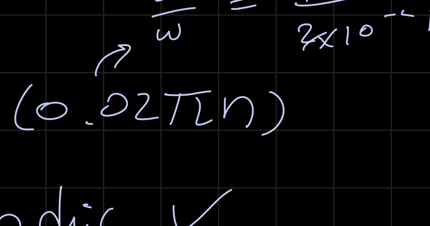
i.e. $K_i = C_i$

$\dfrac{2\pi}{\omega} = \dfrac{2\pi}{2\times10^{-2}\pi} = 100 \Leftarrow T$

(8) $x_1[n] = \sin(0.02\pi n)$

Periodic ✓

(11) $y[n] = \sum_{n=-5}^{5} \sin(2n)\,\delta[n+7]$

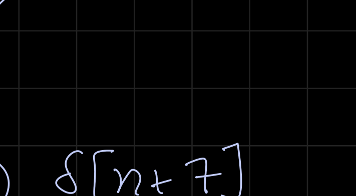


but range is $-5 \to 5$, so ans = 0

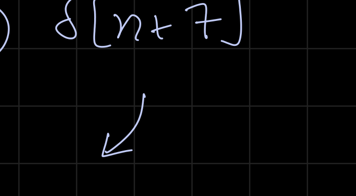
(12) $x(t) = u(t-2) - u(t-4)$

$u(t) \Rightarrow t \geq 0 : 1$

$u(t-2) \Rightarrow t \geq 2 : 1$



$u(t-4) \Rightarrow t \geq 4 : 1$



$\Rightarrow$

but since $\delta(t) = 1$ at $t = 0$

So, ans = 0

rational / irrational ?

$f = \dfrac{0.3}{2\pi}$

(8) (b) $x_2[n] = \cos\left(\dfrac{\pi}{2} + 0.3n\right)$

$\sin = \cos\left(\dfrac{\pi}{2} - \theta\right)$

(c) $x_3[n] = 1 - e^{j2\pi n/5} + e^{j4\pi n/7}$

note: $e^{j\theta} = \cos\theta + j\sin\theta$

$\Rightarrow 1 - \cos\left(\dfrac{2\pi n}{5}\right) - j\sin\left(\dfrac{2\pi n}{5}\right)$

$+ \cos\left(\dfrac{4\pi n}{7}\right) + j\sin\left(\dfrac{4\pi n}{7}\right)$

5 5 7/2 7/2

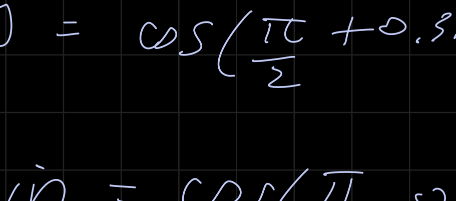
$\Rightarrow \text{LCM}(5, \tfrac{7}{2}) \Rightarrow$

(a) $\dfrac{x(t) + x(-t)}{2} = 0$

$x(t) = -x(-t)$

$\forall t$

(b) $e^{-5t}u(t+2)$



$e^{-5t}u(t+2) = -[e^{5t}u(-t+2)]$

$(e^{-5t})_{(-2,\infty)} + (e^{5t})_{(-\infty,2)}$

$(e^{-5t})_{(-2,2)} + (e^{5t})_{(-2,2)}$

(5) $P = tu(t)$

$\lim_{T\to\infty} \dfrac{1}{2T}\int_{-T}^{T} |t\,u(t)|^2\,dt$

$\lim_{T\to\infty} \dfrac{1}{2T}\int_0^T t^2\,dt$

$\lim_{T\to\infty} \dfrac{1}{2T}\left[\dfrac{t^3}{3}\right]_0^T$

$\lim_{T\to\infty} \dfrac{1}{2T} \times \dfrac{T^3}{3} \Rightarrow \dfrac{T^2}{6}$

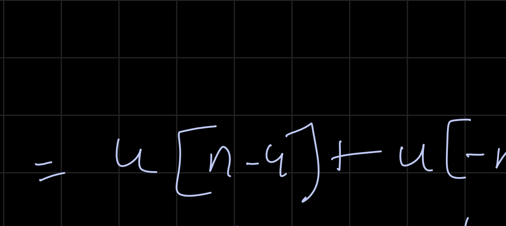
$\to \infty$

neither energy nor power signal

(9) (c) $\dfrac{u[n] - u[n-4] + u[-n] - u[-n-4]}{2} = 0$

$u[n] + u[-n] = u[n-4] + u[-n-4]$

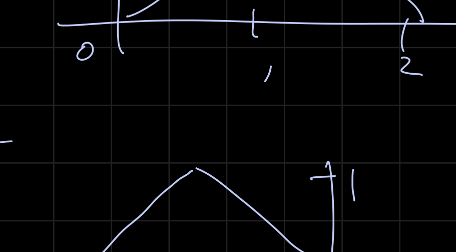
$1 : n \geq 0$ $\quad 1 : n \leq 0$ $\quad n \geq 4$ $\quad n \leq 4$



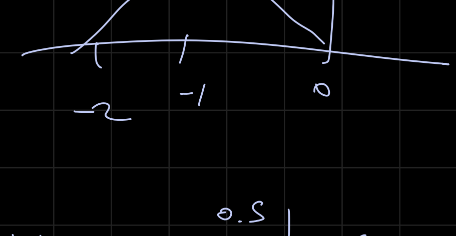
$-3 < n < 3$

$\boxed{|n| < 3}$ ✓

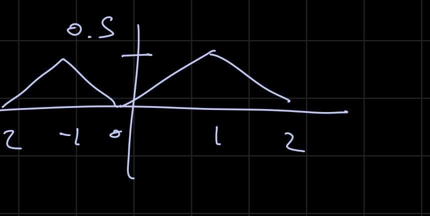
(7) $x(t) =$



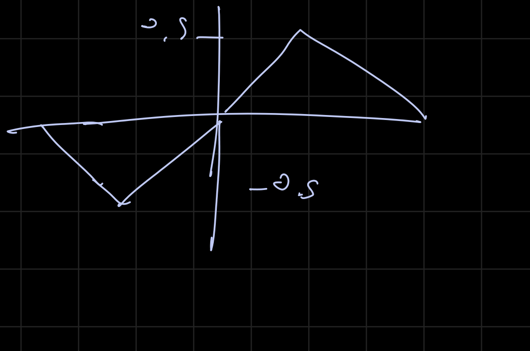
$x(-t) =$



$\text{even} = \dfrac{x(t) + x(-t)}{2} =$



odd =



* Lecture: 5

⇒ Systems and their properties

$\underset{x(t)}{\xrightarrow{\text{input}}} \boxed{\text{system}} \underset{y(t)}{\xrightarrow{\text{output}}}$

① System: with memory or without memory

$x[n] \rightarrow y[n]$

$S_1: \; y(t) = x(t)$ no memory
$S_2: \; y(t) = x^2(t)$ no memory
$S_3: \; y(t) = x(t-2)$ memory ✓ requires past value
$S_4: \; y(t) = \frac{1}{2}\{x(t) + x(t-1)\}$ mem ✓

output depends on just present inputs → no memory required

output needs to store previous outputs → requires memory

$S_5: \; y(t) = x(t+2)$: requires memory

• Invertible / Non-invertible Systems

$\underset{x(t)}{\longrightarrow} \boxed{S_1} \underset{w(t)}{\longrightarrow} \boxed{S_2} \underset{y(t)}{\longrightarrow}$

$y(t) = x(t)$ iff $S_2 = \text{inverse}(S_1)$

ⓐ let $S_1: \; w(t) = 2x(t)$
$S_2: \; y(t) = \frac{1}{2} w(t) = \frac{1}{2} \times 2x(t) = x(t)$

S_1 is invertible and S_2 is the inverse system

ⓑ let $S_1: \; w[n] = \sum_{k=-\infty}^{n} x[n]$

then $S_2: \; y[n] = w[n] - w[n-1]$ invertible ✓

$= \{x[-\infty] + \dots x[n-1] + x[n]\} - \{x[-\infty] + \dots x[n-1]\}$

$= x[n]$

ⓒ let $S_1: \; w[n] = x[n] - x[n-1]$

$S_2: \; y[n] = \sum_{k=-\infty}^{n} w[k] = x[n]$ invertible ✓

$\cancel{x[-\infty]} - x[-\infty-1] + \dots \cancel{x[1]} - \cancel{x[2]} + \cancel{x[2]} - x[3] + x[3] - x[4] + \dots - x[n-1] + x[n-1] - x[n]$

ⓔ $S_1: \; w(t) = x^2(t)$
$S_2: \; y(t) = \sqrt{w(t)}$ } S_1 is not invertible

* System is invertible if and only if distinct inputs lead to distinct outputs

(very casual)

• Causal Systems

$S_1: \; y(t) = x(t - t_0)$: t_0 is a +ve real number

$S_2: \; y[n] = x[n+3]$

System output is depending on past system output

$S_1 \equiv$ causal system
$S_2 \equiv$ non-causal system

Causality: unless there is no cause, there should be no effect.

OR unless there is a non-zero input, there should not be non-zero output

OR a causal system depends on current and past input

eg) Causal system: $y[n] = \sum_{k=-\infty}^{n} x[k]$

image processing: $S_4: \; y[n,m] = \frac{1}{5}\{x[n,m] + x[n-1,m] + x[n+1,m] + x[n,m-1] + x[n,m+1]\}$

non-causal system

• Stability System

A bounded input $\equiv |x(t)| < M_x < \infty$ (finite value $\forall t$)

not on the duration (y input) → but on the amplitude (x axis) ↓

OR $|x[n]| < M_x < \infty$

This bounded input is fed to a system. If the system generates an output which is also bounded, we call it a stable system.

eg) $x(t) = u(t)$; $x[n] = u[n]$
$[0,1]$

unbounded: $x(t) = tu(t)$ OR $x[n] = tu[n]$

if $|y(t)| < M_y < \infty \; \forall t$, for bounded input,
(M_y ↳ finite number)

then the system is stable

eg) $S_1: \; y[n] = \frac{1}{3}\{x[n] - x[n-1] - x[n+1]\}$

$|y[n]| = |\frac{1}{3}x[n] - \frac{1}{3}x[n-1] - \frac{1}{3}x[n+1]|$

since $|x[n]| < M_x < \infty \; \forall n$

and the sum & difference of 3 finite terms is finite, so,

$|y[n]| < M_y < \infty \quad \forall n$

$S_3: \; y[n] = nx[n]$

$|x[n]| < M_x < \infty \; \forall n$
bounded input assumed

$\Rightarrow |y[n]| = |nx[n]| \to \infty$ as $n \to \infty$

unstable system

$S_4: \; y(t) = x(2t)$

no manipulation on the y axis
2t means manipulation on the x-axis

• Time Invariant System

if $x(t) \xrightarrow{\text{system}} y(t)$

then $x(t - t_0) \rightarrow y(t - t_0)$

and $x(t + t_0) \rightarrow y(t + t_0)$

system output does not change if there is a change in time

if $x_1(t) \rightarrow y_1(t)$
and $x_2(t) \rightarrow y_2(t)$
and $x_2(t) = x_1(t - t_0)$

then $y_2(t)$ must be equal to $y_1(t - t_0)$

if it is a time invariant system

eg) $S: \; y(t) = x(2t)$

⇒ let $t_0 = 4$: $y(t - t_0) = x(2t - 8)$

⇒ time variant system

$x_1(t) \rightarrow y_1(t) = x_1(2t)$
$x_2(t) \rightarrow y_2(t) = x_2(2t)$

$y_1(t - t_0) = x_1(2t - 2t_0)$
first scaling : $x_1(2t)$ ($t \to t - t_0$)
then delay : $x_1(2(t - t_0))$

$y_2(t - t_0) = x(2t - t_0)$
first delay : $x(t - t_0)$
then scaling : $x(2t - t_0)$

$S_2: \; y[n] = nx[n]$

$x_1[n] \rightarrow y_1[n] = nx_1[n]$
↓ delayed output
$y_1[n - n_0] = (n - n_0)x[n - n_0]$

$x_2[n] \rightarrow y_2[n] = nx_2[n]$

let $x_2[n] = x_1[n - n_0]$
be delayed input

$y_2[n] = (n)x[n - n_0]$

since $y_1[n] \neq y_2[n]$, system is time varying

note: 1st system had delayed output and we gave delayed input

• Linearity

A system is linear if the properties of additivity & homogeneity holds true

$x_1(t) \rightarrow y_1(t)$
$x_2(t) \rightarrow y_2(t)$

⇒ $ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)$
then system is linear

eg) $y(t) = 2x(t) + 3$

$x_1(t) \rightarrow y_1(t) = 2x_1(t) + 3$

$x_2(t) \rightarrow y_2(t) = 2x_2(t) + 3$

$x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t) = 2(x_1 + x_2) + 2(3)$

but $y(x_1 + x_2) = 2(x_1 + x_2) + 3$

Since, $y_1(t) + y_2(t) \neq y(x_1 + x_2)$

not a linear system

• A system which is linear and time invariant $\equiv$ LTI
↓
linear & time invariant system