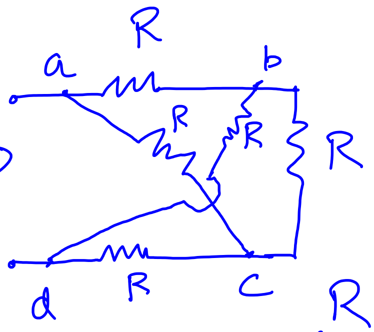
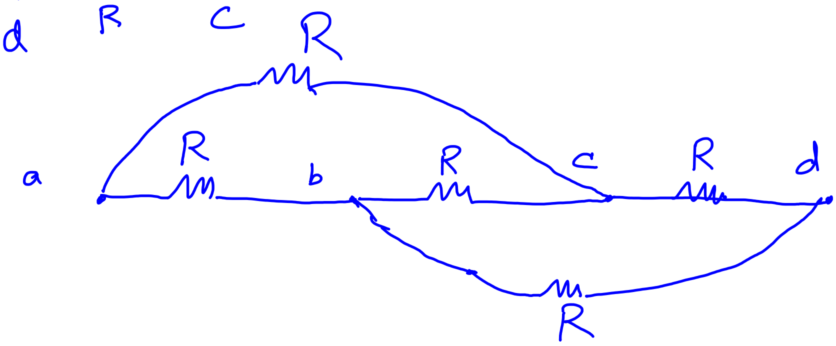
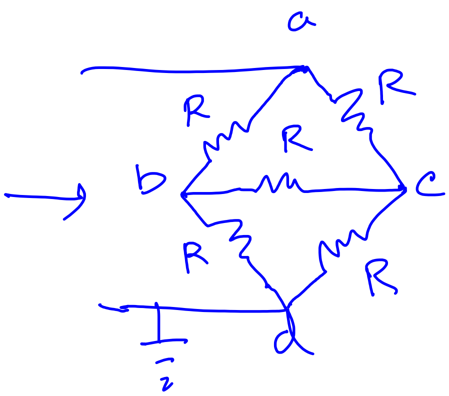
$R_{eq} \rightarrow$





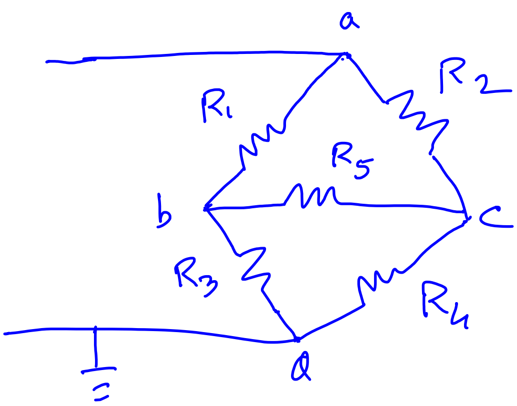
$R_{eq} \longrightarrow$



$v_b = v_c$

$R_{eq} = R$

Wheatstone Bridge



$$\boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}} \Longleftrightarrow i_{R_5} = 0$$

Condition X,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

condition Y,

$$i_{R_5} = 0 \checkmark$$

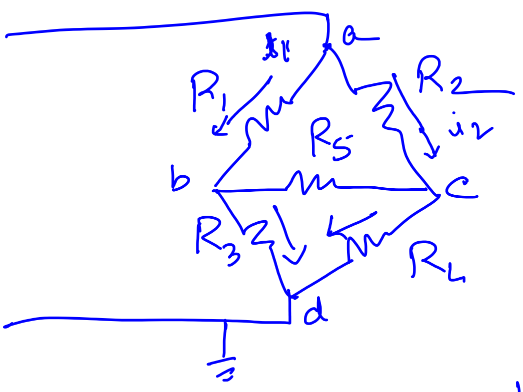
$\boxed{Y \Rightarrow X}$ and $\boxed{X \Rightarrow Y}$

X is a necessary and sufficient condition for Y.

Y is true if and only if X is true.

iff

Proving $Y \Rightarrow X$,



$$i_{R_5} = 0$$

$$\Rightarrow V_b = V_c \checkmark$$

$$V_a - V_b = V_a - V_c$$

$$i_1 R_1 = i_2 R_2$$

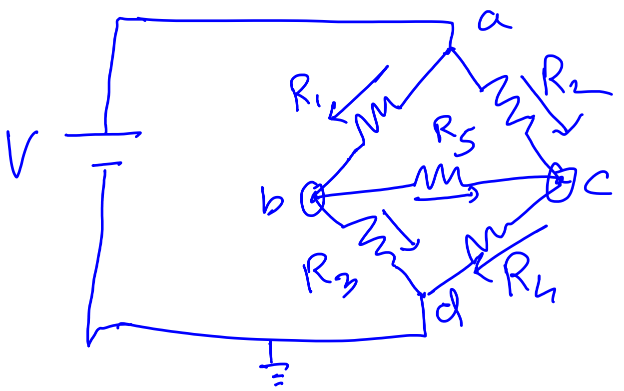
$$\frac{i_1}{i_2} = \frac{R_2}{R_1}$$

$$V_b = i_1 R_3 = i_2 R_4 = V_c$$

$$\frac{i_1}{i_2} = \frac{R_4}{R_3} \implies \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

$$\implies \boxed{Y \implies X}$$

Proving $X \implies Y$,



$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$V_a = V$

KCL at node b,

$$\frac{V_a - V_b}{R_1} = \frac{V_b - V_c}{R_5} + \frac{V_b}{R_3}$$

KCL at node c,

$$\frac{V_a - V_c}{R_2} + \frac{V_b - V_c}{R_5} = \frac{V_c}{R_4}$$

$$V_b\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_c}{R_5} = \frac{V}{R_1} \quad \text{—— (1)}$$

$$\frac{V_b}{R_5} - V_c\left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) = -\frac{V}{R_2} \quad \text{—— (2)}$$

Multiply (I) by R_1 and (II) by R_2,

$$R_1\left(v_b\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_5}\right)-\frac{v_c}{R_5}\right)=$$

$$-R_2\left(\frac{v_b}{R_5}-v_c\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right)\right)$$

$$\Rightarrow \quad v_b\left(1+\frac{R_1}{R_3}+\frac{R_1}{R_5}+\frac{R_2}{R_5}\right)$$

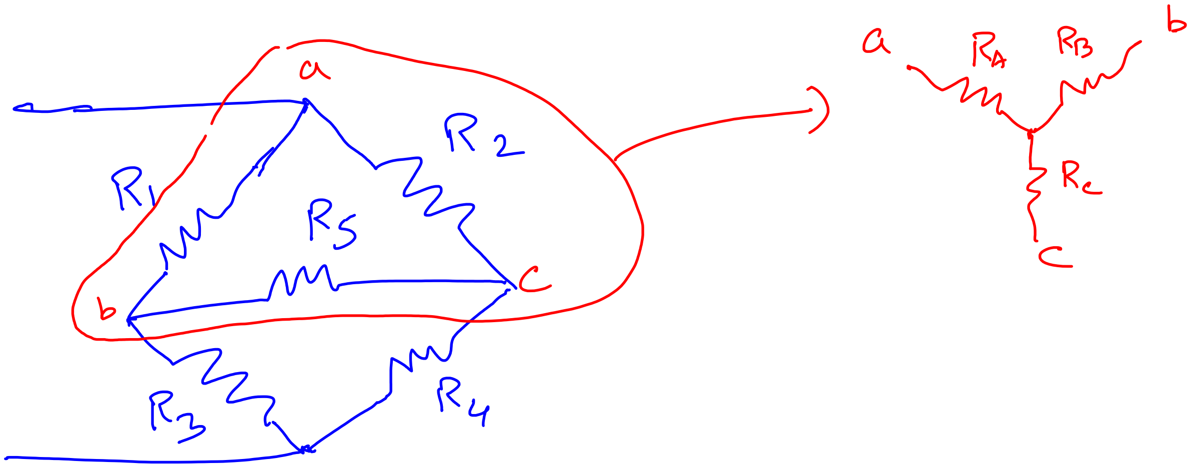
$$= v_c\left(1+\frac{R_2}{R_4}+\frac{R_2}{R_5}+\frac{R_1}{R_5}\right)$$

$$\Rightarrow \quad v_b = v_c$$

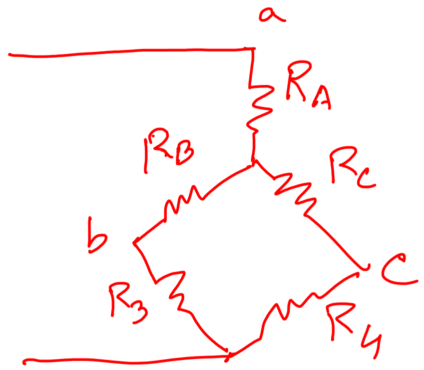
$\Rightarrow \quad i_{R_5} = 0 \quad \Rightarrow$ Y is true.

So $\boxed{X \Rightarrow Y}$

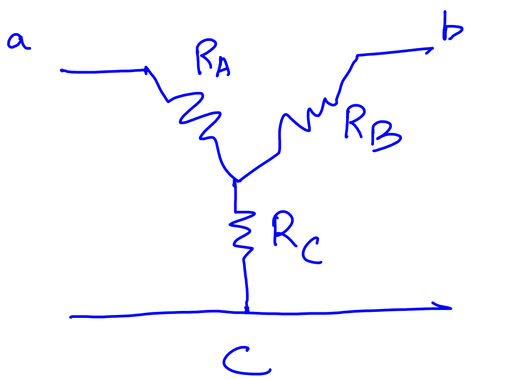
$$\Rightarrow \quad \boxed{X \Leftrightarrow Y}$$



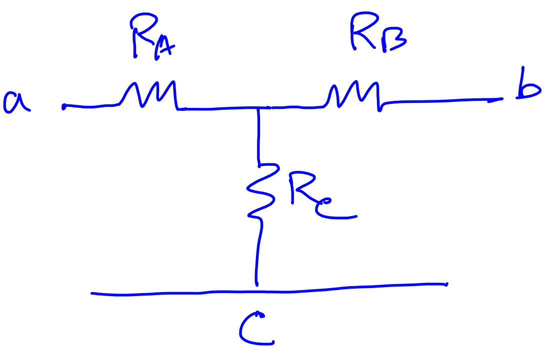
What if $\dfrac{R_1}{R_2} \neq \dfrac{R_3}{R_4}$



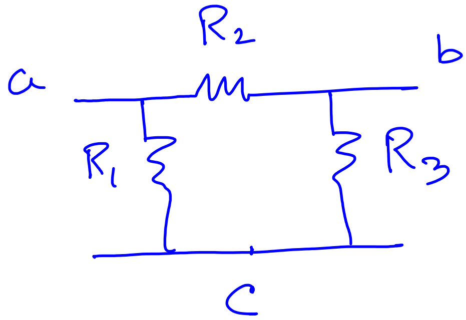
Star - delta Conversion



Y - network



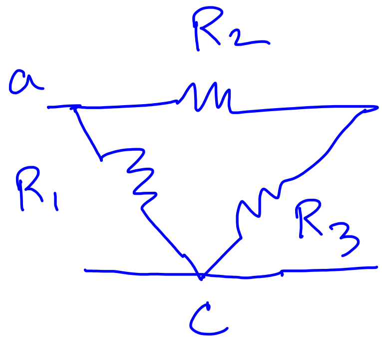
T - network

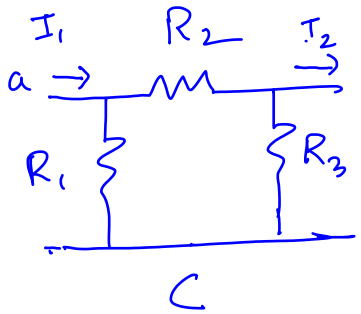


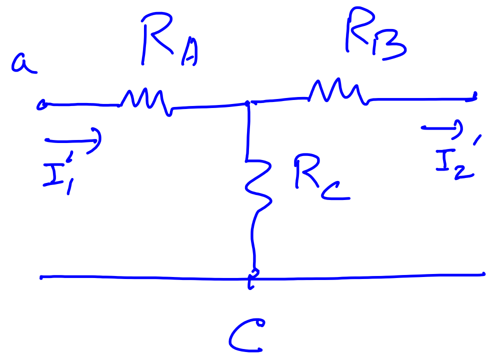
π - network

$\Updownarrow$

Delta - network



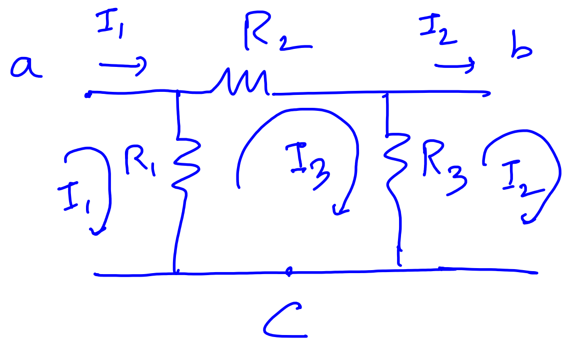




V_{ac}, V_{bc} $\qquad$ V'_{ac}, V'_{bc}

$$\left.\begin{array}{l} I_1 = I_1', \quad I_2 = I_2' \\ \text{and} \quad V_{ac} = V'_{ac}, \quad V_{bc} = V'_{bc} \end{array}\right\} \text{equivalent}$$

For equivalence, what should be the relation among the resistances?



$$V_{ac} = I_1 R_1 - I_3 R_1$$

$$-(I_1 - I_3)R_1 + I_3 R_2 + (I_3 - I_2) R_3 = 0$$

$$V_{bc} = -(I_2 - I_3) R_3$$ ✓

$$I_3 (R_1 + R_2 + R_3) = I_1 R_1 + I_2 R_3$$

$$I_3 = \frac{I_1 R_1 + I_2 R_3}{(R_1 + R_2 + R_3)}$$

$$V_{ac} = I_1 R_1 - \frac{R_1^2}{R_1 + R_2 + R_3} I_1 - \frac{R_3 R_1}{R_1 + R_2 + R_3} I_2$$

$$V_{ac} = \underline{\left(R_1 - \frac{R_1^2}{R_1 + R_2 + R_3} \right)} I_1 - \frac{R_3 R_1}{R_1 + R_2 + R_3} I_2$$

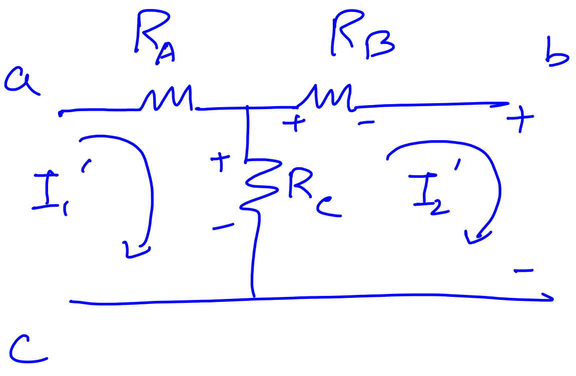
1

$$V_{bc} = - I_2 R_3 + I_3 R_3$$

$$= - I_2 R_3 + R_3 \left(\frac{I_1 R_1 + I_2 R_3}{R_1 + R_2 + R_3} \right)$$

$$V_{bc} = \frac{R_3 R_1}{R_1+R_2+R_3} I_1 - I_2 \left(R_3 - \frac{R_3^{\,2}}{R_1+R_2+R_3} \right)$$

$$\text{(II)}$$



$$v_{R_C} = (I_1' - I_2') R_C$$

$$v_{R_B} = I_2' R_B$$

$$\left.\begin{aligned} v_{ac}' &= I_1' R_A + I_1' R_C - I_2' R_C \\ v_{bc}' &= +(I_1' - I_2') R_C - I_2' R_B \end{aligned}\right\}$$

$$v_{R_C} - v_{R_B} - v_{bc}' = 0$$

$$V_{ac}' = I_1' (R_A + R_C) - R_C I_2'$$

$$\text{(III)}$$

$$V_{bc}' = R_C I_1' - (R_C + R_B) I_2'$$

$$\text{(IV)}$$

<u>Comparing equations (I) and (III)</u>,

$$R_A + R_C = R_1 - \frac{R_1^{\,2}}{R_1+R_2+R_3}$$

$$\checkmark R_C = \frac{R_3 R_1}{R_1+R_2+R_3}$$

similarly derive the other expressions.

$$R_C = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$