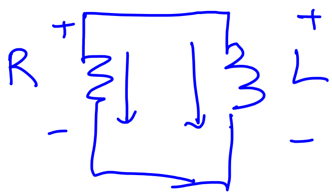
Quick Recap

- → Capacitor
- → RC circuit
- → Inductor
- → RL circuit (natural response)



$$I_L(0) = I_0$$

$$I_L(t) = I_0 e^{-\frac{R}{L}t}, \quad \forall t \geq 0$$

$$V_R = V_L$$

$$I_L + I_R = 0$$

$$V_L = L\frac{dI_L}{dt}$$

$$= I_0 L e^{-\frac{R}{L}t} \times (-R/L)$$

$$= -I_0 R e^{-\frac{R}{L}t}$$

$$E_R = \int_0^\infty V_R I_R \, dt$$

$$= -\int_0^\infty V_L I_L \, dt$$

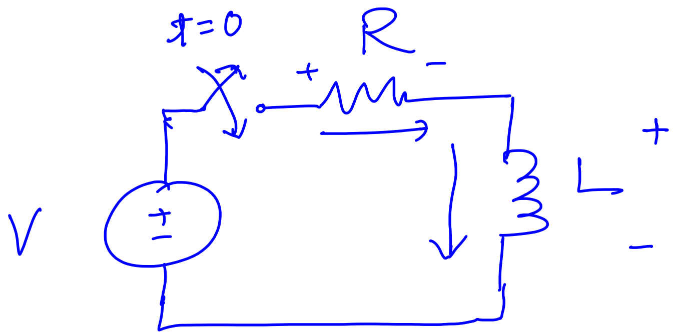
$$= -\int_0^\infty -(I_0 R) e^{-\frac{R}{L}t} \times I_0 e^{-\frac{R}{L}t} \, dt$$

$$= I_0^2 R \int_0^\infty e^{-\frac{2R}{L}t} \, dt$$

$$= \frac{1}{2} L I_0^2$$

$$E_L = \int_0^\infty V_L I_L \, dt = -\frac{1}{2} L I_0^2$$

Forced Response of RL circuit,



$I_L(0) = 0$

$I_R(t) = I_L(t)$

$$V = V_R(t) + V_L(t), \quad \forall t \geq 0$$

$$V = I_L(t) R + L \frac{d I_L(t)}{dt}, \quad \forall t \geq 0$$

$$I_L(t) = I_{L,ss} + I_{L,Tr}(t)$$

$$I_{L,ss} = A_1 \quad (\text{constant})$$

$$I_{L,Tr}(t) = A_2 e^{st}$$

$$I_L(t) = A_1 + A_2 e^{st}$$

$$V = (A_1 + A_2 e^{st}) R + L A_2 s e^{st}, \quad \forall t \geq 0$$

$$V = A_1 R + A_2 e^{st} R + L A_2 s e^{st} \quad \forall t \geq 0$$

2)

$$V = A_1 R$$

and

$$A_2 e^{st} R + L A_2 s e^{st} = 0 \quad \forall t \geq 0$$

$$A_1 = \frac{V}{R}$$

$$A_2 e^{st} (R + Ls) = 0 \quad \forall t \geq 0$$

$$R + Ls = 0$$

$$\Rightarrow \quad s = -\frac{R}{L}$$

$$i_L(0) = A_1 + A_2 e^{s0} = 0$$

$$\Rightarrow \quad A_1 + A_2 = 0$$

$$\Rightarrow \quad A_2 = -A_1 = -\frac{V}{R}$$

$$i_L(t) = A_1 + A_2 e^{st}$$

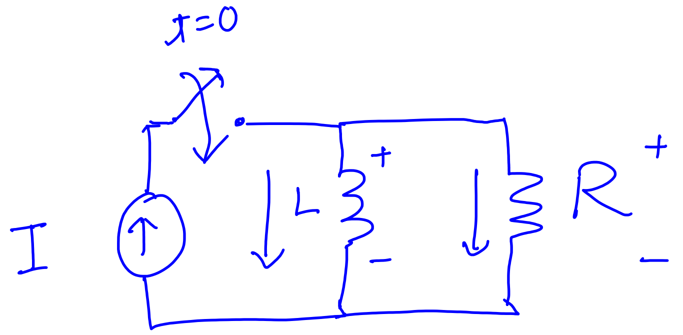
$$= \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$= \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) \quad \forall t \geq 0$$

$$i_L(\infty) = \frac{V}{R}$$

$\Rightarrow$ At steady state, inductor acts as short circuit

$$(V_L(\infty) = 0)$$



$$I_L(0) = I_0$$

$$I_R(\infty) = 0 \quad ?$$

$$V_L(0) =$$

✓ $V_L(t) = V_R(t) \quad \forall t \geq 0$

$$I = I_L(t) + I_R(t) \quad \forall t \geq 0$$

$$\Rightarrow \quad I = I_L(0) + I_R(0)$$

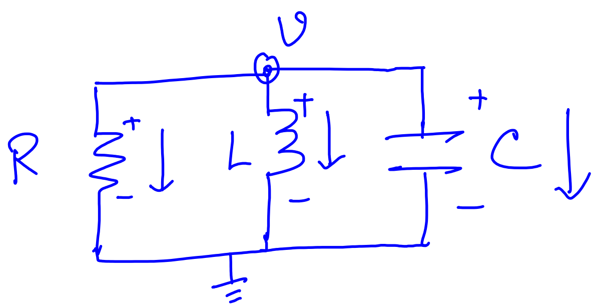
$$= I_0 + I_R(0)$$

$$\Rightarrow \quad I_R(0) = I - I_0$$

$$V_R(0) = R\, I_R(0) = R(I - I_0)$$

$$\Rightarrow \quad V_L(0) = R(I - I_0)$$

RLC circuits



natural response

$$I_L(0) = I_0$$

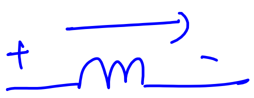
$$V_C(0) = V_0$$

KCL →

$$I_R(t) + I_L(t) + I_C(t) = 0 \quad \forall t \geq 0$$

$$V_R(t) = V_L(t) = V_C(t) = V(t)$$

$$\frac{V(t)}{R} + \underbrace{\frac{1}{L}\int_0^t V_L(s)\,ds + I_L(0)}_{I_L(t)} + C\frac{dv}{dt} = 0 \quad \forall t \geq 0$$



$$v_L = L\frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L}\int_0^t V_L(s)\,ds + i_L(0)$$

Intrego-differential equation,

$$\frac{V(t)}{R} + \frac{1}{L}\int_0^t V(s)\,ds + i_L(0) + C\frac{dv}{dt} = 0$$

Differentiate both sides — — —

$$\frac{1}{R}\frac{dv}{dt} + \frac{v(t)}{L} + C\frac{d^2v}{dt^2} = 0$$ (second-order differential equation)

$$v(t) = Ae^{st}$$ (Assumption)

$$\frac{1}{R}Ase^{st} + \frac{Ae^{st}}{L} + Cs^2Ae^{st} = 0, \quad \forall t \geq 0$$

$$Ae^{st}\left(\frac{1}{R}\cdot s + \frac{1}{L} + Cs^2\right) = 0, \quad \forall t \geq 0$$

$$\Rightarrow \quad Cs^2 + \frac{1}{R}s + \frac{1}{L} = 0$$ (characteristic equation)

$$\Rightarrow \quad C\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right) = 0$$

$$\Rightarrow \quad s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$\Rightarrow \quad \underbrace{s^2 + 2s.\frac{1}{2RC} + \left(\frac{1}{2RC}\right)^2} + \frac{1}{LC} - \left(\frac{1}{2RC}\right)^2 = 0$$

$$\Rightarrow \quad \left(s + \frac{1}{2RC}\right)^2 = \left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}$$

$$\Rightarrow \quad s + \frac{1}{2RC} = \pm\sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$\Rightarrow \quad s = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$s_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

two roots

So, general structure of the natural response

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$