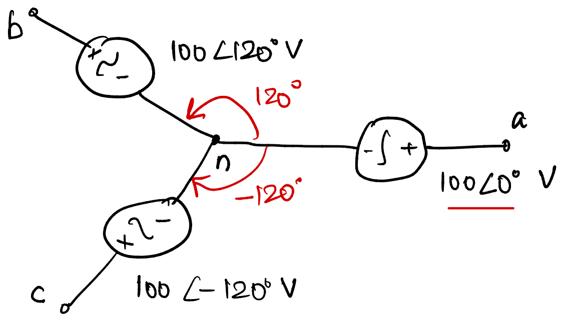
Module 4

Polyphase Circuits

(Chapter 12 of textbook)

Three-phase source



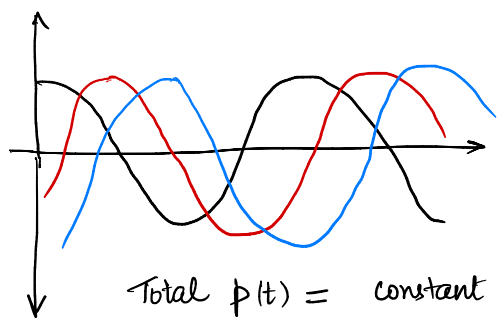
$V_{an} = 100\angle 0^\circ$ V

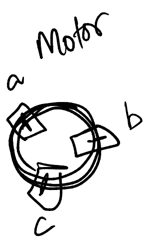
$V_{bn} = 100\angle 120^\circ$ V

$V_{cn} = 100\angle -120^\circ$ V

$$\left.\begin{array}{l} |V_{an}| = |V_{bn}| = |V_{cn}| \\ \rightarrow V_{an} + V_{bn} + V_{cn} = 0 \end{array}\right\} \text{Balanced Source}$$

Freq. Domain	Time Domain
$100\angle 0^\circ$	$100 \cos(\omega t)$
$100\angle 120^\circ$	$100 \cos(\omega t + 120^\circ)$
$100\angle -120^\circ$	$100 \cos(\omega t - 120^\circ)$

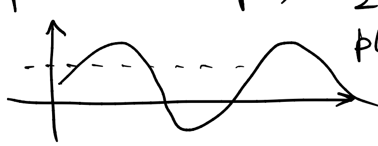




Total $p(t)$ = constant for three-phase sources

For single phase source: $p(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta - \phi)$

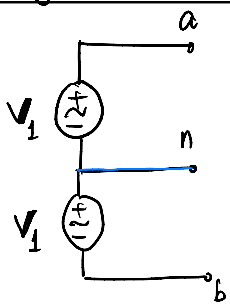
$p(t) \neq$ constant



Advantages of three-phase sources are
- constant power and constant torque to large motors.
- more economical since motors are more efficients.

Single-Phase Three-Wire Source (Two-phase source)

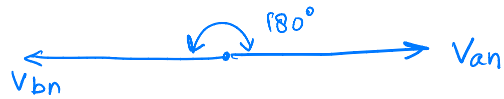
$\theta^\circ,\ \theta^\circ - 180^\circ$



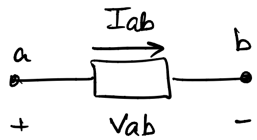
$V_1 = V_m \angle \theta^\circ$

$V_{an} = V_1$
$V_{nb} = V_1$

$$\left\{ \begin{array}{l} V_{an} = V_1 = V_m \angle \theta^\circ \\ V_{bn} = -V_1 = V_m \angle \theta^\circ - 180^\circ \end{array} \right\}$$



Double Script Notations:



V_{an} = voltage at point a w.r.t. voltage at point n

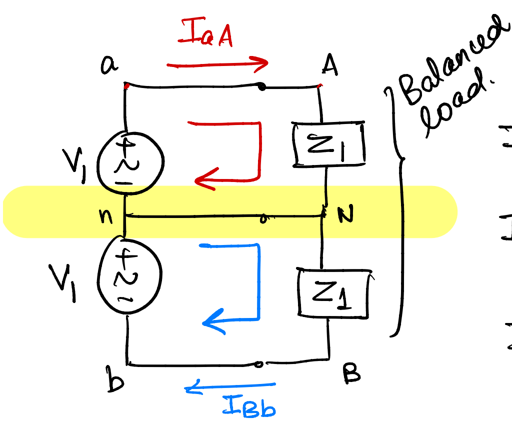
$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$

$V_{bn} = -V_{nb}$

I_{ab} = current flowing from point a to point b.

Balanced Single-Phase Three-Wire Source:

$$\left. \begin{array}{l} |V_{an}| = |V_{bn}| \\ V_{an} + V_{bn} = 0 \end{array} \right\} \text{Balanced source}$$



$V_{an} = V_1$, $V_{bn} = -V_1$

$$I_{aA} = \frac{V_1}{Z_1}$$

$$I_{Bb} = \frac{V_1}{Z_2}$$

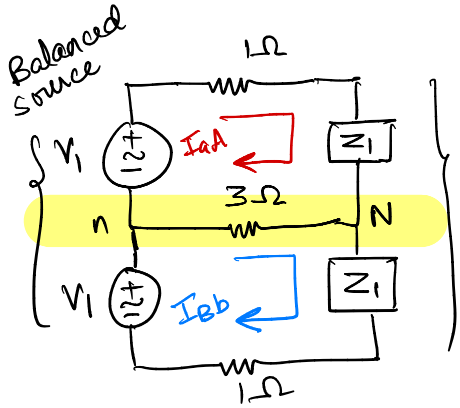
$$I_{nN} = I_{Bb} - I_{aA} = \frac{V_1}{Z_2} - \frac{V_1}{Z_1}$$

Assume $Z_1 = Z_2$ (Balanced load)

$$I_{nN} = \frac{V_1}{Z_1} - \frac{V_1}{Z_1} = 0$$

For balanced source and balanced load } current in neutral line $I_{nN} = 0$

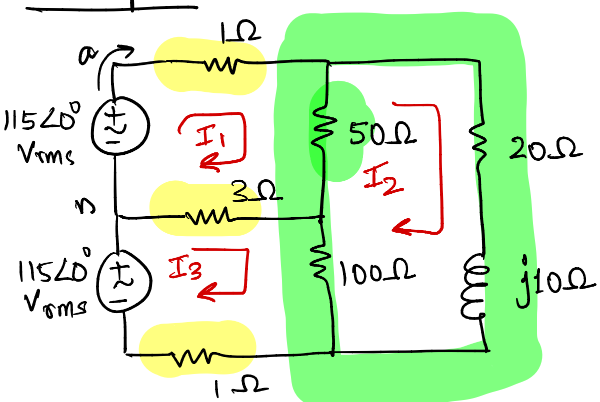
Wires with non-zero resistance



Balanced load

$$I_{nN} = 0 = I_{Bb} - I_{aA}$$

Example



line resistances

loads

Find

1. Average power delivered to each of three loads
2. Avg power lost in outerlines and neutral line.
3. Avg power provided by source.

Using mesh analysis

$$-115\angle 0^\circ + 1(I_1) + 50(I_1 - I_2) + 3(I_1 - I_3) = 0$$

$$50(I_2 - I_1) + (20 + j10)I_2 + 100(I_2 - I_3) = 0$$

$$-115\angle 0^\circ + (I_3 - I_1)3 + (I_3 - I_2)100 + I_3(1) = 0$$

$\longrightarrow$

$$I_1 = 11.24\angle -19.83^\circ \ \text{Arms}$$

$$I_2 = 9.389\angle -24.47^\circ \ \text{Arms}$$

$$I_3 = 10.37\angle -21.80^\circ \ \text{Arms}$$

$I_1 - I_2 = \cancel{11.24} - 9.389$

$P_{avg} = V_{eff} I_{eff} \cos(\theta - \phi)$

$P_{avg} = I_{eff}^2 R$

$P_{avg} = \frac{1}{2} I_m^2 R$

Avg power delivered to loads:

$$P_{avg,\,50\Omega} = |I_2 - I_1|^2 (50)$$
$$= 206 \text{ W}$$

$$P_{avg,\,100\Omega} = 117 \text{ W}$$

$$P_{avg,\,\underbrace{20+j10\Omega}} = I_2^2 (20) = 1763 \text{ W}$$

Total power delivered to loads = 2086 W

Avg power lost in lines

$$P_{avg.\,aA} = |I_1|^2 1 = 126 \text{ W}$$

$$P_{avg,\,Bb} = (I_3)^2 1 = 108 \text{ W}$$

$$P_{avg.\,nN} = |I_3 - I_1|^2 3 = 3 \text{ W}$$

Total power lost in lines = 237 W

Power supplied by sources

$V_{an} = 115\angle 0^\circ$ Vrms

$I_{na} = I_1$

$$P_{avg,\,an} = V_{an} I_{na} \cos(\theta - \phi)$$
$$= 115 \times 11.24 \cos(0 - (-19.83^\circ))$$
$$= 1216 \text{ W}$$

$$P_{avg,\,nb} = 115 \times 10.37 \cos(21.80^\circ)$$
$$= 1107 \text{ W}$$

Total power supplied = 2323 W

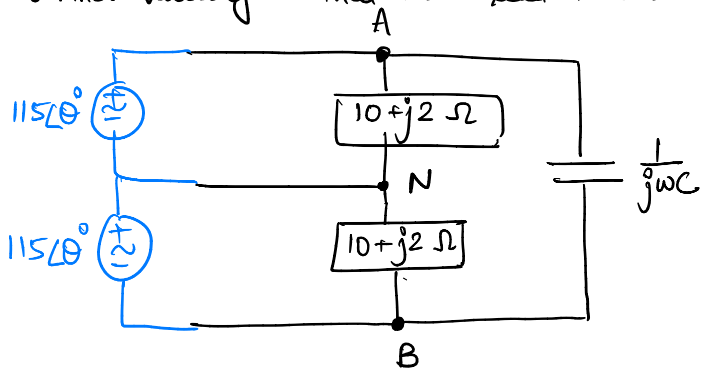
$$\boxed{P_{supplied} = P_{delivered} + P_{loss}}$$

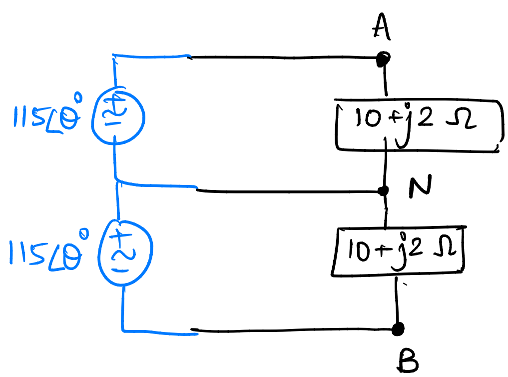
① **Example**: Consider $V_{12} = 9\angle 30^\circ$, $V_{32} = 3\angle 130^\circ$

Find $V_{21} = 9\angle -150^\circ$, $V_{13} = V_{12} + V_{23}$

② Consider a single-phase three-wire balanced source connected to a balanced load. $f = 50\,Hz$, $V_{an} = 115\angle 0^\circ\,V$

- Find power factor of the load if capacitor is omitted.
- Find value of C that will lead to a unity power factor of total load.





$PF = \cos(\theta - \phi)$