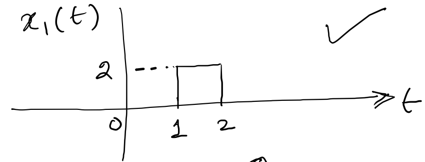
21|8|2024

C.T. Signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

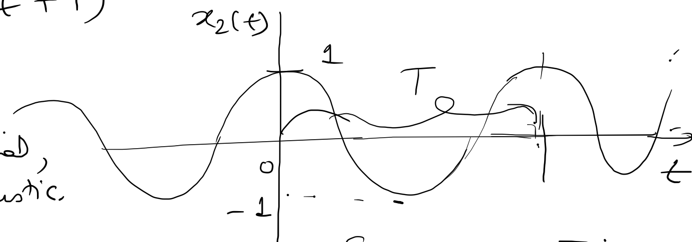


$$= \int_{1}^{2} |2|^2 dt = 4\int_{1}^{2} dt = 4 < \infty$$

↑ Energy Signal

$x_1(t) \equiv$ energy signal, C.T. Signal, aperiodic, deterministic

$x_1(t) \neq x_1(t+T) \longrightarrow$ aperiodic signal

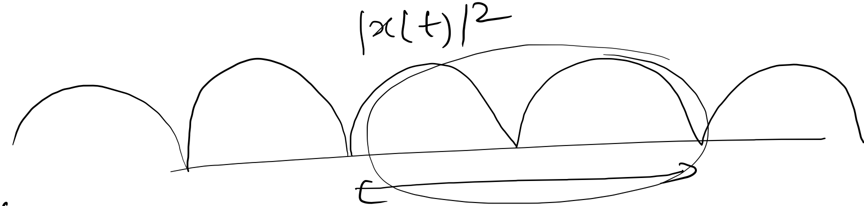


$x_2(t) \equiv$ C.T. Signal, periodic signal, deterministic.

$x_2(t) = \cos \omega_0 t \quad \forall t$

fundamental Time period $\equiv$ smallest positive T

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$



$$= \underbrace{\lim_{T_0 \to \infty} \int_{-T_0}^{T_0} |x(t)|^2 \, dt}_{= \infty}$$

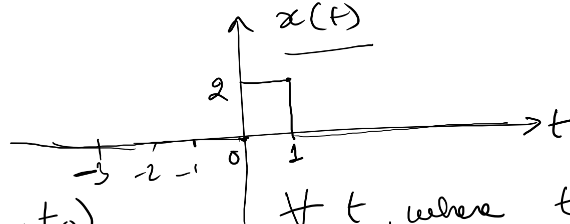
Not an energy signal

$$P = \lim_{T_0 \to \infty} \frac{1}{2T_0} \int_{-T_0}^{T_0} |x(t)|^2 \, dt = \text{finite no.}$$

$$\boxed{0 < P < \infty}$$

Power signal

Transformations of Signals

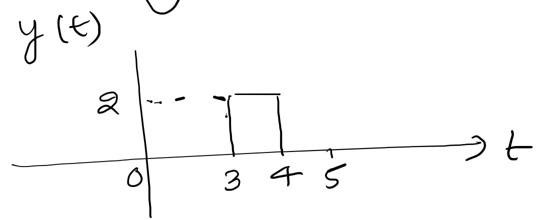


① Translate / Shift a signal

ⓐ Delay $\quad y(t) = x(t - t_0)$ $\quad \forall t$, where t_0 = constant

Delay

Let $t_0 = 3$



$$y(t) = x(t-3)$$

$$y(0) = x(-3) = 0$$

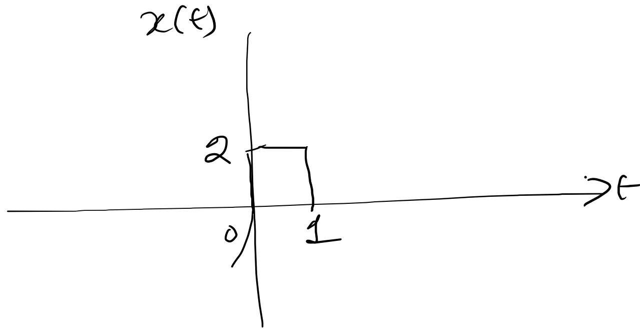
$$y(3) = x(3-3) = x(0) = 2$$

$$y(4) = x(4-3) = x(1) = 2$$

$$y(5) = x(5-3) = x(2) = 0$$

(b) Advance a signal —

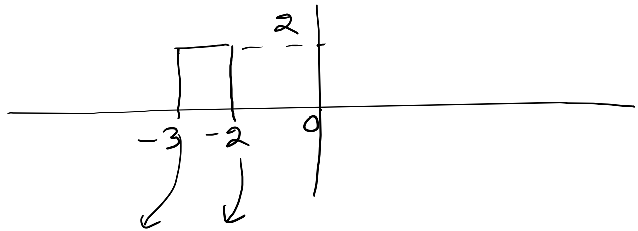
$$y(t) = x(t + t_0)$$



$\underline{t_0 = 3}$

$$y(-3) = x(-3+3) = x(0)$$

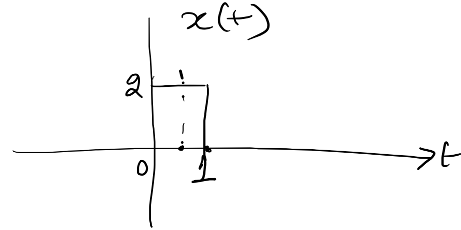
$$y(-2) = x(-2+3) = x(1)$$



2 Scaling of a signal

$$y(t) = x(at)$$

ⓐ $\underline{a > 1}$ $\longrightarrow$ Compression of a signal on the t-axis

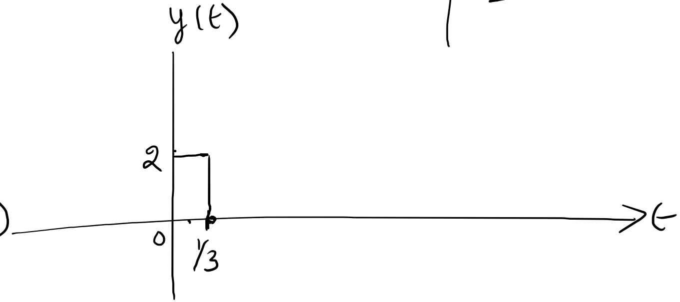


Let $a = 3$

$$y(t) = x(3t)$$

$$y(0) = x(0) = 2$$

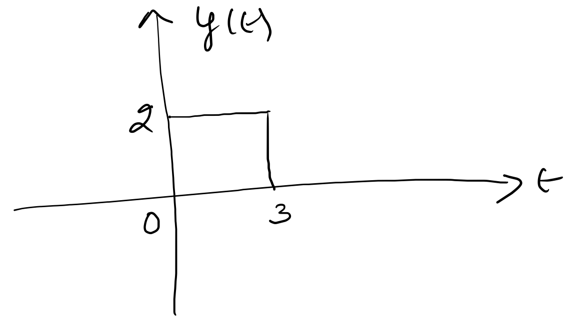
$$y\left(\tfrac{1}{3}\right) = x\left(3 \cdot \tfrac{1}{3}\right) = x(1) = 2$$



$y(t) = 0 \quad t > \frac{1}{3}$

& $t < 0$

(b) $0 < a < 1$ — Dilation on the t-axis

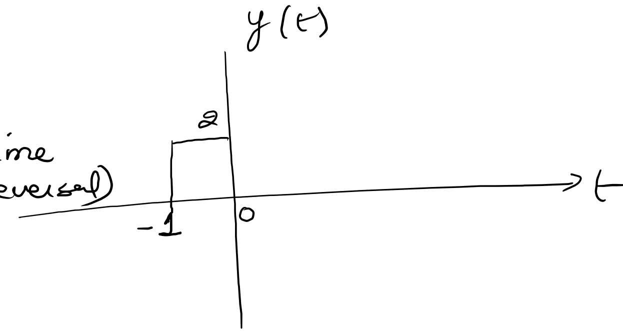


Let $a = 1/3$

$$y(t) = x(at) = x\left(\frac{t}{3}\right)$$

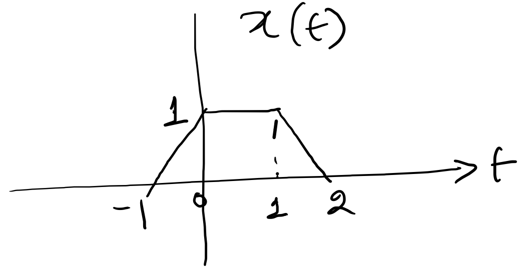
$$y(0) = x(0) = 2$$

$$y(3) = x\left(\frac{3}{3}\right) = x(1) = 2$$

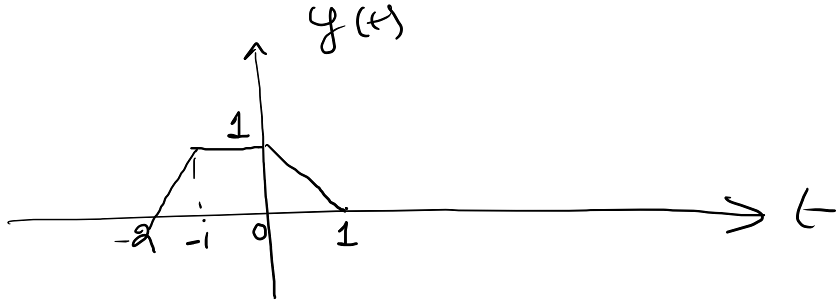


(c) Special case $a = -1$ (time-reversal)

$$y(t) = x(at) = x(-t)$$

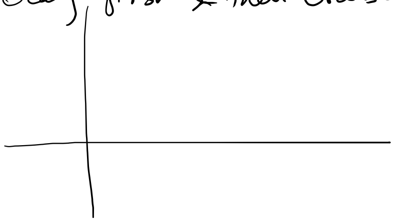


$y(t) = x(-t)$



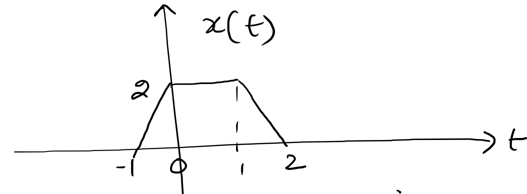
Example $y(t) = x(2t+3)$

Scaling first & Then translation



(a)

$$x(t) \xrightarrow{\text{scale}} x(2t) \xrightarrow{\text{Translate/shift}} x(2(t+3)) = x(2t+6)$$



translation & Then scaling

(b)

$$x(t) \xrightarrow{\text{Translate (Shift)}} x(t+3) \xrightarrow{\text{Scale}} x(2t+3)$$

Fundamental Signals —

C.T. Signals

$$x(t) = C e^{at}$$

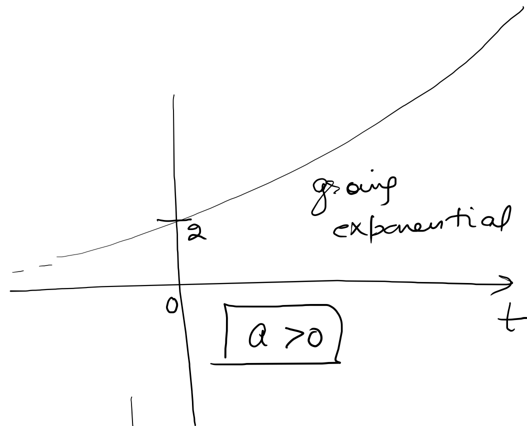
Case - 1

C = Real Constant $= 2$

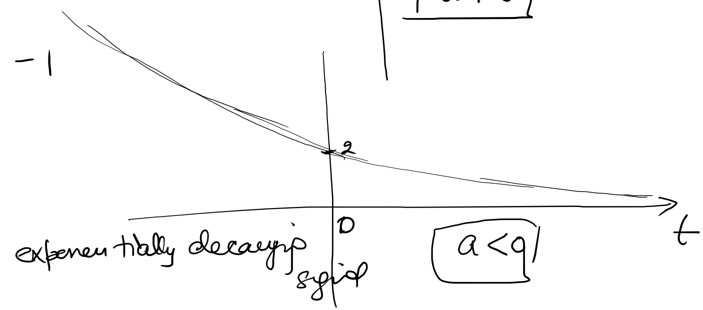
a = real constant

$$x(t) = 2e^{at}$$

(i) Let $a = 1$.



(ii) Let $a = -1$



(iii) $a = 0$

constant signal

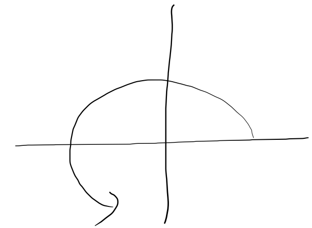
$x(t) = 2 \quad \forall t$

C.T. Signal

Case-2

C = real constant

a = purely imaginary $= j\omega$



$$\boxed{x(t) = e^{j\omega t}} \quad \forall\, t$$

periodic if $x(t) = x(t+T) \quad \forall\, t$

$$x(t+T) = e^{j\omega(t+T)} = e^{j\omega t} \cdot e^{j\omega T} = e^{j\omega t} \quad \text{if } e^{j\omega T} = 1$$

$$= x(t) \quad \forall\, t$$

$$E = \int_{-\infty}^{\infty} \left| e^{j\omega t} \right|^2 dt = \int_{-\infty}^{\infty} 1\, dt = \infty$$

$e^{j\omega T} = 1$ when $\omega T = 2\pi$

$$\boxed{T = \frac{2\pi}{\omega}}$$ fundamental time period

$\omega \equiv$ Units of rad/sec.

Case-3

$C \equiv$ Complex no. $= |c| e^{j\theta} = |c| \cos\theta + j|c| \sin\theta$

$a =$ real no.

$x(t) = |c| e^{j\theta} \cdot e^{at} = |c| \cdot e^{at + j\theta}$

$Re\{x(t)\} = |c| \cos\theta \cdot e^{at}$

$Im\{x(t)\} = |c| \sin\theta \cdot e^{at}$

Case 4

$C =$ Complex no. $\equiv |c| e^{j\theta}$

$a =$ Complex no. $\equiv r + j\omega$

$x(t) = c e^{at} = |c| e^{j\theta} \cdot e^{(r+j\omega)t}$

$= |c| e^{rt} \cdot e^{j(\omega t + \theta)}$

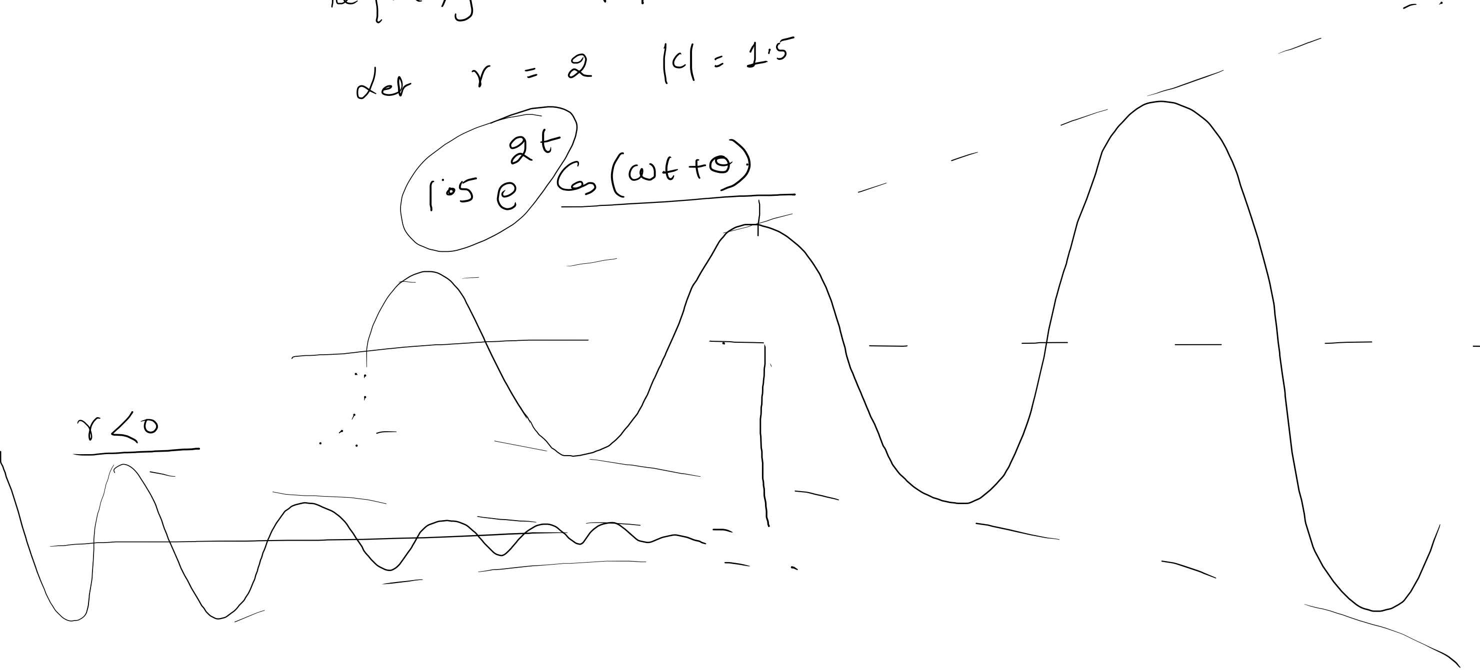
$\Downarrow$

$\cos(\omega t + \theta) + j \sin(\omega t + \theta)$

$$\text{Re}\{x(t)\} = |c|\, e^{rt} \cos(\omega t + \theta) \qquad \text{Im}\{x(t)\} = |c|\, e^{rt} \sin(\omega t + \theta)$$

Let $r = 2 \quad |c| = 1.5$

$$1.5\, e^{2t} \cos(\omega t + \theta)$$



D.T. Signal

$$x[n] = C\alpha^n$$

(1) Case -1

C = real constant

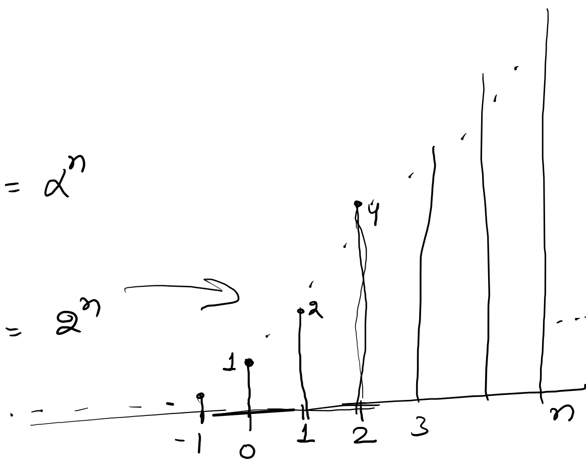
α = real

(i) $\alpha > 1$, $C = 1$

$x[n] = \alpha^n$

let $\alpha = 2$

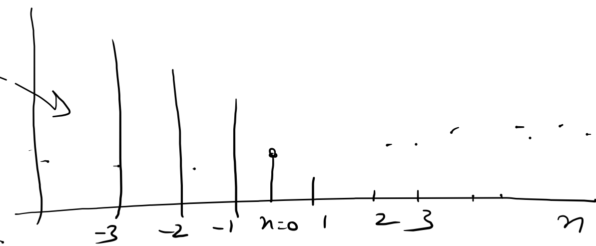
$x[n] = 2^n$



(ii) $0 < \alpha < 1$

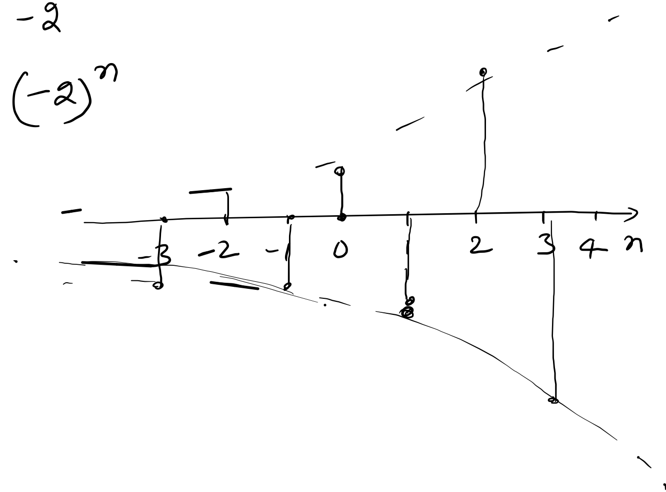
let $\alpha = 0.5$

$x[n] = (0.5)^n = \left(\frac{1}{2}\right)^n$



(iii) $\underline{\alpha < -1}$ Let $\alpha = -2$

$x[n] = (-2)^n$

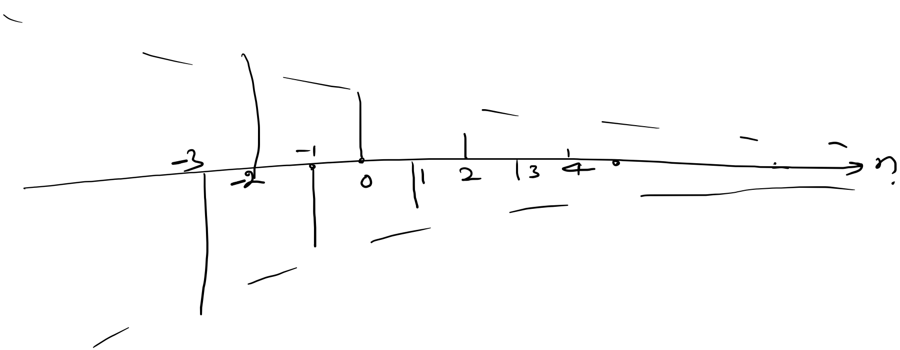


(iv) $\underline{-1 < \alpha < 0}$

Let $\alpha = -\frac{1}{2}$

$x[n] = \left(-\frac{1}{2}\right)^n$

$= (-1)^n \left(\frac{1}{2}\right)^n$



Case -2

C = real

$\alpha = e^{j\omega}$

Let $C=1$

$$\boxed{x[n] = C\alpha^n = e^{j\omega n}}$$

if $x[n] = x[n+N] \quad \forall n$

then $x[n]$ is periodic

$$x[n+N] = e^{j\omega(n+N)} = e^{j\omega n} \cdot e^{j\omega N}$$

$$= e^{j\omega n} \quad \text{if} \quad e^{j\omega N} = 1$$

$$e^{j\omega N} = 1 \implies \omega N = 2\pi K$$

$$N = \frac{2\pi k}{\omega} \equiv \text{has to be an integer}$$

$$\frac{N}{k} \equiv \text{rational no.} = \frac{2\pi}{\omega}$$

$$x[n] = e^{j\frac{n}{6}} = e^{j\omega n}$$

$$\omega = \frac{1}{6}$$

$$\cos\frac{n}{6} + j\sin\frac{n}{6}$$