ECE113: Basic Electronics

WINTER 2024

Dr. Sayan Basu Roy

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Office Hours:

Will be posted in the classroom!

Teaching staffs

Instructors:

Tammam Tillo	Section A
Sayan Basu Roy	Section B

Teaching Fellow:

Awanish Kumar Singh

Mrityunjoy Kumar Ray

Teaching Assistants

Details will be uploaded in the google classroom soon. .

Early History of Semiconductor Devices

ENIAC-The first digital computer

1940's: Vacuum-tube era

- Vacuum tubes were used for radios, television, telephone equipment, and computers

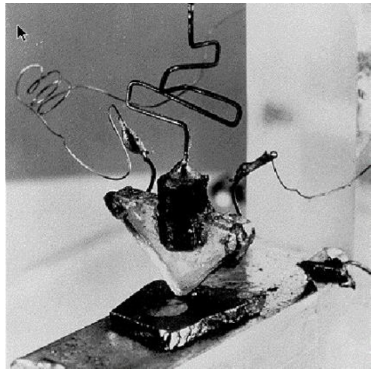
… but they were expensive, bulky, fragile, and energy-hungry

→ Invention of the point-contact transistor

- **Walter Brattain, John Bardeen, and William Shockley, *Bell Labs*, 1947**

  Nobel Prize in Physics 1956

– reproducibility was an issue, however



→ Invention of the bipolar junction transistor (BJT)

- **William Shockley, *Bell Labs*, 1950**

– more stable and reliable; easier and cheaper to make

Discrete Electronic Circuits

- In 1954, *Texas Instruments* produced the first commercial silicon transistor.

~$2.50 each

- Before the invention of the integrated circuit, electronic equipment was composed of discrete components such as transistors, resistors, and capacitors. These components, often simply called "discretes", were manufactured separately and were wired or soldered together onto circuit boards. Discretes took up a lot of room and were expensive and cumbersome to assemble, so engineers began, in the mid-1950s, to search for a simpler approach…

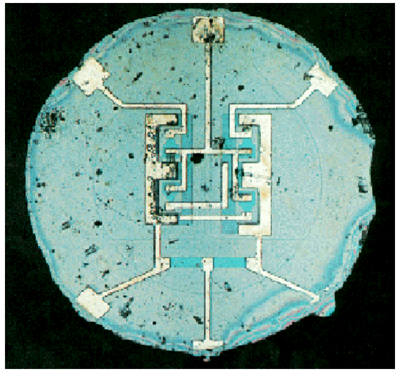
The Integrated Circuit (IC)

- An IC consists of interconnected electronic components in a single piece ("chip") of semiconductor material.

  - In 1958, Jack S. Kilby (Texas Instruments) showed that it was possible to fabricate a simple IC in germanium.

Kilby's first integrated circuit in germanium.

  - In 1959, Robert Noyce (Fairchild Semiconductor) demonstrated an IC made in silicon using SiO_2 as the insulator and Al for the metallic interconnects.

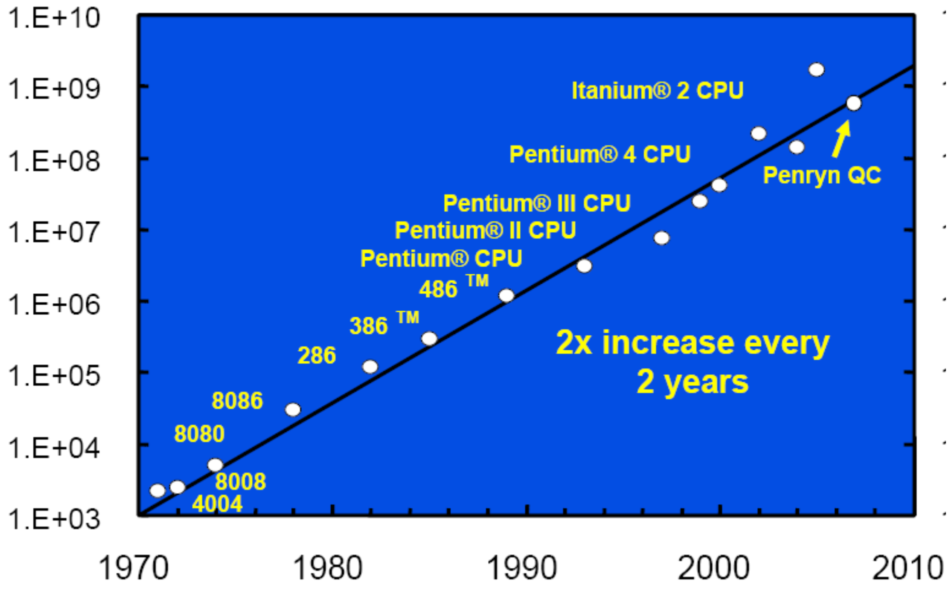


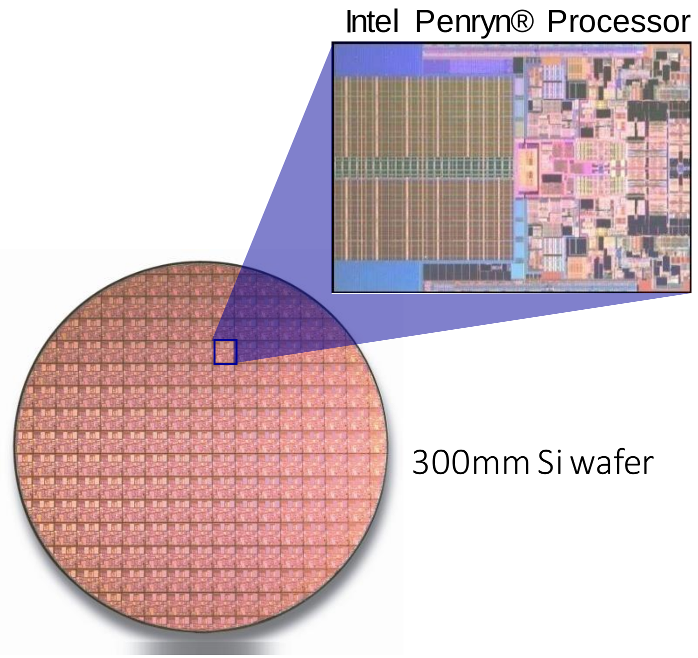
The first planar IC
(actual size: ~1.5mm diameter)

From a Few, to Billions of Components

- By connecting a large number of components, each performing simple operations, an IC that performs complex tasks can be built.
- The degree of integration has increased at an exponential pace over the past ~40 years.
  - The number of devices on a chip doubles every ~2 years, for the same price.

"Moore's Law" still holds today.





The Silicon Revolution

- Steady progress in integrated-circuit technology over 40+ years has had dramatic impact on the way people live, work, and play.

- The semiconductor industry is estimated to have more than $600B/yr in sales in 2024. .

Military

Computers

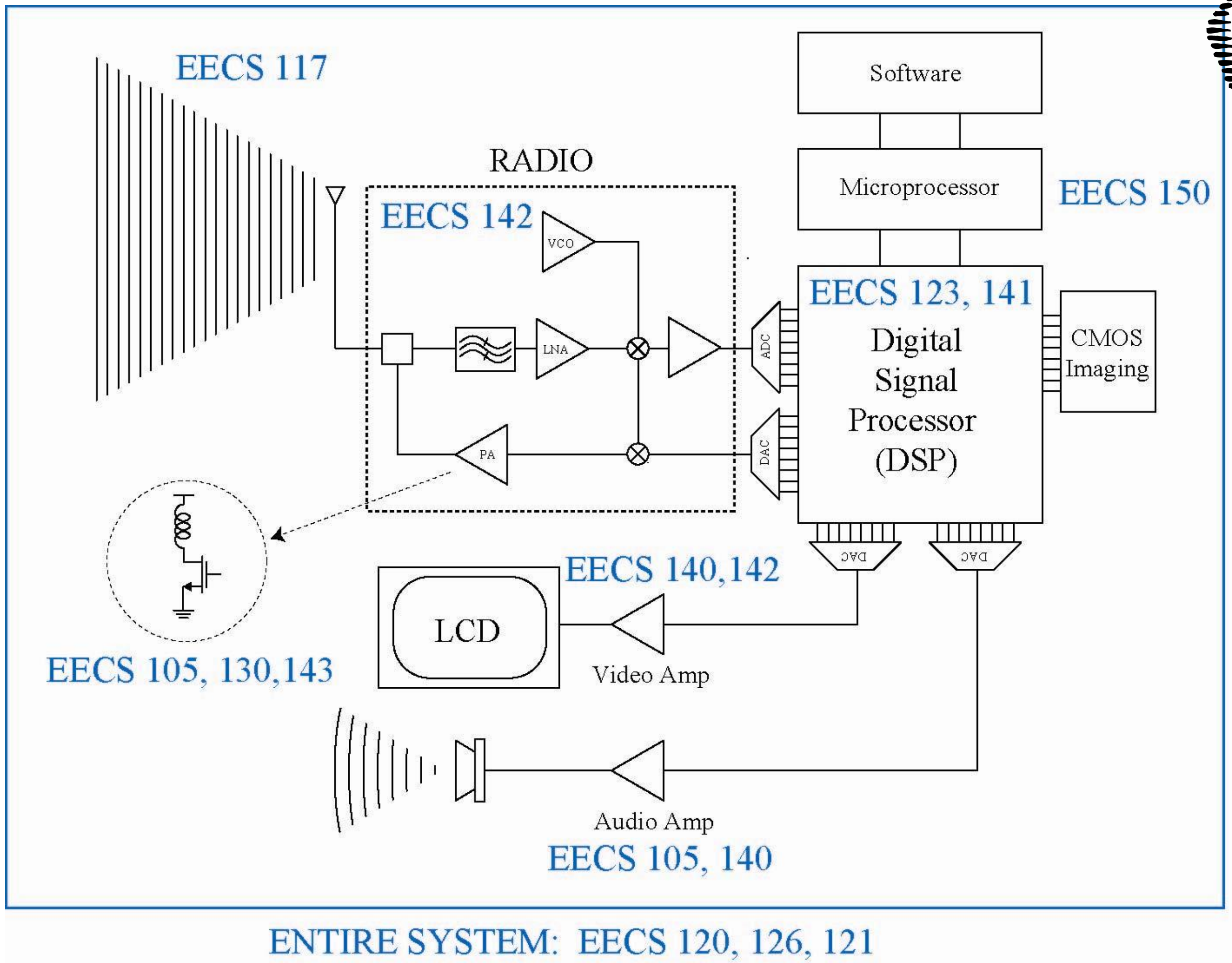
Communications

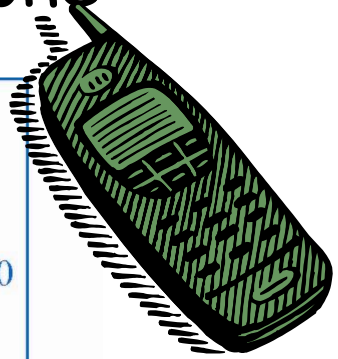
Industrial

Transportation

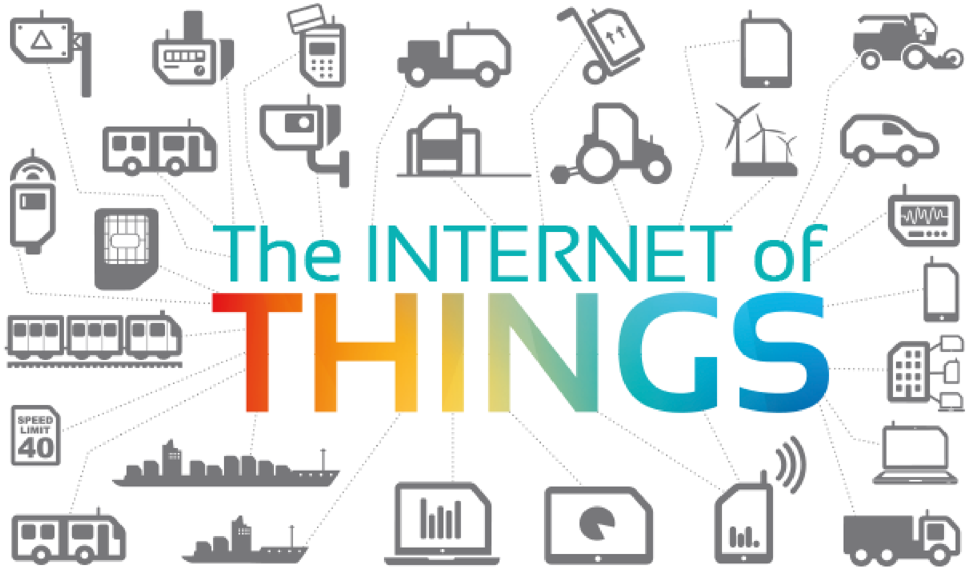
Consumer Electronics

Example electronic system: cell phone





Motivation



Course Outline

- Basic components of Electrical Circuits: Fundamental Electrical variables – charge, current, voltage and power; independent and dependent voltage and current sources, Ideal circuit elements – Resistor, Capacitor and Inductor. Passive and Active Components, Constitutional relationship – Ohms Law;
- Basic concepts of Analysis of Resistive Networks: Nodes, Paths and Loops, Kirchoff's Voltage and Current Laws, Series and Parallel connection of Resistances and division voltage and current. Loop and Node Analysis.
- Network Theorems: Thevenin's Theorem, Norton's Theorem
- Superposition Theorem and Maximum Power Transfer Theorem.
- RC and RL circuits' response
- RLC circuits' response
- Concepts of impedance/admittance, Phasors and Representation
- RC, RL and RLC circuits' response to Alternating inputs (the response to alternating input will be handled only under steady state condition)

Course Outline

- Operational Amplifier – Inverting configuration
- Operational Amplifier – Non-Inverting, Integrating and Differentiating configuration
- Diodes: Basic principle, Clipper, Clamper circuits and application
- LED, Zener Diode and Solar Cell
- Half Wave and Full Wave Rectifiers – Wave form and Ripple & its reduction.

- Text books:

  William H. Hayt Jr., Jack E. Kemmerly and Steven M. Durbin, *Engineering Circuit analysis*. 8th Edition, Tata McGraw Hill

  Smith, Ralph J and Dorf, R C, "Circuits Devices and Systems", 5th Edition, John Wiley and Sons

  Reference book

  There are many text for Principles of Electrical Engineering

Assessment

- Assignments 15% (two assignments)
- Quizzes 15% (two quizzes)
- Laboratory 20% (2% attendance, 8% lab report/file, 10% lab exam)
- Mid Semester 20%
- End Semester 30%

Course communication

- All the announcements, assignments, lectures, lab materials will be shared through **google classroom**.
- No separate communications shall be done using email, unless necessary.
- Consult course site at **google classroom** regularly

How to Join ECE113- Basic Electronics Section B in google classroom?

- Log on to **google classroom** using your IIITD credentials
- Click on the + sign at the top right corner
- Select join class
- Enter this class code: **nxgzhyt**

Laboratory

- Location: We will have laboratory Session in ECE Labs
- Lab materials will be provided through google classroom
- From next week onward: **prepare for the lab beforehand**
- Lab Engineers with the help of TFs and TAs will manage laboratory

Tutorial

- Location: We will have Tutorial Session once in every week
- Tutorial materials will be provided through google classroom
- TFs with the help of TAs will manage Tutorials

Ground rules

- Be on time
- Ask questions and interact with me if some concepts are not clear!
- Post your questions in the google classroom! Use it as an interactive platform to discuss concepts!
- **IIIT Delhi plagiarism policy** will be strictly followed in this class
- Note that your first point of contacts are TAs; followed by TFs; followed by the Instructor.

ECE113: Basic Electronics

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Electrical quantities

- Charge: unit - Coulomb
- Current: unit - Ampere
- Voltage: unit - Volt
- Power: unit – Watt
- Resistance: unit - Ohm

SI units

Charge

- A physical property of matter which interacts with electromagnetic field
  - Experiences a force due to a electromagnetic field
  - Responsible for creating electric field
- The force can be calculated using Coulomb's Law:

$$\boxed{F = \frac{K q_1 q_2}{d^2}}$$

$$F \propto \frac{q_1 q_2}{d^2}$$

- Two types of charges: +ve and –ve
- Same types of charges repel and unlike types attract
- Convention: charge of a proton is +ve and that of an electron is –ve
- The unit of charge is the coulomb (*C*),
- One coulomb is one ampere second (1 coulomb = 6.24×1018 electrons).
- The coulomb is defined as the quantity of electricity which flows past a given point in an electric circuit when a current of one **ampere** is maintained for one second

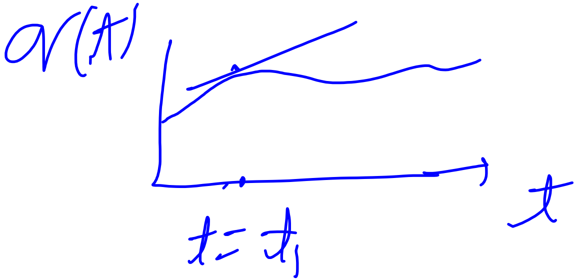
Current

- Electric/ electronic circuits' function depends on flow of charge
- Current is defined as the instantaneous rate at which net positive charge move past a specific point/ cross-section at a specific direction:

$$i = \frac{dq}{dt}$$

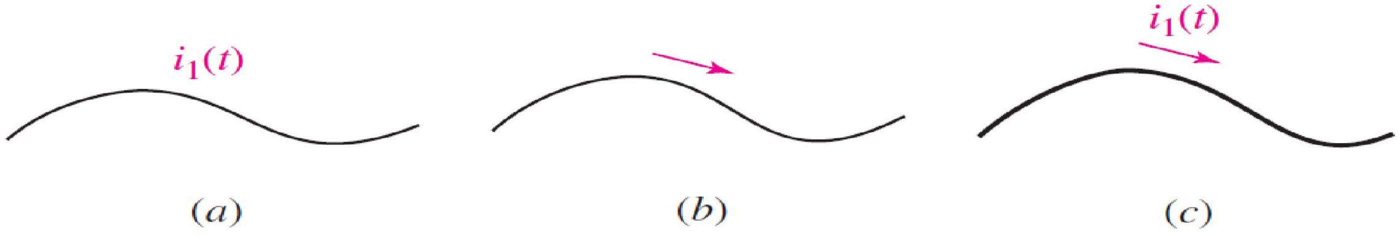
- Unit: Ampere (A): 1 C/s
- Can vary with time (unless DC, in ideal case)
- A current of 5 A flows for 2 minutes in a circuit, find the quantity of electricity transferred.

- Quantity of electricity Q=It coulombs
- I =5 A, t =2×60=120 s
- Hence Q=5×120=**600 C**



Current representation

- Direction is important

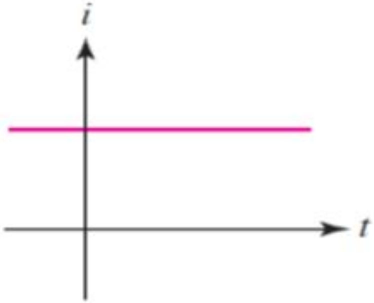


c) Is the correct representation

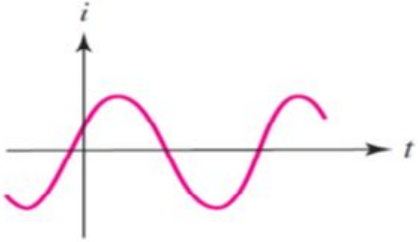
Current representation

Both are
same

Types of current/voltage waveforms

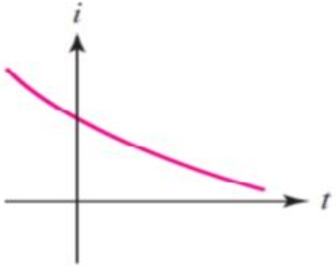


Direct Current (DC)

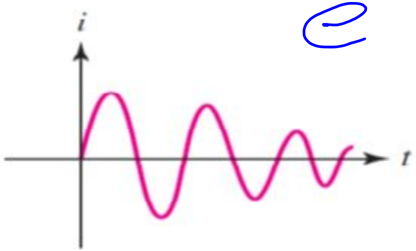


Sinusoidal Current

$e^{-\sigma t} \sin \omega t$



Exponential Current



Damped Sinusoidal Current

Voltage

The unit of electric potential is the volt (V), where one volt is one joule per coulomb. One volt is defined as the difference in potential between two points in a conductor which, when carrying a current of one ampere, dissipates a power of one watt, i.e.

$$\text{volts} = \frac{\boldsymbol{Watt}}{\boldsymbol{amperes}} = \frac{\frac{\boldsymbol{joules}}{\boldsymbol{second}}}{\boldsymbol{ampere}}$$

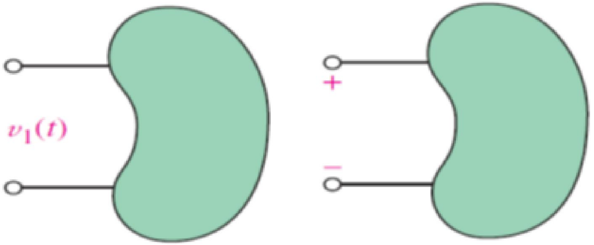
$$= \frac{\boldsymbol{joules}}{\boldsymbol{amperes\ secomds}} = \frac{\boldsymbol{joules}}{\boldsymbol{coulombs}}$$

A change in electric potential between two points in an electric circuit is called a potential difference and its unit is Volt.

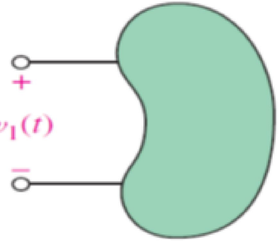
The electromotive force (e.m.f.) provided by a source of energy such as a battery or a generator is measured in volts.

Voltage representation

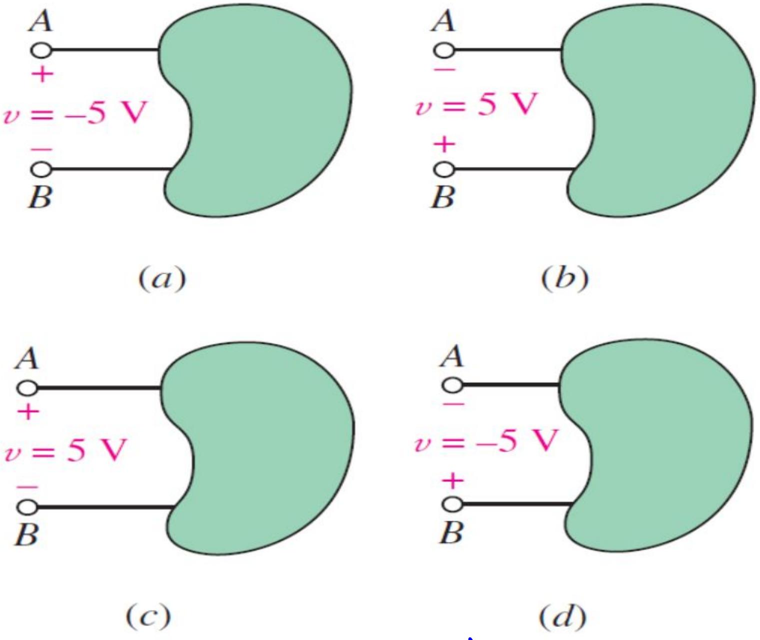
a = b



Incorrect representation



Correct representation



c = d

Electric Power

- When a direct current of I amperes is flowing in an electric circuit and the voltage across the circuit is V volts, then

power in watts $P = VI$

Unit: Watt = 1 J/s

Electrical energy = Power × time

$= V I X t$ joules

$E = P \cdot t$

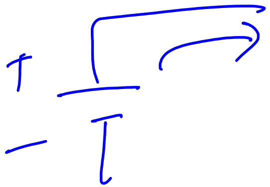
- Although the unit of energy is the joule, when dealing with large amounts of energy the unit used is the **kilowatt hour (kWh)**

where 1kWh = 1000 watt hour

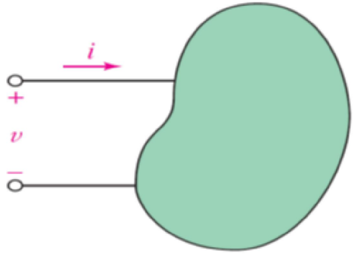
= 1000×3600 watt seconds (joules) = 3600 000 J

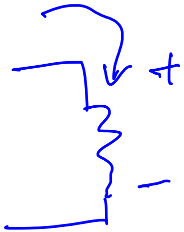
$P = \frac{dE}{dt}$

Passive sign convention



Power absorbed by the element $p = v\,i$
We can also be say that element
generates or supplies a power of $-\,v\,i$





- When the current arrow is directed into the element at the plus-marked terminal, we satisfy the passive sign convention.
- If the current arrow is directed into the "+" marked terminal of an element, then $p = vi$ yields the *absorbed* power. A negative value indicates that power is actually being generated by the element.
- If the current arrow is directed out of the "+" terminal of an element, then $p = vi$ yields the *supplied* power. A negative value in this case indicates that power is being absorbed.

- If a current of 5 A flows for 2 minutes, find the quantity of electricity transferred.

  $Q = I \times t = 5 \times 2 \times 60 = 600\ C$

- An e.m.f. of 250 V is connected across a resistance and the current flowing through the resistance is 4 A. What is the power developed?

- 450 J of energy are converted into heat in 1 minute. What power is dissipated?

- A current of 10 A flows through a conductor and 10 W is dissipated. What p.d. exists across the ends of the conductor?

- A battery of e.m.f. 12 V supplies a current of 5 A for 2 minutes. How much energy is supplied in this time?

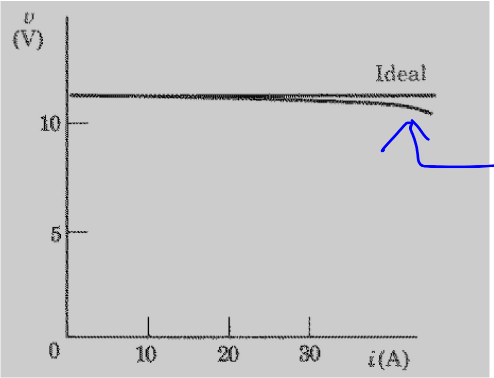
- A d.c. electric motor consumes 36 MJ when connected to a 250 V supply for 1 hour. Find the power rating of the motor and the current taken from the supply.

Energy Sources

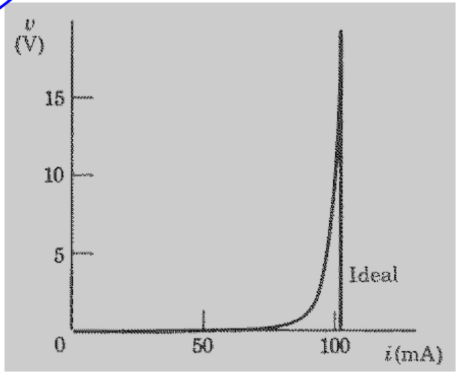
Although every generator supplies both voltage and current, it is desirable to distinguish between two general classes of devices.

If the voltage output is relatively independent of the circuit to which it is connected (as in the battery of Fig. 1), the device is called a "voltage source."

If over wide ranges the output current tends to be independent of the connected circuit (as in the transistor of Fig. 2), it is treated as a "current source."



voltage source
Fig. 1



current source
Fig. 2

Energy Sources

Ideal voltage source: the output voltage is completely independent of current.

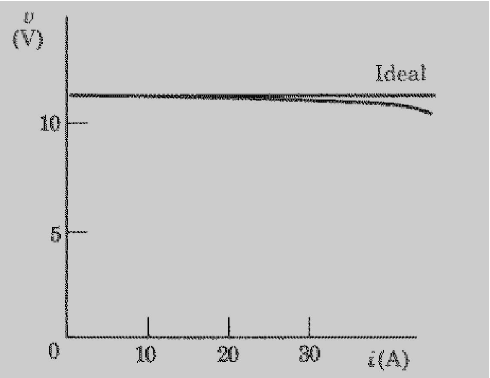
An ideal constant-voltage generator is one with zero internal resistance so that it supplies the same voltage to all loads.

Output voltage is function of time and the voltage is unaffected by changes in circuit configuration. Examples : DC battery, DC power supply

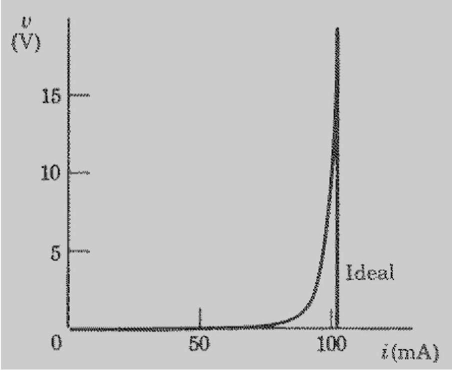
Ideal current source: the output current is completely independent of voltage.

An ideal constant-current generator is one with Infinite internal resistance so that it supplies the same current to all loads.

Output current is also function of time and the current is unaffected by changes in circuit configuration.



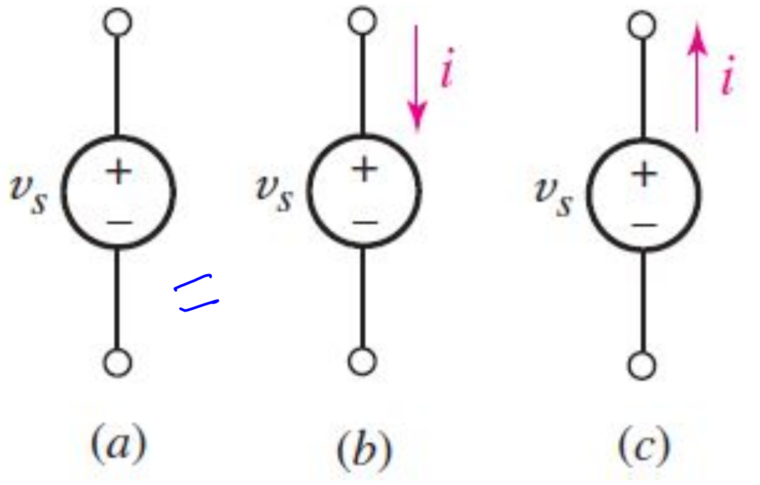
voltage source Fig. 1



current source Fig. 2

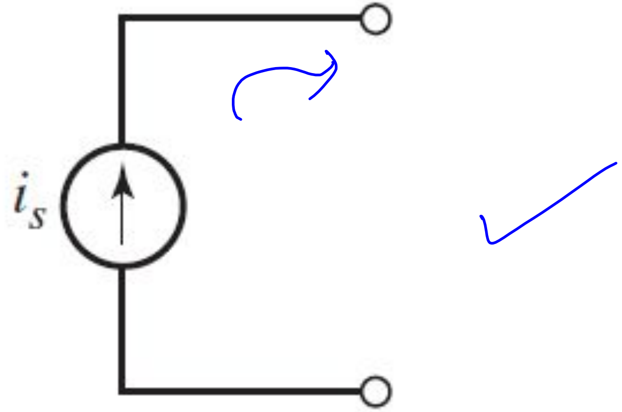
Independent Voltage Source

- Terminal voltage is completely independent of the current
- + sign is a reference, does not necessarily mean that upper terminal is numerically +ve
- This is a theoretical notion, a mathematical model that can be used for further analysis



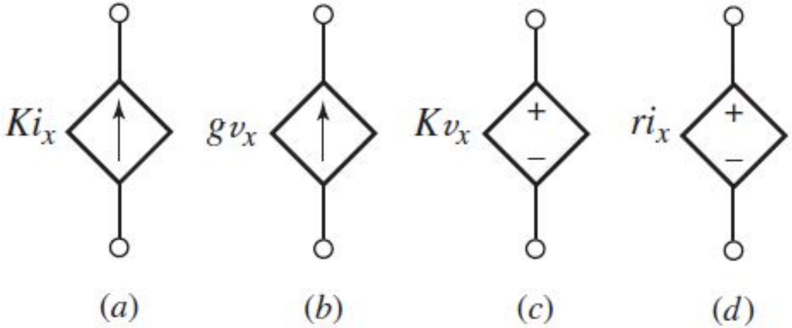
Independent Current Source

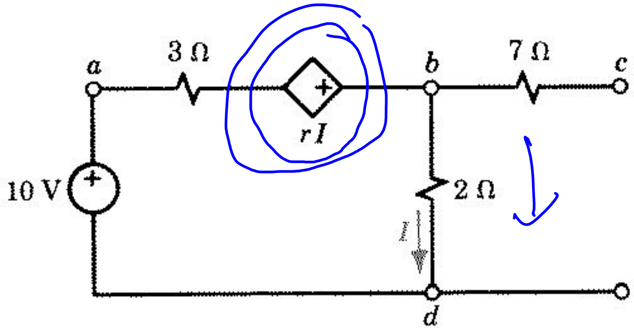
- Current is completely independent of the terminal voltage
- Arrow denotes the direction
- Does not necessarily mean that terminal voltage difference will be 0
- Direction: +ve to –ve is +
- Direction: -ve to +ve is -



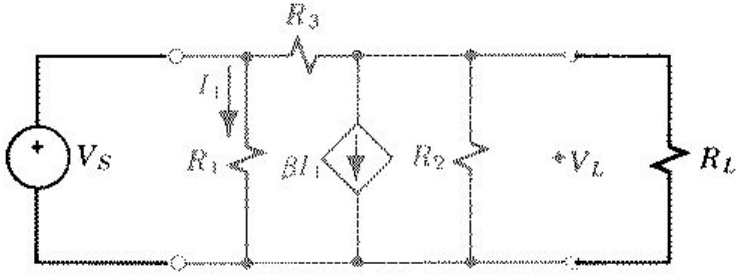
Dependent Voltage or Current Sources

- These are also ideal sources
- Dependent on current or voltage at some other area of the circuit
- Again independent of terminal voltage/current



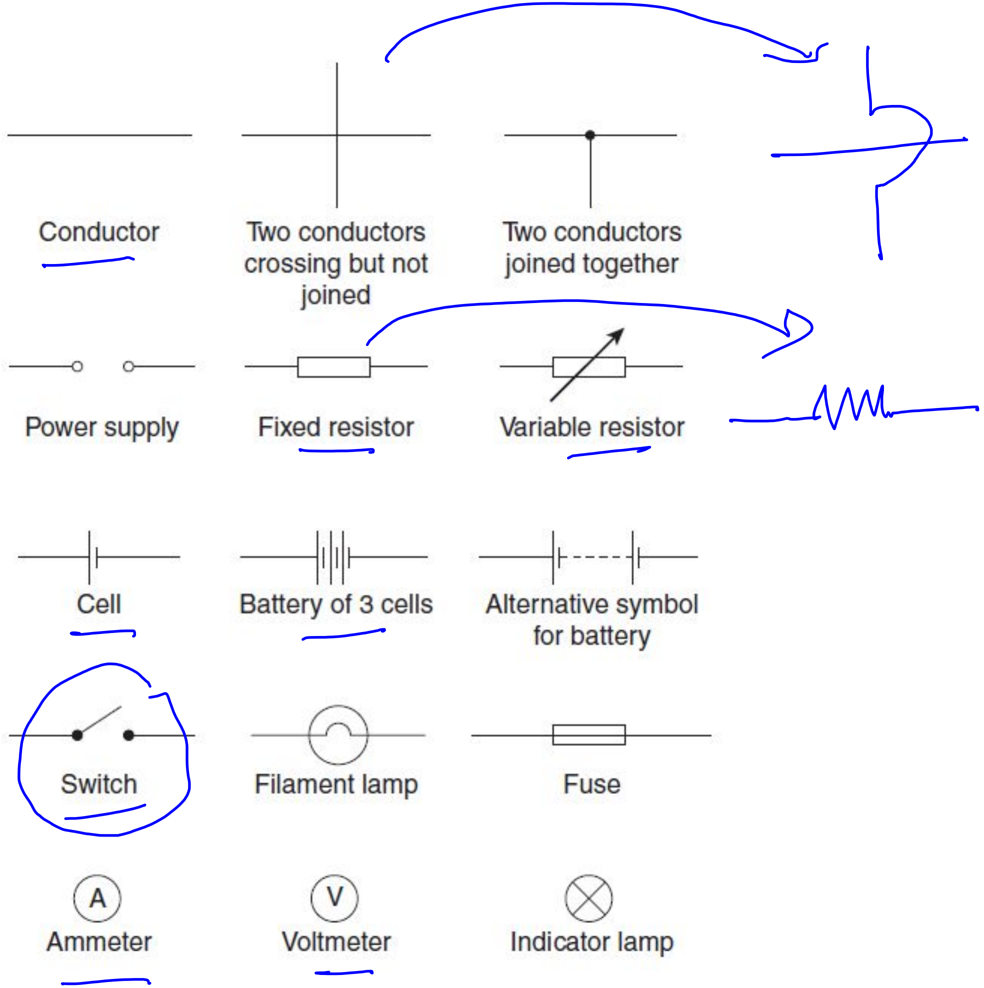


Dependent Voltage Source



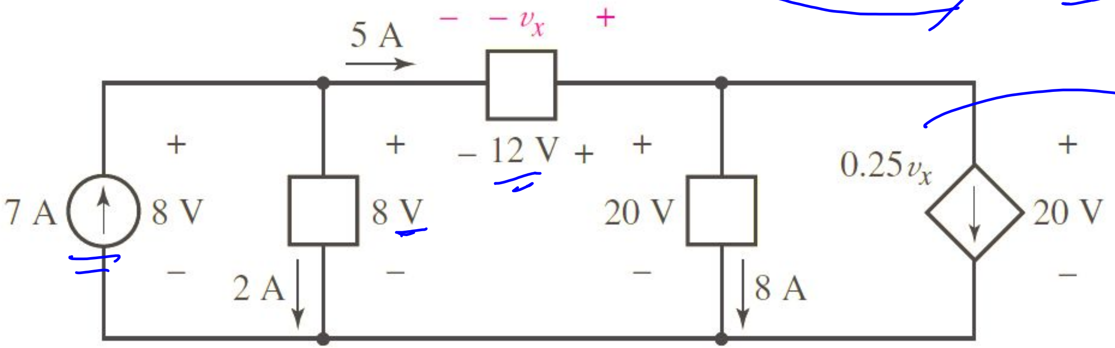
Dependent Current Sources

Standard symbol for electrical components



Example problem

Find the power *absorbed* by each element in the circuit in Fig



$-v_x = 12$

$= -3A$

Ans: (left to right) –56 W; 16 W; –60 W; 160 W; –60 W.

Resistance

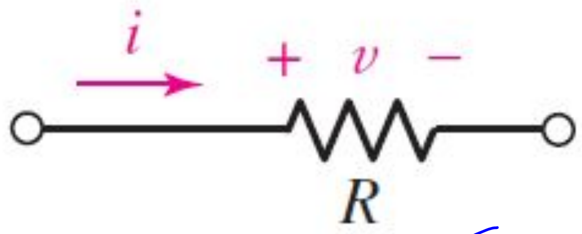
- The flow of electric current is subject to friction. This friction, or opposition, is called **resistance**, ***R***, and is the property of a conductor that limits current.
- The unit of resistance is the **ohm**; 1 ohm is defined as the resistance which will have a current of 1 ampere flowing through it when 1 volt is connected across it.

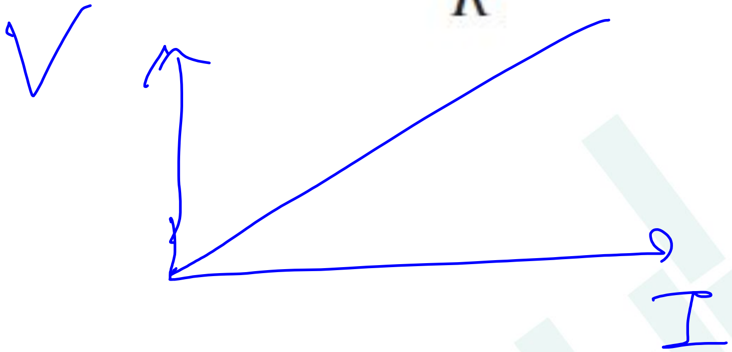
$$\textbf{resistance } \boldsymbol{R} = \frac{\textit{potential difference}}{\textit{current}}$$

- **Ohm's law** states that the current *I* flowing in a circuit is directly proportional to the applied voltage *V* and inversely proportional to the resistance *R*, provided the temperature remains constant. Thus

$$i = \frac{v}{R} \qquad \text{or} \qquad v = I \,.\, R \qquad \text{or} \quad R = \frac{v}{i}$$

- Symbol of resistance is shown un figure





Resistance

Electric Power

- **Power *P*** in an electrical circuit is given by the product of potential difference *V* and current *I*
- The unit of power is the **watt**, **W**.

$$P = V \times I \text{ watts}$$

From Ohm's law $V = I R$, thus

$$P = I^2 R \text{ watts}$$

This is also equivalent to $P = \frac{V^2}{R}$ watts

$P = VI = I^2R$

$= \frac{V^2}{R}$

- A 100 W electric light bulb is connected to a 250 V supply. Determine (a) the current flowing in the bulb, and (b) the resistance of the bulb

Current $I = P / V = 100/250 = 0.4$ amp

Resistance $R = V/I = 250/0.4 = 625\ \Omega$

$R = \varepsilon$

$\varepsilon \to 0$

Resistance and resistivity

The resistance of an electrical conductor depends on

- the length of the conductor
- the cross-sectional area of the conductor
- the type of material and
- the temperature of the material.

Resistance, R,

- directly proportional to length, l, of a conductor, i.e. $R \propto l$.
- inversely proportional to cross-sectional area, a, of a conductor, i.e. $R \propto 1/a$.

Thus resistance $R \propto l/a$

By inserting a constant of proportionality into this relationship the type of material used may be taken into account.

The constant of proportionality is known as the **resistivity** of the material and is given the symbol ρ (Greek rho). Thus

Resistance $R = \frac{\rho\, l}{A}$ ohms

Unit of ρ is ohm meter.

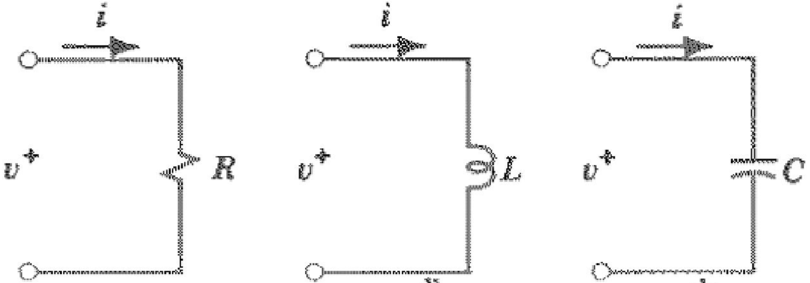
$$R = \frac{\rho l}{A}$$

Problems

- A piece of wire of cross-sectional area 2 mm^2 has a resistance of 300 Ω. Find (a) the resistance of a wire of the same length and material if the cross-sectional area is 5 mm^2, (b) the cross-sectional area of a wire of the same length and material of resistance 750 Ω.
- Calculate the resistance of a 2 km length of aluminium overhead power cable if the cross-sectional area of the cable is 100 mm^2. Take the resistivity of aluminium to be $0.03\times10^{-6}\,\Omega$m.
- Some wire of cross-sectional area 1 mm^2 has a resistance of 20 Ω . Determine (a) the resistance of a wire of the same length and material if the cross-sectional area is 4 mm^2, and (b) the cross- sectional area of a wire of the same length and material if the resistance is 32 Ω.

$$\left.\frac{R_1}{R_2} = \frac{A_2}{A_1}\right\} l \qquad \left.\frac{R_1}{R_2} = \frac{l_1}{l_2}\right\} A$$

Circuit Elements



$$v = Ri \qquad i = \frac{v}{R} = Gv$$

$$v = L\frac{di}{dt} \qquad i = \frac{1}{L}\int_{-\infty}^{t} v\, dt$$

$$i = C\frac{dv}{dt} \qquad v = \frac{1}{C}\int_{-\infty}^{t} i\, dt$$

Resistance Inductance Capacitance

Capacitor

A capacitor is an electrical device that is used to store electrical energy.

Next to the resistor, the capacitor is the most commonly encountered component in electrical circuits.

Capacitors are used extensively in electrical and electronic circuits.

For example, capacitors are used to smooth rectified a.c. outputs, they are used in telecommunication equipment – such as radio receivers – for tuning to the required frequency, they are used in time delay circuits, in electrical filters, in oscillator circuits and in magnetic resonance imaging (MRI) in medical body scanners, to name but a few practical applications.

Every system of electrical conductors possesses capacitance.

For example, there is capacitance between the conductors of overhead transmission lines and also between the wires of a telephone cable.

In these examples the capacitance is undesirable but has to be accepted, minimized or compensated for.

There are other situations where capacitance is a desirable property. Devices specially constructed to possess capacitance are called capacitors (or condensers, as they used to be called). In its simplest form a capacitor consists of two plates which are separated by an insulating material known as a dielectric. A capacitor has the ability to store a quantity of static electricity.

Capacitor

Figure 1(a) shows two parallel conducting plates separated from each other by air. They are connected to opposite terminals of a battery of voltage V volts.

Static electric fields arise from electric charges, electric field lines beginning and ending on electric charges. Thus the presence of the field indicates the presence of equal positive and negative electric charges on the two plates of Figure 1. Let the charge be +Q coulombs on one plate and –Q coulombs on the other. If the voltage is disconnected, the charge persists on capacitor.

The property of this pair of plates which determines how much charge corresponds to a given p.d. between the plates is called their capacitance:

$$C = \frac{Q}{V} \text{ Farad or } Q = C\,V$$

$$Q = CV$$

Symbol of capacitor is shown in Figure 1(b)

If voltage V is constant, no current flows through capacitor. If V is changing with time, there is a current flow through the capacitor and it is given by

$$\boxed{i \cong C\frac{dv}{dt}}$$

$$C = \frac{\varepsilon A}{d}$$

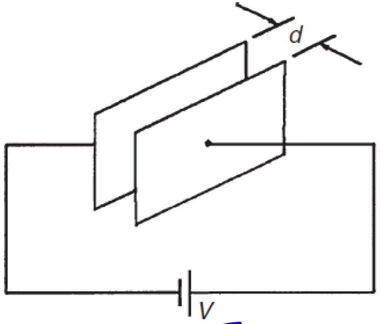


Figure 1(a)

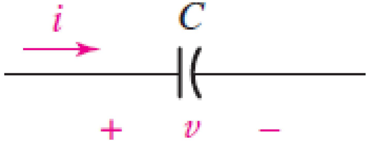


Figure 1(b)

Capacitor

$i = C\frac{dv}{dt}$

Current-voltage relationship

We have $C = Q/V$ or $V = Q.C$

Thus for a current $i(t)$, the voltage $v(t)$ across the capacitor is given as

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt' + v(t_0)$$

where $v(t_0)$ is initial charg

- Find the capacitor voltage that is associated with the current shown graphically in Figure 3. The value of capacitor is 5 μF.

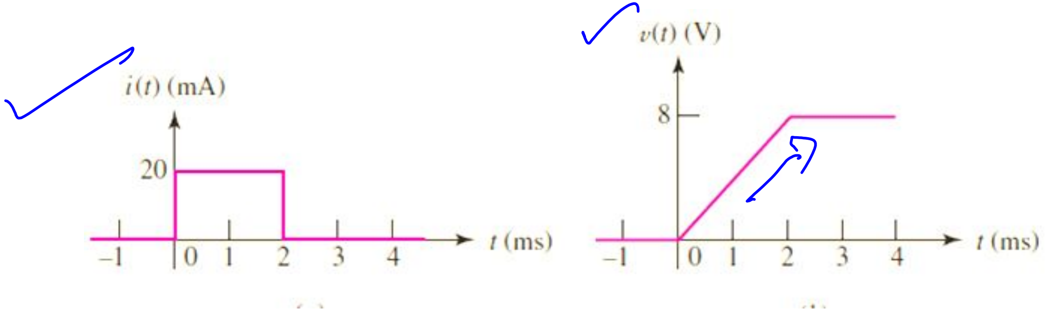


Figure 3

Capacitor

The power across the capacitor is given as

$$\text{Power: } p = vi = vC\frac{dv}{dt}$$

Stored energy can be calculated by integrating power considering zero initial voltage:

$$w_C(t) = \frac{1}{2}Cv(t)^2$$

$$E = \int P\,dt = \int_0^t V I\,dt$$

$$= \int_0^t V \cdot C \frac{dv}{\cancel{dt}} \cdot \cancel{dt}$$

$$= \int_0^{v(t)} V C\,dv = \frac{1}{2} C v(t)^2$$

Inductor

$\phi = LI$

If a conductor is wound as a coil as shown in Figure 5, the coil behaves as an inductance.

Voltage across the coils is approximately proportion to rate of change of current and it is given by following relation:

$$\text{Induced voltage } (v) = L\frac{di}{dt} \text{ volts.}$$

L is constant of proportionality and is measured in henrys.

This relationship can be derived from our knowledge of magnetic fields.

If a magnetic flux (Φ) is obtained by passing a current i, the Inductance (L) is $L = \frac{\Phi}{i}$

(Faraday's Law of induction) and we get,

$$v = \frac{d\Phi}{dt} = L\frac{di}{dt}$$

Given the waveform of the current as shown in Figure 6(a). Plot the voltage waveform.

The voltage output is 3 di/dt and voltage waveform is plotted in

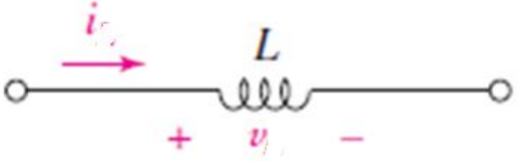


Figure 5

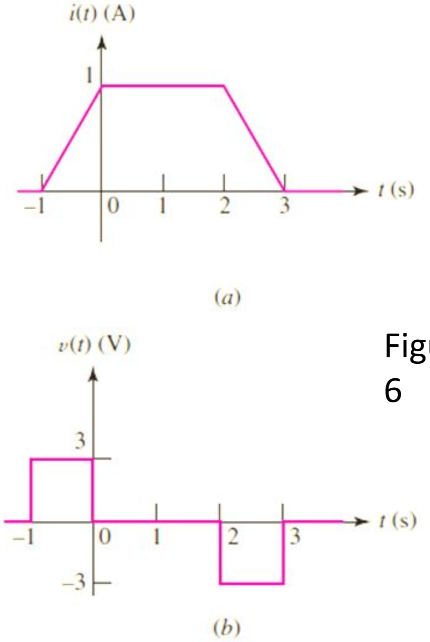
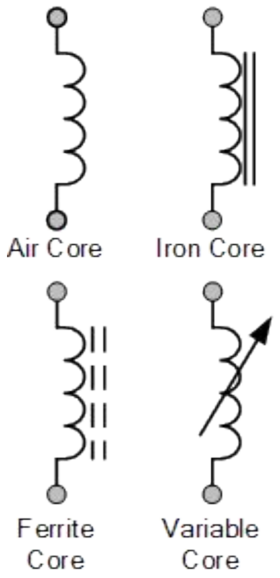
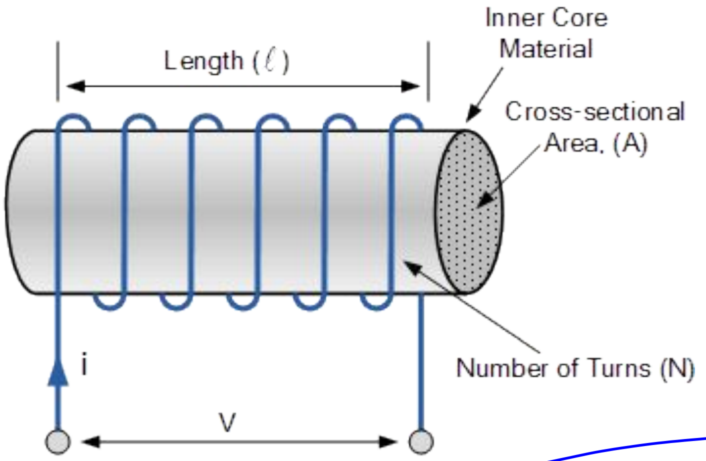


Figure 6

Inductor

$V(t)$ $\quad V \neq \infty$ $\quad I(t)$



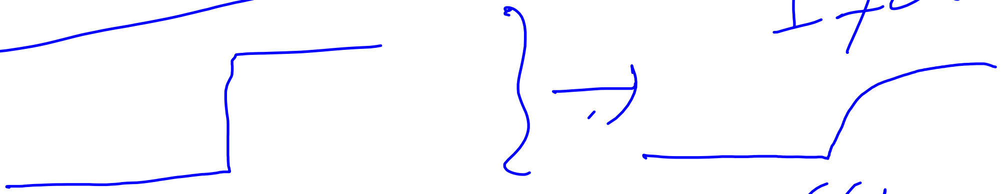


$$\boxed{L = \frac{N^2 \mu A}{l}}$$

μ is the permeability of the core

$I \neq \infty$

Capacitors



$I(t)$ $\quad = V(t)$

Inductor

- The Voltage-current relationship is given as

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt' + i(t_0)}$$

Power $p = vi = Li\frac{di}{dt}$

Energy with zero initial current: $w_L(t) = \frac{1}{2}Li(t)^2$

$$E = \int P\,dt = \int V I\,dt$$

$$= \int L I . \frac{dI}{\cancel{dt}} . \cancel{dt}$$

$$= \frac{1}{2} L I^2$$

Energy stored in linear elements

If the *v-i* characteristics of a device is known, energy = $\int v\, i\, dt$

Inductance

We know v = $L\, di/dt$ volts and I = 0 at t = 0,

and

This equation t[illegible] $w_L = \int_0^T L \frac{di}{dt} i\, dt = \int_0^I Li\, di = \frac{1}{2} LI^2$ nd it is not dissipated like resistance.

Capacitance

In capacitance, we know $i = C\, dv/dt$ and v across the capacitor is zero at $t = 0$.

and

In capacitance also $w_C = \int_0^T vC \frac{dv}{dt} dt = \int_0^V Cv\, dv = \frac{1}{2} CV^2$

Resistance

In resistance, we have v = R i and I = 0 at t = 0 and

$$w_R = \int_0^T Ri\, i\, dt = \int_0^T Ri^2\, dt = Ri^2 T$$

Capacitor and Inductor

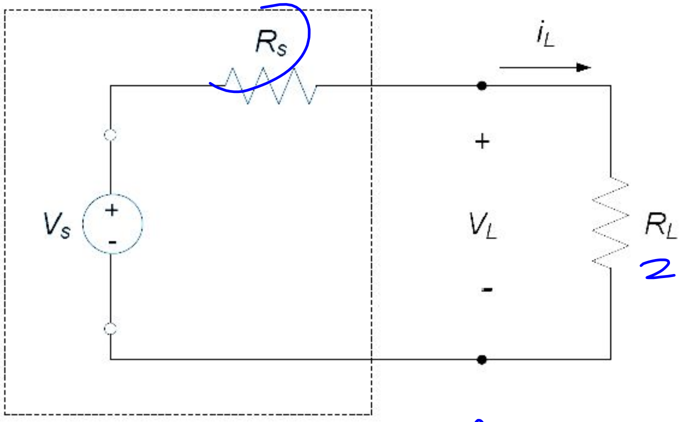
- These elements store and deliver finite amount of energy
- Passive linear circuit elements
- The current-voltage relationships for these new elements are time dependent
- Ideal Capacitors and Inductors can neither generate nor dissipate energy

Applicability of KCL and KVL

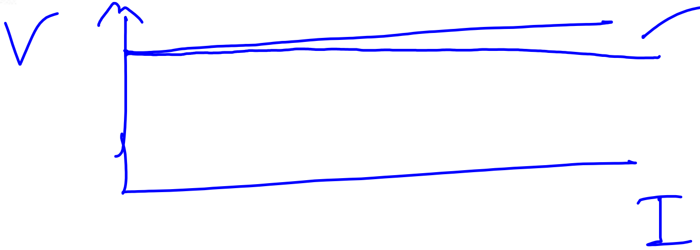
- L and C are linear elements
- Kirchhoff's laws can be applied (are not restricted to only resistance)

Non-ideal Voltage Source

trade-off



$R_L >> R_s$



non-ideal source

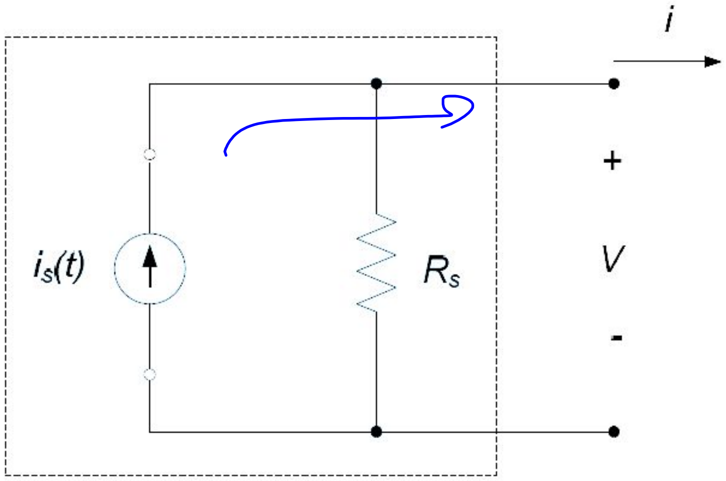
- Modelled as an ideal voltage source V_S and internal resistance R_S
- When load resistance R_L draws large current, supply voltage V_S decreases and the converse
- It does not provide infinite power
- Ideal voltage source has zero internal resistance R_S

Problem:

- The no load or open circuit voltage of voltage source is 16 Volts. When a load resistance of 10 ohm is applied, the output voltage V_L is 8 Volts. Find the source resistance R_S.
- Twelve cells, each with an internal resistance of 0.24 and an e.m.f. of 1.5 V are connected (a) in series, (b) in parallel. Determine the e.m.f. and internal resistance of the batteries so formed.

Non-ideal current source

- Modelled as an ideal current source I_S and internal resistance R_S in parallel
- Higher terminal voltage results in lower supply current
- Ideal current source means a source with infinite internal resistance

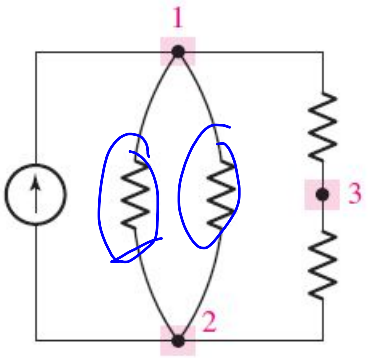


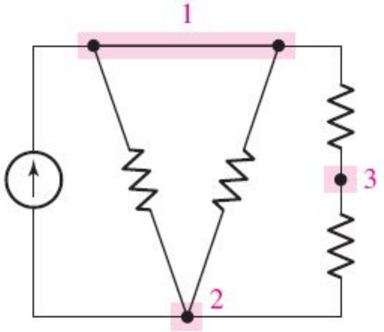
Ideal case ← $R_s \approx \infty$

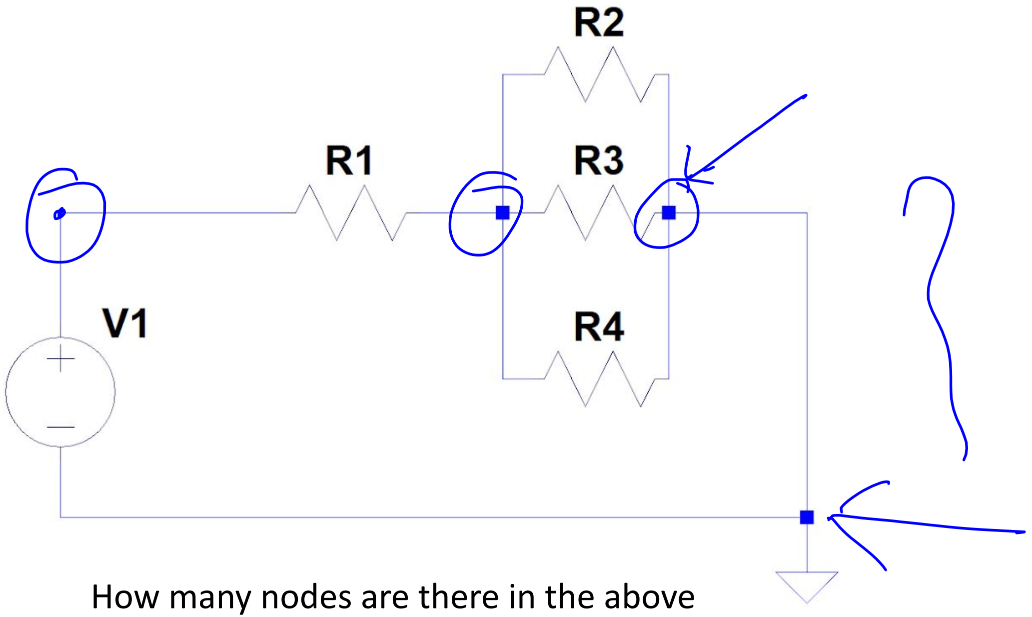
Nodes

branches

- Node: A point at which two or more circuit elements have a common connection.



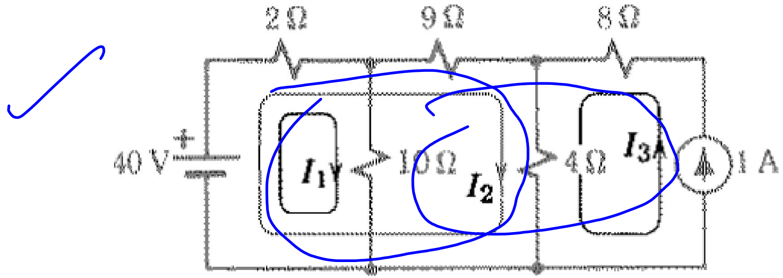


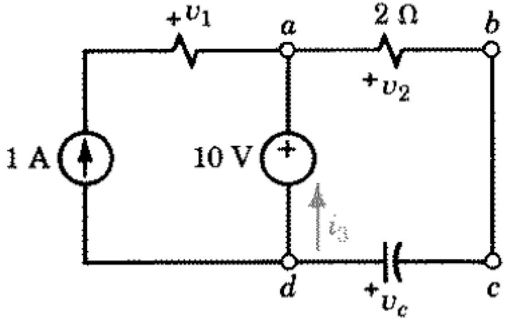


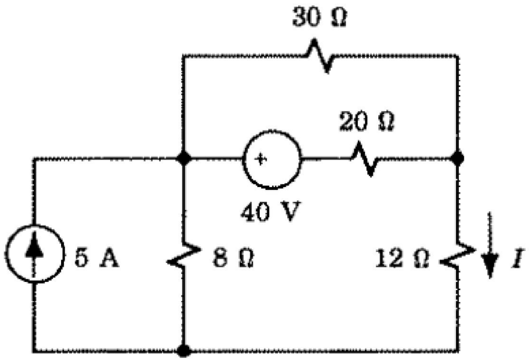
How many nodes are there in the above circuit?

Loops

- Loop: If a set of nodes and elements are traversed in such a way that no node is encountered more than once, then the set of nodes and elements are called a path. If the start and end point of a path is the same node, then it is called a loop.





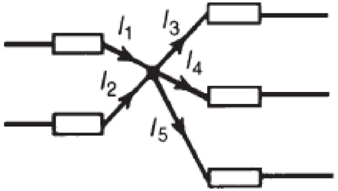


Kirchhoff's Laws

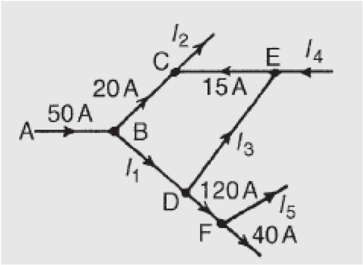
- Kirchhoff's Current Law (KCL):
- At any junction in an electric circuit the total current flowing towards that junction is equal to the total current flowing away from the junction, i.e. ∑ I =0
- Thus current entering a node is equal to the current exiting the node
- For the given example

$$I_1 + I_2 = I_3 + I_4 + I_5 \quad \text{or}$$

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$



- Find the unknown currents marked in Figure



Kirchhoff's Laws - Examples

Use Kirchhoff's laws to determine the currents flowing in each branch of the network shown in Figure 1.

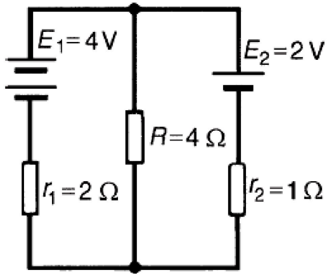


Fig. 1

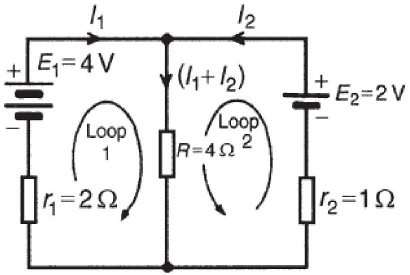


Fig. 2

Procedure Use Kirchhoff's current law and label current directions on the original circuit diagram. The directions chosen are arbitrary, but it is usual, as a starting point, to assume that current flows from the positive terminals of the batteries. This is shown in Figure 2, where the three branch currents are expressed in terms of *I*1 and *I*2 only, since the current through *R* is *I*1+*I*2.

Kirchhoff's Laws - Examples

- Divide the circuit into two loops and apply Kirchhoff's voltage law to each. From loop 1 of Figure 2, and moving in a clockwise direction as indicated (the direction chosen does not matter), gives

$$E_1 = I_1 r_1 + (I_1 + I_2)R, \text{ i.e. } 4 = 2I_1 + 4(I_1 + I_2)$$

$$\text{i.e. } 6I_1 + 4I_2 = 4 \qquad (1)$$

- From loop 2 of Figure 2, and moving in an anticlockwise direction as indicated (once again, the choice of direction does not matter; it does not have to be in the same direction as that chosen for the first loop), gives:

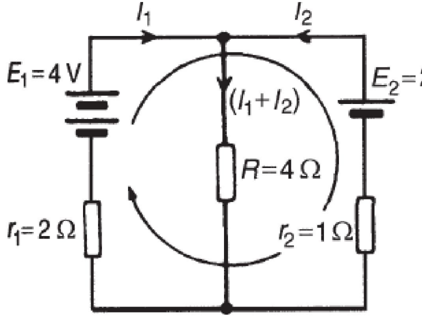
$$E_2 = I_2 r_2 + (I_1 + I_2)R, \text{ i.e. } 2 = I_2 + 4(I_1 + I_2)$$

$$\text{i.e. } 4I_1 + 5I_2 = 2 \qquad (2)$$

- From equ. 1 and 2, we get

  I1 = 0.867 A and I2 = -0.286 A and I1 + I2 = 0.571 A

- Note that a third loop is possible, as shown in Figure 3, giving a third equation which can be used as a check. How?



Kirchhoff's Laws - Examples

For the bridge network shown in Figure 4 determine the currents in each of the resistors.
Let the current in the 2 resistor be $i1$, then by Kirchhoff's current law, the current in the 14 resistor is ($i - i1$). Let the current in the 32 resistor be $i2$ as shown in Figure 5. Then the current in the 11 resistor is ($i1 - i2$) and that in the 3 resistor is ($i - i1 + i2$). Applying Kirchhoff's voltage law to loop 1 and moving in a clockwise direction as shown in Figure 5 gives:

$$13\, i1 - 11\, i2 = 54 \qquad (1)$$

Applying Kirchhoff's voltage law to loop 2 and moving in an anticlockwise direction as shown in Figure 5 gives:

$$16\, i1 + 32\, i2 = 112 \qquad (2)$$

Using equ. (1) and (2), find all currents.

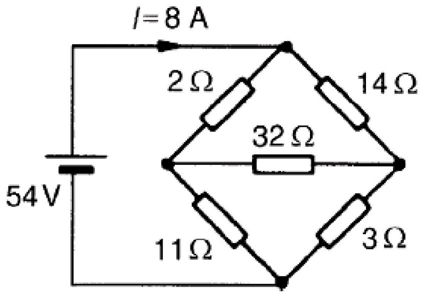
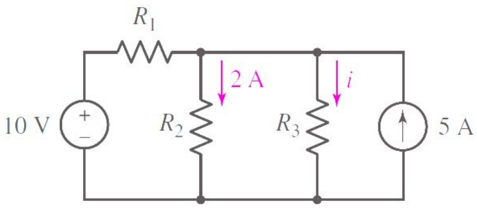


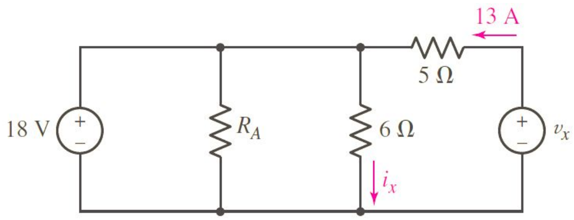
Fig. 4

Problems

For the circuit in the Fig, compute the current through resistor R3 if it is known that the voltage source supplies a current of 3 A.



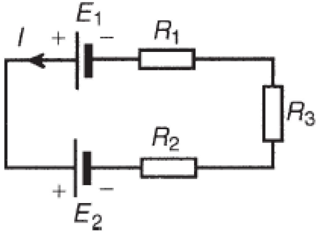
If ix = 3 A and the 18 V source delivers 8 A of current, what is the value of RA?



Kirchhoff's Laws

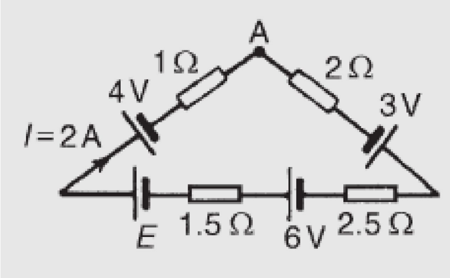
Kirchhoff's Voltage Law (KVL):

In any closed loop in a network, the algebraic sum of the voltage drops (i.e. products of current and resistance) taken around the loop is equal to the resultant e.m.f. acting in that loop.



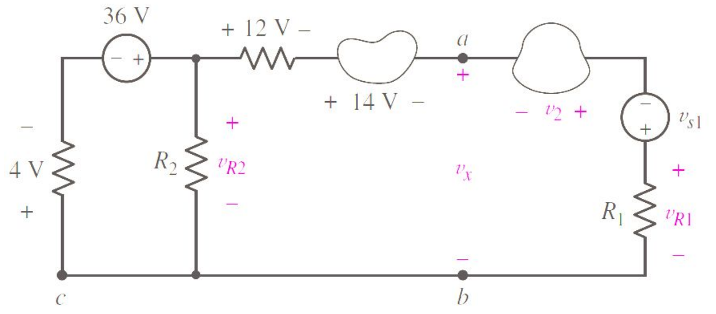
Thus $E_1 - E_2 = IR_1 + IR_2 + IR_3$

Determine the value of e.m.f. E in Figure

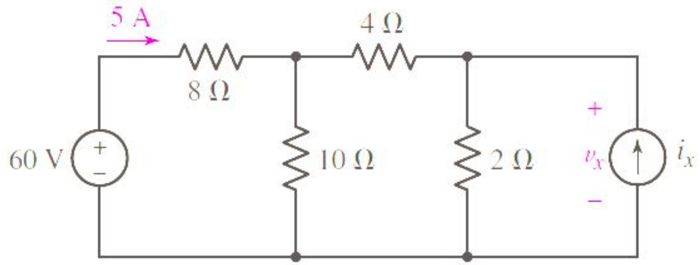


Problems

Find v_{R2} (the voltage across R2) and the voltage labeled v_x.



Determine v_x



Quick Recap

Resistance (R)
Inductance (L)
Capacitance (C)

} Passive Elements

Volatage source
Current source

} Active Elements

Independent source
Dependent source

Current-controlled

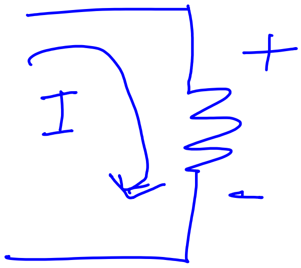
Voltage controlled

$$R = \frac{\rho l}{A}$$ ✓

Algebraic Sum

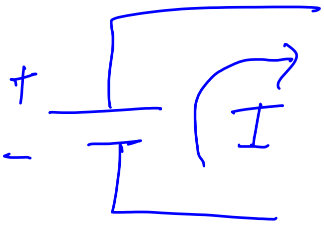
KCL →

KVL →



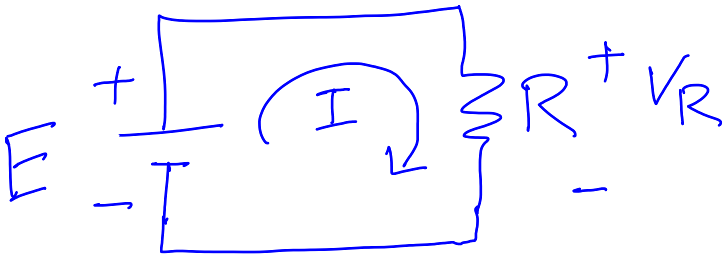
✓

Power absorbed



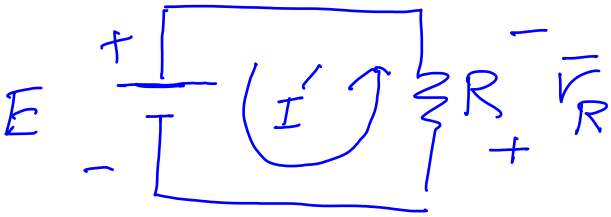
Power generated

KCL and KVL are also applicable to circuits having C and L.



$$E - V_R = 0 \Rightarrow E = V_R = IR$$

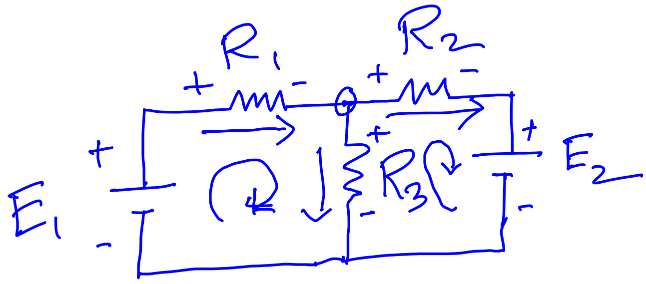
$$\underline{\underline{I = E/R}}$$



$$E + \bar{V}_R = 0 \Rightarrow I' = -E/R$$

Initial choice of current direction can be arbitrary ....

$I = -I'$ (in the above example)



I_1, I_2, I_3
$\downarrow \quad \downarrow \quad \downarrow$
$R_1 \quad R_2 \quad R_3$

$$\begin{cases} I_1 = I_2 + I_3 & \text{--- (1) (KCL)} \\ E_1 = I_1 R_1 + I_3 R_3 & \text{--- (2) (KVL)} \\ I_3 R_3 = I_2 R_2 + E_2 & \text{--- (3) (KVL)} \end{cases}$$

System of Linear Equations . .

$$\underbrace{A}_{3\times 3}\, x = \underbrace{b}_{3\times 1} \checkmark$$

$$x = \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{3\times 1}$$

$$A = \begin{bmatrix} 1 & -1 & -1 \\ R_1 & 0 & R_3 \\ 0 & -R_2 & R_3 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ E_1 \\ E_2 \end{bmatrix}$$

Check for $\det(A) \neq 0$

for unique sol.

$$x = A^{-1} b$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{b}$$

C_1 C_2

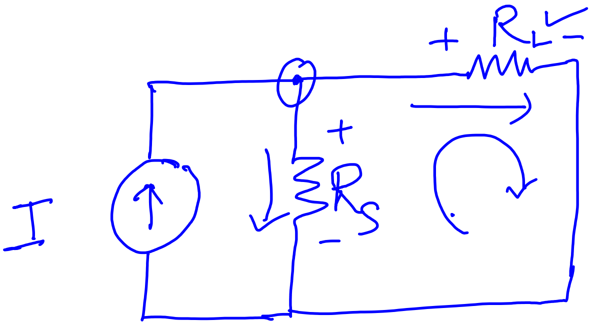
infinitely many solutions $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{b_1}{b_2}$

$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} \longrightarrow$ no solution (parallel lines)

$\alpha_1 C_1 + \alpha_2 C_2 = 0 \Rightarrow \alpha_1 = 0$ and $\alpha_2 = 0$

C_1, C_2 are linearly independent.

Same concept can be extended to "n" equations. . .



$I_S \, , \; I_L$

$I = I_S + I_L$ (KCL)

$I_S R_S = I_L R_L$ (KVL)

$I_L = \dfrac{R_S}{R_L} I_S$

$I_L \left(1 + \dfrac{R_S}{R_L}\right) = I$

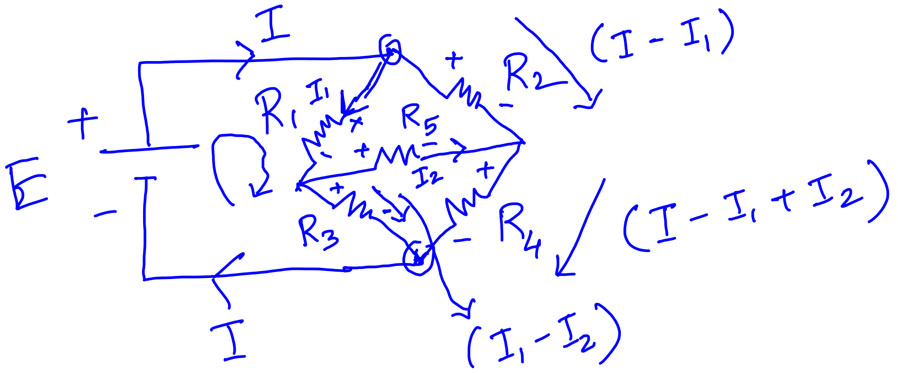
$$I_L = \frac{R_L}{R_S + R_L} I$$

$A = \begin{bmatrix} 1 & 1 \\ R_S & -R_L \end{bmatrix}$

$b = \begin{bmatrix} I \\ 0 \end{bmatrix}$

$x = A^{-1} b$

$x = \begin{bmatrix} I_S \\ I_L \end{bmatrix}$



$$Ax = b$$

$$E = I_1 R_1 + (I_1 - I_2) R_3$$

$$I_1 R_1 + I_2 R_5 = (I - I_1) R_2 \checkmark$$

$$I_2 R_5 + (I - I_1 + I_2) R_4 = (I_1 - I_2) R_3$$

$$x = \begin{bmatrix} I \\ I_1 \\ I_2 \end{bmatrix}$$

$A \rightarrow 3 \times 3$

$b \rightarrow 3 \times 1$

A circuit having "b" no. of branches

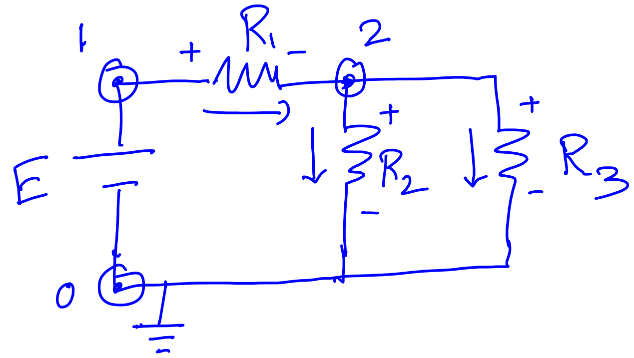
How many minimum no. variables you need to compute "b" branch voltages and "b" branch currents?

b no. of unknowns!

Let's use the following notations throughout...

b = no. of branches/elements

n = no. of nodes (in a circuit)



$n = 3$ $\qquad b = 4$

V_1 and V_2

$V_1 \rightarrow$ voltage of node 1 w.r.t node 0.

$V_2 \rightarrow$ .. ———— –

V_{R_1}
V_{R_2}
V_{R_3}
V_E

branch voltages

$\rightarrow \left\{ V_1, V_2 \right\}$ node-to-reference voltages

$V_1 = E$

$$\frac{V_1 - V_2}{R_1} = \frac{V_2}{R_2} + \frac{V_2}{R_3}$$

$\rightarrow$ Independent variables

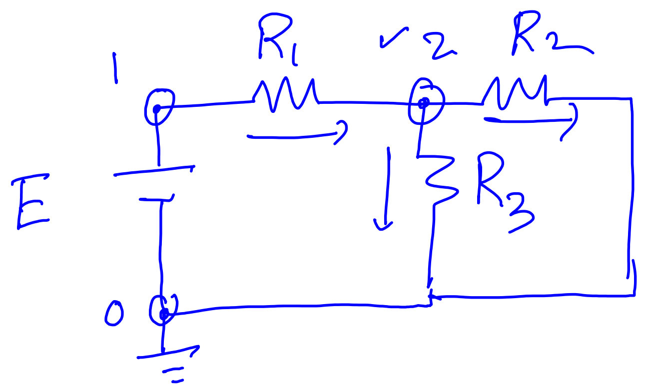
$\rightarrow$ KCL

Here, node "0" is considered as reference node.

$(n-1)$ voltage variables to solve for a linear circuit problem.

(node-to-reference voltages)
(datum)

Node Variable Analysis



$$\left.\begin{array}{l} \underline{b} = 4 \\ \underline{n} = 3 \end{array}\right\}$$

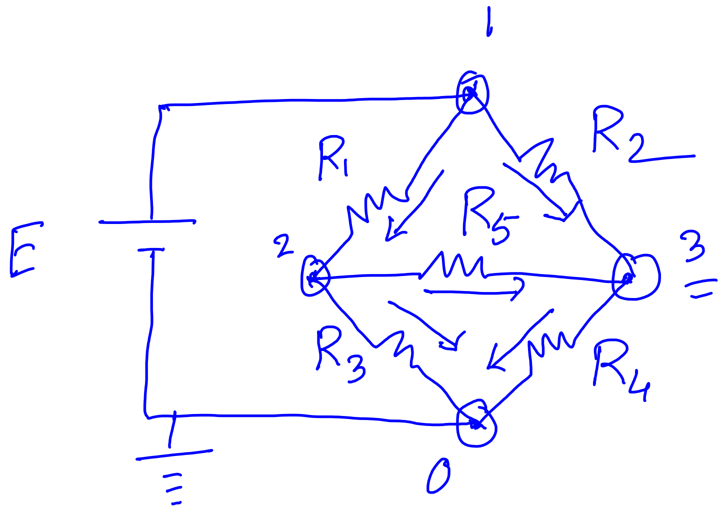
$\underline{(n-1)}$

$V_1, \; V_2$

$$\frac{V_1 - V_2}{R_1} = \frac{V_2}{R_3} + \frac{V_2}{R_2}$$

$V_1 = E$

Solve for V_2



$b = 6$

$n =$

$V_1, \; \underline{V_2}, \; \underline{V_3}$

$V_1 = E$

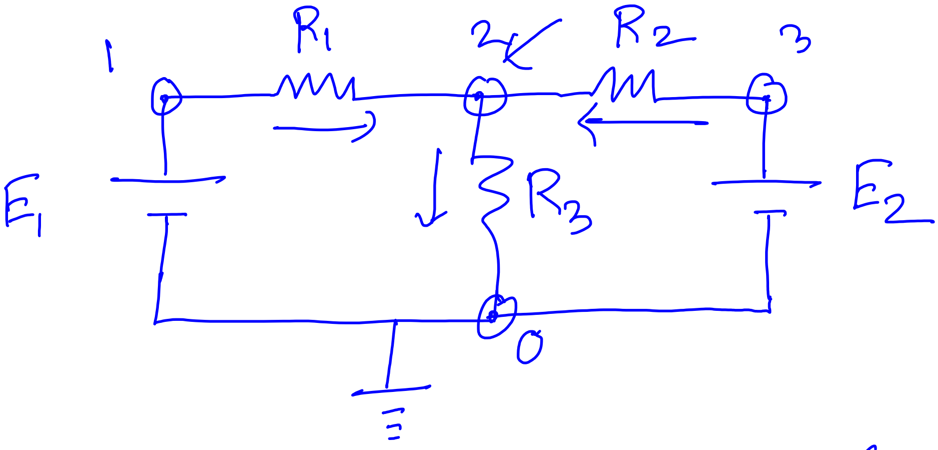
KCL at node 2,

$$\frac{V_1 - V_2}{R_1} = \frac{V_2 - V_3}{R_5} + \frac{V_2}{R_3}$$

KCL at node 3,

$$\frac{V_1 - V_3}{R_2} + \frac{V_2 - V_3}{R_5} = \frac{V_3}{R_4}$$

solve for V_2 and V_3



$n =$

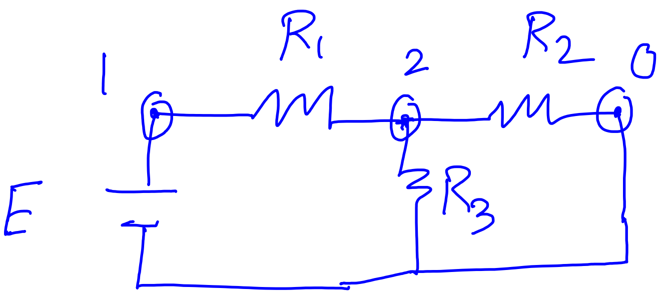
V_1, V_2, V_3

$V_1 = E_1$, $V_3 = E_2$

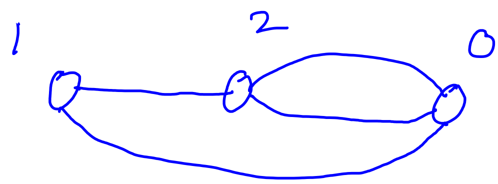
$$\frac{V_1 - V_2}{R_1} + \frac{V_3 - V_2}{R_2} = \frac{V_2}{R_3}$$

Loop Variable Analysis

Topological description of a circuit



circuit



(graph)

$G = \{V, E\}$

set of nodes (vertices)

set of Edges (links/branches)

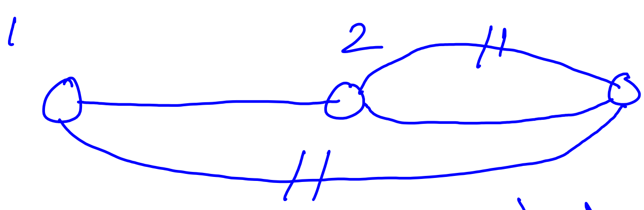
$V = \{0, 1, 2\}$

$E = \{\{1, 2\}, \text{—}\}$

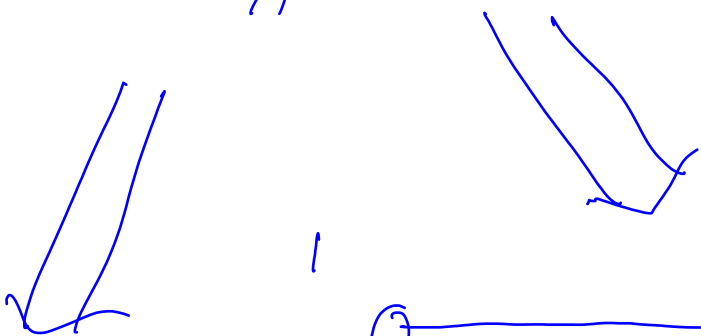
From a graph, if we remove few edges, we can create a sub-graph.

Tree is a special type of Graph, which has the following properties.

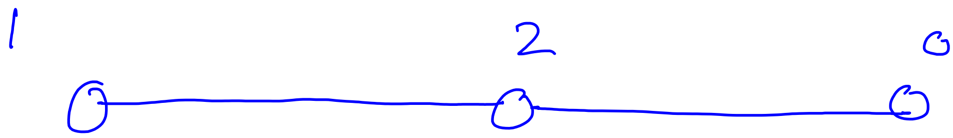
1> There are no closed-paths (loops)
2> It has exactly $(n-1)$ links/edges
3> No node is kept isolated.

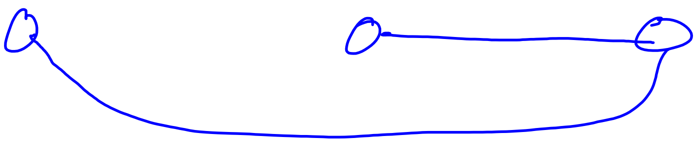


(original graph)



(tree)





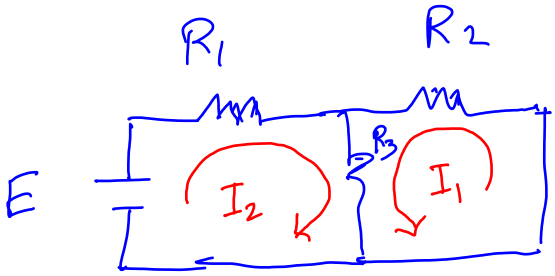
(tree)

Chords = branches that you remove from the graph to construct a tree.

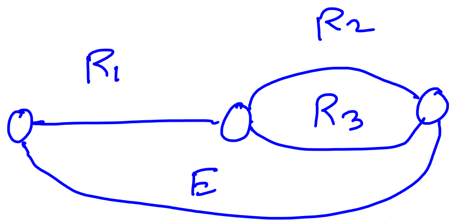
no. of chords = no. of branches in the original graph − no. of branches in the tree

$= b - (n-1)$

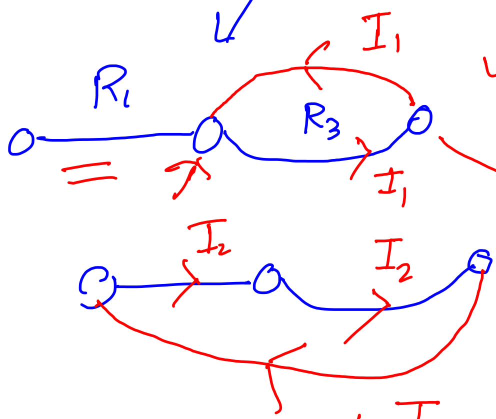
$= b - n + 1$ ✓



(circuit)

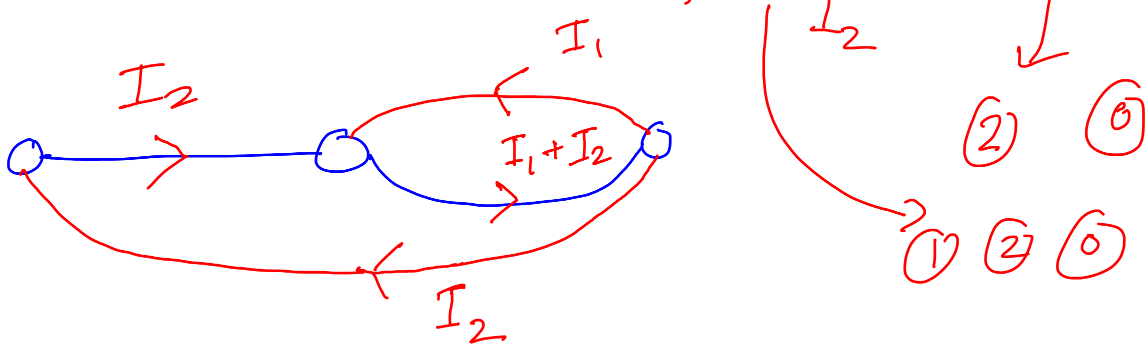


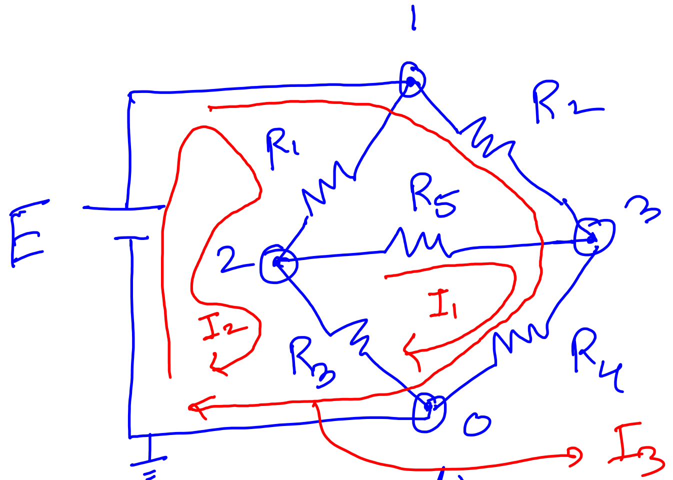
(graph)



(tree)

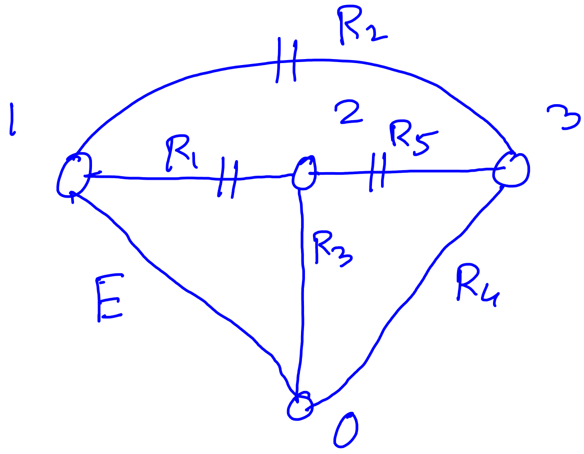
(E, R_2) → chords



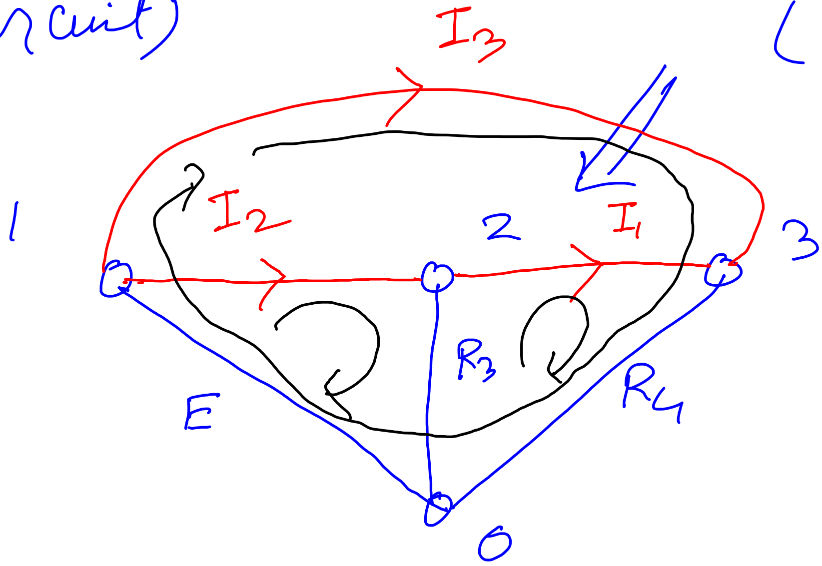


(circuit)

$\Rightarrow$



(graph)



$$I_1 R_5 + (I_1 + I_3) R_4 + (I_1 - I_2) R_3 = 0$$

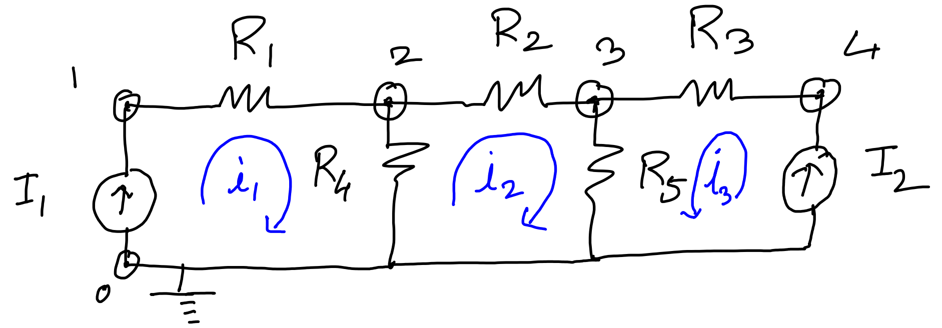
$$I_2 R_1 + (I_2 - I_1) R_3 = E$$

$$I_3 R_2 + (I_1 + I_3) R_4 = E$$

Quick Recap

"b" → no. of branches
"n" → no. of nodes

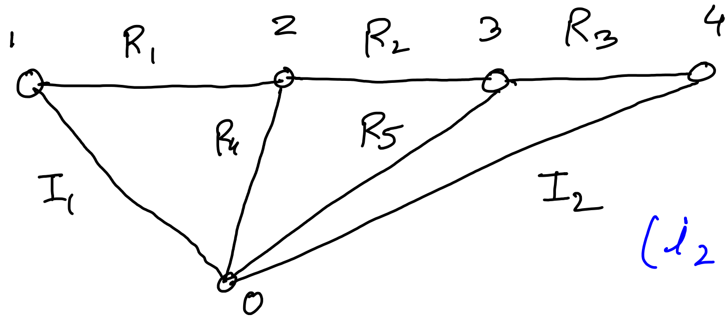
$(n-1)$ variables to do node analysis
$(b-n+1)$ variables to do loop analysis



(circuit)

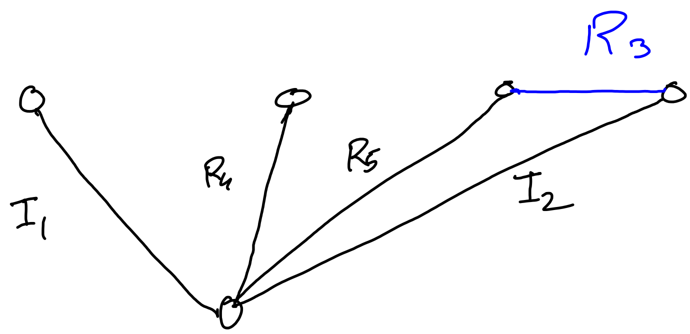
$i_1 = I_1$

$i_3 = I_2$

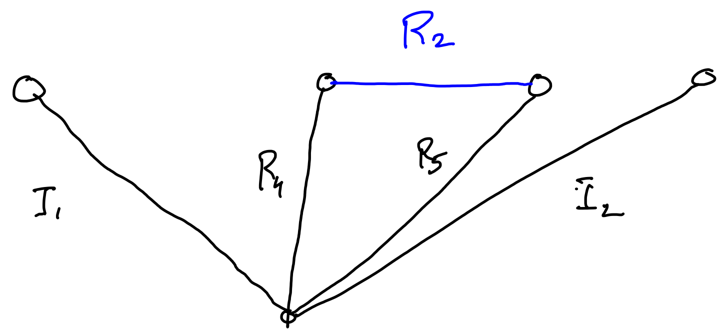


(graph)

$$(i_2 - i_1) R_4 + i_2 R_2 + (i_2 + i_3) R_5 = 0$$



(tree)

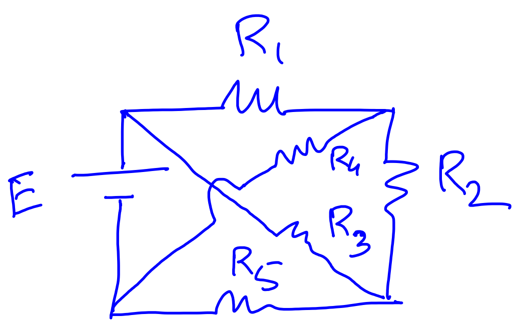


Mesh → a loop which does not contain any other loop.

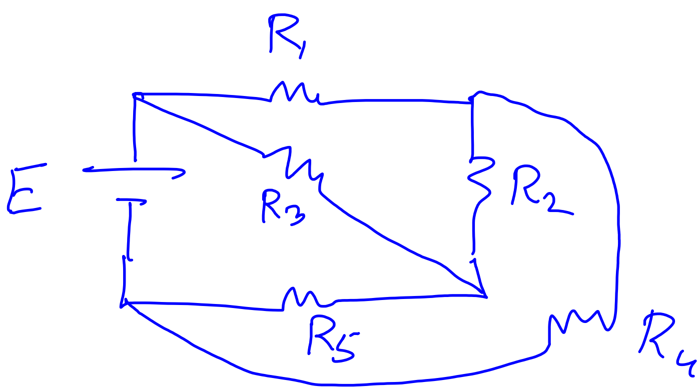
Loops → 6

Meshs → 3

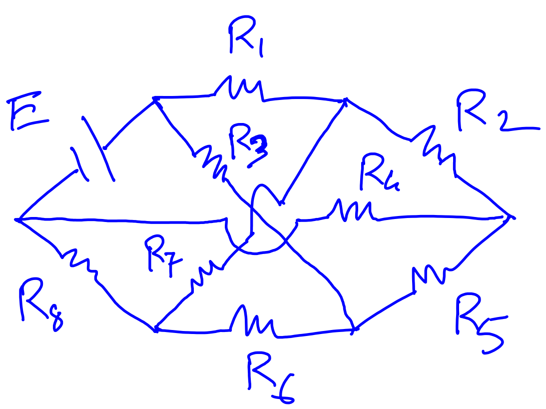
For planar circuits, use Mesh currents as independent variables - -



⇒

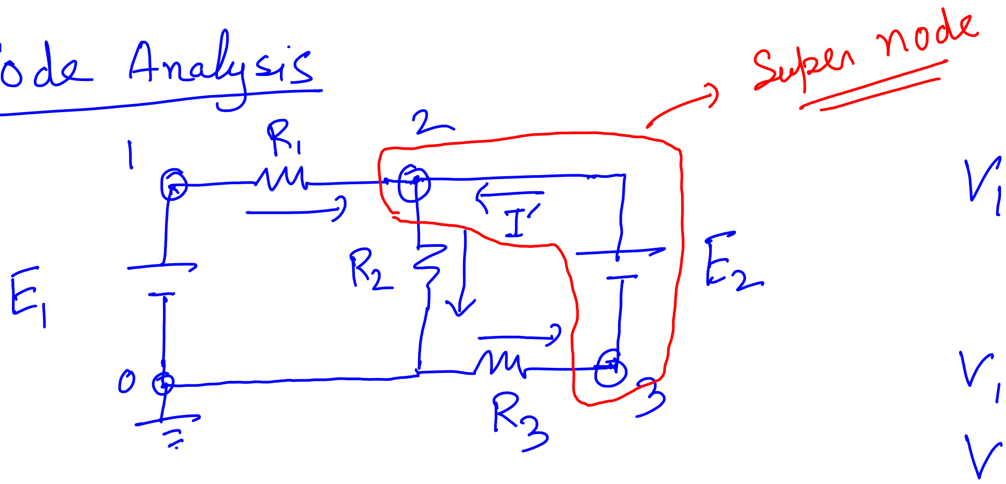


(planar)



(non-planar)

Node Analysis



$V_1,\ V_2,\ V_3$

$V_1 = E_1$

$V_2 - V_3 = E_2$

$$\frac{V_1 - V_2}{R_1} + I' = \frac{V_2}{R_2}$$

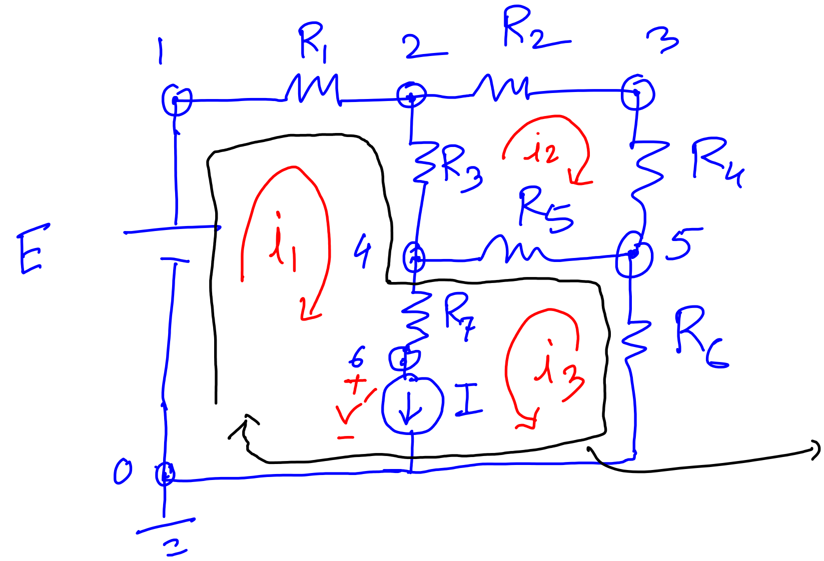
$$-\frac{V_3}{R_3} = I'$$

Combine

$$\frac{V_1 - V_2}{R_1} - \frac{V_3}{R_3} = \frac{V_2}{R_2}$$

(eq. using super-node)

Mesh Analysis



$b = 9$

$n = 7$

$b - n + 1 = 3$

(Super Mesh) KVL will lead to eq. (5).

KVL at Mesh 1,

$$E = i_1 R_1 + (i_1 - i_2) R_3 + (i_1 + i_3) R_7 + V' \quad \text{—— (1)}$$

$$(i_2 - i_1) R_3 + i_2 R_2 + i_2 R_4 + (i_2 + i_3) R_5 = 0 \quad \text{—— (2)}$$

$$(i_3+i_1)R_7 + V' + i_3 R_6 + (i_3+i_2)R_5 = 0 \quad \text{--- } (3)$$

Adding (1) and (3),

$$E + i_3 R_6 + (i_3+i_2)R_5 = i_1 R_1 + (i_1 - i_2)R_3 \quad \text{--- } (5)$$

$$i_1 + i_3 = I \quad \text{--- } (4)$$

Use (2), (4) and (5) to solve for i_1, i_2 and i_3.

Quick Recap

R, L, C (V-I characteristics)

source → voltage source, current source

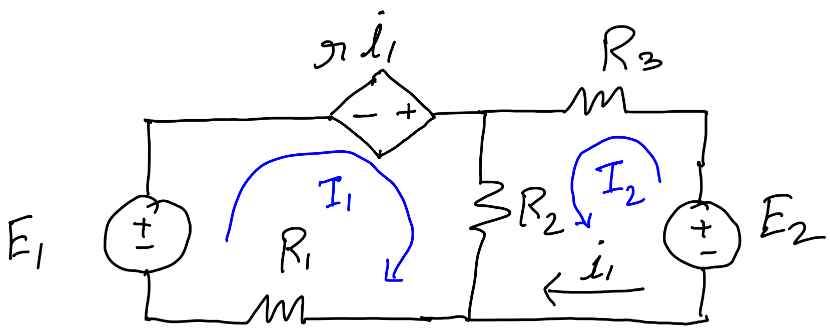
KCL (conservation of charge) ✓

KVL (conservation of energy) ✓

Node variable analysis

Loop " " → Mesh (planar circuit)

Circuits with dependent sources



Mesh Analysis

KVL for Mesh 1,

$i_1 = -I_2$

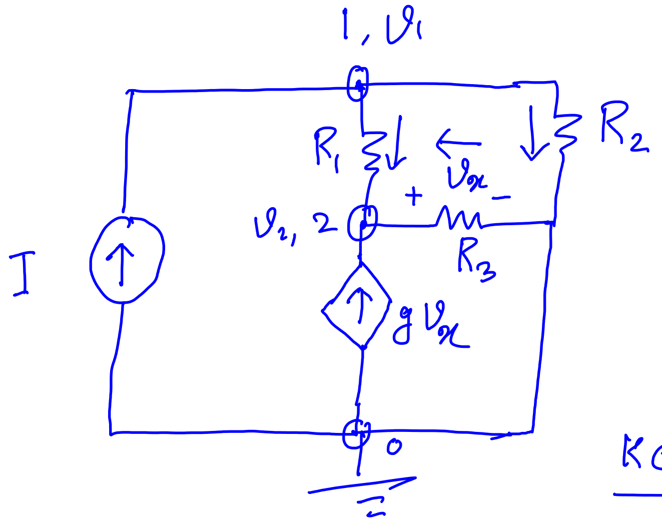
$$E_1 + r i_1 - (I_1 + I_2) R_2 - I_1 R_1 = 0$$

KVL for Mesh 2,

$$E_2 - I_2 R_3 - (I_1 + I_2) R_2 = 0$$

$$Ax = b \qquad x = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$A = \begin{bmatrix} -R_1 - R_2 & -r - R_2 \\ -R_2 & -R_2 - R_3 \end{bmatrix} \qquad b = \begin{bmatrix} -E_1 \\ -E_2 \end{bmatrix}$$



(Node Analysis)

<u>KCL at node 1,</u>

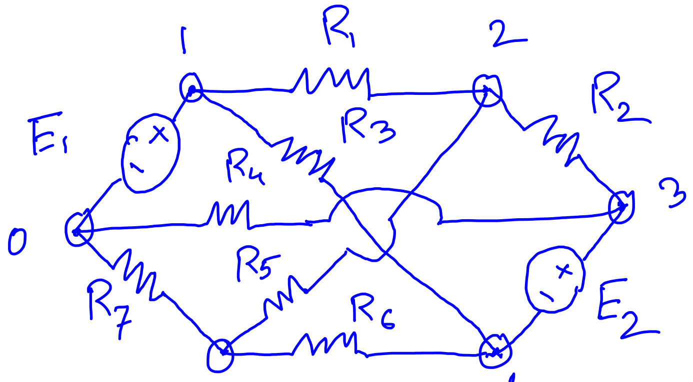
$$I = \frac{v_1 - v_2}{R_1} + \frac{v_1}{R_2}$$

<u>KCL at node 2,</u>

$$\frac{v_1 - v_2}{R_1} - \frac{v_2}{R_3} + g v_x = 0$$

$$v_x = v_2$$

<u>Non-planar Circuit</u>



(circuit)

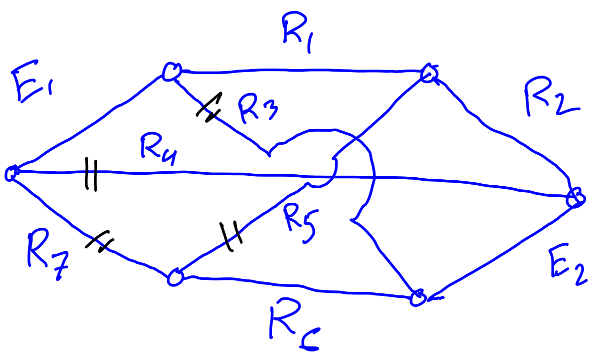
$$n = 6$$

$$b = 9$$

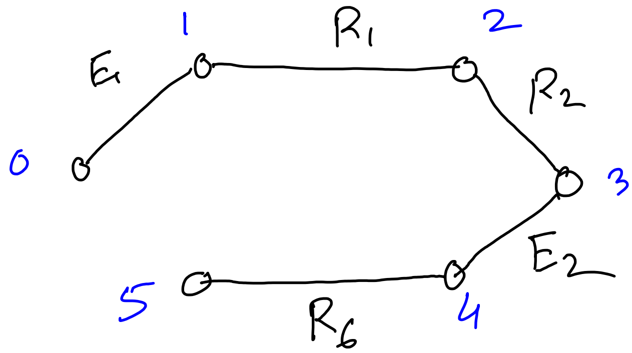
$$\text{no. of chords} = b - n + 1$$

$$= 9 - 6 + 1$$

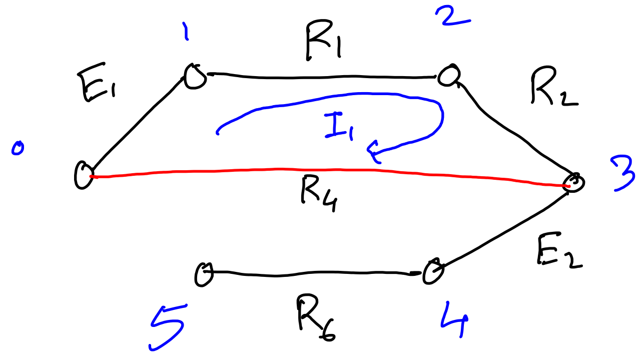
$$= 4$$



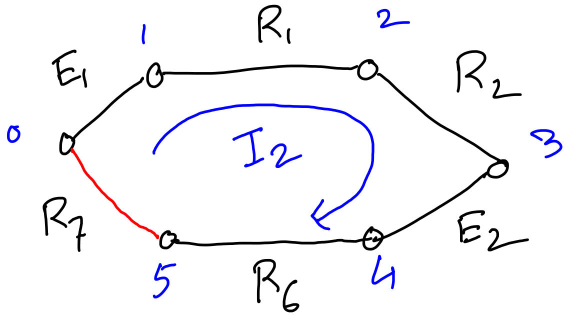
(graph)



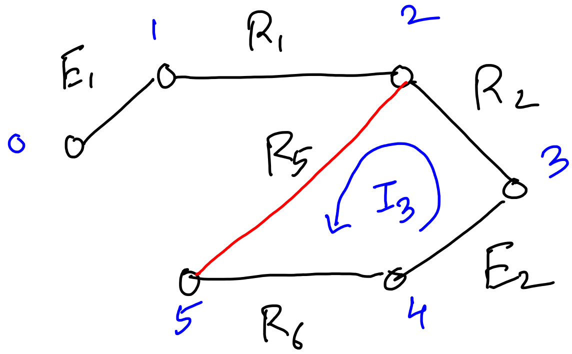
(tree)



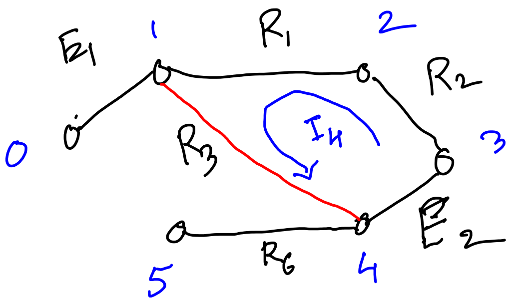
(loop 1)



(loop 2)



(loop 3)



(loop 4)

<u>branch</u>	<u>Magnitude</u>	<u>Direction</u>
R_1	$(I_1 + I_2 - I_4)$	$1 \longrightarrow 2$

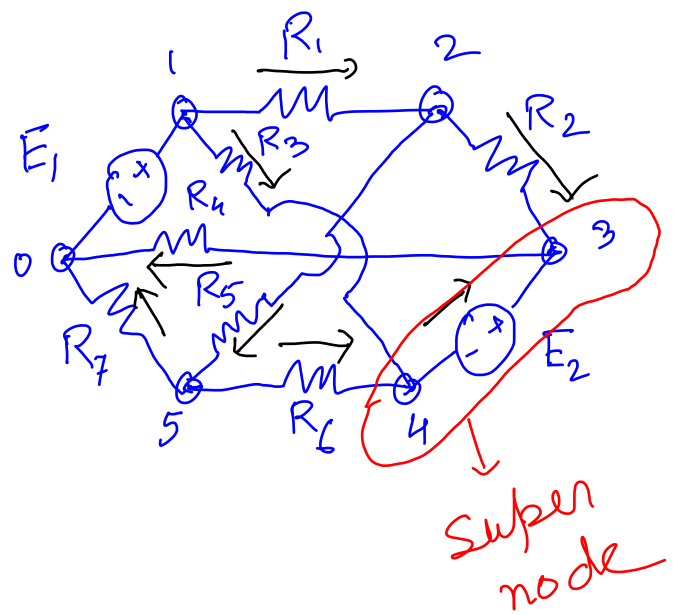
R_2	$(I_1 + I_2 - I_3 - I_4)$	$2 \longrightarrow 3$
E_2	$(I_3 + I_4 - I_2)$	$4 \longrightarrow 3$
R_6	$(I_2 - I_3)$	$4 \longrightarrow 5$
E_1	$(I_1 + I_2)$	$0 \longrightarrow 1$

KVL for loop 1,

$$E_1 = (I_1 + I_2 - I_4) R_1 + (I_1 + I_2 - I_3 - I_4) R_2 + I_1 R_4$$

KVL for loop 2,

$$E_1 = (I_1 + I_2 - I_4) R_1 + (I_1 + I_2 - I_3 - I_4) R_2 + E_2 + (I_2 - I_3) R_6 + I_2 R_7$$



(Node Analysis)

$$V_1 = E_1$$

KCL at node 2,

$$\frac{V_1 - V_2}{R_1} = \frac{V_2 - V_3}{R_2} + \frac{V_2 - V_5}{R_5}$$

KCL at node 5,

$$\frac{V_2 - V_5}{R_5} = \frac{V_5}{R_7} + \frac{V_5 - V_4}{R_6}$$

KCL at super node,

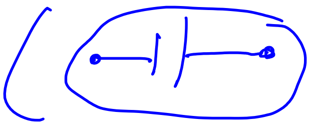
$$\frac{V_5 - V_4}{R_6} + \frac{V_1 - V_4}{R_3} + \frac{V_2 - V_3}{R_2} = \frac{V_3}{R_4}$$

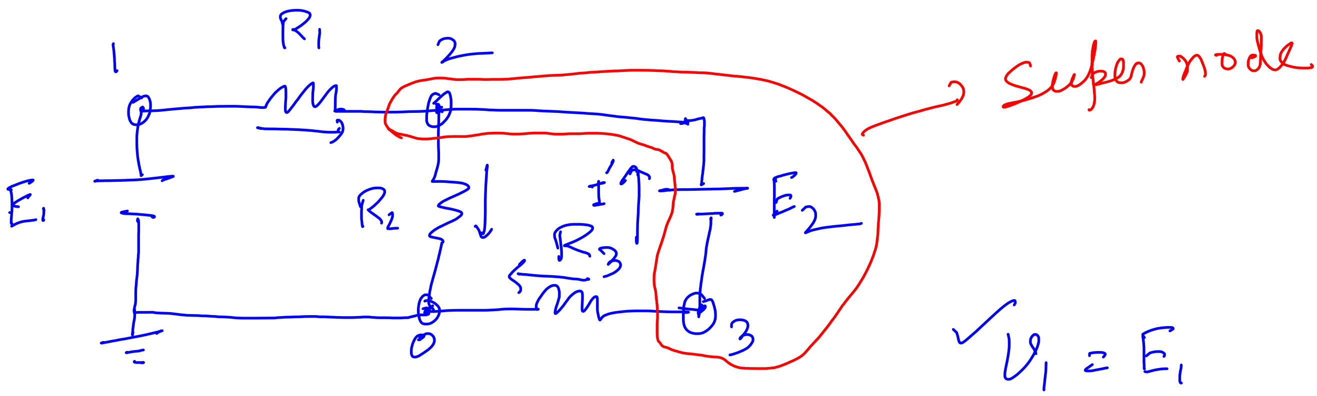
$$V_3 - V_4 = E_2$$

Quick Recap

Node Analysis (using node-to-reference voltages) $\rightarrow (n-1)$

Mesh Analysis (Mesh currents) $\rightarrow (b-n+1)$

Super node ()



✓ $V_1 = E_1$

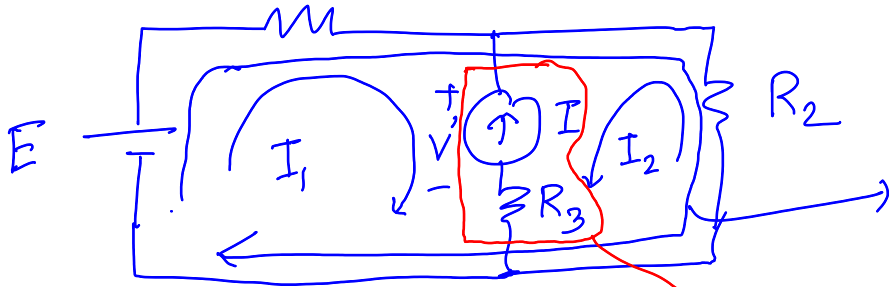
$$\frac{V_1 - V_2}{R_1} + I' = \frac{V_2}{R_2}$$ (KCL at node 2)

$$0 = \frac{V_3}{R_3} + I'$$ (KCL at node 3)

✓ $V_2 - V_3 = E_2$

$$\Rightarrow \checkmark \ \frac{V_1 - V_2}{R_1} - \frac{V_3}{R_3} = \frac{V_2}{R_2}$$ (KCL at super node)

Super Mesh



super Mesh

KVL for loop 1,

$$E = I_1 R_1 + V' + (I_1 + I_2) R_3$$

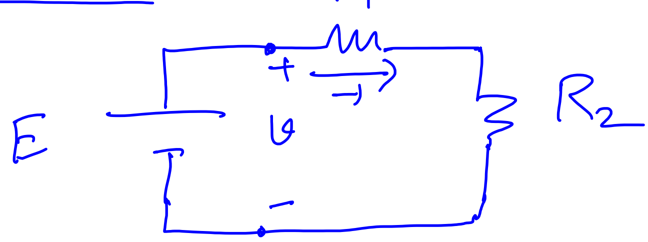
KVL for loop 2,

$$0 = I_2 R_2 + V' + (I_1 + I_2) R_3$$

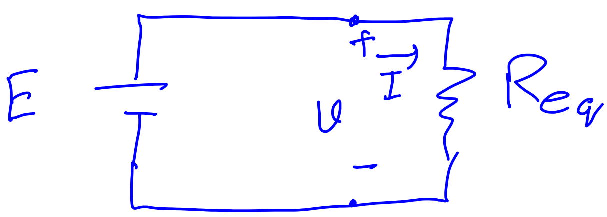
$$E = I_1 R_1 - I_2 R_2$$ → (KVL for Super Mesh)

$$I = -I_1 - I_2$$

Series Resistances

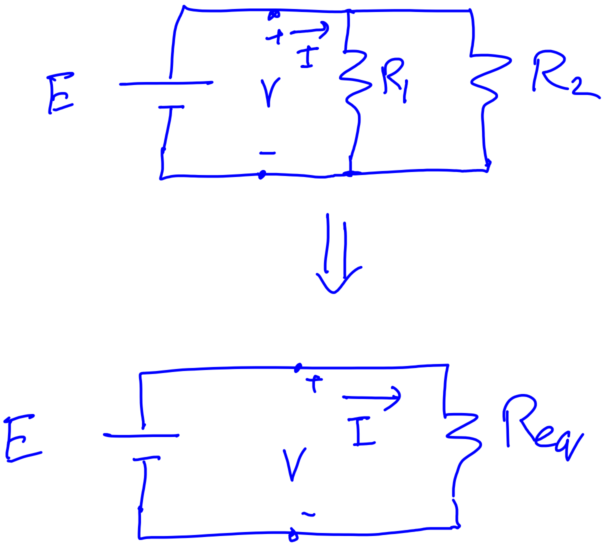


⇓

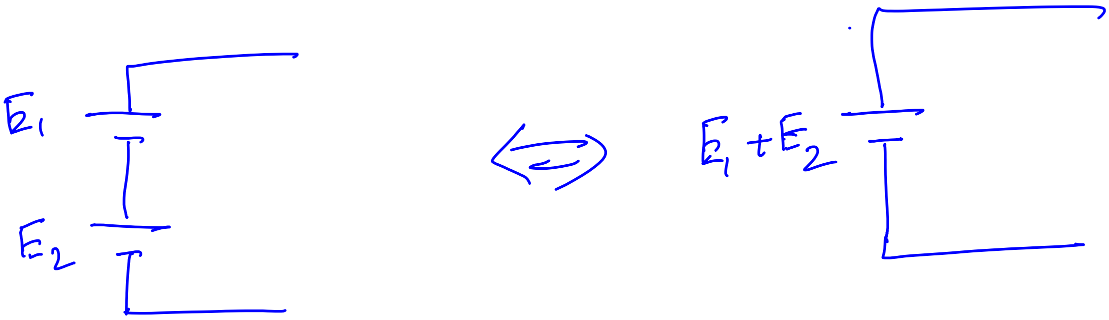


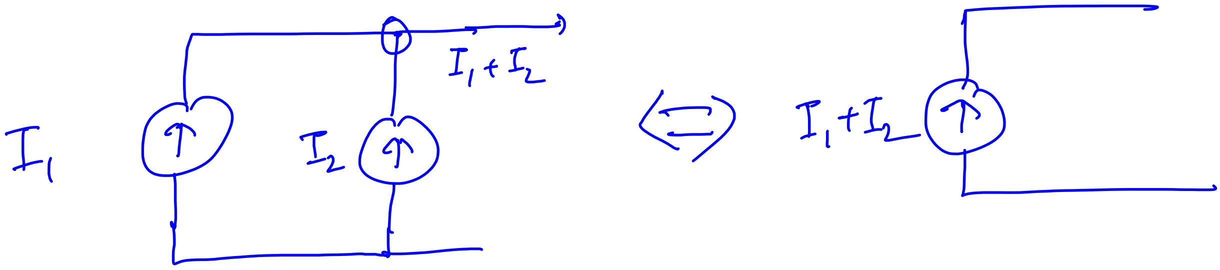
$$R_{eq} = R_1 + R_2$$

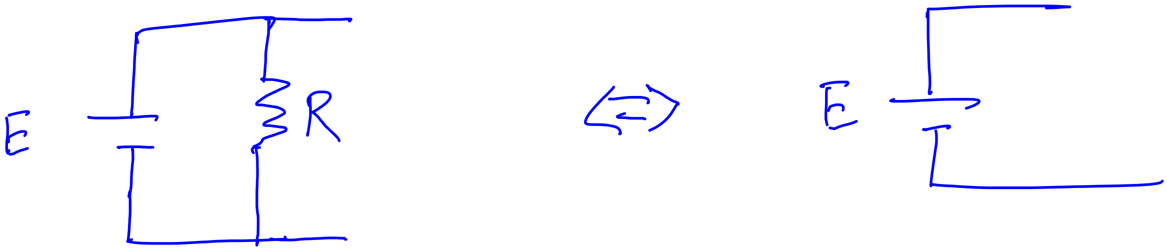
Parallel Resistances

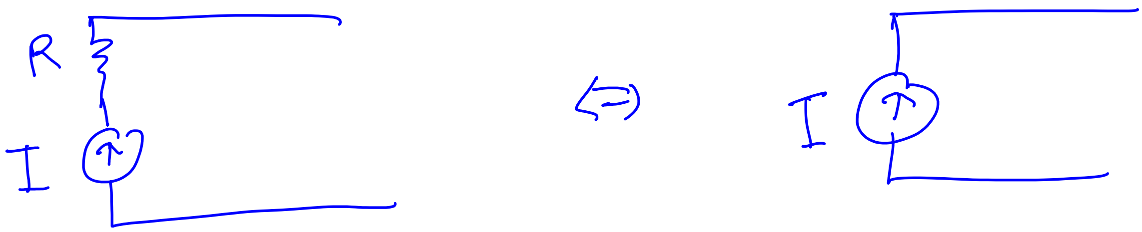


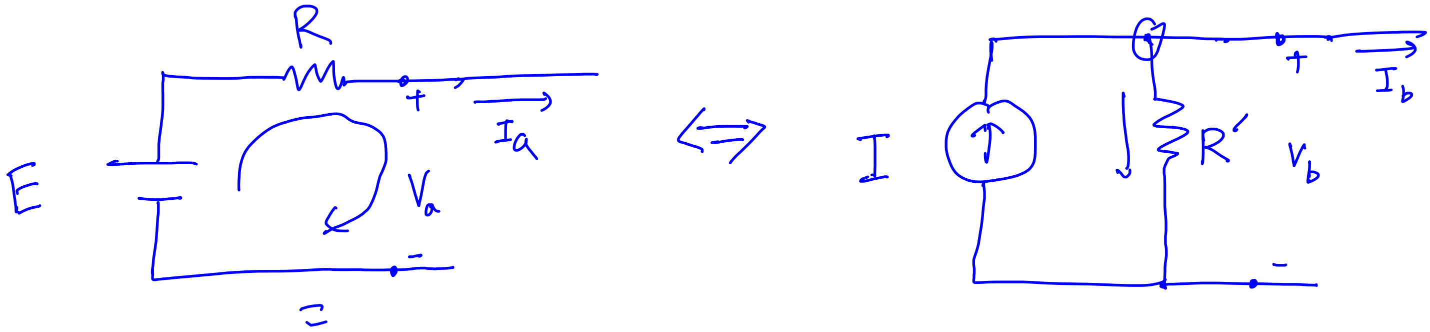
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$







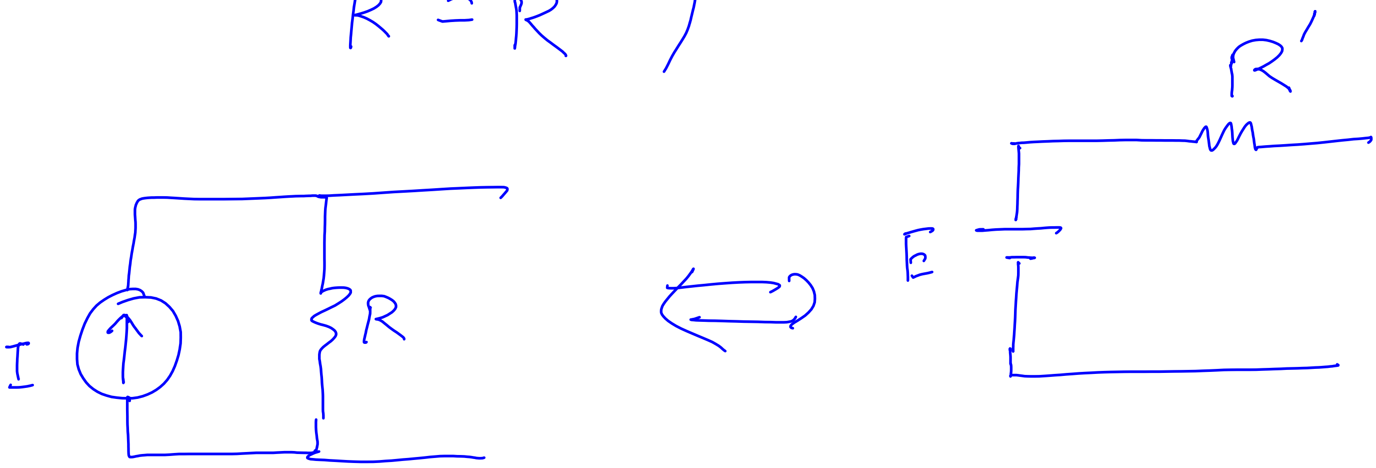




$$E = I_a R + V_a \qquad\qquad I = \frac{V_b}{R'} + I_b$$

For equivalence $\quad I_a = I_b$ and $V_a = V_b$

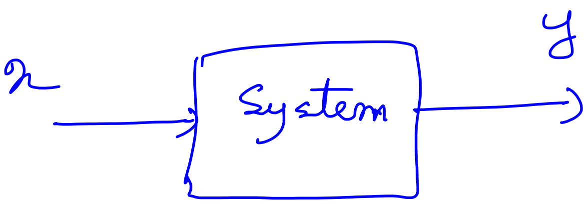
$$\left.\begin{aligned} I &= E/R \\ R' &= R \end{aligned}\right\}$$



$$R' = R$$

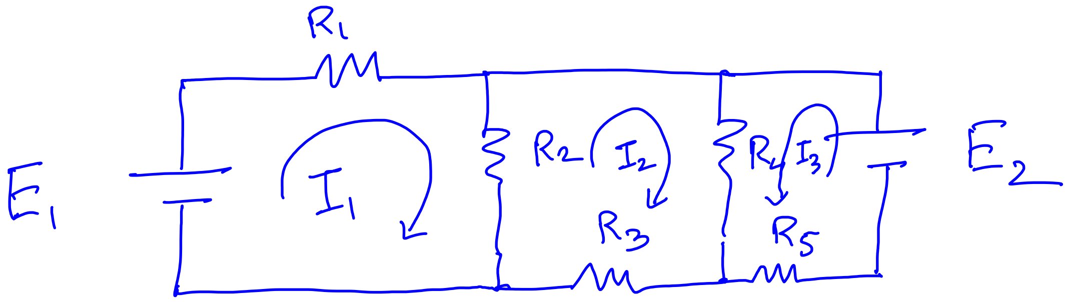
$$E = IR$$

Superposition Principle



$$\left.\begin{aligned} x_1 &\longrightarrow y_1 \\ x_2 &\longrightarrow y_2 \end{aligned}\right\} \Rightarrow \alpha_1 x_1 + \alpha_2 x_2 \longrightarrow \alpha_1 y_1 + \alpha_2 y_2$$

The system is linear



$$E_1 = I_1 R_1 + (I_1 - I_2) R_2$$

$$0 = -(I_1 - I_2) R_2 + (I_2 + I_3) R_4 + I_2 R_3$$

$$E_2 = (I_2 + I_3) R_4 + I_3 R_5$$

$$Ax = b \quad , \quad A \in \mathbb{R}^{3\times 3}$$

$$x = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \qquad x \in \mathbb{R}^{3\times 1}$$

$$b \in \mathbb{R}^{3\times 1}$$

$$A = \begin{bmatrix} R_1+R_2 & -R_2 & 0 \\ -R_2 & R_2+R_3+R_4 & R_4 \\ 0 & R_4 & R_4+R_5 \end{bmatrix} \qquad b = \begin{bmatrix} E_1 \\ 0 \\ E_2 \end{bmatrix}$$

$$b = \begin{bmatrix} E_1 \\ 0 \\ E_2 \end{bmatrix} = \underbrace{\begin{bmatrix} E_1 \\ 0 \\ 0 \end{bmatrix}}_{b_1} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ E_2 \end{bmatrix}}_{b_2}$$

$$Ax = b = b_1 + b_2$$

$$\left.\underbrace{x = A^{-1}b_1}_{x_1} + \underbrace{A^{-1}b_2}_{x_2}\right\}$$ superposition holds

$x_1 \to$ contribution from E_1

$x_2 \to$ Contribution from E_2

$$Ax = b$$

there are "p" no. of independent sources.

$$b = \sum_{i=1}^{p} b_p$$

Example

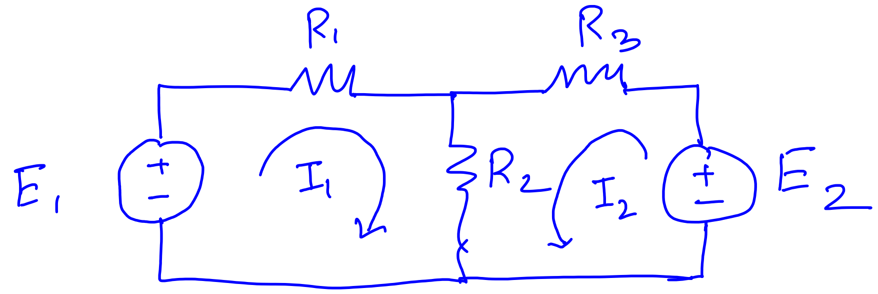
$$b = \begin{bmatrix} E_1 - E_2 \\ 0 \\ E_3 \end{bmatrix} = \underbrace{\begin{bmatrix} E_1 \\ 0 \\ 0 \end{bmatrix}}_{b_1} - \underbrace{\begin{bmatrix} E_2 \\ 0 \\ 0 \end{bmatrix}}_{b_2} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix}}_{b_3}$$

Superposition Theorem

Each branch current or voltage can be decomposed into contributions from individual independent current or voltage sources.

Quick Recap

Superposition Theorem $\rightarrow$



$$E_1 = I_1 R_1 + (I_1 + I_2) R_2$$

$$E_2 = I_2 R_3 + (I_1 + I_2) R_2$$

$$A x = b$$

$$A = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

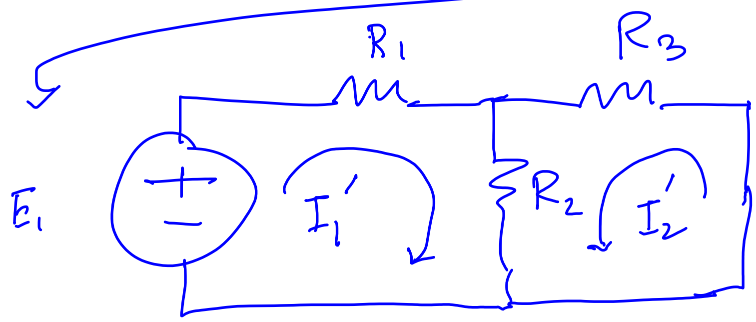
$$x = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$b = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

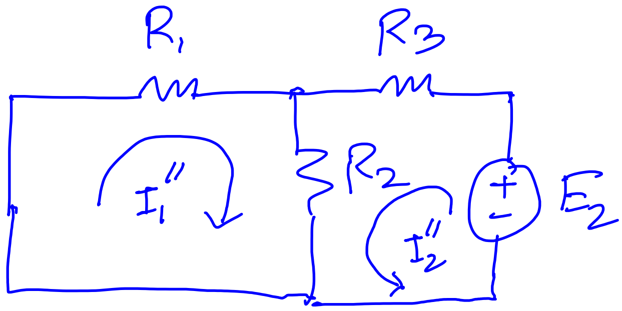
$$b = \underbrace{\begin{bmatrix} E_1 \\ 0 \end{bmatrix}}_{b_1} + \underbrace{\begin{bmatrix} 0 \\ E_2 \end{bmatrix}}_{b_2}$$

$$A x = b = b_1 + b_2$$

$$x = \underbrace{A^{-1} b_1}_{x_1} + \underbrace{A^{-1} b_2}_{x_2}$$

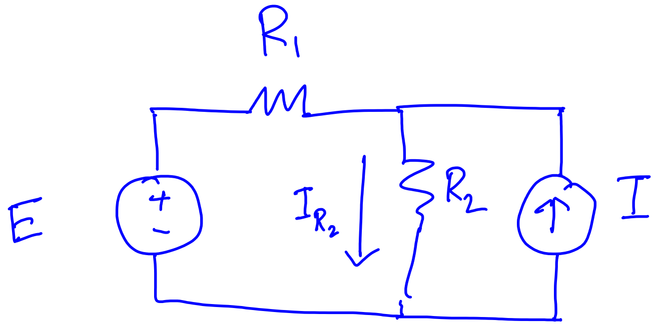


$$x_1 = \begin{bmatrix} I_1' \\ I_2' \end{bmatrix}$$

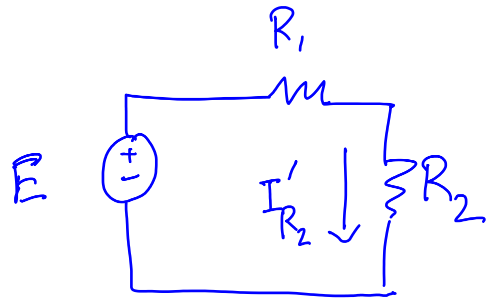


$$x_2 = \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix}$$

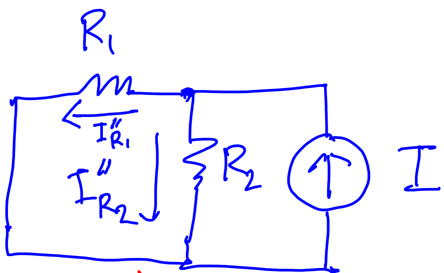
$$\left.\begin{aligned} I_1 &= I_1' + I_1'' \\ I_2 &= I_2' + I_2'' \end{aligned}\right\}$$



$$I_{R_2} = ?$$



$$I'_{R_2} = \frac{E}{R_1 + R_2}$$



$$I = I''_{R_1} + I''_{R_2}$$

$$I''_{R_1} \cdot R_1 = I''_{R_2} \cdot R_2$$

$$I''_{R_2} + \frac{R_2}{R_1} I''_{R_2} = I$$

$$\Rightarrow \quad I''_{R_2} = \frac{R_1 I}{R_1 + R_2}$$

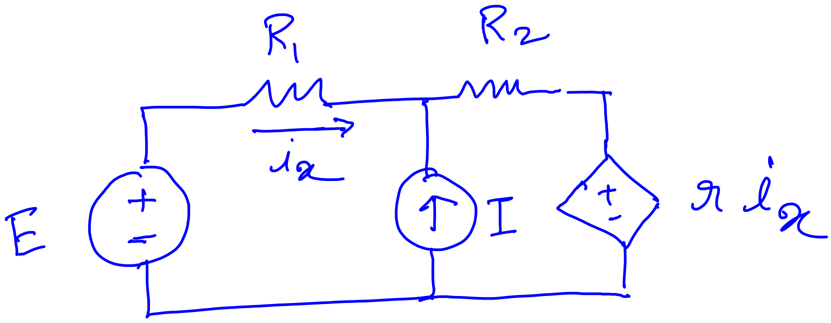
Total current

Current due to E

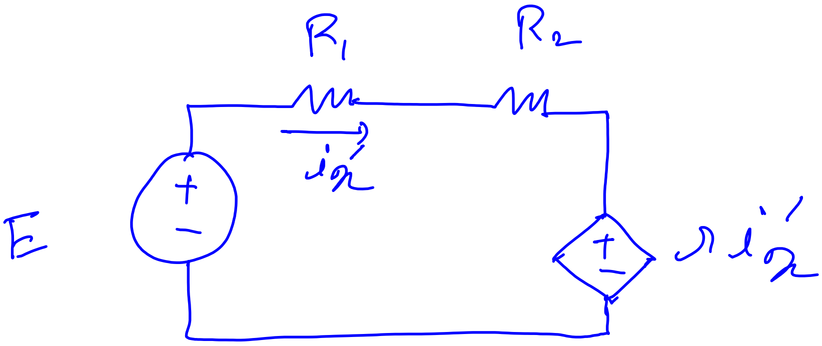
Current due to I

$$I_{R_2} = I'_{R_2} + I''_{R_2}$$

$$= \frac{E}{R_1 + R_2} + \frac{R_1 I}{R_1 + R_2}$$



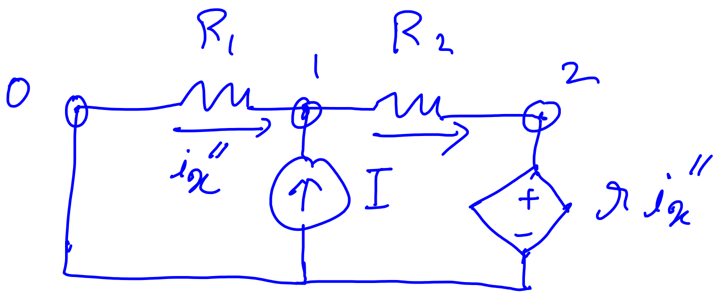
$r i_x = ?$



$$i_x' = \frac{E - r i_x'}{R_1 + R_2}$$

$\Downarrow$

solve for i_x'



KCL at node 1

$$i_x'' + I = \frac{v_1 - v_2}{R_2}$$

$$i_x'' = -\frac{v_1}{R_1}$$

$$v_2 = r i_x''$$

solve for i_x''

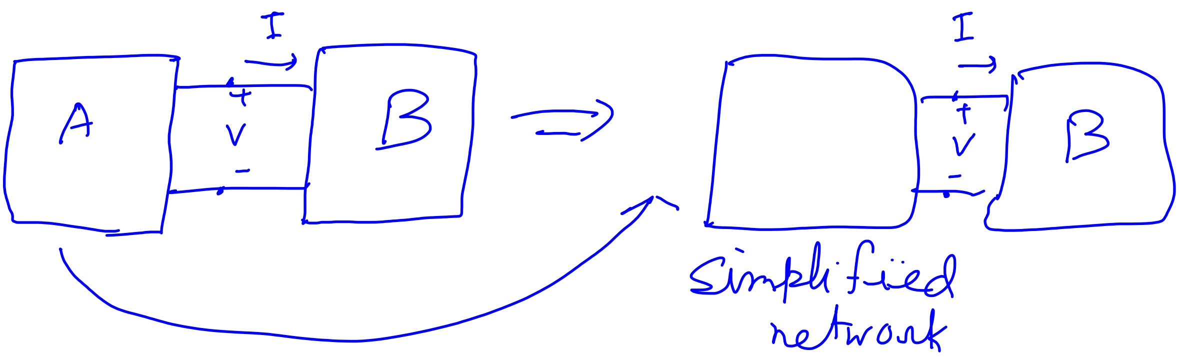
Total current

current due to E

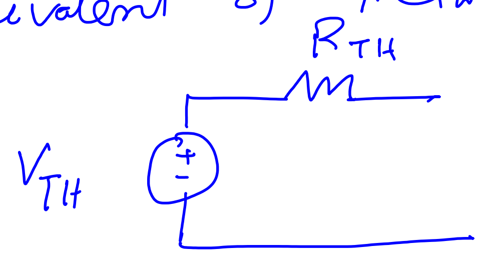
current due to I

$$i_x = i_x' + i_x''$$

Thevnin's Theorem / Norton's Theorem

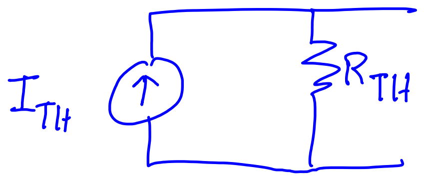


Thevnin Equivalent of Network A



Norton Equivalent of Network A

$V_{TH} = I_{TH} \cdot R_{TH}$



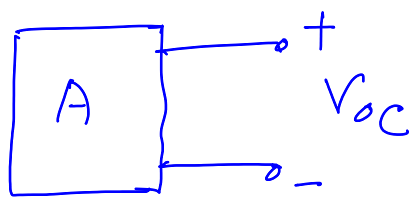
Properties of Network A

1) It should be a linear network
2) Voltage sources or current sources can be present.
3) There should be no controlled-source coupling with network B.

Properties of Network B

1) It can be linear or non-linear
2) There should not be any coupling via controlled sources with network A.

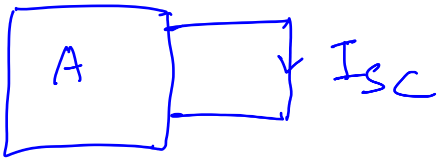
How to Compute V_{TH}



Remove network B and compute V_{OC}

$V_{TH} = V_{OC}$

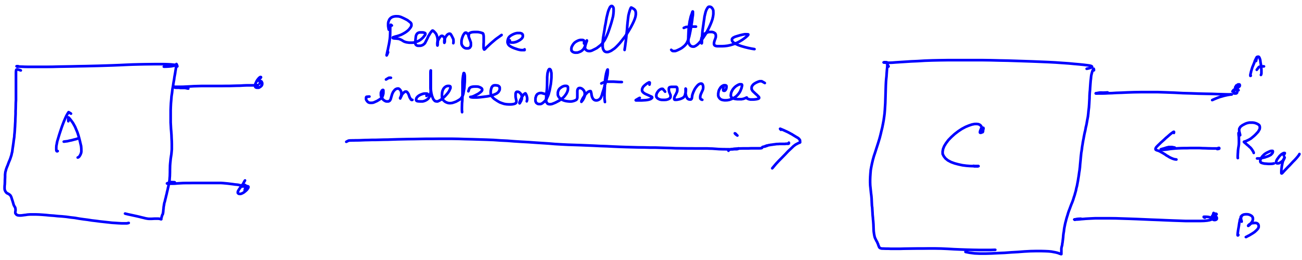
How to Compute I_{TH}



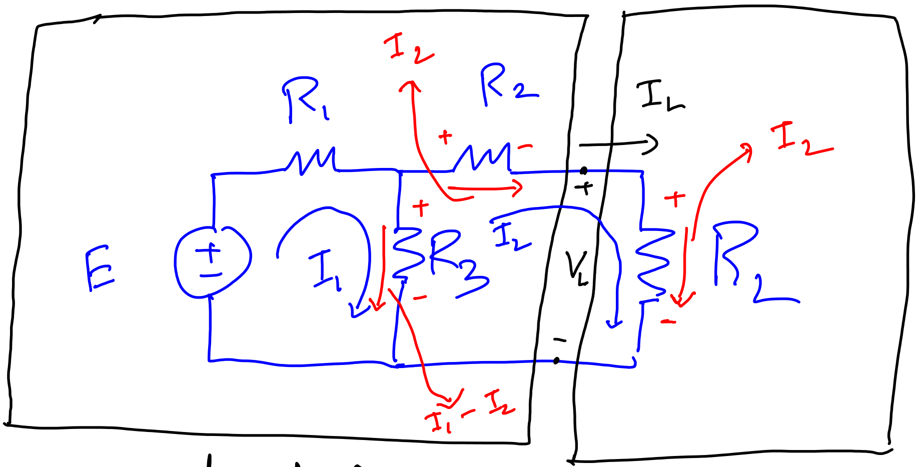
Remove network B, short-circuit the terminal and compute I_{sc}

$$I_{TH} = I_{sc}$$

How to compute R_{TH}

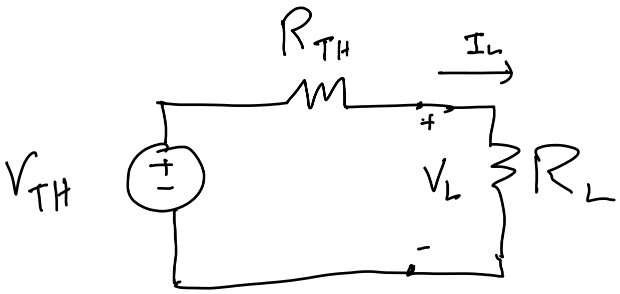


$$R_{TH} = R_{eq}$$



network A

network B



$$E = I_1 R_1 + (I_1 - I_2) R_3$$

$$(I_1 - I_2) R_3 = I_2 (R_2 + R_L)$$

$$I_L = I_2$$

$$I_1 R_3 = I_2 (R_3 + R_2 + R_L)$$

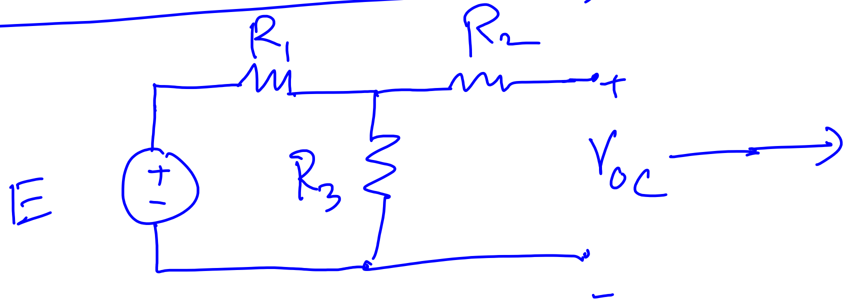
$$E = I_1 (R_1 + R_3) - I_2 R_3$$

$$= \frac{R_1 + R_3}{R_3} \cdot (R_3 + R_2 + R_L) \cdot I_2 - I_2 R_3$$

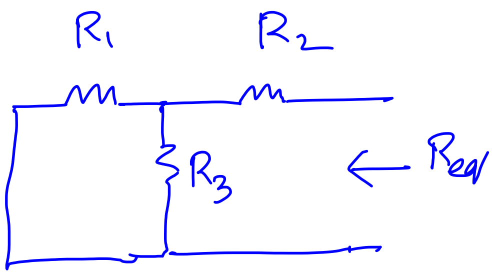
$$I_2\left(\frac{(R_1+R_3)(R_3+R_2+R_L)}{R_3} - R_3\right) = E$$

$$I_L = I_2 = \left.\frac{E\cdot R_3}{R_1R_3+R_1R_2+R_1R_L+R_3R_2+R_3R_L}\right\}$$

Thevnin Equivalent Circuit,



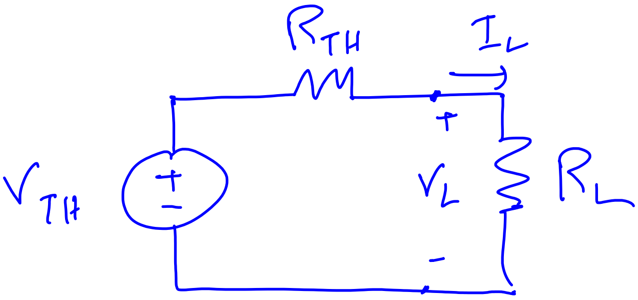
network A



network C

$$R_{TH} = R_2 + \frac{R_1R_3}{R_1+R_3}$$

$$V_{TH} = V_{oc} = \frac{R_3E}{R_1+R_3} \checkmark$$

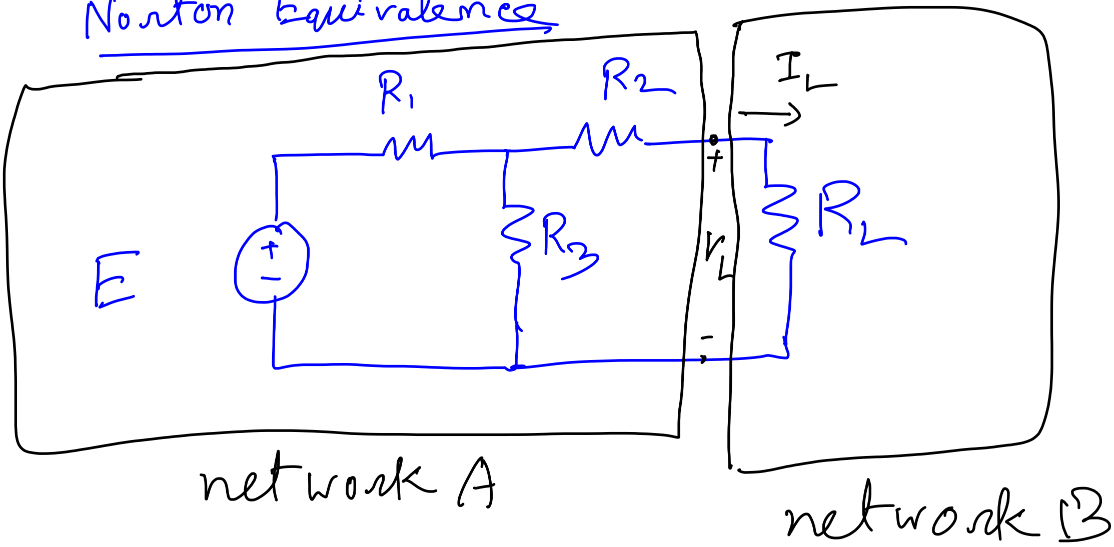


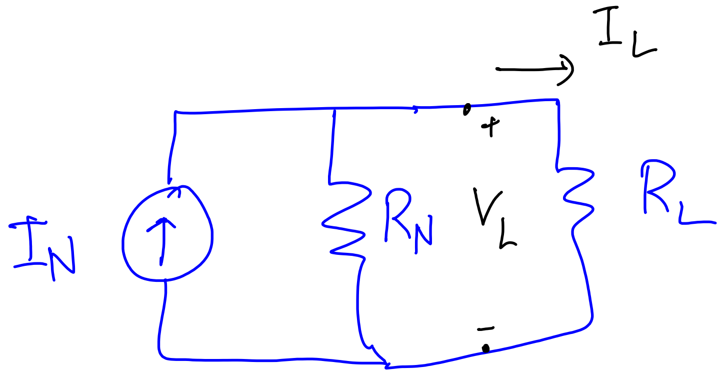
$$I_L = \frac{V_{TH}}{R_{TH}+R_L}$$

$$= \frac{ER_3/R_1+R_3}{R_2+\frac{R_1R_3}{R_1+R_3}+R_L}$$

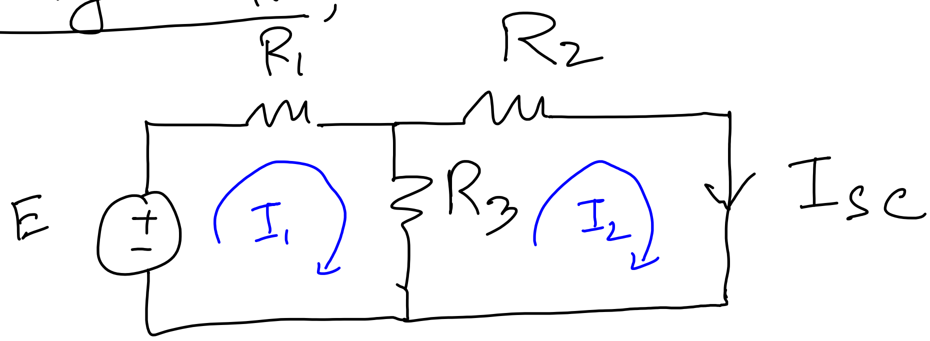
$$= \frac{ER_3}{R_1R_2+R_2R_3+R_1R_3+R_1R_L+R_3R_L}$$

Norton Equivalence





Calculating I_N,



$$E = I_1 R_1 + (I_1 - I_2) R_3$$

$$I_{sc} = I_2$$

$$(I_1 - I_2) R_3 = I_2 R_2$$

$$\Rightarrow \quad I_1 R_3 = (R_2 + R_3) I_2$$

$$\Rightarrow \quad I_1 = \frac{R_2 + R_3}{R_3} I_2$$

$$E = I_1 (R_1 + R_3) - I_2 R_3$$

$$E = \frac{(R_1+R_3)(R_2+R_3)}{R_3} I_2 - I_2 R_3$$

$$= I_2 \left[\frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3} \right]$$

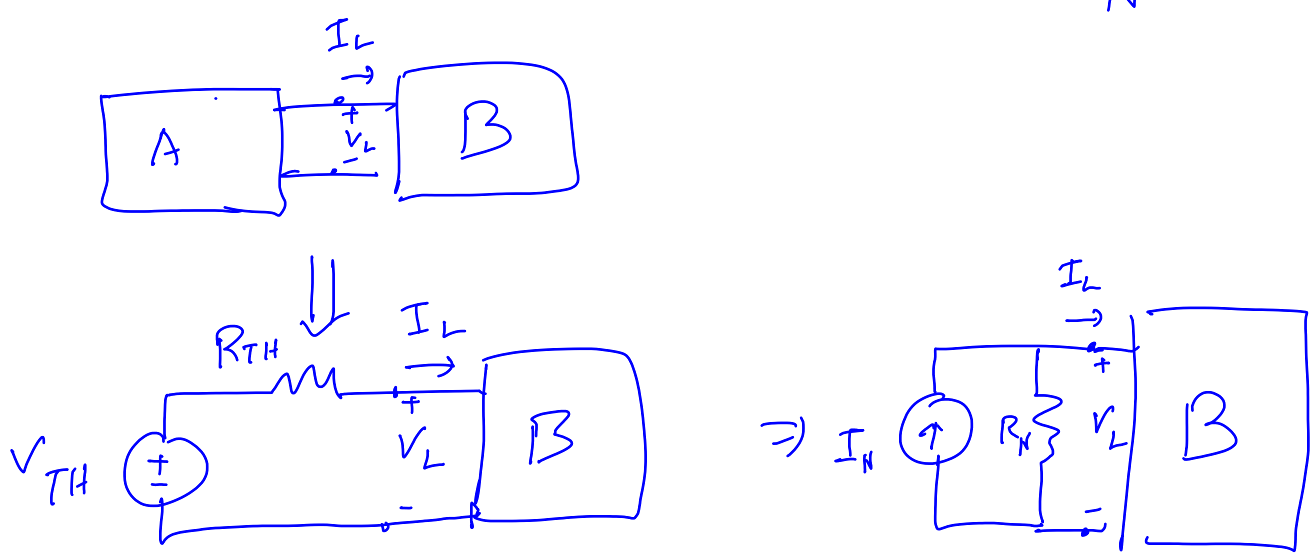
$$I_N = I_{SC} = I_2 = \frac{R_3 E}{R_1R_2 + R_2R_3 + R_1R_3} \Bigg\}$$

$$V_{TH} = I_N . R_{TH}, \qquad R_N = R_{TH}$$

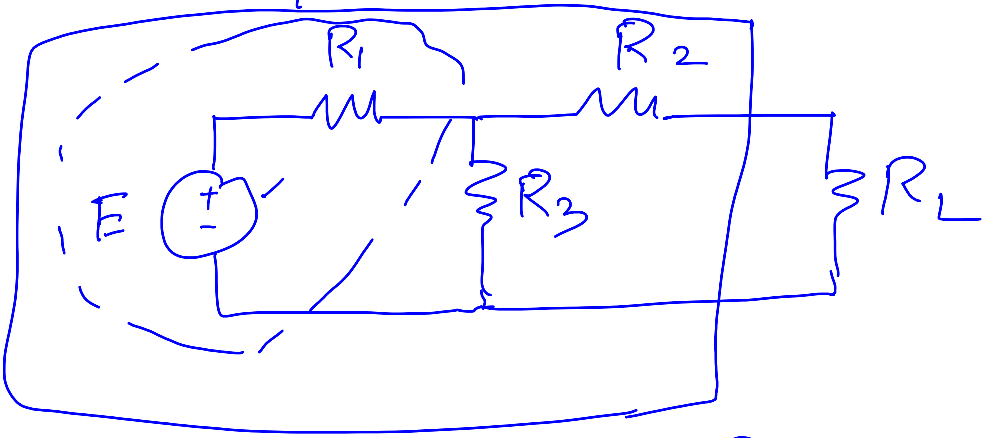
$$V_{TH} = \frac{R_3 E}{R_1 + R_3} \qquad R_{TH} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

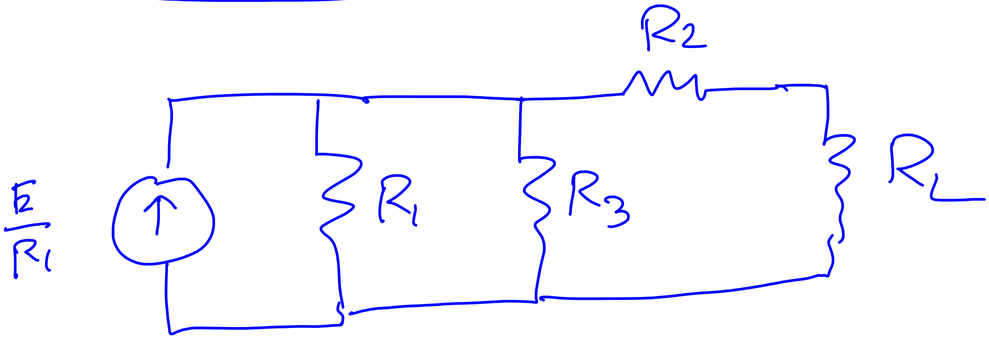
$$\frac{V_{TH}}{R_{TH}} = \frac{R_3 E/(R_1+R_3)}{R_2 + \frac{R_1R_3}{R_1+R_3}} = \frac{R_3 E}{R_1R_2 + R_2R_3 + R_1R_3}$$

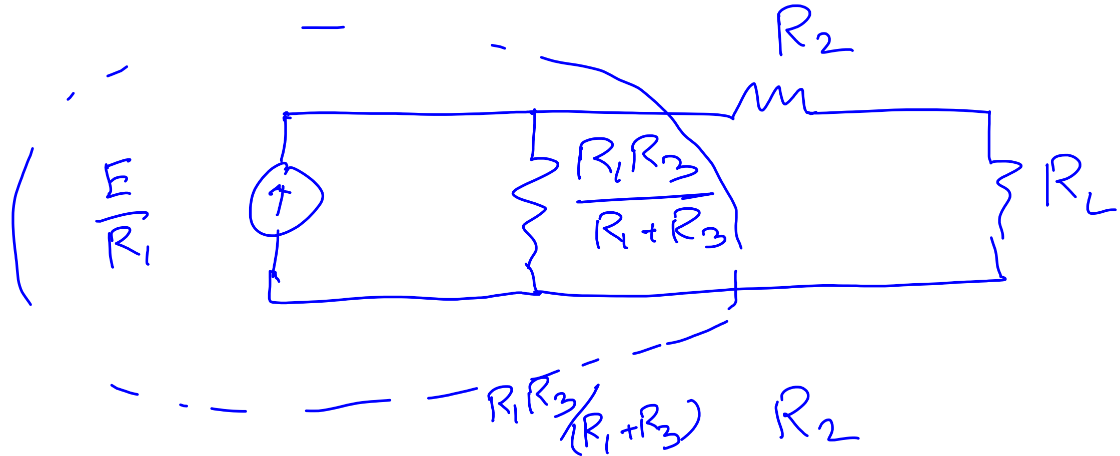
$$= I_N$$

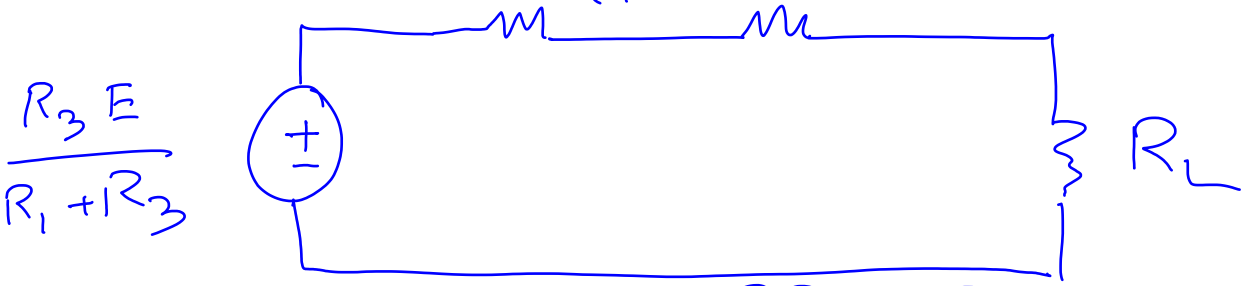


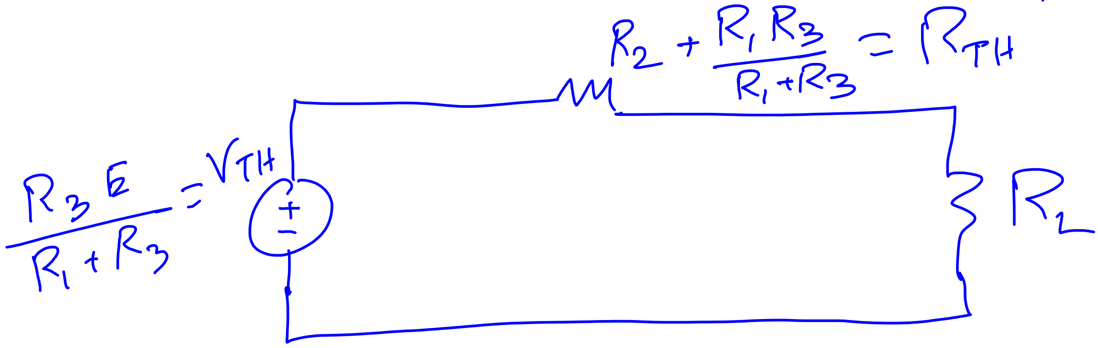
Alternative Approach to figure out Thevenin / Norton Equivalent network

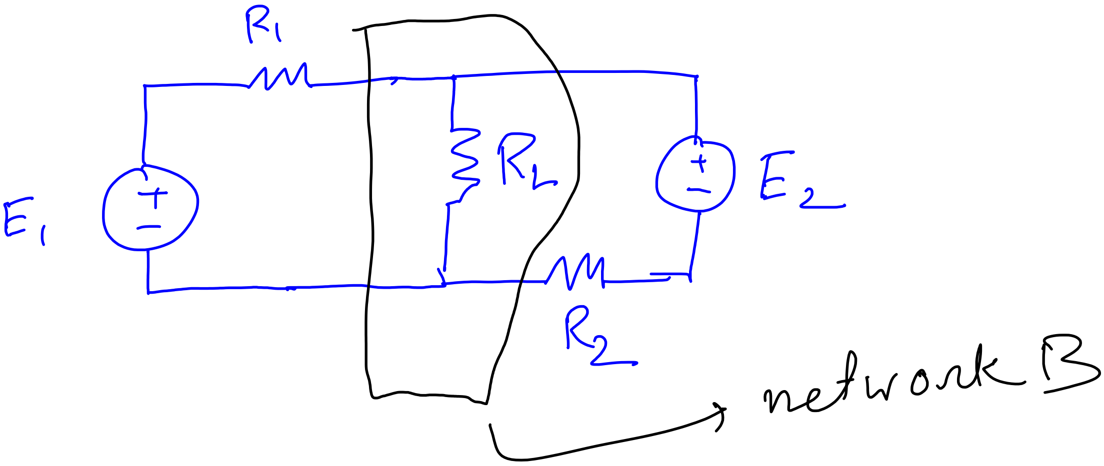




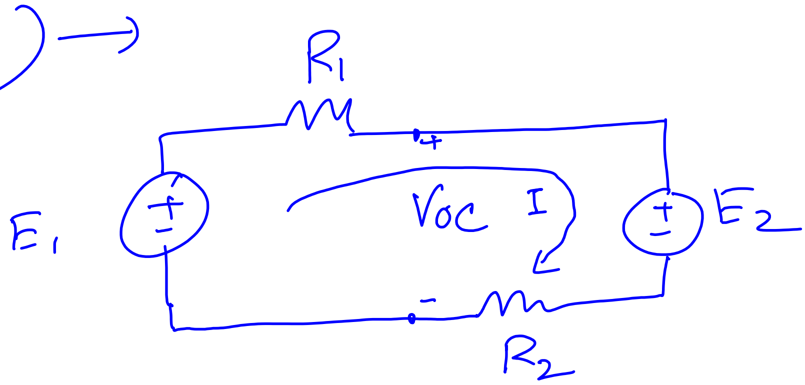








V_{TH}

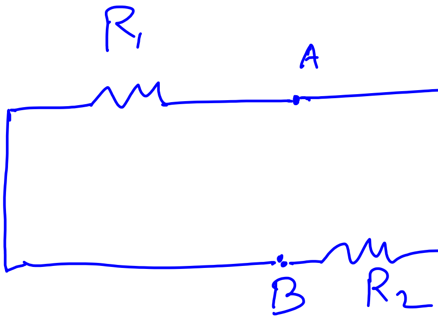


$$E_1 - E_2 = I\,(R_1 + R_2)$$

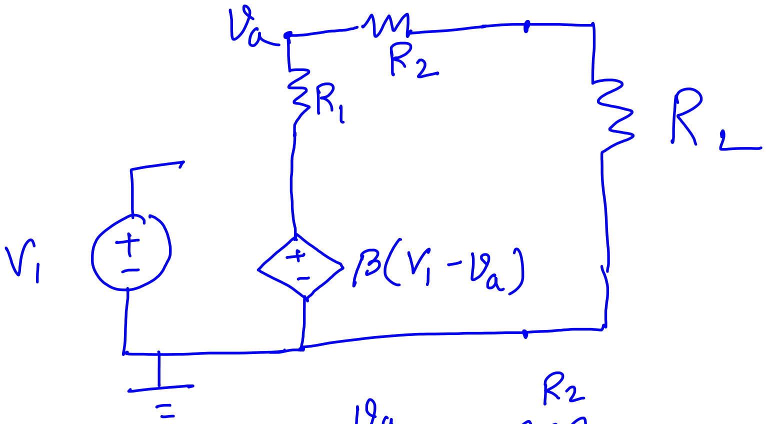
$$V_{TH} = V_{oc} = E_1 - I R_1$$

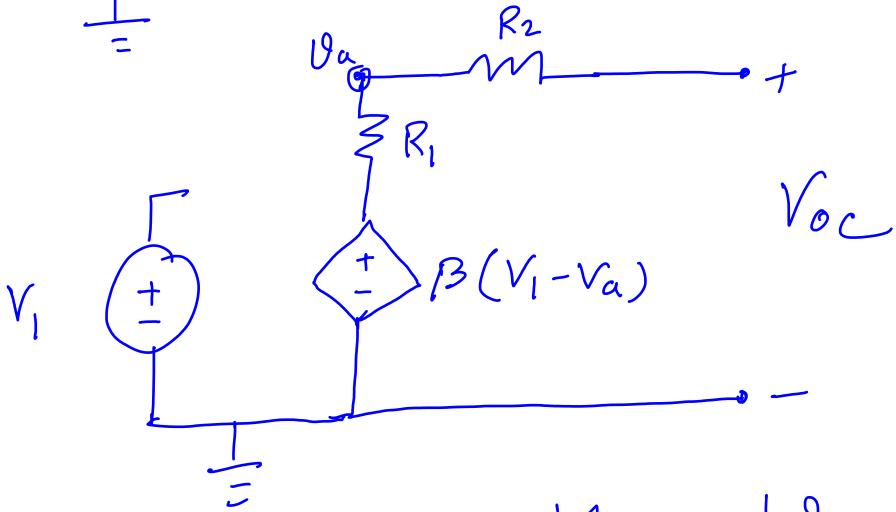
$$= E_1 - \frac{E_1 - E_2}{R_1 + R_2} \cdot R_1 \quad ✓$$

R_{TH}



$$R_{TH} = R_{AB} = \frac{R_1 R_2}{R_1 + R_2}$$





$$V_{oc} = v_a = \beta(V_1 - v_a)$$

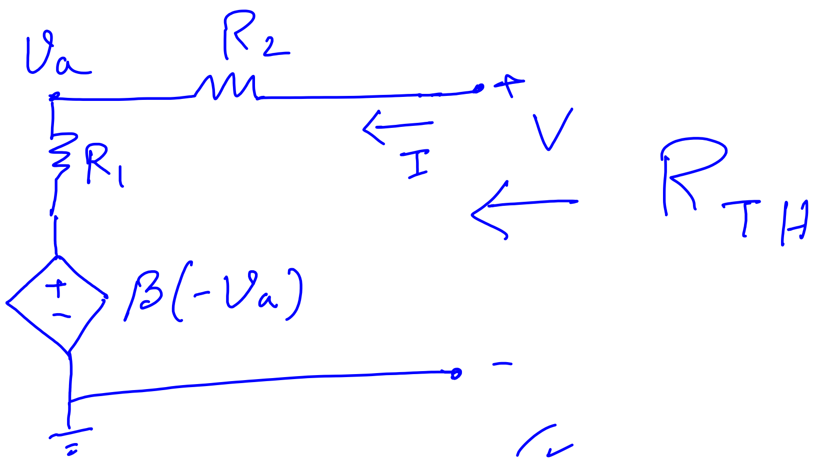
$$v_a = \beta(V_1 - v_a)$$

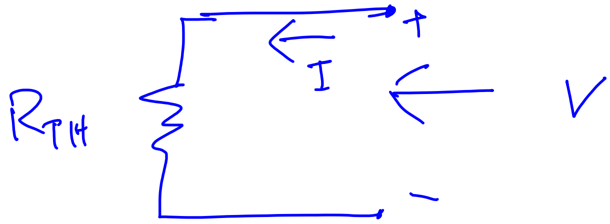
$$(1+\beta)\, v_a = \beta V_1$$

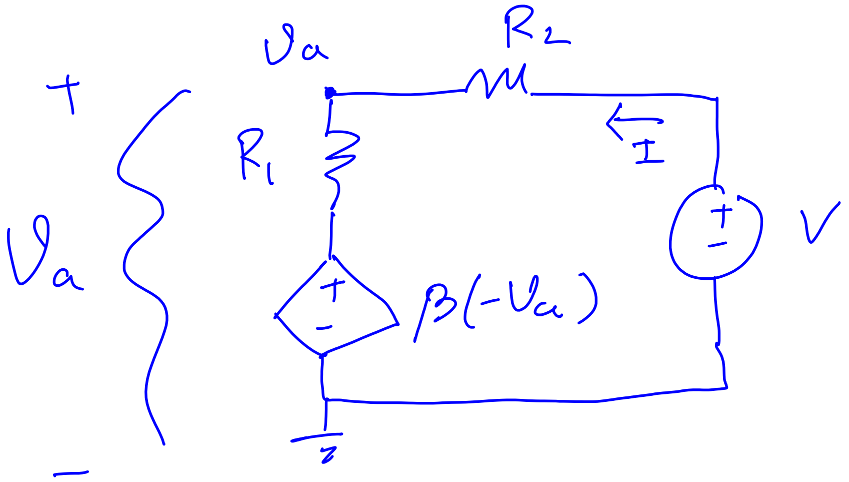
$$v_a = \frac{\beta}{1+\beta} V_1$$

$$V_{TH} = V_{oc} = \frac{\beta}{1+\beta} V_1$$

$R_{TH} \rightarrow$







$$V = I R_2 + I R_1 - \beta v_a$$

$$v_a = I R_1 - \beta v_a$$

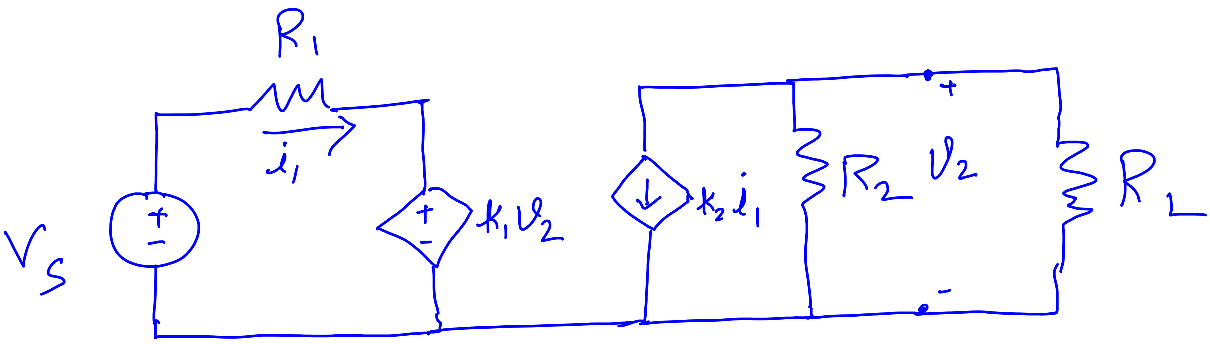
$$(1+\beta) v_a = I R_1$$

$$V = I(R_1 + R_2) - \frac{\beta}{1+\beta} I R_1$$

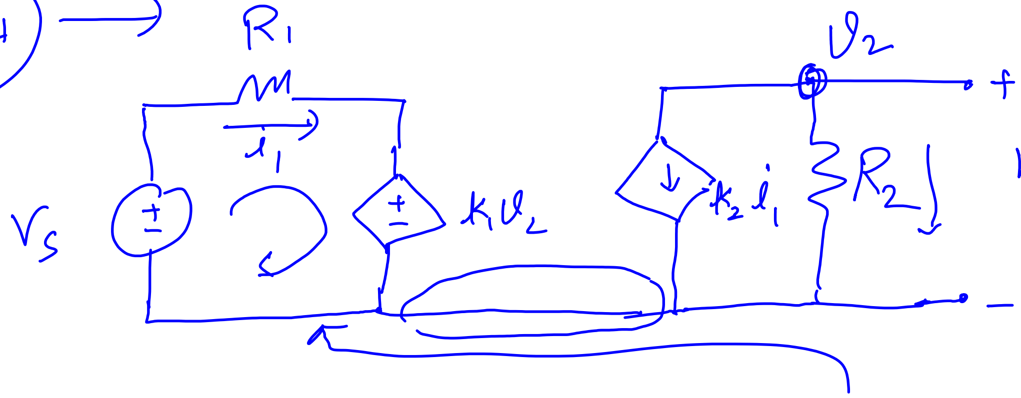
$$= I\left[R_1 + R_2 - \frac{\beta}{1+\beta} R_1\right]$$

$$= I\left[R_2 + \frac{1}{1+\beta} R_1\right]$$

$$\Rightarrow \quad R_{TH} = \frac{V}{I} = \left(R_2 + \frac{1}{1+\beta} R_1\right)$$



V_{TH} →



$V_{oc} = v_2$

$$V_S = i_1 R_1 + k_1 v_2 \qquad (KVL)$$

$$k_2 i_1 + \frac{v_2}{R_2} = 0 \qquad (KCL \text{ at node } 2)$$

$$i_1 = -\frac{1}{k_2 R_2} v_2$$

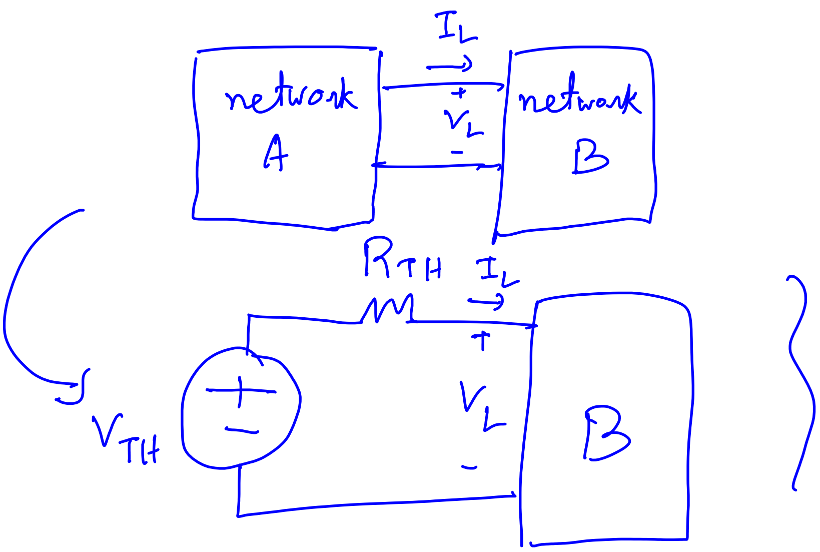
$$V_S = -\frac{R_1}{k_2 R_2} v_2 + k_1 v_2$$

$$\Rightarrow \quad v_2 = \frac{V_S}{k_1 - \frac{R_1}{k_2 R_2}}$$

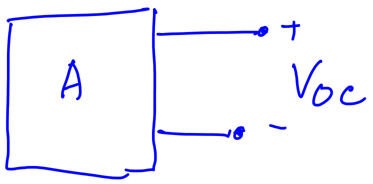
$$V_{TH} = V_{oc} = v_2$$

Quick Recap

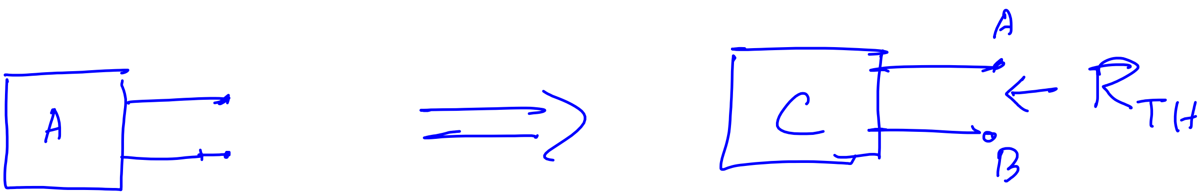
Thevnin's Theorem



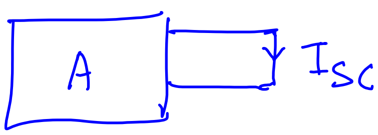
Thevnin equivalent circuit



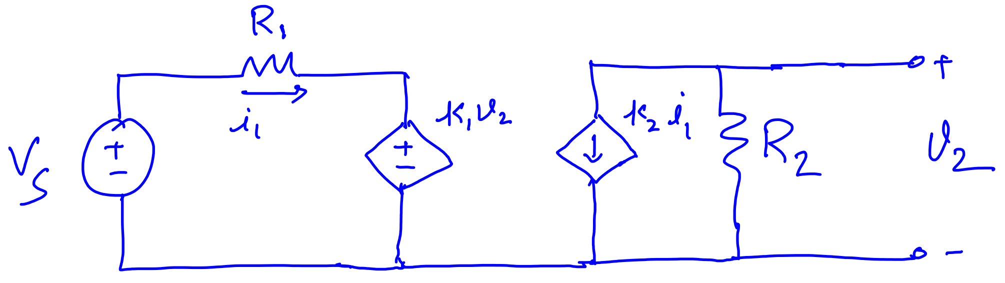
$$V_{TH} = V_{OC}$$



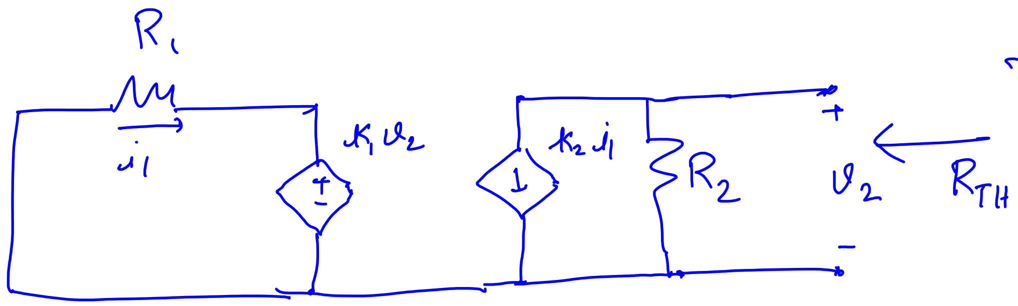
(switch off all the independent sources)



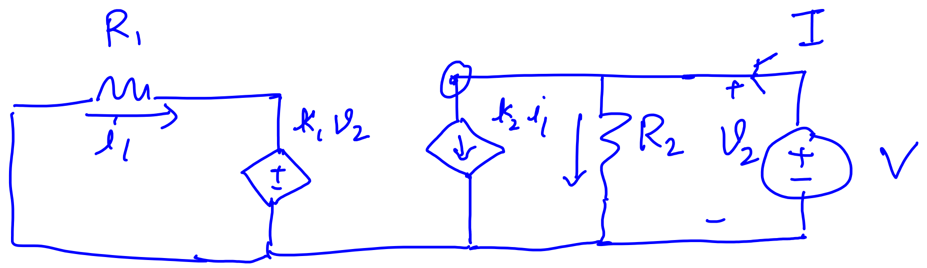
$$I_N = I_{SC}$$



network A



network C



$$i_1 R_1 + k_1 v_2 = 0 \quad \text{(KVL)}$$

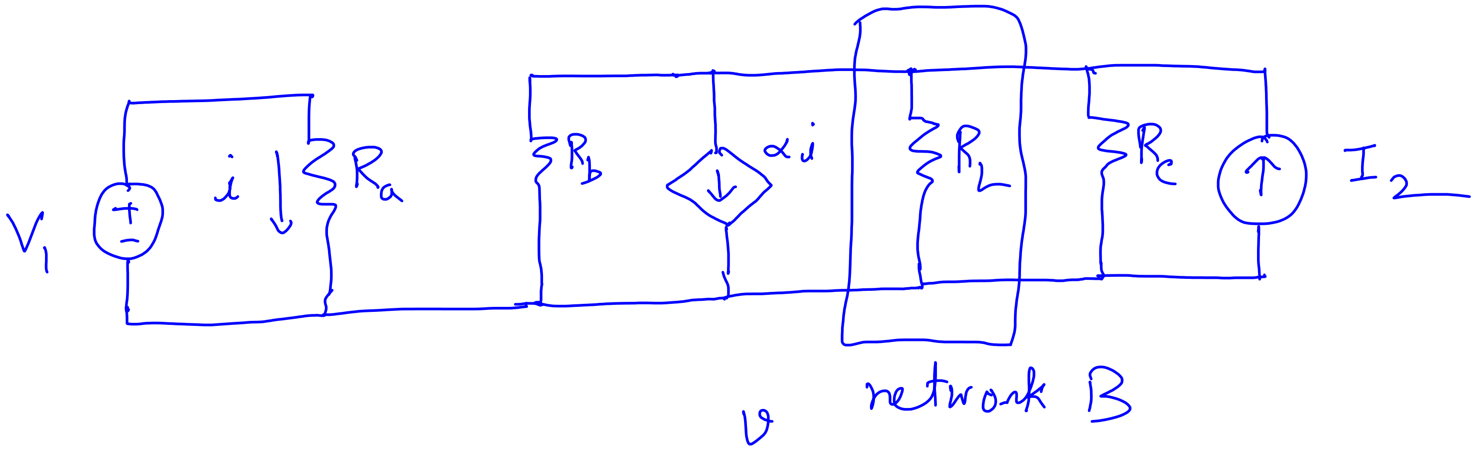
$$I = k_2 i_1 + \frac{v_2}{R_2} \quad \text{(KCL)} \qquad v_2 = V$$

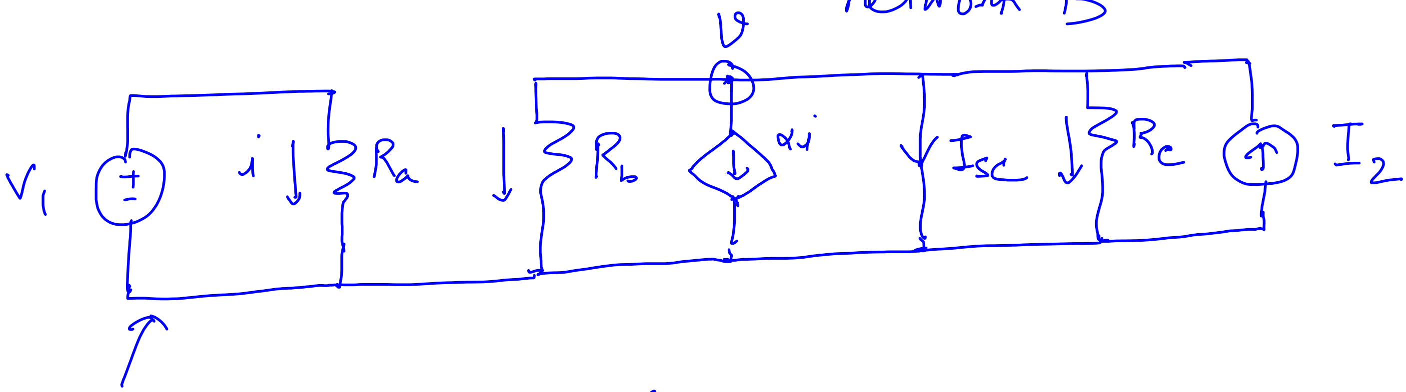
$$i_1 = -\frac{k_1 v_2}{R_1}$$

$$I = -\frac{k_1 k_2}{R_1} v_2 + \frac{v_2}{R_2}$$

$$= V\left(-\frac{k_1 k_2}{R_1} + \frac{1}{R_2}\right)$$

$$R_{TH} = \frac{V}{I} = \frac{1}{\left(-\frac{k_1 k_2}{R_1} + \frac{1}{R_2}\right)}$$



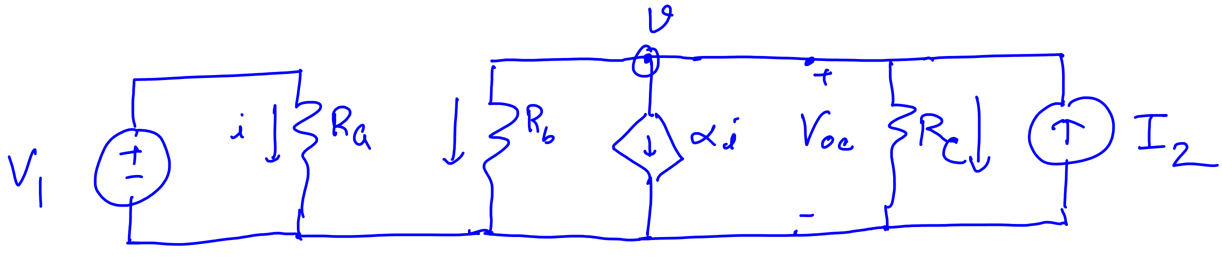


$$V_1 = i R_a \quad (\text{KVL})$$

$$I_2 = \frac{v}{R_b} + \frac{v}{R_c} + I_{sc} + \alpha i \quad (\text{KCL})$$

$$I_{sc} = I_2 - \alpha i$$

$$I_N = I_{sc} = I_2 - \alpha \frac{V_1}{R_a}$$



$$v = V_{oc}$$

$$V_1 = i R_a \quad (\text{KVL})$$

$$I_2 = \frac{v}{R_b} + \frac{v}{R_c} + \alpha i \quad (\text{KCL})$$

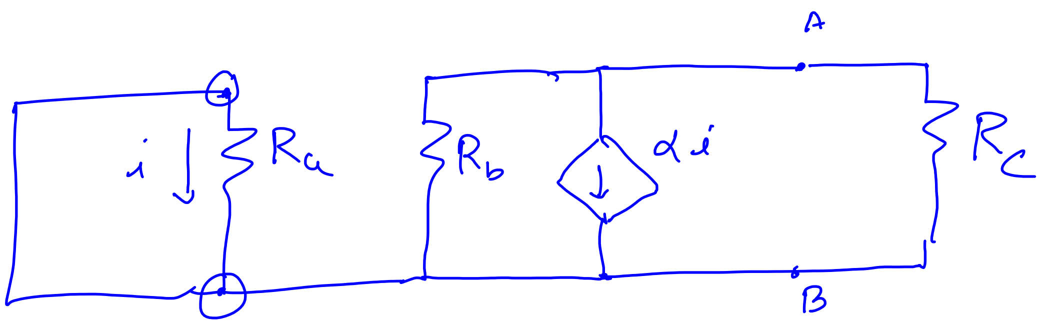
$$I_2 = \frac{v}{R_b} + \frac{v}{R_c} + \frac{\alpha V_1}{R_a}$$

$$I_2 - \frac{\alpha V_1}{R_a} = v\left(\frac{1}{R_b} + \frac{1}{R_c}\right)$$

$$I_N$$

$$V_{TH} = V_{OC} = v = \frac{I_2 - \frac{\alpha V_1}{R_a}}{\left(\frac{1}{R_b} + \frac{1}{R_c}\right)}$$

2) $$\frac{V_{TH}}{I_N} = \frac{1}{\left(\frac{1}{R_b} + \frac{1}{R_c}\right)} = R_{TH}$$



network C

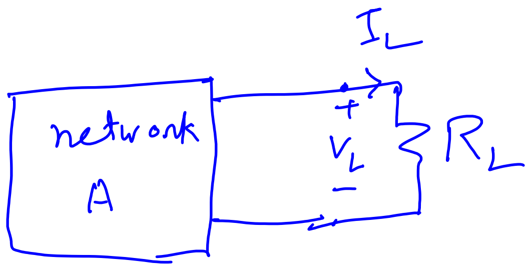
R_{AB}

$i = 0$

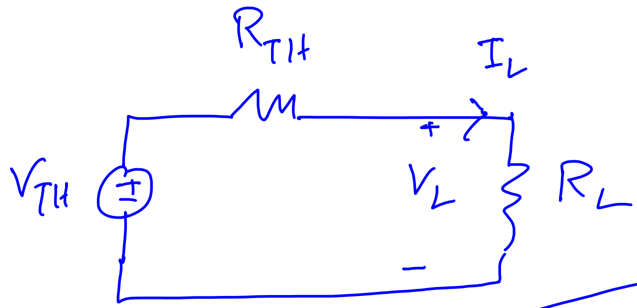
$$R_{AB}^{-1} = R_b^{-1} + R_c^{-1}$$

$$R_{TH} = R_{AB} = \frac{1}{\frac{1}{R_b} + \frac{1}{R_c}}$$

Maximum Power Transfer Theorem



$$P_L = V_L I_L$$



$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

$$P_L = I_L^2 R_L$$

$$P_L = \frac{V_{TH}^2}{(R_{TH} + R_L)^2} R_L$$

$$\frac{dP_L}{dR_L} = V_{TH}^2 \left[\frac{-2R_L}{(R_{TH}+R_L)^3} + \frac{1}{(R_{TH}+R_L)^2} \right]$$

$$= V_{TH}^2 \left[\frac{R_{TH} - R_L}{(R_{TH}+R_L)^3} \right]$$

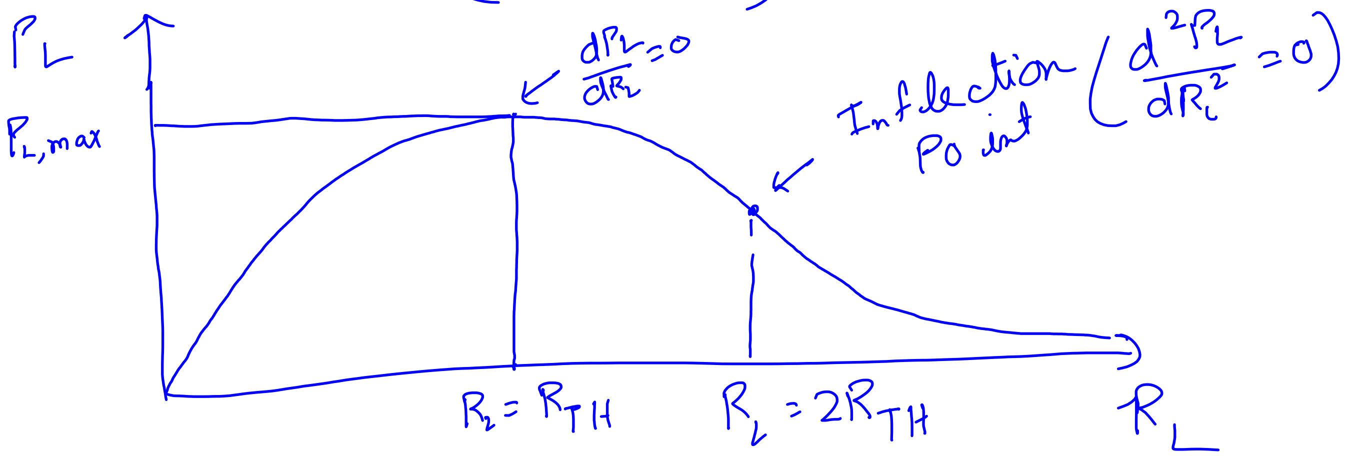
$$\frac{dP_L}{dR_L} = 0 \Rightarrow R_L = R_{TH}$$

$$\frac{d^2P_L}{dR_L^2} = V_{TH}^2 \left[\frac{-3(R_{TH}-R_L)}{(R_{TH}+R_L)^4} - \frac{1}{(R_{TH}+R_L)^3} \right]$$

$$= V_{TH}^2 \left[\frac{2R_L - 4R_{TH}}{(R_{TH}+R_L)^4} \right]$$

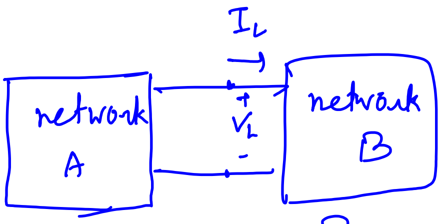
$$\left.\frac{d^2P_L}{dR_L^2}\right|_{R_L=R_{TH}} < 0 \Rightarrow R_L = R_{TH}$$ is a maxima.

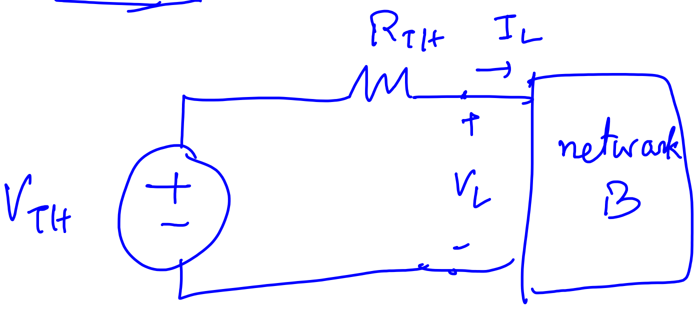
$$P_L = V_{TH}^2 \frac{R_L}{(R_{TH}+R_L)^2}$$

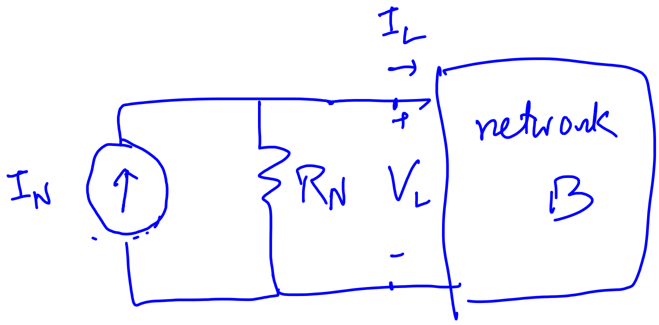


$$P_{L,max} = V_{TH}^2 \frac{R_{TH}}{(R_{TH}+R_{TH})^2} = \frac{V_{TH}^2}{4R_{TH}}$$

Quick Recap

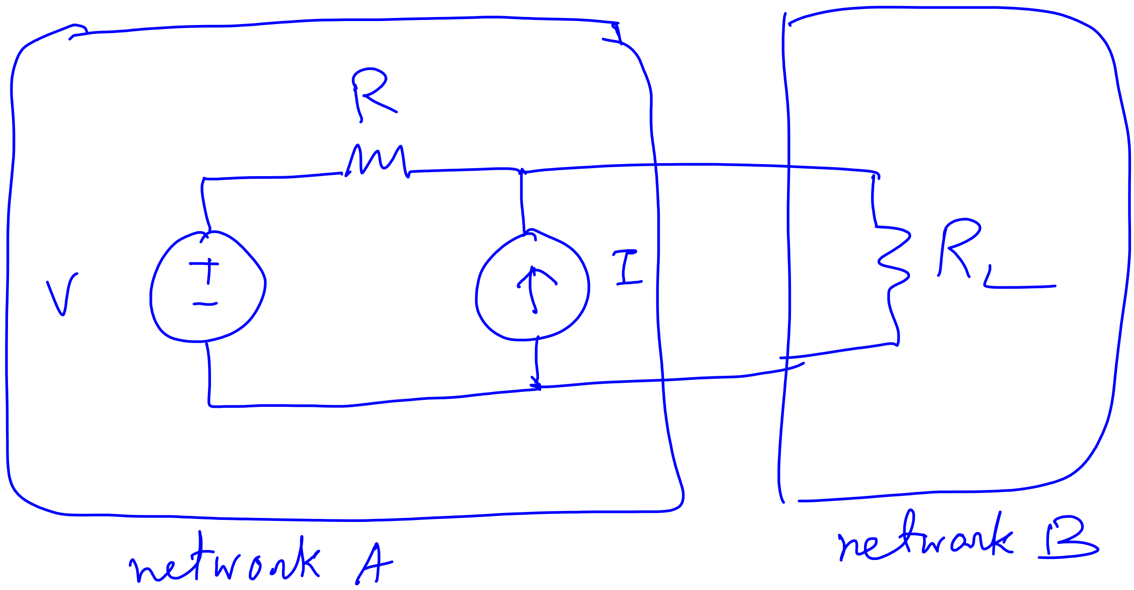


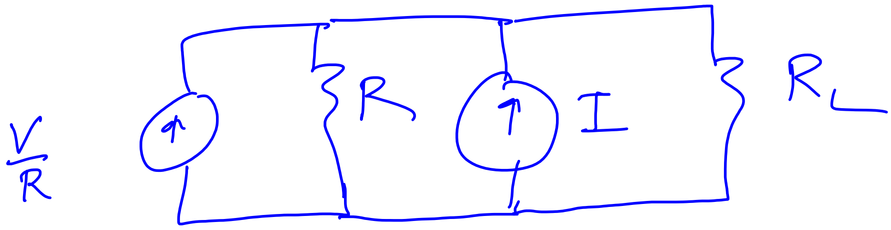


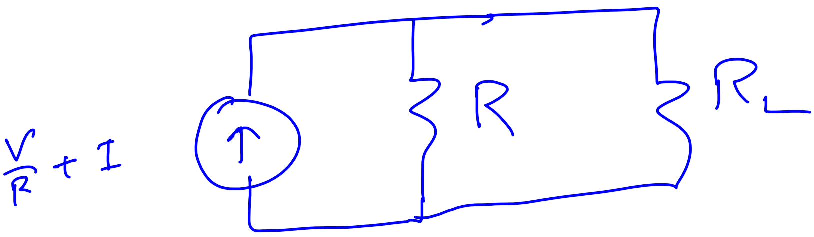


$$V_{TH} = I_N R_{TH}$$

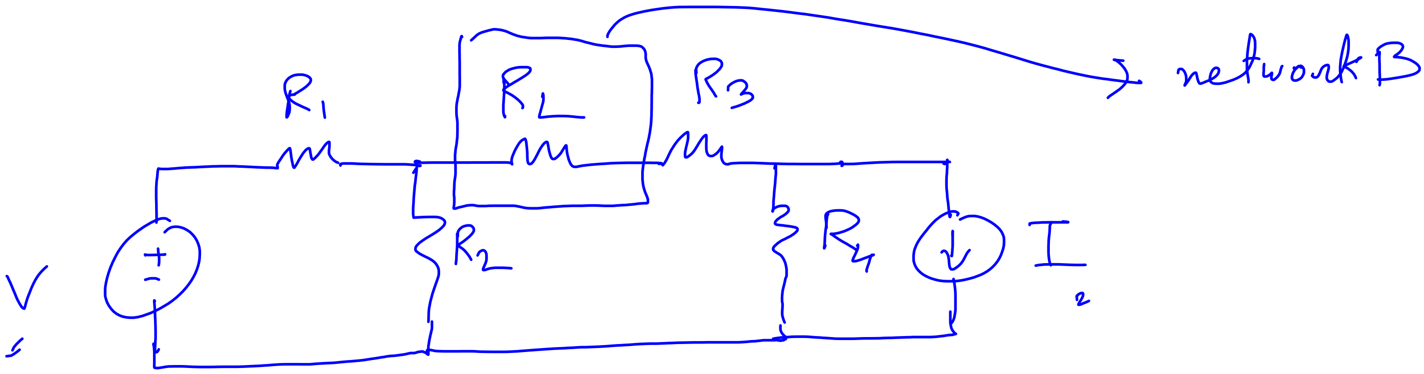
$$R_N = R_{TH}$$



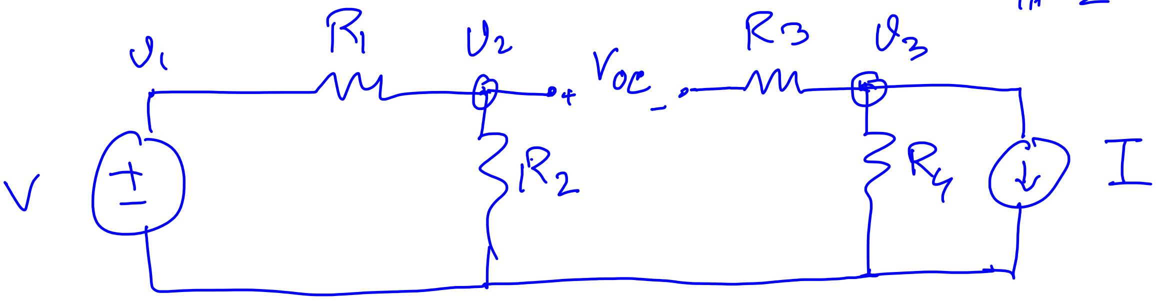




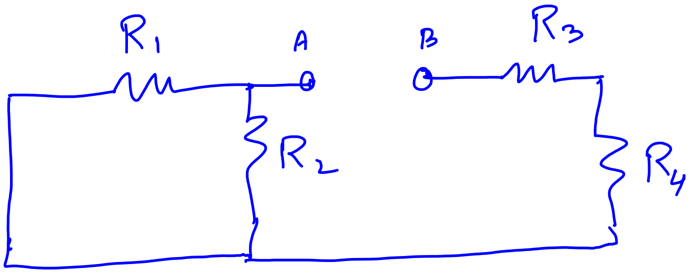
$$\left.\begin{aligned} I_N &= \frac{V}{R} + I \\ R_N &= R \end{aligned}\right\}$$



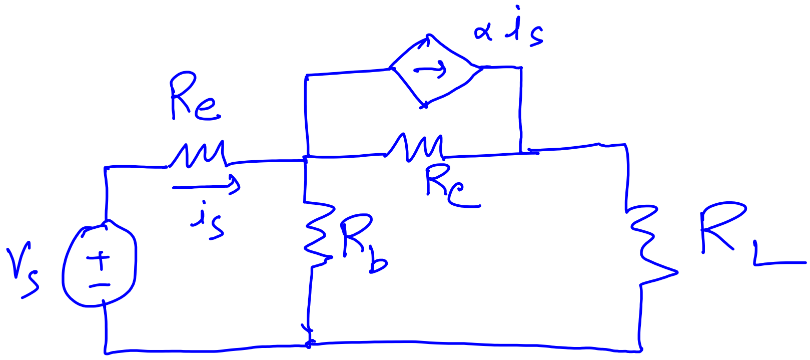
$$V_{TH} = V_{OC} = v_2 - v_3$$

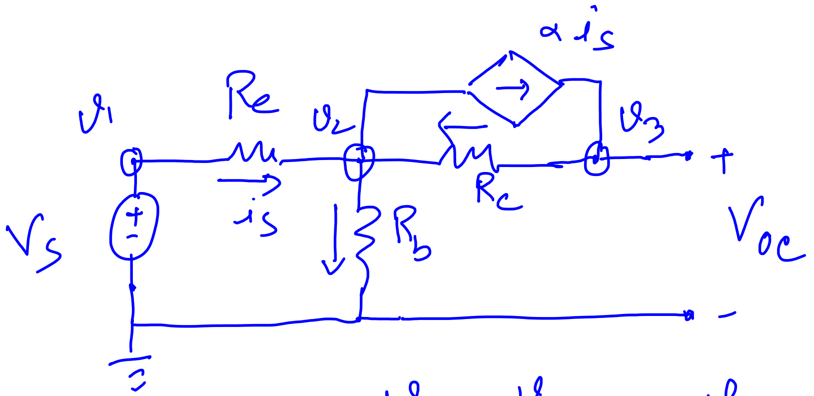


$$R_{TH} = (R_1 \parallel R_2) + R_3 + R_4$$



R_{AB}



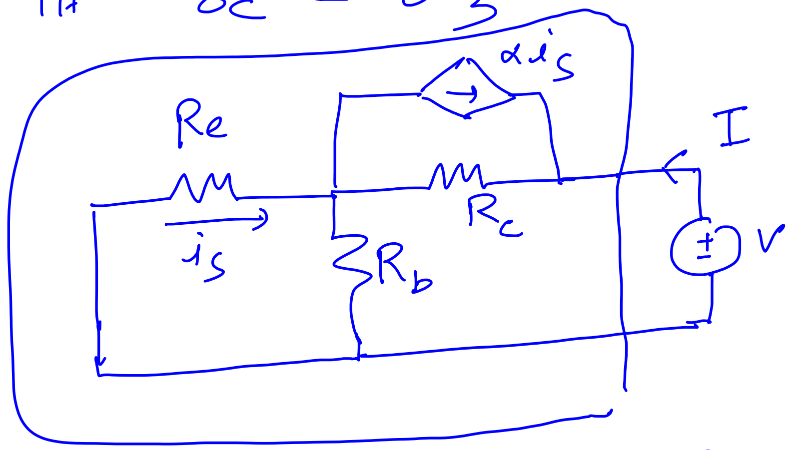


$$v_1 = V_s$$

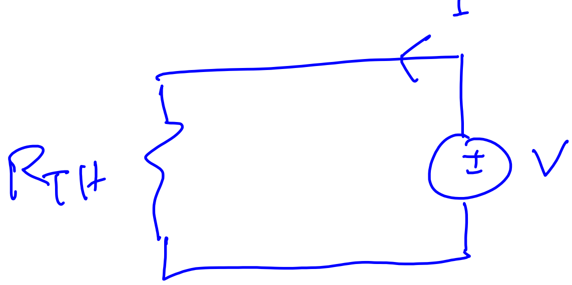
$$\frac{v_1 - v_2}{R_e} + \frac{v_3 - v_2}{R_c} = \frac{v_2}{R_b} + \alpha\left(\frac{v_1 - v_2}{R_e}\right)$$

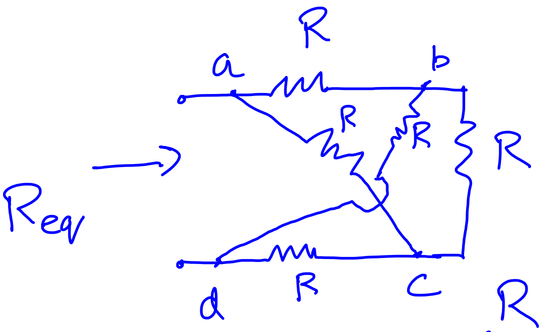
$$\frac{v_3 - v_2}{R_c} = \alpha\left(\frac{v_1 - v_2}{R_e}\right)$$

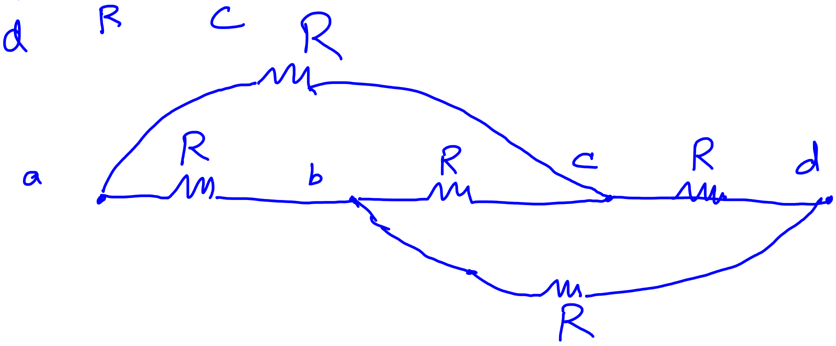
$$V_{TH} = V_{oc} = v_3$$

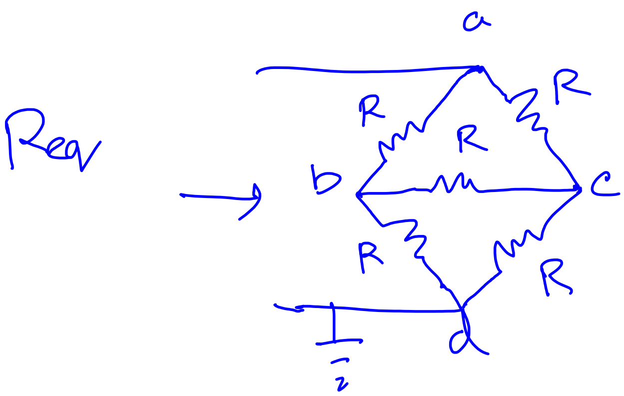


$$R_{TH} = \frac{V}{I}$$





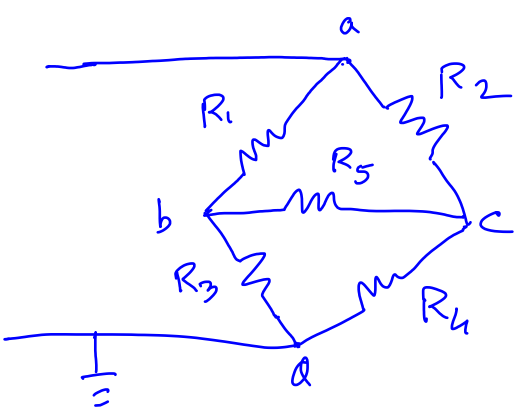




$v_b = v_c$

$R_{eq} = R$

Wheatstone Bridge



$$\boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$$

$$\Updownarrow$$

$$i_{R_5} = 0$$

Condition X,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

condition Y,

$$i_{R_5} = 0 \checkmark$$

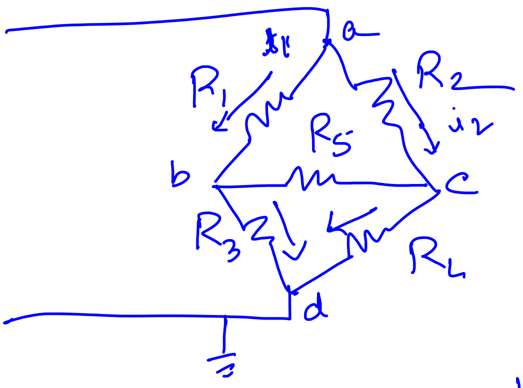
$\boxed{Y \Rightarrow X}$ and $\boxed{X \Rightarrow Y}$

X is a necessary and sufficient condition for Y.

Y is true if and only if X is true.

iff

Proving $Y \Rightarrow X$,



$$i_{R_5} = 0$$

$$\Rightarrow V_b = V_c \checkmark$$

$$V_a - V_b = V_a - V_c$$

$$i_1 R_1 = i_2 R_2$$

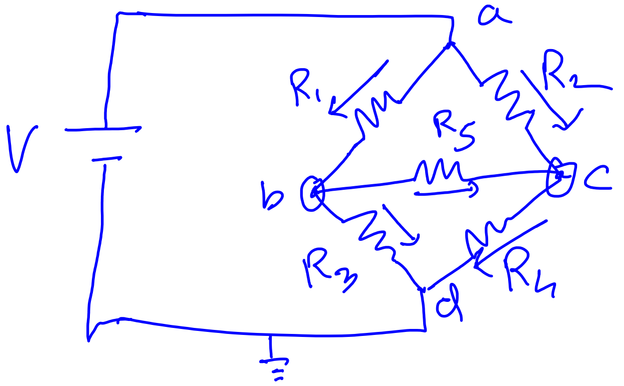
$$V_b = i_1 R_3 = i_2 R_4 = V_c$$

$$\frac{i_1}{i_2} = \frac{R_2}{R_1}$$

$$\frac{i_1}{i_2} = \frac{R_4}{R_3} \Rightarrow \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

$$\Rightarrow \boxed{Y \Rightarrow X}$$

Proving $X \Rightarrow Y$,



$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$V_a = V$$

KCL at node b,

$$\frac{V_a - V_b}{R_1} = \frac{V_b - V_c}{R_5} + \frac{V_b}{R_3}$$

KCL at node c,

$$\frac{V_a - V_c}{R_2} + \frac{V_b - V_c}{R_5} = \frac{V_c}{R_4}$$

$$V_b\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_c}{R_5} = \frac{V}{R_1} \quad \text{--- (1)}$$

$$\frac{V_b}{R_5} - V_c\left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) = -\frac{V}{R_2} \quad \text{--- (2)}$$

Multiply (I) by R_1 and (II) by R_2,

$$R_1\left(v_b\left(\frac{1}{R_1}+\frac{1}{R_3}+\frac{1}{R_5}\right)-\frac{v_c}{R_5}\right)=$$

$$-R_2\left(\frac{v_b}{R_5}-v_c\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right)\right)$$

$$\Rightarrow\quad v_b\left(1+\frac{R_1}{R_3}+\frac{R_1}{R_5}+\frac{R_2}{R_5}\right)$$

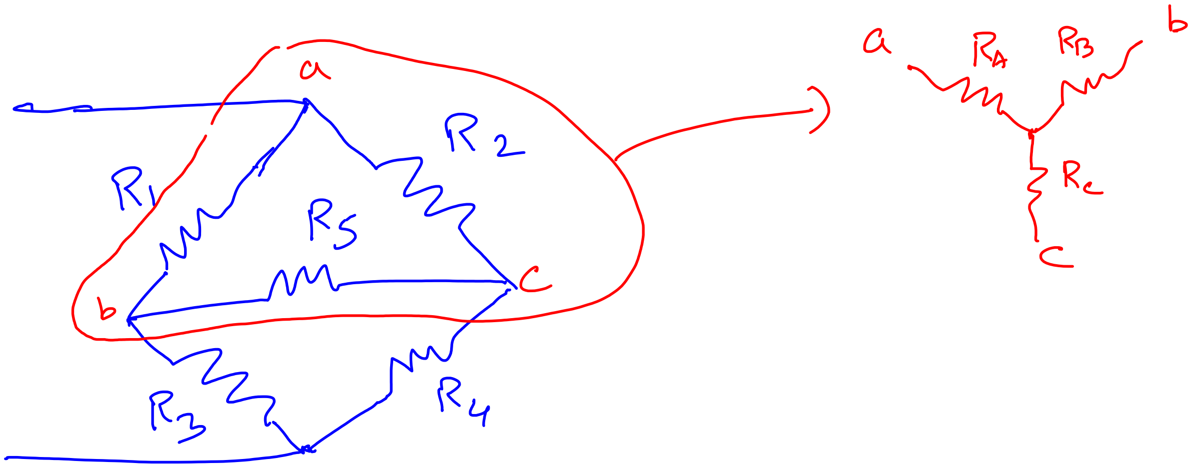
$$=v_c\left(1+\frac{R_2}{R_4}+\frac{R_2}{R_5}+\frac{R_1}{R_5}\right)$$

$$\Rightarrow\quad v_b=v_c$$

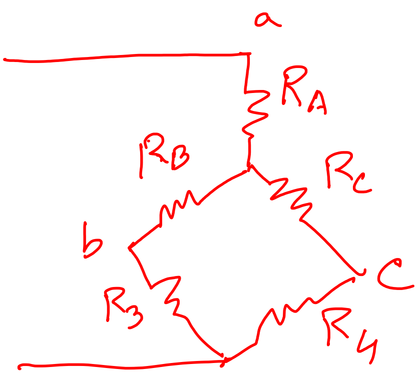
$$\Rightarrow\quad i_{R_5}=0\quad\Rightarrow\quad Y \text{ is true.}$$

So $\boxed{X \Rightarrow Y}$

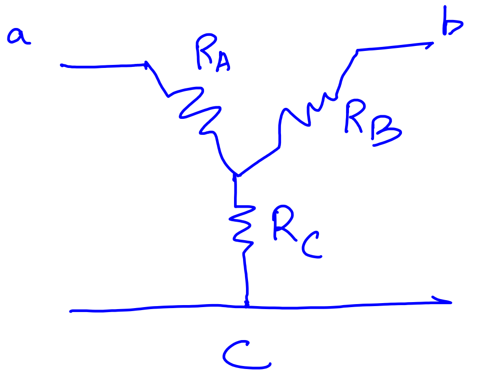
$$\Rightarrow\quad \boxed{X \Leftrightarrow Y}$$



What if $\frac{R_1}{R_2} \neq \frac{R_3}{R_4}$

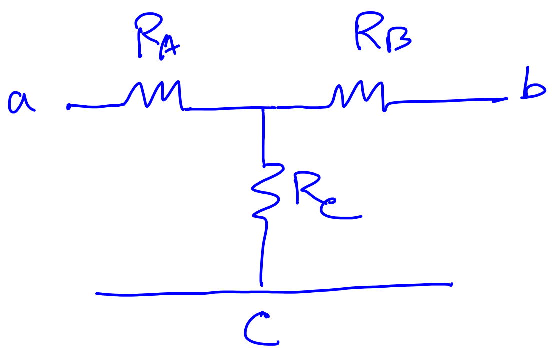


Star - delta Conversion

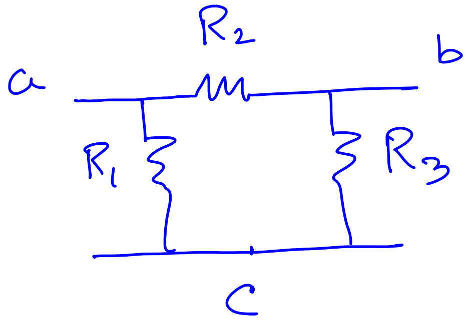


Y - network

$\Updownarrow$



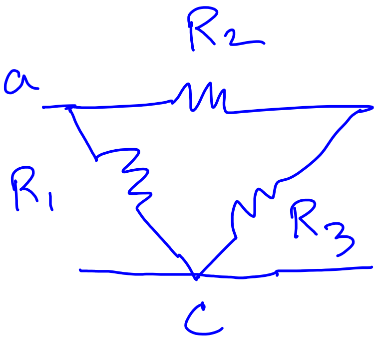
T - network

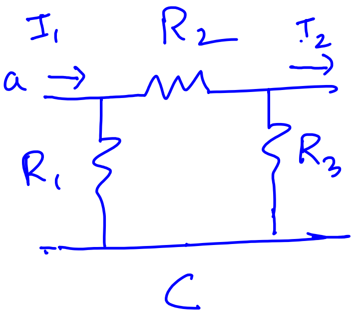


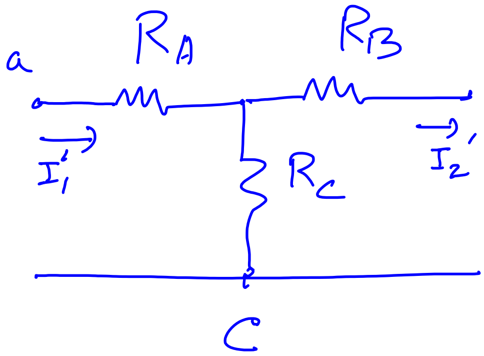
Π - network

$\Updownarrow$

Delta - network



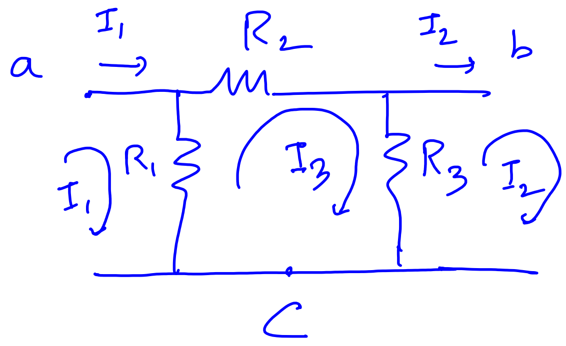




V_{ac}, V_{bc} $\qquad\qquad$ V_{ac}', V_{bc}'

$$\left.\begin{aligned} &I_1 = I_1', \quad I_2 = I_2' \\ \text{and} \quad &V_{ac} = V_{ac}', \quad V_{bc} = V_{bc}' \end{aligned}\right\} \text{equivalent}$$

For equivalence, what should be the relation among the resistances?



$$V_{ac} = I_1 R_1 - I_3 R_1$$

$$-(I_1 - I_3)R_1 + I_3 R_2 + (I_3 - I_2)R_3 = 0$$

$$V_{bc} = -(I_2 - I_3)R_3$$

$$I_3(R_1 + R_2 + R_3) = I_1 R_1 + I_2 R_3$$

$$I_3 = \frac{I_1 R_1 + I_2 R_3}{(R_1 + R_2 + R_3)}$$

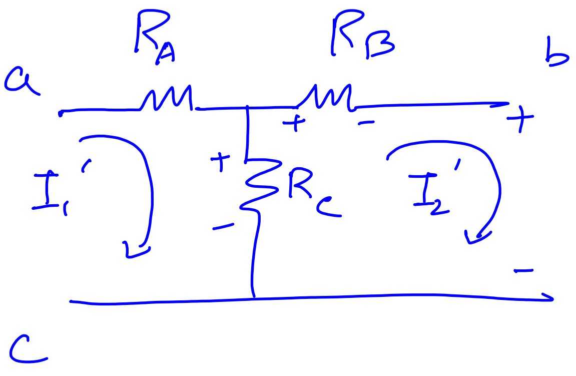
$$V_{ac} = I_1 R_1 - \frac{R_1^2}{R_1 + R_2 + R_3} I_1 - \frac{R_3 R_1}{R_1 + R_2 + R_3} I_2$$

$$V_{ac} = \underline{\left(R_1 - \frac{R_1^2}{R_1 + R_2 + R_3} \right)} I_1 - \underbrace{\frac{R_3 R_1}{R_1 + R_2 + R_3}}_{1} I_2$$

$$V_{bc} = -I_2 R_3 + I_3 R_3$$

$$= -I_2 R_3 + R_3 \left(\frac{I_1 R_1 + I_2 R_3}{R_1 + R_2 + R_3} \right)$$

$$v_{bc} = \frac{R_3 R_1}{R_1+R_2+R_3} I_1 - I_2\left(R_3 - \frac{R_3^2}{R_1+R_2+R_3}\right)$$ — (ii)



$$v_{R_c} = (I_1' - I_2')R_c$$

$$v_{R_B} = I_2' R_B$$

$$v_{ac}' = I_1' R_A + I_1' R_c - I_2' R_c$$

$$v_{bc}' = +(I_1' - I_2')R_c - I_2' R_B$$

$$v_{R_c} - v_{R_B} - v_{bc}' = 0$$

$$v_{ac}' = I_1'(R_A + R_c) - R_c I_2'$$ — (iii)

$$v_{bc}' = R_c I_1' - (R_c + R_B) I_2'$$ — (iv)

<u>Comparing equations (i) and (iii)</u>,

$$R_A + R_c = R_1 - \frac{R_1^2}{R_1+R_2+R_3}$$

$$\checkmark\ R_c = \frac{R_3 R_1}{R_1+R_2+R_3}$$

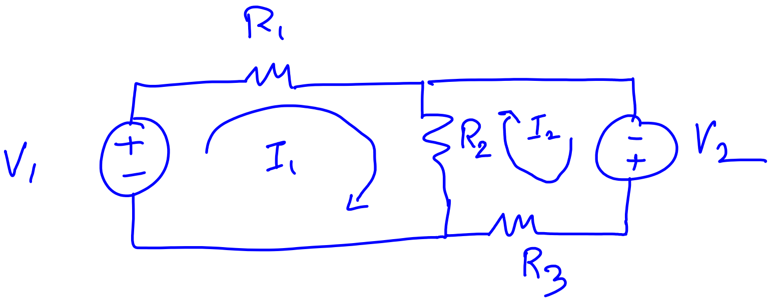
similarly derive the other expressions.

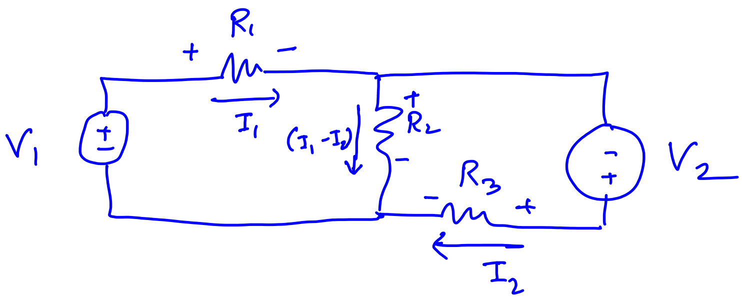
$$R_C = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

Quick Recap





$$V_{R_1} = I_1 R_1$$
$$V_{R_2} = (I_1 - I_2) R_2$$
$$V_{R_3} = I_2 R_3$$

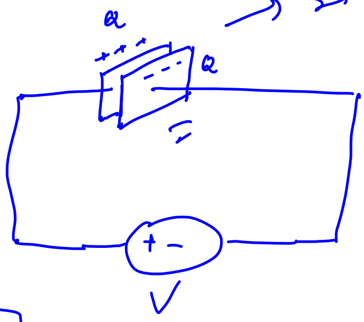
$$V_1 - V_{R_1} - V_{R_2} = 0$$

$$V_2 - V_{R_3} + V_{R_2} = 0$$

Capacitors

$Q = CV$

$C \rightarrow$ capacitance



$\rightarrow$ store charge

$$\boxed{I = \frac{dQ}{dt} = C\frac{dV}{dt}}$$

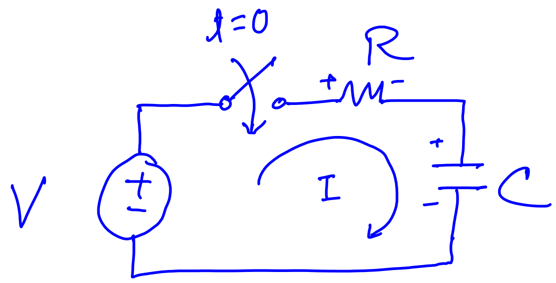
$$C = \frac{\varepsilon A}{d}$$

$\varepsilon \rightarrow$ permittivity

$A \rightarrow$ area of the plate

$d \rightarrow$ distance between the plates unit of capacitance Farad = $\frac{\text{Coulomb}}{\text{volt}}$ }

Capacitor Charging →



$V_R = IR$

$V = V_R + V_C$

$I = C\frac{dV_C}{dt}$

$V = IR + V_C$

$= CR\frac{dV_C}{dt} + V_C$

$V = RC\frac{dV_C}{dt} + V_C$

$$\Rightarrow \int_0^{V_C} \frac{dV_C}{V - V_C} = \int_0^t \frac{1}{RC}\, dt$$

$$\Rightarrow -\ln\left(\frac{V - V_C}{V}\right) = \frac{t}{RC}$$

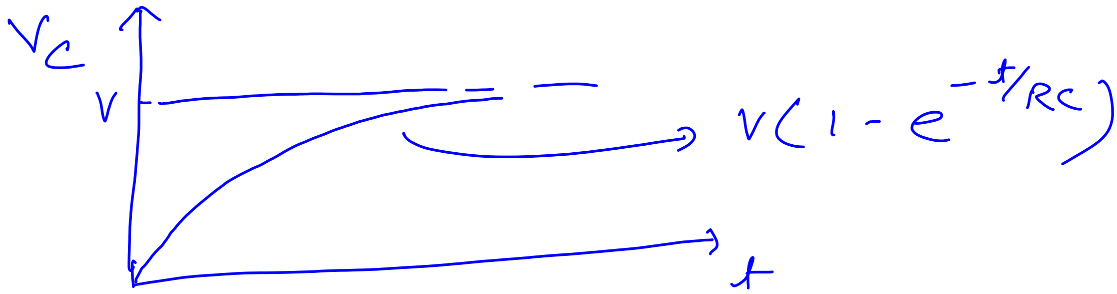
$$\Rightarrow \frac{V - V_C}{V} = e^{-t/RC}$$

$$\Rightarrow V_C = V - Ve^{-t/RC}$$

$$V_C = V(1 - e^{-t/RC}) \quad \forall t \geq 0$$

at $t = 0$ $V_C = 0$

at $t \to \infty$ $V_C \to V$ at $t \to \infty$ the capacitor is fully charged

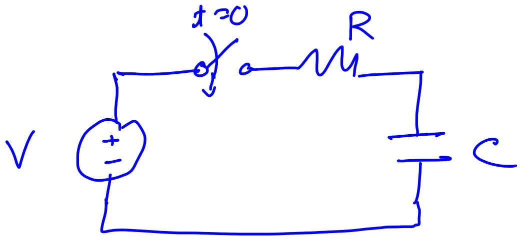


$$I = C\frac{dV_c}{dt} = \frac{CV}{RC}e^{-t/RC} = \frac{V}{R}e^{-t/RC}$$

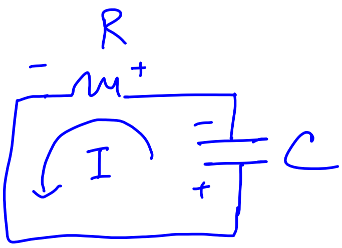
at $t=0$ the discharged capacitor acts as a short circuit

at $t\to\infty$ the capacitor acts as an open circuit.

Capacitor discharging



After a very long time remove the voltage source and connect the R and C together.



$$V_R + V_c = 0$$

$$V_R = IR \quad , \quad I = C\frac{dV_c}{dt}$$

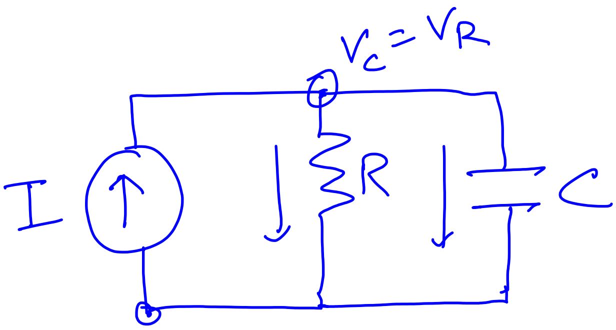
$$IR + V_c = 0$$

$$RC\frac{dV_c}{dt} + V_c = 0$$

$$\int_{V_c(0)=V}^{V_c} \frac{dV_c}{V_c} = \int_0^t -\frac{1}{RC}\,dt$$

$$\ln \frac{V_c}{V} = -\frac{t}{RC}$$

$$\Rightarrow \quad V_c = V e^{-t/RC} \quad \text{(discharging)}$$



$$I = \frac{V_c}{R} + C\frac{dV_c}{dt}$$

$$\Rightarrow \int_0^{V_c} \frac{dV_c}{IR - V_c} = \int_0^t \frac{1}{RC}\,dt$$

$$\Rightarrow \quad -\ln \frac{IR - V_c}{IR} = \frac{t}{RC}$$

$$\Rightarrow \quad IR - V_c = IR\, e^{-t/RC}$$

$$\Rightarrow \quad V_c = IR\left(1 - e^{-t/RC}\right)$$

$$I_c = C\frac{dV_c}{dt} = \frac{IRC}{RC} e^{-t/RC} = I e^{-t/RC}$$

Quick Recap

Capacitors (C)

$$C = \frac{\epsilon A}{d}$$

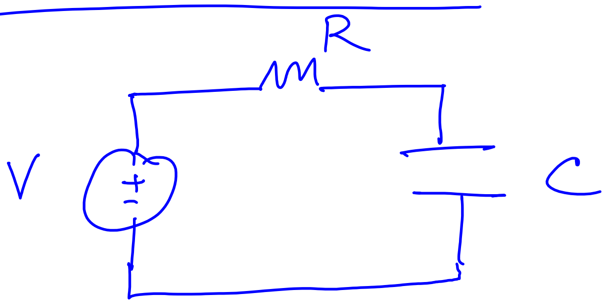
$$E = \int_0^t P\,dt = \int_0^t VI\,dt = \int_0^t V \cdot C \frac{dV}{dt} \cdot dt$$

Energy Power

$$= \int_{V(0)}^{V(t)} CV\,dV$$

$$= \frac{1}{2} CV^2(t) - \frac{1}{2} CV^2(0)$$

Charging of a capacitor



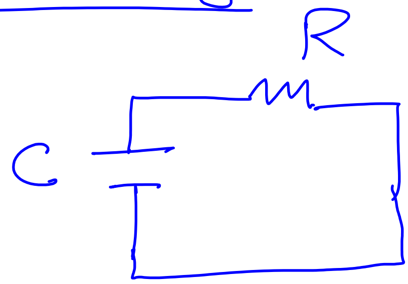
$$\boxed{V_C(0) = 0}$$

$$V_C(t) = V\left(1 - e^{-t/RC}\right) \quad \forall t \geq 0$$

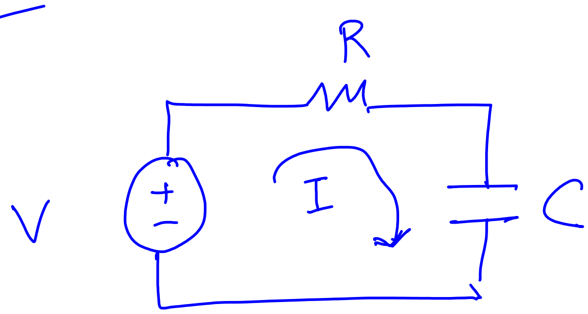
$$\boxed{V_C(\infty) = V}$$

Capacitor discharging

$V_c(0) = V$



$$V_c(t) = V e^{-t/RC} \quad \forall t \geq 0$$



$, \quad V_c(0) = 0$

$$V = \underbrace{IR} + \underbrace{\frac{1}{C}\int I\,dt}$$

$$0 = \frac{dI}{dt} \cdot R + \frac{1}{C} I$$

$$\int_{I(0)}^{I(t)} \frac{dI}{I} = -\frac{1}{RC}\int_0^t dt$$

$$\ln \frac{I(t)}{I(0)} = -\frac{1}{RC} t$$

$$\Rightarrow \quad I(t) = I(0)\, e^{-t/RC}$$

$$V = V_R(t) + V_C(t) \quad \forall t \geq 0$$

$$V = V_R(0) + V_c(0) \quad \Rightarrow \quad V_R(0) = V$$

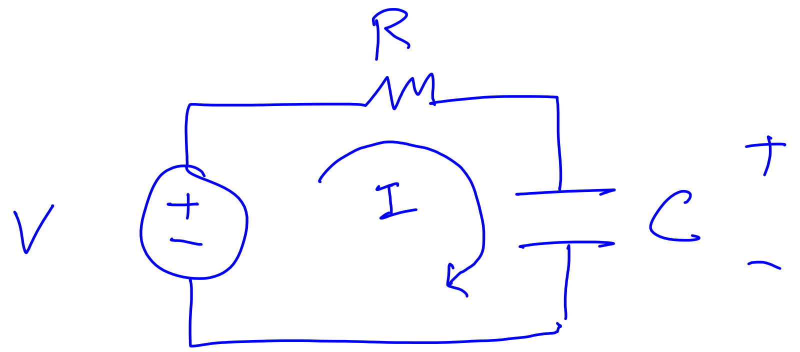
$$I(t) = \frac{V_R(t)}{R} \qquad \forall\, t \geq 0$$

$$I(0) = \frac{V_R(0)}{R} = \frac{V}{R}$$

$$I(t) = I(0)\, e^{-t/RC}$$

$$= \frac{V}{R}\, e^{-t/RC}$$

//



$$V_c(0) = V_0$$

$$V = IR + V_c$$

$$= C\frac{dV_c}{dt}\cdot R + V_c$$

$$V - V_c = RC\frac{dV_c}{dt}$$

$$\Rightarrow \int_{V_0}^{V_c(t)} \frac{dV_c}{V - V_c} = \int_0^t \frac{dt}{RC}$$

$$-\ln\frac{V - V_c(t)}{V - V_0} = \frac{t}{RC}$$

$$\frac{V - V_c(t)}{V - V_0} = e^{-t/RC}$$

$$V - V_c(t) = V e^{-t/RC} - V_0 e^{-t/RC}$$

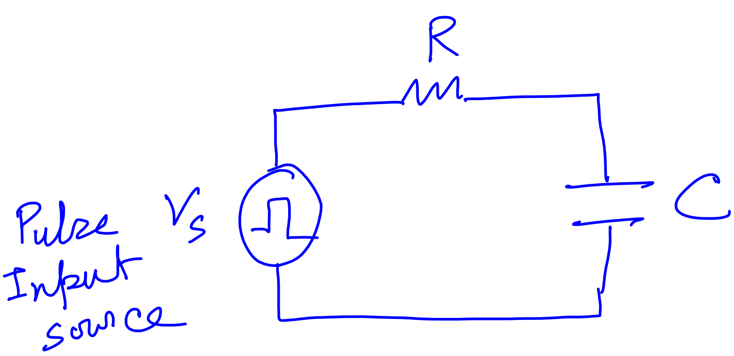
$$V_c(t) = \underbrace{V\left(1 - e^{-t/RC}\right)}_{\text{Forced Response}} + \underbrace{V_0 e^{-t/RC}}_{\text{Natural Response}}$$

Due to the initial conditions

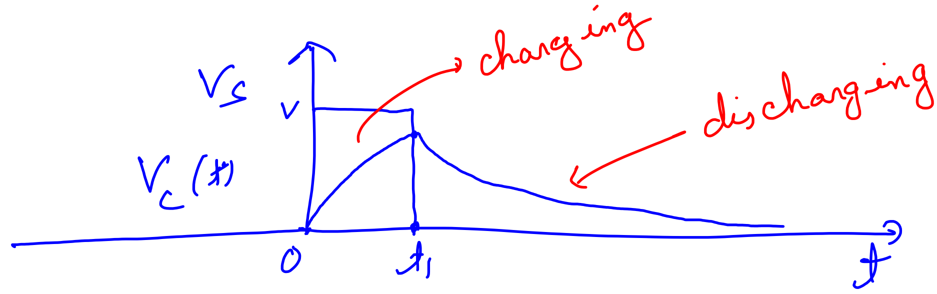
$$= \underbrace{V}_{\text{steady state}} \underbrace{- V e^{-t/RC} + V_0 e^{-t/RC}}_{\text{transient response}}$$

Due to the external source

(*) Superposition principle can also be applied to derive the above.



$$\rightarrow V_s = \begin{cases} V & 0 \le t \le t_1 \\ 0 & t > t_1 \end{cases}$$

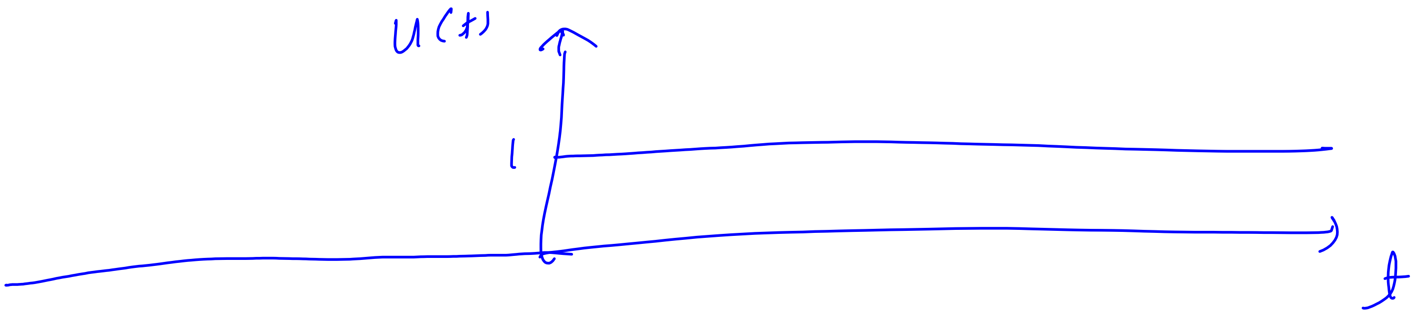


$$V_c(t) = V\left(1 - e^{-t/RC}\right) \qquad 0 \le t \le t_1$$

$$= \underbrace{V\left(1 - e^{-t_1/RC}\right)}_{V_c(t_1)} e^{-(t-t_1)/RC} \qquad t \geq t_1$$

Unit Step Function

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$



$$V_s(t) = \underbrace{V u(t)}_{\downarrow} - \underbrace{V u(t - t_1)}_{\downarrow}$$

$$V_c(t) = V\left(1 - e^{-t/RC}\right) u(t) - V\left(1 - e^{-(t-t_1)/RC}\right) u(t-t_1).$$

for $0 \le t \le t_1$

$$V_C(t) = V\left(1 - e^{-t/RC}\right)$$

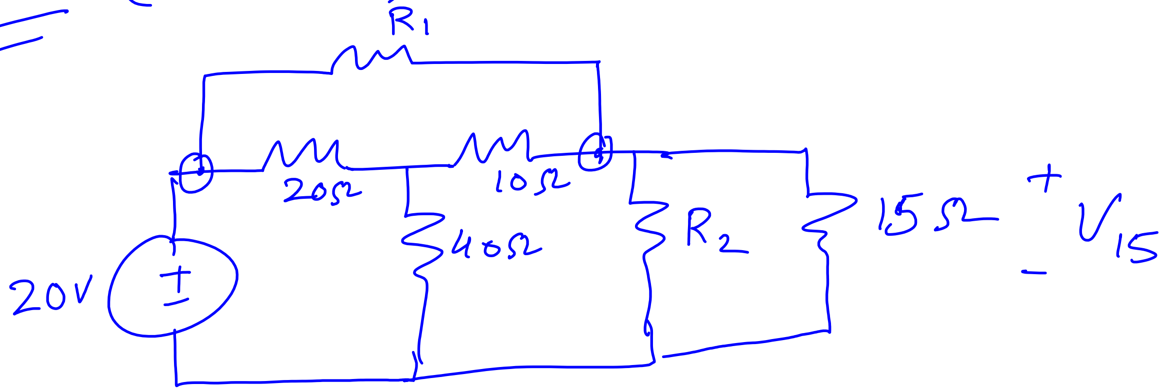
$t \geq t_1$

$$V_C(t) = V\left(1 - e^{-t/RC}\right) - V\left(1 - e^{-(t-t_1)/RC}\right)$$

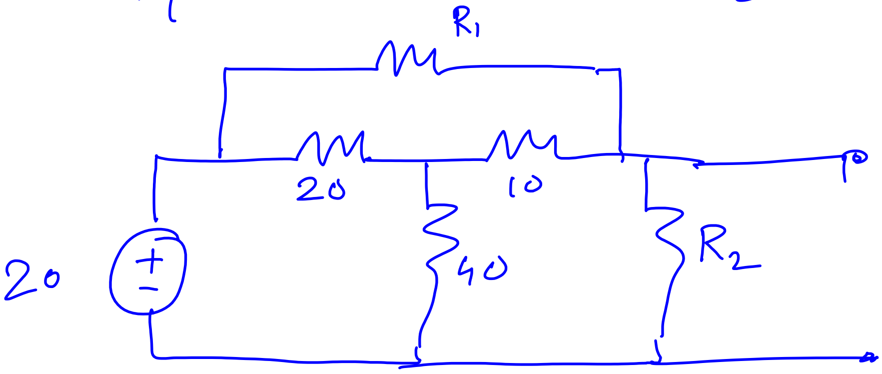
$$= V\left(e^{-(t-t_1)/RC} - e^{-t/RC}\right)$$

$$= V e^{-(t-t_1)/RC}\left(1 - e^{-t_1/RC}\right)$$

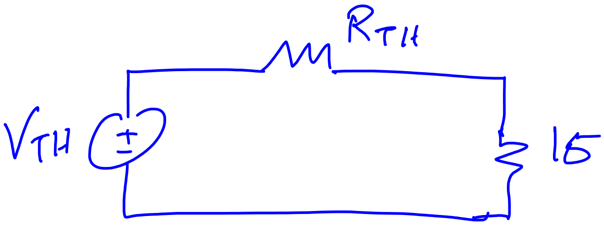
Q.1 (Midsem)



$R_1 = 0 \Rightarrow V_{15} = 20$



(R_2 can be any value, since V_{15} is independent of R_2 when $R_1 = 0$)



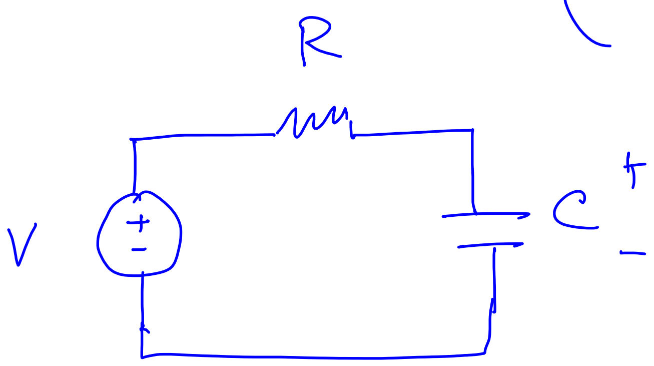
$$\Rightarrow P_{15} = \frac{V_{TH}^2}{(R_{TH}+15)^2} \times 15$$

Quick Recap

RC circuit $\rightarrow$ charging, discharging

Unit step function

$$u(t) = \begin{cases} 0 & \forall\, t < 0 \\ 1 & \forall\, t \geq 0 \end{cases}$$



$v_c(0) = 0$

$$V_c(t) = V\left(1 - e^{-t/RC}\right) \quad \forall\, t \geq 0$$

$RC \rightarrow$ time constant (τ)

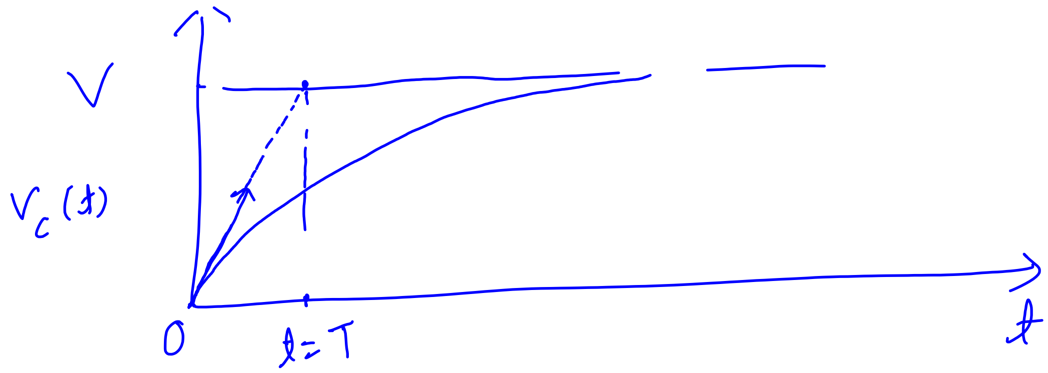
$\tau = RC$

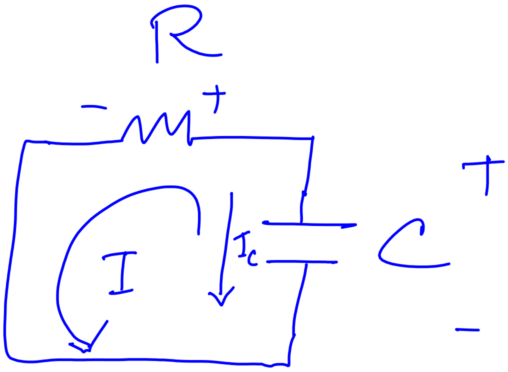
at $t = \tau$,

$$V_c(t) \approx 66\%\, V$$

$$\left.\frac{dV_c}{dt}\right|_{t=0} = \left.\frac{V}{RC} e^{-t/RC}\right|_{t=0} = \frac{V}{RC} = \frac{V}{T}$$

If the rate of change of V_c would have remained same as $\left.\frac{dV_c}{dt}\right|_{t=0}$ for $t>0$, the capacitor voltage would have reached V at $t=T$.

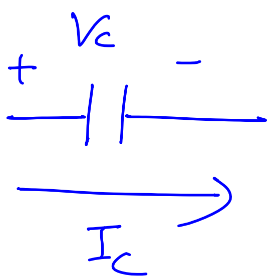




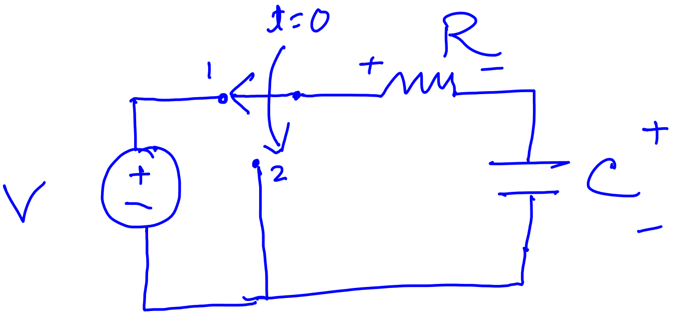
$v_c(0) = V$

$$V_c(t) = V e^{-t/RC}$$

$$I_c = C\frac{dV_c}{dt} = -I$$



$$I_c = C\frac{dV_c}{dt}$$



$V_C(0)$

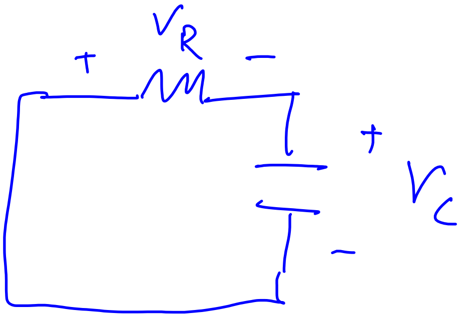
$$V_C(t) = V e^{-t/RC} \quad \forall t \geq 0$$

$$V_C(0^-) = V \Rightarrow V_C(0^+) = V \quad \text{if } I_C \neq \infty$$

$$\lim_{\varepsilon \to 0} V_C(0-\varepsilon) = \lim_{\varepsilon \to 0} V_C(0+\varepsilon) \quad \text{(continuity property)}$$

$$V_R(0^-) = 0 \qquad V_R(0^+) = -V \quad \text{(discontinuous)}$$

$t \geq 0$



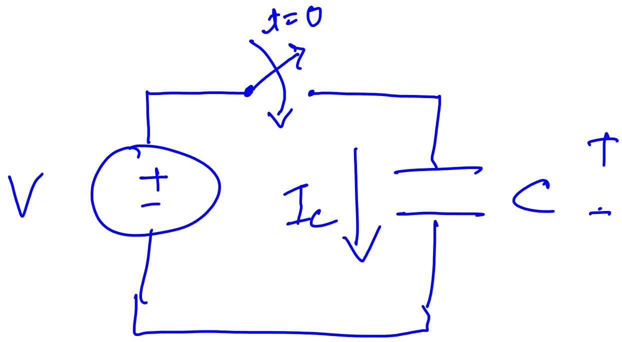
$$V_C(t) + V_R(t) = 0 \quad \forall t \geq 0$$

$$V_C(0) + V_R(0) = 0$$

$$\Rightarrow V_R(0) = -V_C(0)$$

$$= -V$$

Let's consider the following circuit,

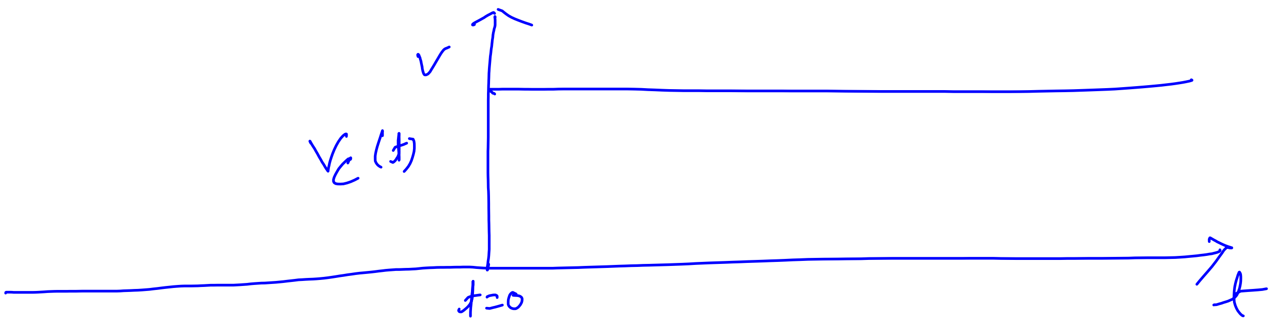


$V_C(0^-) = 0$

$v_c(0^+) = V$ (using KVL)

$$I_c = C \frac{dv_c}{dt}$$

$$v_c(t) = V u(t)$$

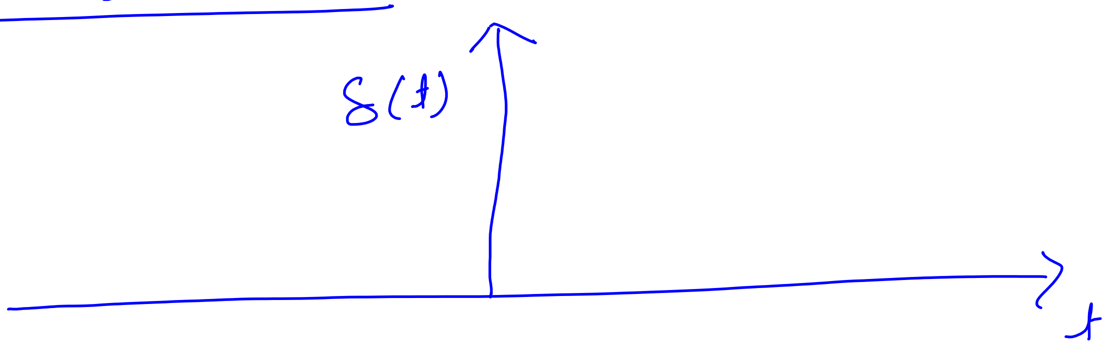


1) $\frac{dv_c}{dt} = 0 \quad \forall\, t \neq 0$

2) $\frac{dv_c}{dt}$ is not defined at $t=0$

3) $$\int_{-\infty}^{\infty} \frac{dv_c}{dt}\, dt = V$$

Dirac-delta function



1) $\delta(t) = 0 \quad \forall\, t \neq 0$

2) $\delta(t)$ is not defined at $t = 0$

3) $\displaystyle\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

$$\Rightarrow \quad \frac{d\,u(t)}{dt} = \delta(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(t)\, dt$$

if $t < 0$,

$$\boxed{u(t) = \int_{-\infty}^{t} \delta(t)\, dt = 0}$$

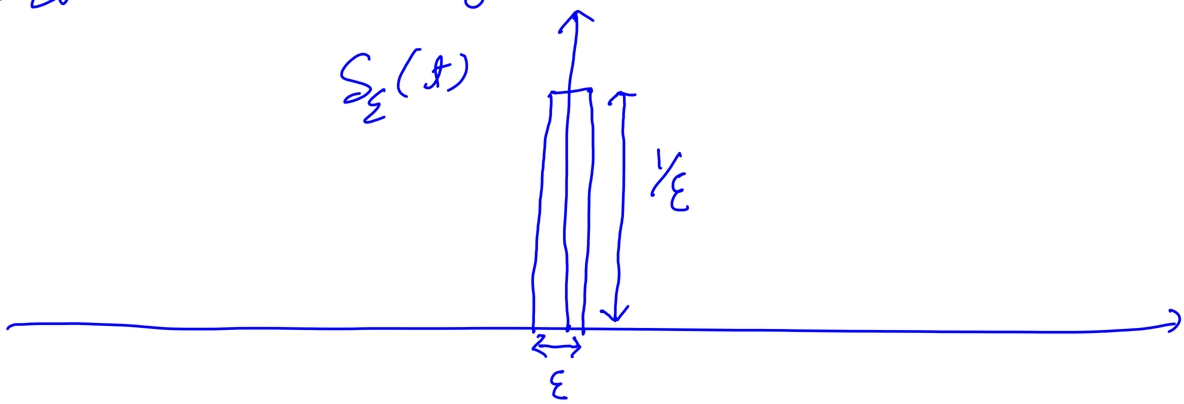
if $t \geq 0$,

$$u(t) = \int_{-\infty}^{t} \delta(t)\, dt = \int_{-\infty}^{\infty} \delta(t)\, dt - \int_{t}^{\infty} \delta(t)\, dt$$

$$= 1 - 0 = 1$$

$$\int_{-\infty}^{+\infty} \delta(t)\,dt = \int_{0^-}^{0^+} \delta(t)\,dt = 1$$

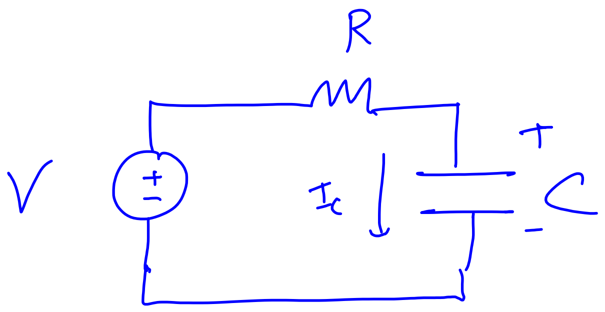
$\delta_\varepsilon(t)$



$$\delta_\varepsilon(t) = 0 \qquad \forall\, t > \varepsilon/2 \text{ and } \forall\, t < -\varepsilon/2$$

$$= 1/\varepsilon \qquad -\varepsilon/2 \le t \le \varepsilon/2$$

$$\delta(t) = \lim_{\varepsilon \to 0} \delta_\varepsilon(t)$$



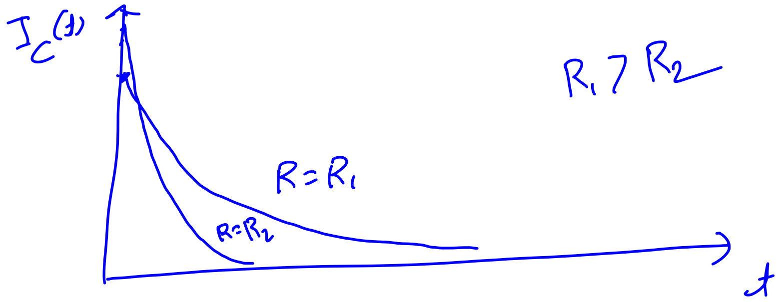
$, V_c(0^+) = 0$

$$V_c(t) = V\left(1 - e^{-t/RC}\right) \qquad \forall\, t \geq 0$$

$$I_c(t) = C\,\frac{dV_c}{dt} = \frac{V}{R}\,e^{-t/RC}$$

$\lim_{R \to 0} I_c(t) = \delta(t) \cdot C$ (some constant)

Try to derive the expression of C ...



$R_1 > R_2$

Quick Recap

Capacitors

$$C = \frac{\varepsilon A}{d}$$

$\varepsilon \rightarrow$ permitivity

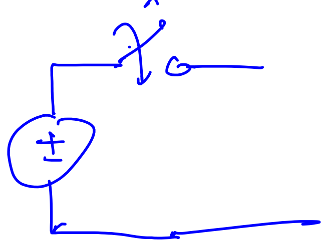
$\varepsilon_0 \rightarrow$ permitivity of free space
(8.854 pF/m)

$\varepsilon = \varepsilon_r \varepsilon_0$

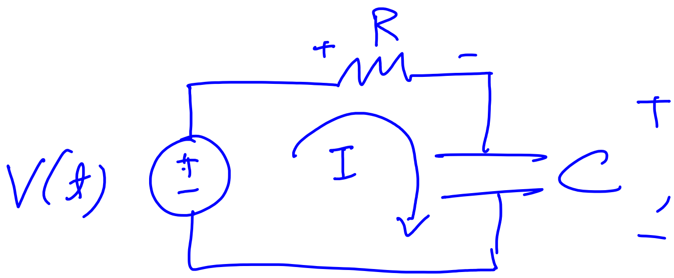
$\varepsilon_r \rightarrow$ Relative permitivity

RC circuits → charging
→ discharging

unit step function



Dirac-delta (Impulse function)



$V_c(0) = 0$

$$V(t) = V_R(t) + V_c(t)$$

$$= I(t)R + \frac{1}{C}\int_0^t I(t)\,dt$$

$$V(t) = RC\frac{dV_c}{dt} + V_c(t)$$

general structure of the problem,

$$\frac{dx}{dt} + Px = Q(t), \quad \forall t \geq 0$$

$$\boxed{x(0) = x_0}$$

$$\underbrace{e^{Pt}\frac{dx}{dt} + Pe^{Pt}x} = e^{Pt}Q(t)$$

$$x_d(t) = e^{Pt}\cdot x(t)$$

$$\frac{dx_d(t)}{dt} = \underbrace{e^{Pt}\frac{dx}{dt} + Pe^{Pt}x(t)}$$

$$= e^{Pt}Q(t)$$

$$\frac{dx_d}{dt} = e^{Pt} Q(t)$$

$$x_d(t) = x_d(0) + \int_0^t e^{Pt} Q(t)\, dt$$

$$x(t) = \underbrace{x_d(0)\, e^{-Pt}}_{\text{natural response}} + \underbrace{e^{-Pt} \int_0^t e^{P\tau} Q(\tau)\, d\tau}_{\text{forced response}}$$

$$x_d(t) = e^{Pt} x(t)$$

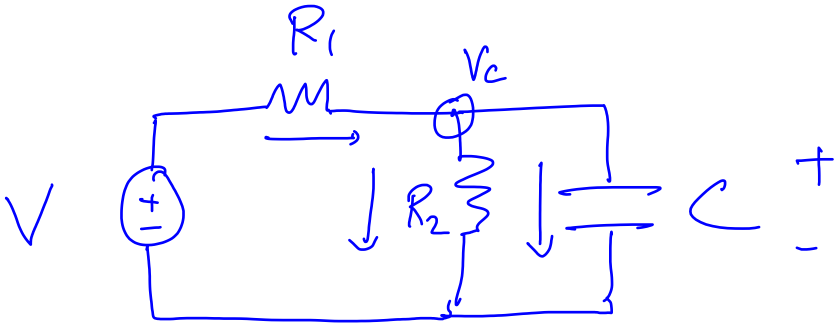
$$x_d(0) = x(0)$$

✓ $$\frac{dx}{dt} + P x(t) = Q(t) \qquad x(0) = x_0$$

$$x(t) = \underbrace{x_n(t)}_{\text{natural response}} + \underbrace{x_f(t)}_{\text{forced response}}$$

$$\frac{dx_n}{dt} + P x_n(t) = 0 \qquad x_n(0) = x_0$$

$$\frac{dx_f}{dt} + P x_f(t) = Q(t) \qquad x_f(0) = 0$$



$$V_c(0) = 0$$

$$\frac{V - V_c}{R_1} = \frac{V_c}{R_2} + C\frac{dV_c}{dt}$$

$$\frac{V}{R_1} = V_c\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + C\frac{dV_c}{dt}$$

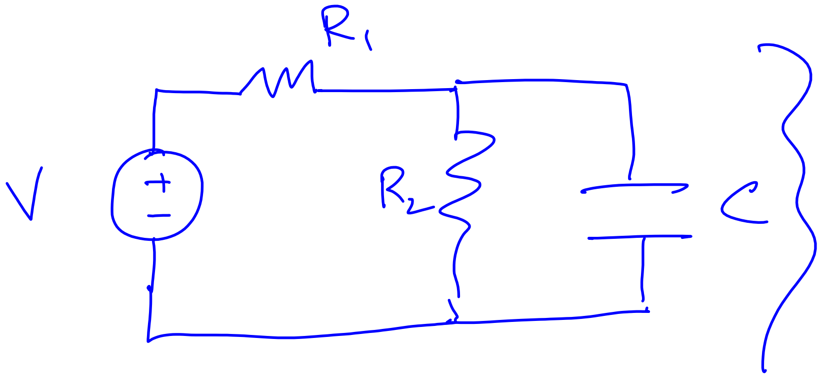
$$\frac{V}{R_1} = V_c \cdot \frac{1}{R_{eq}} + C\frac{dV_c}{dt}$$

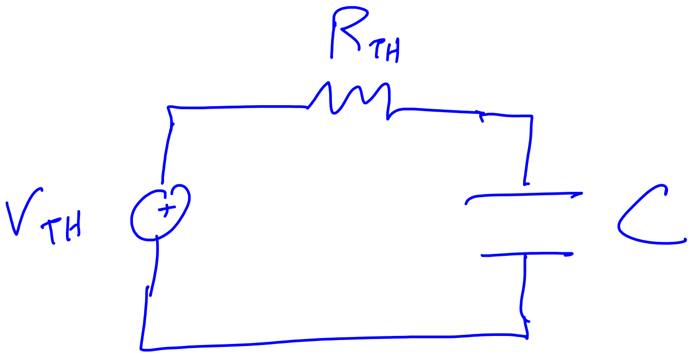
$$\frac{R_{eq}V}{R_1} = V_c + R_{eq}C\frac{dV_c}{dt}$$

$$V_c(t) = \frac{R_{eq}V}{R_1}\left(1 - e^{-t/R_{eq}C}\right)$$

$$= \frac{V}{R_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}\left(1 - e^{-t/R_{eq}C}\right)$$

$$= \underbrace{\frac{R_2 V}{R_1 + R_2}}_{V_{TH}}\left(1 - e^{-t/\underbrace{R_{eq}}_{R_{TH}}C}\right)$$



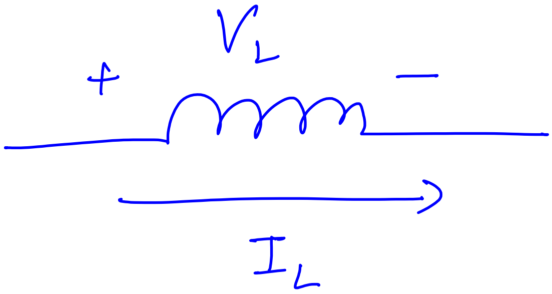


$$V_{TH} = \frac{R_2 V}{R_1 + R_2}$$

$$R_{TH} = (R_1 \parallel R_2)$$

$$= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Inductor (L)



$$V_L = L \frac{dI_L}{dt}$$

Inductance (Henry)

Faraday's Law

$$V \propto \frac{d\psi}{dt}$$

$\psi \rightarrow$ magnetic flux linkage

$$\Psi = N\phi$$

$N \to$ no. of turns

$\phi \to$ magnetic flux

$$\Psi = L\,i$$

$$L = \frac{N^2 \mu A}{l}$$ Solenoid (thin and very long coil)

$\mu \to$ permeability

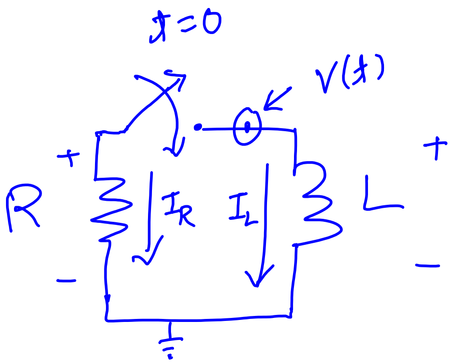
$A \to$ area of the core

$l \to$ length

$$\mu = \mu_r \mu_0$$

$\mu_0 \to 4\pi \times 10^{-7}$ H/m (free space)

Natural Response of a RL circuit



$$\boxed{V_L = V_R = V}$$

$$I_L(0^-) = I_0$$

$$V_C(t^-) = V_C(t^+) \quad \text{if} \quad I_C(t) \neq \infty$$

$$I_L(t^-) = I_L(t^+) \quad \text{if} \quad V_L(t) \neq \infty$$

(continuity of Inductor current)

Applying KVL,

$$I_L(t) + I_R(t) = 0 \qquad \forall\ t \geq 0$$

$$I_L(t) + \frac{V(t)}{R} = 0$$

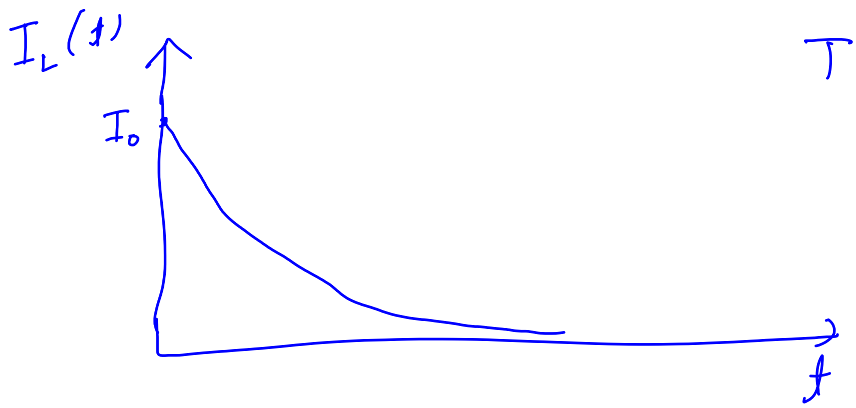
$$I_L(t) + \frac{L}{R}\frac{d I_L(t)}{dt} = 0$$

$$I_L(t) = I_L(0)\, e^{-\frac{R}{L}t}$$

$$= I_0\, e^{-\frac{R}{L}t}$$

$$= I_0\, e^{-t/T}$$

$$T = \frac{L}{R} \quad (\text{time constant})$$



$$V_L(t) = L\frac{d i_L(t)}{dt} = -\frac{L I_0}{T} e^{-t/T}$$

$$V_L(t) = -(R I_0)\, e^{-t/T}$$

$$I_L(0^-) = I_0 \quad \Rightarrow \quad I_L(0) = I_0$$

$$I_L(t) + I_R(t) = 0 \qquad \forall\, t \geq 0$$

$$\Rightarrow \qquad I_R(0) = -I_L(0)$$

$$= -I_0$$

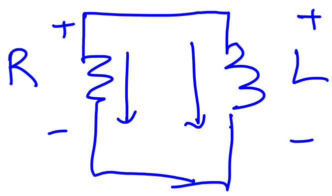
$$V_R(0) = R\, I_R(0) = -I_0 R$$

$$V_L(t) = V_R(t) \qquad \forall\, t \geq 0$$

$$\boxed{V_L(0) = -I_0 R}$$

Quick Recap

- → Capacitor
- → RC circuit
- → Inductor
- → RL circuit (natural response)



$$I_L(0) = I_0$$

$$I_L(t) = I_0 e^{-\frac{R}{L}t}, \quad \forall t \geq 0$$

$$E_R = \int_0^{\infty} V_R I_R \, dt$$

$$V_R = V_L$$

$$I_L + I_R = 0$$

$$= -\int_0^{\infty} V_L I_L \, dt$$

$$V_L = L \frac{dI_L}{dt}$$

$$= I_0 L \, e^{-\frac{R}{L}t} \times (-R/L)$$

$$= -\int_0^{\infty} -(I_0 R) e^{-\frac{R}{L}t} \times I_0 e^{-\frac{R}{L}t} \, dt$$

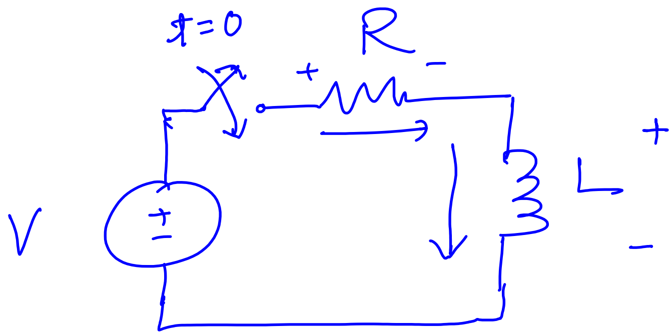
$$= -I_0 R \, e^{-\frac{R}{L}t}$$

$$= I_0^2 R \int_0^{\infty} e^{-\frac{2R}{L}t} \, dt$$

$$= \frac{1}{2} L I_0^2$$

$$E_L = \int_0^\infty V_L I_L \, dt = -\frac{1}{2} L I_0^2$$

Forced Response of RL circuit,



$I_L(0) = 0$

$I_R(t) = I_L(t)$

$$V = V_R(t) + V_L(t), \quad \forall t \geq 0$$

$$V = I_L(t) R + L \frac{d I_L(t)}{dt}, \quad \forall t \geq 0$$

$$I_L(t) = I_{L,ss} + I_{L,Tr}(t)$$

$I_{L,ss} = A_1$ (constant)

$I_{L,Tr}(t) = A_2 e^{st}$

$$I_L(t) = A_1 + A_2 e^{st}$$

$$V = (A_1 + A_2 e^{st}) R + L A_2 s e^{st}, \quad \forall t \geq 0$$

$$V = A_1 R + A_2 e^{st} R + L A_2 s e^{st} \quad \forall t \geq 0$$

2)

$$V = A_1 R$$

and

$$A_2 e^{st} R + L A_2 s e^{st} = 0 \quad \forall t \geq 0$$

$$A_1 = \frac{V}{R}$$

$$A_2 e^{st} (R + Ls) = 0 \quad \forall t \geq 0$$

$$R + Ls = 0$$

$$\Rightarrow \quad s = -\frac{R}{L}$$

$$i_L(0) = A_1 + A_2 e^{s0} = 0$$

$$\Rightarrow \quad A_1 + A_2 = 0$$

$$\Rightarrow \quad A_2 = -A_1 = -\frac{V}{R}$$

$$i_L(t) = A_1 + A_2 e^{st}$$

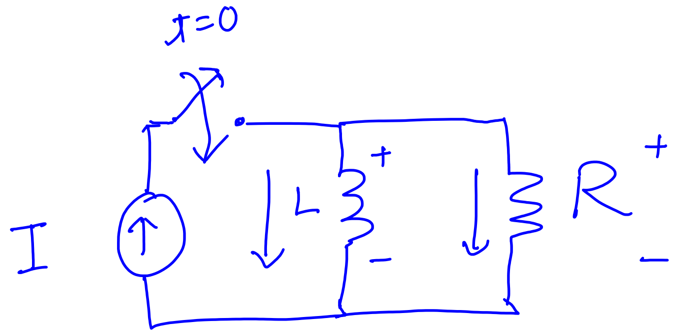
$$= \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$= \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) \quad \forall t \geq 0$$

$$i_L(\infty) = \frac{V}{R}$$

$\Rightarrow$ At steady state, inductor acts as short circuit

$(V_L(\infty) = 0)$



$I_L(0) = I_0$

$I_R(\infty) = 0$?

$V_L(0) =$

$\checkmark$ $V_L(t) = V_R(t) \quad \forall t \geq 0$

$I = I_L(t) + I_R(t) \quad \forall t \geq 0$

$\Rightarrow \quad I = I_L(0) + I_R(0)$

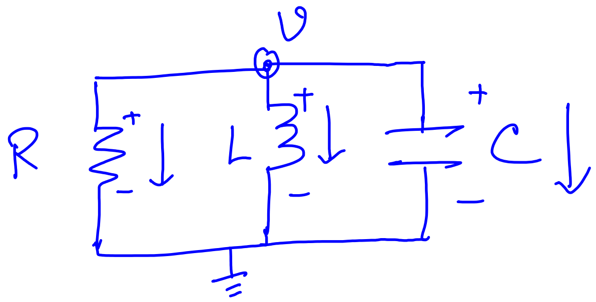
$= I_0 + I_R(0)$

$\Rightarrow \quad I_R(0) = I - I_0$

$V_R(0) = R\, I_R(0) = R(I - I_0)$

$\Rightarrow \quad V_L(0) = R(I - I_0)$

RLC circuits



natural response

$I_L(0) = I_0$

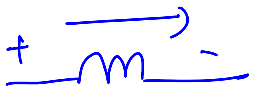
$V_C(0) = V_0$

KCL →

$$I_R(t) + I_L(t) + I_C(t) = 0 \quad \forall t \geq 0$$

$$V_R(t) = V_L(t) = V_C(t) = V(t)$$

$$\frac{V(t)}{R} + \underbrace{\frac{1}{L}\int_0^t V_L(s)\,ds + I_L(0)}_{I_L(t)} + C\frac{dv}{dt} = 0 \quad \forall t \geq 0$$



$$v_L = L\frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L}\int_0^t V_L(s)\,ds + i_L(0)$$

Intrego-differential equation,

$$\frac{V(t)}{R} + \frac{1}{L}\int_0^t V(s)\,ds + i_L(0) + C\frac{dv}{dt} = 0$$

Differentiate both sides — — —

$$\frac{1}{R}\frac{dv}{dt} + \frac{v(t)}{L} + C\frac{d^2v}{dt^2} = 0$$ (second-order differential equation)

$$v(t) = \underline{\underline{Ae^{st}}}$$ (Assumption)

$$\frac{1}{R}Ase^{st} + \frac{Ae^{st}}{L} + Cs^2Ae^{st} = 0, \quad \forall t \geq 0$$

$$Ae^{st}\left(\frac{1}{R}\cdot s + \frac{1}{L} + Cs^2\right) = 0, \quad \forall t \geq 0$$

$$\Rightarrow \quad Cs^2 + \frac{1}{R}s + \frac{1}{L} = 0$$ → (characteristic equation)

$$\Rightarrow \quad C\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right) = 0$$

$$\Rightarrow \quad s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$\Rightarrow \quad \underbrace{s^2 + 2s\cdot\frac{1}{2RC} + \left(\frac{1}{2RC}\right)^2} + \frac{1}{LC} - \frac{1}{(2RC)^2} = 0$$

$$\Rightarrow \quad \left(s + \frac{1}{2RC}\right)^2 = \left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}$$

$$\Rightarrow \quad s + \frac{1}{2RC} = \pm\sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$\Rightarrow \quad s = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

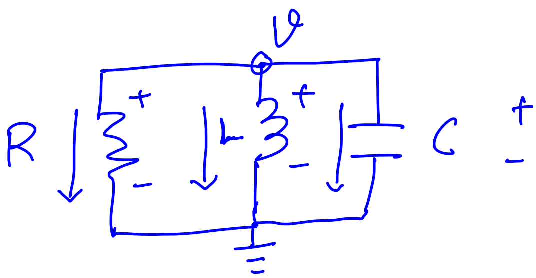
$$\left.\begin{aligned} s_1 &= -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \\ s_2 &= -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \end{aligned}\right\} \text{two roots}$$

So, general structure of the natural response

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Quick Recap

- natural response of a parallel RLC circuit



$$V_R = V_C = V_L = v, \quad \forall t \geq 0$$

$$\underline{I_R} + \underline{I_L} + \underline{I_C} = 0 \quad \forall t \geq 0$$

$$V_C(0) = V_0, \quad I_L(0) = I_0$$

$$C\frac{d^2 v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$

$$v(t) = Ae^{st} \quad \text{(Assumption)}$$

$$Cs^2 + \frac{1}{R}s + \frac{1}{L} = 0 \quad \text{(characteristic eq.)}$$

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$\alpha \rightarrow$ decay rate , $\omega_0 \rightarrow$ resonant frequency

$\alpha = \frac{1}{2RC}$, $\omega_0^2 = \frac{1}{LC}$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(general structure)}$$

$A_1, A_2, S_1, S_2 \longrightarrow$ (complex numbers)

$(\alpha > \omega_0) \rightarrow$ Overdamped System

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$\Rightarrow$ real roots $\quad S_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} < 0$

$$\alpha > \sqrt{\alpha^2 - \omega_0^2} \Rightarrow S_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} < 0$$

negative real roots

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \checkmark$$

$\Rightarrow$ A_1 and A_2 are real numbers.

$$v(0) = V_C(0) = V_0$$

$t=0$

$$V_0 = A_1 + A_2 \quad \text{———} \quad (1)$$

$$\frac{dv}{dt} = A_1 S_1 e^{s_1 t} + A_2 S_2 e^{s_2 t}$$

$$I_R(t) + I_L(t) + I_C(t) = 0 \quad \forall t \geq 0$$

$$I_R(0) + I_L(0) + I_C(0) = 0$$

$$I_R(0) = \frac{V_R(0)}{R} = \frac{v(0)}{R} = \frac{V_0}{R}$$

$$I_C(0) = -I_R(0) - I_L(0)$$

$$= -\frac{V_0}{R} - I_0$$

$$\frac{dv(0)}{dt} = \frac{dV_c(0)}{dt} = \frac{1}{C} I_C(0) = -\frac{1}{C}\underbrace{\left(I_0 + \frac{V_0}{R}\right)}$$

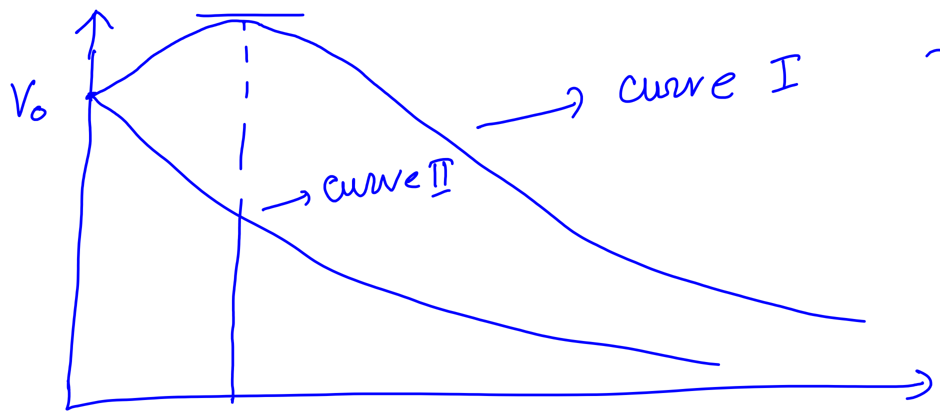
$$\frac{dv}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$$

$t = 0$

$$-\frac{1}{C}\left(I_0 + \frac{V_0}{R}\right) = A_1 s_1 + A_2 s_2 \quad \text{——— (ii)}$$

Solve for A_1 and A_2 from (i) and (ii).

$v(\infty) = 0$ because $s_1, s_2 < 0$



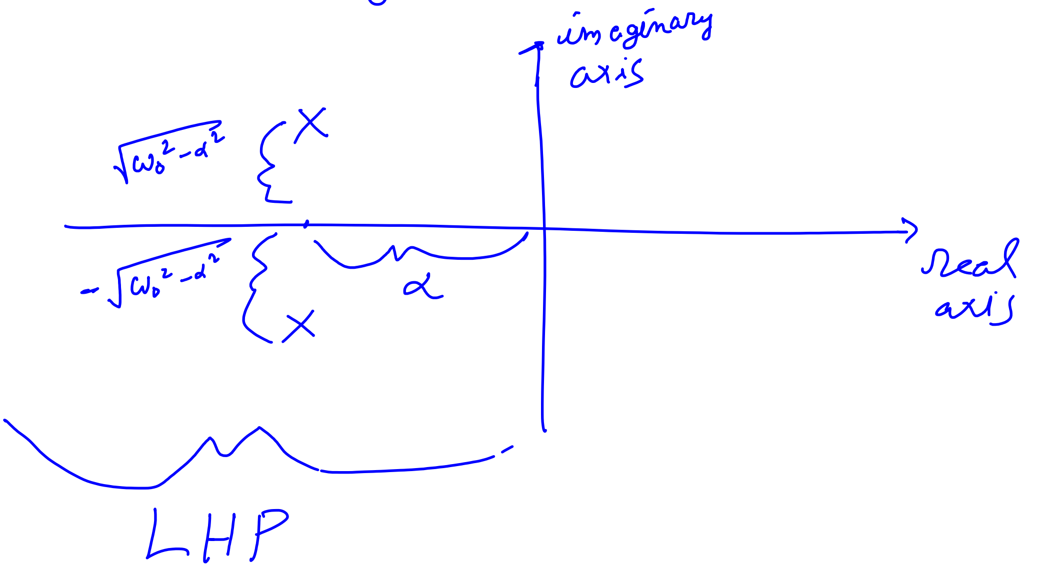
possibilities

$(\alpha < \omega_0) \longrightarrow$ Underdamped System

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$= -\alpha \pm \sqrt{(-1)(\omega_0^2 - \alpha^2)}$$

$$= -\alpha \pm j\sqrt{(\omega_0^2 - \alpha^2)} \quad , \quad (j = \sqrt{-1})$$



$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\boxed{e^{j\omega t} = \cos\omega t + j\sin\omega t}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$\Rightarrow \quad S_{1,2} = -\alpha \pm j\omega_d$

$\Rightarrow \quad v(t) = e^{-\alpha t}\left(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}\right)$

$$A_1 = \gamma_1 + j\delta_1$$

$$A_2 = \gamma_2 + j\delta_2$$

$$v(t) = e^{-\alpha t}\left((\gamma_1 + j\delta_1)e^{j\omega_d t} + (\gamma_2 + j\delta_2)e^{-j\omega_d t}\right)$$

$$= e^{-\alpha t}\Big((\gamma_1 + j\delta_1)(\cos\omega_d t + j\sin\omega_d t)$$
$$+ (\gamma_2 + j\delta_2)(\cos\omega_d t - j\sin\omega_d t)\Big)$$

$$v(0) = (\gamma_1 + j\delta_1) + (\gamma_2 + j\delta_2)$$

$$= (\gamma_1 + \gamma_2) + j(\delta_1 + \delta_2)$$

$$\Rightarrow \quad \delta_1 + \delta_2 = 0 \quad \checkmark$$

$$\delta_1 = \delta \quad \therefore \quad \delta_2 = -\delta$$

$$v(t) = e^{-\alpha t}\left(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}\right)$$

$$\frac{dv(t)}{dt} = -\alpha e^{-\alpha t}\left(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}\right)$$
$$+ e^{-\alpha t}\left(A_1 j\omega_d e^{j\omega_d t} - A_2 j\omega_d e^{-j\omega_d t}\right)$$

$$\frac{dv(0)}{dt} = -\alpha(A_1 + A_2) + A_1 j\omega_d - A_2 j\omega_d$$

$$= -\alpha\left(\gamma_1 + \cancel{j\delta} + \gamma_2 - \cancel{j\delta}\right) + (\gamma_1 + j\delta).j\omega_d$$
$$- (\gamma_2 - j\delta) j\omega_d$$

$$= -\alpha(\gamma_1 + \gamma_2) + j\omega_d(\gamma_1 - \gamma_2)$$
$$- \delta\omega_d - \delta\omega_d$$

$$= -\alpha(\gamma_1 + \gamma_2) - 2\delta\omega_d + j\omega_d(\gamma_1 - \gamma_2)$$

$$\Rightarrow \quad \gamma_1 - \gamma_2 = 0$$

$$\Rightarrow \quad \gamma_1 = \gamma_2 = \gamma$$

$$\left.\begin{aligned} A_1 &= \gamma + j\delta \\ A_2 &= \gamma - j\delta \end{aligned}\right\} \text{complex conjugates}$$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$
$$= e^{-\alpha t}\left(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}\right)$$
$$= e^{-\alpha t}\left(A_1 \cos\omega_d t + j A_1 \sin\omega_d t\right.$$
$$\left. + A_2 \cos\omega_d t - j A_2 \sin\omega_d t\right)$$

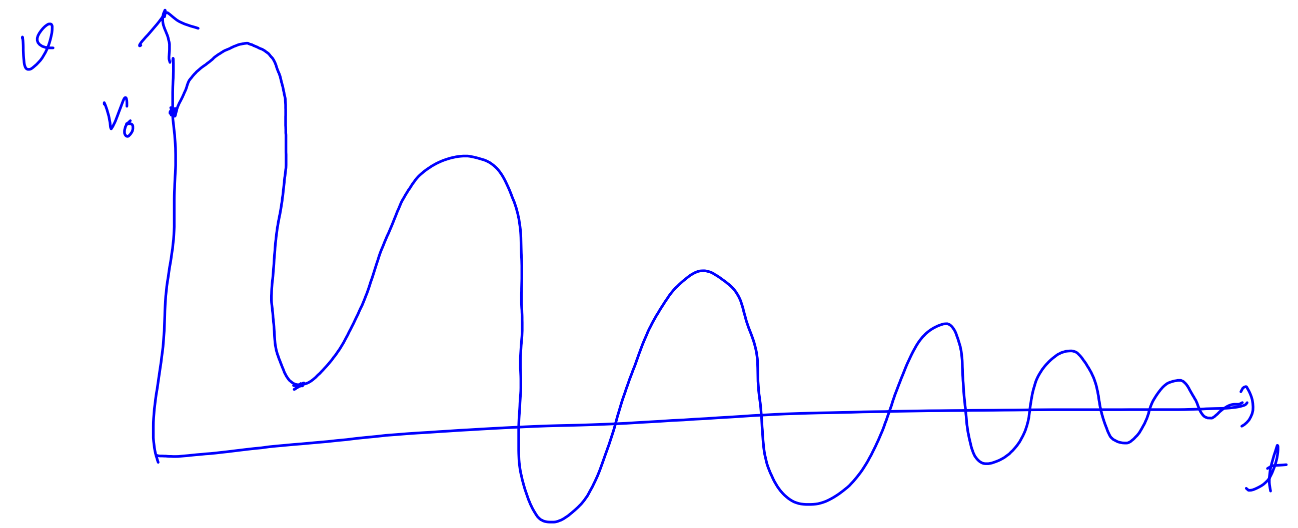
$$= e^{-\alpha t}\left(\underbrace{(A_1 + A_2)}_{2\gamma}\cos\omega_d t + j\underbrace{(A_1 - A_2)}_{-2\delta}\sin\omega_d t\right)$$

$$v(t) = e^{-\alpha t}\left(\underbrace{A_d \cos \omega_d t + B_d \sin \omega_d t}\right)$$

A_d, B_d are real numbers

$(A_d = 2Y,\ B_d = -2\delta)$

$$v(\infty) = 0$$



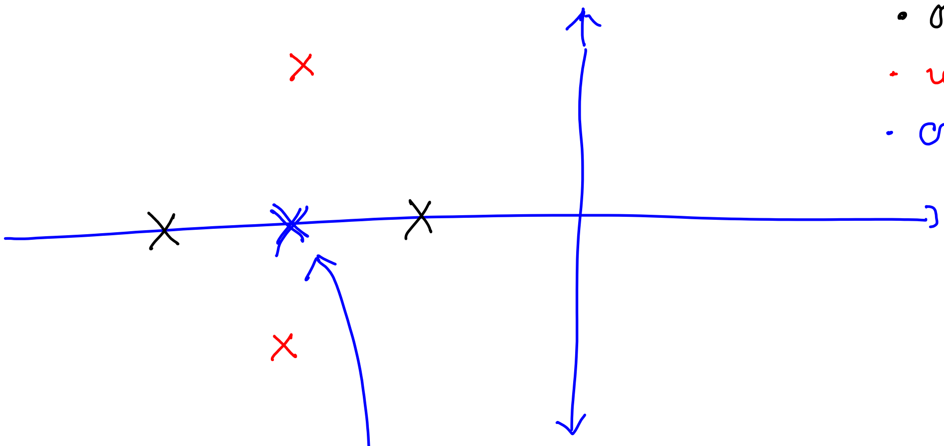
$(\alpha = \omega_0) \rightarrow$ (Critically damped)

$$C \frac{d^2 v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L} v = 0$$

$$\frac{d^2 v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC} v = 0$$

$$\alpha = \frac{1}{2RC}, \quad \omega_0^2 = \frac{1}{LC}$$

$$\frac{d^2 v}{dt^2} + 2\alpha \frac{dv}{dt} + \alpha^2 v = 0$$



$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$= -\alpha \qquad (\text{since } \alpha = \omega_0)$$

Let's assume,

$$v(t) = A_2 e^{st}$$ It's a valid solution

but, only unknown A_2, however, we have two initial conditions.

This is also a valid solution

$$v(t) = A_1 t e^{st} \quad (\text{Assumption})$$

$$\frac{dv(t)}{dt} = A_1 t s e^{st} + A_1 e^{st}$$

$$\frac{d^2 v}{dt^2} = A_1 t s^2 e^{st} + A_1 s e^{st} + A_1 s e^{st}$$

$$\frac{d^2 v}{dt^2} + 2\alpha \frac{dv}{dt} + \alpha^2 v$$

$$= A_1 t s^2 e^{st} + A_1 s e^{st} + A_1 s e^{st}$$
$$+ 2\alpha\left(A_1 t s e^{st} + A_1 e^{st}\right) + \alpha^2 A_1 t e^{st}$$

$$= A_1 t \underline{\alpha^2} e^{-\alpha t} - A_1 \alpha \cancel{e^{-\alpha t}} - A_1 \alpha \cancel{e^{-\alpha t}}$$
$$+ 2\alpha\left(\underline{-A_1 t \alpha} e^{-\alpha t} + A_1 \cancel{e^{-\alpha t}}\right) + \alpha^2 A_1 \underline{t} e^{-\alpha t}$$

$$= 0$$

general solution

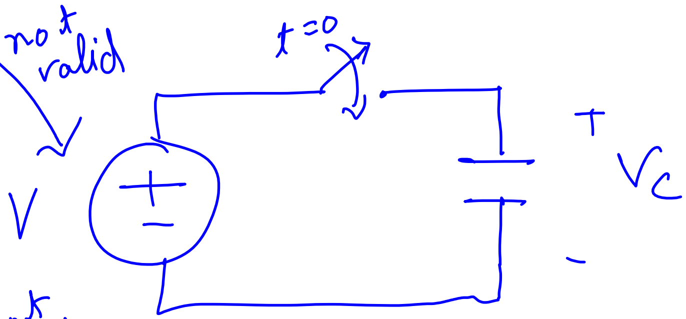
$$v(t) = A_1 t e^{st} + A_2 e^{st}$$
$$= e^{st}\left(A_1 t + A_2\right)$$
$$= e^{-\alpha t}\left(A_1 t + A_2\right)$$

Quick Recap

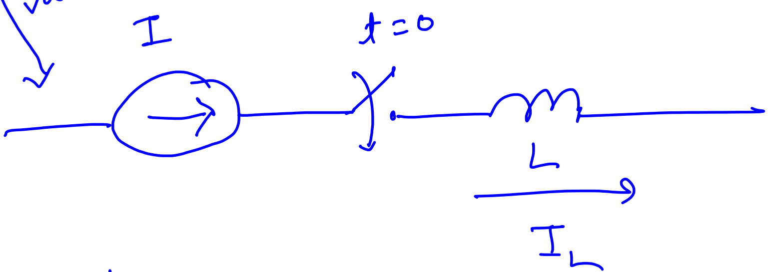
$V_C(t^-) = V_C(t^+)$, provided $I_C(t) \neq \infty$

$I_L(t^-) = I_L(t^+)$, provided $V_L(t) \neq \infty$

not valid



not valid



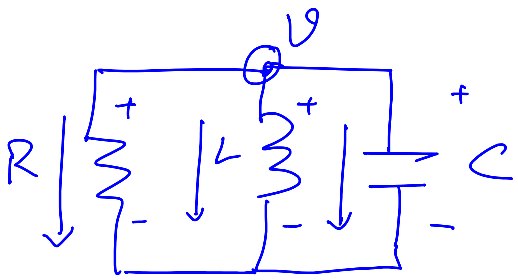
circuits involving dc sources

$$I_C(\infty) = 0 \quad \left(\lim_{t\to\infty} I_C(t) = 0 \right)$$

$$V_L(\infty) = 0$$

RLC circuit (parallel)

└→ natural response $\left\{\begin{array}{l}\text{overdamped} \\ \text{underdamped} \\ \text{critically damped}\end{array}\right.$



$V_C(0) = V_0$

$I_L(0) = I_0$

natural response,

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(overdamped)}$$

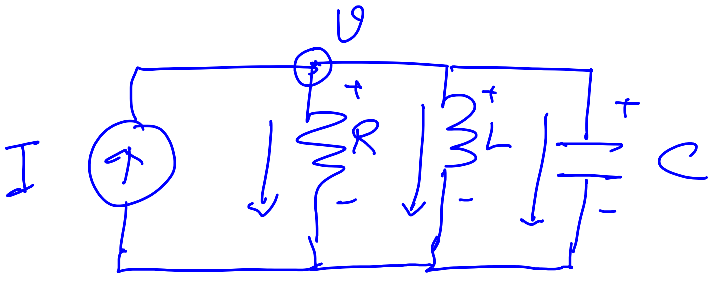
$$v(t) = e^{-\alpha t}(A_d \sin\omega_d t + B_d \cos\omega_d t)$$

(underdamped)

$$v(t) = A_1 e^{st} \cdot t + A_2 e^{st} = e^{st}(A_1 t + A_2)$$

(critically damped system)

Forced Response of RLC circuit,



$\left.\begin{array}{l}V_C(0) = V_0 \\ I_L(0) = I_0\end{array}\right\}$

$$v(t) = v_f(t) + v_n(t)$$

$$\frac{d^2 v}{dt^2} + 2\alpha \frac{dv}{dt} + \omega_0^2 v = \bar{V}$$

$$\left.\begin{aligned} v(0) &= v_0 \\ \frac{dv(0)}{dt} &= \dot{v}_0 \end{aligned}\right\}$$

$$\frac{d^2 v_f}{dt^2} + 2\alpha \frac{dv_f}{dt} + \omega_0^2 v_f = \bar{V}$$

$$v_f(0) = 0$$

$$\frac{dv_f}{dt}(0) = 0$$

$$\frac{d^2 v_n}{dt^2} + 2\alpha \frac{dv_n}{dt} + \omega_0^2 v_n = 0$$

$$v_n(0) = v_0$$

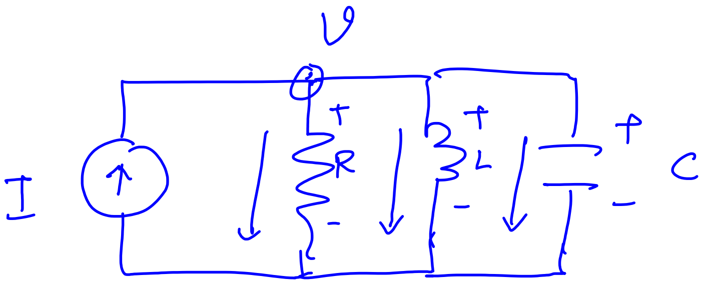
$$\frac{dv_n(0)}{dt} = \dot{v}_0$$

$$v(t) = v_f(t) + v_n(t)$$

$$= V_{ss} + v_T(t) \rightarrow \left\{\begin{aligned} &A_1 e^{s_1 t} + A_2 e^{s_2 t} \\ &A_1 t e^{st} + A_2 e^{st} \\ &\vdots \end{aligned}\right.$$

$$v(t) = V_{ss} + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ (overdamped case)

✓ $V_{ss} = 0$ since ($v(\infty) = 0$)



$V_C(0) = V_0$
$I_L(0) = I_0$

$$I = I_R(t) + I_L(t) + I_C(t) \quad \forall t \geq 0$$

$$= \frac{v(t)}{R} + \frac{1}{L}\int_0^t v(\tau)\,d\tau + I_L(0) + C\frac{dv}{dt}$$

$$0 = \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v + C\frac{d^2v}{dt^2}$$

$$\Rightarrow V_{ss} = 0$$

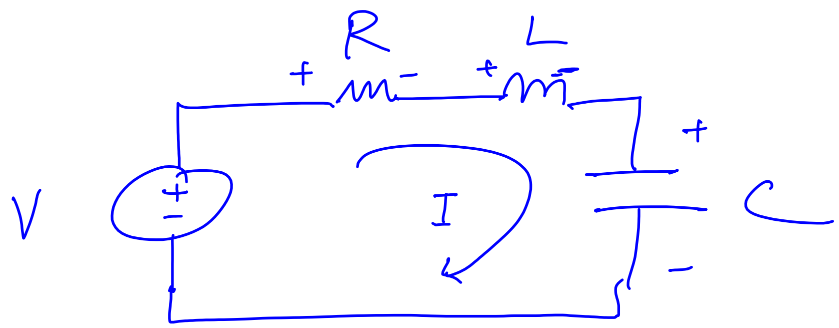
Total response, (general structure)

$$v(t) = V_{ss} + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ (overdamped)

$$= V_{ss} + e^{-\alpha t}(A_d \sin\omega_d t + B_d \cos\omega_d t)$$ (underdamped)

$$= V_{ss} + e^{-\alpha t}(A_1 t + A_2)$$ (critically damped)

Series RLC circuit,



$V_C(0) = V_0$

$I_L(0) = I_0$

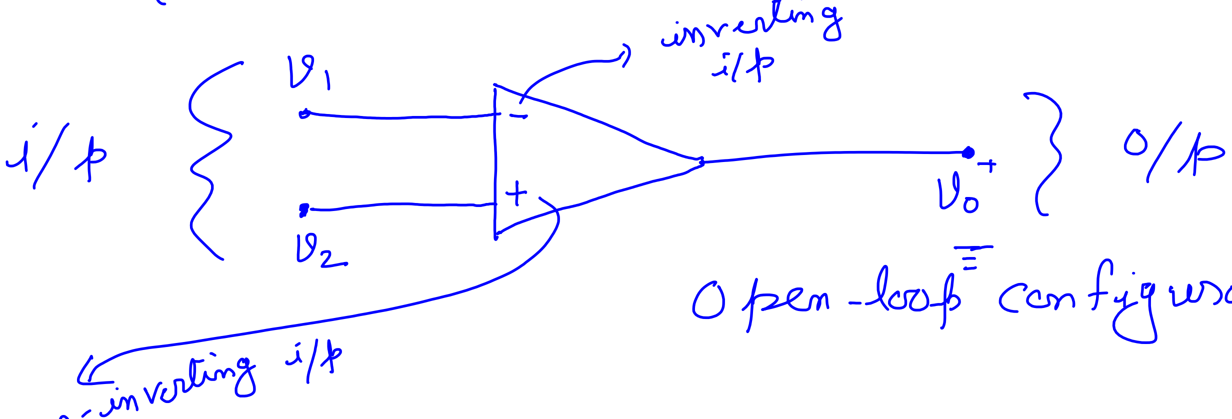
$$V = V_R(t) + V_L(t) + V_c(t) \qquad \forall\, t \geq 0$$

$$I_C(t) = I_L(t) = I_R(t) = I(t)$$

$$V = I(t)R + L\frac{dI(t)}{dt} + \frac{1}{C}\int_0^t I(\tau)\,d\tau + V_c(0)$$

$$0 = \frac{dI}{dt}R + L\frac{d^2I}{dt^2} + \frac{1}{C}I$$

Operational Amplifier (Active element)
(OpAmp)



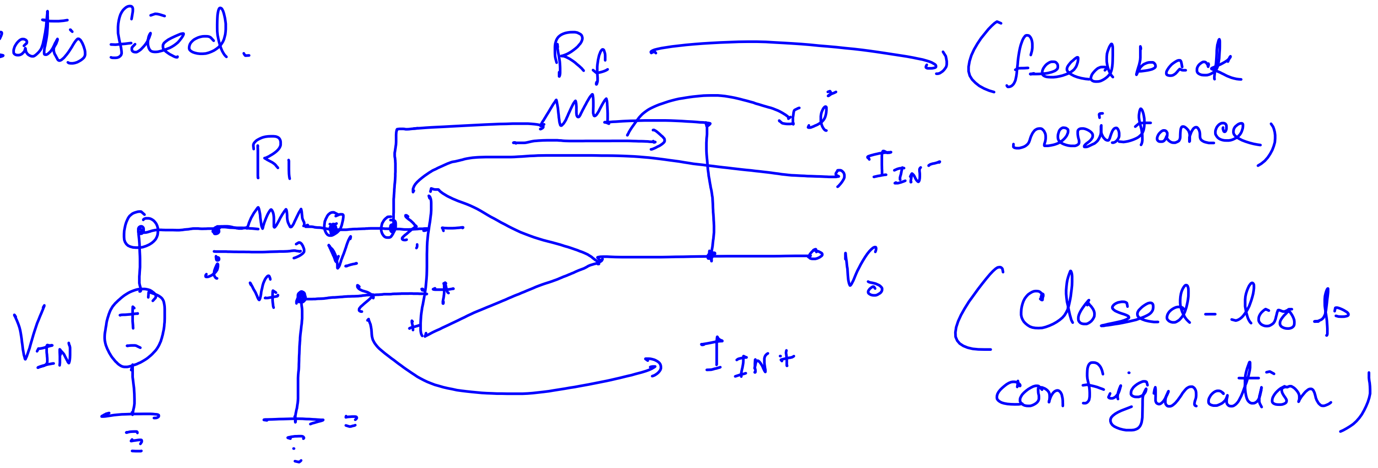
Open-loop configuration

$$\underbrace{V_o = A(V_2 - V_1)}_{\text{(linear property, only valid in a certain range of operating condition)}} \qquad \text{(differential amplifier)}$$

For an ideal OpAmp ($A \longrightarrow \infty$)

$$(v_2 - v_1) \simeq 0$$

In closed-loop, this property is automatically satisfied.



In closed-loop, an ideal OpAmp follows the subsequently defined properties.

1) $V_+ = V_-$ (virtual short circuit)

2) $I_{IN^-} = I_{IN^+} = 0$

$$V_+ = 0 = V_-$$

$$i = \frac{V_{IN}}{R_1} = \frac{-V_o}{R_f} \Rightarrow \boxed{V_o = -\frac{R_f}{R_1} V_{IN}}$$

(Inverting amplifier)

Quick Recap

RLC → total response

$$v(t) = v_f(t) + v_n(t)$$

$$= V_{ss} + \underbrace{v_T(t)}$$

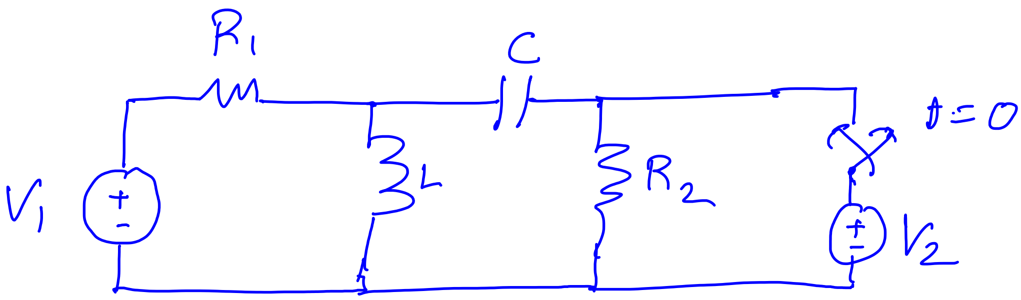
$A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$A_1 t e^{st} + A_2 e^{st}$

$e^{-\alpha t}(A_d \sin\omega_d t + B_d \cos\omega_d t)$

$I_L(t^-) = I_L(t^+) \qquad (V_L(t) \neq \infty)$

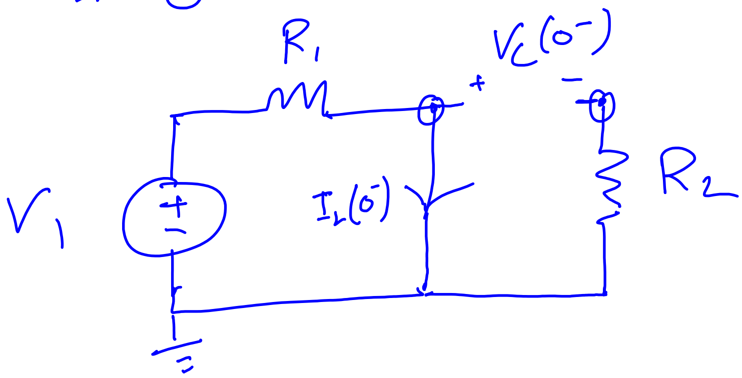
$V_C(t^-) = V_C(t^+) \qquad (I_C(t) \neq \infty)$



$I_L(0^-) = ?$

$V_C(0^-) = ?$

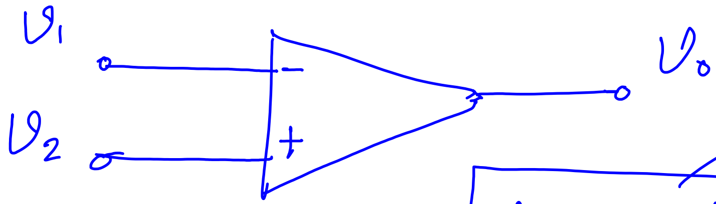
at $t = 0^-$



$$I_L(0^-) = \frac{V_1}{R_1}$$

$$V_C(0^-) = 0$$

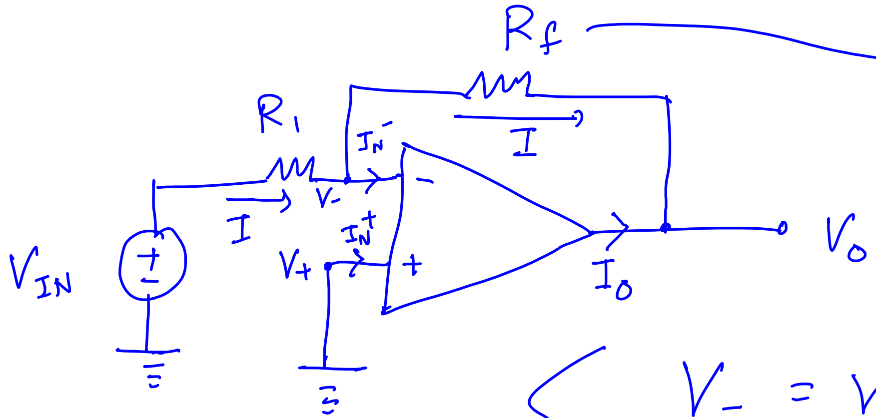
open-loop OpAmp



open-loop gain

$$\boxed{v_o = A(v_2 - v_1)}$$

closed-loop OpAmp



feedback resistance

Ideal OpAmp $\begin{cases} V_- = V_+ & \text{(virtual short)} \\ I_N^- = I_N^+ = 0 \end{cases}$

$$\frac{V_{IN}}{R_1} = -\frac{V_o}{R_f} \Rightarrow V_o = -\frac{R_f}{R_1} V_{IN}$$

$$I + I_o = 0 \Rightarrow I_o = -I = -\frac{V_{IN}}{R_1}$$

$I_{IN} = 0$, however, $I_o \neq 0$. This is possible because of the power supplies ($+V_{cc}$, $-V_{cc}$). OpAmp is an active element.

Equivalent circuit of OpAmp ✓

Fundamental questions

$$\frac{0}{0} = ?$$

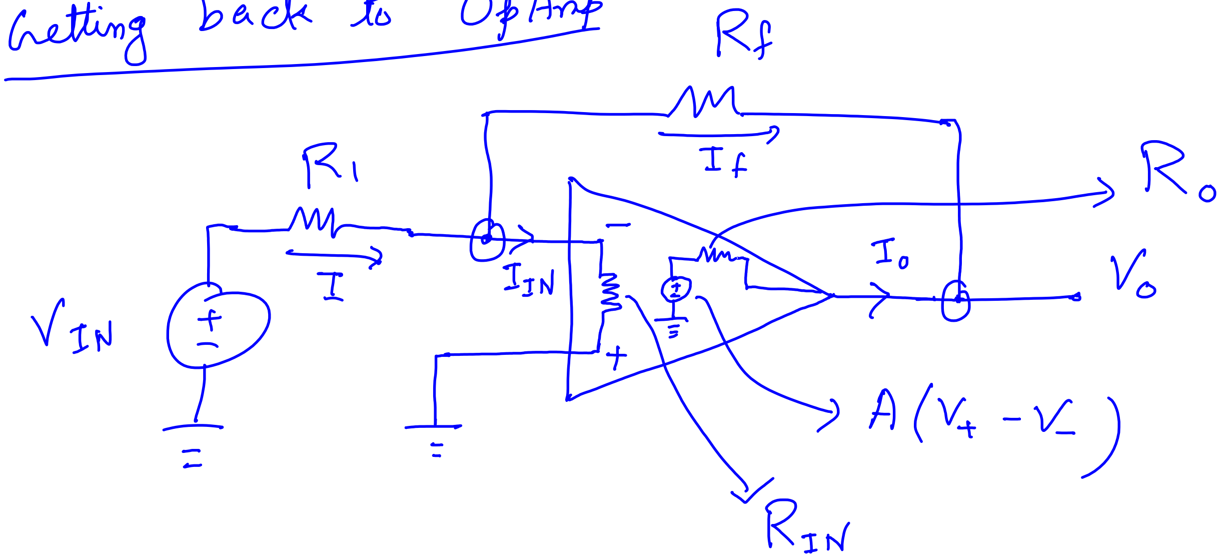
$$\infty \times 0 = ?$$

$$\frac{0}{0} = \underline{m} \Rightarrow 0 = m \times 0$$

any real number

$$\lim_{t\to\infty} \frac{e^{-pt}}{e^{-qt}} \begin{cases} = 0 & \text{if } p>q \\ = \infty & \text{if } p<q \\ = 1 & \text{if } p=q \end{cases}$$

Getting back to OpAmp



Ideal Op Amp

$$\left.\begin{array}{l} A = \infty \\ R_{IN} = \infty \\ R_o = 0 \end{array}\right\}$$

$$I = I_f + I_{IN} \quad \text{—— (I)}$$

$$I_f + I_o = 0 \quad \text{—— (II)}$$

$$\frac{V_{IN} - V^-}{R_1} = \frac{V^- - V_0}{R_f} + \frac{V^-}{R_{IN}} \quad \text{—— (I)}$$

$$\frac{V^- - V_0}{R_f} + \frac{A(V^+ - V^-) - V_0}{R_0} = 0 \quad \text{—— (II)}$$

$\left(\frac{0}{0}\right)$

$\underline{\underline{V^+ = 0}}$

since $R_{IN} = \infty$

$$\Rightarrow \quad \frac{V_{IN} - V^-}{R_1} = \frac{V^- - V_0}{R_f}$$

since $R_0 = 0 \Rightarrow \boxed{A(V^+ - V^-) = V_0}$

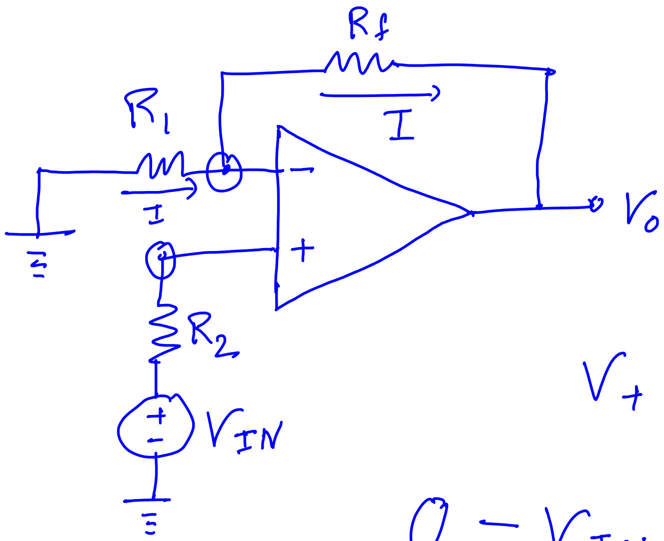
$$A(V^+ - V^-) = V_0$$

$$A(0 - V^-) = V_0$$

since $A = \infty \Rightarrow V^- = 0$ and V_0 is finite.

$\Rightarrow \quad V^+ = V^- = 0$ ✓

$I_{IN} = 0$ ✓



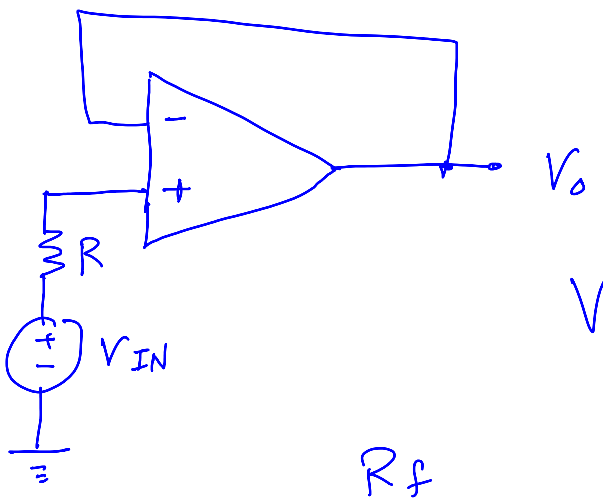
$$V_+ = V_- = V_{IN}$$

$$\frac{0 - V_{IN}}{R_1} = \frac{V_{IN} - V_o}{R_f}$$

$$\Rightarrow \quad \frac{V_o}{R_f} = \left(\frac{1}{R_1} + \frac{1}{R_f}\right) V_{IN}$$

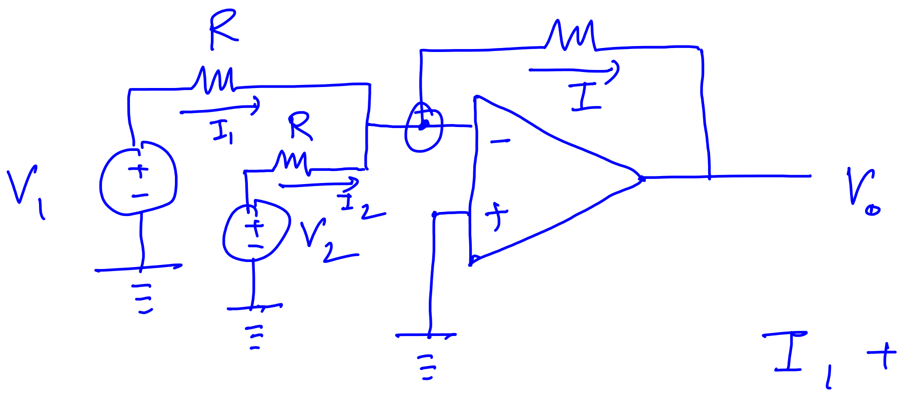
$$\Rightarrow \quad V_o = \left(1 + \frac{R_f}{R_1}\right) V_{IN}$$

(non-inverting amplifier)



$$V_o = V_{IN}$$

(Buffer)



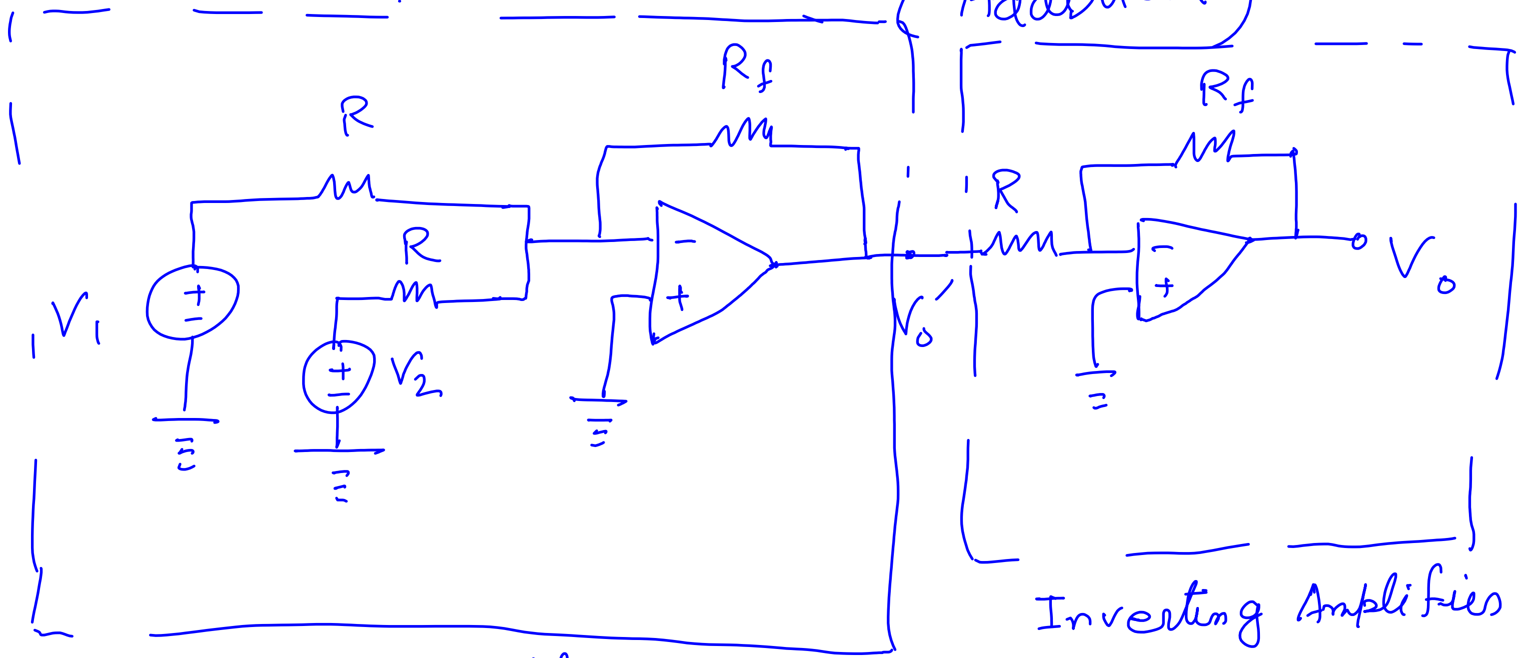
$$I_1 + I_2 = I$$

$$(V^+ = V^- = 0)$$

$$\frac{V_1}{R} + \frac{V_2}{R} = -\frac{V_o}{R_f}$$

$$V_o = -\frac{R_f}{R}(V_1 + V_2)$$

(Addition)



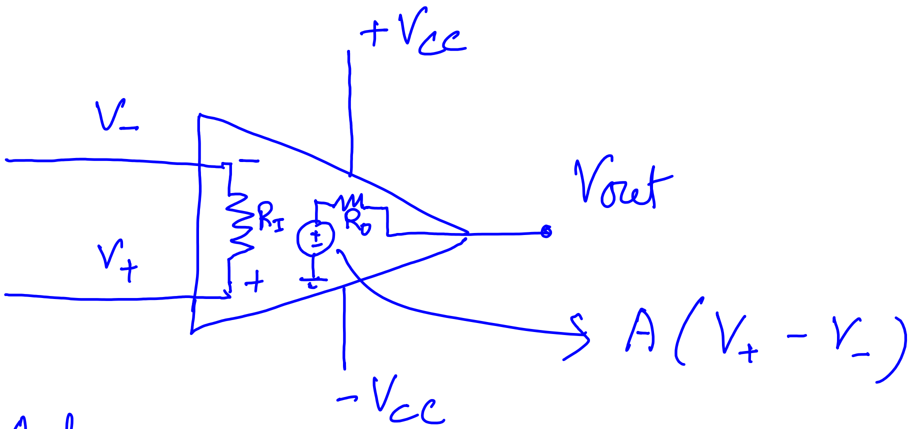
Inverting Adder

$$V_o' = -\frac{R_f}{R}(V_1 + V_2)$$

$$V_o = -\frac{R_f}{R} V_o' = +\frac{R_f^2}{R^2}(V_1 + V_2)$$

Quick Recap

Operational Amplifier (OpAmp)

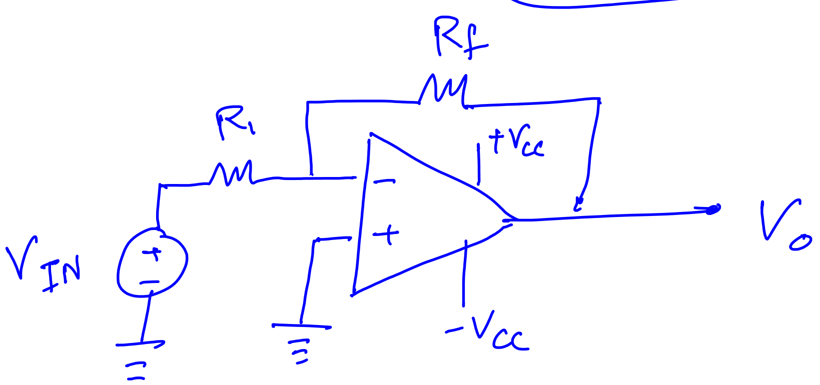


Ideal OpAmp

$A \rightarrow \infty$, $R_I \rightarrow \infty$, $R_o \approx 0$ ✓

$V_- = V_+$ (In closed-loop)

$$\underbrace{(-V_{cc} < V_{out} < V_{cc})}$$ → linear operating zone



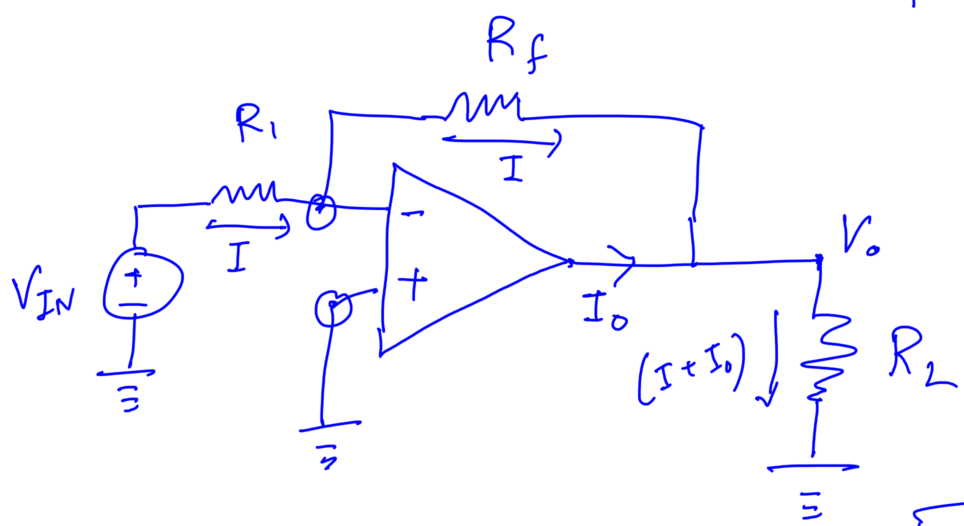
$$V_o = -\frac{R_f}{R_1} V_{IN} \quad (-V_{CC} \leq V_o \leq V_{CC})$$

Ex

$R_f = 10\,\Omega$, $R_1 = 1\,\Omega$

$V_{IN} = 1\,V$ $\qquad$ $V_{CC} = 5\,V$

$V_o = -5\,V$ (saturation property)



$$\boxed{V_- = V_+ = 0}$$

✓ $$\frac{V_{IN}}{R_1} = \frac{-V_o}{R_f} = I$$

$$V_o = -\frac{R_f}{R_1} V_{IN}$$

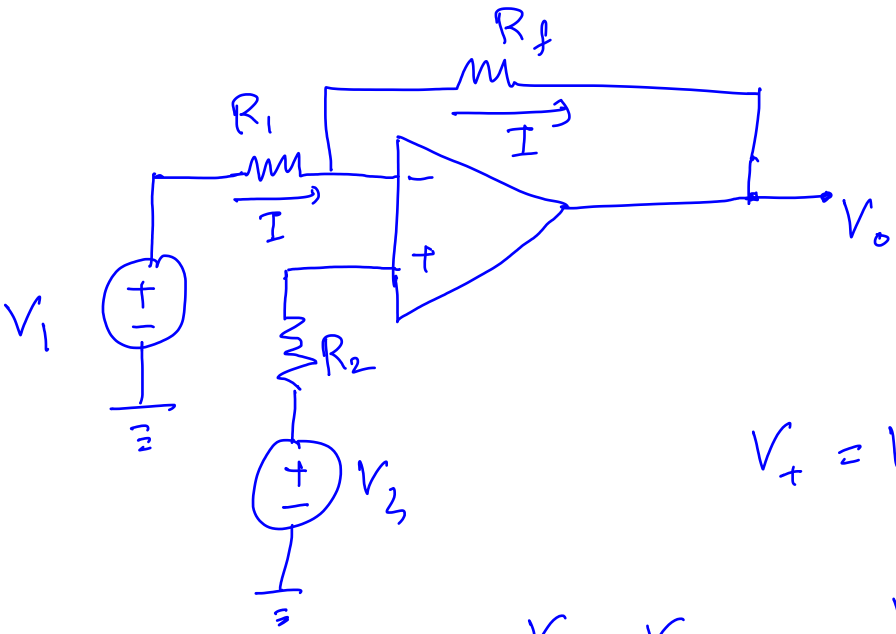
$$I + I_o = \frac{V_o}{R_L} \Rightarrow I_o = \frac{V_o}{R_L} - I$$

$$= \frac{V_o}{R_L} + \frac{V_o}{R_f}$$

$$= V_o \left(\frac{1}{R_L} + \frac{1}{R_f} \right)$$

$$= -\frac{R_f}{R_1} V_{IN} \left(\frac{1}{R_L} + \frac{1}{R_f} \right)$$

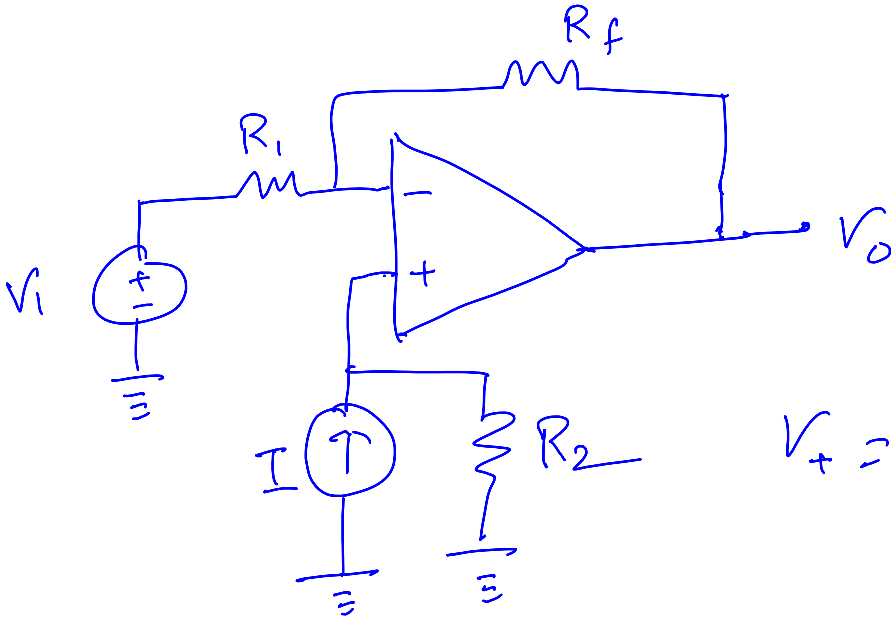
$$I_o = -V_{IN}\frac{R_2+R_f}{R_1 R_2}$$ (dependent on the load resistance)



$$V_+ = V_2 = V_-$$

$$\frac{V_1 - V_2}{R_1} = \frac{V_2 - V_o}{R_f}$$

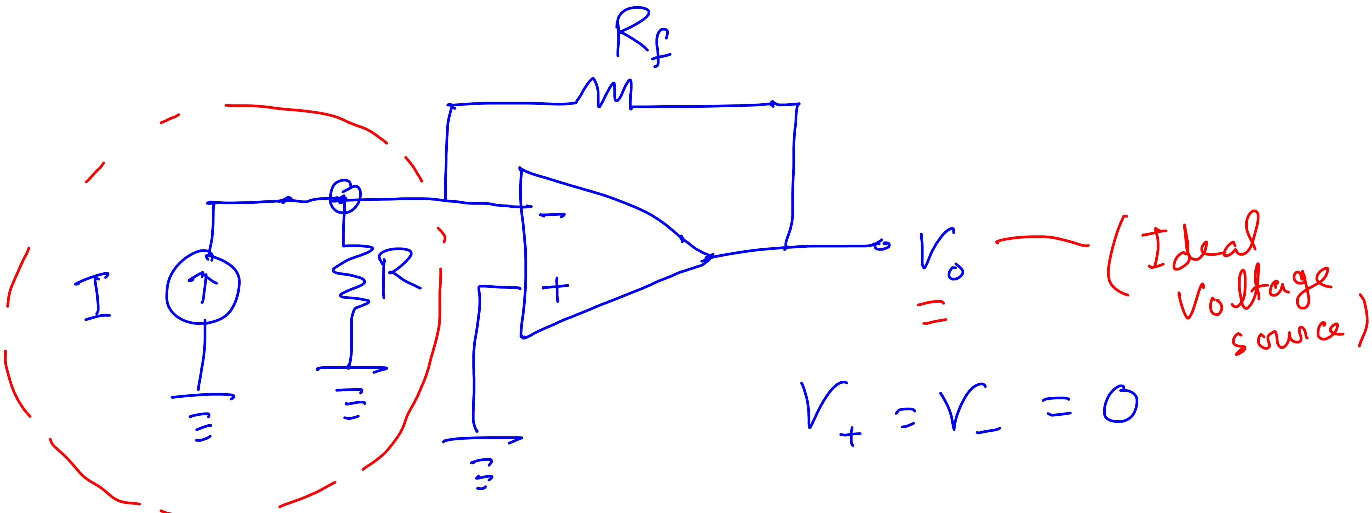
$$V_o = \underbrace{-\frac{R_f}{R_1}V_1}_{\text{contribution from inverting amplification}} + \underbrace{V_2\left(1+\frac{R_f}{R_1}\right)}_{\text{contribution from non-inverting amplification}}$$



$$V_+ = IR_2 = V_-$$

$$\frac{V_1 - IR_2}{R_1} = \frac{IR_2 - V_o}{R_f}$$

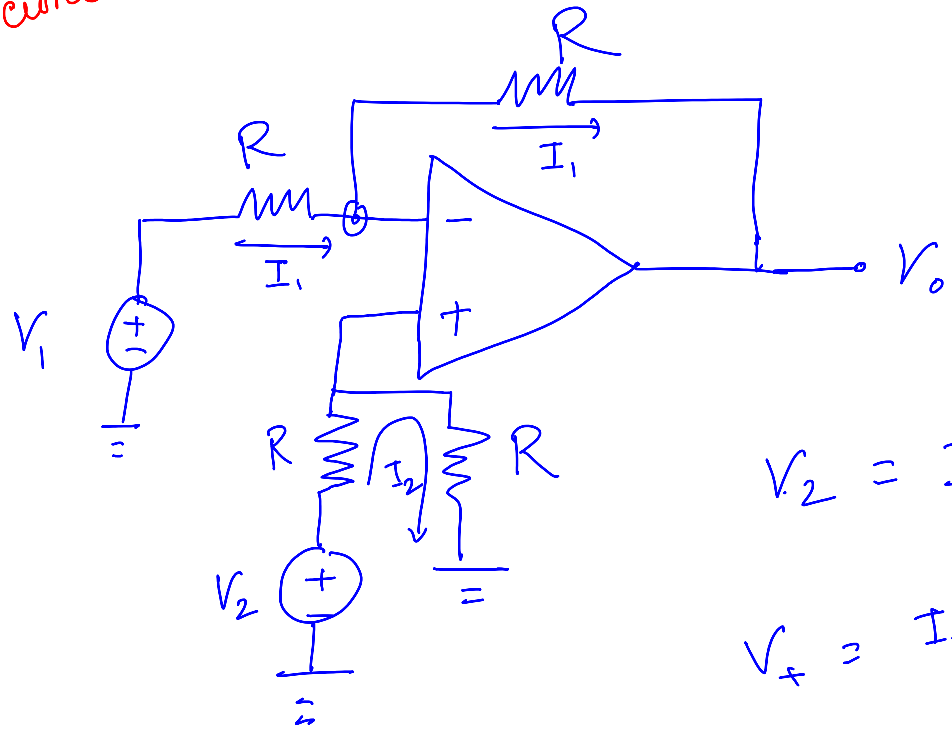
$$V_o = -\frac{R_f}{R_1}V_1 + IR_2\left(1+\frac{R_f}{R_1}\right)$$



practical current source

$$V_+ = V_- = 0$$

$$V_o = -IR_f$$



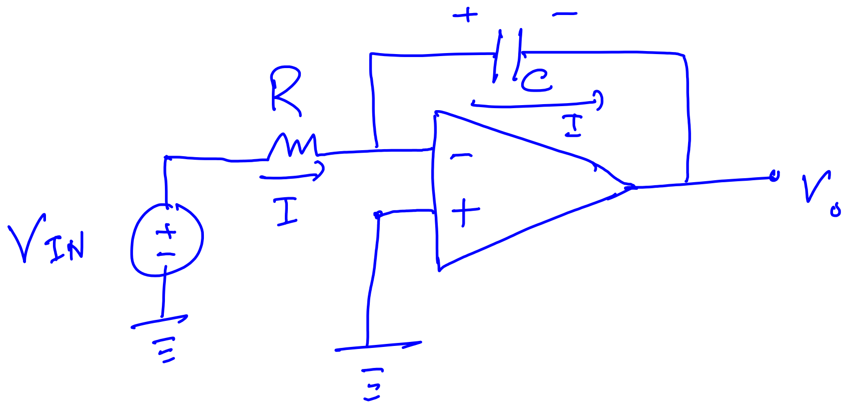
$$V_2 = I_2 2R$$

$$V_+ = I_2 R = \frac{V_2}{2R}R = \frac{V_2}{2}$$

$$\Rightarrow V_- = \frac{V_2}{2}$$

$$\frac{V_1 - \frac{V_2}{2}}{R} = \frac{\frac{V_2}{2} - V_o}{R}$$

$$V_o = V_2 - V_1$$ (substraction)



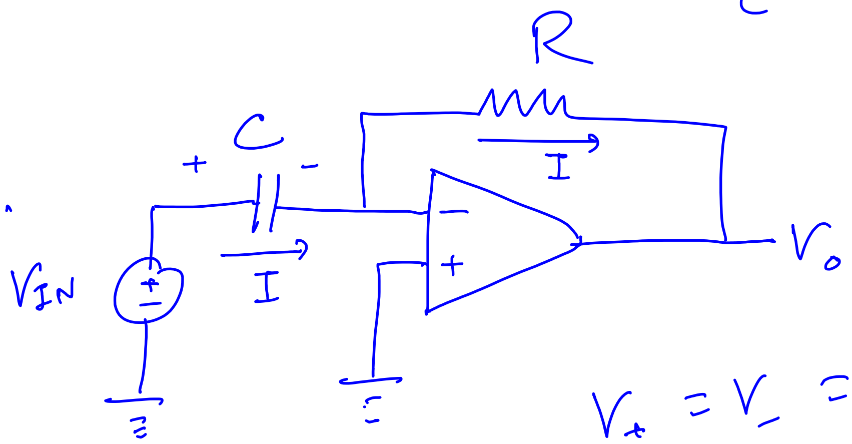
$$V_C(0) = 0$$

$$V_+ = V_- = 0$$

$$\frac{V_{IN}}{R} = -C\frac{dV_o}{dt}$$

$$V_o(t) = -\frac{1}{RC}\int_0^t V_{IN}(\tau)\,d\tau$$

(Integrator)



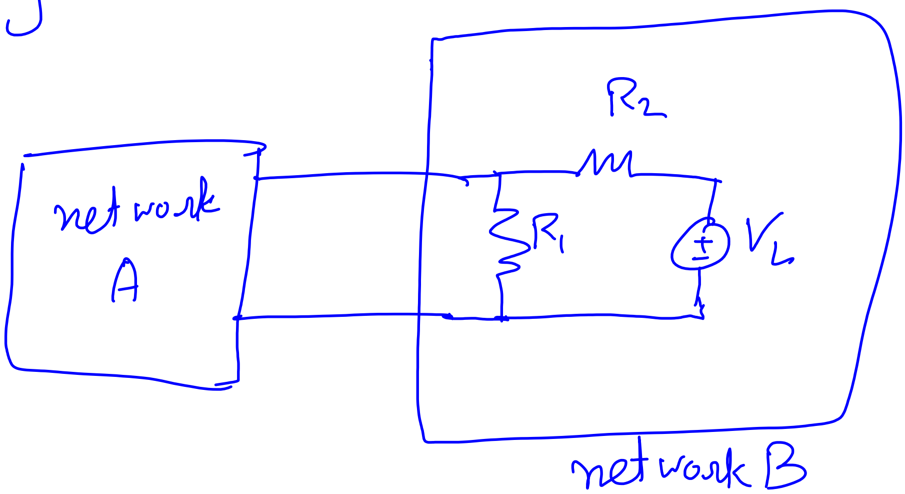
$$V_C(0) = 0$$

$$V_+ = V_- = 0$$

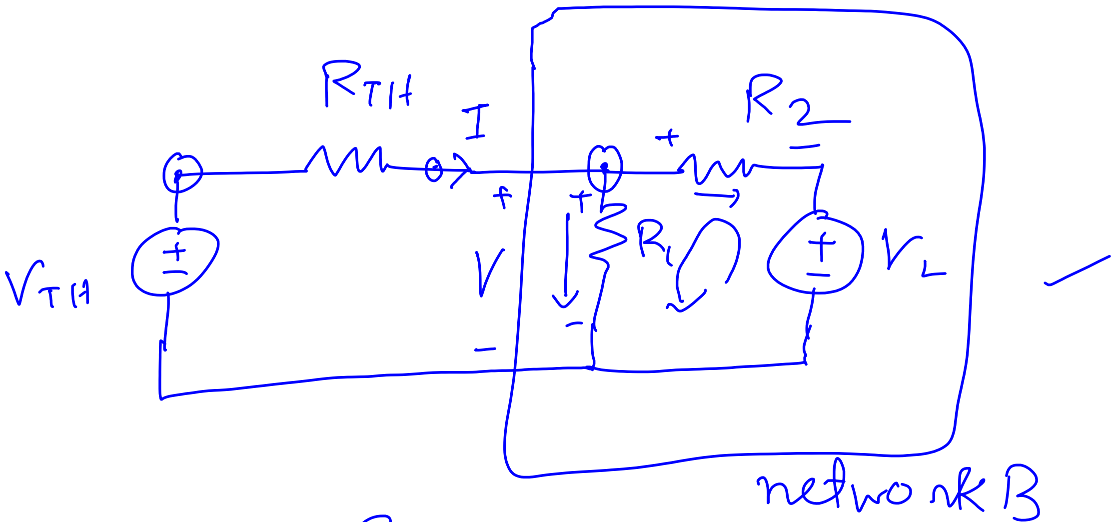
$$C\frac{dV_{IN}}{dt} = -\frac{V_o}{R}$$

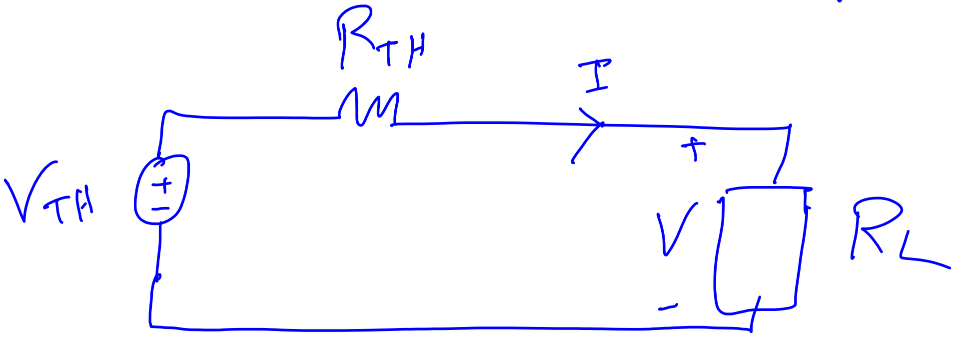
$$V_o = -RC\frac{dV_{IN}}{dt}$$ (Differentiator)

Q.3) (Assignment 1)



What should be the magnitude of V_L such that maximum power is delivered to network B?





From maximum power transfer $V = \frac{V_{TH}}{2}$.

$\Rightarrow I = \frac{V_{TH}}{2R_{TH}}$, $I_{R1} = \frac{V_{TH}}{2R_1}$

$$I = I_{R1} + I_{R2}$$

$$I_{R2} = I - I_{R1}$$

$$= \frac{V_{TH}}{2}\left(\frac{1}{R_{TH}} - \frac{1}{R_1}\right)$$

$$-V_L - V_{R2} + V_{R1} = 0$$

$$V_L = V_{R1} - V_{R2}$$

$$= \frac{V_{TH}}{2} - I_{R2} \cdot R_2$$

$$= \frac{V_{TH}}{2} - \frac{V_{TH}}{2}\left(\frac{1}{R_{TH}} - \frac{1}{R_1}\right) \cdot R_2$$