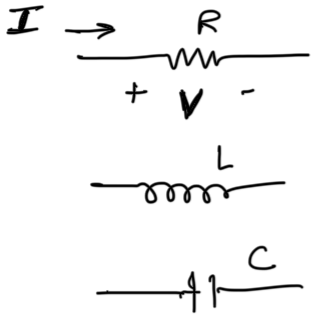
Phasor

$$i(t) = I_m \cos(\omega t + \phi) = Re\{ I_m e^{j\omega t + j\phi} \}$$

↓ Remove $e^{j\omega t}$. $Re\{\}$ ↑

$$\underbrace{I_m e^{j\phi}}_{\text{phasor}} = I_m \angle\phi$$



R	$V = IR$	$Z_R = R$	voltage & current in phase
L	$V = j\omega L I$	$Z_L = j\omega L$	voltage leads current by 90°
C	$V = \frac{1}{j\omega C} I$	$Z_C = \frac{1}{j\omega C}$	voltage lags current by 90°

Impedance

$$Z = \frac{V}{I} \quad \Omega$$

$$Z = \underbrace{X}_{\text{resistance}} + \underbrace{jY}_{\text{reactance}}$$

Admittance

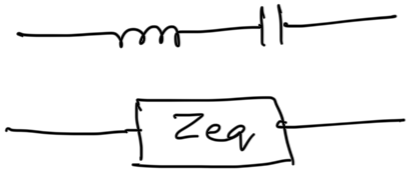
$$Y = \frac{1}{Z} \quad ; \quad Re\{Y\} = \text{conductance}, \quad Im\{Y\} = \text{susceptance}$$

Impedance Combinations

Series : $Z_{eq} = Z_1 + Z_2 + \cdots Z_N$

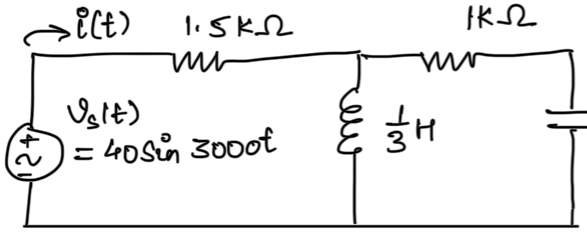
Parallel : $\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots \frac{1}{Z_N}$

Example: find impedance of a series combination of an inductor L (L = 5mH) and capacitor C (C = 100 μF)
$\omega = 10^4$ rad/s



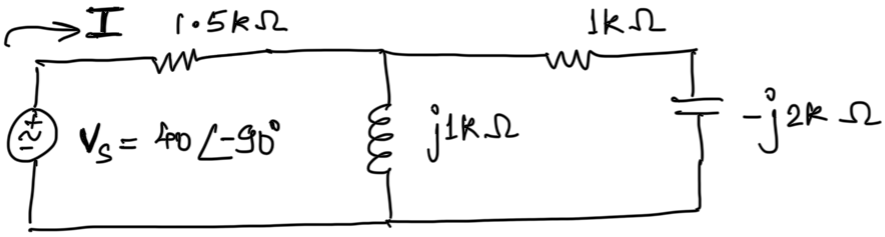
$$Z_{eq} = Z_L + Z_C$$
$$= j50 - j$$
$$= j49 = 0 + j49$$
$$= 49 e^{j\pi/2}$$

Example :–



find i(t)

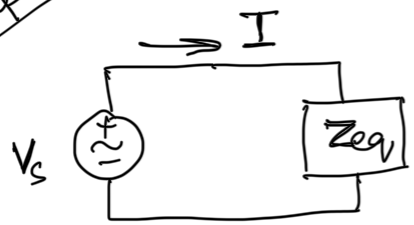
Step 1 Convert to freq. domain (write in terms of phasor & Z)



Sin wt
= cos(90°-wt)
= cos(wt-90°)

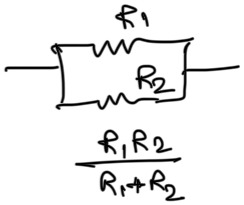
$$Z_C = \frac{1}{j\omega C}$$
$$= \frac{1}{j\,3000 \times \frac{1}{6} \times 10^{-6}}$$
$$= \frac{2 \times 10^6}{j \times 10^3}$$
$$= -j\,2k\Omega$$

Step 2



$$I = \frac{V_S}{Z_{eq}}$$

Find Z_{eq}.



$\frac{R_1 R_2}{R_1 + R_2}$

$$Z_{eq} = 1.5 + \frac{(j1)(1-j2)}{1-j1} \quad k\Omega$$
$$= 2.5 \angle 36.87^\circ \quad k\Omega$$

$$I = \frac{V_S}{Z_{eq}} = \frac{40 \angle -90^\circ}{2.5 \times 10^3 \angle 36.87^\circ} = 16 \angle -126.87^\circ \text{ mA}$$

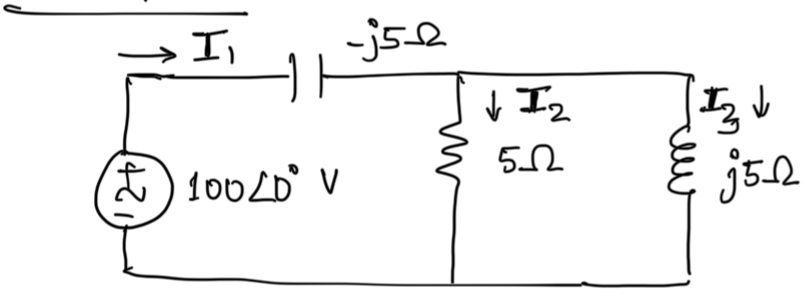
$$i(t) = Re\{16 e^{-j126.87^\circ} e^{j\omega t}\}$$
$$= 16 \cos(\omega t - 126.87^\circ) \text{ mA}$$
$$\omega = 3000$$

Complex Number $< \begin{matrix} x + iy \\ r\angle\theta \end{matrix}$

$r = \sqrt{x^2+y^2}$, $x = r\cos\theta$

$\theta = \tan^{-1}(y/x)$, $y = r\sin\theta$

Example

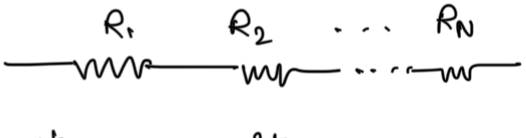


Find I_1, I_2, I_3.

Review

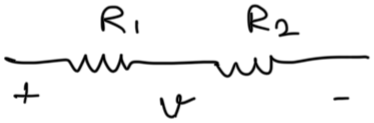
Voltage Division

$$v_k = v \frac{R_k}{R_1 + \cdots R_N}$$



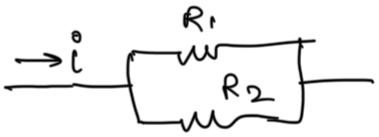
Current Division

$$i_k = i \frac{1/R_k}{1/R_1 + 1/R_2 + \cdots 1/R_N}$$



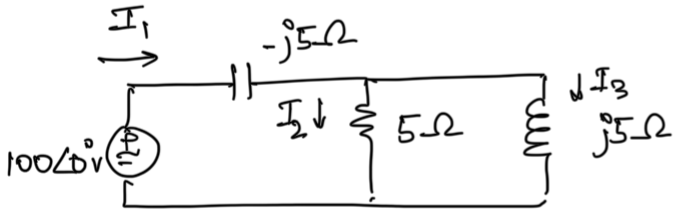
$$v_1 = v \frac{R_1}{R_1 + R_2}$$

$$v_2 = v \frac{R_2}{R_1 + R_2}$$



$$i_1 = i \frac{R_2}{R_1 + R_2}$$

$$i_2 = i \frac{R_1}{R_1 + R_2}$$



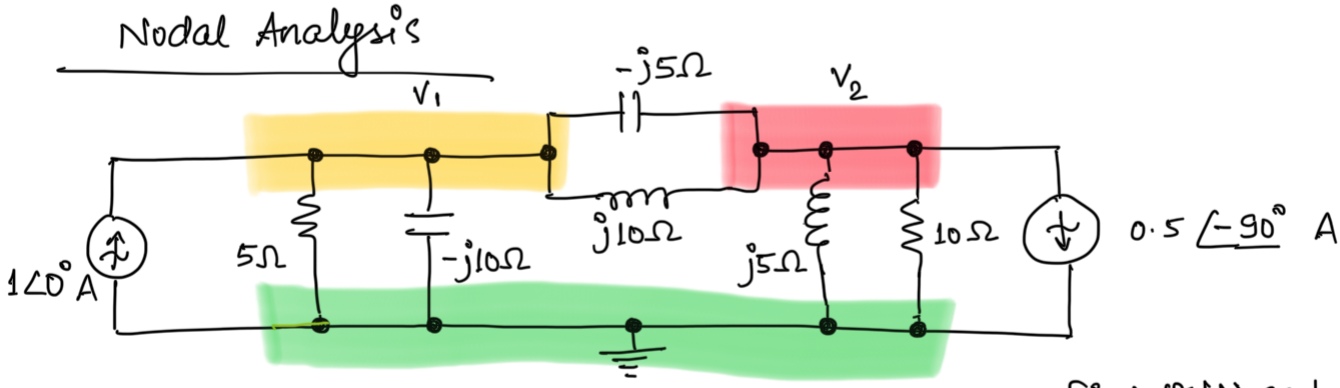
find I_1, I_2, I_3

$$I_1 = \frac{100\angle 0^\circ}{Z_{eq.}}, \quad I_2 = I_1\left(\frac{j5}{5+j5}\right), \quad I_3 = I_1\left(\frac{5}{5+j5}\right)$$

$$Z_{eq.} = 3.53\angle -45^\circ\ \Omega = 2.5 - j2.5\ \Omega$$

$$I_1 = 28.3\angle 45^\circ\ A$$

Nodal Analysis



Find $v_1(t)$ and $v_2(t)$

at node 1 ;

$$1\angle 0^\circ = \frac{V_1}{5} + \frac{V_1}{-j10} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j10}$$

$$\Rightarrow (0.2 + j0.2)V_1 - j\,0.1\,V_2 = 1 \quad \text{——— (1)}$$

at node 2 ;

$$-0.5\angle -90^\circ = \frac{V_2}{10} + \frac{V_2}{j5} + \frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j10}$$

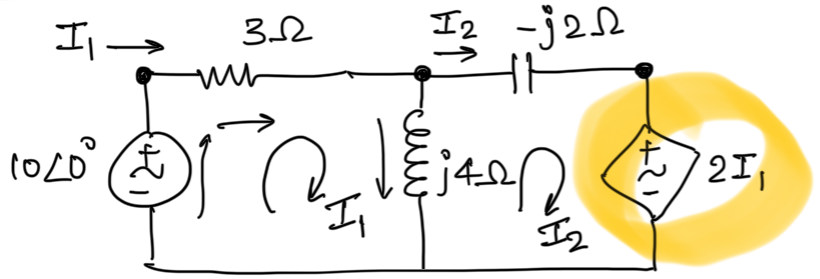
$$\Rightarrow -j0.1\,V_1 + (0.1 - j0.1)V_2 = j\,0.5 \quad \text{——— (2)}$$

$$V_1 = 1 - j2 = 2.24\angle -63.4^\circ\ V$$

$$V_2 = -2 + j4 = 4.47\angle 116.6^\circ\ V$$

$$v_1(t) = 2.24 \cos(\omega t - 63.4^\circ) \quad V$$
$$v_2(t) = 4.47 \cos(\omega t + 116.6^\circ) \quad V$$

Mesh Analysis



find I_1, I_2.

$$-10\angle 0^\circ + 3I_1 + (I_1 - I_2)j4 = 0$$
$$(I_2 - I_1)j4 + (-j2)I_2 + 2I_1 = 0$$

$$I_1 = 1.24\angle 29.7^\circ \quad A$$
$$I_2 = 2.77\angle 56.3^\circ \quad A$$

$$i_1(t) = 1.24 \cos(\omega t + 29.7^\circ) \quad A$$
$$i_2(t) = 2.77 \cos(\omega t + 56.3^\circ) \quad A$$