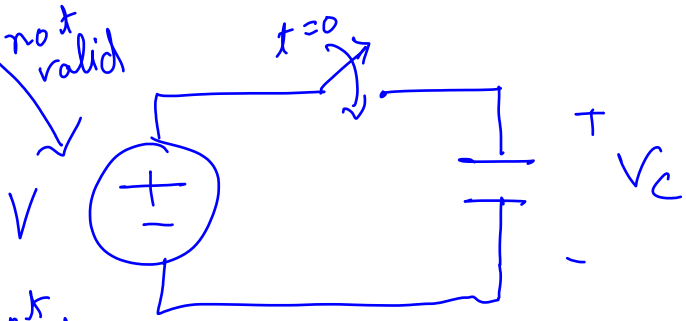
Quick Recap

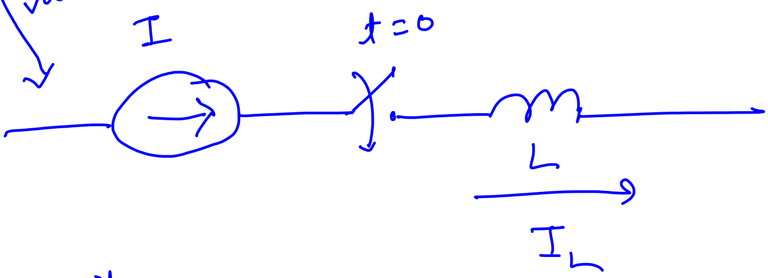
$V_C(t^-) = V_C(t^+)$, provided $I_C(t) \neq \infty$

$I_L(t^-) = I_L(t^+)$, provided $V_L(t) \neq \infty$

not valid



not valid



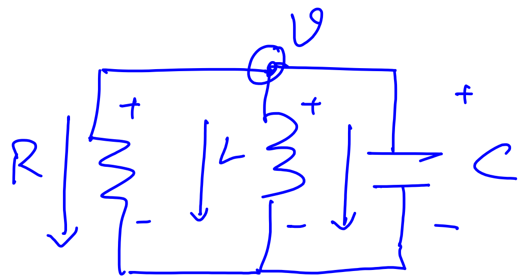
circuits involving dc sources

$$I_C(\infty) = 0 \qquad \left(\lim_{t\to\infty} I_C(t) = 0 \right)$$

$$V_L(\infty) = 0$$

RLC circuit (parallel)

└→ natural response $\begin{cases} \text{overdamped} \\ \text{underdamped} \\ \text{critically damped} \end{cases}$



$V_C(0) = V_0$

$I_L(0) = I_0$

natural response,

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(overdamped)}$$

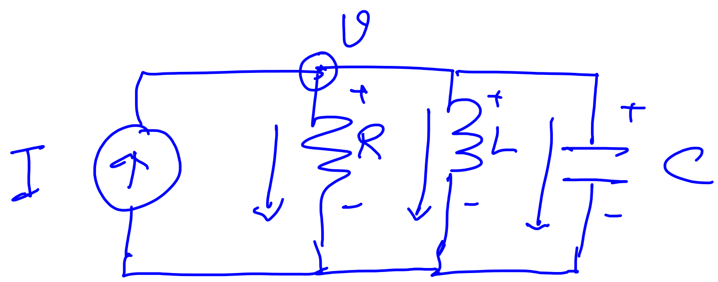
$$v(t) = e^{-\alpha t}\left(A_d \sin \omega_d t + B_d \cos \omega_d t\right)$$

(underdamped)

$$v(t) = A_1 e^{st} \cdot t + A_2 e^{st} = e^{st}(A_1 t + A_2)$$

(critically damped system)

Forced Response of RLC circuit,



$\left.\begin{array}{l} V_C(0) = V_0 \\ I_L(0) = I_0 \end{array}\right\}$

$$v(t) = v_f(t) + v_n(t)$$

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = V$$

$$\left.\begin{array}{l} v(0) = v_0 \\ \frac{dv(0)}{dt} = \dot{v}_0 \end{array}\right\}$$

$$\frac{d^2v_f}{dt^2} + 2\alpha\frac{dv_f}{dt} + \omega_0^2 v_f = V$$

$$v_f(0) = 0$$

$$\frac{dv_f}{dt}(0) = 0$$

$$\frac{d^2v_n}{dt^2} + 2\alpha\frac{dv_n}{dt} + \omega_0^2 v_n = 0$$

$$v_n(0) = v_0$$

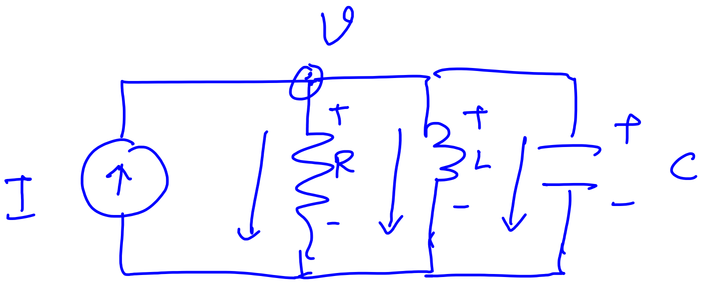
$$\frac{dv_n(0)}{dt} = \dot{v}_0$$

$$v(t) = v_f(t) + v_n(t)$$

$$= V_{ss} + v_T(t) \rightarrow \begin{cases} A_1 e^{s_1 t} + A_2 e^{s_2 t} \\ A_1 t e^{st} + A_2 e^{st} \\ \vdots \end{cases}$$

$$v(t) = V_{ss} + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ (overdamped case)

✓ $V_{ss} = 0$ since ($v(\infty) = 0$)



$V_C(0) = V_0$
$I_L(0) = I_0$

$$I = I_R(t) + I_L(t) + I_C(t) \quad \forall t \geq 0$$

$$= \frac{v(t)}{R} + \frac{1}{L}\int_0^t v(\tau)\,d\tau + I_L(0) + C\frac{dv}{dt}$$

$$0 = \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v + C\frac{d^2v}{dt^2}$$

$$\Rightarrow V_{ss} = 0$$

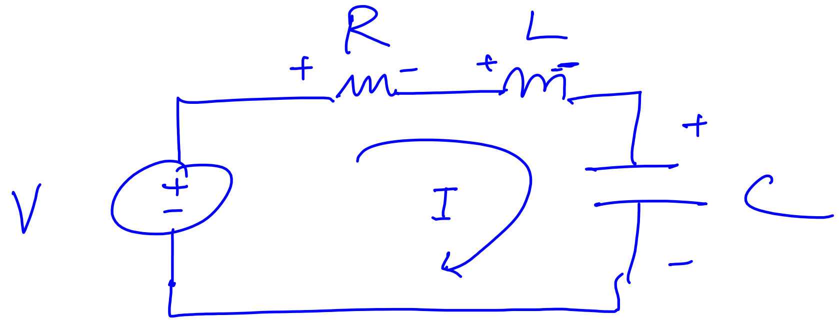
Total response, (general structure)

$$v(t) = V_{ss} + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ (overdamped)

$$= V_{ss} + e^{-\alpha t}(A_d \sin\omega_d t + B_d \cos\omega_d t)$$ (underdamped)

$$= V_{ss} + e^{-\alpha t}(A_1 t + A_2)$$ (critically damped)

Series RLC circuit,



$V_C(0) = V_0$

$I_L(0) = I_0$

$$V = V_R(t) + V_L(t) + V_C(t) \quad \forall\, t \geq 0$$

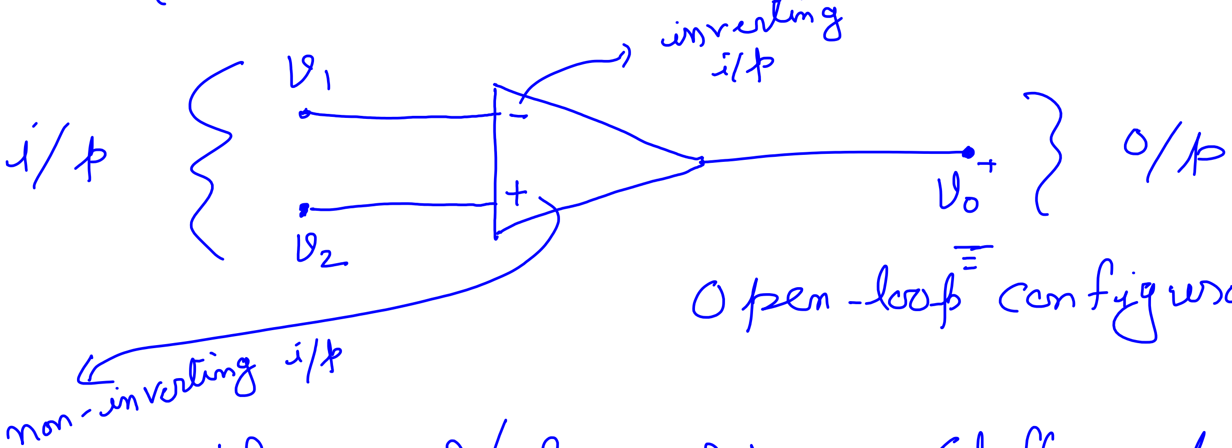
$$I_C(t) = I_L(t) = I_R(t) = I(t)$$

$$V = I(t)R + L\frac{dI(t)}{dt} + \frac{1}{C}\int_0^t I(\tau)\,d\tau + V_C(0)$$

$$0 = \frac{dI}{dt}R + L\frac{d^2 I}{dt^2} + \frac{1}{C}I$$

Operational Amplifier (Active element)

(OpAmp)



Open-loop configuration

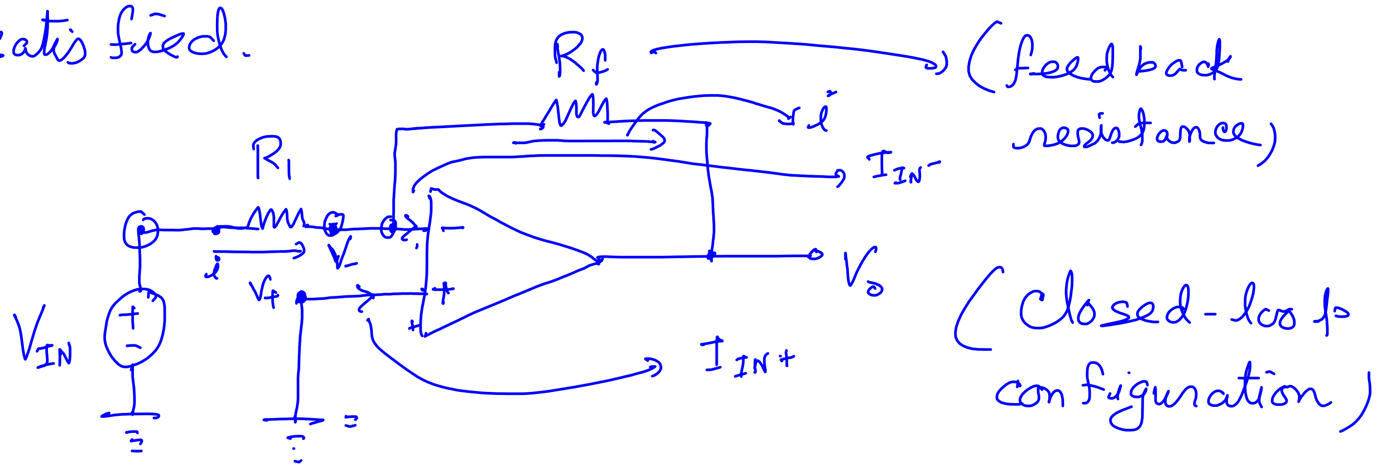
$$\underbrace{v_0 = A(v_2 - v_1)}_{}$$ (differential amplifier)

(linear property, only valid in a certain range of operating condition)

For an ideal OpAmp $(A \longrightarrow \infty)$

$$(V_2 - V_1) \simeq 0$$

In closed-loop, this property is automatically satisfied.



In closed-loop, an ideal OpAmp follows the subsequently defined properties.

1) $V_+ = V_-$ (virtual short circuit)

2) $I_{IN^-} = I_{IN^+} = 0$

$$V_+ = 0 = V_-$$

(Inverting amplifier)

$$i = \frac{V_{IN}}{R_1} = \frac{-V_o}{R_f} \Rightarrow \boxed{V_o = -\frac{R_f}{R_1} V_{IN}}$$