Date

* Multivariate Calculus {Lecture 1}

⇒ Functions of "n" variables OR vector functions

gives scalar values ~~even though argument~~ can be vector

let $\bar{x} = (x_1, x_2 \ldots x_n)$

then $f(\bar{x})$ is a function of n variables or a vector function {or scalar valued vector function}

Domain of f : $\mathbb{R}^n = R \times R \times R \times \ldots R$
Range of f : $\mathbb{R}$

distance function from origin

eg : $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

domain : $x^2+y^2+z^2 > 0 \Rightarrow$ always $\mathbb{R}$

① $w(x,y) = \sqrt{y-x^2}$

domain: $y - x^2 \geq 0$
range : $[0, \infty)$

② $\sin(xy)$

domain : $\mathbb{R}^2$ range : $[-1, 1]$

③ $xy \log z$

domain : $R \times R \times R^+$ range : R

CHP 14

Practice Problems } 11, 15, 27, 29, 32, 40, 43, 45, 46

14.1

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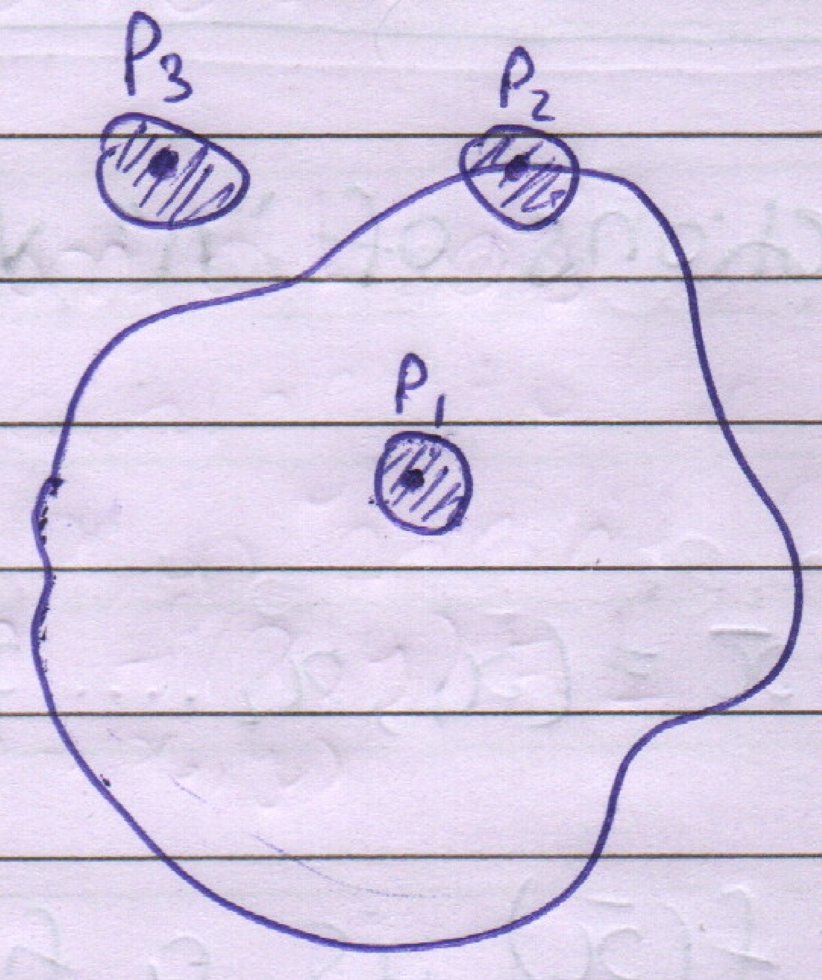
* Points: Interior / Boundary / Exterior



consider a region K in $\mathbb{R}^n$,

① ~~an~~ interior point (P_1) if a disk with center P_1 can be constructed where all points inside the disk lies inside K.

② boundary point (P_2): if every disk with the center P_2 contains points inside and outside

③ exterior point (P_3): find a disk with center P_3 containing all points outside K.

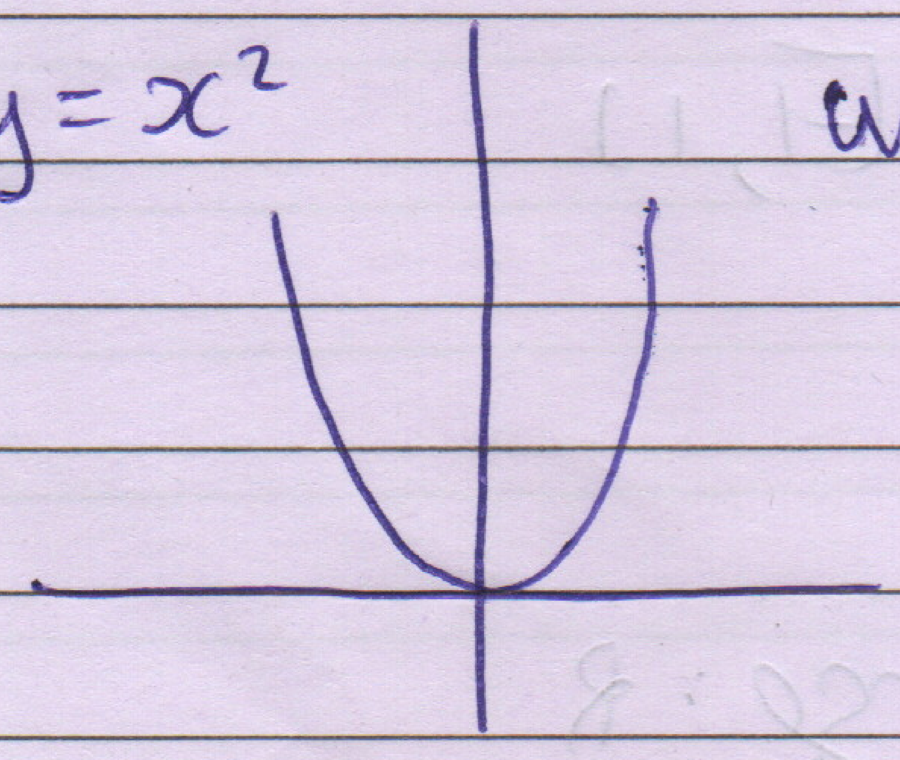
* REGION: open / closed / bounded

OPEN: all points are interior points

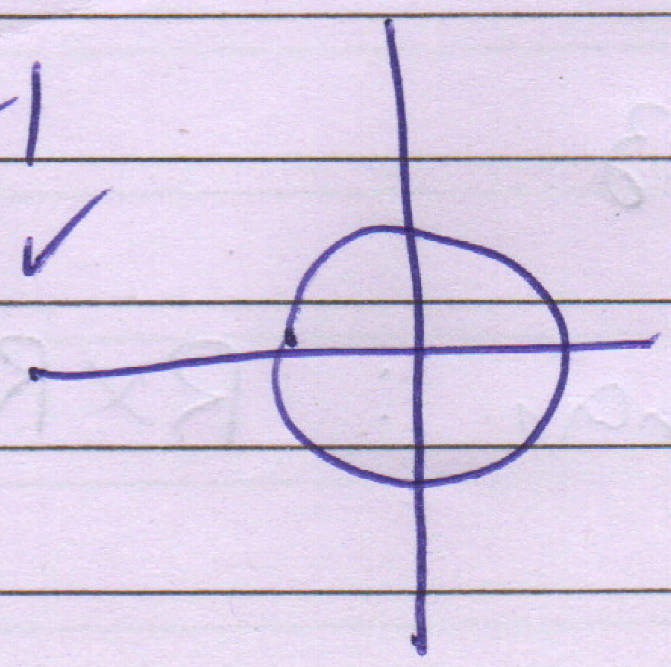
CLOSED: all points are either interior or boundary points

BOUNDED: region which lies inside a sufficiently large disk of fixed / finite radius

eg: $y = x^2$ unbounded ✓



$x^2+y^2=1$
bounded ✓



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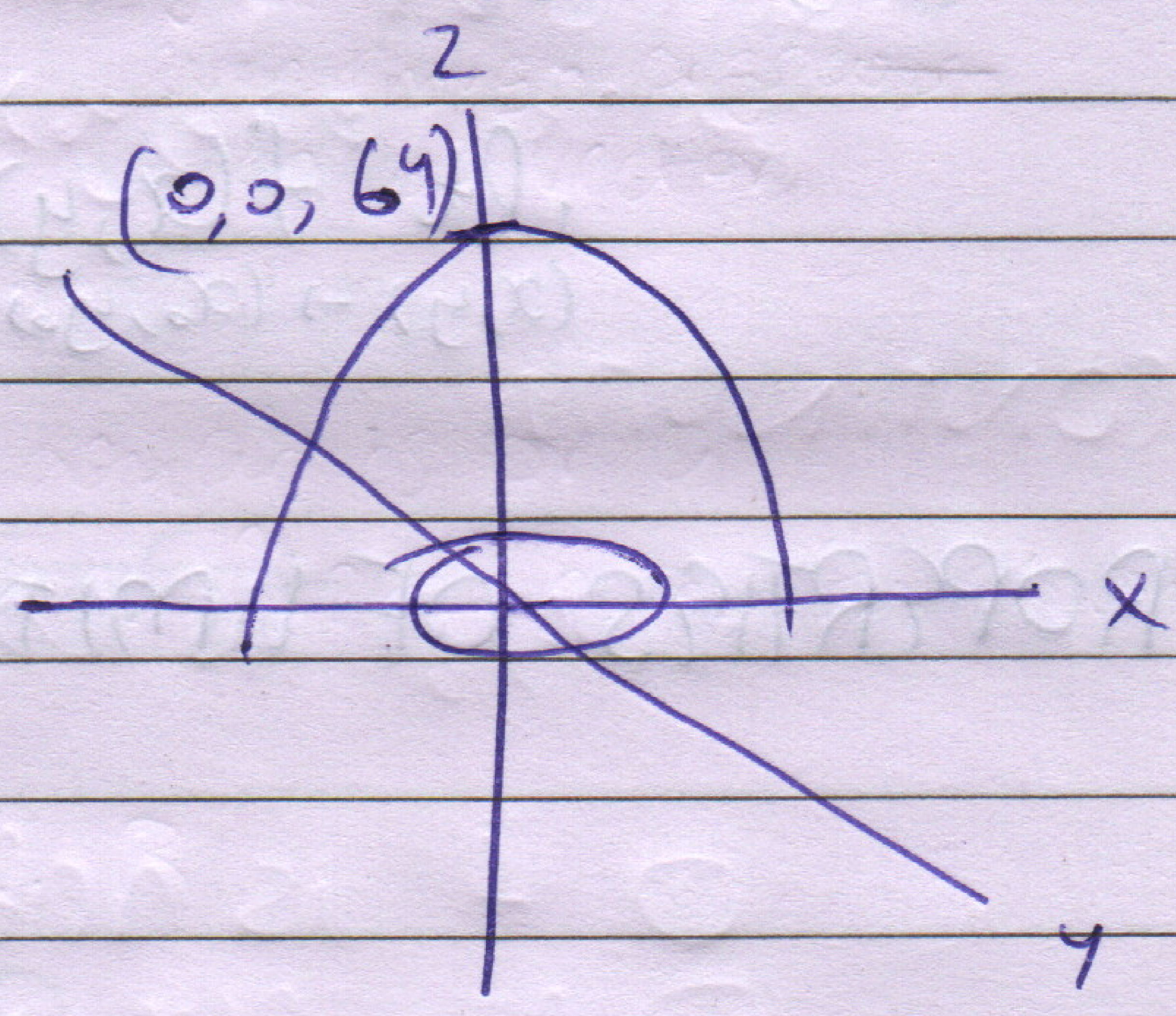
* LEVEL CURVES OF A FUNCTION

⇒ set of all points in the plane (2d) where the function $f(x,y)$ has constant value

$\Rightarrow f(x,y) = c$

eg. $f(x,y) = 64 - x^2 - y^2$



$f(x,y) = c = 64 - x^2 - y^2$

$x^2 + y^2 = 64 - c$

since we eliminated z, we can say this is a projection of the xy plane

ez defn → level curves remove a single dimension from the function

* FUNCTIONS OF 3 VARIABLES

everything is same in points* and region* except

disk → sphere

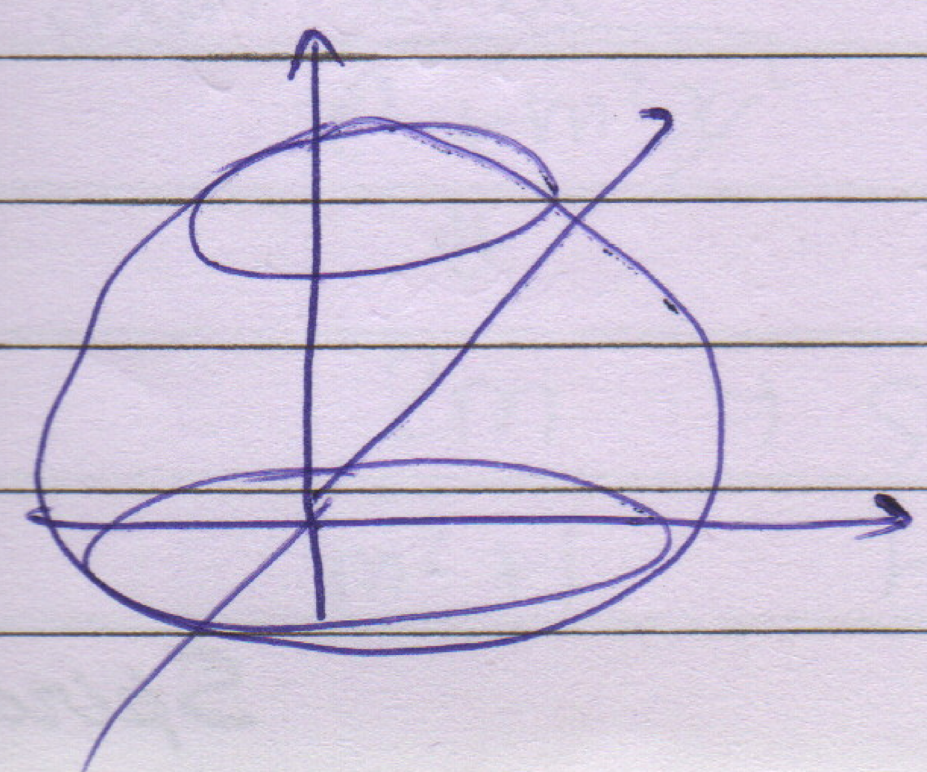
but...

↗ because 3d → 2d

⇒ LEVEL ~~CURVE~~ SURFACES

eg: $w = \sqrt{x^2+y^2+z^2} = c \Rightarrow x^2+y^2+z^2 = c^2$

(hollow) sphere at origin and radius $= c$



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* Limits and continuity in higher dimensions

we say that $f(x,y)$ approaches a limit L as (x,y) approaches (x_0,y_0) if

$$\underset{(x,y)\to(x_0,y_0)}{\mathrm{lt}} f(x,y) = L \quad \Big| \quad \lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

⇒ PROPERTIES OF LIMITS $\quad \lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$

$$\lim_{(x,y)\to(x_0,y_0)} g(x,y) = M$$

① SUM : $\lim_{(x,y)\to(x_0,y_0)} (f \pm g) = L \pm M$
DIFF

② PRODUCT : $\lim (f \cdot g) = L \cdot M$

③ Scalar multiple : $\lim (kf) = kL$

④ Quotient : $\lim \left(\frac{f}{g}\right) = \frac{L}{M}$: $M \neq 0$

⑤ Power : $\lim (f^{\frac{r}{s}}) = L^{\frac{r}{s}}$

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eg① ⇒ $\lim\limits_{(x,y)\to(0,1)} \dfrac{x-xy+3}{x^2y+5xy-y^3} = -3$

just put the values (0,1) and check if alright

eg② ⇒ $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-xy}{\sqrt{x}-\sqrt{y}} \times \cancel{\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}}$

$\Rightarrow \cancel{\dfrac{(x^2-xy)(\sqrt{x}+\sqrt{y})}{x-y}}$

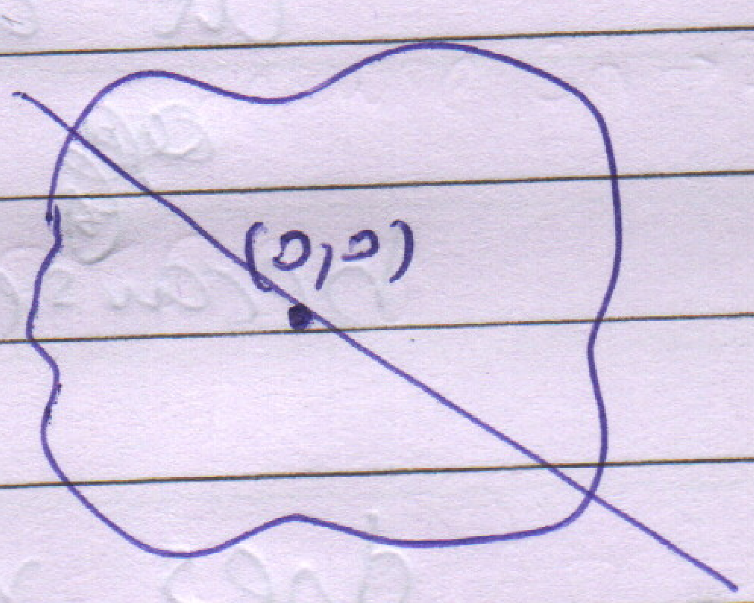
$\dfrac{x\cancel{(\sqrt{x}-\sqrt{y})}(\sqrt{x}+\sqrt{y})}{\cancel{\sqrt{x}-\sqrt{y}}} \Rightarrow \underline{0}$

eg③: $f(x,y) = \lim\limits_{(x,y)\to(0,0)} \dfrac{2xy}{x^2+y^2}$

Note: if limit exists, it is unique.
So, if you can find 2 limits, it doesn't exist

Remember: approach from left/right

let region:



let us approach points on this line i.e. $y=mx$
so

$f(x,y) = \lim \dfrac{2xy}{x^2+y^2}$

$f(x,mx) = \lim \dfrac{2mx^2}{x^2+m^2x^2}$

$\Downarrow$

$\dfrac{2m}{m^2+1}$ ⇒ so, lim exists for any m
but this can't happen. Hence lim doesn't exist

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* TWO PATH TEST FOR NON EXISTENCE OF LIMIT

if $f(x,y)$ has different limits for two different paths both approaching to the point (x_0, y_0), then $\lim f(x,y)$ doesn't exist

* CONTINUITY

$f(x,y)$ is continuous at (x_0, y_0) if

① f is defined at (x_0, y_0)

② $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y)$ exists

③ $\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0, y_0)$

eg: $f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2} & : (x,y) \neq (0,0) \\ 0 & : (x,y) = (0,0) \end{cases}$

is f continuous?
↓
point wise continuous

function is continuous at all point except (0,0) perhaps because ② $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist

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* CONTINUITY OF COMPOSITE FUNCTIONS

if f and g are continuous then

$$h = f \circ g = f(g(\overset{x,y}{\;}))\text{ is continuous}$$

eg: $g = xy$ $\Rightarrow h = f(g(x,y)) = \cos(xy)$
$f = \cos z$

These are trigonometric functions
hence continuous

we can conclude similarly for polynomial function

Thomas Calculus 11th edition
engineering maths by Crazik 9th edition

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