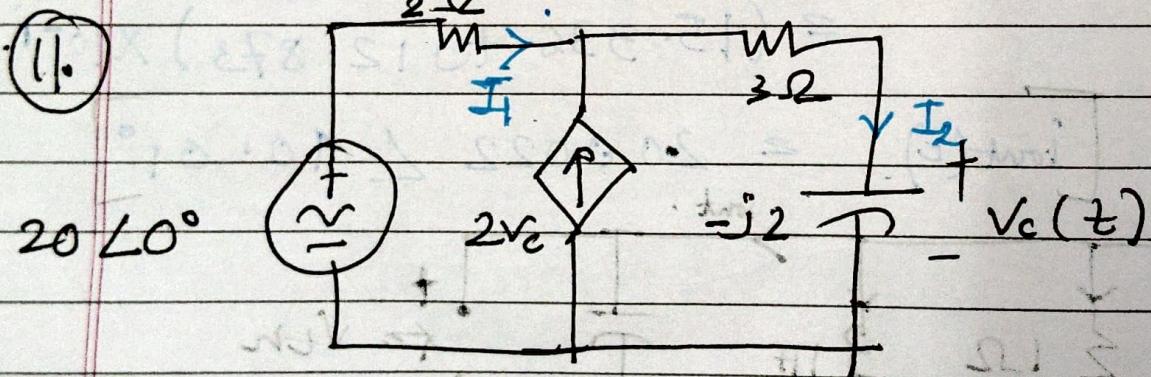


$$= 20 \cdot 032 \angle 38 \cdot 10^\circ \times 10^4$$



$$20 \angle 0^\circ$$

$$v_c(t) = I_2 \times (-j2). \quad I_2 = \frac{v_c}{-j2};$$

Super Mesh:

$$-20 \angle 0^\circ - I_1 \times 2 + 3 \times \left(\frac{v_c}{-j2} \right) + v_c = 0$$

Nodal:

$$I_1 + 2v_c = I_2 \Rightarrow I_1 + 2v_c = \frac{v_c}{-j2} \quad -\text{(11)}$$

(1) \rightarrow

$$-20 - I_1 \times 2 + \frac{3V_c}{2} \angle 90^\circ + V_c = 0.$$

$$-20 - I_1 \times 2 + j1.5V_c + V_c = 0$$

~~Ans~~

$$2I_1 = V_c (1 + j1.5) - 20.$$

$$(11) \rightarrow I_1 = j0.5V_c - 2V_c$$

putting value $\Rightarrow jV_c - 4V_c = V_c + j1.5V_c - 20$

$$\Rightarrow 5V_c + j0.5V_c = 20 + j0.$$

$$\Rightarrow V_c = \frac{20}{(5 + j0.5)} = (3.960 - j0.3960)$$

$$V_c = 3.979 \angle -5.710^\circ$$

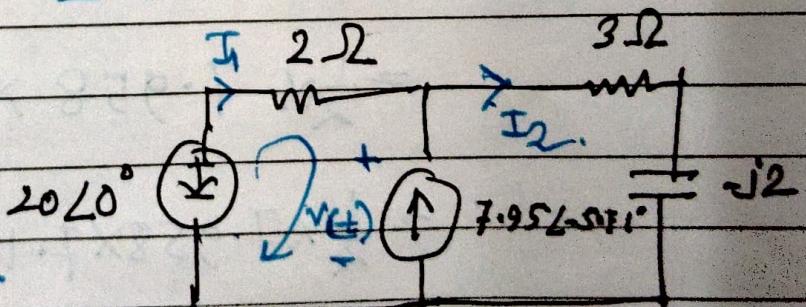
$$2V_c = 7.958 \angle -5.710^\circ$$

$$I_1 = \frac{V_c}{-j2}$$

$$= \frac{7.958 \angle -5.710^\circ}{2 \angle -90^\circ}$$

$$I_2 = 3.979 \angle 84.29^\circ$$

$$I_1 = I_2 + 2V_c(j) = 3.979 \angle 84.29^\circ + 7.958 \angle -5.710^\circ$$



$$I_1 = 8.3114 + j3.1677$$

$$I_1 = 8.894 \angle 20.86^\circ$$

kN,

$$-20\angle 0^\circ + 2I_1 + v(t) = 0$$

$$v(t) = 20\angle 0^\circ - \cancel{17.78} (17.788 \angle 20.86^\circ)$$

$$= \cancel{2.234} - j8.875 \times 10^{-1}$$

$$= 3.377 - j6.334$$

$$v(t) = 7.178 \angle -61.93^\circ$$

$$P_{avg} = \frac{1}{2} \operatorname{Im} V_m \cos \phi$$

$$= \frac{1}{2} \times 7.958 \times 7.178 \left(\cos(-\frac{51.93}{51.10}) \right)$$

$$= \frac{1}{2} \times 7.958 \times 7.178 \cos(-56.22^\circ)$$

$$P_{avg} = 15.88 \text{ Watt}$$

28 (a) $R.M.S = \frac{3}{\sqrt{2}}$

(b) $4 \sin 2\omega t \cos 1\omega t$

$$= 2 (\sin(3\omega t) + \sin(\omega t))$$

$$\Rightarrow \sqrt{RMS_1^2 + RMS_2^2}$$

$$= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$$

$$= 2.$$

(c) $2 - \sin 1\omega t$

$$= x(t) \text{ let}$$

$$RMS = \sqrt{\frac{1}{T} \int_0^T |x(t)|^2 dt}$$

$$= \int_0^T (4 - 4 \sin 1\omega t + \sin^2 1\omega t) dt$$

$$= [4T - 0 + \frac{T}{2}] = 9T/2$$

$$\therefore RMS = \sqrt{\frac{1}{T} \times \frac{9T}{2}} = 2.121.$$

(d) $RMS = \sqrt{\frac{1}{T} \left(\int_0^{T/2} A^2 dt + \int_{T/2}^T 0^2 dt \right)}$

$$RMS = \sqrt{\frac{1}{T} \left(\int_1^3 (2.82)^2 dt + \int_3^4 0 dt \right)}$$

$$= \sqrt{\frac{1}{T} \times (2.82)^2 \times 2}$$

$$T = 3.$$

$$= 2.302.$$

350

$$I = 119 \angle 3^\circ$$

$$14 \angle 32^\circ + 22 \angle 0^\circ = 0.338 - j0.557 \text{ A}$$

$$I = 0.6515 \angle -58.74^\circ$$

$$\text{Power} = \frac{1}{2} V_m I_m \cos \phi$$

$$= \frac{1}{2} \times 119 \times 0.6515 \times \cos(3^\circ - (-58.74))$$

$$= 18.353 \text{ Watt. } 18.353 \text{ watt.}$$

$$S = VI^*$$

$$= V (0.338 + j0.557)$$

$$= 119 \angle 3^\circ \times 0.6515 \angle 58.74^\circ$$

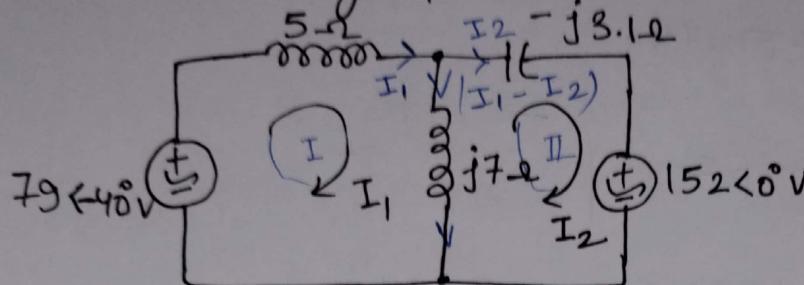
$$= 36.707 + j68.287 \text{ VA}$$

$$S = 77.527 \angle 61.74^\circ \text{ VA.}$$

$$\text{power factor} = \frac{P}{S} = \frac{18.353}{77.527}$$

$$\cos \theta = 0.236$$

(5) With Regards to the two-mesh circuit depicted in fig. determine the average power observed by each passive element, and the average power supplied by each source, and verify that the total supplied average power = the total absorbed average power



$$\begin{aligned} \underline{\text{KVL I}} \quad & I_1 - 79\angle -40^\circ + 5I_1 + 7j(I_1 - I_2) = 0 \\ & -(60.51 - 50.78j) + 5I_1 + 7jI_1 - 7jI_2 = 0 \end{aligned}$$

$$\Rightarrow [(5+7j)I_1 - 7jI_2 = 60.51 - 50.78j] \quad \text{--- (1)}$$

$$\begin{aligned} \underline{\text{KVL II}} \quad & -3.1jI_2 + 152 - 7j(I_1 - I_2) = 0 \\ & -7jI_1 + 3.9jI_2 = -152 \neq 0 \quad \text{--- (2)} \end{aligned}$$

Solving (1) and (2) By Cramer's Rule.

$$\begin{aligned} \Delta &= \begin{vmatrix} 5+7j & -7j \\ -7j & 3.9j \end{vmatrix} \\ &= 3.9j(5+7j) - 4.9j \\ &= -27.3 + 19.5j - 4.9j \\ \boxed{\Delta = -27.3 - 14.5j} \end{aligned}$$

$$\Delta_1 = \begin{vmatrix} 60.51 - 50.78j & -7j \\ -152 & 3.9j \end{vmatrix}$$

$$\Delta_1 = 198.04 + 235.98j - 1064j = 198.04 - 828.02j$$

$$\Delta_2 = \begin{vmatrix} 5+7j & 60.51 - 50.78j \\ -7j & -152 \end{vmatrix}$$

$$= -760 - 1.64j + 355.46 + 423.57j$$

$$\Delta_2 = -404.54 - 640.43j$$

$$I_1 = \frac{198.04 - 828.02j}{-27.3 - 29.5j} \Rightarrow 21.18 \angle -56.23^\circ$$

$$I_2 = \frac{-404.54 - 640.43j}{-27.3 - 29.5j} \Rightarrow 18.8 \angle 10.5^\circ$$

Power by Voltage Sources $P_1 = \text{Re}[V I_1^*]$

$$P_1 = \text{Re}\{(79 \angle -40^\circ)(21.18 \angle -56.23^\circ)\} \Rightarrow P_1 = -181.577\text{W}$$

$$P_2 = \text{Re}\{(152 \angle 0^\circ)(18.8 \angle 10.5^\circ)\} \Rightarrow P_2 = 2809.74\text{W}$$

Power by passive element

$$P_3 = |I_1|^2 R = 5 \times (21.18)^2 = 2376.2\text{W}$$

$$P_4 = \text{Re}\{V I^*\} = \text{Re}\{|I|^2 Z\} = \text{Re}\{(18.8)^2 (-j3.1)\} = 0\text{W}$$

$$P_5 = \text{Re}\{(I_1 - I_2)^2 Z\}$$

$$= \text{Re}\{(15.68)^2 \times 7j\}$$

$$= 0\text{W}$$

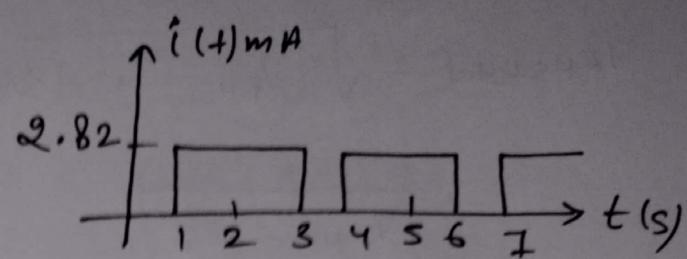
Q6. Compute the effective value of $i(t)$

(a) $i(t) = 3 \sin 4t + A$

(b) $v(t) = 4 \sin 2\omega t \cos \omega t$

(c) $i(t) = 2 - \sin \omega t \text{ mA}$

(d) the waveform plotted in fig.



$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

(a) $T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\begin{aligned} I_{rms} &= \sqrt{\frac{2}{\pi} \int_0^{\pi/2} (3 \sin 4t)^2 dt} \\ &= \sqrt{\frac{2}{\pi} \int_0^{\pi/2} 9 \sin^2 4t dt} \quad [\because \cos 2A = 1 - 2\sin^2 A] \\ &= 3 \sqrt{\frac{2}{\pi} \int_0^{\pi/2} \frac{1}{2} (1 - \cos 8t) dt} \\ &= 3 \sqrt{\frac{1}{\pi} \left[t \right]_0^{\pi/2} - \left[\frac{\sin 8t}{8} \right]_0^{\pi/2}} \\ &= 3 \sqrt{\frac{1}{\pi} \left[\frac{\pi}{2} \right]} = \frac{3}{\sqrt{2}} = 2.12 \text{ A} \\ &\boxed{I_{rms} = 2.12 \text{ A}} \end{aligned}$$

(b) $v(t) = 4 \sin 2\omega t \cos \omega t$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$v(t) = \frac{4}{2} (\sin 3\omega t + \sin \omega t)$$

$$= \underbrace{2(\sin 3\omega t)}_{(1)} + \underbrace{\sin \omega t}_{(2)}$$

$$V_{RMS_1} = \frac{2}{\sqrt{2}} = \sqrt{2}, \quad V_{RMS_2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$V_{RMS\text{total}} = \sqrt{(V_{RMS_1})^2 + (V_{RMS_2})^2} = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$$

$$\boxed{V_{RMS} = 2V}$$

$$= \sqrt{4} = 2V$$

$$(c) i(t) = 2 - \sin(10t) \text{ mA}$$

$$I_{RMS} = \sqrt{(\text{DC Value})^2 + \left(\frac{\text{AC sinusoidal}}{\sqrt{2}}\right)^2}$$

$$= \sqrt{(2)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{4+0.5} = \sqrt{4.5}$$

$$\boxed{I_{RMS} = 2.12 \text{ mA}}$$

$$(d) I_{RMS} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

$$= \sqrt{\frac{1}{3} \int_1^3 (2 - \sin t)^2 dt}$$

$$= \sqrt{\frac{1}{3} \int_1^3 (2 - 2\sin t)^2 dt}$$

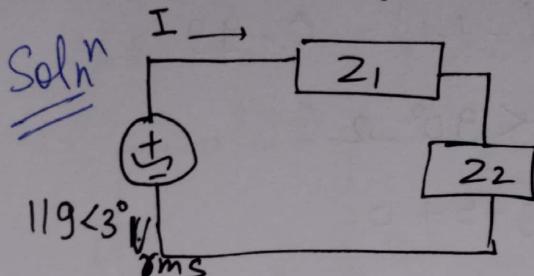
$$= \sqrt{\frac{(2.82)^2}{3} \int_1^3 dt}$$

$$= 2.82 \sqrt{\frac{1}{3} \times (3-1)}$$

$$= 2.82 \cdot \sqrt{\frac{2}{3}}$$

$$\boxed{I_{RMS} = 2.30 \text{ A}}$$

Q7 for the circuit of fig. Compute the average power delivered to each load, the apparent power supplied by the source and the power factor of the combined loads if (a) $Z_1 = 14 \angle 32^\circ \Omega$ and $Z_2 = 22 \Omega$, (b) $Z_1 = 2 \angle 0^\circ \Omega$ and $Z_2 = 6 - j \Omega$ (c) $Z_1 = 100 \angle 70^\circ \Omega$ and $Z_2 = 75 \angle 90^\circ \Omega$



$$(a) \quad Z_1 = 11.87 + j 7.41$$

$$Z_2 = 22 + j 0$$

$$\text{Equivalent impedance } Z_{\text{total}} = Z_1 + Z_2 = 14 \angle 32^\circ + 22 \Omega$$

$$= 33.87 + j 7.41$$

$$= 34.67 \angle 12.3^\circ$$

$$\text{Current, } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z_{\text{total}}} = \frac{119 \angle 30^\circ}{(34.67 \angle 12.3^\circ)}$$

$$= 3.42 \angle -9.4^\circ A$$

Power delivered to each load

$$P_1 = (I)^2 R_e(Z_1) = (3.42)^2 \cdot 11.87 \Rightarrow 138.8 W$$

$$P_2 = (I)^2 R_e(Z_2) = (3.42)^2 \cdot 22 \Rightarrow 257.32 W$$

Apparent Supplied Power.

$$S = V_s I = (119)(3.42) = 406.98 \text{ VA}$$

$$\text{Power factor (PF)} = \cos(\theta_{\text{total}}) = \cos(12.3^\circ) = 0.977$$

$$(b) \quad Z_1 = 2 \angle 0^\circ \Omega, \quad Z_2 = 6 - j \Omega$$

$$Z_{\text{total}} = 2 + (6 - j) = 8 - j \Omega = 8.06 \angle -7.1^\circ$$

$$\text{Current} \rightarrow \frac{V_{\text{rms}}}{Z_{\text{total}}} = \frac{119 \angle 30^\circ}{8.06 \angle -7.1^\circ} \Rightarrow I = 14.76 \angle 10.1^\circ A$$

Power delivered to each load

$$P_1 = |I|^2 \operatorname{Re}(Z_1) = (14.76)^2 (2) = 435.7 \text{ W}$$

$$P_2 = |I|^2 \operatorname{Re}(Z_2) = (14.76)^2 (6) = 1307.14 \text{ W}$$

Apparent Power (S) = $|V_s| |I| = (119)(14.76)$

$$= 1756.44 \text{ VA}$$

Power factor $\Rightarrow PF = \cos(\theta_{\text{total}}) = \cos(-7.1) = 0.99$

(C) $Z_1 = 100 \angle 70^\circ \Omega$ and $Z_2 = 75 \angle 90^\circ \Omega$

$$Z_1 = 34.2 + j93.97 \Omega \quad Z_2 = j75 + 0$$

$$Z_{\text{total}} = (34.2 + j93.97) + (j75) = 34.2 + j168.97 \Omega$$

$$Z_{\text{total}} = 172.9 \angle 78.5^\circ$$

Current $I = \frac{V_{\text{rms}}}{Z_{\text{total}}} = \frac{119 \angle 3^\circ}{172.9 \angle 78.5} = 0.69 \angle -75.5^\circ A$

Power delivered to each load

$$P_1 = |I|^2 \operatorname{Re}(Z_1) = (0.69)^2 (34.2) = 16.2 \text{ W}$$

$$P_2 = |I|^2 \operatorname{Re}(Z_2) = (0.69)^2 (10) = 6 \text{ W}$$

Apparent Power $S = |V_s| |I| = (119)(0.69) = 82.11 \text{ VA}$

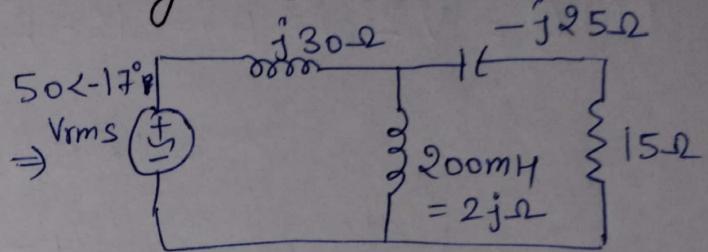
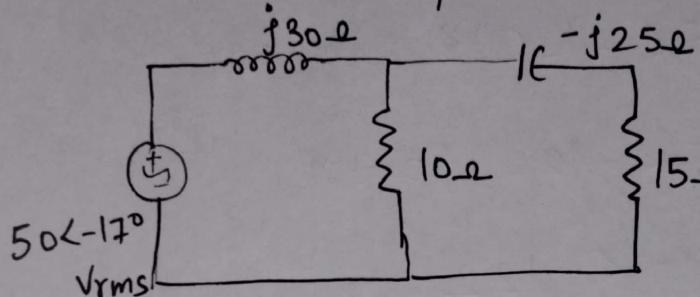
Power factor (PF) $= \cos(\theta_{\text{total}}) = \cos(78.5) = 0.19$

Q8. Replace the 10Ω resistor in the circuit of Fig. with a $200mH$ inductor assume an operating frequency of 10 rad/s and calculate :

(a) the PF of the Source

(b) the apparent power supplied by the source.

(c) the reactive power delivered by the source.



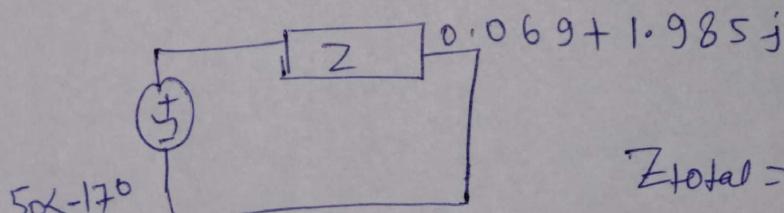
$$X_L = j\omega L$$

$$\begin{aligned} &= j \times 10 \times 200 \times 10^{-3} \\ &= 2j\Omega \end{aligned}$$

$$15 + (-j25) = 15 - j25$$

$$Z_{\text{equivalent}} \Rightarrow \frac{1}{Z_{\text{total}}} = \frac{1}{30j} + \frac{1}{2j} + \frac{1}{(15-j25)}$$

$$Z_{\text{eq}} = 0.069 + 1.985j$$



$$Z_{\text{total}} = 1.986 \angle 88.0^\circ$$

$$I = \frac{V_{\text{rms}}}{Z_{\text{total}}} = \frac{50 \angle -17^\circ}{1.986 \angle 88^\circ}$$

$$I_{\text{rms}} = 25.17 \angle -105^\circ \text{ A}$$

$$(a) \text{Power factor (PF)} = \cos(\theta_{\text{total}}) = \cos(88^\circ) \\ = 0.0348$$

$$(b) |S| = \sqrt{I} = (50 \angle -17^\circ) (25.17 \angle -105^\circ) \\ = 1258.5 \angle -122^\circ$$

$$(c) Q = \sqrt{I} \sin \theta = (1258.5 \angle -122^\circ) \sin(88^\circ) \\ = 1245.90 \angle -122^\circ$$