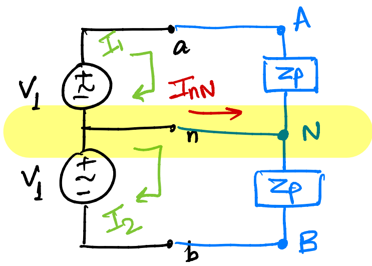
Review

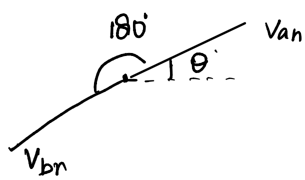
Single-phase Three-wire sources (Two-phase)



$V_1 = V_p \angle \theta^\circ$

$V_{an} = V_a - V_n = V_1 = V_p \angle \theta^\circ$

$V_{bn} = -V_1 = -V_p \angle \theta^\circ = V_p \angle \theta^\circ - 180^\circ$



Balanced source $\begin{cases} |V_{an}| = |V_{bn}| = V_p \\ V_{an} + V_{bn} = 0 \end{cases}$

$I_{nN} = 0 \leftarrow$ when source and load are balance.

Time-domain	Freq. domain
$v_{an}(t) = V_p \cos(\omega t + \theta^\circ)$	$V_{an} = V_1 = V_p \angle \theta^\circ$
$v_{bn}(t) = V_p \cos(\omega t + \theta - 180^\circ)$	$V_{bn} = -V_1 = V_p \angle \theta^\circ - 180^\circ$

$p(t) = v(t)\, i(t)$

$P_{avg} = V \cdot I$ ✗

$P_{avg} = \frac{1}{2} \mathrm{Re}\{ \underbrace{V \cdot I^*} \}$ ✓

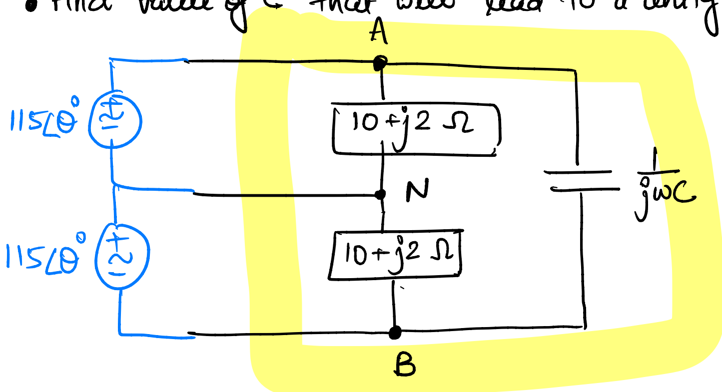
① **Example**: Consider $V_{12} = 9\angle 30^\circ$, $V_{32} = 3\angle 130^\circ$

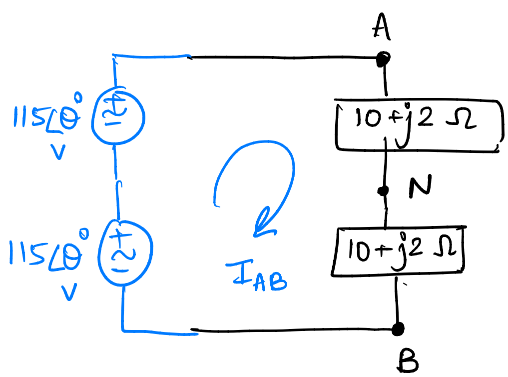
Find $V_{21} = 9\angle -150^\circ$, $V_{13} = V_{12} + V_{23}$

② Consider a single-phase three-wire balanced source connected to a balanced load. $f = 50\,Hz$, $V_{an} = 115\angle 0^\circ\,V$

→ • Find power factor of the load if capacitor is omitted.

• Find value of C that will lead to a unity power factor of total load.





$$PF = \cos(\theta - \phi)$$

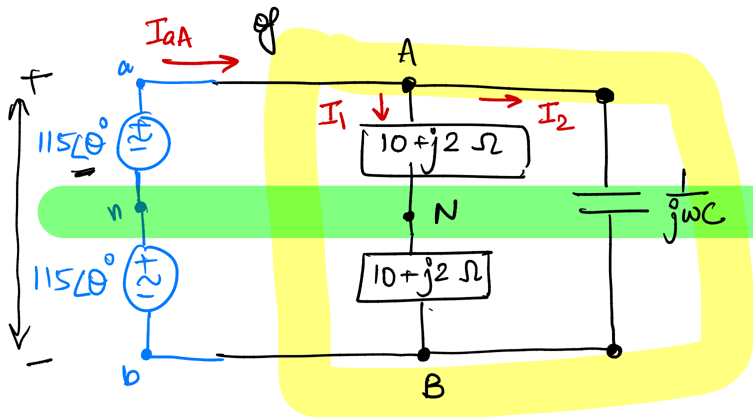
$$I_{AB} = \frac{115\angle 0^\circ + 115\angle 0^\circ}{2(10+j2)} = \frac{230\angle 0^\circ}{2(10+j2)}$$

$$= 11.27\angle 0^\circ - 11.3^\circ \; A$$

$$\phi = 0^\circ - 11.3^\circ$$

$$PF = \cos(\underbrace{\theta - \theta + 11.3^\circ}) = 0.98 \text{ lagging}$$

$$PF = 1 \Rightarrow \cos(\theta - \phi) = 1 \Rightarrow \theta - \phi = 0 \Rightarrow \theta = \phi$$



$\theta = \phi$

$$I_{aA} = I_1 + I_2 = \frac{115\angle\theta^\circ}{10+j2} + \frac{230\angle\theta^\circ}{\left(\frac{1}{j\omega C}\right)} =$$

S = Complex power of total load $= \frac{1}{2} V I^*$

$$PF = \frac{Re\{S\}}{|S|} \Rightarrow PF = 1 \Rightarrow |S| = Re\{S\} \Rightarrow \boxed{Im\{S\} = 0}$$

$$\boxed{Z_P = 10 + j2} \qquad \boxed{Z_C = \frac{1}{j\omega C}}$$

$$S = \frac{1}{2} V_{an} I_{AN}^* + \frac{1}{2} V_{nb} I_{NB}^* + \frac{1}{2} V_{ab} I_{AB}^*$$

$$= \frac{1}{2} V_{an} \left(\frac{V_{an}}{Z_P}\right)^* + \frac{1}{2} V_{nb} \left(\frac{V_{nb}}{Z_P}\right)^* + \frac{1}{2} V_{ab} \left(\frac{V_{ab}}{Z_C}\right)^*$$

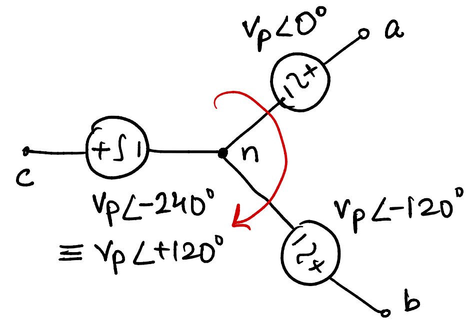
$$= \underline{\frac{1}{2} \frac{|V_{an}|^2}{|Z_P|^2} Z_P} + \underline{\frac{1}{2} \frac{|V_{nb}|^2}{|Z_P|^2} Z_P} + \frac{1}{2} \frac{|V_{ab}|^2}{|Z_C|^2} Z_C$$

$$= \frac{|V_{an}|^2}{|Z_P|^2} Z_P + \frac{1}{2} \frac{|V_{ab}|^2}{|Z_C|^2} Z_C$$

$$S = \frac{115^2}{104} (10 + 2j) + \frac{1}{2} \frac{(230)^2}{(1/\omega C)^2} \left(\frac{-j}{\omega C}\right)$$

$$Im(S) = 0 \Rightarrow C = \frac{1}{\omega \times 104} = \frac{1}{100\pi \times 104} = 30.6\ \mu F$$

Three-phase Sources

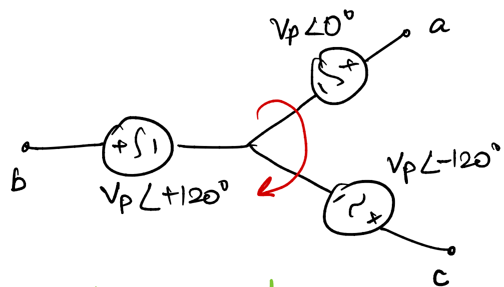


$$V_{an} = V_p\angle 0^\circ$$
$$V_{bn} = V_p\angle -120^\circ$$
$$V_{cn} = V_p\angle 120^\circ$$

$$\left\{\begin{array}{l} |V_{an}| = |V_{bn}| = |V_{cn}| = V_p \\ V_{an} + V_{bn} + V_{cn} = 0 \end{array}\right.$$

↳ Balanced source.

→ abc or positive sequence

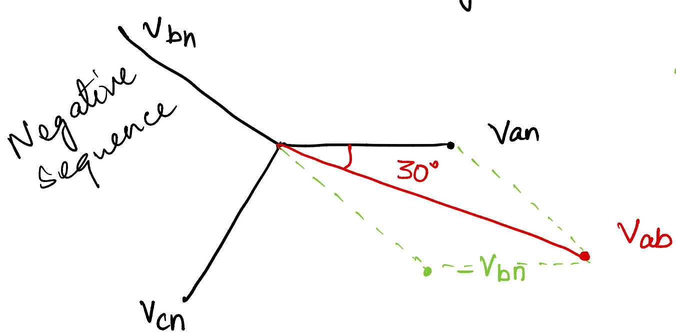


Balanced source

cba or negative sequence

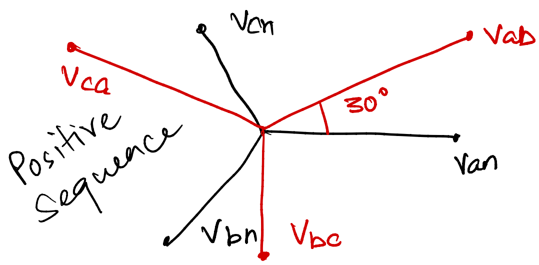
Phase voltages: V_{an}, V_{bn}, V_{cn}

Line-to-line or line voltages: V_{ab}, V_{bc}, V_{ca}



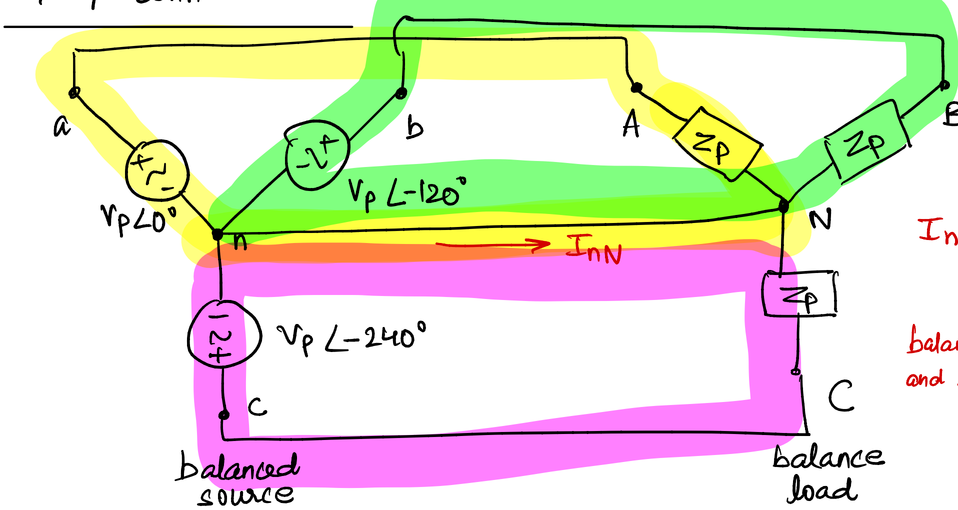
$$V_{ab} = V_{an} - V_{bn} = V_{an} + (-V_{bn})$$

$$\boxed{V_L = \sqrt{3}\,V_p}$$

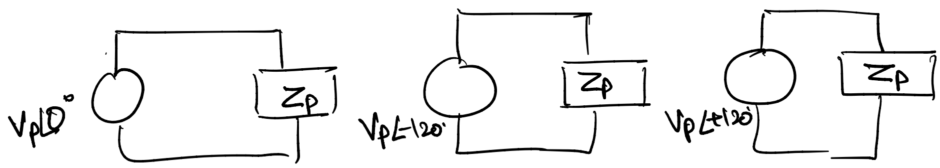


$$V_{ab} = V_{an} + V_{nb}$$
$$= V_{an} - V_{bn}$$
$$= \sqrt{3}\,V_p\angle 30^\circ$$
$$V_{bc} = \sqrt{3}\,V_p\angle -90^\circ$$
$$V_{ca} = \underbrace{\sqrt{3}\,V_p}_{V_L}\angle -210^\circ$$

Y-Y connection



$I_{nN} = 0$
for
balanced source
and balanced load



$\boxed{\text{" per-phase " analyses}}$

$$I_{aA} = \frac{V_{an}}{Z_p}$$

$$I_{bB} = \frac{V_{bn}}{Z_p} = \frac{V_{an}\angle -120°}{Z_p} = I_{aA}\angle -120°$$

$$I_{cC} = \frac{V_{cn}}{Z_p} = \frac{V_{an}\angle +120°}{Z_p} = I_{aA}\angle +120°$$

$$I_{nN} = I_{aA} + I_{bB} + I_{cC} = 0$$

$V_{an} = V_p\angle 0°$

$V_{bn} = V_p\angle -120°$

$= V_{an}\angle -120°$

$= V_{an}\,(1\angle -120°)$

Example : Consider three-phase balanced Y-Y connected system.

$V_{an} = 200\angle 0°$ V rms

$Z_p = 100\angle 60°\ \Omega$

Find phase and line voltages.

Find phase and line currents.

Find total average power delivered to load.

Phase Voltages:

$V_{an} =$

$V_{bn} = 200 \angle -120^\circ$ Vrms

$V_{cn} = 200 \angle +120^\circ$ Vrms

Line voltages: $V_{ab} = \sqrt{3} V_p \angle 30^\circ = 200\sqrt{3} \angle 30^\circ$ Vrms

$V_{bc} = 200\sqrt{3} \angle -90^\circ$ Vrms

$V_{ca} = 200\sqrt{3} \angle -210^\circ$ Vrms

Line currents:

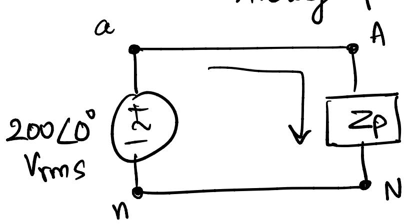
$$I_{aA} = \frac{V_{an}}{Z_p} = \frac{200\angle 0^\circ}{100\angle 60^\circ} = 2\angle -60^\circ \text{ A rms}$$

$I_{bB} = 2 \angle -180^\circ$ A rms

$I_{cC} = 2 \angle -300^\circ$ A rms

Total Average power:

Average power in phase A



$$P_{avg,A} = \frac{1}{2}\,\text{Re}\{V I^*\} = \text{Re}\{\underline{V_{eff} I_{eff}^*}\}$$

$$= \text{Re}\{V_{an} I_{AN}\}$$

$$= \text{Re}\{200\angle 0^\circ \times 2\angle -60^\circ\} \text{ W}$$

$$= \text{Re}\{400 \angle -60^\circ\}$$

$$= 400 \cos(-60^\circ)$$

$$= 200 \text{ W}$$

$P_{avg,B} = 200$ W

$P_{avg,C} = 200$ W

P_{avg} total $= 600$ W.