Linear Circuit

x (input) $\longrightarrow$ [system] $\longrightarrow$ y (Output)

system is linear if kx (input) $\longrightarrow$ [ ] $\longrightarrow$ ky (Output), k constant

Resistor "R" : $v = iR$

Inductor "L" $v = L\frac{di}{dt}$

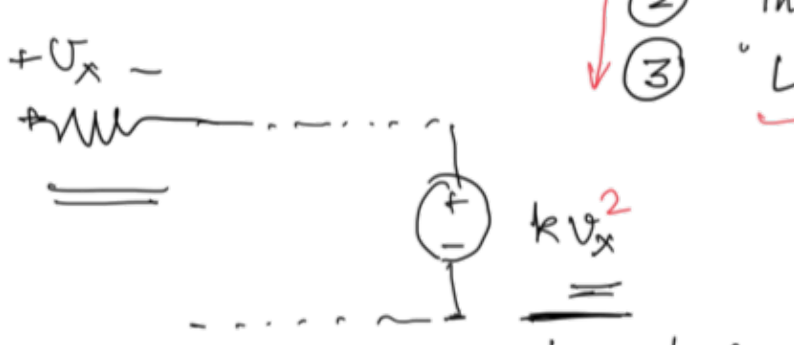
Capacitor "C" $i = C\, dv/dt$

$v_1 \;,\; i_1$
$kv_1 \;,\; ki_1$

$v = A\left(\frac{di}{dt}\right)^2$

Linear Electric Circuits

consists of
1. R, L, C linear elements
2. Independent voltage/current sources
3. "Linear" dependent sources.



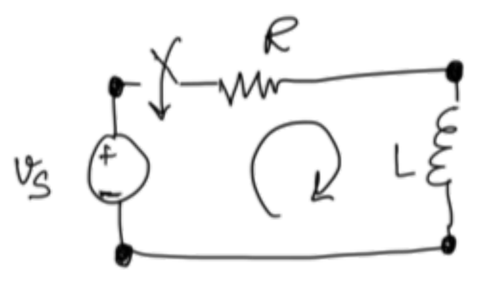
$V_s = k\,V_x$

Response of a linear circuit

→ full response = transient + steady-state

transient:
- due to sudden change
- natural response
- R, L, C

steady-state:
- due to source
- forced response
- V_s, i_s



$$-V_s + iR + L\frac{di}{dt} = 0$$

$$i = i_{transient} + i_{steady\text{-}state}$$

$t \to \infty$ $\quad i = i_{steady\text{-}state}$

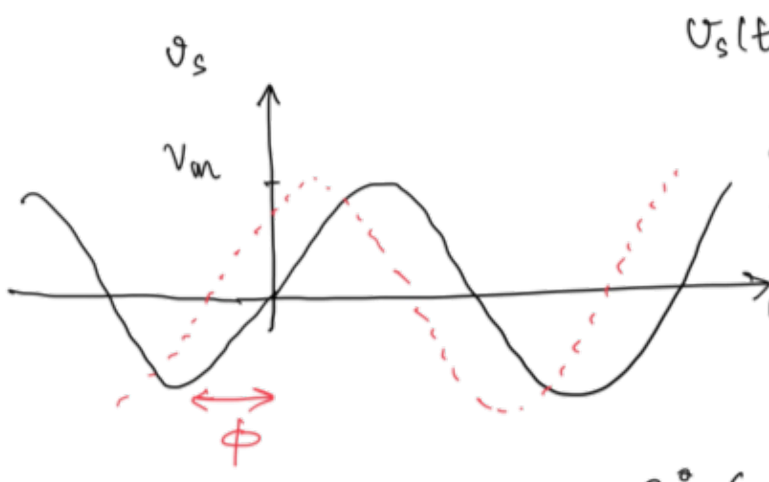
Sinusoids and complex number

Sinusoidally varying voltage source

$V_s(t) = V_m \cos(\omega t)$

$V_s(t) = V_m \sin(\omega t)$

$V_s(t) = V_m \sin(\omega t + \phi)$



$V_m \sin(\omega t)$
$V_m \sin(\omega t+\phi)$ is leading $V_m \sin(\omega t)$

$V_m \sin(\omega t)$ is lagging $V_m \sin(\omega t+\phi)$ by angle ϕ.

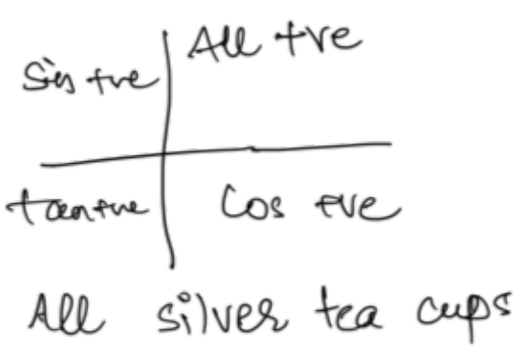
$\sin(\underset{\text{radian}}{\omega t} + \underset{\text{degrees}}{45^\circ})$ $\quad \times \frac{\pi}{180^\circ}$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = +\cos\theta$

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$



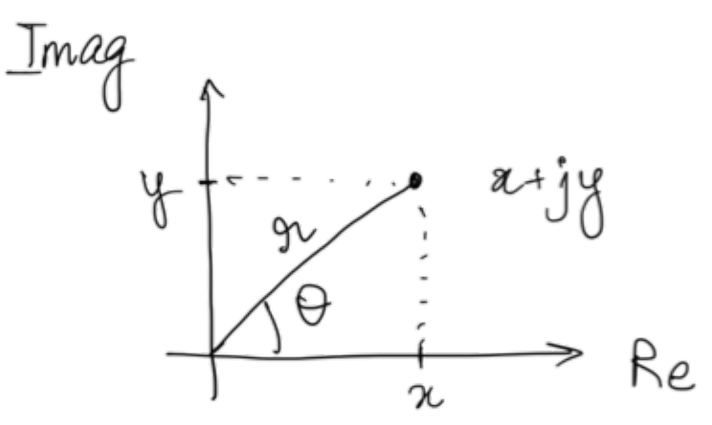
Complex Number

Real : x

Complex numbers : $\underbrace{x+iy \equiv x+jy}_{\text{rectangular form}}$; $j = \sqrt{-1}$

Euler's identity: $\underbrace{e^{j\theta}}_{\text{exp}} = \underbrace{\cos\theta + j\sin\theta}_{\text{rect.}}$

Complex No.
- rectangular: $x+jy$
- exponential: $re^{j\theta}$
- (polar form : $r\angle\theta$)



$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}(y/x)$

$$(r_1 e^{j\theta_1}) \times (r_2 e^{j\theta_2}) = (r_1 \cdot r_2)e^{j(\theta_1+\theta_2)}$$

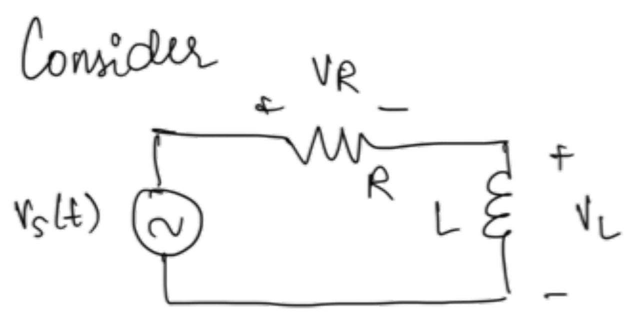
$$\frac{A+jB}{C+jD} \times \frac{C-jD}{C-jD} = \frac{(\;-\;)(\;-\;)}{C^2+D^2}$$

Module 1

Sinusoidal steady-state Analysis

assuming transient has died out long back.

Consider



$V_s(t) = V_m \cos(\omega t)$

Find $i(t)$

$$V_s(t) = iR + L\frac{di}{dt}$$

$$\Rightarrow V_m \cos(\omega t) = iR + L\frac{di}{dt}$$

General form solution: $i = I_1 \cos\omega t + I_2 \sin \omega t$

$$\Rightarrow V_m \cos(\omega t) = (\qquad\qquad)R + L(\qquad\qquad)$$

$$\Rightarrow \underbrace{(I_1R + \omega L I_2 - V_m)}_{0} \cos\omega t + \underbrace{(RI_2 - \omega L I_1)}_{0} \sin\omega t = 0$$

$$I_1 R + \omega L I_2 - V_m = 0$$
$$R I_2 - \omega L I_1 = 0$$