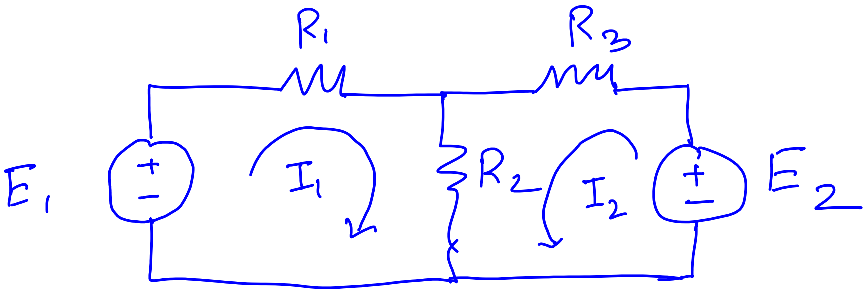
Quick Recap

Superposition Theorem $\rightarrow$



$$E_1 = I_1 R_1 + (I_1 + I_2) R_2$$

$$E_2 = I_2 R_3 + (I_1 + I_2) R_2$$

$$Ax = b$$

$$A = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

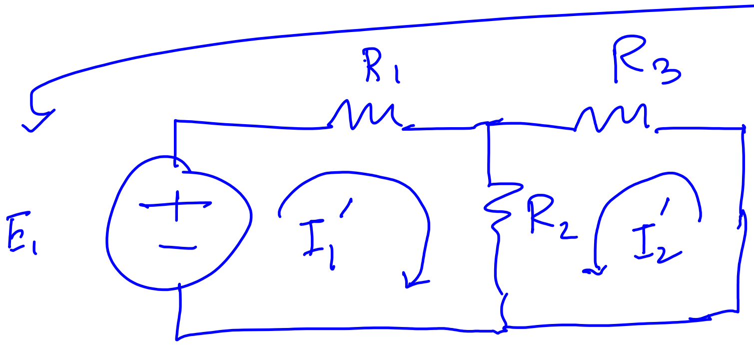
$$x = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$b = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

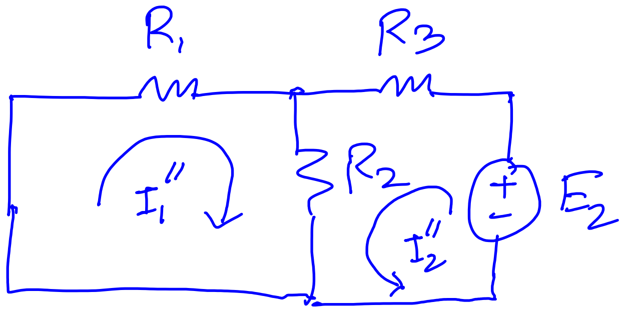
$$b = \underbrace{\begin{bmatrix} E_1 \\ 0 \end{bmatrix}}_{b_1} + \underbrace{\begin{bmatrix} 0 \\ E_2 \end{bmatrix}}_{b_2}$$

$$Ax = b = b_1 + b_2$$

$$x = \underbrace{A^{-1} b_1}_{x_1} + \underbrace{A^{-1} b_2}_{x_2}$$

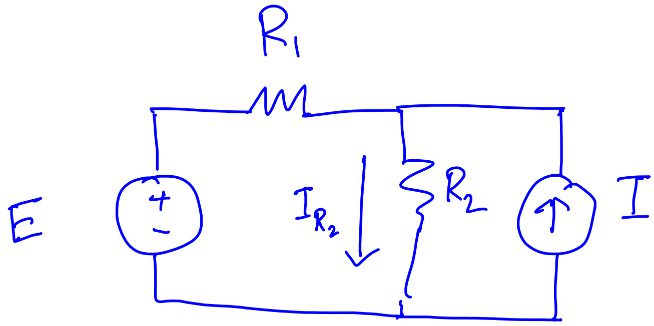


$$x_1 = \begin{bmatrix} I_1' \\ I_2' \end{bmatrix}$$

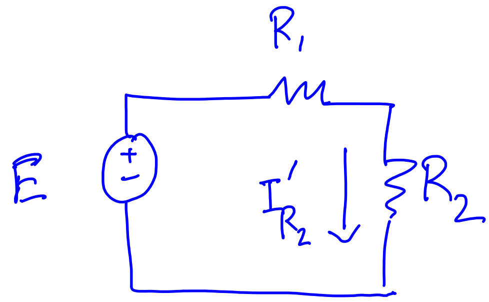


$$x_2 = \begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix}$$

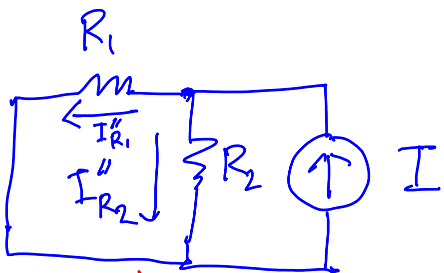
$$\left.\begin{aligned} I_1 &= I_1' + I_1'' \\ I_2 &= I_2' + I_2'' \end{aligned}\right\}$$



$$I_{R_2} = ?$$



$$I'_{R_2} = \frac{E}{R_1 + R_2}$$



$$I = I''_{R_1} + I''_{R_2}$$

$$I''_{R_1} \cdot R_1 = I''_{R_2} \cdot R_2$$

$$I''_{R_2} + \frac{R_2}{R_1} I''_{R_2} = I$$

$$\Rightarrow \quad I''_{R_2} = \frac{R_1 I}{R_1 + R_2}$$

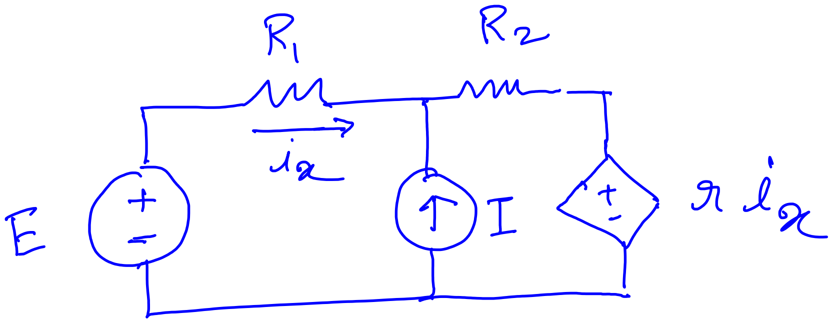
Total current

Current due to E

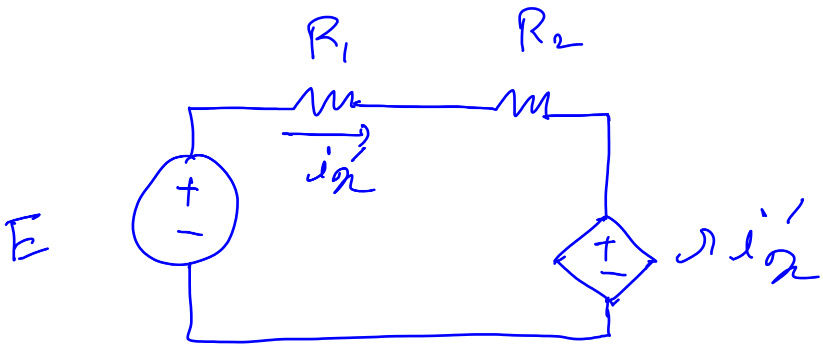
Current due to I

$$I_{R_2} = I'_{R_2} + I''_{R_2}$$

$$= \frac{E}{R_1 + R_2} + \frac{R_1 I}{R_1 + R_2}$$



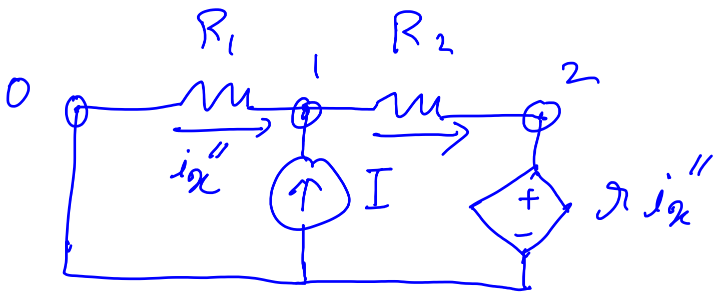
$r\, i_x = ?$



$$i_x' = \frac{E - r\, i_x'}{R_1 + R_2}$$

$\Downarrow$

solve for i_x'



KCL at node 1

$$i_x'' + I = \frac{v_1 - v_2}{R_2}$$

$$i_x'' = -\frac{v_1}{R_1}$$

$$v_2 = r\, i_x''$$

solve for i_x''

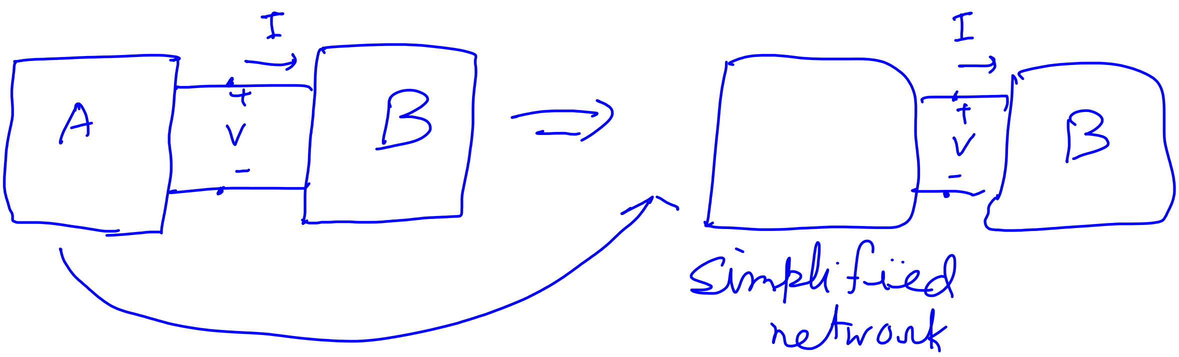
Total current

Current due to E

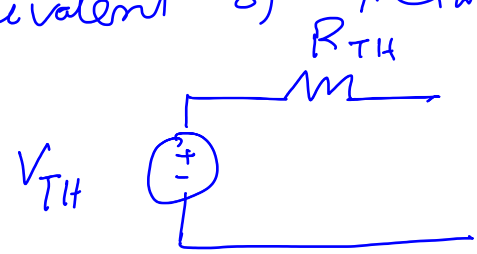
current due to I

$$i_x = i_x' + i_x''$$

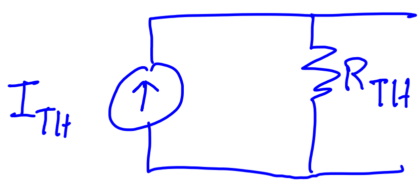
Thevnin's Theorem / Norton's Theorem



Thevnin Equivalent of Network A



Norton Equivalent of Network A



$$V_{TH} = I_{TH} \cdot R_{TH}$$

Properties of Network A

1) It should be a linear network

2) Voltage sources or current sources can be present.

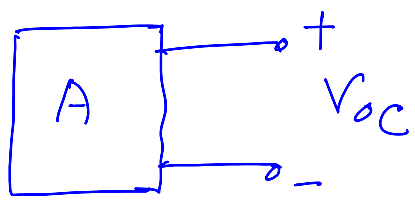
3) There should be no controlled-source coupling with network B.

Properties of Network B

1) It can be linear or non-linear

2) There should not be any coupling via controlled sources with network A.

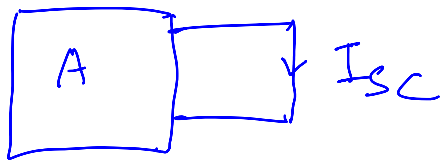
How to Compute V_{TH}



Remove network B and compute V_{oc}

$$V_{TH} = V_{oc}$$

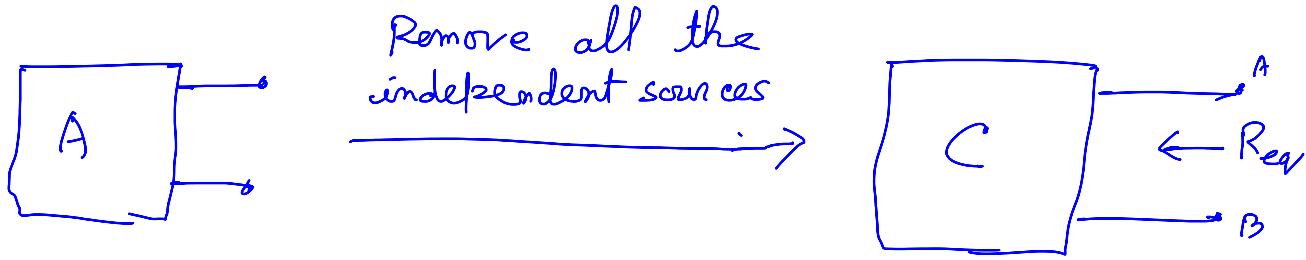
How to Compute I_{TH}



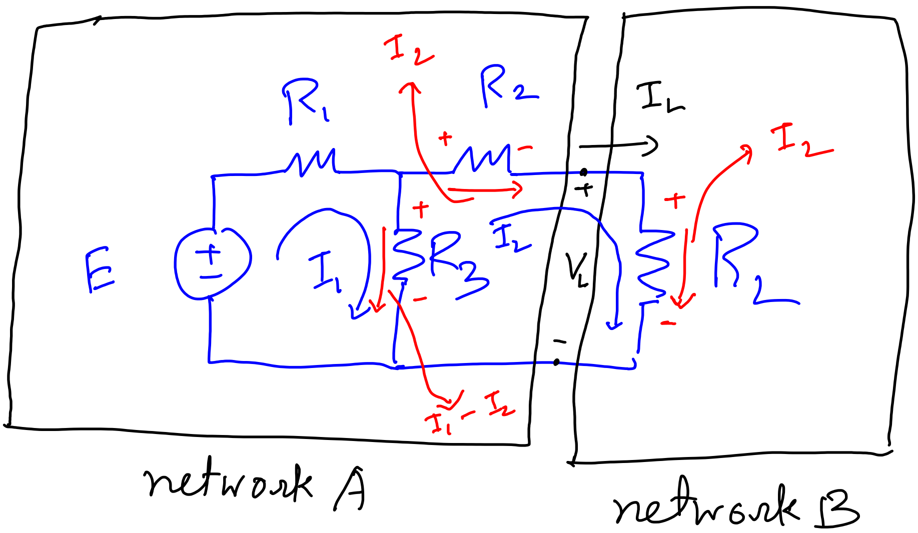
Remove network B, short-circuit the terminal and compute I_{SC}

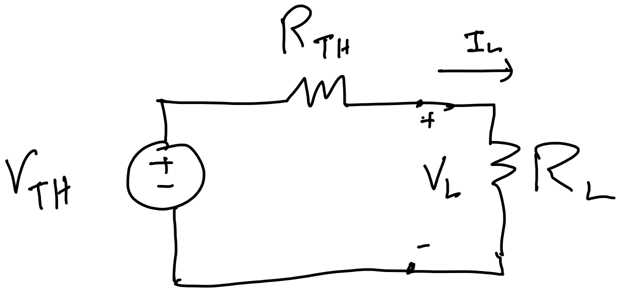
$$I_{TH} = I_{SC}$$

How to compute R_{TH}



$$R_{TH} = R_{eq}$$





$$E = I_1 R_1 + (I_1 - I_2) R_3$$

$$(I_1 - I_2) R_3 = I_2 (R_2 + R_L)$$

$$I_L = I_2$$

$$I_1 R_3 = I_2 (R_3 + R_2 + R_L)$$

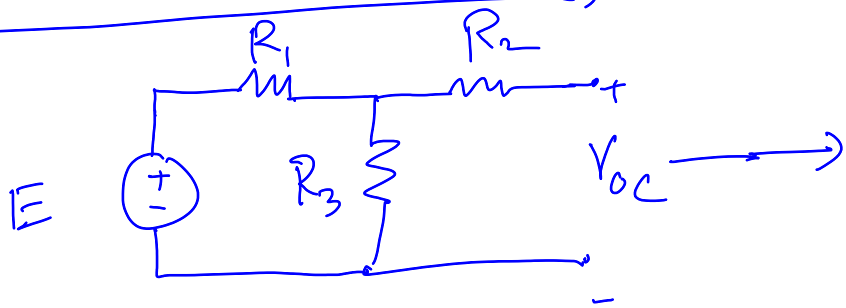
$$E = I_1 (R_1 + R_3) - I_2 R_3$$

$$= \frac{R_1 + R_3}{R_3} \cdot (R_3 + R_2 + R_L) \cdot I_2 - I_2 R_3$$

$$I_2\left(\frac{(R_1+R_3)(R_3+R_2+R_L)}{R_3} - R_3\right) = E$$

$$I_L = I_2 = \frac{E.\ R_3}{R_1R_3+R_1R_2+R_1R_L+R_3R_2+R_3R_L}$$

Thevnin Equivalent circuit,



network A

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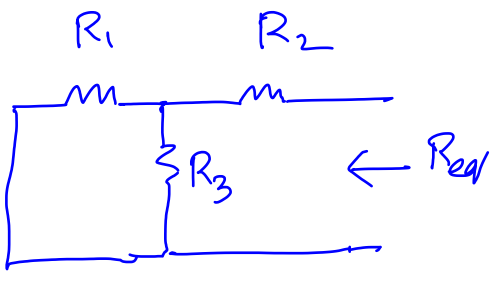

network C

$$R_{TH} = R_2 + \frac{R_1R_3}{R_1+R_3}$$

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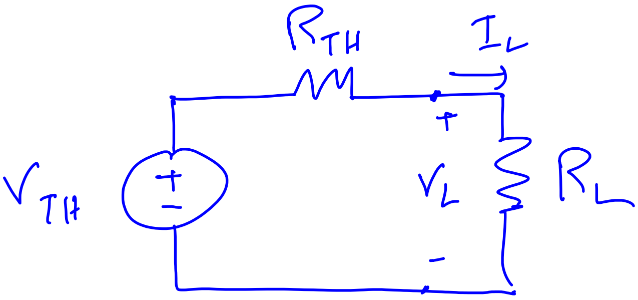
$$V_{TH} = V_{oc} = \frac{R_3 E}{R_1+R_3} \checkmark$$



$$I_L = \frac{V_{TH}}{R_{TH}+R_L}$$

$$= \frac{E R_3 / R_1+R_3}{R_2 + \frac{R_1R_3}{R_1+R_3} + R_L}$$

$$= \frac{E R_3}{R_1R_2+R_2R_3+R_1R_3+R_1R_L+R_3R_L}$$