Signals and Systems (Revision)

* Signal energy and Power

CTS : $$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

DTS : $$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

eg: $x[n] = 4$

$$E = \sum_{n=-\infty}^{\infty} |4|^2 = \sum_{n=-\infty}^{\infty} 16 = \infty \text{ J}$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |4|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \times (N-(-N)+1) \times 16$$

$$= \lim_{N \to \infty} \frac{2N+1}{2N+1} \times 16 = \underline{16 \text{ W}}$$

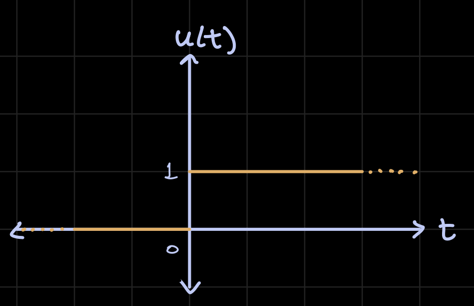
if $E < \infty$: then energy signal
else if $P < \infty$: then power signal
else : neither

* Fundamental Signals

① Unit Step function $u(t)$ OR $u[n]$

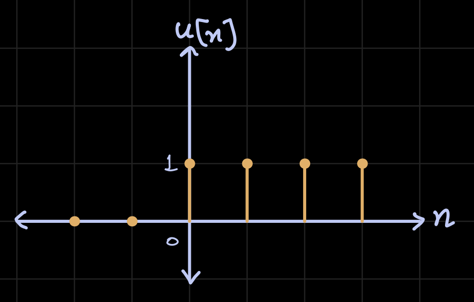
CTS:

$$u(t) = \begin{cases} 1 & : t \geq 0 \\ 0 & : t < 0 \end{cases}$$



DTS:

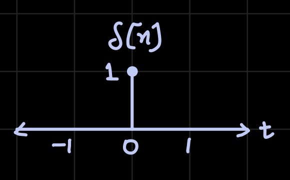
$$u[n] = \begin{cases} 1 & : n \geq 0 \\ 0 & : n < 0 \end{cases}$$



② Unit Impulse Function $\delta(t)$ OR $\delta[n]$

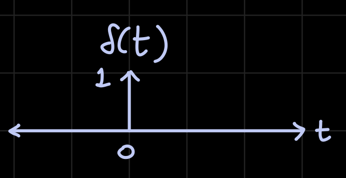
DTS: also called unit sampling sequence

$$\delta[n] = \begin{cases} 1 : n = 0 \\ 0 : n \neq 0 \end{cases}$$



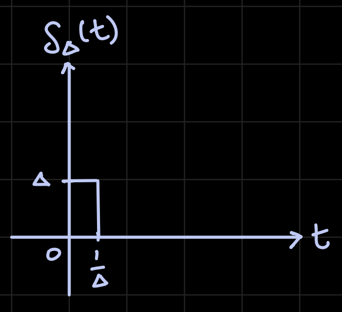
CTS: also called Dirac Delta function and singularity function as well

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \qquad \delta(t) = \begin{cases} 1 & : t = 0 \\ 0 & : t \neq 0 \end{cases}$$



$$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & : 0 \leq t < \Delta \\ 0 & : \text{else} \end{cases}$$

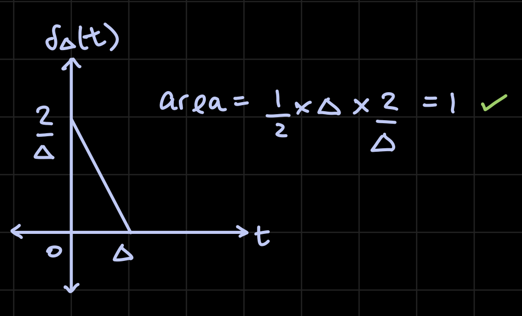
infinitesimally small

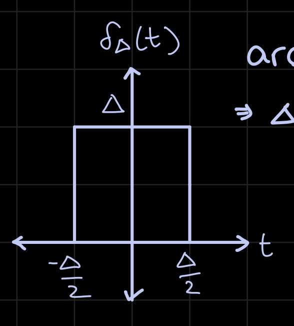


just note that the area under the curve should be equal to one, i.e. $\int_{-\infty}^{\infty} \delta(t) = 1$

and so, the graph drawn above is just a form of unit impulse function.

The following are unit impulse functions as well:





area = $\Rightarrow \Delta \times \left(\frac{\Delta}{2} + \frac{\Delta}{2} \right)$

$\Rightarrow 1$ ✓

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$

* PROPERTIES

$$u(t) = \int_{-\infty}^{\infty} \delta(t)dt$$

$$\delta(t) = \frac{d(u(t))}{dt}$$

- Discrete Time Signal
  - $\delta[n] = u[n] - u[n-1]$
  - $u[n] = \sum_{k=-\infty}^{n} \delta[k]$
  - $u[n] = \sum_{k=0}^{\infty} \delta[n-k]$

- CTS
  - even function : $\delta(t) = \delta(-t)$
  - $\int x(t)\delta(t)dt = x(0)$
  - $\int x(t)\delta(t-t_0)dt = x(t_0)$
  - $\delta(at) = \frac{1}{|a|}\delta(t)$
  - $x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$

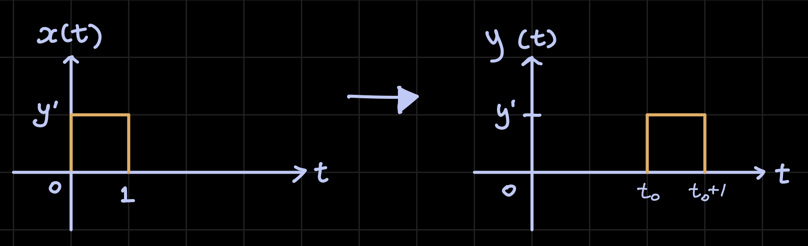
* OPERATIONS ON SIGNAL

① TRANSLATION
- Delay
- Advance

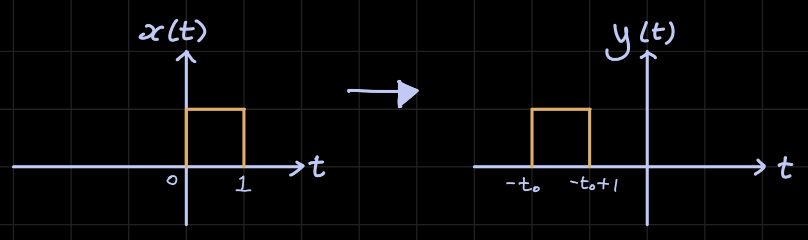
② SCALING
- Compression
- Dilation
- Reversal

⇒ TRANSLATION $y(t) = x(t \pm t_0)$

- Delay $y(t) = x(t - t_0)$

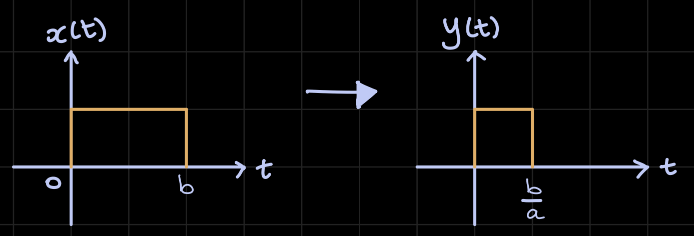


- Advance $y(t) = x(t + t_0)$

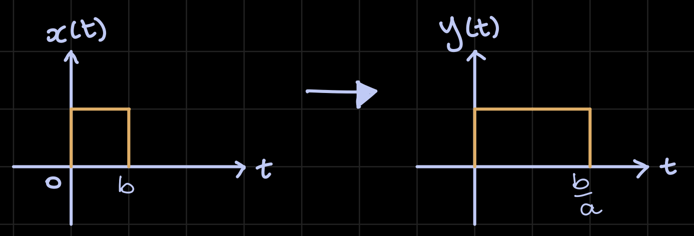


⇒ SCALING $y(t) = x(at)$

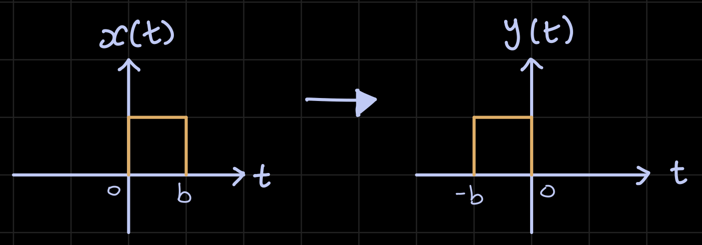
- Compression $a > 1$



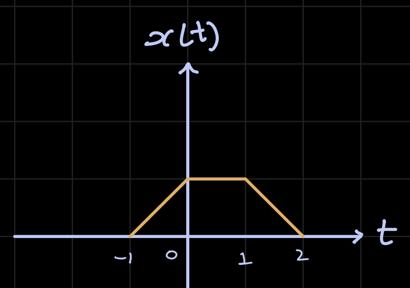
- Dilation $0 < a < 1$



- Reversal $a = -1$



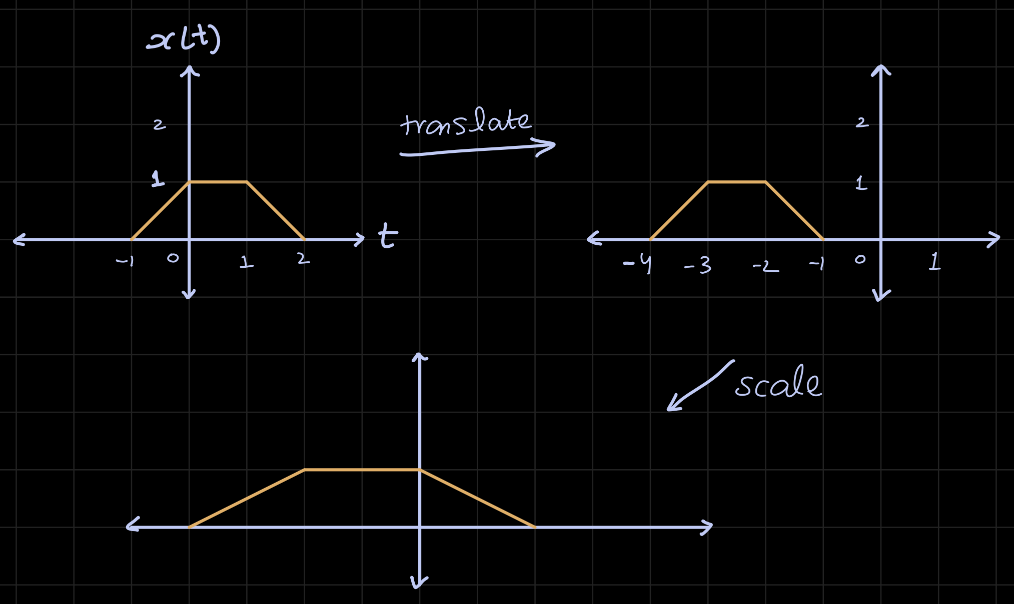
eg} if $y(t) = x(2t+3)$ and $x(t)$ is given :



Ans} initial thought I got :

$\underset{2t}{\text{scale}} \rightarrow \underset{2(t+3) = 2t+6}{\text{translate}}$ ✗

$\underset{t+3}{\text{translate}} \rightarrow \underset{2t+3}{\text{scale}}$ ✓



* SNS Assignment

1) $P_{avg} = \lim_{T \to 0} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$\Rightarrow \lim_{T \to 0} \frac{1}{2T} \int$