tr5frbj 12/08/24

Circuit Theory and Devices

tut as well

Lab: LT Spice (2nd week onwards)

book: Engineering circuit Analysis (9th Edition)

Relative grading

Labs	20%
Quiz (every 2 weeks (n-1))	20%
Mid sem	30%
End sem	30%

scientific calc ⇐ Mid sem, End sem

Course: 9 modules chapter 10 onwards
{continuation of BE}

* Lecture : 1

⇒ Linear Circuits

$x \rightarrow \boxed{\text{system}} \rightarrow y$

input   output

system is linear if

$Kx \rightarrow \boxed{\text{sys}} \rightarrow Ky$

where k is constant

"R" Resistor : linear element ✓
$V = iR$

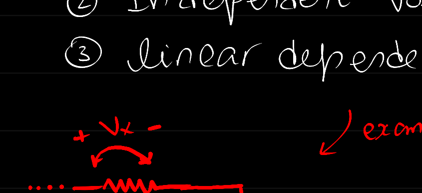
"L" Inductor : linear element ✓
$V = L\frac{di}{dt}$

"C" Capacitor : linear element ✓
$i = C\frac{dv}{dt}$

* Linear Electric circuits :

consists of ⇒

1. R, L, C → linear elements
2. Independent voltage & current sources
3. linear dependent sources



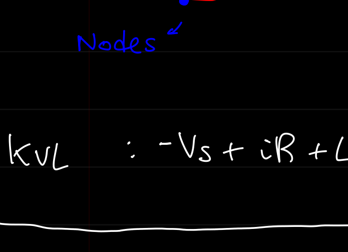
example
would not have been linear if $V_s = KV_x^2$

NOTE: diode and transistors are non-linear elements

* Response of a linear circuit

① full response = transient response + Steady State

transient response	Steady State
temporary	persists $t \to \infty$
due to sudden change	due to source
also called natural response	also called forced response
depends on linear elements R, L, C	depends on current/voltage source



KVL : $-V_s + iR + L\frac{di}{dt} = 0$

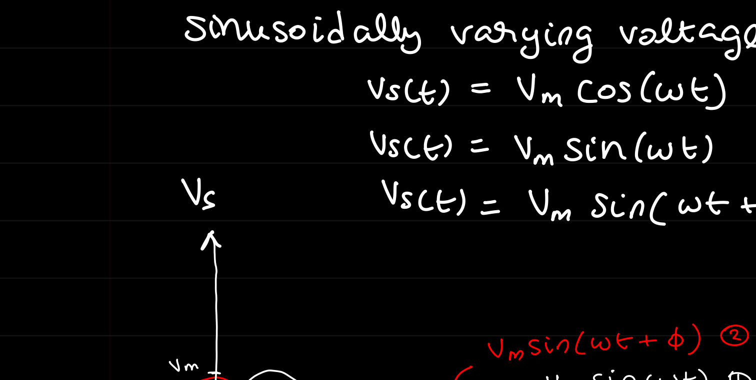
$i_{final} = i_{transient} + i_{steady\ state}$

as $t \to \infty$, $i_{final} = i_{steady\ state}$

* Sinusoids & complex numbers

sinusoidally varying voltage source

$V_s(t) = V_m\cos(\omega t)$
$V_s(t) = V_m\sin(\omega t)$
$V_s(t) = V_m\sin(\omega t + \phi)$



② is leading ①
① is lagging ②
} by angle φ

* TRIGO

$\sin(-\theta) = -\sin(\theta)$
$\cos(-\theta) = \cos(\theta)$
$\sin(A+B) = \sin A\cos B + \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$

sin +ve	All +ve
tan +ve	cos +ve

* Complex Numbers

Real : x
complex : $x + iy$

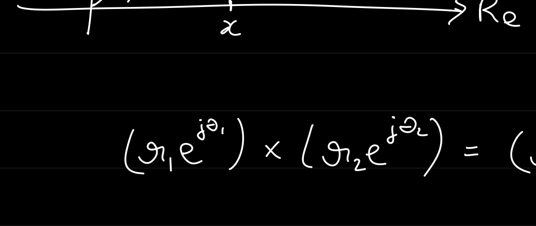
rectangular form
real part, imaginary part
$i = \sqrt{-1}$ a.k.a. j

⇒ Euler's identity : $\underbrace{e^{j\theta}}_{\text{exponential form}} = \underbrace{\cos\theta + j\sin\theta}_{\text{rectangular form}}$

Complex Number

rectangular: $x + iy$

exponential: $re^{j\theta}$ (≡ polar form : $r\angle\theta$)



$r = \sqrt{x^2 + y^2}$
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$(r_1 e^{j\theta_1}) \times (r_2 e^{j\theta_2}) = (r_1 \cdot r_2)\, e^{j(\theta_1+\theta_2)}$

AC	DC
• we can use transformers to vary voltage	• appliances need constant current
• transported easily over large distances	

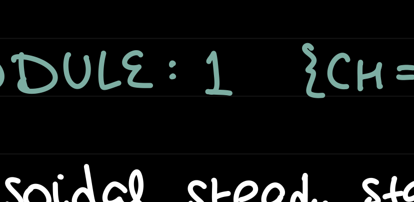
AC → DC : using rectifiers

* MODULE : 1 {CH = 10} 10:30 am

⇒ Sinusoidal steady state analysis

assuming transient has died out long back

Consider,



$V_s(t) = V_m\cos(\omega t)$
find $i(t)$

$V_s(t) = iR + L\frac{di}{dt}$

$\Rightarrow V_m\cos(\omega t) = iR + L\frac{di}{dt}$

both will take cos(wt) form

General form Solution : $i = I_1\cos\omega t + I_2\sin\omega t$

$\Rightarrow V_m\cos(\omega t) = (I_1\cos\omega t + I_2\sin\omega t)R + L\frac{di}{dt}$

$\frac{di}{dt} = \frac{d}{dt}(I_1\cos\omega t + I_2\sin\omega t)$
$\Rightarrow \omega(I_1(-\sin\omega t) + I_2\cos\omega t)$
$\Rightarrow \omega(I_2\cos\omega t - I_1\sin\omega t)$

$V_m\cos(\omega t) = (I_1\cos\omega t + I_2\sin\omega t)R + (I_2\cos\omega t - I_1\sin\omega t)L\omega$

$\Rightarrow \cos\omega t\cdot(I_1R + I_2L) + \sin\omega t\cdot(I_2R - I_1L)$

$\Rightarrow (I_1R + I_2L - V_m)\cos(\omega t) + (I_2R - I_1L)\sin(\omega t) = 0$

* at $t = 0$,
$(I_1R + I_2L - V_m)\cdot 1 + (I_2R - I_1L)\cdot 0 = 0$

$\boxed{I_1R + I_2L = V_m}$

* at $t = \frac{\pi}{2\omega}$, $(I_1R + I_2L - V_m)\cdot 0 + (I_2R - I_1L)\cdot 1 = 0$

$I_1 = \frac{V_mLR}{R^2+L}$

$\boxed{I_1 = I_2\cdot\frac{R}{L}}$

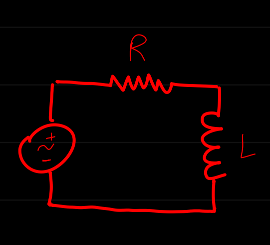
$I_1R + I_2L = V_m \Rightarrow I_2\frac{R^2}{L} + I_2L = V_m$
$\Rightarrow I_2\left(\frac{R^2+L}{L^2}\right) = V_m \Rightarrow I_2 = \frac{V_mL^2}{R^2+L}$

14/08/24 # Lecture : 2 9:30 Am

Wednesday → Quiz : Chp 1, 2, 3

* Sinusoidal Steady state analysis

AC sources ↙ ↳ not considering any transient response



angular freq

$V_s = V_m \cos(\omega t)$

$freq\ (hz) = \frac{\omega}{2\pi}$

$V_s(t) = i(t) R + L \frac{di(t)}{dt}$

general soln: $i = I_1 \cos(\omega t) + I_2 \sin(\omega t)$

put i from gen soln in above eqn

$I_1 = \frac{R V_m}{R^2 + \omega^2 L^2}$, $I_2 = \frac{\omega L V_m}{R^2 + \omega^2 L^2}$

$i(t) = \frac{R V_m}{R^2 + \omega^2 L^2} \cos(\omega t) + \frac{\omega L V_m}{R^2 + \omega^2 L^2} \sin(\omega t)$

Note: use $\cos(A+B) = \cos A \cos B - \sin A \sin B$

⇒ So, $i(t) = A\cos(\omega t + \phi)$: general soln

becomes ⇒ $i(t) = A[\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)]$

$\frac{R V_m}{R^2 + \omega^2 L^2} \cos(\omega t) + \frac{\omega L V_m}{R^2 + \omega^2 L^2} \sin(\omega t) = A\cos(\omega t)\cos(\phi) - A\sin(\omega t)\sin(\phi)$

⇓

$A\cos(\phi) = \frac{R V_m}{R^2 + \omega^2 L^2}$, $A\sin(\phi) = \frac{-\omega L V_m}{R^2 + \omega^2 L^2}$

$A^2\cos^2(\phi) + A^2(\sin^2\phi) = A^2(1) = A^2 = \frac{R^2 V_m^2 - \omega^2 L^2 V_m^2}{R^2 + \omega^2 L^2}$

⇓

$\frac{V_m^2 (R^2 - \omega^2 L^2)}{R^2 + \omega^2 L^2}$

$A = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$

$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$

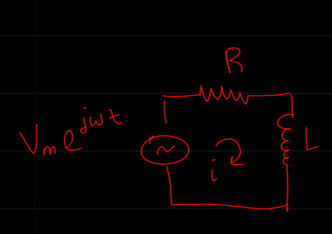
$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

⇒ Consider a complex source

$V_m \cos(\omega t) + j V_m \sin(\omega t) = V_m e^{j\omega t}$

Real Voltage source $= V_m \cos(\omega t) = Re\{V_m e^{j\omega t}\}$

Note: Euler's formula → $e^{j\theta} = \cos\theta + j\sin\theta$



$V_m e^{j\omega t} = iR + L\frac{di}{dt}$

dont integrate ✗
general soln ✓

Assume $i = I_m e^{j(\omega t + \phi)}$
put eqn

$V_m e^{j\omega t} = I_m e^{j\omega t + j\phi} R + L(j\omega) I_m e^{j\omega t + j\phi}$

⇒ $I_m e^{j\omega t + j\phi} = \frac{V_m e^{j\omega t}}{R + j\omega L}$

is this the real current? No
current cannot be complex

Real $i(t) = Re\{I_m e^{j\omega t + j\phi}\} = Re\left\{\frac{V_m}{R + j\omega L} e^{j\omega t}\right\}$

find the real part of this complex number by simplifying
{multiply by the conjugate ≡ factorize}

$i(t) = \frac{V_m}{R + j\omega L} \cdot \left(\frac{R - j\omega L}{R - j\omega L}\right) e^{j\omega t} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} e^{-j\tan^{-1}\left(\frac{\omega L}{R}\right)} \cdot e^{j\omega t}$

⇒ Note: rect form → polar form

$x + jy \rightleftarrows r e^{j\theta}$

$x = r\cos\theta$, $y = r\sin\theta$

$r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Real $i(t) = Re\left\{\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cdot e^{j\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)}\right\}$

* $i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

Same answer as when we were not considering complex numbers

* PHASOR

$i(t) = I_m \cos(\omega t + \phi) = Re\{I_m e^{j\omega t + j\phi}\}$

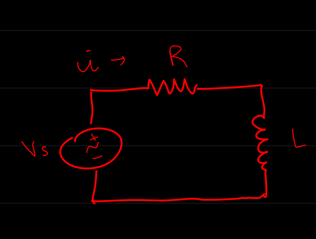
⇓ remove $e^{j\omega t}$, $Re\{\dots\}$

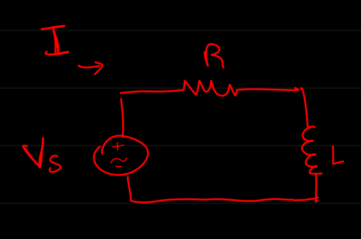
not changing with time

complex func ✗
complex number ✓

$\underbrace{I_m e^{j\phi}}_{\text{phasor}} = I_m \angle \phi$

time domain	phasor representation
$i(t) = I_m \cos(\omega t + \phi)$	$I_m \angle \phi$
$v(t) = V_m \cos(\omega t + \theta)$	$V_m \angle \theta$
⇒ real function of t	⇒ complex, constant
⇒ notations: small letters	⇒ Bold, capital letters





$i(t) = I_m \cos(\omega t + \phi)$ ✓
$i(t) = Re\{I_m e^{j\omega t + j\phi}\}$ ✓
$i(t) = I_m \angle \phi$ ✗ complex constant
$i(t) = I_m e^{j(\omega t + \phi)}$ ✗ complex
$i(t) = I_m \angle \omega t + \phi$ ✗ complex phasor notation

phasor notation

but we can write ⇒ phasor notation of $i(t) = I_m \angle \phi = I_m \angle(\omega t + \phi)$

Note: $\angle\phi = e^{j\phi}$ or $\angle \omega t + \phi = e^{j\omega t + j\phi}$

$\underbrace{V_m e^{j\theta}}_{\mathbf{V}} = (R + j\omega L) \underbrace{I_m e^{j\phi}}_{\mathbf{I}}$

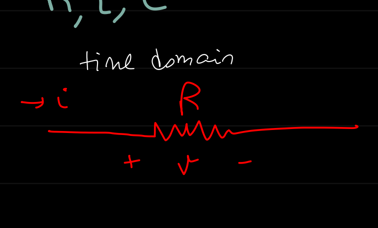
$\boxed{\mathbf{V} = (R + j\omega L)\mathbf{I}}$

↳ phasor notation

* phasor representation of relationship between i and v for R, L, C

① RESISTOR

time domain



$v = iR$

consider complex voltage & current

$v = V_m e^{j(\omega t + \theta)}$
$i = I_m e^{j(\omega t + \phi)}$

$V_m e^{j\omega t + j\theta} = R \cdot I_m e^{j\omega t + j\phi}$

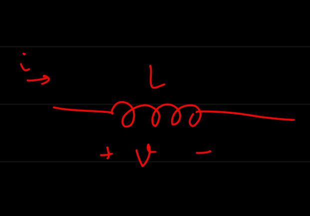
$V_m \cancel{e^{j\omega t}} \cdot e^{j\theta} = R \cdot I_m \cancel{e^{j\omega t}} e^{j\phi}$

$V_m e^{j\theta} = R \cdot I_m e^{j\phi}$

$\underbrace{V_m \angle \theta}_{\mathbf{V}} = \underbrace{I_m \angle \phi}_{\mathbf{I}} \cdot R$

$\boxed{\mathbf{V} = \mathbf{I}R}$

② INDUCTOR



$v = L\frac{di}{dt}$

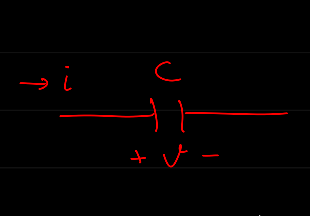
$v \to V_m e^{j(\omega t + \theta)}$ $i \to I_m e^{j\omega t + j\phi}$

$V_m e^{j\omega t} e^{j\theta} = L\, I_m e^{j\omega t} \cdot e^{j\phi} (j\omega)$

$\underbrace{V_m e^{j\theta}}_{\mathbf{V}} = j\omega L \cdot \underbrace{I_m e^{j\phi}}_{\mathbf{I}}$

$\boxed{\mathbf{V} = j\omega L \cdot \mathbf{I}}$

③ CAPACITOR



$i = C\frac{dv}{dt}$

$i \to I_m e^{j(\omega t + \phi)}$
$v \to V_m e^{j(\omega t + \theta)}$

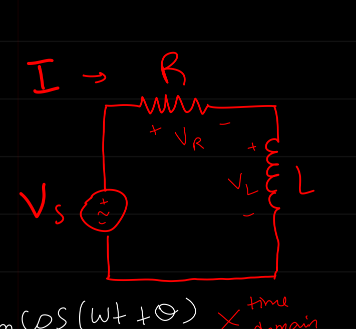
$I_m \cancel{e^{j\omega t}} \cdot e^{j\phi} = C \cdot V_m \cancel{e^{j\omega t}} e^{j\theta} \cdot j\omega$

$\underbrace{I_m e^{j\phi}}_{\mathbf{I}} = C \cdot j\omega \cdot \underbrace{V_m e^{j\theta}}_{\mathbf{V}}$

$\mathbf{I} = j\omega C \cdot \mathbf{V}$

$\boxed{\mathbf{V} = \frac{1}{j\omega C} \cdot \mathbf{I}}$

* PHASOR Representation using KVL



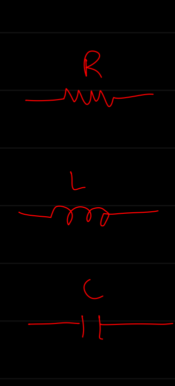
$V_s = V_m \cos(\omega t + \theta)$ ✗ time domain
$V_s = V_m e^{j(\omega t + \theta)}$ ✓ phasor notation

$\mathbf{V}_s = \mathbf{V}_R + \mathbf{V}_L$

$= \mathbf{I}R + (j\omega L)\mathbf{I}$

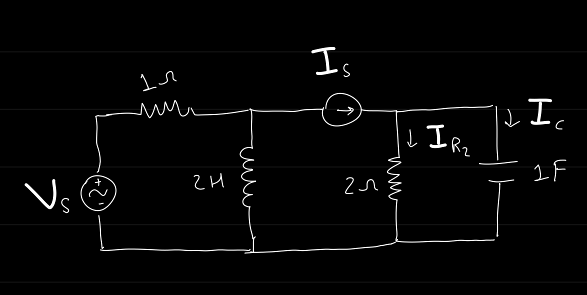
$= \mathbf{I}(R + j\omega L)$

$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L} = \frac{V_m \angle \theta}{R + j\omega L}$



Time domain	Phasor (Frequency domain)
$v = iR$	$\mathbf{V} = \mathbf{I}R$
$v = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\, \mathbf{I}$
$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$

⇒ Example



① Given: both AC sources, V_s & I_s operate at $\omega = 2\ rad/s$

② Given: $I_c = 2\angle 28°\ A$

Find (i) V_c
(ii) I_{R_2}
(iii) I_s

① $V_c = \frac{I_c}{j\omega C} = \frac{1}{j(2)(1)} \cdot 2\angle 28°$

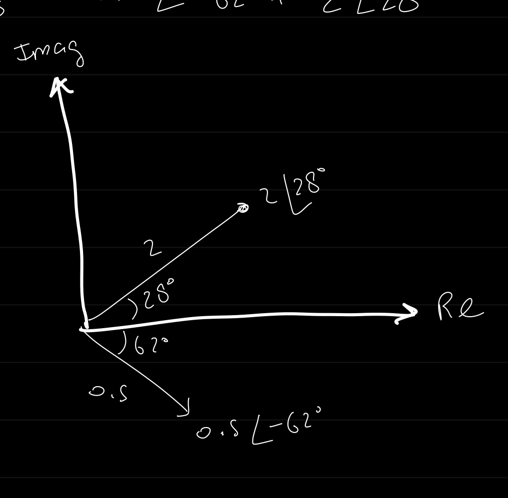
Note: $\frac{1}{j} = -j = -90°$

we add the angles directly which are being multiplied

$V_c = (0.5\angle -90°)(2\angle 28°) = 1\angle -62°\ V$

② $I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_{R_2}}{2} = \frac{1\angle -62°}{2} = 0.5\angle -62°\ A$

③ $I_s = 0.5\angle -62° + 2\angle 28°$



$2.06\angle 14°\ A$

* Lecture : 3 19/08/24 9:30AM

- Phasor : $i(t) = I_m \cos(\omega t + \phi) = Re\{I_m e^{j(\omega t + \phi)}\}$

$\downarrow$ Remove $e^{j\omega t}$ and $Re\{...\}$

$I_m e^{j\phi} = I_m \angle \phi$ } Phasor

exponential form — polar form

time domain $\rightleftarrows$ freq. domain

$V_R = IR \qquad V_L = j\omega L I \qquad V_c = \frac{1}{j\omega C} I$

* IMPEDANCE

$$Z = \frac{V}{I} = \frac{1}{R}$$

$Z_R = R$ — voltage and current in the same phase

$Z_L = j\omega L$ — voltage leads current by 90°

$Z_C = \frac{1}{j\omega C}$ — voltage lags current by 90°

$Z = x + jy$ (resistance, reactance)

* Admittance

$$Y = \frac{1}{Z}$$

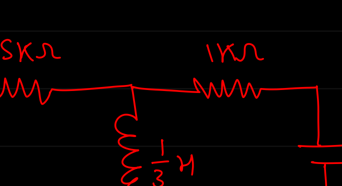
$Y = x + jy$ (conductance, susceptance)

* Impedance combination

Series : $Z_{eq} = Z_1 + Z_2 + \dots Z_N$

Parallel : $Z_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots \frac{1}{Z_N}$

eg} Find impedance of series combination of an inductor L (L=5mH) and capacitor (C=100μF) $\omega = 10^4$ rad/s

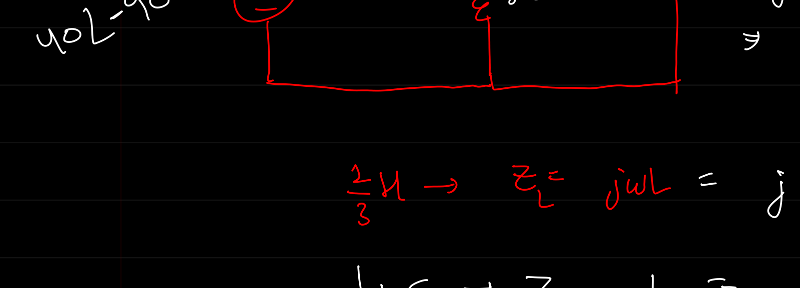


$\Rightarrow Z_{eq} = Z_L + Z_C = j\omega L + \frac{1}{j\omega C} = 10^4 \cdot j \cdot 5\times10^{-3} + \frac{10^{-4}\times10^{3}}{j \cdot 5\times 100 \times 10^{-6}}$

Note: $\frac{1}{j} = -j$ $\qquad \Rightarrow 50j - j = 49j$

Note: $e^{j\theta} = \cos\theta + j\sin\theta = e\angle\theta$ $\qquad \Rightarrow 0 + 49j = 49e^{j\cdot\frac{\pi}{2}}$

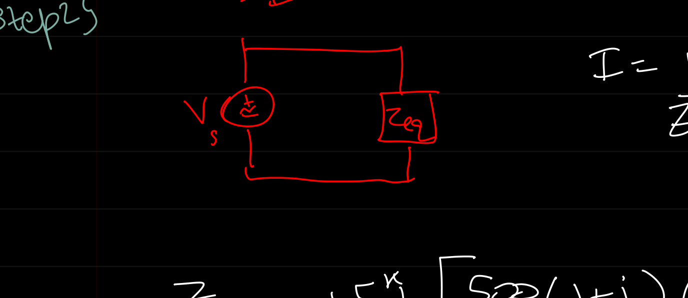
eg:



Find i(t)

$\frac{10^3(1-2j)(1+j)}{2}$ $\qquad 500(1-2j)(1+j)$ $\qquad 10^3\frac{(1-2j)}{1-j}$

step1} Convert to freq. domain (write in terms of phasor and Z)



$2000 - 2000j \parallel 2000j$

$\left(\frac{1}{1000-2000j} + \frac{1}{1000j}\right)^{-1}$

$-j \cdot 2K\Omega$ $\qquad \frac{1-j}{10^3 - 2\cdot10^3 j}$

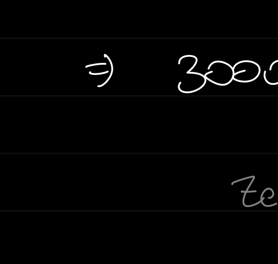
$\Rightarrow \frac{1000j + 1000 - 2000j}{(10^3 - 2\cdot10^3 j)10^3 \cdot j}$

$\frac{1}{3}H \rightarrow Z_L = j\omega L = j \cdot 3000 \cdot \frac{1}{3} \Rightarrow j \cdot 1k$

$\frac{1}{6}\mu F \rightarrow Z_c = \frac{1}{j\omega C} = \frac{1}{j \cdot 3000 \times \frac{1}{6} \times 10^{-6}} \Rightarrow \frac{2\cdot10^3}{j}$

$\Rightarrow -j \cdot 2K\Omega$

step2}



$$I = \frac{V_s}{Z_{eq}}$$

$Z_{eq} = 1.5^k + [500(1+j)(1-2j)]$

$\Rightarrow 1500 + 500[1 - 2j + j + 2)$

$\Rightarrow 500[3 + 3 - j] \Rightarrow 500[6 - j]$

$\Rightarrow 3000 - 500j$

$\frac{j(1-2j)(1+j)}{2} \Rightarrow \frac{(j+2)(1+j)}{2} \Rightarrow \frac{j + j^2 + 2 + 2j}{2} \Rightarrow \frac{3j+1}{2}$

PPT} $\qquad Z_{eq} = 1.5 + \frac{(j1)(1-j2)}{1-j}$ kΩ

$\Rightarrow \frac{4+3j}{2}$

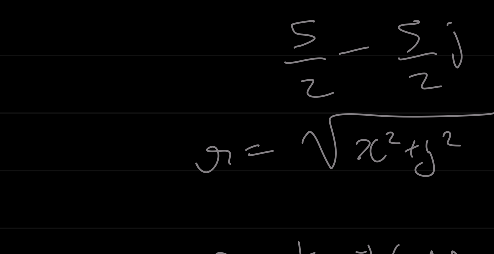
Note: complex number $<$ $x+iy$ / $r\angle\theta$

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$I_3 + I_2 = I_1$

eg3



find I_1, I_2, I_3

$Z_{eq}^{(1)} = \left(\frac{1}{5} + \frac{1}{5j}\right)^{-1} = \left(\frac{5j+5}{25j}\right)^{-1} = \frac{5j}{j+1}$

$Z_{eq} \Rightarrow^{(2)} -5j + \frac{5j}{j+1} \Rightarrow \frac{+5 - 5j + 5j}{j+1} = \frac{5}{j+1}$

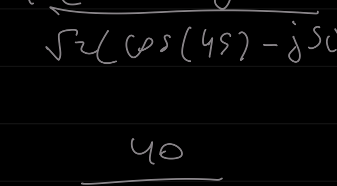
$\Rightarrow \frac{5(j-1)}{-2} \Rightarrow \frac{5-5j}{2}$

$\frac{5}{2} - \frac{5}{2}j$

$r = \sqrt{x^2+y^2} = \sqrt{\frac{25}{4} + \frac{25}{4}} = \frac{\sqrt{50}}{2}$ $\qquad \frac{5\sqrt{2}}{2}$

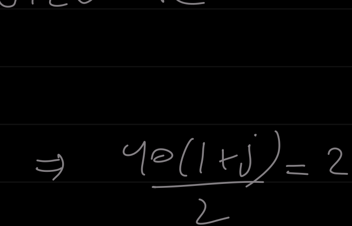
$\theta = \tan^{-1}(-1) \Rightarrow -45°$

Voltage division



$V_1 = V \cdot \frac{R_1}{R_1 + R_2}$

Current division



$i_1 = i \cdot \frac{R_2}{R_1 + R_2}$

$I_1 = \frac{V}{Z_{eq}} = \frac{\cancel{100}^{20}\angle 0°}{\frac{5\sqrt{2}}{2}\angle -45°} \Rightarrow \frac{40\angle 0°}{\sqrt{2}\angle -45°}$

$r\angle\theta = re^{j\theta} =$

$\frac{40(\cos 0 + j\sin 0)}{\sqrt{2}(\cos(45) - j\sin(45))}$

$\frac{40}{\sqrt{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)} \Rightarrow \frac{40}{1-j} \Rightarrow \frac{40(1+j)}{2} = 20(1+j)$

$20 + 20j$

$r = \sqrt{x^2+y^2} \Rightarrow \sqrt{400+400}$

$\Rightarrow 20\sqrt{1+1} = 20\sqrt{2}$

$\theta = \tan^{-1}(1) = 45°$

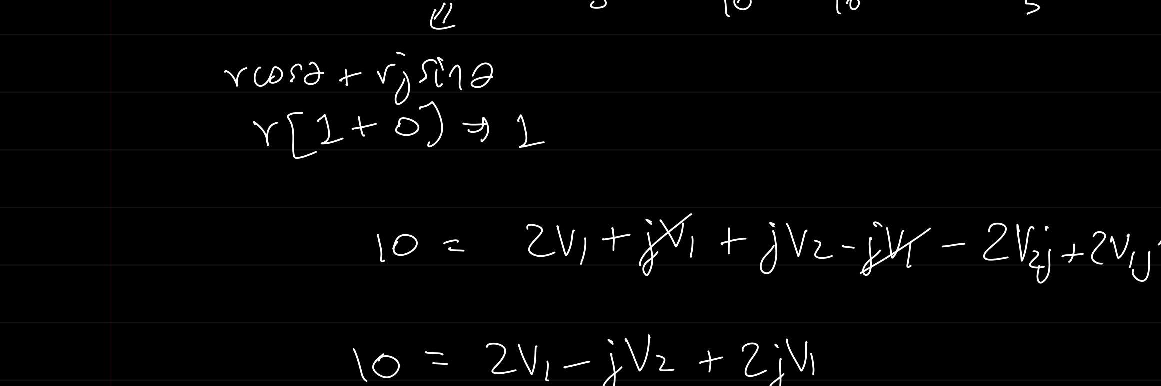
$I_1 = 20\sqrt{2}\angle 45°$

$I_2 = I_1 \times \frac{j5}{5+j5} \Rightarrow \cancel{20}^{10}\sqrt{2}\angle 45° \times \frac{j(1-j)}{\cancel{2}}$

$\Rightarrow 10\sqrt{2}\angle 45° \times (j+1)$

* NODAL ANALYSIS

Node ≡ Same voltage



at node 1:

$1\angle 0° = \frac{V_1}{5} + j\frac{V_1}{10} + \frac{(V_2 - V_1)j}{10} - \frac{(V_2 - V_1)j}{5}$

$r\cos\theta + rj\sin\theta$

$r[1 + 0) \Rightarrow 1$

$10 = 2V_1 + \cancel{jV_1} + jV_2 - \cancel{jV_1} - 2V_2 j + 2V_1 j$

$10 = 2V_1 - jV_2 + 2jV_1$

$10 = 2V_1 + j(2V_1 - V_2)$ ①

at node 2:

$\frac{V_2 - V_1}{j10} + \frac{V_2 - V_1}{-j5} + \frac{V_2}{j5} + \frac{V_2}{10} = 0.5\angle -90°$

$0.5\cancel{\cos 90°} - 0.5j\sin 90°$
$\Rightarrow -0.5j$

$V_2 - V_1 + 2V_1 - \cancel{2V_2} + \cancel{2V_2} + V_2 j = 5$

$3V_2 - V_1 + V_2 j = 5 \Rightarrow V_1 = 3V_2 - 5 + V_2 j$

Put value of V_1 in ①

$10 = 6V_2 - 10 + 2V_2 j + j(6V_2 - 10 + 2V_2 j - V_2)$

$10 = 6V_2 - 10 + 2V_2 j + 6V_2 j - 10j - 2V_2 - V_2 j$

$20 = 4V_2 + 7V_2 j - 10j$ $\qquad \frac{30-20j}{13}$

$20 + 10j = V_2(4 + 7j)$ $\qquad \frac{150 - 100j}{65}$

$V_2 = \frac{(20+10j)(4-7j)}{16+49} = \frac{80 - 140j + 40j + 70}{65}$

$V_2 = \frac{30}{13} - j\left(\frac{20}{13}\right)$ $\qquad r = \sqrt{\frac{900+400}{169}} = \frac{10}{\sqrt{13}}$

$\theta = \tan^{-1}\left(\frac{-2}{3}\right)$

$V_2 = \frac{10\sqrt{13}}{13}\angle -33.69°$

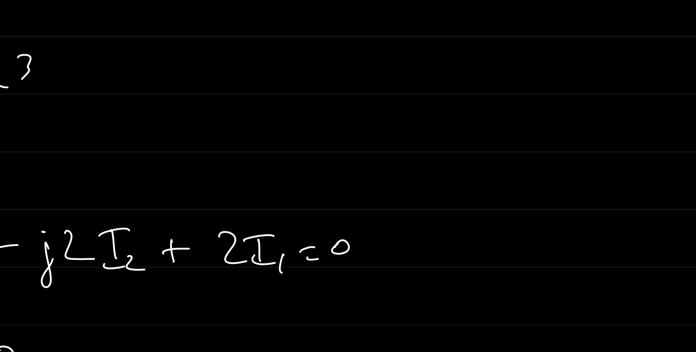
$= -\tan\left(\frac{2}{3}\right) =$
$= -33.69$

$V_1 = 3V_2 - 5 + jV_2$

$\Rightarrow V_2(3 + j) - 5 = \frac{30}{13} - \frac{20j}{13}(3+j) - 5$

$\Rightarrow \frac{30 - 60j + 20 - 65}{13} = \frac{-60j - 15}{13} \Rightarrow V_1 = \frac{15\sqrt{17}}{13}\angle -104°$

* mesh Analysis



2 meshes

$10\angle 0° = 3I_1 + j4(I_1 - I_2)$

$(I_2 - I_1)j4 - I_2(j2) + 2I_1 = 0$

$10\angle 0° = 10 = 3I_1 + j4(I_1 - I_2) = 3I_1 + j4I_1 - j4I_2$

$j4I_2 + 10 = I_1(3$

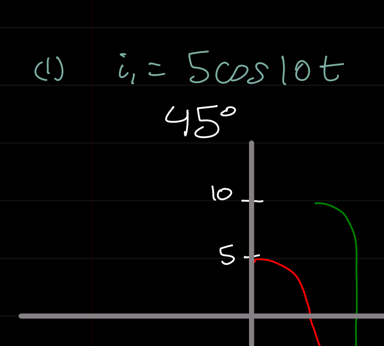
$j4I_2 - j4I_1 - j2I_2 + 2I_1 = 0$

$j4I_2 - j4(10 -$

CTD Tutorial : 1 19/08/24 1:00 – 3:00

Q1) Determine the angle by which v_1 leads i_2 if $v_1 = 10\cos(10t - 45^\circ)$ and i_1 is equal to

(1) $i_1 = 5\cos 10t$
45°



(3) $i_1 = 5\cos(10t - 40^\circ)$
5°

(2) $i_1 = 5\cos(10t - 80^\circ)$
35°

(4) $i_1 = 5\cos(10t + 40^\circ)$
85°

(5) $5\sin(10t - 19^\circ)$
$= 5\cos(10t - 19^\circ - 90^\circ)$
$= 5\cos(10t - 109^\circ)$
Ans: 64°

Q2) Determine which waveform in each of the following pairs is lagging

(1) $\cos 4t$ leads $\sin 4t$ by $\frac{\pi}{2}$ $\{90^\circ\}$

(2) $\cos(4t - 80^\circ)$ lags $\cos(4t)$ by 80°

(3) $\cos(4t + 80^\circ)$ leads $\cos(4t)$ by 80°

(4) $\sin(5t)$ lags $\cos(5t + 2^\circ)$ by 92°

(5) $\sin 5t + \cos 5t$

Q3) Perform the indicated operation {express in rectangular & polar form}

(a) $\dfrac{2+j3}{1+8\angle 90^\circ} - 4$

$8\angle 90^\circ$
$\hookrightarrow r\cos\theta + jr\sin\theta = 8\cdot 0 + 8\cdot j\cdot 1 = 8j$

$\Rightarrow \dfrac{2+j3}{1+8j} - 4 \Rightarrow \dfrac{2+j3-4-j32}{1+8j} \times \dfrac{1-8j}{1-8j}$

$\Rightarrow \dfrac{(-2+29j)(1-8j)}{8} = -2+16j+29j+(29\times 8)$

$\Rightarrow 230 + 45j$

$\Rightarrow 46 + 9j$

$\dfrac{2+j3}{1+8\angle 90^\circ} - 4$

$8\angle 90^\circ = 8\cdot\cos(90) + 8\cdot j\sin(90) = 8j$

$\dfrac{2+j3}{1+j8} \times \dfrac{1-8j}{1-8j} - 4$

$\Rightarrow \dfrac{2-16j+j3+24}{65} - 4 \Rightarrow \dfrac{26-13j}{65} - 4$

$\Downarrow$

$\dfrac{-234-13j}{65} \Leftarrow \dfrac{26-13j-260}{65}$

x y $\boxed{\dfrac{-18}{5} - \dfrac{1}{5}\cdot j}$ rectangular form

$\Downarrow$

$r = \sqrt{x^2+y^2} = \sqrt{\dfrac{18^2}{25} + \dfrac{1}{25}}$

$\Rightarrow \sqrt{\dfrac{325}{25}} = \sqrt{13}$

$\theta = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{1}{18}\right) = 3.179^\circ$

polar form $\boxed{\sqrt{13}\angle 3.179^\circ}$

(b) $\left(\dfrac{10\angle 25^\circ}{5\angle -10^\circ} + \dfrac{3\angle 15^\circ}{3-5j}\right) j2$

Q4) Transform into Phasor form

(a) $11\sin 100t$

(b) $11\cos 100t$

(c) $11\cos(100t - 90^\circ)$

(d) $3\cos 100t - 3\sin 100t$

Q5) Determine equivalent Admittance assuming $w = 1000$ rad/s

(a) 25Ω in series with $20mH$

$Z_L = jwL = j\times 1000\times 20\times 10^{-3}$
$= j\cdot 20$

Admittance $(Y) = \dfrac{1}{Z_{eq}}$

$Z_{eq} = 25 + j20 \nearrow Y = \dfrac{1}{25+j20} \times \dfrac{25-j20}{25-j20}$

$\boxed{\dfrac{1}{41} - \dfrac{4}{205}\cdot j} \Leftarrow \dfrac{5-j4}{205} \Leftarrow \dfrac{25-j\cdot 20}{625+400} \Rightarrow \dfrac{25-j\cdot 20}{1025}$

(b) 25Ω in parallel with $20mH$

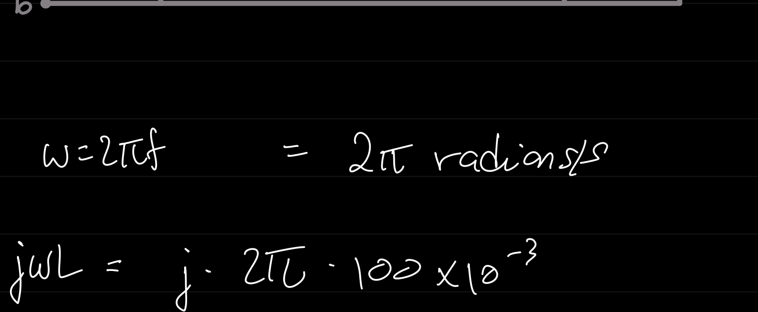
admittance $= \dfrac{1}{25} + \dfrac{1}{20j} = \dfrac{\overset{5}{\cancel{25}} + j\overset{4}{\cancel{20}}}{\underset{100}{\cancel{500}}j}$

$\dfrac{-j\cdot 5 - 4}{100} \Leftarrow \dfrac{-j(5+j\cdot 4)}{100} \Leftarrow \dfrac{5+j\cdot 4}{100j}$

$\hookrightarrow \dfrac{-1}{25} + \left(\dfrac{-j}{20}\right)$

Q6) Calculate the equivalent impedance as seen at the open terminals if the freq is ...

(a) 1Hz
(b) 1KHz
(c) 1MHz
(d) 1GHz



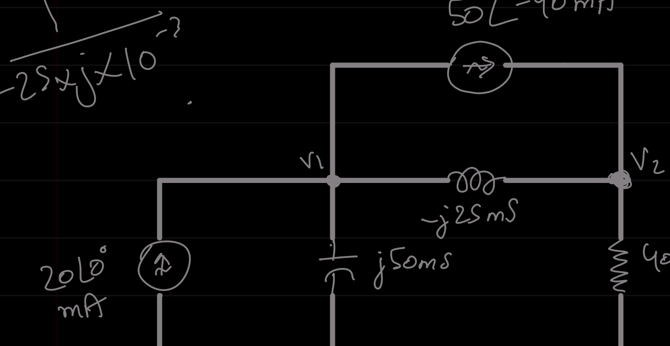
(a) $f = 1Hz$ $w = 2\pi f$ $= 2\pi$ radians/s

$Z_L = jwL = j\cdot 2\pi\cdot 100\times 10^{-3}$

$\Rightarrow \dfrac{j\cdot 2\pi}{\cancel{10}5} \Rightarrow \dfrac{j\pi}{5} \Rightarrow$

Q7) Use nodal analysis to find V_1 and V_2

$\dfrac{1}{-25\times j\times 10^{-3}}$



$\boxed{V1}$

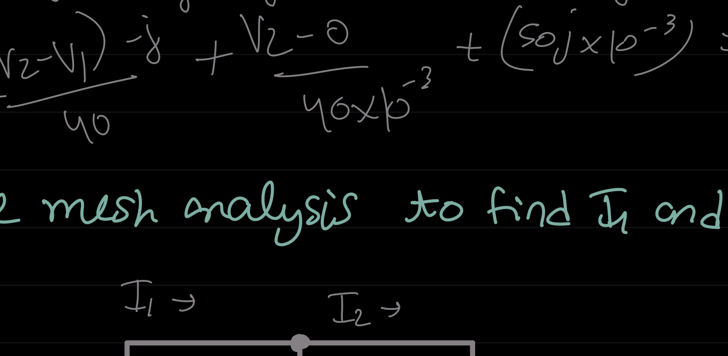
$\dfrac{V_1\cdot j}{20} - \dfrac{(V_1 - V_2)j}{40} + (-50j\times 10^{-3}) + 20\times 10^{-3} = 0$

$\Rightarrow \dfrac{5(V_1 j + V_2 j)}{10} - \dfrac{10j}{10} + \dfrac{4}{10} = 0$

$\Rightarrow 5V_1 j + 5V_2 j - 10j + 4 = 0 \Rightarrow 4 + j(5V_1 + 5V_2 - 10) = 0$

② $\dfrac{(V_2 - V_1)\cdot j}{40} + \dfrac{V_2 - 0}{40\times 10^{-3}} + (50j\times 10^{-3}) = 0$

Q8) Use mesh analysis to find I_1 and I_2



Apply KVL for ①:

$3(I_1 - I_2) + 15\angle 90^\circ - 10\angle 0^\circ + I_1(j5) = 0$

$3I_1 - 3I_2 + 15\cancel{\cos 90^\circ} + 15j\sin 90^\circ - 10\cos 0^\circ - 10j\cancel{\sin 0^\circ} + I_1 5\cdot j = 0$

$3I_1 - 3I_2 + 15j - 10 + 5j I_1 = 0$

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

Apply KVL for ②

$-4I_2\cdot j + 20\angle 0^\circ - 15\angle 90^\circ + (I_2 - I_1)3 = 0$

$-4I_2\cdot j + 20 - 15j + 3I_2 - 3I_1 = 0$

$3I_2 - 3I_1 + 20 + j(-15 - 4I_2) = 0$

$3I_1 = 3I_2 + 20 + j(-15 - 4I_2)$

Put ② in ①

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

$\cancel{3I_2} + 20 + j(-15 - 4I_2) - \cancel{3I_2} - 10 + j(15 + 5I_1) = 0$

$10 + j(-15 - 4I_2) + j\left(15 + 5\left[\dfrac{3I_2 + 20 - 15j - 4I_2\cdot j}{3}\right]\right) = 0$

$30 - 45j - 12I_2 j + j(45 + 15I_2 + 100 - 75j - 20I_2 j) = 0$

$30 - \cancel{45}j - 12I_2 j + \cancel{45}j + 15I_2 j + 100j + 75 + 20I_2 = 0$

$105 + 20I_2 + j(3I_2 + 100) = 0$

$105 + 20I_2 + j\cdot 3I_2 + j\cdot 100 = 0$

$I_2(20 + j\cdot 3) = -105 - j\cdot 100$

$I_2 = \dfrac{-105 - j\cdot 100}{20 + j\cdot 3} \times \dfrac{20 - j\cdot 3}{20 - j\cdot 3}$

$I_2 = \dfrac{-2100 + 315\cdot j - 2000j - 300}{409}$

$I_2 = \dfrac{-2400 - 1685j}{409}$

Home work:1 19/08/24

* Section 10.1

Q1) (a) $5\sin(5t-9°)$

$t=0 \Rightarrow 5\sin(-9°) = -5\sin(9°) = -0.782$

$t=0.01 \Rightarrow 5\sin(0.05-9°) = -5\sin(8.95°)$ X

$\Downarrow$
radians

$5\sin\left(\left[0.05\times\frac{180}{\pi}\right)-9°\right) = 5\sin(2.86°-9°)$

$\Rightarrow -5\sin(6.14) = -0.534$

$t=0.1s \Rightarrow 5\sin(0.5-9°) = 5\sin(28.64-9) = 5\sin(19.64)$

$\Rightarrow 1.6805$

(b) $4\cos 2t$

$t=0 \Rightarrow 4\cos(0) = 4$

$t=1 \Rightarrow 4\cos(2) = 3.997$

$t=1.5 \Rightarrow 4\cos(3) = 3.994$

(c) $3.2\cos(6t+15°)$

$t=0 \Rightarrow 3.2\cos(15°) = 3.09$

$t=0.01 \Rightarrow 3.2\cos(0.06+15°) = 3.2\cos(3.43°+15°)$

$\Rightarrow 3.2\cos(18.43°)$

$\Rightarrow 3.035$

$t=0.1 \Rightarrow 3.2\cos(0.6+15°) = 3.2\cos(34.3°+15°)$

$= 3.2\cos(49.3)$

$= 2.086$

Q2) (a) $300\sin(628t) \rightarrow 300\cdot\cos(628t-90°)$

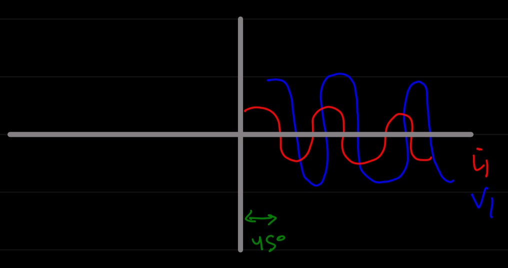
$4\sin(3\pi t+30°) \rightarrow 4\cos(3\pi t-60°)$

$14\sin(50t-5°) - 10\cos 50t \rightarrow$

Q3) $V_1 = 10\cos(10t-45°)$

(a) $i_1 = 5\cos 10t$

$-45°$

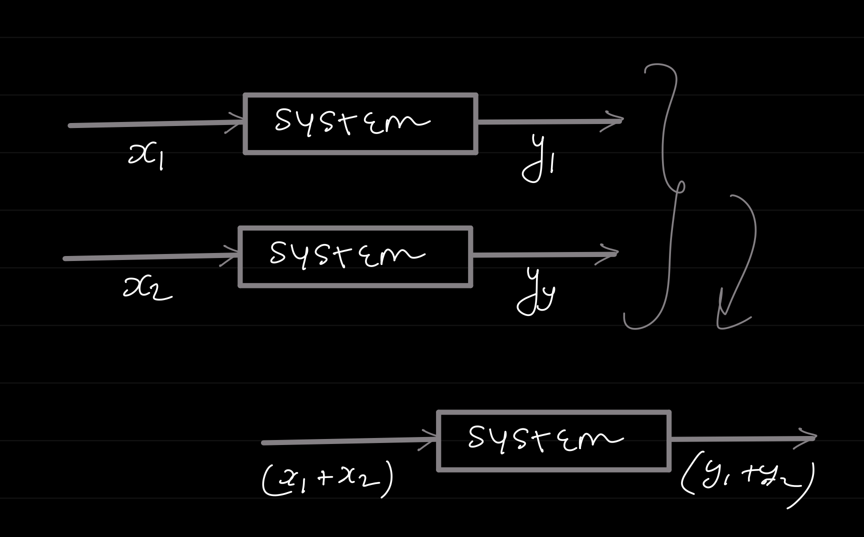


9:30 – 11:00AM
21/08/24

LECTURE: 4

* Superposition principle

active elements : provide energy eg: volt src/ curr src
passive element : consume/ store energy
eg: R/L/C

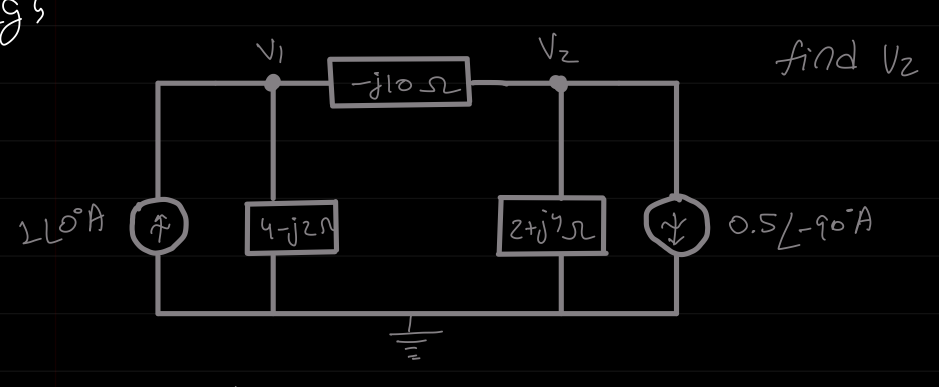


* Multiple independent sources

- consider separate independent sources "acting alone"
- add all responses

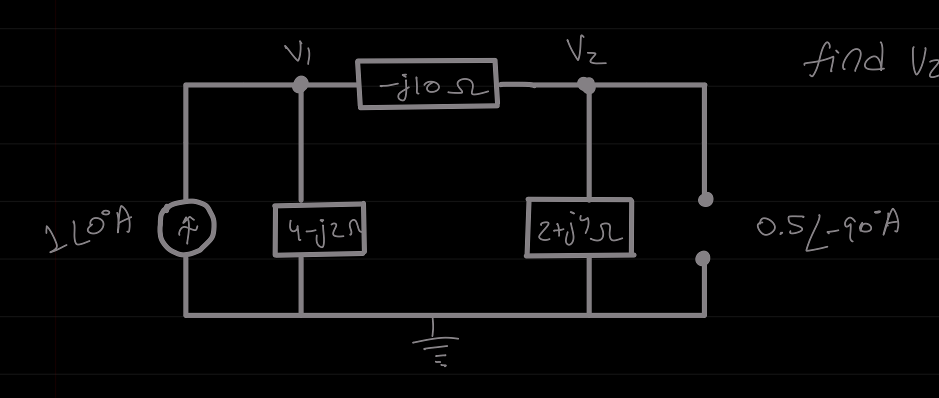
eg)

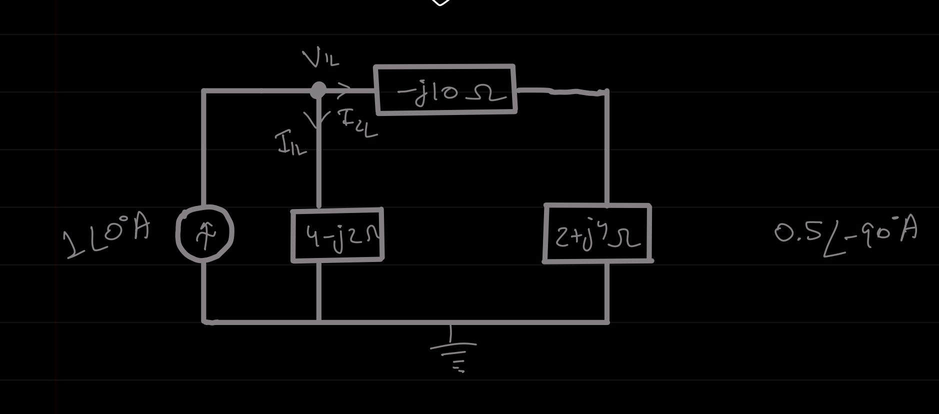
find V_2



consider left side source
so we need to open the circuit at the right source

find V_2





$V_{1L} = I_{1L}(4-j2)$

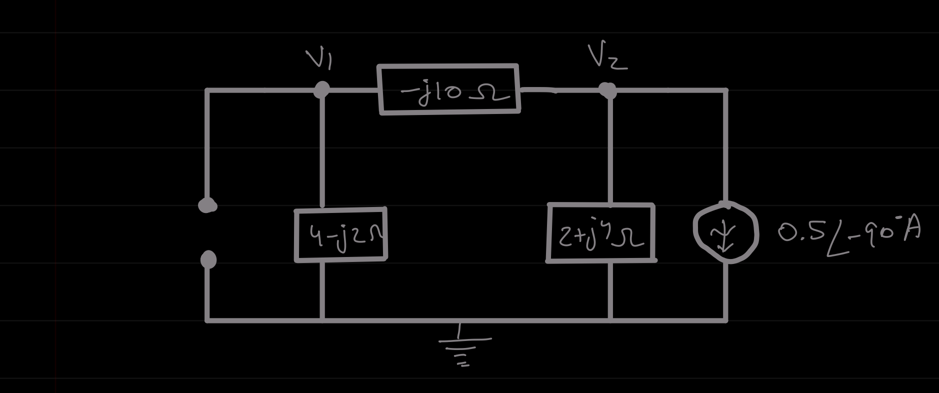
$\Rightarrow \dfrac{-j10+2+j4}{(4-j2)+(-j10+2+j4)}(4-j2)$

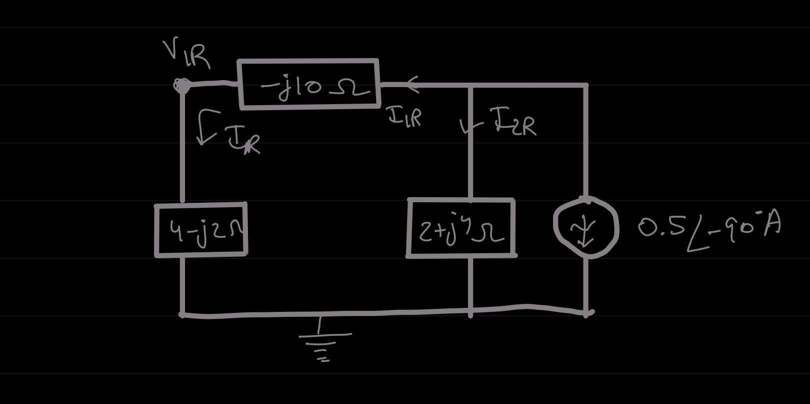
$\Rightarrow \dfrac{(2-j6)(4-j2)}{6-j8} \Rightarrow \dfrac{8-j4-24j-12}{6-8j}$

$\Rightarrow \dfrac{-4-28j}{6-8j} = 2-j2\,V$

$I_{1L}+I_{2L} = 1\angle 0^\circ = 1\cos 0^\circ + j\,1\sin 0^\circ = 1A$

consider right src





$V_{1R} = I_{1R}(4-j2)$

$I_{1R} = \dfrac{(-0.5\angle -90^\circ A)(2+j4)}{-j10+4-2j+2+j4}$

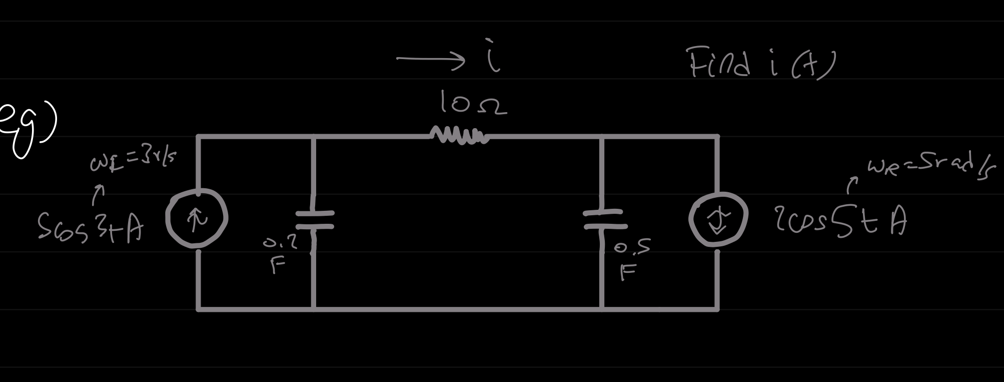
$V_{1R} = \dfrac{(j\cdot 0.5)(2+j4)(4-j2)}{6-j8}$

$\Rightarrow \dfrac{(j-2)(4-j2)}{6-j8} = \dfrac{j4+2-8+j4}{6-j8} = \dfrac{j8-6}{6-j8}$
$= -1V$

$V_1 = V_{1L}+V_{1R} = 2-j2-1 = \boxed{1-j2\,V}$

eg)

Find $i(t)$

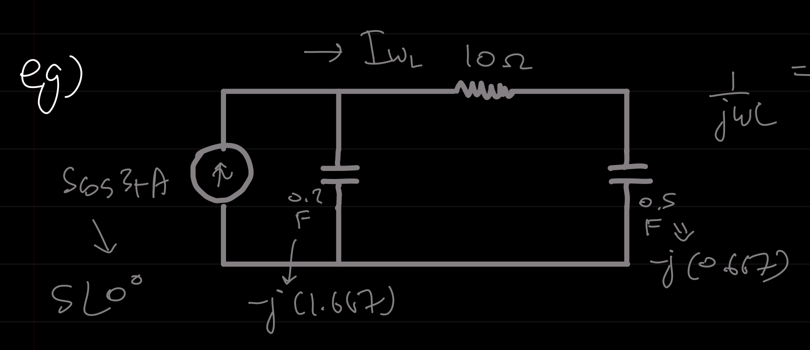


Note: we have 2 sources operating at 2 different frequencies (ω)

① consider left src $\omega_L = 3rad/s$

eg)

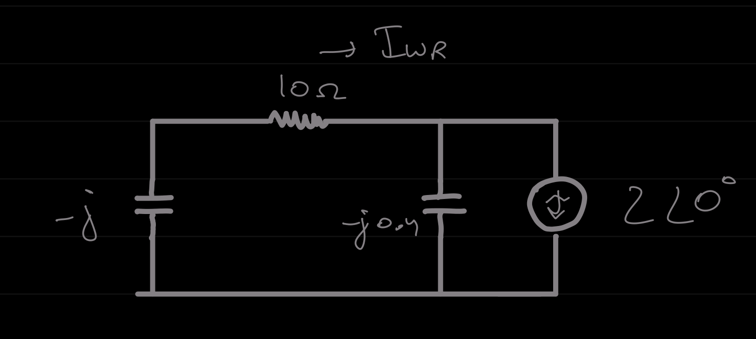
$\dfrac{1}{j\omega C} = \dfrac{1}{j(3)0.2} = \dfrac{-j}{0.6}$



$I_{\omega_1} = 5\angle 0^\circ \left(\dfrac{-j(1.667)}{-j(1.667)+10-j(0.667)}\right)$

$\Rightarrow 811.7\angle -76.86^\circ mA$

② consider right source $\omega_R = 5rad/s$



$I_{\omega R} = 2\angle 0^\circ \left(\dfrac{-j\cdot 0.4}{-j0.4+10+-j}\right)$

$\Rightarrow \dfrac{-j(0.8)}{-j(1.4)+10} \Rightarrow \dfrac{-j(0.8)[10+j(1.4)]}{100+1.96}$

$\dfrac{-j8+1.12}{101.96} \Rightarrow$

$\Rightarrow 0.0792\angle -82.03^\circ A = 79.2\angle -82.03^\circ mA$

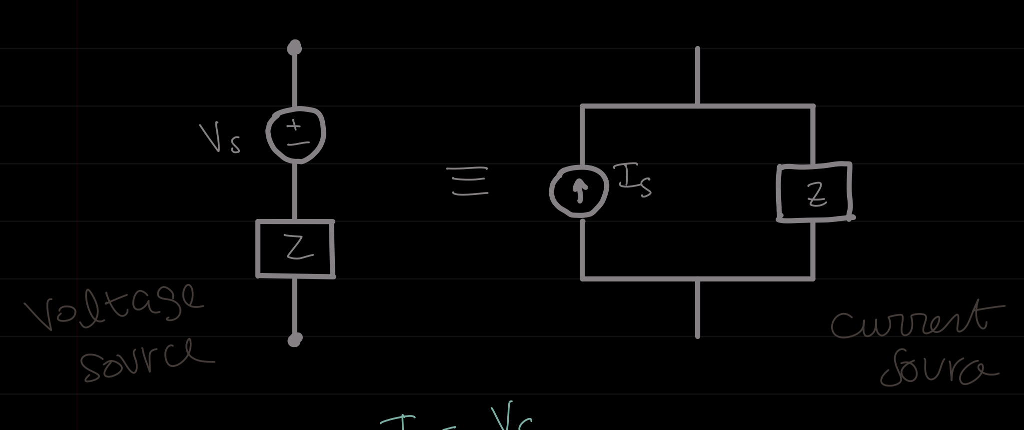
Applying superposition

$I = I_{\omega L} + I_{\omega R}$ ✗

Phasors can only be added when they correspond to same frequency

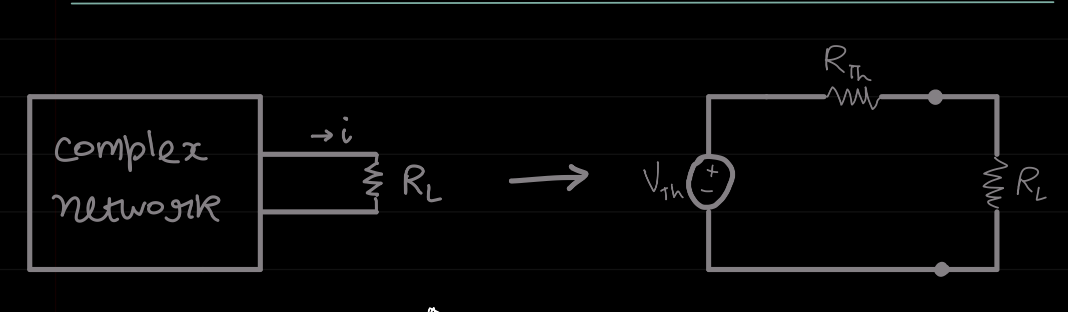
$i(t) = [811.7\cos(3t-76.86^\circ) + 79.2\cos(5t-82.03^\circ)]\ mA$

* SOURCE TRANSFORMATION

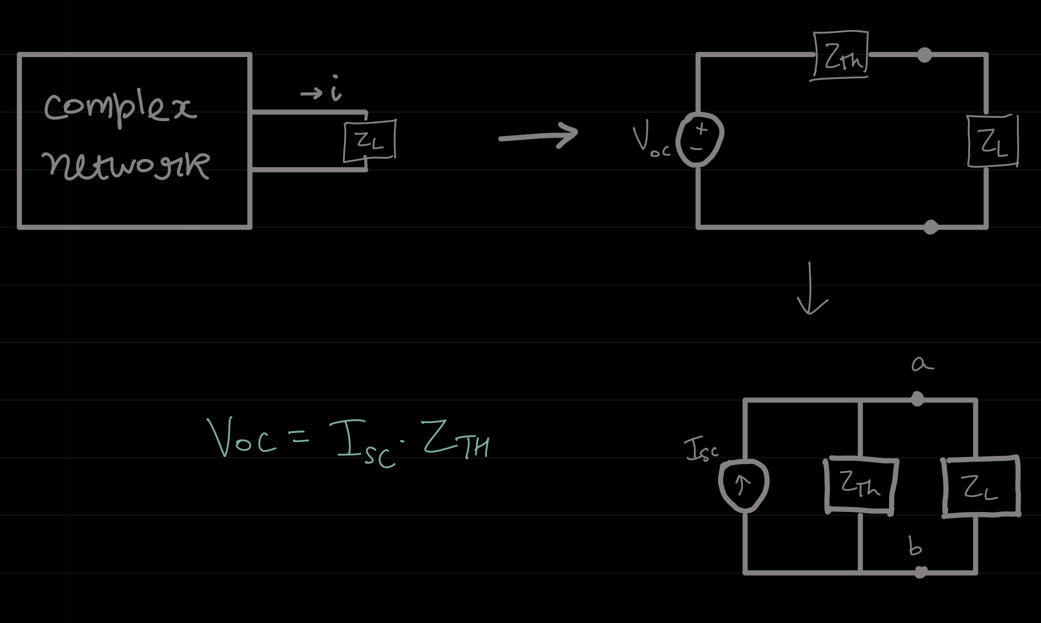


$I_s = \dfrac{V_s}{Z}$

* THEVENIN & NORTON EQUIVALENT CKTS



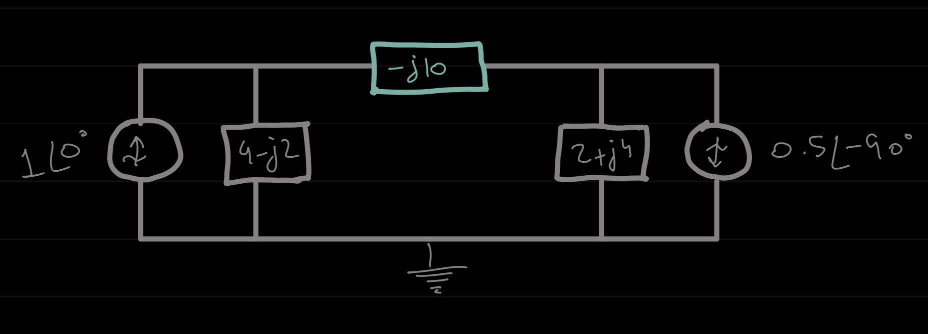
Normally



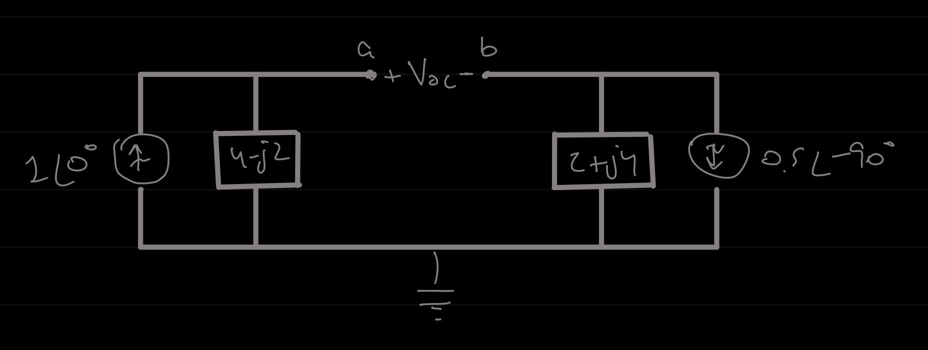
$V_{oc} = I_{sc}\cdot Z_{TH}$

	by open circuiting V_{oc}	by short circuiting I_{sc}	Z_{TH} ratio of V_{oc} & I_{sc}
① Independent sources	✓	✓	✓
② Independent + dependent sources	✓	✓	✓
③ Dependent sources only	we need to consider test sources eg: 1A / 1V and then find $V_{oc} \Rightarrow I_{sc} \Rightarrow Z_{Th} = \dfrac{V_{oc}}{I_{sc}}$		

eg) find Thevenin equivalent seen by $-j10\Omega$



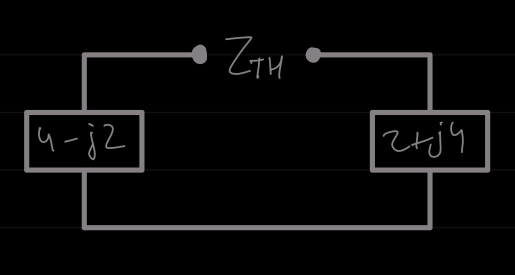
- : complex network
- : R_L



$V_{oc} = V_a - V_b = 1\angle 0^\circ(4-j2) - 0.5\angle -90^\circ(2+j4)$
$= 4-j2+2-j1 = 6-j3\,V$

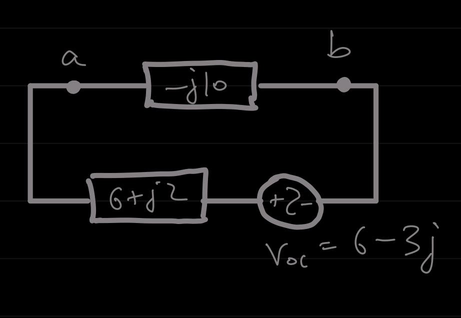
Find Z_{Th}

all volt src → short circuit
all curr src → open circuit



$Z_{TH} = 4-j2+2+j4 = 6+j2\ \Omega$

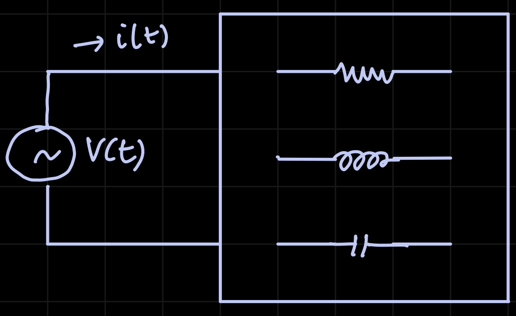
Equivalent circuit



$I_{ab} = \dfrac{V_{oc}}{6+j2-j10} = \dfrac{V_{oc}}{6-j8}$

* Lecture : 5 28/08/24

⇒ AC Power Analysis {Module : 3}



$\cos(A)\cos(B) = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

instantaneous power : $p(t) = v(t) \cdot i(t)$

$$v(t) = V_m \cdot \cos(\omega t + \theta)$$
$$i(t) = I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = V_m \cdot \cos(\omega t + \theta) \cdot I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = \underbrace{\frac{V_m I_m}{2}\cos(\theta - \phi)}_{\substack{\text{constant} \\ \text{(DC term)}}} + \underbrace{\frac{V_m I_m}{2}\cos(2\omega t + \theta + \phi)}_{\substack{\text{twice frequency} \\ \text{(harmonic)}}}$$

• Average Power

$$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos(\theta - \phi)\,dt + \underbrace{\frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos(2\omega t + \theta + \phi)}_{0}$$

$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)}$$

$$P_{avg} = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$$

real number ✓
phasor ✗

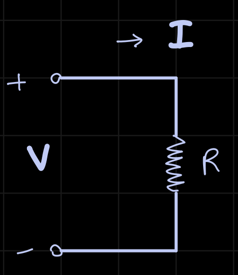
$\mathbf{I} = I_m e^{j\phi}$
$\mathbf{I}^* = I_m e^{-j\phi}$
(conjugate)

* Avg. Power absorbed

• by a purely resistive element

$$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$$

here, phase diff = 0°

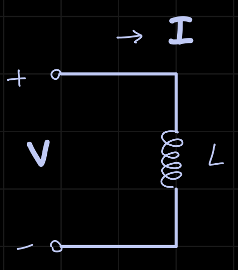


$$P_{avg} = \frac{V_m \cdot I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

• by a purely inductive element

$$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$$

here, phase diff = 90°

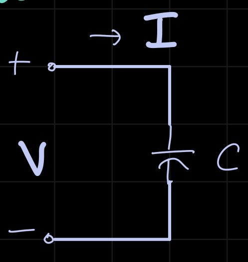


So,

$P_{avg} = 0$ for inductor

• by a purely capacitive element

$$P_{avg} = \frac{V_m \cdot I_m}{2} \cdot \cos(-90°)$$



$P_{avg} = 0$ for capacitor

* Lecture : 6

$V(t) = V_m \cos(\omega t + \theta)$ ← Revision

↓

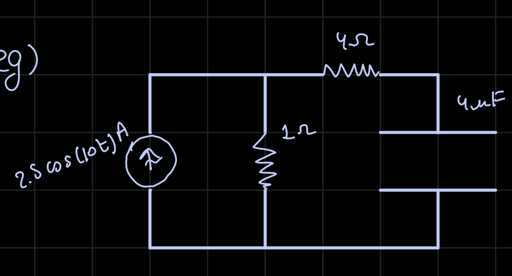
$V(t) = Re\{V_m e^{j\omega t} e^{j\theta}\}$

↓ $- e^{j\omega t}$

$V_m e^{j\theta} = V_m \angle \theta$

exponential polar

• eg)

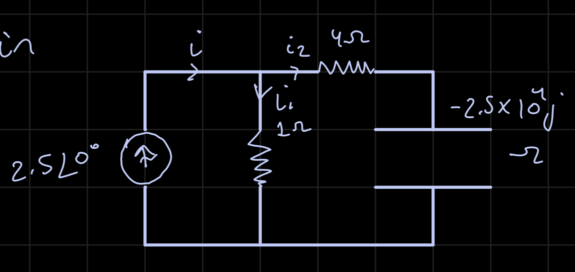


① find power delivered to each element at $t = 0, 10, 20ms$

② Find P_{avg} to each element

Ans) convert to freq. domain

$V_{1\Omega} = V_{4\Omega} + V_C$



① $P_{inst} = \frac{V_m I_m}{2} \cos(\theta - \phi) + \frac{V_m I_m}{2} \cos(2\omega t + \phi + \theta)$

$I_1 = -2.5 \times \left(\frac{4 - j2.5 \times 10^4}{1 + 4 - j\,2.5 \times 10^4} \right) \simeq \underline{\underline{-2.5A}}$ imaginary part is way larger than real part

$\Rightarrow \frac{-10 + j625 \times 10^{\cancel{0}}}{5 - j\,25 \times 10^3} \Rightarrow \frac{-2 + j125}{1 - j50}$

$\Rightarrow \frac{-2 + j \cdot 125}{1 - j50} \times \frac{1 + j50}{1 + j50} \Rightarrow$

$I_{1\Omega}(t) = 2.5 \cos(10t)$

$V_{1\Omega} = (I_{1\Omega})(1\Omega) \Rightarrow V_{1\Omega}(t) = 2.5 \cos(10t)$

$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{4}$

$p_{1\Omega}(t) = 2.5 \cos(10t) \cdot 2.5 \cos(10t)$ W

$p_{1\Omega}(t=0) = 6.25W$

$p_{1\Omega}(t = 10ms) = 6.1877W$

$p_{1\Omega}(t = 20ms) = 6.033W$

$(P_{avg})_{1\Omega} = 3.125W$

⇒ P delivered to 4Ω

$P_{4\Omega}(t) = V_{4\Omega}(t) \times I_{4\Omega}(t)$

$I_{4\Omega} = I_S \times \frac{1}{5 - j2.5 \times 10^4} = 10^{-4} \angle 90° A$

$V_{4\Omega} = 4 \times I_{4\Omega} = 4 \times 10^{-4} \angle 90°$ V

convert to time domain

$p_{4\Omega}(t) = 2 \times 10^{-8} + 2 \times 10^{-8} \cos(20t)$ W

$t = 0 : 4 \times 10^{-8}$ W

$t = 10ms = 3.96 \times 10^{-8}$ W

$t = 20ms = 3.87 \times 10^{-8}$ W

$P_{avg\,4\Omega} = 2 \times 10^{-8}$ W

⇒ P delivered to capacitor

$p_C(t) = V_C(t) \cdot I_C(t)$

$(p_{avg})_C = 0$ since phase diff $= 90°$

$I_C = I_2 = 10^{-4} \angle 90° A \Rightarrow I_C(t) = 10^{-4} \cos(10t + 90°)$

$V_C = (-j\,2.5 \times 10^{-4}) \times 10^{-4} \angle 90$

$-j \cdot 25 \times 10^3 \times (+j10^{-4})$

$\underline{\underline{2.5 V}}$

$p_C(t) = 1.25 \times 10^{-4} \cos(20t + 90°)$ W

$t = 0 \Rightarrow p = 0W$

$t = 10ms \Rightarrow p = 2.48 \times 10^{-5}$ W

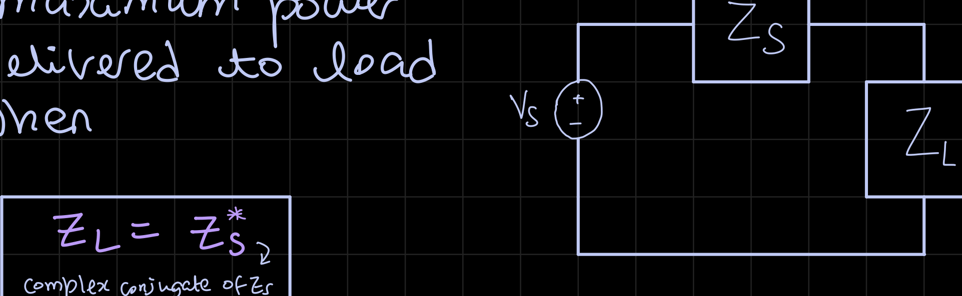
Note: we cannot multiply I_C and V_C in phasor form and then convert to time domain for getting P_C (power) because power does not has a phasor part. It is a real value.

* Correct or not ?

depends on t ✗ ① $p_{1\Omega}(t) + p_{4\Omega}(t) + p_C(t) = \text{constant}1$

✓ ② $p_{avg\,1\Omega} + p_{avg\,4\Omega} + p_{avg\,C} = \text{constant}2 \equiv P_{avg}$ source

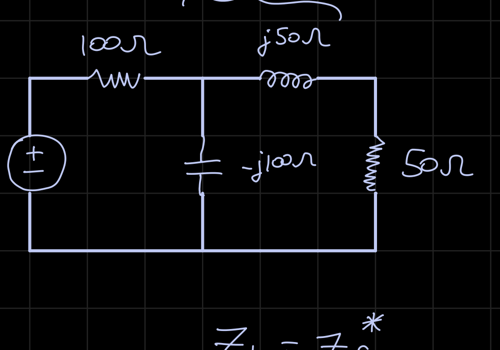
active sign convention passive sign convention



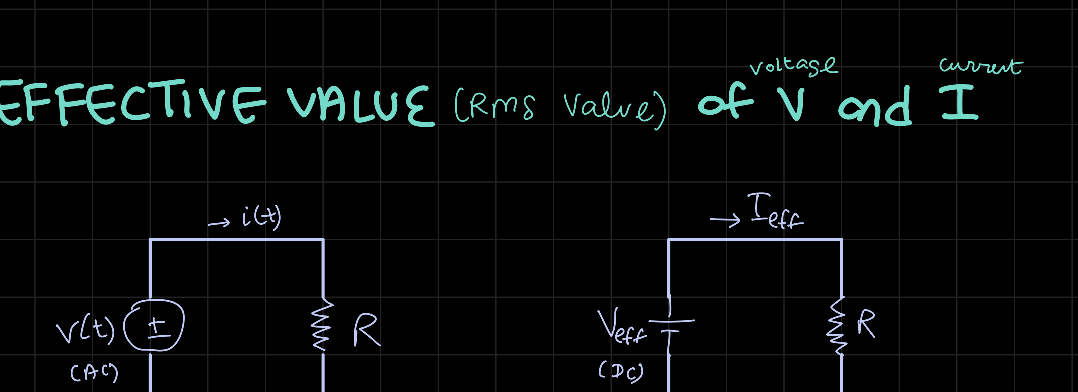
* maximum Power Transfer Theorem

maximum power delivered to load when

$\boxed{Z_L = Z_S^*}$ complex conjugate of Z_S

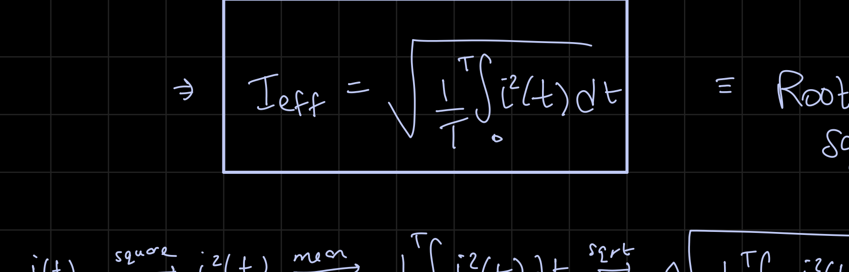


⇒ Impedance matching



$Z_L = Z_S^*$

* EFFECTIVE VALUE (Rms value) of V (voltage) and I (current)



if $P_{avg_1} = P_{avg_2}$

$\Rightarrow \frac{1}{T} \int_0^T i^2(t) R\, dt = I_{eff}^2 R$

$\Rightarrow \boxed{I_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2(t)\, dt}} \equiv$ Root mean square

$i(t) \xrightarrow{\text{square}} i^2(t) \xrightarrow{\text{mean}} \frac{1}{T} \int_0^T i^2(t)\, dt \xrightarrow{\text{sqrt}} \sqrt{\frac{1}{T} \int_0^T i^2(t)\, dt}$

• Sinusoidally varying voltage

$i(t) = I_m \cos(\omega t + \phi)$

$\cos^2 x = \frac{2\cos x + 1}{2}$

$I_{eff} = \sqrt{\frac{1}{T} \int_0^T I_m^2 \cos^2(\omega t + \phi)\, dt}$

$\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$ current rms value unit: A_{rms}

$\boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$ voltage rms value unit: V_{rms}

not a phasor it is real number ≡ magnitude of current / voltage

eg: $I = 10 \angle 90° A \Rightarrow I = 5\sqrt{2} \angle 90° A$ rms

OR $V = 100\sqrt{2} \angle -60° V \Rightarrow V = 100 \angle -60° V_{rms}$

* Apparent Power

$P_{apparent} = I_{eff} V_{eff}$ unit: VA (volt ampere), not W

* Power factor

$PF = \frac{\text{avg power}}{\text{apparent power}} = \frac{(V_m \cdot I_m)/2 \cos(\theta - \phi)}{V_{eff} \cdot I_{eff}}$

$PF = \cos(\theta - \phi)$ power factor angle

* Lecture : 7

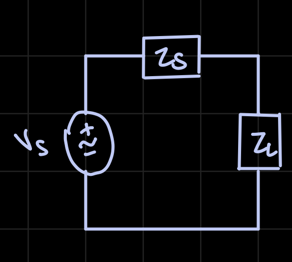
- Instantaneous Power : $p(t) = v(t)\, i(t)$
- Average Power : $P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$
- $P_{avg \cdot sources} = \Sigma P_{avg \cdot elements}$

* Note : $\overset{\text{impedence}}{Z} = \underset{\text{resistive}}{R} + j\overset{\text{reactive}}{X}$

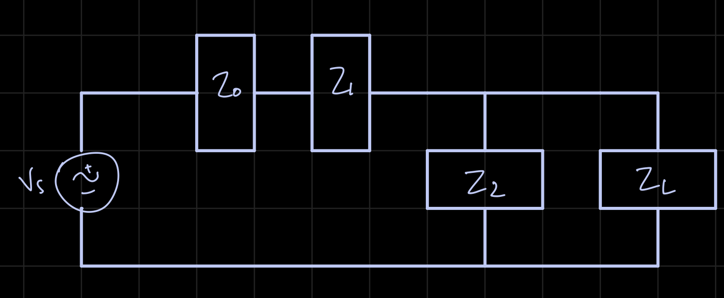
$P_{avg})_{resistive}$: $P \neq 0$

$P_{avg})_{reactive}$: $P = 0$

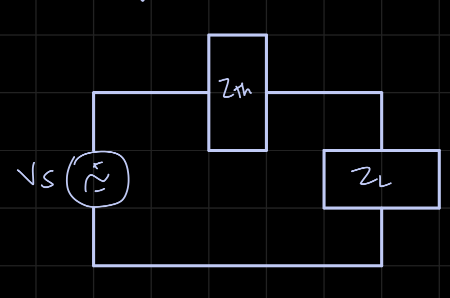
* Max Power Transfer :



Circuit has max. power when $Z_S = Z_L^*$ (complex conjugate)



⇓ apply thevenin's theorem



$Z_L = Z_{Th}^*$

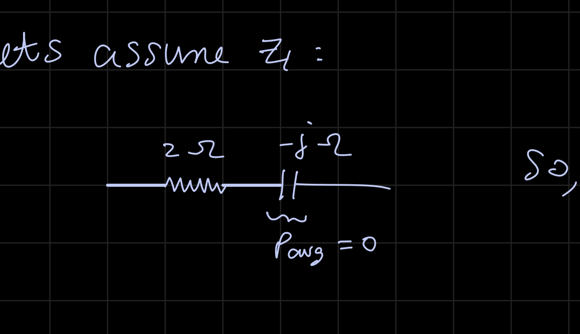
* Power Factor

$PF = \cos(\overset{\text{angle of voltage phasor}}{\theta} - \overset{\text{angle of current phasor}}{\phi}) = \frac{P_{averase}}{P_{apparent}}$

for purely resistive load : PF = 1 {Max} $\theta - \phi = \pm 90°$
for purely reactive load : PF = 0 {min}

Note → PF = 0.5 leading → capacitive $(\theta - \phi) < 0°$
PF = 0.5 lagging → inductive $(\theta - \phi) > 0°$

eg)



find
1. Average power delivered to each load
2. $P_{avg \cdot source} = ?$
3. $P_{apparent \cdot src} = ?$
4. PF of combined load = ?

Ans) $P_{avg\, z_1} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$

voltage across Z_1, not the src

lets assume Z_1 :

2Ω $-j\,\Omega$ ($P_{avg} = 0$)

So, $P_{avg\, z_1}$ = Power delivered to 2Ω resistor

$= I_{eff}^2 R$

find $\mathbf{I}$ by KVL :

$-60\angle 0° + \mathbf{I}(2 - j1 + 1 + 5) = 0$

$\mathbf{I} = \frac{60\angle 0°}{3 + j4} = \underbrace{12\angle -53.13°}_{I_{rms}}$ Arms

$i(t) = 12\cos(\omega t - 53°)$ A rms

$\underset{(A_{rms})}{I_{eff}} \rightarrow \underset{(A)}{I_m}$: $I_{eff} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = I_{eff}\sqrt{2}$

$I_{eff} = 12\, A_{rms} \Rightarrow I_m = 12\sqrt{2}$ A

$i(t) = 12\sqrt{2} \cos(\omega t - 53°)$ A

$P_{avg\,(z_1)} = (12)^2 \times 2 = 288$ W

note: $P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$

Z_2 : I_{eff} same : series circuit

$I_{eff} = 12$

and $P_{avg})_{Z_2} = I_{eff}^2 R = (144)(1) = 144$ W

2. $P_{avg})_{src} = V_{eff} I_{eff} \cos(\theta - \phi)$

$\mathbf{V}_{src} = 60\angle 0°$ Vrms ⇒ $V_{eff} = 60$ Vrms

$\mathbf{I}_{src} = 12\angle -53.13°$ Vrms ⇒ $I_{eff} = 12$ Arms

$\theta = 0°$, $\phi = -53.13°$

$P_{avg})_{src} = [60 \times 12] \cos(53.13°) = 432$W

we can observe that $288 + 144 = 432$
and hence,

$P_{avg \cdot sources} = \Sigma P_{avg \cdot elements}$ holds

3. $P_{apparent} = V_{eff} \cdot I_{eff} = (60)(12) = 720$ W

4. PF of combined loads = PF of source

$PF = \cos(\theta - \phi) = \cos(\underbrace{0 + 53.13°}) = 0.6$ lagging

$\theta - \phi > 0$, lagging

* $P_{avg} = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$

$\mathbf{V} = V_m \angle\theta$
$\mathbf{I} = I_m \angle\phi$

$\mathbf{V}_{eff} = V_{eff}\angle\theta$
$\mathbf{I}_{eff} = I_{eff}\angle\phi$

$P_{avg} = Re\{\mathbf{V}_{eff}\mathbf{I}_{eff}\}$

* Complex Power

$S = \mathbf{V}_{eff}\mathbf{I}_{eff}^*$ Volt Ampere {VA}

this is a complex number not a phasor of power

$S = (V_{eff}\angle\theta)(I_{eff}\angle\phi)$
$= V_{eff} \cdot I_{eff}\, e^{j(\theta - \phi)}$

$\Rightarrow S = \underbrace{V_{eff} I_{eff} \cos(\theta - \phi)}_{\text{average power } P} + j\underbrace{V_{eff} I_{eff} \sin(\theta - \phi)}_{\text{reactive power } Q}$

(volt ampere) VA (watt) W (volt ampere reactance) VAR

$\boxed{S = P + jQ}$

$|S| = V_{eff} I_{eff} = P_{apparent}$ (volt amp)

note: $Z = R + jX$

Purely resistive load : $Z = R$, $X = 0$

$S = \underset{V_{eff} I_{eff} \cos(0)}{P} + j\overset{V_{eff} I_{eff} \sin(\theta - \phi)}{Q}$ ($\sin(\theta-\phi) = 0$)

$P \neq 0$
$Q = 0$

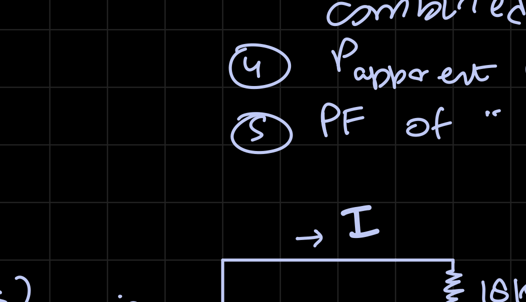
Purely reactive load

$S = \underset{V_{eff} I_{eff} \cos(90°)}{P} + j\overset{V_{eff} I_{eff} \sin(\theta - \phi)}{Q}$ ($\theta - \phi = 90°$)

$P = 0$, $Q \neq 0$

Q signifies energy flow rate into or out of reactive component of load

eg)

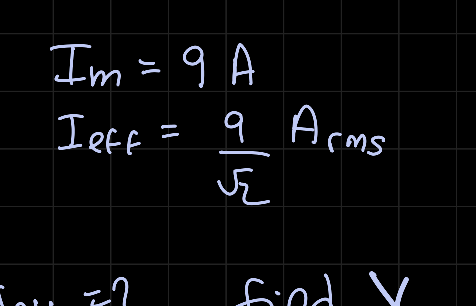


assume $C = 1\mu F$, $\omega = 45$ rad/s

find:

1. complex Power provided by src
2. Time average power absorbed by combined load
3. Reactive Power absorbed by combined load
4. $P_{apparent}$ absorbed " "
5. PF of " " "

Ans)



$9e^{j9}$ A ≡ $9\cos(9°) + j9\sin(9°)$

$\frac{1}{j\omega c} = \frac{1}{j \times 45 \times 1 \times 10^{-6}} = -j\frac{10^6}{45}$

1. $S_{source} = \overbrace{V_{eff} \times I_{eff}}^{\text{source}}$

$I_m = 9$ A
$I_{eff} = \frac{9}{\sqrt{2}}$ Arms

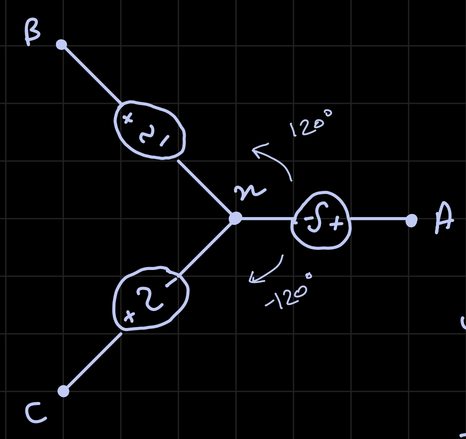
$V_{eff} = ?$, find V

$\mathbf{V} = \mathbf{I}(Z_{eq}) = 9\angle 9°\left(18 \times 10^3 - j\frac{10^6}{45}\right)$

∗ Lecture : 8

• Polyphase Circuits (Chp 12)

⇒ Three Phase Source



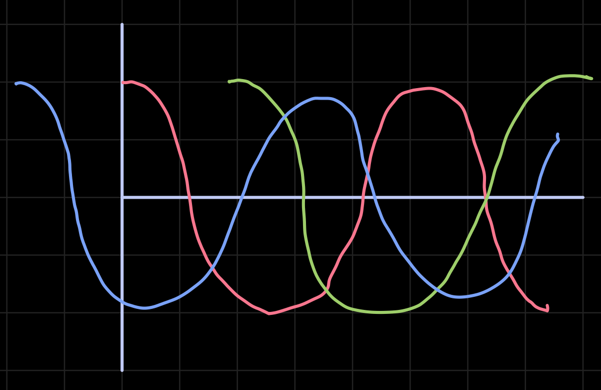
$V_{an} = 100\angle 0°\ V$
$V_{bn} = 100\angle 120°\ V$
$V_{cn} = 100\angle -120°\ V$

if $|V_{an}| = |V_{bn}| = |V_{cn}|$
& $V_{an} + V_{bn} + V_{cn} = 0$
then it is a

Balanced Source

total instantaneous power = constant

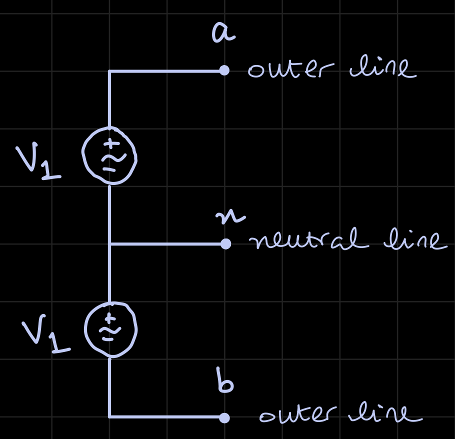
freq. domain	time domain
$100\angle 0°$	$100\cos(\omega t)$
$100\angle 120°$	$100\cos(\omega t + 120°)$
$100\angle -120°$	$100\cos(\omega t - 120°)$



total $p(t)$ → constant

∗ Single Phase Three-wire Source

≡ Two phase source



$V_1 = V_m\angle 0°$

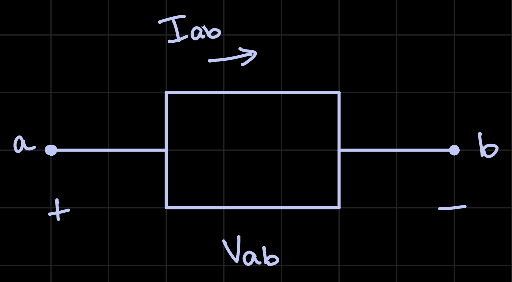
$V_{an} = V_1 = V_m\angle 0°$
$V_{bn} = -V_1 = V_m\angle 0° - 180°$

$V_{bn} \leftarrow \overset{180°}{\frown} \rightarrow V_{an}$

double script notation

$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$
$V_{bn} = -V_{nb}$

$V_{an} = V_A$ wrt V_n

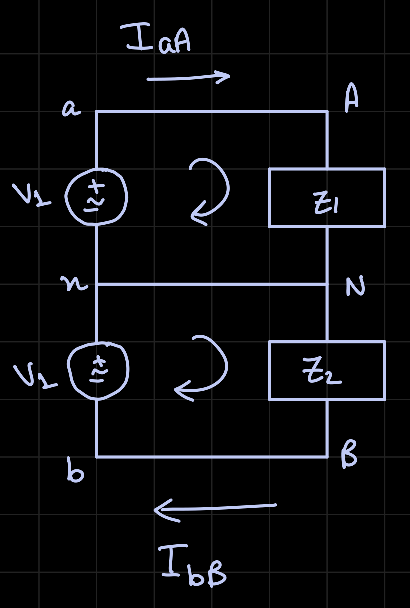


I_{ab}: current flowing from a to b

• Balanced single phase three wire source

Conditions:

$$\left.\begin{array}{l} |V_{an}| = |V_{bn}| \\ V_{an} + V_{bn} = 0 \end{array}\right\} \Rightarrow \text{Balanced Src}$$



$I_{aA} = \frac{V_1}{Z_1}$

$I_{bB} = \frac{V_1}{Z_2}$

$I_{nN} = -I_{aA} + I_{bB}$

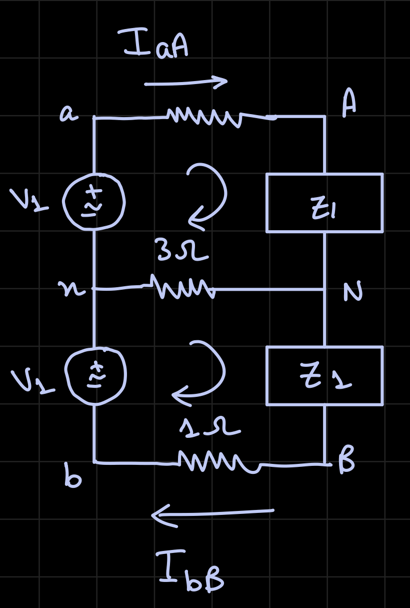
$\Rightarrow \frac{V_1}{Z_2} - \frac{V_1}{Z_1}$

Assume $Z_1 = Z_2$

$I_{nN} = \frac{V_1}{Z_1} - \frac{V_2}{Z_1} = 0$

when both srcs and loads are equal

Balanced Load : current in neutral line is equal to zero.



all terms are phasors

even with resistance
$I_{nN} = 0 = I_{bB} - I_{aA}$
due to
① Balanced Load
≡ Symmetry

* Lecture : 9

09/09/24

time domain | freq. domain

$V_p\cos(\omega t+\theta^\circ)$ — $V_p\angle\theta^\circ$

$V_p\cos(\omega t+\theta^\circ-180^\circ)$ — $V_p\angle\theta-180^\circ$

we cannot directly say,

$$P_{avg} = VI \quad \times$$

but $P_{avg} = \frac{1}{2}\mathrm{Re}\{VI^*\}$ ✓

② a) $I_{AB} = \dfrac{115\angle\theta^\circ + 115\angle\theta^\circ}{2(10+j2)} = \dfrac{230\angle\theta^\circ}{2(10+j2)}$

$= 11.27\angle(\theta^\circ - 11.3^\circ)A$

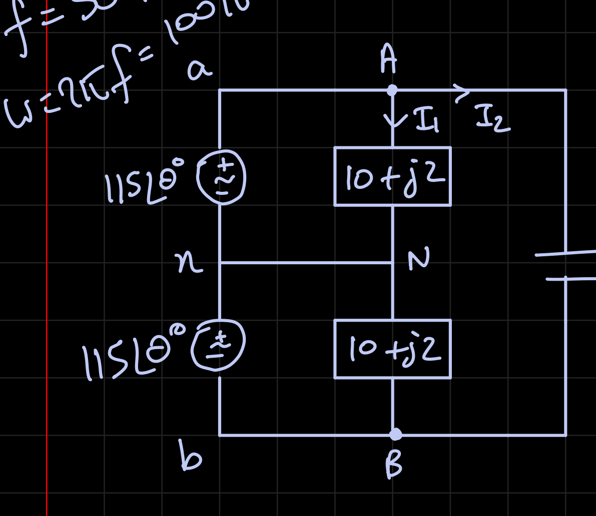
$\phi = \theta - 11.3^\circ$

PF $= \cos(\theta - \theta + 11.3^\circ) = 0.98$ Lagging

because PF angle is +ve

b) PF $=1 \Rightarrow \cos(\theta-\phi)=1 \Rightarrow \theta-\phi=0$

$\Rightarrow \theta = \phi$



$I_{aA} = I_1 + I_2$

$\Rightarrow \dfrac{115\angle\theta^\circ}{10+j2} + \dfrac{230\angle\theta^\circ}{\left(\frac{1}{j\omega C}\right)}$

$\Rightarrow$ Lot of calculation alternative method $\Rightarrow$

Note that this is still a balanced load, so we can remove nN since $I_{nN}=0$

using complex power

$S \equiv$ complex power of total load

$$S = \frac{1}{2}VI^*$$

$$PF = \frac{\mathrm{Re}\{S\}}{|S|} = 1 \Rightarrow \mathrm{Re}\{S\} = |S|$$

so, $\mathrm{Im}\{S\} = 0$ here

$$S = \frac{1}{2}V_{an}I_{AN}^* + \frac{1}{2}V_{nb}I_{NB}^* + \frac{1}{2}V_{ab}I_{AB}^*$$

$$\Rightarrow \frac{1}{2}V_{an}\left(\frac{V_{an}}{Z_p}\right)^* + \frac{1}{2}V_{nb}\left(\frac{V_{nb}}{Z_p}\right)^* + \frac{1}{2}V_{ab}\left(\frac{V_{ab}}{Z_c}\right)^*$$

$$\Rightarrow \frac{1}{2}\frac{|V_{an}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{nb}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{ab}|^2}{|Z_c|^2}\times Z_c$$

$\mathrm{Re}\{115\angle 0\}$

$$\Rightarrow \frac{|V_{an}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{ab}|^2}{|Z_c|^2}\times Z_c$$

$\sqrt{10^2+2^2}$ $\quad$ $\mathrm{Re}\{\frac{1}{j\omega C}\} = \frac{1}{j\omega C}\times\frac{-1}{j\omega C}$

$$\Rightarrow \frac{115^2}{104}(10+j2) + \frac{1}{2}\left(\frac{230^2}{\left(\frac{1}{\omega C}\right)^2}\right)\left(\frac{-j}{\omega C}\right) \quad \frac{-1}{-1(\omega C)^2}$$

$$\Rightarrow 115^2\left(\frac{\cancel{10}+j2}{104} - 2j\omega C\right)$$

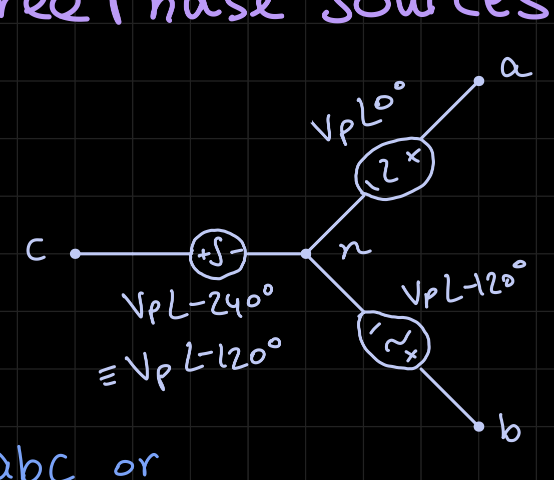
$$\Rightarrow 2\times115^2\left(\frac{j}{104} - j\omega C\right)$$

$$\Rightarrow 2\times115^2\left(\frac{1}{104} - \omega C\right) = 0$$

$$\frac{1}{104} - 100\pi C = 0$$

$$C = \frac{1}{10400\pi} = 30.60\,\mu F \checkmark$$

* Three Phase Sources



$V_{an} = V_p\angle 0^\circ$

$V_{bn} = V_p\angle -120^\circ$

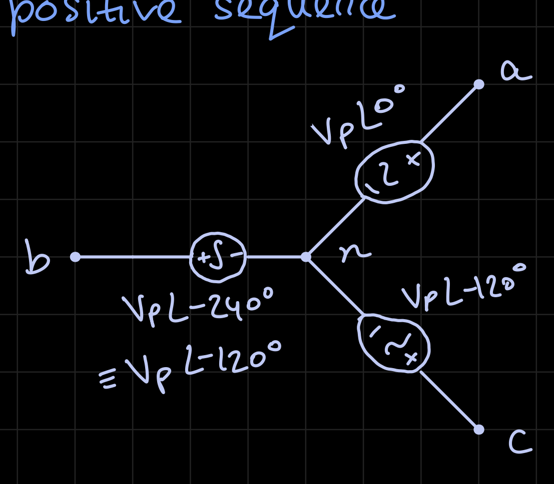
$V_{cn} = V_p\angle 120^\circ)$

$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$

$V_{an} + V_{bn} + V_{cn} = 0$

↳ balanced source

abc or positive sequence



cba or negative sequence

just a naming thingy

Phase voltages $\quad V_{an}, V_{bn}, V_{cn}$

Line-to-line Voltages OR line voltages $\quad V_{ab}, V_{bc}, V_{ac}$

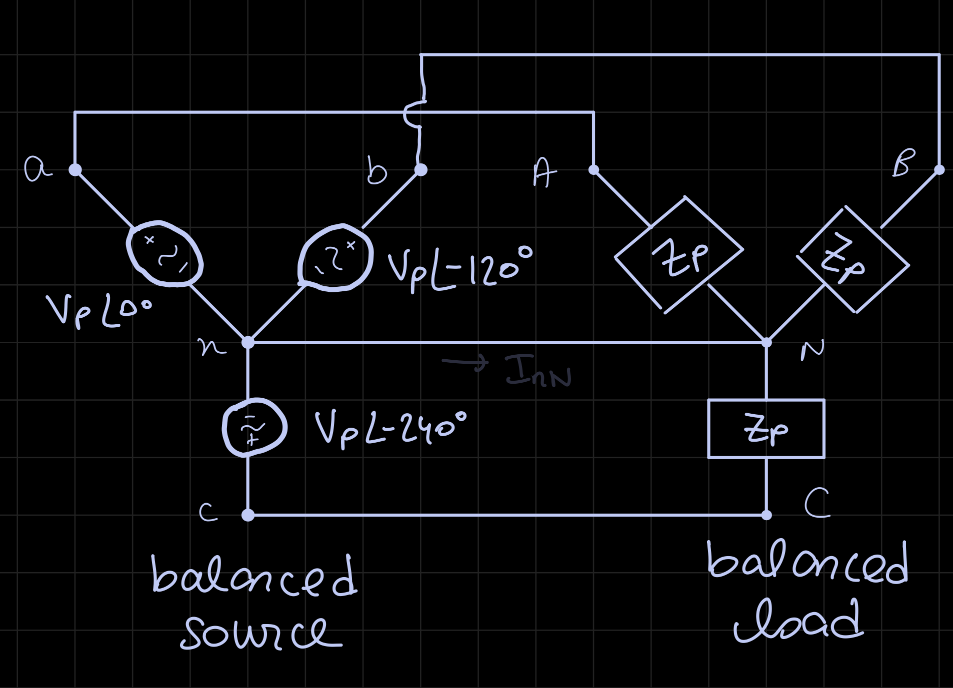
$$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$$

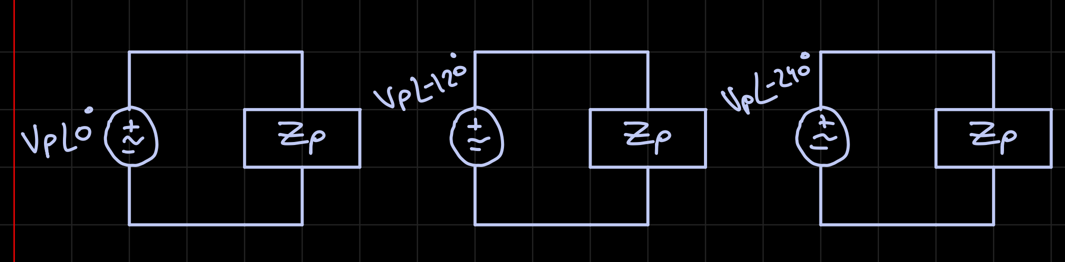
$$V_{ab} = \sqrt{3}V_p\angle 30^\circ$$

$$V_{bc} = \sqrt{3}V_p\angle -90^\circ$$

$$V_{ca} = \sqrt{3}V_p\angle -210^\circ$$

* Y-Y connection





"Per-phase"

used for easier analysis

eg. to find total power by ckt, just add the three individual power obtained from above ckts.

going back to the Y-Y connection ⇒

$$I_{aA} = \frac{V_{an}}{Z_p}$$

$$I_{bB} = \frac{V_{bn}}{Z_p} = \frac{V_{an}\angle -120^\circ}{Z_p} = I_{aA}\angle -120^\circ$$

$$I_{cC} = \frac{V_{cn}}{Z_p} = \frac{V_{an}\angle +120^\circ}{Z_p} = I_{aA}\angle +120^\circ$$

$$I_{nN} = I_{aA} + I_{bB} + I_{cC} = 0$$

eg} consider three phase balanced Y-Y connected system

$$V_{an} = 200\angle 0^\circ \ V_{rms}$$
$$Z_p = 100\angle 60^\circ\ \Omega$$

find phase and line voltage & current and total avg power delivered to load

Ans}

$V_{an} = 200\angle 0^\circ\ V_{Rms}$

$V_{bn} = 200\angle -120^\circ\ V_{Rms}$

$V_{cn} = 200\angle +120^\circ\ V_{Rms}$

line voltage:

$$V_{ab} = \sqrt{3}V_p\angle 30^\circ = 200\sqrt{3}\angle 30^\circ\ V_{Rms}$$

$$V_{bc} = 200\sqrt{3}\angle -90^\circ\ V_{Rms}$$

$$V_{ca} = 200\sqrt{3}\angle -210^\circ = 200\sqrt{3}\angle 150^\circ\ V_{rms}$$

line currents:

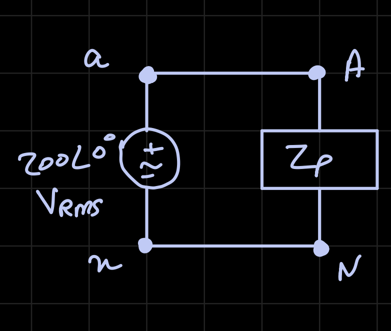
$$I_{aA} = \frac{V_{an}}{Z_p} = \frac{200\angle 0^\circ}{100\angle 60^\circ} = 2\angle -60^\circ A_{rms}$$

$$I_{bB} = 2\angle -180^\circ\ A_{rms}$$

$$I_{cC} = 2\angle -300^\circ\ A_{rms} = 2\angle 60^\circ A_{rms}$$

total avg. power: $P_{avg} = \frac{1}{2}\mathrm{Re}\{VI^*\}$

in phase A :



$P_{avg})_A = \frac{1}{2}\mathrm{Re}\{VI^*\}$

$= \mathrm{Re}\{V_{eff}I_{eff}\}$

$= \mathrm{Re}\{200\angle 0^\circ \times 2\angle -60^\circ\}$

$\Rightarrow \mathrm{Re}\{400\angle -60^\circ\} \Rightarrow 400\cos(-60) = 200W$

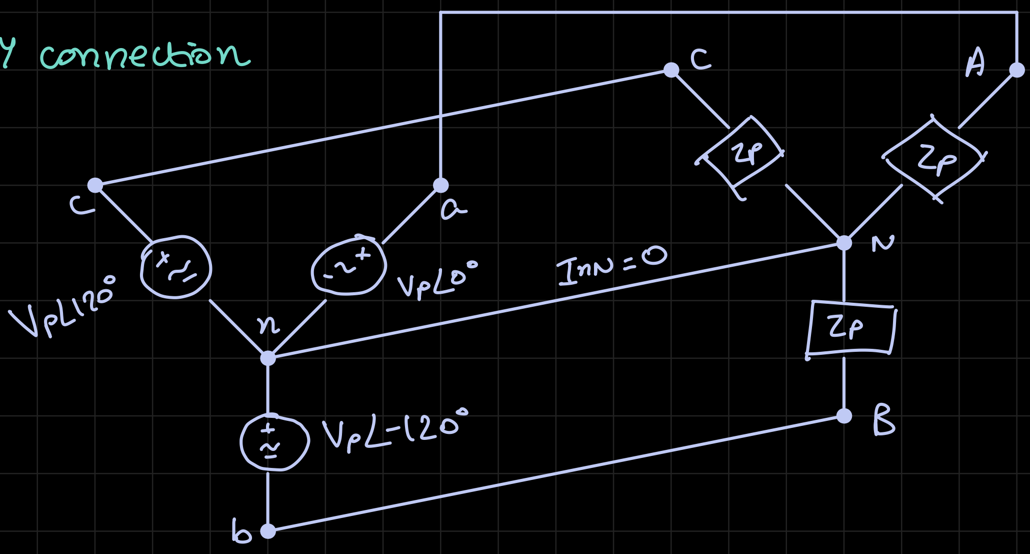
$P_{avg})_B = 200W$

$P_{avg})_C = 200W$

$\}\ P_{avg})_{total} = 600W$

* lecture : 10 11/09/24

→ Y-Y connection



balanced load : all load are same

balanced src : all src magnitudes are equal

line : lines connecting load to src
aA cC
bB nN

Phase Voltages: $V_{AN} = V_{an}$, $V_{BN} = V_{bn}$, $V_{CN} = V_{cn}$

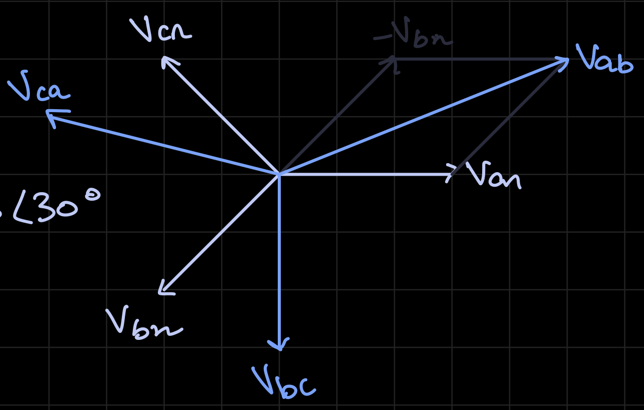
line voltages : V_{aA}, V_{bB}, V_{cC}

line currents : I_{aA}, I_{bB}, I_{cC}

phase currents : $I_{AN} = I_{aA}$, $I_{BN} = I_{bB}$, $I_{CN} = I_{cC}$

$V_{an} = V_p \angle 0°$
$V_{bn} = V_p \angle -120°$
$V_{cn} = V_p \angle -240°$

$V_{ab} = V_{an} - V_{bn}$



$$\underbrace{\begin{aligned} V_{ab} &= V_{an} - V_{bn} = \sqrt{3} V_p \angle 30° \\ V_{bc} &= \sqrt{3} V_p \angle -90° \\ V_{ca} &= \sqrt{3} V_p \angle -210° \end{aligned}}_{\text{line voltages}}$$

$V_{line} = \sqrt{3} V_{phase}$

line currents = phase currents

* Total Instantaneous Power

$$P_{total}(t) = p_A(t) + p_B(t) + p_C(t)$$

Classes & Attribs

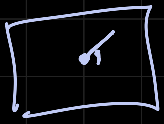
interface (entity)

- Bird
  - dmg (hits)
- Pig
  - health (hits)
- User
  - save stage
  - score
  - level_status
- Level mng
  - max_score
  - no. of pigs
  - ↳ blocks
    - type (wood glass steel)
  - ↳ pigs

(entry point)
GAME
update score

Slingshot
angle
stretch

V=10



W/A

S,D

$V\cos\theta, V\sin\theta$

per second / rate

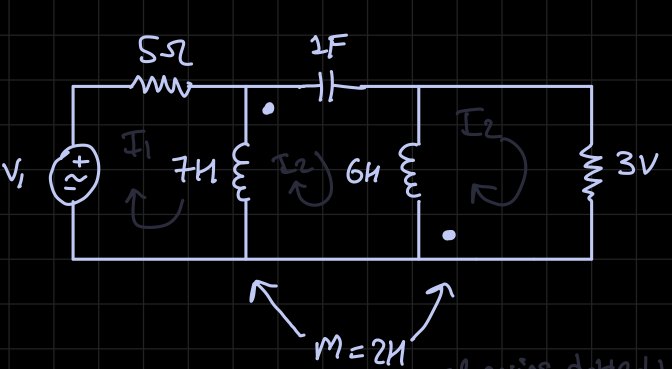
$-1 \to 0$

V

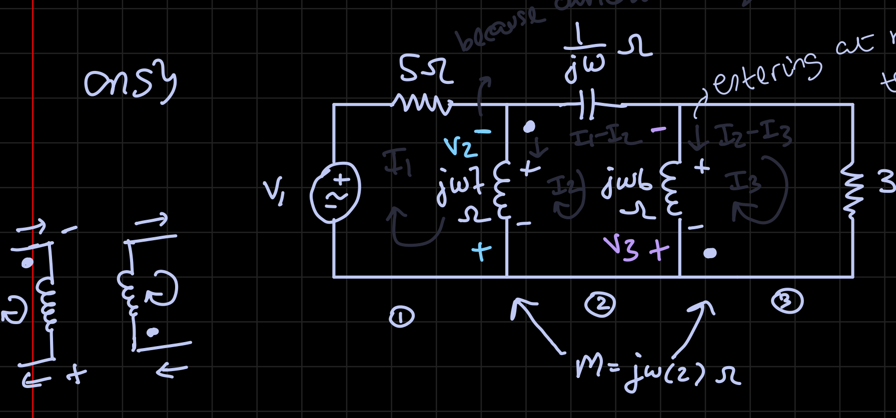
TODO : revise lecture 11 (missed due to SNS quiz)

* lecture 12:

eg3



ans}



① $-V_1 + 5I_1 - V_2 = 0$

$$V_2 = (j\omega 7)(I_1 - I_2)(-1)$$

because same sign of V_2 & V_3

$$V_2 = -j\omega 7(I_1 - I_2) + j2\omega(I_2 - I_3)$$

③ $3I_3 + V_3 = 0$

self + mutual inductance

same polarity

$$V_3 = -j6\omega(I_2 - I_3) + j2\omega(I_1 - I_2)$$

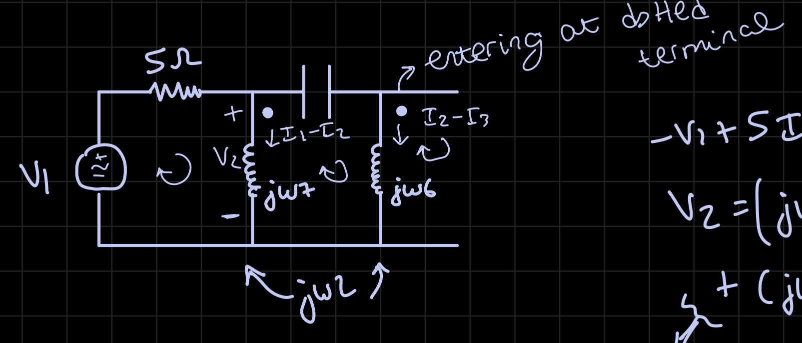
② $V_2 + \frac{1}{j\omega} I_2 - V_3 = 0$

$$-j\omega 7(I_1 - I_2) + j2\omega(I_2 - I_3) + j6\omega(I_2 - I_3) - j2\omega(I_1 - I_2) + \frac{I_2}{j\omega} = 0$$

$$I_1(-j5\omega) + I_2(j17\omega) + I_3(-j8\omega) + \frac{I_2}{j\omega} = 0$$

$$I_1(-j5\omega) + I_2\left(j17\omega + \frac{1}{j\omega}\right) + I_3(-j8\omega) = 0$$

eg3



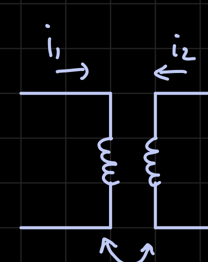
$-V_1 + 5I_1 + V_2 = 0$

$V_2 = (j\omega 7)(I_1 - I_2) + (j\omega 2)(I_2 - I_3)$

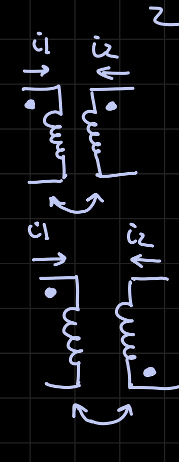
we donot need to care abt this sign if we use V_2, V_3 method.

* ENERGY STORED

$w(t) = \frac{1}{2} L (i(t))^2$ for only one inductor



$$w(t) = \frac{1}{2} L(i_1)^2 + \frac{1}{2} L(i_2)^2 \pm M i_1(t) i_2(t)$$



$\boxed{+ve}$ sign will occur iff both i_1 and i_2 are entering either dotted or undotted

$\boxed{-ve}$ sign iff both enter different (dotted/undotted)

• coupling coefficient (K)

$$M \leq \sqrt{L_1 L_2}$$

$$K = \frac{M}{\sqrt{L_1 L_2}} \Rightarrow 0 \leq K \leq 1$$

$\boxed{K \to 0}$ poor coupling or no coupling

$\boxed{K \to 1}$ strong coupling (very close to each other)

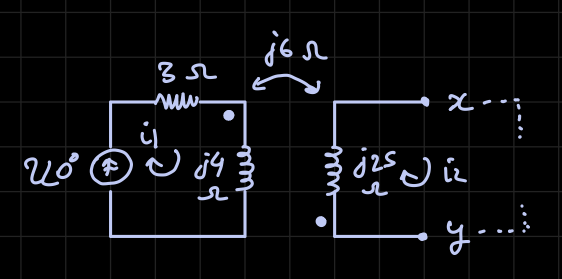
⇒ K depends on: distance btw coils ; size of coils ; ferrite core

$y \propto \frac{1}{x}$ $y \propto x$ $y \propto x$

uses: wireless power transfer (inductively coupled)
- electric vehicle charging
- magsafe charging

eg3 let $i_s = 2\cos(10t)$ A. find total energy stored in network at $t = 0$ if $k = 0.6$ and

(a) xy terminals are open circuited
(b) xy are short circuited



Ans} $w(t) = \frac{1}{2} L_1 (i_1)^2 + \frac{1}{2} L_2 (i_2) \pm M i_1(t) i_2(t)$

$$j\omega L_1 = j4 \Rightarrow L_1 = \frac{4}{\omega} = 0.4\ H$$

(a) if xy are open: $i_2 = 0$

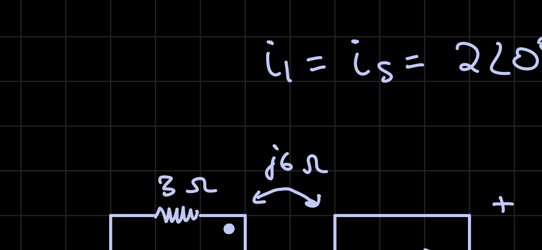
$$\text{so, } w(t) = \frac{1}{2} \times 0.4 \times (2\cos 10t)^2$$

$$w(t) = 0.8\cos^2(10t)$$

$$\text{at } t = 0 \Rightarrow w(t) = 0.8\ J$$

(b) $w(t) = \frac{1}{2} L_1 \cdot i_1^2 + \frac{1}{2} L_2 i_2^2 - M \cdot i_1 \cdot i_2$

$$i_1 = i_s = 2\angle 0°$$



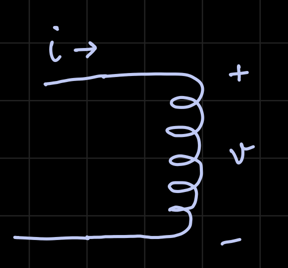
$i_2 = \frac{V_x}{j25}$ and $V_x = -j\omega M I_1$

$$i_2 = \frac{-0.6(2\angle 0°)}{2.5} = -0.48A$$

$$w(t=0) = \frac{1}{2}(0.4)(2^2) + \frac{1}{2}(\cdots$$

* Lecture : 11 {Revision} [absent] 24/9/24

• MODULE 5: Magnetically coupled circuits



$$v = L\frac{di}{dt}$$

current → magnetic flux

$i \propto \phi$ — ①

magnetic flux → induced voltage

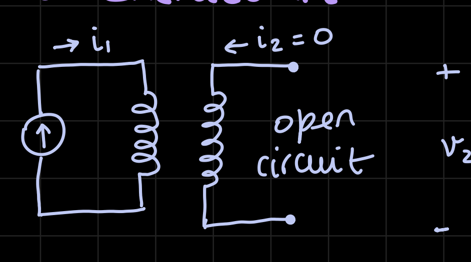
$v \propto \frac{d\phi}{dt}$ — ②

① and ② → $V = L\frac{di}{dt}$

for DC source: current is constant

hence $\frac{di}{dt} = 0$ and $\therefore v = 0$ (induced)

• Mutual Inductance

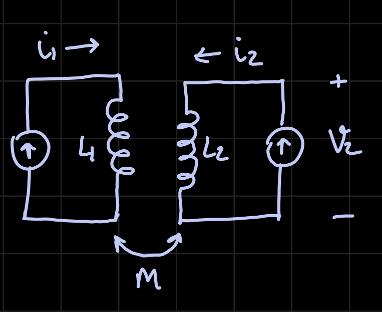


$v_2 \propto \frac{di_1}{dt}$

$V_2 = M\frac{di_1}{dt}$

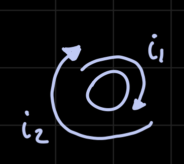
L: self inductance

mutual inductance

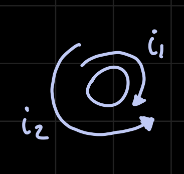


$$v_2 = \underbrace{L\frac{di_2}{dt}}_{\text{self inductance}} + \underbrace{M\frac{di_1}{dt}}_{\text{mutual inductance}}$$

* Additive / Substractive Property {two different coils}



$$v_2 = L\frac{di_2}{dt} + M\frac{di_1}{dt}$$

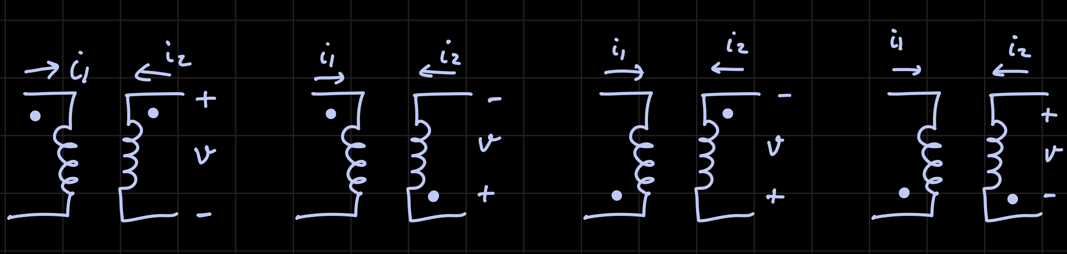


$$v_2 = L\frac{di_2}{dt} - M\frac{di_1}{dt}$$

• Dotted Notation

current entering _ terminal means +ve voltage reference at _

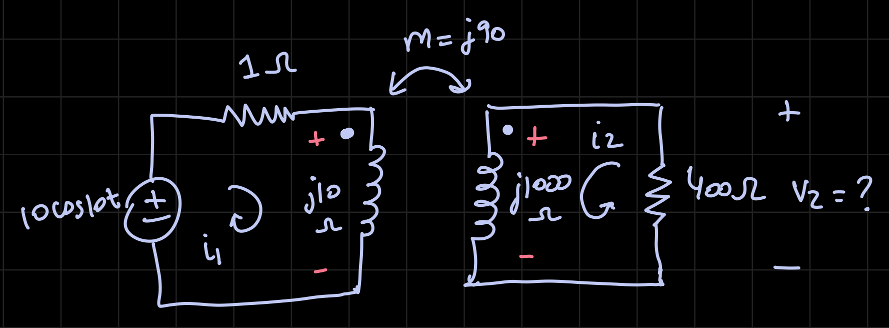
dotted/undotted



$v = L_1\frac{di}{dt} + M\frac{di_2}{dt}$ time domain

$V = j\omega L_1 I_1 + j\omega M I_2$ phasor domain

eg}



① $-10\angle 0° + I_1 + j10 I_1 + j90 I_2 = 0$

$I_1(1 + j10) + I_2(j90) = 10$

② $400 I_2 + j1000 I_2 + j90 I_1 = 0$

$I_1(j90) + I_2(400 + j1000) = 0$

CRAMMERS :

$$\Delta = \begin{vmatrix} 1+j10 & j1000 \\ j90 & 400+j1000 \end{vmatrix}$$

$= (1+j10)(400+j1000) + 90000$

$= 400 + j1000 + j4000 - 10.000 + 90.000$

$= j5000 + 80{,}400 = 80555\angle 3.55°$

$$\Delta_1 = \begin{vmatrix} j1000 & 10 \\ 400+j1000 & 0 \end{vmatrix} = -4000 - j10.000$$

$= 10770\angle -111.8°$

$$\Delta_2 = \begin{vmatrix} 10 & 1+j10 \\ 0 & j90 \end{vmatrix} = j900 = 900\angle 90°$$

$I_1 = \frac{\Delta}{\Delta_1} = -3.2 + 6.75j$

full cramers part wrong because I took $M = L_2 = j1000$ above

$$D_0 = \begin{vmatrix} 1+j10 & j90 \\ j90 & 400+j1000 \end{vmatrix}$$

$= 400 + j1000 + j4000 - 10.000 + 8100$

$= j5000 - 1500 = 5220\angle 106.69°$

---

energy stored in the circuit

$$w(t) = \frac{1}{2}L_1 i_1(t)^2 + \frac{1}{2}L_2 i_2(t)^2 \pm M i_1(t) i_2(t)$$

+: current entering same: • and • OR _ and _

−: current entering different: • and _ OR _ and •

COUPLING COEFFICIENT (K)

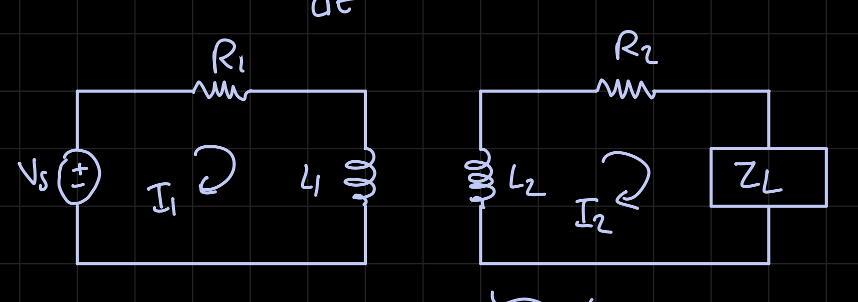
$$K = \frac{M}{\sqrt{L_1 L_2}}$$: note ∴ $M \leq \sqrt{L_1 L_2}$

So, $0 \leq K \leq 1$

* lecture 13 25/09/24

- **LINEAR TRANSFORMER**

when $v \propto \frac{di}{dt}$ $\Rightarrow$ no magnetic core



primary secondary

$$-V_s + I_1R_1 + (j\omega L_1)I_1 - (j\omega M)I_2 = 0$$
$$(j\omega L_2)I_2 + R_2I_2 + Z_LI_2 - (j\omega M)I_1 = 0$$

$$\Rightarrow \quad V_s = \underbrace{(R_1 + j\omega L_1)}_{Z_{11}} I_1 - j\omega M I_2$$
$$0 = (-j\omega M)I_1 + \underbrace{(R_2 + j\omega L_2 + Z_L)}_{Z_{22}} I_2$$

$$\left.\begin{aligned} V_s &= Z_{11}I_1 - j\omega M I_2 \\ 0 &= (-j\omega M)I_1 + Z_{22}I_2 \end{aligned}\right\} \Rightarrow \frac{I_2}{I_1} = \frac{j\omega M}{Z_{22}}$$

Input Impedance : $Z_{in} = \frac{V_s}{I_1} = Z_{11} - j\omega M\frac{I_2}{I_1}$

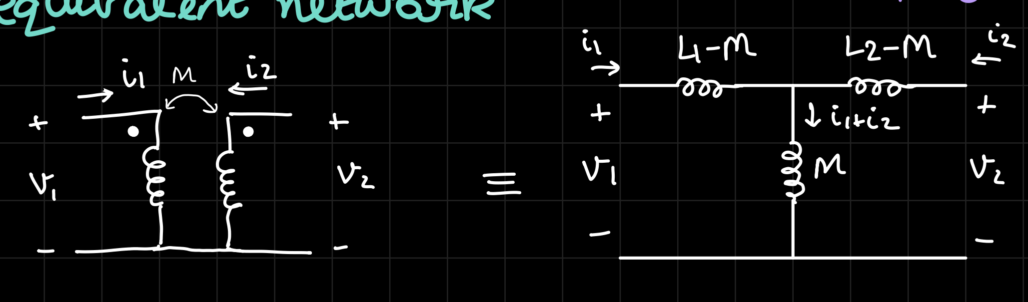
$$\boxed{Z_{in} = Z_{11} + \underbrace{\frac{\omega^2M^2}{Z_{22}}}_{\text{Reflective Impedence}}}$$

where

$$Z_{11} = R_1 + j\omega L_1 \quad , \quad Z_{22} = R_2 + j\omega L_2 + Z_L$$

- if $K = \frac{M}{\sqrt{L_1L_2}} = 0 \Rightarrow Z_{in} = Z_{11}$ i.e. no coupling

- **T equivalent network** (no mutual coupling)



$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$
$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

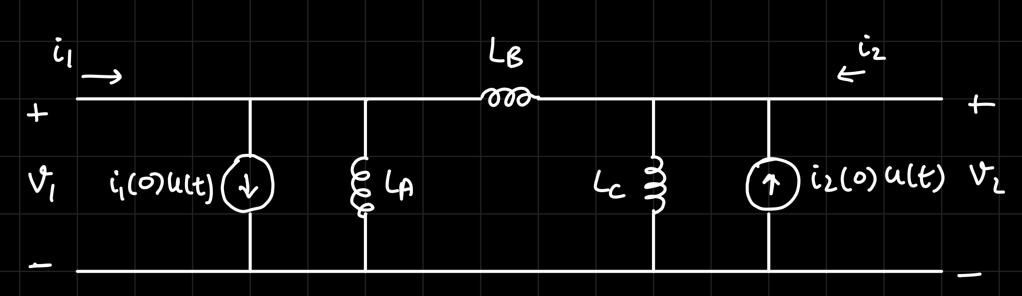
Note: no mutual coupling since there is no arrow for it

$$V_1 = (L_1 - M)\frac{di_1}{dt} + M\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right)$$
$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

similarly, we get V_2 same as we got in the left circuit. Hence, both circuits are equivalent.

Note: if dots are on opposite sides, replace M with $-M$

- **π EQUIVALENT NETWORK**



$$L_A = \frac{L_1L_2 - M^2}{L_2 - M}$$

$$L_B = \frac{L_1L_2 - M^2}{M}$$

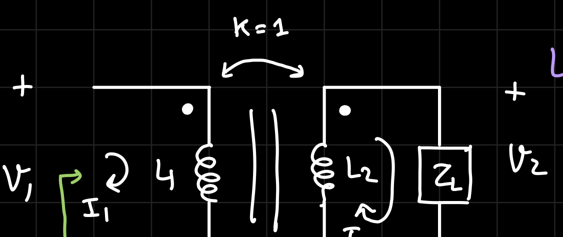
$$L_C = \frac{L_1L_2 - M^2}{L_1 - M}$$

use nodal analysis to prove this

$$v = L\frac{di}{dt} \Rightarrow i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$$
$$i = i(0)u(t) + \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

if dots are on opposite side : $M \to -M$

- **Ideal Transformer** $K = 1 \Rightarrow$ achieved only with ferrite core (iron) and $L_1 \,\&\, L_2 \gg Z_L$



$$V_1 = j\omega L_1 I_1 - j\omega M I_2$$
$$0 = -j\omega M I_1 + (j\omega L_2 + Z_L)I_2$$

$$Z_{in} = \frac{V_1}{I_1} = j\omega L_1 + \frac{\omega^2M^2}{Z_L + j\omega L_2}$$

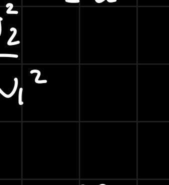
$$Z_{in} = \frac{Z_L}{\left(\frac{Z_L}{j\omega L_1}\right) + a^2}$$ and since $j\omega L_1 \gg Z_L \Rightarrow$

$$Z_{in} = \frac{Z_L}{a^2}$$

$$\left.\begin{aligned} &\text{Current ratio} : \frac{I_2}{I_1} = \frac{1}{a} \\ &\text{Voltage ratio} : \frac{V_2}{V_1} = a \end{aligned}\right\}$$ replace a by $-a$ if dots are on opp. side

- **TURN RATIO**

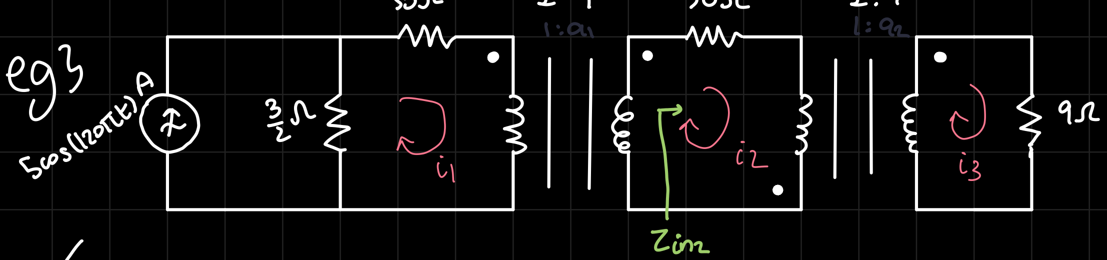
let it be 1:a



then

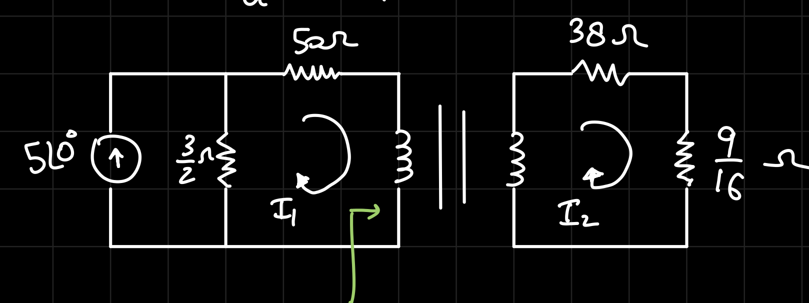
$$a = \frac{L_2}{L_1} = \frac{N_2^2}{N_1^2}$$

eg3

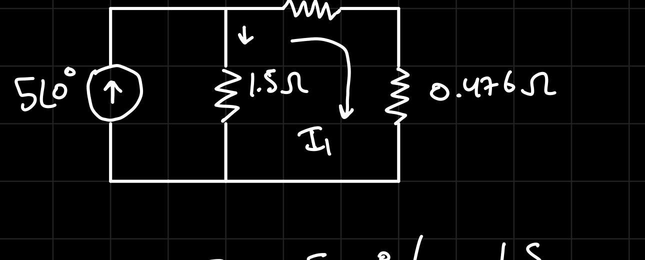


calculate power delivered to each resistor

$$Z_{in2} = \frac{Z_L}{a^2} = \frac{9}{16}\,\Omega$$



$$Z_{in1} = \frac{(9/16) + 38}{a_1^2} = 0.476\,\Omega$$



$$I_1 = 5\angle 0°\left(\frac{1.5}{1.5 + 50 + 0.476}\right) = 0.144\angle 0°\,A$$

We know $\frac{I_1}{I_2} = \frac{1}{a_1} \Rightarrow I_2 = \frac{0.144\angle 0°}{9} = 0.016\angle 0°\,A$

$$\frac{I_3}{I_2} = \frac{-1}{a_2} \Rightarrow I_3 = -\frac{0.016\angle 0°}{4} = -4\angle 0°\,mA$$

$$P_{avg\ 1.5\Omega} = \frac{1}{2}(5 - 0.144)^2 \times 1.5 = 17.67\,W$$

$$P_{avg\ 50\Omega} = \frac{1}{2}(0.144)^2 \times 50 = 0.518\,W$$

$$P_{avg\ 38\Omega} = \frac{1}{2}(0.016)^2 \times 38 = 0.0048\,W$$

$$P_{avg\ 9\Omega} = \frac{1}{2}(-4)^2 \times 10^{-6} \times 9 = 72\times10^{-6}\,W$$

* lecture : 14

• module : 6 → Circuit Analysis in S-domain
(textbook Chp 14)

$V_m \cos(\omega t+\theta) = Re\{V_m e^{j\omega t+\theta}\}$

$\Downarrow$

to phasor how?

↓ Remove Re{ } , $e^{j\omega t}$

$V_m e^{j\theta} = V_m \angle\theta$

in phasor form, we can eliminate the differential equations and converted to algebraic equations

We cannot use phasors if not sinusoidal signals ??

let $v(t) = V_m e^{\sigma t}\cos(\omega t+\theta)$

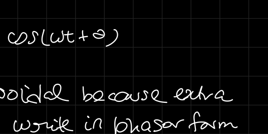
V(t) is not sinusoidal because extra term but we can write in phasor form using laplace transform

put $\sigma = 0,\ \omega = 0 \Rightarrow v(t) = V_m$ DC source

$\sigma = 0,\ \omega \neq 0 \Rightarrow v(t) = V_m\cos(\omega t+\theta)$ AC source
sinusoidally varying ✓

$\sigma \neq 0,\ \omega = 0 \Rightarrow v(t) = V_m e^{\sigma t}\cos(\theta) = V_1 e^{\sigma t}$
exponential signal

$\sigma \neq 0,\ \omega \neq 0 \Rightarrow$



$v(t) = V_m e^{\sigma t}\cos(\omega t+\theta)$

$= V_m e^{\sigma t}\left[\dfrac{e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}}{2}\right]$

$= \dfrac{V_m e^{j\theta}}{2}\left[e^{\overbrace{(\sigma+j\omega)}^{s}t} + e^{\overbrace{(\sigma-j\omega)}^{s^*}t}\right]$

$S \equiv$ complex frequency

$S = \sigma + j\omega$

σ = Neper Frequency (neper/sec)
ω = Radian frequency $= 2\pi f$ (rad/s)

• Laplace Transform

$f(t) \longrightarrow \boxed{\text{Laplace Transform}} \longrightarrow F(S)$

LT : invertible ✓

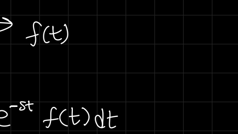
$F(S) \xrightarrow[\text{Transform}]{\text{Laplace}} f(t)$

one sided laplace transform $F(S) = \mathcal{L}(f(t)) = \int_{0^-}^{\infty} e^{-st} f(t)\,dt$

$f(t) = \dfrac{1}{2\pi j}\int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{st} F(S)\,ds$

$f(t) \Longleftrightarrow F(S)$

• Unit Step Function : u(t)



$\mathcal{L}(u(t)) = \int_{0^-}^{\infty} e^{-st} u(t)\,dt$

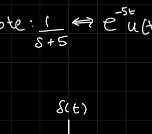
$= \int_{0}^{\infty} e^{-st}\,dt = \left[\dfrac{e^{-st}}{-s}\right]_0^{\infty} = -\dfrac{e^{-\infty}}{s} + \dfrac{e^{0}}{s}$

$= \dfrac{1}{s}$ ✓

$\boxed{u(t) \rightleftharpoons \dfrac{1}{s}}$

note : $\dfrac{1}{s+5} \Leftrightarrow e^{-5t}u(t)$

* Unit Impulse function $\delta(t)$



$\mathcal{L}(\delta(t)) = \int_{0^-}^{\infty} e^{-st}\delta(t)\,dt$

$\delta(t) = 1$ at $t = 0$

$\boxed{\delta(t) \rightleftharpoons 1}$

* Exponential

$e^{-\alpha t}u(t) \Longleftrightarrow \dfrac{1}{s+\alpha}$

$te^{-\alpha t}u(t) \Longleftrightarrow \dfrac{1}{(s+\alpha)^2}$

• Laplace Transform Theorems

① linearity theorems

$\mathcal{L}\{f_1(t)+f_2(t)\} = \mathcal{L}\{f_1(t)\} + \mathcal{L}\{f_2(t)\}$

summation in time domain ≡ summation in phasor domain

$\mathcal{L}\{kf(t)\} = k\mathcal{L}\{f(t)\}$

k is a constant
multiplication in time ≡ multiplication in phasor

eg} $V(S) = \dfrac{7}{S} - \dfrac{31}{S+17}$ find $v(t) = ?$

$v(t) = \mathcal{L}^{-1}\{V(S)\} = 7\mathcal{L}^{-1}\left\{\dfrac{1}{S}\right\} - 31\,\mathcal{L}^{-1}\left\{\dfrac{1}{S+17}\right\}$

$v(t) = 7u(t) - 31e^{-17t}u(t) = (7 - 31e^{-17t})\,u(t)$

• ZEROES & POLES

for some functions : $V(S) = \dfrac{N(S)}{D(S)}$

Zeroes : Values of S that lead to numerator, $N(S) = 0$

Poles : Values of S that lead to denominator, $D(S) = 0$

eg : $V(S) = \dfrac{S-3}{S(S+6)}$

Zeroes : $S = 3$
Poles : $S = 0, -6$

eg} $V(S) = \dfrac{1}{(S+\alpha)(S+\beta)}$ find $v(t)$

convert to : $V(S) = \dfrac{A}{S+\alpha} + \dfrac{B}{S+\beta}$
↳ method of residue

$\dfrac{1}{(S+\alpha)(S+\beta)} = \dfrac{A}{S+\alpha} + \dfrac{B}{S+\beta}$

$1 = A(S+\beta) + B(S+\alpha)$

when $S = -\alpha \Rightarrow 1 = A(\beta-\alpha)$

$A = \dfrac{1}{\beta-\alpha}$

when $S = -\beta \Rightarrow 1 = B(\alpha-\beta)$

$B = \dfrac{-1}{\beta-\alpha}$

$V(S) = \left(\dfrac{\frac{1}{\beta-\alpha}}{S+\alpha}\right) + \left(\dfrac{\frac{-1}{\beta-\alpha}}{S+\beta}\right)$

$v(t) = \dfrac{1}{(\beta-\alpha)}e^{-\alpha t}u(t) - \dfrac{1}{(\beta-\alpha)}e^{-\beta t}u(t)$

$= \dfrac{1}{(\beta-\alpha)}\left(e^{-\alpha t} - e^{-\beta t}\right)u(t)$

eg} $I(S) = \dfrac{7s+5}{s^2+s}$ $i(t) = ?$

zeroes : $\dfrac{-5}{7}$

poles : 0, −1

$I(S) = \dfrac{7S+5}{S(S+1)} = \dfrac{A}{S} + \dfrac{B}{S+1}$

$A = \dfrac{1}{\beta-\alpha} = \dfrac{1}{1}$, $B = \dfrac{-1}{\beta-\alpha} = -1$

$7S+5 = A(S+1) + B(S)$

$7s+5 = (A+B)s + A$

$A+B = 7 \Rightarrow B = 2$
$A = 5$

$I(S) = \dfrac{5}{S} + \dfrac{2}{S+1}$

$\mathcal{L}^{-1}(I(S)) = 5u(t) + 2e^{-t}u(t)$

⇒ Repeated Poles

$F(S) = \dfrac{2}{S^3+12S^2+36S} = \dfrac{2}{S(S^2+12S+36)}$

$F(S) = \dfrac{2}{S(S+2)^2}$

$F(S) = \dfrac{A}{S} + \dfrac{B}{S+2} + \dfrac{C}{S+2}$

if we try to solve like earlier, we get inconsistent eqns like $2 = 0$

for repeated poles :

$V(S) = \dfrac{N(S)}{(S-p)^n}$

$V(S) = \dfrac{a_n}{(S-p)^n} + \dfrac{a_{n-1}}{(S-p)^{n-1}} + \dfrac{a_{n-2}}{(S-p)^{n-2}} + \dots \dfrac{a_1}{(S-p)}$

$F(S) = \dfrac{A}{S} + \dfrac{B}{(S+6)^2} + \dfrac{C}{(S+6)}$

$2 = A(S+6)^2 + B(S) + C(S)(S+6)$

when $S = 0 \Rightarrow 2 = 36A \Rightarrow A = \dfrac{1}{18}$

when $S = -6 \Rightarrow 2 = -6B \Rightarrow B = -\dfrac{1}{3}$

differentiate to find C

$2 = A(S+6)^2 + BS + C\,S(S+6)$

$\dfrac{d(2)}{dS} \Rightarrow 0 = 2A(S+6) + B + C(S+6) + CS$

$\Rightarrow S = -6 \Rightarrow 0 = B + C(-6)$

$C = \dfrac{B}{6} = \dfrac{-1}{18}$

So, $A = \dfrac{1}{18}$, $B = \dfrac{-1}{3}$, $C = \dfrac{-1}{18}$

$F(t) = \dfrac{1}{18}u(t) - \dfrac{1}{3}\left(te^{-6t}u(t)\right) - \dfrac{1}{18}e^{-6t}u(t)$

• THEOREMS :

① Time differentiation theorem

$f(t) \rightleftharpoons F(S)$

$\boxed{\dfrac{d}{dt}f(t) \rightleftharpoons sF(S) - f(0^-)}$

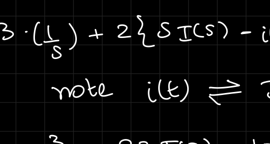
$L\dfrac{di}{dt} \to j\omega L I = SLI$
↳ when $\sigma = 0$

② Time Integration theorem

$f(t) \rightleftharpoons F(S)$

$\int_{0^-}^{t} f(t)\,dt \rightleftharpoons \dfrac{F(S)}{S}$

eg}



DC src + switch

$i(t) = ?$

KVL? ⇒ $-3u(t) + 2\dfrac{di}{dt} + 4i(t) = 0$

then integrate
but better way → Laplace

we cant use phasors because source ≠ sinusoidal
and we have a transient from $i(0^-) = 5A$

$\Rightarrow -3\cdot\left(\dfrac{1}{s}\right) + 2\{SI(S) - i(0^-)\} + 4I(S) = 0$

note $i(t) \rightleftharpoons I(S)$

$-\dfrac{3}{S} + 2S\,I(S) - 10 + 4I(S) = 0$

$I(S) = \dfrac{\frac{3}{S}+10}{2S+4} = \dfrac{10S+3}{2S^2+4S}$

$= \dfrac{A}{2S} + \dfrac{B}{S+2}$

$10S+3 = AS + 2A + 2BS$

$10S+3 = S(A+2B) + 2A$

$A = \dfrac{3}{2}$, $A+2B = 10$

$B = \dfrac{17}{4}$

$\Rightarrow I(S) = \dfrac{3/2}{2S} + \dfrac{17/4}{S+2} \Rightarrow i(t) = ?$

$i(t) = \dfrac{3}{4}u(t) + \dfrac{17}{4}e^{-2t}u(t)$ ✓

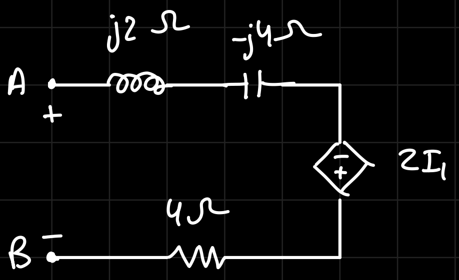
* CTD Practice

① Quiz : 1 (Set A)

mesh analysis: current
nodal analysis: voltage

• midsem : 2023

②



thevenin's voltage :

* lecture 15

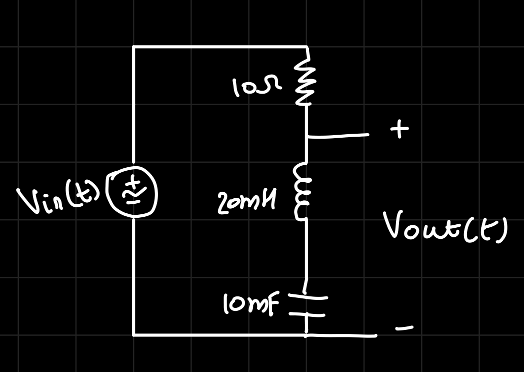
13/10/24

• Phasor vs S-domain

Assumptions:
zero initial conditions
sinusoidally varying source
"steady state"

<u>time domain</u>	<u>freq. domain</u>
$v(t) = V_m \cos(\omega t + \theta)$ $= Re\{V_m e^{j\omega t} e^{j\theta}\}$	$V = V_m e^{j\theta}$
$v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$ $= Re\{V_m e^{\sigma t} e^{j\theta} e^{j\omega t}\}$	$V = V_m e^{j\theta}$ where $s = \sigma + j\omega$ complex frequency
R	R
L	$L(\sigma + j\omega) = sL$
C	$\frac{1}{(\sigma + j\omega)C} = \frac{1}{sC}$

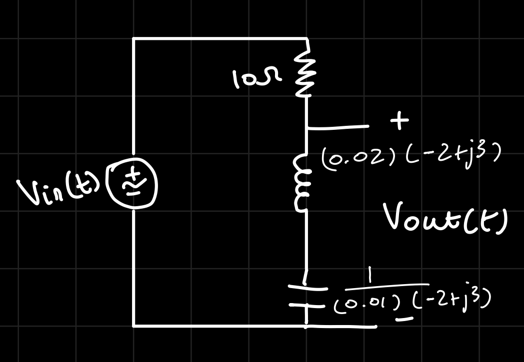
eg}



$V_{in}(t) = 5e^{-2t}\cos(3t + 45°)$

$S = \sigma + j\omega$
$S = -2 + j3$

$V_{in} = 5e^{j45} = 5\angle 45° \text{ V}$



Voltage divider for finding Vout

$$V_{out} = \frac{5\angle 45° \times (0.02)(-2+j3) \times \frac{1}{(0.01)(-2+j3)}}{10 + (0.02)(-2+j3) + \frac{1}{(0.01)(-2+j3)}}$$

$$= 5.86 \angle -24.4$$

$$V_{out}(t) = Re\{5.86 e^{j(-24.4)} e^{(\sigma + j\omega)t}\}$$

$$= 5.86 e^{-2t}\cos(3t - 24.4) \text{ V}$$

for zero initial condition, we can use phasors for $e^{\sigma t}\cos(\omega t + \theta)$

Can we still use phasors for $\delta(t)$, $u(t)$, $\sin(\omega t)u(t)$
NO
but laplace transform works ✓

$$\mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t)\, dt \Rightarrow F(s)$$

variable ✓

$\delta(t)$	1	$f_1(t) \pm f_2(t)$	$F_1(t) \pm F_2(t)$
$u(t)$	$1/s$	$kf(t)$	$KF(t)$
$tu(t)$	$1/s^2$	$\frac{d}{dt}f(t)$	$sF(s) - f(0^-)$
$e^{-at}u(t)$	$1/(s+a)$		
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\int_{0^-}^{t} f(t)dt$	$F(s)/s$
$\sin(\omega t)u(t)$	$\frac{\omega}{s^2+\omega^2}$	$f_1(t) * f_2(t)$	$F_1(t) \cdot F_2(t)$
$\cos(\omega t)u(t)$	$\frac{s}{s^2+\omega^2}$	$f(t-a)u(t-a)$	$e^{-as}F(s)$
$e^{-at}\sin(\omega t)u(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$f(t)e^{-at}$	$F(s+a)$

$$e^{-at}\cos(\omega t)u(t) = \frac{s}{(s+a)^2 + \omega^2}$$

Initial and final value theorems

$$f(0^+) = \lim_{s \to \infty} sF(s)$$
$$f(\infty) = \lim_{s \to 0} sF(s)$$

all poles of $sF(s)$ lie in left half plane

proof: $\mathcal{L}\{\sin(\omega t)u(t)\}$

$$= \int_{0^-}^{\infty} e^{-st}\left\{\frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)u(t)\right\}dt$$

$$= \frac{1}{2j}\int_{0^-}^{\infty} e^{t(j\omega - \sigma - j\omega)}u(t) - e^{-t(j\omega - \sigma - j\omega)}\, dt$$

$$= \frac{1}{2j}\int_{0^-}^{\infty} e^{-\sigma t}u(t) - e^{\sigma t}u(t)\, dt$$

$$= \frac{1}{2j}\mathcal{L}\{e^{j\omega t}\} - \mathcal{L}\{e^{-j\omega t}\}$$

$$= \frac{1}{2j}\left[\left(\frac{1}{s - j\omega}\right) - \left(\frac{1}{s + j\omega}\right)\right]$$

$$= \frac{1}{2j}\,\frac{2j\omega}{s^2 + \omega^2} = \frac{\omega}{s^2 + \omega^2}$$

example

① find $f(0^+)$ and $f(\infty)$ for $F(s) = \frac{4e^{-2s}(s+50)}{s}$

$$f(0^+) = \lim_{s \to \infty} sF(s) = \lim_{s \to \infty} \frac{4e^{-2s}(s+50)}{\cancel{s}}\cancel{s}$$

$$= \lim_{s \to \infty} \frac{4(s+50)}{e^{2s}}$$

$\frac{\infty}{\infty}$ form

use L'Hôpital's rule

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{f'(x)}{g'(x)}$$

$$\Rightarrow f(0^+) = \lim_{s \to \infty} \frac{4}{2e^{2s}} = \frac{1}{\infty} \text{ form} = \underline{0}$$

final value : $f(\infty) = \lim_{s \to 0} 4e^{-2s}(s+50)$

$$= 4(50) = \underline{200}$$

② find $f(0^+)$ for $F(s) = \frac{5s^2 + 10}{2s(s^2 + 3s + 5)}$

$$\Rightarrow f(0^+) = \lim_{s \to \infty} sF(s) = \frac{5s^2 + 10}{2s^2 + 6s + 10}$$

$$\lim_{s \to \infty} \frac{10s}{4s + 6} \Rightarrow \lim_{s \to \infty} = \frac{10}{4} = \frac{5}{2}$$

$$f(\infty) = \lim_{s \to 0} \frac{5s^2 + 10}{2(s^2 + 3s + 5)} = 1$$

example : find ILT of $F(s) = \frac{s^2 + 6}{s^2 + 7}$

$$F(s) = 1 - \frac{1}{s^2 + 7} = 1 - \frac{\sqrt{7}}{\sqrt{7}(s^2 + (\sqrt{7})^2)}$$

$$f(t) = \delta(t) - \frac{1}{\sqrt{7}}\sin(\sqrt{7}t)u(t)$$

eg} find ILT of $\frac{s+2}{s^2 + 2s + 4}$

$$\Rightarrow \frac{1}{s+2} \Rightarrow f(t) = e^{-2t}u(t) \quad \times$$

$$\Rightarrow \frac{(s+1) + 1}{(s+1)^2 + (\sqrt{3})^2} \Rightarrow \frac{s+1}{(s+1)^2 + (\sqrt{3})^2} + \frac{1}{s^2 + s + 5}$$

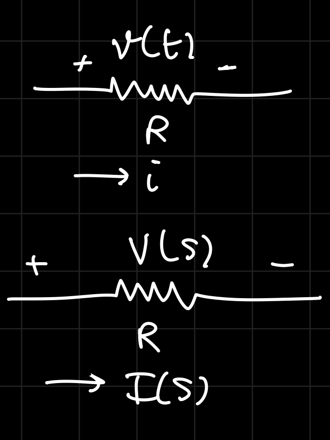
$$\Downarrow$$

$$\cos(\sqrt{3}t)u(t)$$

$$s^2 + 2s + 4$$
$$\Downarrow$$
$$s = -1 \pm j\sqrt{3}$$

Impedances in S-domain

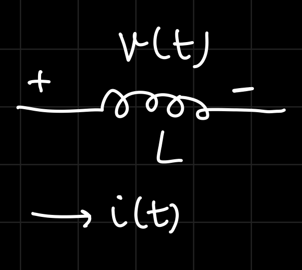
Resistor :



$V(t) = iR$

$V(s) = I(s)R$

Inductor :

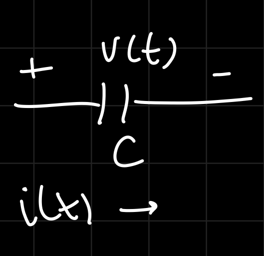


$$v(t) = L\frac{di(t)}{dt}$$

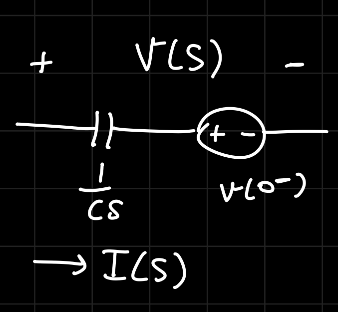
$$V(s) = L(sI(s) - i(0^-))$$

$$V(s) = sL\,I(s) - L\,i(0^-)$$

Capacitor :



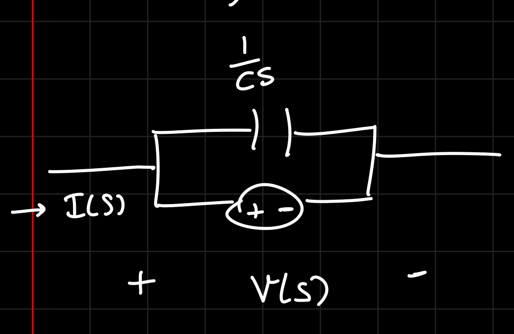
$$i(t) = C\frac{dv}{dt}$$



$$I(s) = C[sV(s) - v(0^-)]$$

$$I(s) = CsV(s) - Cv(0^-)$$

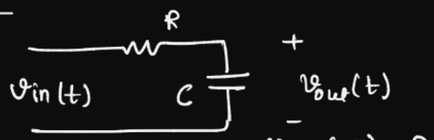
$$V(s) = \frac{1}{Cs}I(s) + \frac{v(0^-)}{s}$$



* Lecture 16 {missed}

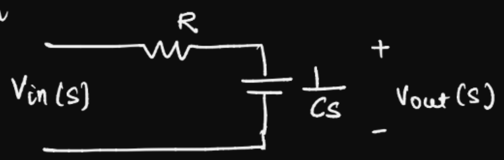
Transfer function :

Lets consider :



$v_{out}(0^-) = 0$

freq. domain



$$V_{out}(s) = \frac{1/Cs}{R + 1/Cs} V_{in}(s)$$

$$\Rightarrow \underbrace{\frac{V_{out}(s)}{V_{in}(s)}}_{\text{transfer function. } H(s)} = \frac{1}{sRC+1}$$

$$v_{out}(t) = \mathcal{L}^{-1}\{V_{out}(s)\} = \mathcal{L}^{-1}\{\underbrace{H(s)}_{\text{defines the system.}} \cdot V_{in}(s)\}$$

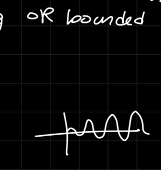
* lecture: 17 — 21/10/24

• Stability

Stable: bounded o/p for bounded i/p
unstable: unbounded o/p for bounded i/p
marginally stable: oscillating OR bounded offset

let: $e^{\sigma t}\cos(\omega_o t)\,u(t)$

① $\sigma > 0$: unstable
② $\sigma < 0$: stable
③ $\sigma = 0$: marginally stable

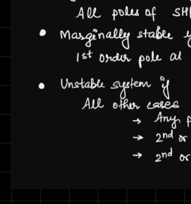


* **Stability Criterion**

System transfer fn $H(S) = k\dfrac{(S-z_1)(S-z_2)(S-z_3)\cdots(S-z_m)}{(S-p_1)(S-p_2)(S-p_3)\cdots(S-p_n)}$

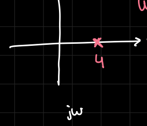
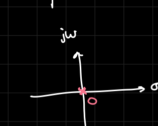
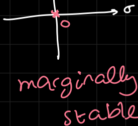
- System is stable if
  All poles of $SH(S)$ lie on LHS of complex plane
- Marginally stable if
  1st order pole at $S=0$ OR 1st order complex conjugate pole pair at $j\omega$-axis
- Unstable system if
  All other cases
  → Any poles on RHS OR
  → 2nd or higher order pole at $S=0$ OR
  → 2nd or higher order conjugate pole pair at $j\omega$-axis

order of poles ≡ order of S ($S = \sigma + j\omega$)



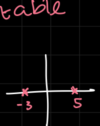
at $j\omega$ axis:
could be unstable
or marginally stable

	$f(t)$	$sF(s)$	
①	$e^{-2t}u(t)$	$s\cdot\frac{1}{s+2}$	stable
②	$e^{4t}u(t)$	$s\frac{1}{s-4}$	unstable
③	$u(t)$	$s\cdot\frac{1}{s}$	marginally stable

(③: don't cancel — we have one zero and one pole)

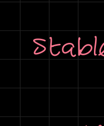
④ $e^{-3t}u(t) + e^{+5t}u(t)$ $\quad s\left\{\frac{1}{s+3} + \frac{1}{s-5}\right\}$ unstable



⑤ $s\cdot\dfrac{8}{(s+1)(s+2)(s+3)}$ stable

⑥ $s\cdot\dfrac{14}{(s+1-\sqrt{2})(s-1+\sqrt{2})}$ unstable

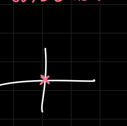
⑦ $tu(t)$ $\quad \dfrac{1}{s^2}$
↳ poles = 2
2nd order pole at $s=0$
unstable



⑧ $e^{-3t}\sin(\omega_o t)\,u(t)$ $\quad s\,\dfrac{\omega_o}{(s+3)^2+\omega_o^2}$

stable

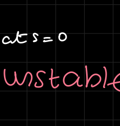
poles:
$(s+3)^2 + \omega_o^2 = 0$
$(s+3)^2 = -\omega_o^2$
$s+3 = \pm\sqrt{-1}\,\omega_o$
$s = -3 \pm j\omega_o$



⑨ $\sin(\omega_o t)\,u(t)$ $\quad s\cdot\dfrac{\omega_o}{s^2+\omega_o^2}$

marginally stable

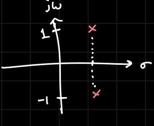
$s^2 = -\omega_o^2$
$s = \pm j\omega_o$



⑩ $e^{3t}\sin(\omega_o t)\,u(t)$ $\quad s\cdot\dfrac{\omega_o}{(s-3)^2+\omega_o^2}$

unstable (RHS)

$s-3 = \pm j\omega_o$
$s = 3 \pm j\omega_o$



⑪ $t\cos(\omega_o t)\,u(t)$

note: $f(t) \rightleftharpoons F(s)$
$tf(t) \rightleftharpoons -\frac{d}{ds}F(s)$

$\cos(\omega_o t)u(t) \rightleftharpoons \dfrac{s}{s^2+\omega_o^2} = s(s^2+\omega_o^2)^{-1}$

$\frac{d}{ds} = (s^2+\omega_o^2)^{-1} - s(s^2+\omega_o^2)^{-2}(2s)$

$= \dfrac{1}{s^2+\omega_o^2} - \dfrac{2s^2}{(s^2+\omega_o^2)^2}$

$= \dfrac{s^2+\omega_o^2-2s^2}{(s^2+\omega_o^2)^2} = \dfrac{\omega_o^2 - s^2}{(s^2+\omega_o^2)^2}$

$s = \pm j\omega_o$ (2nd order)

unstable

• Final Value Theorem

$$f(t\to\infty) = \lim_{s\to 0} sF(s)$$

↳ when all poles are on LHS (condition)

• Initial Value theorem

$$f(0) = \lim_{s\to\infty} sF(s) \quad \text{(no condition)}$$

eg} $f(t) = \sin(\omega_o t)\,u(t)$

final value: $f(\infty) = \lim_{s\to 0} sF(s)$

$= \lim_{s\to 0} s\,\dfrac{\omega_o}{\omega_o^2+s^2} = 0$ ✗

indeterminant because condition not met
(all poles are not on LHS)

• Chapter 15 {frequency response}

assuming zero initial conditions:

capacitor impedance $= Z_C = \dfrac{1}{sC}$

where $s = \sigma + j\omega$

we represent capacitor as open circuit in case of DC input voltage

Transfer function:

$H(s) = \dfrac{V_{out}(s)}{V_{in}(s)} \Rightarrow V_{out}(s) = \underbrace{V_{in}(s)}_{\text{input}} \cdot \underbrace{H(s)}_{\text{system}}$

let $\sigma = 0$ since we are only considering frequency
$\Rightarrow s = j\omega$

$H(j\omega) = \dfrac{V_{out}(j\omega)}{V_{in}(j\omega)}$ Voltage gain V/V

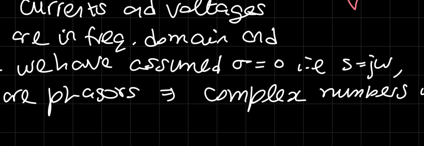
$H(j\omega) = \dfrac{I_{out}(j\omega)}{I_{in}(j\omega)}$ current gain A/A

$H(j\omega) = \dfrac{V_{out}(j\omega)}{I_{in}(j\omega)}$ transfer impedance $\frac{V}{A}$ or Ω

$H(j\omega) = \dfrac{I_{out}(j\omega)}{V_{in}(j\omega)}$ transfer admittance $\frac{A}{V}$ or Ω^{-1}

note: currents and voltages here are in freq. domain and since we have assumed $\sigma = 0$ i.e $s = j\omega$, they are phasors ⇒ complex numbers ✓

$H(j\omega) = |H(j\omega)|\,\angle H(j\omega)$



here these are low pass filters

• DECIBELS SCALE (dB)

$$H_{dB} = 20\log_{10}|H(j\omega)|$$

note: $\log_{10}1 = 0$
$\log_{10}10 = 1$
$\log_{10}2 = 0.3$
$\log_{10}(0.1) = -1$
$\log_{10}(10^{-k}) = -k$
$\log_{10}(0) = \text{undefined}$

find H_{dB} at $\omega = 146$ rad/s if $H(s) = 20(s+100)$

⇒ $H_{dB} = 20\log_{10}|H(j\omega)|$
$= 20\log_{10}|20j\omega + 2000|$
$= 20\log_{10}\left(20\sqrt{\omega^2+100^2}\right)$ ↳ $\omega = 146$
$= 70.97$ dB

---

let ....
$H(s) = K\dfrac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$

$H_{dB} = 20\log|K| + 20\log|s-z_1| + 20\log|s-z_2| + \cdots 20\log|s-z_m| - 20\log|s-p_1| - 20\log|s-p_2| - \cdots 20\log|s-p_n|$

let $H(j\omega) = H_1(j\omega)\cdot H_2(j\omega)$

$20\log_{10}|H(j\omega)| = 20\log_{10}|H_1(j\omega)| + 20\log|H_2(j\omega)|$

$\angle H(j\omega) = \angle H_1(j\omega) + \angle H_2(j\omega)$

⇒ consider: $H(s) = 1 + \dfrac{s}{a}$ → simple zero

$H(j\omega) = 1 + \dfrac{j\omega}{a}$

$H_{dB} = 20\log_{10}\left|1 + j\left(\frac{\omega}{a}\right)\right|$
$= 20\log_{10}\left(\sqrt{1^2 + \frac{\omega^2}{a^2}}\right)$

① let $\omega \lll a$
⇒ $\frac{\omega}{a}$ → negligible
$H_{dB} = 20\log_{10}(1) = \underline{0\text{ dB}}$

② if $\omega \ggg a$
⇒ 1^2 term becomes negligible
$H_{dB} = 20\log_{10}\left(\frac{\omega}{a}\right)$ dB

③ if $\omega = 10a$
$H_{dB} = 20\log(10) = 20$ dB

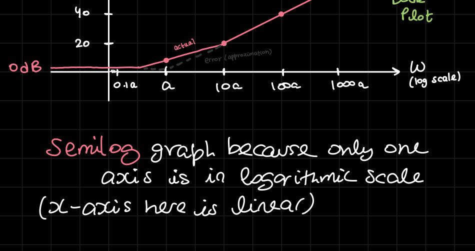
④ $\omega = 100a$
$H_{dB} = 20\log(100) = 40$ dB

⑤ $\omega = 1000a$
$H_{dB} = 60$ dB

⑥ $\omega = 10000a$
$H_{dB} = 80$ dB

Observation: $\omega \times 10 \to H_{dB} + 20$ dB

using log scale instead of ω directly



Semilog graph because only one axis is in logarithmic scale
(x-axis here is linear)

now let $\omega = a \Rightarrow H_{dB} = 20\log_{10}(\sqrt{2})$
$= 20 \times \frac{1}{2}\log_{10}(2)$
$= 3$ dB

corner frequency / break freq at $\omega = a$

- Lecture 18 {Bode Plots}

Missed

23/10/24

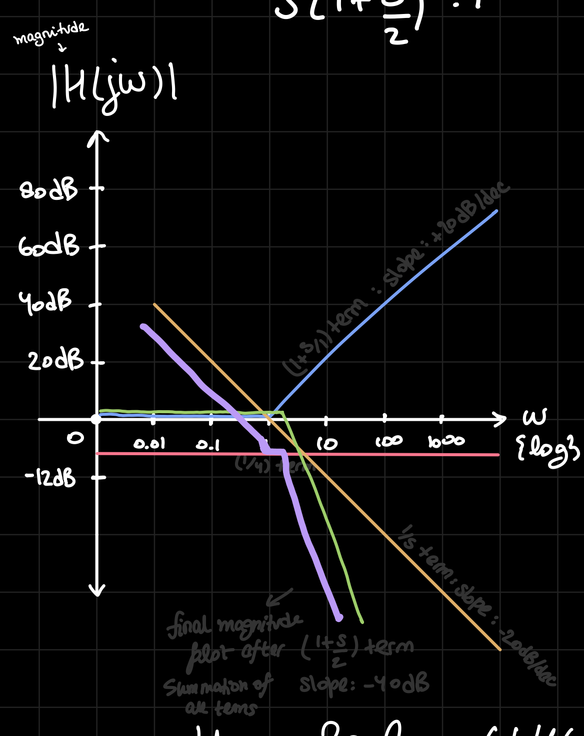
Lecture 19 04/11/24

eg3 draw bode plots of $H(s) = \frac{s+1}{s(s+2)^2}$

zeroes : 1 → $s = -1$
poles : 3 → $s = 0, -2, -2$

$$H(s) = \frac{(1+s/1)}{s(1+\frac{s}{2})^2 \cdot 4} = \frac{1}{4} \cdot \frac{1}{s} \cdot (1+\frac{s}{1}) \cdot \frac{1}{(1+\frac{s}{2})^2}$$

simple pole case so, we have corner frequency at $w = a$ i.e. at $w=1$ & $w=2$ respectively



$$H_{dB} = 20\log_{10}(|H(jw)|)$$

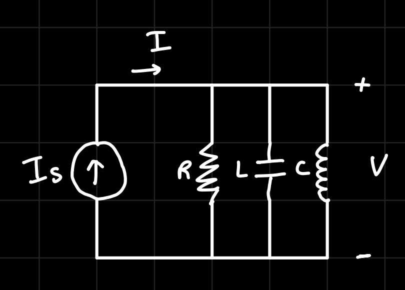
$$= 20\log_{10}(\tfrac{1}{4}) + 20\log_{10}(|\tfrac{1}{jw}|)$$

$-20\log(w)$ straight line no break frequency

$$20\log_{10}(1+\tfrac{s}{1}) + 40\log_{10}(1+\tfrac{s}{2})$$

todo: phase plot for ↑

- **Resonance** : A network is in resonance when V and I at network input terminals are in phase



① Assume : $I_s = I_o \cos(wt + \phi)$

$$Y(jw) = \frac{1}{Z(jw)} = \frac{1}{R} + jwC + \frac{1}{jwL}$$

$$= \frac{1}{R} + j(wC - \frac{1}{wL})$$

for resonance ⇒ V and I are in phase (purely resistive circuits)

i.e. $wC - \frac{1}{wL} = 0$

⇒ $w = \frac{1}{\sqrt{LC}} = w_0$ "resonant frequency"

② Assume $I_s = I_o e^{\sigma t} \cos(wt + \phi)$

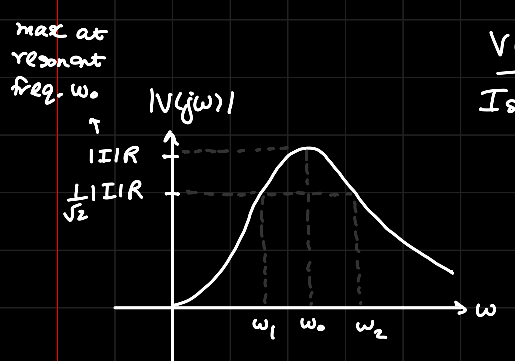
$$Y(s) = \frac{1}{Z(s)} = \frac{1}{R} + sC + \frac{1}{sL}$$

$$= \frac{s^2 + s(1/RC) + (1/LC)}{(1/C)s}$$

$V_{out} = V$
$I_{in} = I_s$ ⇒ transfer function?

$$\frac{V(s)}{I_s(s)} = Z(s) = \frac{1}{Y(s)}$$

transfer impedance



w_1, w_2 : half power frequency

height of this curve depends only on value of Resistance (R)

Width of the curve depends on L & C as well

There will be some current flowing in capacitor & inductor but it is equal and opposite so it cancels out and net total current flows only through the resistor

At resonance :

$I_s = I_R$
$I_C = -I_L$ ⇒ $I_C + I_L = 0$

quality factor

$$Q = 2\pi \frac{\text{max. energy stored}}{\text{total energy lost per time period}}$$

$$= 2\pi \frac{[w_L(t) + w_C(t)]_{max}}{P_R \cdot T}$$

P_R : avg power lost in resistor
$= \frac{1}{2}|I|^2 R$ WATTS

T : time period (sec)
$w_L(t)$: $\frac{1}{2} L i_L^2(t)$ (J)
$w_C(t)$: $\frac{1}{2} C v_C^2(t)$ (J)

$$Q = w_0 RC = R\sqrt{\frac{C}{L}}$$

α = damping factor $= \frac{1}{2RC} = \frac{w_0}{2Q_0}$

w_d : damped frequency $= \sqrt{w_0^2 - \alpha^2}$
$= w_0 \sqrt{1 - (\frac{1}{2Q_0})^2}$

- **HALF POWER BANDWIDTH (BW)**

$$BW = w_2 - w_1 = \frac{w_0}{Q_0}$$

where $w_0 = \sqrt{w_1 w_2}$

note: $BW \propto \frac{1}{Q_0}$

note: if $Q_0 > 5$: high-Q circuit