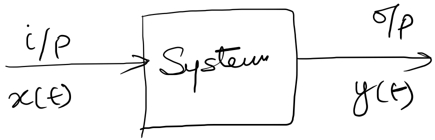
30/8/2024

Systems & Their properties



$x[n] \longrightarrow y[n]$

① System : with memory or without memory

$S_1 : y(t) = x(t)$

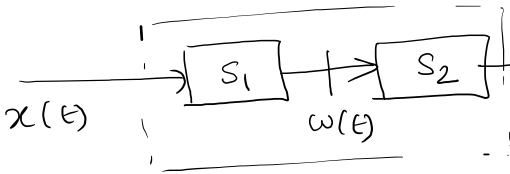
$S_2 : y(t) = x^2(t)$

$S_3 : y(t) = x(t-2)$

$S_4 : y(t) = \frac{1}{2}\left\{ x(t) + x(t-1) \right\}$

$S_5 : y(t) = x(t+2)$

② Invertible / Non-invertible Systems



$y(t) = x(t)$ if $S_2 = inv(S_1)$

ⓐ

$S_1 : \quad w(t) = 2x(t)$

$S_2 : \quad y(t) = \frac{1}{2} w(t) = \frac{1}{2} \times 2x(t) = x(t)$

S_1 is invertible & the inverse system is S_2 :

ⓑ

$S_1 : \quad w[n] = \sum_{k=-\infty}^{n} x[k]$

$S_2 : \quad y[n] = w[n] - w[n-1]$

$= \sum_{k=-\infty}^{n} x[k] - \sum_{k=-\infty}^{n-1} x[k]$

$= \cancel{\sum_{k=-\infty}^{n-1} x[k]} + x[n] - \cancel{\sum_{k=-\infty}^{n-1} x[k]}$

$= x[n]$

(c) $x[n] \longrightarrow \boxed{S_1} \xrightarrow{w[n]} \boxed{S_2} \longrightarrow y[n]$

$$S_1 : w[n] = x[n] - x[n-1]$$

$$S_2 : y[n] = \sum_{k=-\infty}^{n} w[k]$$

Invertible

(d)

$$\left.\begin{aligned} S_1 &: w(t) = x^2(t) \\ S_2 &: y(t) = \sqrt{w(t)} \end{aligned}\right\}$$ S_1 is not invertible

If distinct i/ps lead to distinct o/ps, the system is invertible, else not.

3 Causal Systems —

$S_1 : \quad y(t) = x(t - t_0)$ $\quad$ t_0 is +ve real no.

$S_2 : \quad y[n] = x[n+3]$

$S_1 \equiv$ causal system

$S_2 \equiv$ non-causal system

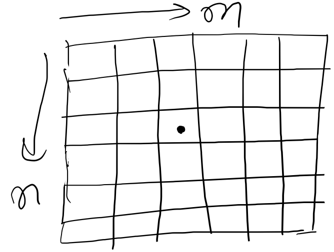
Causality: Unless there is a cause, there should be no effect.

Or unless there is a non-zero i/p, there should not be a non-zero o/p.

Or a causal system depends on current & past i/p.

Accumulator: $\quad S_3 : \quad y[n] = \sum_{k=-\infty}^{n} x[k]$ causal.

S_4: $$y[n,m] = \frac{1}{5}\Big\{ x[n,m] + x[n-1,m] + x[n+1,m] + x[n,m-1] + x[n,m+1] \Big\}$$



Non-causal System

④ **Stability** —

BIBO Stability

Stable System : A Bounded i/p should lead to a bounded o/p.

is on amplitude

Bounded i/p $\equiv$ $|x(t)| < M_x < \infty$

$\hookrightarrow$ finite value

$\forall\, t$

Or $|x[n]| < M_x < \infty$

$\hookrightarrow$ finite value for all n.

this bounded i/p is fed to a system.

If The system generates an o/p, That is also bounded

Then, we call it a stable system —

Examples of bounded i/ps.

$x(t) = u(t)$; $x[n] = u[n]$

unbounded signals

$x(t) = t\,u(t)$ or $x[n] = t\,u[n]$

if $|y(t)| \leq M_y < \infty \quad \forall t$

$\hookrightarrow$ finite no.

for bounded i/p, then the system is stable.

S_1: $y[n] = \frac{1}{3}\left\{ x[n] + x[n-1] + x[n+1] \right\}$

$|y[n]| = \left| \frac{1}{3} x[n] + \frac{1}{3} x[n-1] + \frac{1}{3} x[n+1] \right|$

$\leq \frac{1}{3}|x[n]| + \frac{1}{3}|x[n-1]| + \frac{1}{3}|x[n+1]| \quad \forall n$

if $|x[n]| < M_x \quad \forall n$

$|y[n]| =$ bounded for all n.

S_2 : $y[n] = \frac{1}{3}\left\{ x[n] - x[n-1] - x[n-2] \right\}$

$$|y[n]| = \frac{1}{3}\left| x[n] - x[n-1] - x[n-2] \right|$$

Since $|x[n]| < M_x < \infty \; \forall n$

Sum & difference of 3 finite nos. will again be a finite no.

So $|y[n]| < M_y < \infty \quad \forall n$

Stable System.

S_3 : $y[n] = n\,x[n]$

$|x[n]| < M_x < \infty \quad \forall n$

bounded i/p

$|y[n]| = |n\,x[n]| \longrightarrow \infty$ as $n \longrightarrow \infty$

Unstable system

S_4 : $y(t) = x(2t)$ Stable system

(5) Time - Invariant Systems

$$x(t) \xrightarrow{S} y(t)$$

$$x(t-t_0) \longrightarrow y(t-t_0)$$

$$x(t+t_0) \longrightarrow y(t+t_0)$$

$$x_1(t) \rightarrow \boxed{\quad} \rightarrow y_1(t)$$

If $\quad x_2(t) \longrightarrow y_2(t)$

if $\quad x_2(t) = x_1(t-t_0) \longrightarrow$ Is $y_2(t) = y_1(t-t_0)$?

If yes, it is a time-invariant system

else, not.

$y_2(t) =$

$S: \; y(t) = x(2t)$

$x_1(t) \longrightarrow y_1(t) \; \boxed{= x_1(2t)}$

$x_2(t) \longrightarrow y_2(t) \; \boxed{= x_2(2t)}$

$\boxed{y_1(t - t_0) = x_1(2t - 2t_0)}$ —— ⓐ I o/p

$x_2(t) = x_1(t - t_0) \Rightarrow y_2(t) = x_1(2t - t_0)$ ⓑ $\Rightarrow$ II o/p

ⓐ $\neq$ ⓑ

$y_2(t) \neq y_1(t - t_0)$

So, "S" is a time-varying system

$S_2 : y[n] = n\,x[n]$

$x_1[n] \longrightarrow y_1[n] = n\,x_1[n]$

$\downarrow$ delayed o/p

$y_1[n-n_0] = (n-n_0)\,x_1[n-n_0]$ — (a)

$x_2[n] \longrightarrow y_2[n] = n\,x_2[n]$

Let $x_2[n] = x_1[n-n_0]$

delayed i/p

$y_2[n] = n\,x_1[n-n_0]$ — (b)

But $y_2[n] \neq y_1[n-n_0]$

Time-varying

(5) Linearity —

A system is linear if the properties of additivity & homogeneity holds true.

$$x_1(t) \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow y_2(t)$$

$$a x_1(t) + b x_2(t) \longrightarrow a y_1(t) + b y_2(t) \longrightarrow \text{System is Linear}$$

$$\forall\, a, b \in \mathbb{R}$$

Additivity : $x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$

homogeneity : $a x_1(t) \longrightarrow a y_1(t)$

$$S: \quad y(t) = 2x(t) + 3 = 2(i/p) + 3$$

$$x_1(t) \longrightarrow y_1(t) = 2x_1(t) + 3$$

$$x_2(t) \longrightarrow y_2(t) = 2x_2(t) + 3$$

$$\boxed{\overset{i/p}{a x_1(t) + b x_2(t)}} \longrightarrow 2\left(i/p \right) + 3$$

$$= 2\left(a x_1(t) + b x_2(t) \right) + 3 \quad \text{——— (a)}$$

$$= 2a x_1(t) + 2b x_2(t) + 3$$

$$a y_1(t) + b y_2(t) = a\left(2x_1(t) + 3\right) + b\left(2x_2(t) + 3\right)$$

$$= 2a x_1(t) + 3a + 2b x_2(t) + 3b \quad \text{(b)}$$

Not Linear

A system that is both linear & time-invariant is called an LTI system

LTI $\equiv$ Linear & Time Invariant