28/8/2024

C.T. Signals — frequencies are unique

$0 \longrightarrow \infty$

D.T. Signals — $[0, 2\pi]$ unique frequencies

$$x[n] = e^{j\omega_0 n}$$

Let $\omega_0 = 0$ $\quad = e^{j0n} = e^{j0} = \underbrace{\cos 0}_{=1} + j\underbrace{\sin 0}_{=0}$

$= 1$

$\omega_0 = 2\pi$ $\quad = e^{j2\pi . n} = \cos 2\pi n + j \sin 2\pi n \quad \forall n$

$= 1$

4π

6π

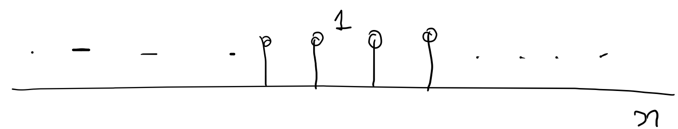
$\vdots$

-2π

$-4\pi \ldots$

$\omega_0 = 2\pi K$

$x[n]$



$w_0 = \pi$

$$x[n] = e^{jw_0 n} = e^{j\pi n}$$

$$= \cos \pi n + j \sin \pi n$$

$$= (-1)^n$$

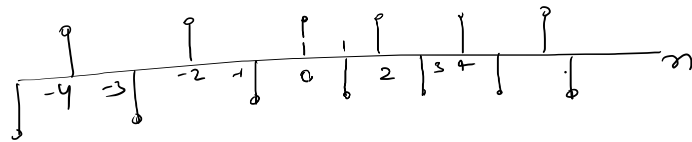
$w_0 = (2k+1)\pi$

$[-\pi, \pi]$

$[0, 2\pi]$

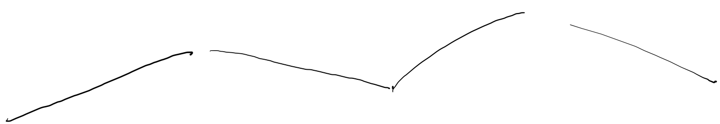
$\ldots -3\pi = -\pi = \pi = 3\pi = 5\pi \ldots$

$x[n]$



$[0 \text{ — } 2\pi]$

$[0 \text{ — } \pi \text{ — } 2\pi]$

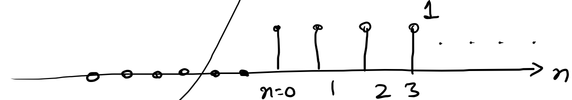


D.T. Signals

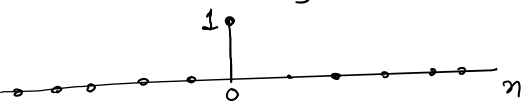
Unit Step signal $u[n]$ | unit Impulse fn. $\delta[n]$

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

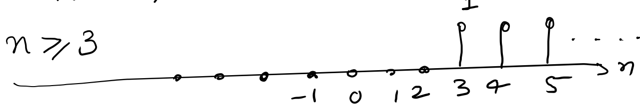


$\delta[n]$



$u[n-3]$ $n-3 \geq 0$

$n \geq 3$



$\delta[n-3]$, $\delta[n-5]$, $\delta[n+1]$

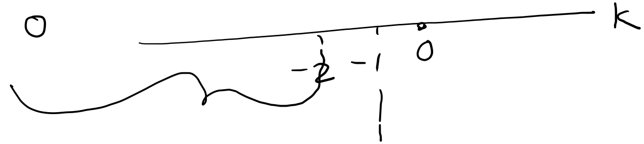
Prove ~~that~~ $\boxed{\delta[n] = u[n] - u[n-1]}$

first order difference equation

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

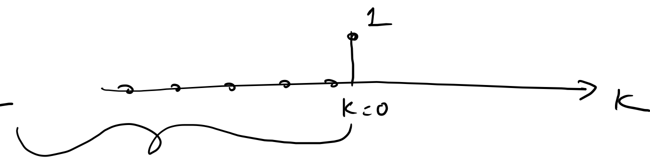
$u[n] = 0$ for all $n < 0$

$$u[-2] = \sum_{k=-\infty}^{-2} \delta[k] = 0$$

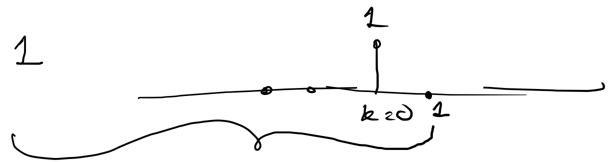


$$u[-1] = \sum_{k=-\infty}^{-1} \delta[k] = 0$$

$$u[0] = \sum_{k=-\infty}^{0} \delta[k] = 1$$



$$u[1] = \sum_{k=-\infty}^{1} \delta[k] = 1$$



$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

$\delta[\theta] = 1$

$= 0$

$$u[-2] = \sum_{k=0}^{\infty} \delta[-2-k]$$

$\Downarrow$

$-2-k=0$

$k = -2$

$= 0 + 0 + \dots$

$= \delta[-2] + \delta[-3] + \delta[-4] + \dots$

$= 0 + 0 + \dots$

$= 0$

$$u[1] = \sum_{k=0}^{\infty} \delta[1-k] = \underbrace{\delta[1]}_{0} + \underbrace{\delta[0]}_{=1} + \underbrace{\delta[-1]}_{0} + \underbrace{\delta[-2]}_{0} + \dots$$

$= 1$

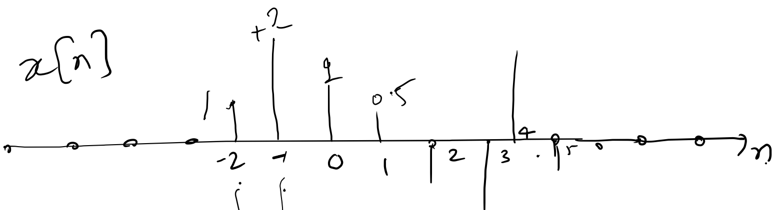
$$x[n]\,\delta[n] = ? = x[0]\,\delta[n] \neq x[0]$$

L.H.S $\Rightarrow$ δn

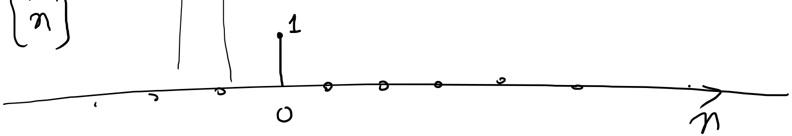
R.H.S $\Rightarrow$ Scalar

$$x[n]\,\delta[n-n_0] = ?$$

$x[n]$



$\delta[n]$



$$x[n]\,\delta[n] = \begin{cases} x[0] & n=0 \\ 0 & \text{otherwise} \end{cases}$$

$$x[n]\,\delta[n] = x[0]\,\delta[n]$$

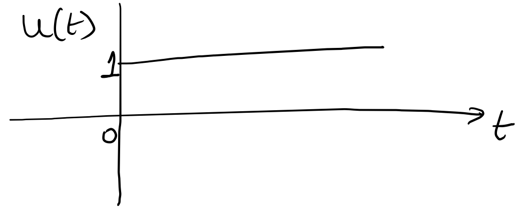
Sifting

Sampling

Unit Step fn —

C.T

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$



$$\delta(t) = \frac{d}{dt} u(t)$$

another variation

$$u(t) = \begin{cases} 1 & t > 0 \\ 1/2 & t = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta(t) = 0 \qquad t \neq 0$$

Unit Impulse fn.

⇓

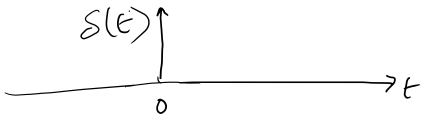
Singularity fn.
or
Generalized fn

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

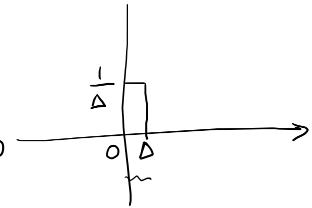
Operational definition

$$\delta(t) = 0 \qquad t \neq 0$$

& $$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



$\delta_\Delta(t)$

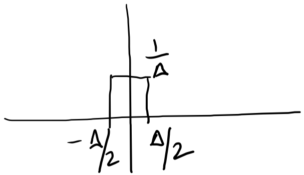


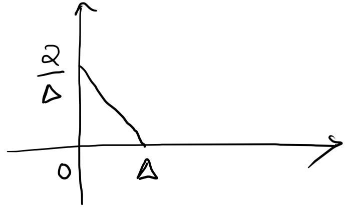
$$\delta_\Delta(t) = \begin{cases} 0 & t \geq \Delta \ \& \ t < 0 \\ \frac{1}{\Delta} & \text{otheri} \end{cases}$$

infinetesimally small duratin Δ

$$\int_{-\infty}^{\infty} \delta_\Delta(t)\, dt = \int_0^{\Delta} \delta_\Delta(t)\, dt = \frac{1}{\Delta} \int_0^{\Delta} dt = \frac{1}{\Delta}\, t \Big|_0^{\Delta} = 1$$

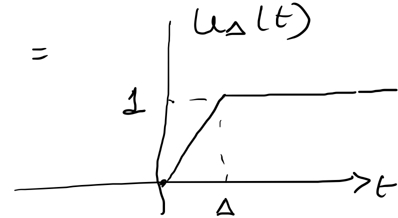
$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$





$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

$$u_\Delta(t) =$$



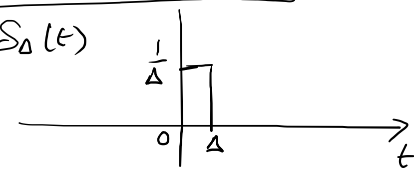
$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

$$\boxed{\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t) = \frac{d}{dt} u_\Delta(t)}$$

$$u(t) = \int_{0}^{\infty} \delta(t-\tau)\, d\tau$$

$\delta_\Delta(t)$



System