tr5frbj 12/08/24

Circuit Theory and Devices
↗ tut as well

Lab: LT Spice (2nd week onwards)

book: Engineering Circuit Analysis (9th Edition)

Relative grading

Labs	20%
Quiz (every 2 weeks (n−1))	20%
Mid sem	30%
End sem	30%

Scientific calc ⇐ (Mid sem, End sem)

Course: 9 modules — chapter 10 onwards
{continuation of BE}

∗ Lecture : 1

⇒ Linear Circuits

x (input) → [system] → y (output)

system is linear if

$Kx \rightarrow \boxed{sys} \rightarrow Ky$

where k is constant

"R" Resistor: linear element ✓
$V = iR$

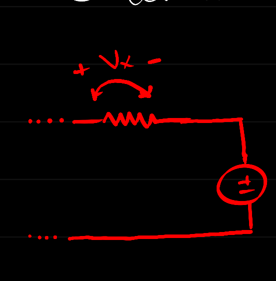
"L" Inductor: linear element ✓
$V = L\frac{di}{dt}$

"C" Capacitor: linear element ✓
$i = C\frac{dv}{dt}$

∗ Linear Electric circuits:

consists of ⇒

1. R, L, C → linear elements
2. Independent voltage & current sources
3. linear dependent sources



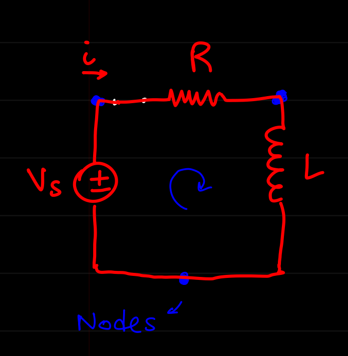
example — would not have been linear if $V_s = KV_x^2$

NOTE: diode and transistors are non-linear elements

∗ Response of a linear circuit

① full response = transient response + Steady State

transient response	Steady State
temporary	persists $t \to \infty$
due to sudden change	due to source
also called natural response	also called forced response
depends on linear elements R, L, C	depends on current/voltage source



KVL : $-V_s + iR + L\frac{di}{dt} = 0$

$i_{final} = i_{transient} + i_{steady\ state}$

as $t \to \infty$, $i_{final} = i_{steady\ state}$

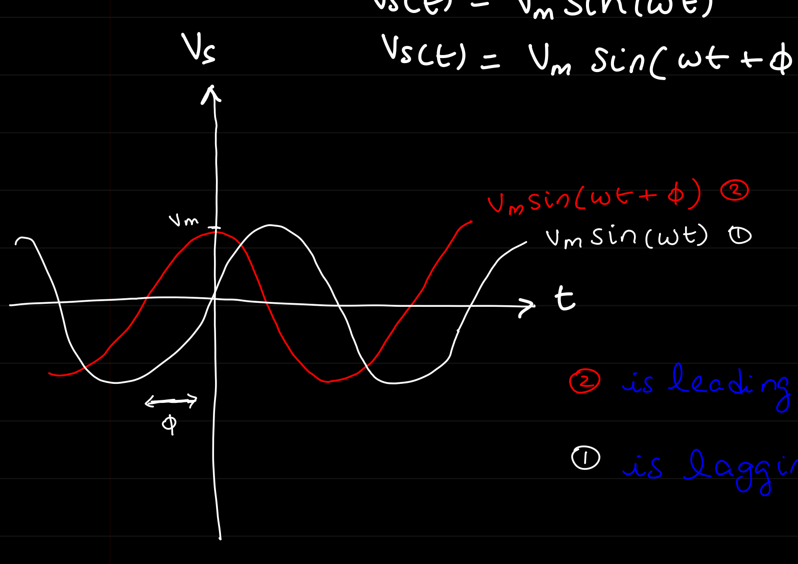
∗ Sinusoids & complex numbers

sinusoidally varying voltage source

$V_s(t) = V_m \cos(\omega t)$

$V_s(t) = V_m \sin(\omega t)$

$V_s(t) = V_m \sin(\omega t + \phi)$



② is leading ①
① is lagging ②
} by angle φ

∗ TRIGO

$\sin(-\theta) = -\sin(\theta)$

$\cos(-\theta) = \cos(\theta)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

sin +ve	All +ve
tan +ve	cos +ve

∗ Complex Numbers

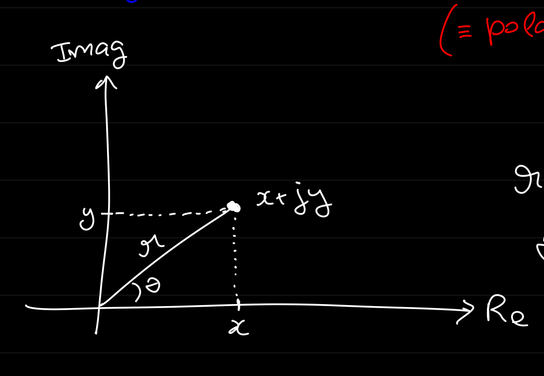
Real : x

complex : $\underbrace{x + iy}_{\text{rectangular form}}$

real part (x), imaginary part (y), $i = \sqrt{-1}$ a.k.a. j

⇒ Euler's identity : $\underbrace{e^{j\theta}}_{\text{exponential form}} = \underbrace{\cos\theta + j\sin\theta}_{\text{rectangular form}}$

Complex Number
- rectangular: $x + iy$
- exponential: $re^{j\theta}$ (≡ polar form : $r\angle\theta$)



$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$(r_1 e^{j\theta_1}) \times (r_2 e^{j\theta_2}) = (r_1 \cdot r_2)\, e^{j(\theta_1 + \theta_2)}$

AC	DC
• we can use transformers to vary voltage	• appliances need constant current
• transported easily over large distances	

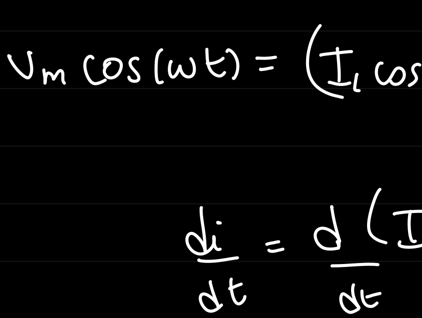
AC → DC : using rectifiers

∗ MODULE : 1 {CH = 10} 10:30 am

⇒ Sinusoidal steady state analysis

assuming transient has died out long back

Consider,



$V_s(t) = V_m \cos(\omega t)$
find $i(t)$

$V_s(t) = iR + L\frac{di}{dt}$

$\Rightarrow V_m \cos(\omega t) = iR + L\frac{di}{dt}$

both will take cos(wt) form

General form Solution : $i = I_1 \cos\omega t + I_2 \sin\omega t$

$\Rightarrow V_m \cos(\omega t) = (I_1 \cos\omega t + I_2 \sin\omega t)R + L\frac{di}{dt}$

$\frac{di}{dt} = \frac{d}{dt}(I_1 \cos\omega t + I_2 \sin\omega t)$

$\Rightarrow \omega(I_1(-\sin\omega t) + I_2 \cos\omega t)$

$\Rightarrow \omega(I_2 \cos\omega t - I_1 \sin\omega t)$

$V_m \cos(\omega t) = (I_1 \cos\omega t + I_2 \sin\omega t)R + (I_2 \cos\omega t - I_1 \sin\omega t)L\omega$

$\Rightarrow \cos\omega t \cdot (I_1 R + I_2 L) + \sin\omega t \cdot (I_2 R - I_1 L)$

$\Rightarrow (I_1 R + I_2 L - V_m)\cos(\omega t) + (I_2 R - I_1 L)\sin(\omega t) = 0$

∗ at $t = 0$,
$(I_1 R + I_2 L - V_m) \cdot 1 + (I_2 R - I_1 L) \cdot 0 = 0$

$\boxed{I_1 R + I_2 L = V_m}$

∗ at $t = \frac{\pi}{2\omega}$, $(I_1 R + I_2 L - V_m) \cdot 0 + (I_2 R - I_1 L) \cdot 1 = 0$

$\boxed{I_1 = I_2 \cdot \frac{R}{L}}$

$I_1 R + I_2 L = V_m \Rightarrow I_2 \frac{R^2}{L} + I_2 L = V_m$

$\Rightarrow I_2\left(\frac{R^2 + L}{L}\right) = V_m \Rightarrow I_2 = \frac{V_m L^2}{R^2 + L}$

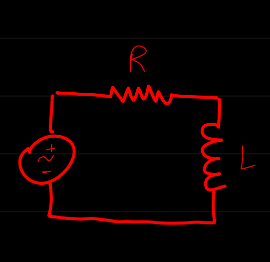
$I_1 = \frac{V_m L R}{R^2 + L}$

14/08/24 **Lecture : 2** 9:30 AM

Wednesday → Quiz : Chp 1, 2, 3

* Sinusoidal steady state analysis

AC sources ↙ ↳ not considering any transient response



$V_s = V_m \cos(\omega t)$ (angular freq)

$\text{freq}(hz) = \frac{\omega}{2\pi}$

$V_s(t) = i(t)R + L\frac{di(t)}{dt}$

general solⁿ: $i = I_1 \cos(\omega t) + I_2 \sin(\omega t)$

put i from gen solⁿ in above eqⁿ

$I_1 = \frac{RV_m}{R^2+\omega^2L^2}$, $I_2 = \frac{\omega L V_m}{R^2+\omega^2L^2}$

$i(t) = \frac{RV_m}{R^2+\omega^2L^2}\cos(\omega t) + \frac{\omega L V_m}{R^2+\omega^2L^2}\sin(\omega t)$

Note: use $\cos(A+B) = \cos A \cos B - \sin A \sin B$

⇒ So, $i(t) = A\cos(\omega t + \phi)$: general solⁿ

becomes ⇒ $i(t) = A[\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)]$

$\frac{RV_m}{R^2+\omega^2L^2}\cos(\omega t) + \frac{\omega L V_m}{R^2+\omega^2L^2}\sin(\omega t) = A\cos(\omega t)\cos(\phi) - A\sin(\omega t)\sin(\phi)$

⇓

$A\cos(\phi) = \frac{RV_m}{R^2+\omega^2L^2}$, $A\sin(\phi) = \frac{-\omega L V_m}{R^2+\omega^2L^2}$

$A^2\cos^2(\phi) + A^2(\sin^2\phi) = A^2(1) = A^2 = \frac{R^2V_m^2 - \omega^2L^2V_m^2}{R^2+\omega^2L^2}$

⇓

$\frac{V_m^2(R^2-\omega^2L^2)}{R^2+\omega^2L^2}$

$A = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}$

$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$

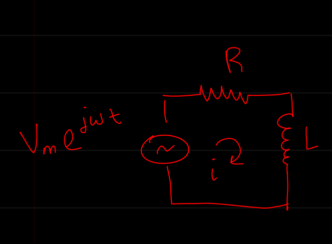
$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

⇒ Consider a complex source

$V_m\cos(\omega t) + jV_m\sin(\omega t) = V_m e^{j\omega t}$

Real Voltage source $= V_m\cos(\omega t) = \text{Re}\{V_m e^{j\omega t}\}$

Note: Euler's formula → $e^{j\theta} = \cos\theta + j\sin\theta$



$V_m e^{j\omega t} = iR + L\frac{di}{dt}$

dont integrate ✗
general solⁿ ✓

Assume $i = I_m e^{j(\omega t+\phi)}$
put eqⁿ

$V_m e^{j\omega t} = I_m e^{j\omega t + j\phi}R + L(j\omega)I_m e^{j\omega t + j\phi}$

⇒ $I_m e^{j\omega t + j\phi} = \frac{V_m e^{j\omega t}}{R + j\omega L}$

is this the real current? No
current cannot be complex

Real $i(t) = \text{Re}\{I_m e^{j\omega t + j\phi}\} = \text{Re}\left\{\frac{V_m}{R+j\omega L}e^{j\omega t}\right\}$

find the real part of this complex number by simplifying
{multiply by the conjugate ≡ factorize}

$i(t) = \frac{V_m}{R+j\omega L}\left(\frac{R-j\omega L}{R-j\omega L}\right)e^{j\omega t} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}e^{-j\tan^{-1}\left(\frac{\omega L}{R}\right)}\cdot e^{j\omega t}$

⇒ Note: rect form → polar form

$x + jy \rightleftarrows re^{j\theta}$

$x = r\cos\theta$, $y = r\sin\theta$

$r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Real $i(t) = \text{Re}\left\{\frac{V_m}{\sqrt{R^2+\omega^2L^2}} \cdot e^{j\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)}\right\}$

* $i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

same answer as when we were not considering complex numbers

* PHASOR

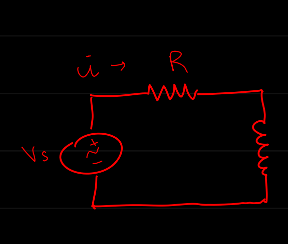
$i(t) = I_m\cos(\omega t + \phi) = \text{Re}\{I_m e^{j\omega t + j\phi}\}$

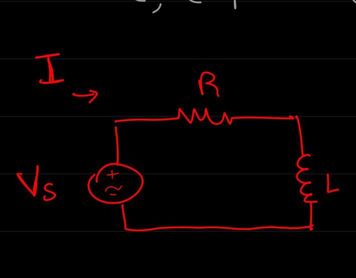
⇓ remove $e^{j\omega t}$, Re{...}

not changing with time
complex func ✗
complex number ✓

$\underbrace{I_m e^{j\phi}}_{\text{phasor}} = I_m\angle\phi$

time domain	phasor representation
$i(t) = I_m\cos(\omega t + \phi)$	$I_m\angle\phi$
$v(t) = V_m\cos(\omega t + \theta)$	$V_m\angle\theta$
⇒ real function of t	⇒ complex, constant
⇒ notations: small letters	⇒ Bold, capital letters





$i(t) = I_m\cos(\omega t + \phi)$ ✓
$i(t) = \text{Re}\{I_m e^{j\omega t + j\phi}\}$ ✓
$i(t) = I_m\angle\phi$ ✗ complex constant
$i(t) = I_m e^{j(\omega t+\phi)}$ ✗ complex
$i(t) = I_m\angle\omega t+\phi$ ✗ complex phasor notation
(phasor notation)

but we can write ⇒ phasor notation of $i(t) = I_m\angle\phi = I_m\angle(\omega t+\phi)$

Note: $\angle\phi = e^{j\phi}$ or $\angle\omega t+\phi = e^{j\omega t + j\phi}$

$\underbrace{V_m e^{j\theta}}_{\mathbf{V}} = (R + j\omega L)\underbrace{I_m e^{j\phi}}_{\mathbf{I}}$

$\boxed{\mathbf{V} = (R + j\omega L)\mathbf{I}}$

↳ phasor notation

* phasor representation of relationship between i and v for R, L, C

① RESISTOR

time domain
→ i R, + v −

$v = iR$

consider complex voltage & current

$v = V_m e^{j(\omega t+\theta)}$
$i = I_m e^{j(\omega t+\phi)}$

$V_m e^{j\omega t + j\theta} = R \cdot I_m e^{j\omega t + j\phi}$

$V_m \cancel{e^{j\omega t}} \cdot e^{j\theta} = R \cdot I_m \cancel{e^{j\omega t}} e^{j\phi}$

$V_m e^{j\theta} = R \cdot I_m e^{j\phi}$

$\underbrace{V_m\angle\theta}_{\mathbf{V}} = \underbrace{I_m\angle\phi}_{\mathbf{I}} \cdot R$

$\boxed{\mathbf{V} = \mathbf{I}R}$

② INDUCTOR

i → L, + v −

$v = L\frac{di}{dt}$

$v \to V_m e^{j(\omega t+\theta)}$ $i \to I_m e^{j\omega t + j\phi}$

$V_m e^{j\omega t}e^{j\theta} = L\, I_m e^{j\omega t} \cdot e^{j\phi}(j\omega)$

$\underbrace{V_m e^{j\theta}}_{\mathbf{V}} = j\omega L \cdot \underbrace{I_m e^{j\phi}}_{\mathbf{I}}$

$\boxed{\mathbf{V} = j\omega L \cdot \mathbf{I}}$

③ CAPACITOR

→ i C, + v −

$i = C\frac{dv}{dt}$

$i \to I_m e^{j(\omega t+\phi)}$
$v \to V_m e^{j(\omega t+\theta)}$

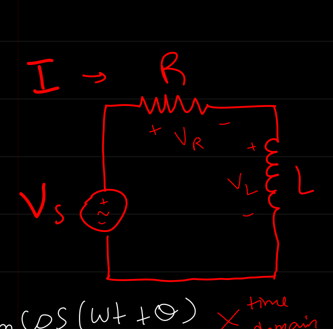
$I_m \cancel{e^{j\omega t}} \cdot e^{j\phi} = C \cdot V_m \cancel{e^{j\omega t}} e^{j\theta} \cdot j\omega$

$\underbrace{I_m e^{j\phi}}_{\mathbf{I}} = C \cdot j\omega \cdot \underbrace{V_m e^{j\theta}}_{\mathbf{V}}$

$\mathbf{I} = j\omega C \cdot \mathbf{V}$

$\boxed{\mathbf{V} = \frac{1}{j\omega C} \cdot \mathbf{I}}$

* PHASOR Representation using KVL



$V_s = V_m\cos(\omega t + \theta)$ ✗ time domain
$V_s = V_m e^{j(\omega t+\theta)}$ ✓ phasor notation

$\mathbf{V}_s = \mathbf{V}_R + \mathbf{V}_L$

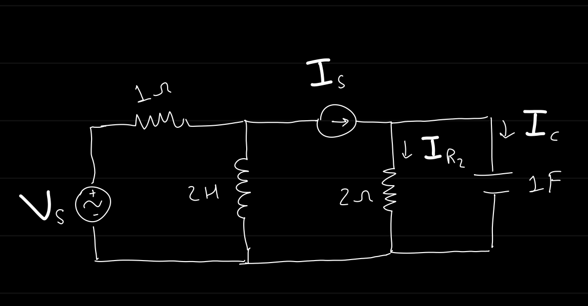
$= \mathbf{I}R + (j\omega L)\mathbf{I}$

$= \mathbf{I}(R + j\omega L)$

$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L} = \frac{V_m\angle\theta}{R + j\omega L}$

Time domain		Phasor (Frequency domain)
R	$v = iR$	$\mathbf{V} = \mathbf{I}R$
L	$v = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\,\mathbf{I}$
C	$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$

⇒ Example



① Given: both AC sources, V_s & I_s operate at $\omega = 2\ rad/s$

② Given: $I_c = 2\angle 28°\ A$

Find (i) V_c
(ii) I_{R_2}
(iii) I_s

① $V_c = \frac{I_c}{j\omega C} = \frac{1}{j(2)(1)} \cdot 2\angle 28°$

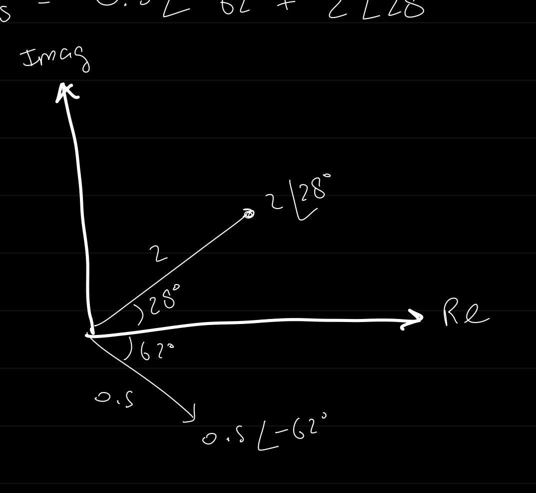
Note: $\frac{1}{j} = -j = -90°$

we add the angles directly which are being multiplied

$V_c = (0.5\angle -90°)(2\angle 28°) = 1\angle -62°\ V$

② $I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_{R_2}}{2} = \frac{1\angle -62°}{2} = 0.5\angle -62°\ A$

③ $I_s = 0.5\angle -62° + 2\angle 28°$



$2.06\angle 14°\ A$

* Lecture: 3 — 19/08/24 9:30AM

- Phasor: $i(t) = I_m \cos(\omega t + \phi) = Re\{I_m e^{j(\omega t+\phi)}\}$

Remove $e^{j\omega t}$ and $Re\{..\}$

$$I_m e^{j\phi} = I_m \angle \phi \quad \} \text{ Phasor}$$

exponential form — polar form

time domain ⇌ freq. domain

$V_R = IR \qquad V_L = j\omega L\, I \qquad V_C = \frac{1}{j\omega C} I$

* IMPEDANCE

$$Z = \frac{V}{I} = \frac{1}{R}$$

$Z_R = R$ — voltage and current in the same phase

$Z_L = j\omega L$ — voltage leads current by 90°

$Z_C = \frac{1}{j\omega C}$ — voltage lags current by 90°

$Z = x + jy$ — resistance, reactance

* Admittance

$$Y = \frac{1}{Z}$$

$Y = x + jy$ — conductance, susceptance

* Impedance combination

Series : $Z_{eq} = Z_1 + Z_2 + \dots Z_N$

Parallel : $Z_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots \frac{1}{Z_N}$

eg) Find impedance of series combination of an inductor L (L=5mH) and capacitor (C=100μF) $\omega = 10^4$ rad/s

5mH 100μF

$\Rightarrow Z_{eq} = Z_L + Z_C = j\omega L + \frac{1}{j\omega C} = 10^4 \cdot j \cdot 5\times10^{-3} + \frac{10^{-4}\times 10^{3}}{j\cdot 5\times 100 \times 10^{-6}}$

Note: $\frac{1}{j} = -j$ $\qquad \Rightarrow 50j - j = 49j$

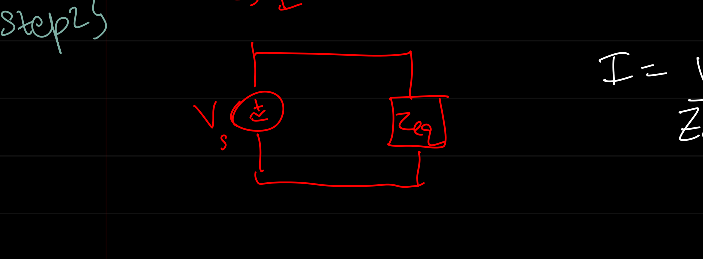
Note: $e^{j\theta} = \cos\theta + j\sin\theta = e\angle\theta$ $\qquad \Rightarrow 0 + 49j = 49e^{j\cdot\frac{\pi}{2}}$

eg:

Find i(t)

$\frac{10^3(1-2j)(1+j)}{2}$ $\quad 500(1-2j)(1+j) \quad \frac{10^3(1-2j)}{1-j}$

step1) Convert to freq. domain (write in terms of phasor and Z)



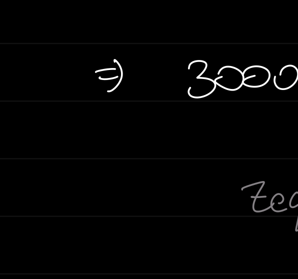
$2000 - 2000j \;||\; 1000j$ $\quad \left(\frac{1}{1000-2000j} + \frac{1}{1000j}\right)^{-1}$ $\quad \frac{1-j}{1000j + 1000 - 2000j}$ $\quad (10^3 - 2\cdot10^3 j)10^3 j$

$\frac{1}{3}H \to Z_L = j\omega L = j\cdot 3000 \cdot \frac{1}{3} \Rightarrow j\cdot 1k$

$\frac{1}{6}\mu F \to Z_C = \frac{1}{j\omega C} = \frac{1}{j\cdot 3000 \times \frac{1}{6} \times 10^{-6}} \Rightarrow \frac{2\cdot10^3}{j}$

$\Rightarrow -j\cdot 2k\Omega$

step2)



$I = \frac{V_s}{Z_{eq}}$

$Z_{eq} = 1.5k + [500(1+j)(1-2j)]$

$\Rightarrow 1500 + 500[1 - 2j + j + 2]$

$\Rightarrow 500[3 + 3 - j] \Rightarrow 500[6 - j]$

$\Rightarrow 3000 - 500j$

PPT) $\quad Z_{eq} = 1.5 + \frac{(j1)(1-j2)}{1-j}$ $k\Omega$ $\quad \frac{j(1-2j)(1+j)}{2} \Rightarrow \frac{(j+2)(1+j)}{2} \Rightarrow \frac{j+j^2+2+2j}{2}$ $\quad \frac{3j+1}{2}$

$\Rightarrow \frac{4+3j}{2}$

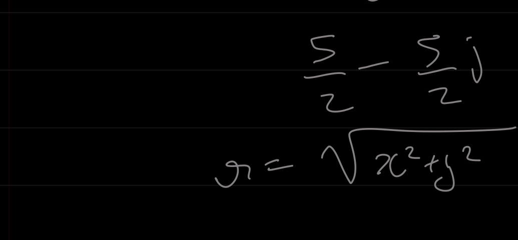
Note: complex number $< \begin{matrix} x+iy \\ r\angle\theta \end{matrix}$

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$I_3 + I_2 = I_1$

eg)



find I_1, I_2, I_3

$Z_{eq}^{(1)} = \left(\frac{1}{5} + \frac{1}{5j}\right)^{-1} = \left(\frac{5j+5}{25j}\right)^{-1} = \frac{5j}{j+1}$

$Z_{eq} \Rightarrow^{(2)} -5j + \frac{5j}{j+1} \Rightarrow \frac{+5 - 5j + 5j}{j+1} = \frac{5}{j+1}$

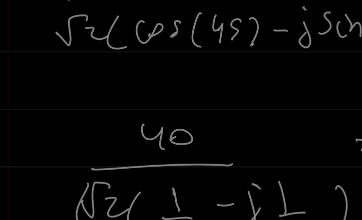
$\Rightarrow \frac{5(j-1)}{-2} \Rightarrow \frac{5-5j}{2}$

$\frac{5}{2} - \frac{5}{2}j$ $\qquad \frac{5\sqrt{2}}{2}$

$r = \sqrt{x^2+y^2} = \sqrt{\frac{25}{4} + \frac{25}{4}} = \frac{\sqrt{50}}{2}$

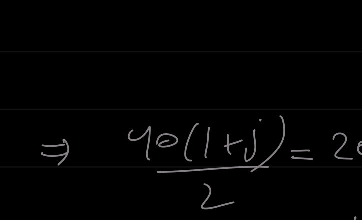
$\theta = \tan^{-1}(-1) \Rightarrow -45°$

Voltage division



$V_1 = V\cdot\frac{R_1}{R_1+R_2}$

Current division



$i_1 = i\cdot\frac{R_2}{R_1+R_2}$

$I_1 = \frac{V}{Z_{eq}} = \frac{\cancel{100}^{20}\angle 0°}{\frac{5\sqrt{2}}{2}\angle -45°} \Rightarrow \frac{40\angle 0°}{\sqrt{2}\angle -45°}$

$\frac{40(\cos 0 + j\sin 0)}{\sqrt{2}(\cos(45) - j\sin(45))}$ $\qquad r\angle\theta = re^{j\theta} =$

$\frac{40}{\sqrt{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)} \Rightarrow \frac{40}{1-j} \Rightarrow \frac{40(1+j)}{2} = 20(1+j)$

$20 + 20j$

$r = \sqrt{x^2+y^2} \Rightarrow \sqrt{400+400}$ $\Rightarrow 20\sqrt{1+1} = 20\sqrt{2}$

$\theta = \tan^{-1}(1) = 45°$

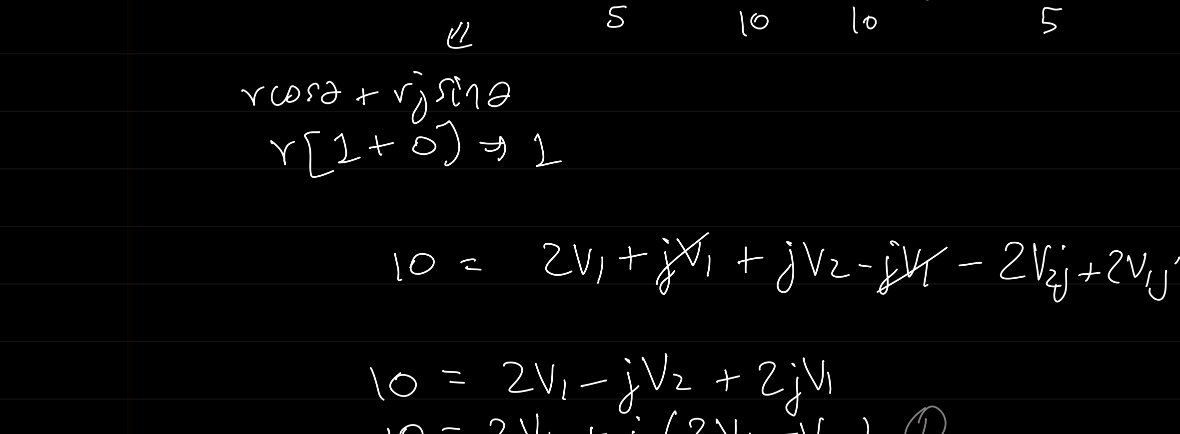
$I_1 = 20\sqrt{2}\angle 45°$

$I_2 = I_1 \times \frac{j5}{5+j5} \Rightarrow \cancel{20}^{10}\sqrt{2}\angle 45° \times \frac{j(1-j)}{2}$

$\Rightarrow 10\sqrt{2}\angle 45° \times (j+1)$

* NODAL ANALYSIS

Node ≡ Same voltage



at node 1:

$1\angle 0° = \frac{V_1}{5} + \frac{jV_1}{10} + \frac{(V_2-V_1)j}{10} - \frac{(V_2-V_1)j}{5}$

$r\cos\theta + rj\sin\theta$

$r[1+0] \Rightarrow 1$

$10 = 2V_1 + \cancel{jV_1} + jV_2 - \cancel{jV_1} - 2V_2 j + 2V_1 j$

$10 = 2V_1 - jV_2 + 2jV_1$

$10 = 2V_1 + j(2V_1 - V_2)$ ①

at node 2:

$\frac{V_2 - V_1}{j10} + \frac{V_2 - V_1}{-j5} + \frac{V_2}{j5} + \frac{V_2}{10} = 0.5\angle -90°$

$0.5\cancel{\cos 90°} - 0.5j\sin 90°$ $\Rightarrow -0.5j$

$V_2 - V_1 + 2V_1 - \cancel{2V_2} + \cancel{2V_2} + V_2 j = 5$

$3V_2 - V_1 + V_2 j = 5 \Rightarrow V_1 = 3V_2 - 5 + V_2 j$

Put value of V_1 in ①

$10 = 6V_2 - 10 + 2V_2 j + j(6V_2 - 10 + 2V_2 j - V_2)$

$10 = 6V_2 - 10 + 2V_2 j + 6V_2 j - 10j - 2V_2 - V_2 j$

$20 = 4V_2 + 7V_2 j - 10j$ $\qquad \frac{30-20j}{13}$

$20 + 10j = V_2(4 + 7j)$ $\qquad \frac{150-100j}{65}$

$V_2 = \frac{(20+10j)(4-7j)}{16+49} = \frac{80 - 140j + 40j + 70}{65}$

$V_2 = \frac{30}{13} - j\left(\frac{20}{13}\right)$ $\qquad r = \sqrt{\frac{900+400}{169}} = \frac{10}{\sqrt{13}}$

$\theta = \tan^{-1}\left(-\frac{2}{3}\right)$

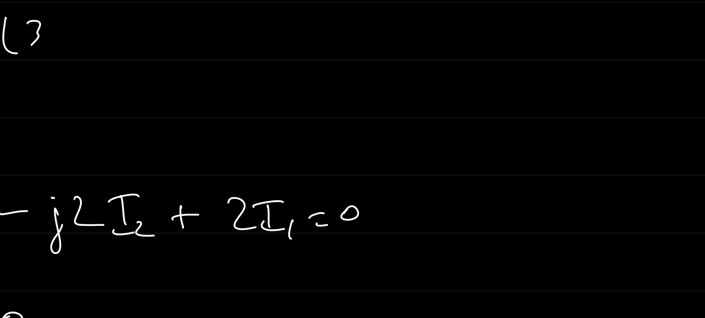
$V_2 = \frac{10\sqrt{13}}{13}\angle -33.69°$ $\qquad = -\tan\left(\frac{2}{3}\right) = -33.69$

$V_1 = 3V_2 - 5 + jV_2$

$\Rightarrow V_2(3+j) - 5 = \frac{30}{13} - \frac{20j}{13}(3+j) - 5$

$\Rightarrow \frac{30 - 60j + 20 - 65}{13} = \frac{-60j - 15}{13} \Rightarrow V_1 = \frac{15\sqrt{17}}{13}\angle -104°$

* Mesh Analysis



2 meshes

$10\angle 0° = 3I_1 + j4(I_1 - I_2)$

$(I_2 - I_1)j4 - I_2(j2) + 2I_1 = 0$

$10\angle 0° = 10 = 3I_1 + j4(I_1 - I_2) = 3I_1 + j4I_1 - j4I_2$

$j4I_2 + 10 = I_1(3$

$j4I_2 - j4I_1 - j2I_2 + 2I_1 = 0$

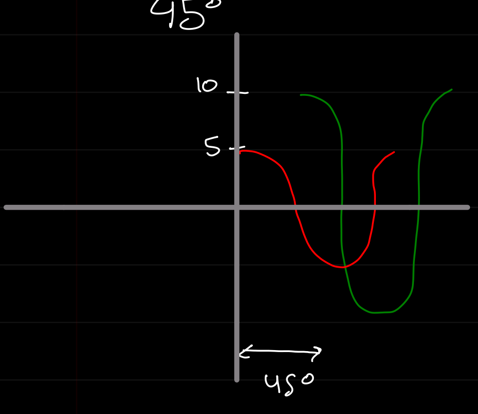
$j4I_2 - j4(10 -$

* CTD Tutorial : 1

19/08/24   1:00 – 3:00

Q1) Determine the angle by which v_1 leads i_2 if $V_1 = 10\cos(10t - 45°)$ and i_1 is equal to

(1) $i_1 = 5\cos 10t$
$45°$



(3) $i_1 = 5\cos(10t - 40°)$
$5°$

(2) $i_1 = 5\cos(10t - 80°)$
$35°$

(4) $i_1 = 5\cos(10t + 40°)$
$85°$

(5) $5\sin(10t - 19°)$
$= 5\cos(10t - 19° - 90°)$
$= 5\cos(10t - 109°)$
Ans: $64°$

Q2) Determine which waveform in each of the following pairs is lagging

(1) $\cos 4t$ leads $\sin 4t$ by $\frac{\pi}{2}$ {90°}

(2) $\cos(4t - 80°)$ lags $\cos(4t)$ by $80°$

(3) $\cos(4t + 80°)$ leads $\cos(4t)$ by $80°$

(4) $\sin(5t)$ lags $\cos(5t + 2°)$ by $92°$

(5) $\sin 5t + \cos 5t$

Q3) Perform the indicated operation {express in rectangular & polar form}

(a) $\dfrac{2+j3}{1+8\angle 90°} - 4$

$8\angle 90°$
$\hookrightarrow j\cos\theta + j\sin\theta = 8\cdot 0 + 8\cdot j \cdot 1 = 8j$

$\Rightarrow \dfrac{2+j3}{1+8j} - 4 \Rightarrow \dfrac{2+j3-4-j32}{1+8j} \times \dfrac{1-8j}{1-8j}$

$\Rightarrow \dfrac{(-2+29j)(1-8j)}{8} = -2+16j+29j+(29\times 8)$

$\Rightarrow 230 + 45j$

$\Rightarrow 46 + 9j$

$\dfrac{2+j3}{1+8\angle 90°} - 4$

$8\angle 90° = 8\cdot\cos(90) + 8\cdot j\sin(90) = 8j$

$\dfrac{2+j3}{1+j8} \times \dfrac{1-8j}{1-8j} - 4$

$\Rightarrow \dfrac{2-16j+j3+24}{65} - 4 \Rightarrow \dfrac{26-13j}{65} - 4$

$\Downarrow$

$\dfrac{-234-13j}{65} \Leftarrow \dfrac{26-13j-260}{65}$

x y $\boxed{\dfrac{-18}{5} - \dfrac{1}{5}\cdot j}$ rectangular form

$\Downarrow$

$r = \sqrt{x^2+y^2} = \sqrt{\dfrac{18^2}{25} + \dfrac{1}{25}}$

$\Rightarrow \sqrt{\dfrac{325}{25}} = \sqrt{13}$

$\theta = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{1}{18}\right) = 3.179°$

polar form $\boxed{\sqrt{13}\angle 3.179°}$

(b) $\left(\dfrac{10\angle 25°}{5\angle -10°} + \dfrac{3\angle 15°}{3-5j}\right) j2$

Q4) Transform into Phasor form

(a) $11\sin 100t$

(b) $11\cos 100t$

(c) $11\cos(100t - 90°)$

(d) $3\cos 100t - 3\sin 100t$

Q5) Determine equivalent Admittance assuming $w = 1000$ rad/s

(a) 25Ω in series with $20mH$

$Z_L = jwL = j \times 1000 \times 20 \times 10^{-3}$
$= j\cdot 20$

Admittance $(Y) = \dfrac{1}{Z_{eq}}$

$Z_{eq} = 25 + j20 \nearrow Y = \dfrac{1}{25+j20} \times \dfrac{25-j20}{25-j20}$

$\boxed{\dfrac{1}{41} - \dfrac{4}{205}\cdot j} \Leftarrow \dfrac{5-j4}{205} \Leftarrow \dfrac{25-j\cdot 20}{625+400} \Rightarrow \dfrac{25-j\cdot 20}{1025}$

(b) 25Ω in parallel with $20mH$

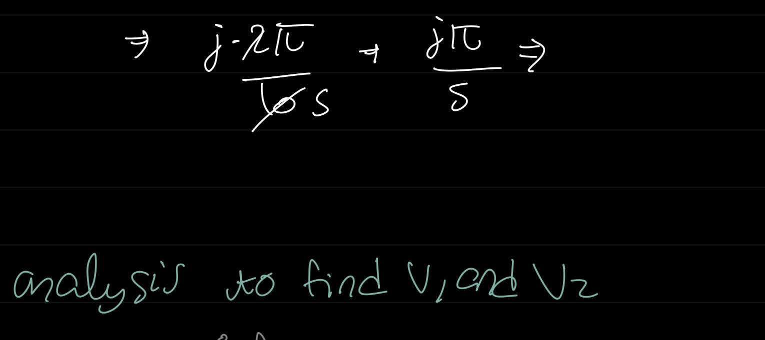
admittance $= \dfrac{1}{25} + \dfrac{1}{20j} = \dfrac{\overset{5}{\cancel{25}} + j\overset{4}{\cancel{20}}}{\underset{100}{\cancel{500}}j}$

$\dfrac{-j\cdot 5 - 4}{100} \Leftarrow \dfrac{-j(5+j\cdot 4)}{100} \Leftarrow \dfrac{5+j\cdot 4}{100j}$

$\hookrightarrow \dfrac{-1}{25} + \left(\dfrac{-j}{20}\right)$

Q6) Calculate the equivalent impedance as seen at the open terminals if the freq is ...

(a) 1Hz
(b) 1KHz
(c) 1MHz
(d) 1GHz



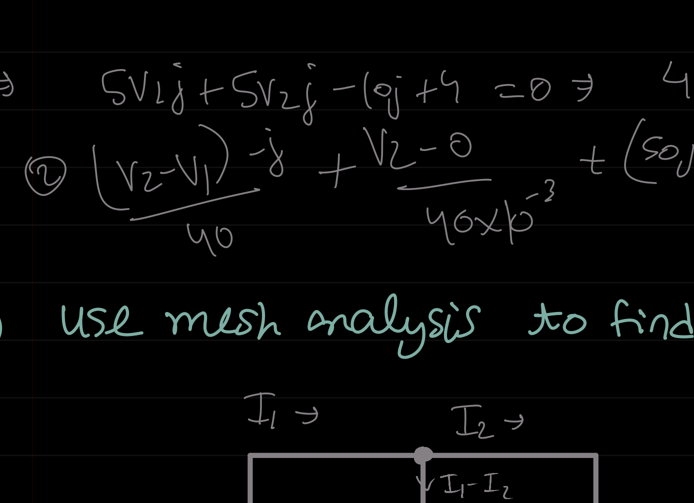
(a) $f = 1Hz$ $w = 2\pi f = 2\pi$ radians/s

$Z_L = jwL = j\cdot 2\pi \cdot 100 \times 10^{-3}$

$\Rightarrow \dfrac{j\cdot 2\pi}{\cancel{10}5} \Rightarrow \dfrac{j\pi}{5} \Rightarrow$

Q7) Use nodal analysis to find V_1 and V_2

$\dfrac{1}{-25\times j \times 10^{-3}}$



$\boxed{V1}$

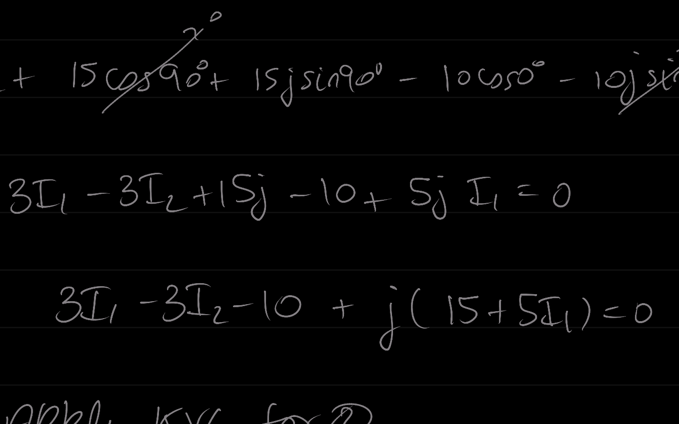
$\dfrac{V_1\cdot j}{20} - \dfrac{(V_1-V_2)j}{40} + (-50j\times 10^{-3}) + 20\times 10^{-3} = 0$

$\Rightarrow \dfrac{5(V_1 j + V_2 j)}{10} - \dfrac{10j}{10} + \dfrac{4}{10} = 0$

$\Rightarrow 5V_1 j + 5V_2 j - 10j + 4 = 0 \Rightarrow 4 + j(5V_1 + 5V_2 - 10) = 0$

② $\dfrac{(V_2 - V_1)\cdot j}{40} + \dfrac{V_2 - 0}{40\times 10^{-3}} + (50j\times 10^{-3}) = 0$

Q8) Use mesh analysis to find I_1 and I_2



Apply KVL for ①:

$3(I_1 - I_2) + 15\angle 90° - 10\angle 0° + I_1(j5) = 0$

$3I_1 - 3I_2 + 15\cancel{\cos 90°} + 15j\sin 90° - 10\cos 0° - 10j\cancel{\sin 0°} + I_1 5\cdot j = 0$

$3I_1 - 3I_2 + 15j - 10 + 5j I_1 = 0$

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

Apply KVL for ②

$-4I_2\cdot j + 20\angle 0° - 15\angle 90° + (I_2 - I_1)3 = 0$

$-4I_2\cdot j + 20 - 15j + 3I_2 - 3I_1 = 0$

$3I_2 - 3I_1 + 20 + j(-15 - 4I_2) = 0$

$3I_1 = 3I_2 + 20 + j(-15 - 4I_2)$

Put ② in ①

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

$\cancel{3I_2} + 20 + j(-15 - 4I_2) - \cancel{3I_2} - 10 + j(15 + 5I_1) = 0$

$10 + j(-15 - 4I_2) + j\left(15 + 5\left[\dfrac{3I_2 + 20 - 15j - 4I_2\cdot j}{3}\right]\right) = 0$

$30 - 45j - 12I_2 j + j(45 + 15I_2 + 100 - 75j - 20I_2 j) = 0$

$30 - \cancel{45}j - 12I_2 j + \cancel{45}j + 15I_2 j + 100j + 75 + 20I_2 = 0$

$105 + 20I_2 + j(3I_2 + 100) = 0$

$105 + 20I_2 + j\cdot 3I_2 + j\cdot 100 = 0$

$I_2(20 + j\cdot 3) = -105 - j\cdot 100$

$I_2 = \dfrac{-105 - j\cdot 100}{20 + j\cdot 3} \times \dfrac{20 - j\cdot 3}{20 - j\cdot 3}$

$I_2 = \dfrac{-2100 + 315\cdot j - 2000j - 300}{409}$

$I_2 = \dfrac{-2400 - 1685j}{409}$

Home work: 1 19/08/24

* Section 10.1

Q1) (a) $5\sin(5t-9^\circ)$

$t=0 \Rightarrow 5\sin(-9^\circ) = -5\sin(9^\circ) = -0.782$

$t=0.01 \Rightarrow 5\sin(0.05-9^\circ) = -5\sin(8.95^\circ)$ X

$\Downarrow$ radians

$5\sin\left(\left[0.05\times\frac{180}{\pi}\right)-9^\circ\right) = 5\sin(2.86^\circ-9^\circ)$

$\Rightarrow -5\sin(6.14) = -0.534$

$t=0.1s \Rightarrow 5\sin(0.5-9^\circ) = 5\sin(28.64-9) = 5\sin(19.64)$

$\Rightarrow 1.6805$

(b) $4\cos 2t$

$t=0 \Rightarrow 4\cos(0) = 4$

$t=1 \Rightarrow 4\cos(2) = 3.997$

$t=1.5 \Rightarrow 4\cos(3) = 3.994$

(c) $3.2\cos(6t+15^\circ)$

$t=0 \Rightarrow 3.2\cos(15^\circ) = 3.09$

$t=0.01 \Rightarrow 3.2\cos(0.06+15^\circ) = 3.2\cos(3.43^\circ+15^\circ)$

$\Rightarrow 3.2\cos(18.43^\circ)$

$\Rightarrow 3.035$

$t=0.1 \Rightarrow 3.2\cos(0.6+15^\circ) = 3.2\cos(34.3^\circ+15^\circ)$

$= 3.2\cos(49.3)$

$= 2.086$

Q2) (a) $300\sin(628t) \rightarrow 300\cdot\cos(628t-90^\circ)$

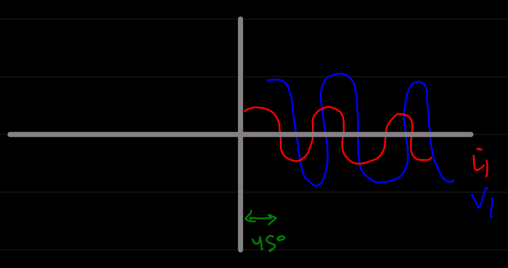
$4\sin(3\pi t+30^\circ) \rightarrow 4\cos(3\pi t-60^\circ)$

$14\sin(50t-5^\circ) - 10\cos 50t \rightarrow$

Q3) $V_1 = 10\cos(10t-45^\circ)$

(a) $i_1 = 5\cos 10t$

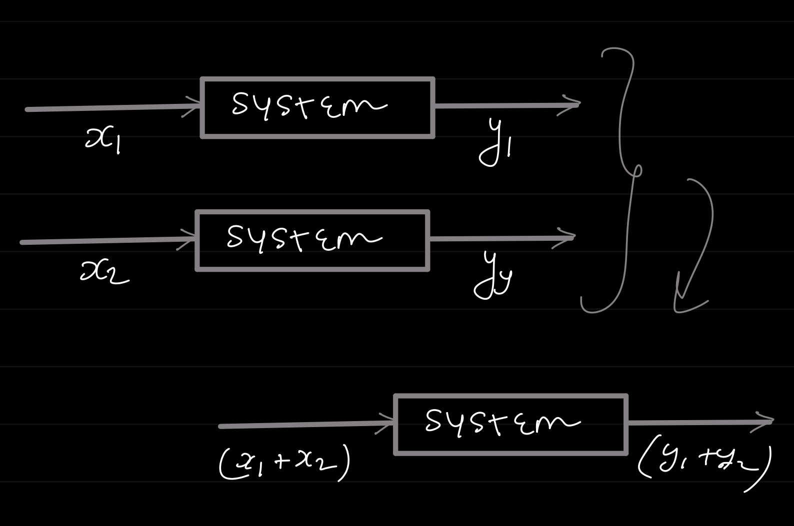
-45°



LECTURE: 4

* Superposition principle

active elements : provide energy eg: volt src / curr src
passive element : consume / store energy
eg: R / L / C

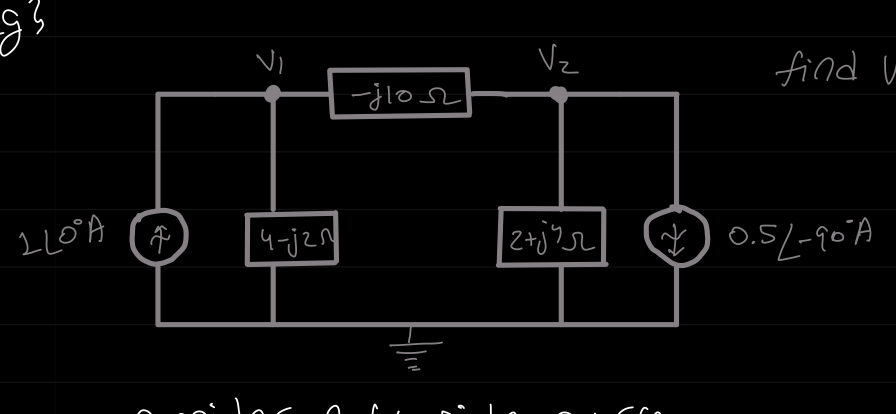


* Multiple independent sources

- consider separate independent sources "acting alone"
- add all responses

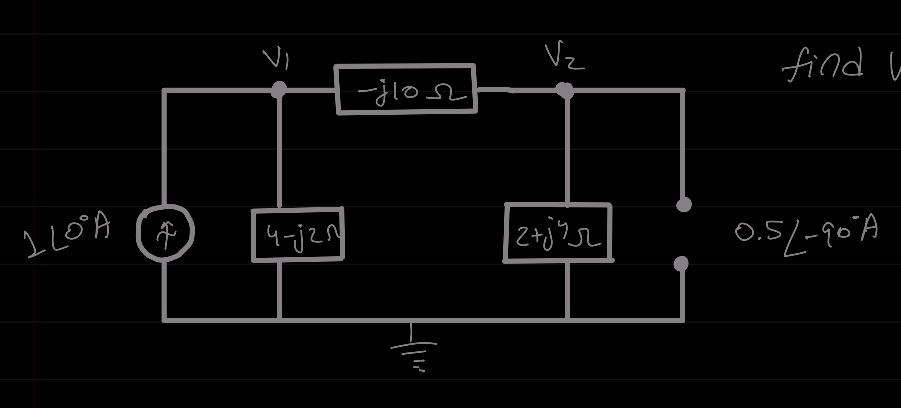
eg)

find V_2

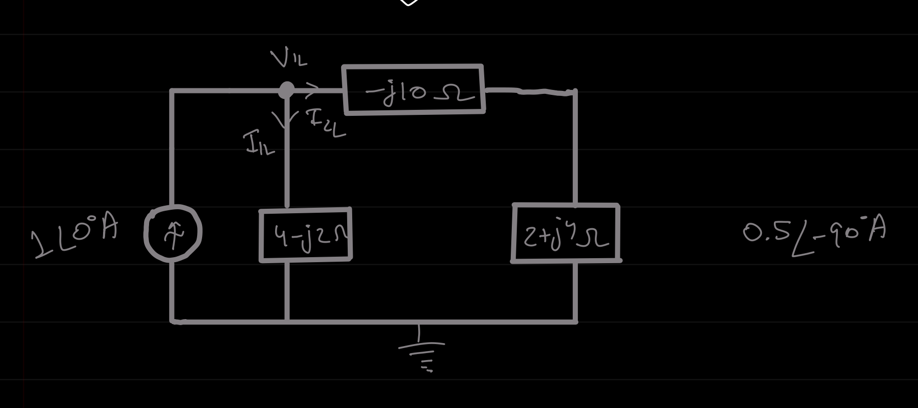


consider left side source
so we need to open the circuit at the right source

find V_2



⇓



$V_{1L} = I_{1L}(4-j2)$

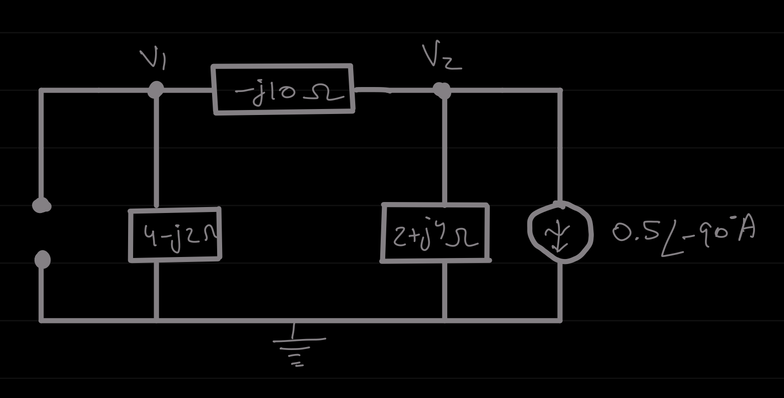
$$\Rightarrow \frac{-j10+2+j4}{(4-j2)+(-j10+2+j4)}(4-j2)$$

$$\Rightarrow \frac{(2-j6)(4-j2)}{6-j8} \Rightarrow \frac{8-j4-24j-12}{6-8j}$$

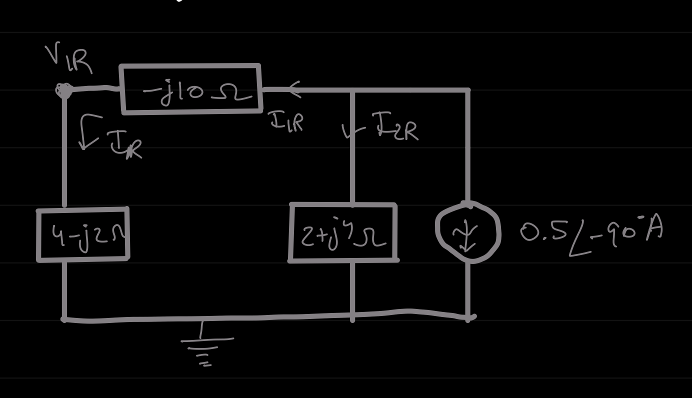
$$\Rightarrow \frac{-4-28j}{6-8j} = 2-j2\,V$$

$I_{1L}+I_{2L} = 1\angle 0° = 1\cos 0° + j1\sin 0° = 1A$

consider right src



⇓



$V_{1R} = I_{1R}(4-j2)$

$$I_{1R} = \frac{(-0.5\angle -90°A)(2+j4)}{-j10+4-2j+2+j4}$$

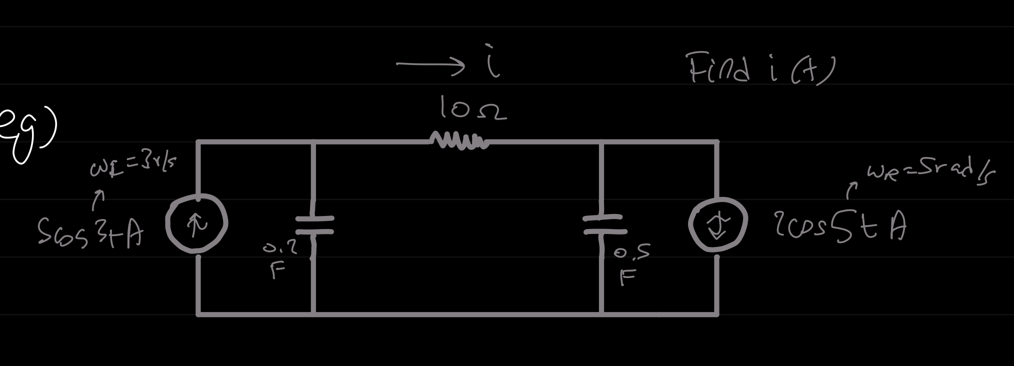
$$V_{1R} = \frac{(j\cdot 0.5)(2+j4)(4-j2)}{6-j8}$$

$$\Rightarrow \frac{(j-2)(4-j2)}{6-j8} = \frac{j4+2-8+j4}{6-j8} = \frac{j8-6}{6-j8} = -1V$$

$V_1 = V_{1L}+V_{1R} = 2-j2-1 = \boxed{1-j2\,V}$

eg)

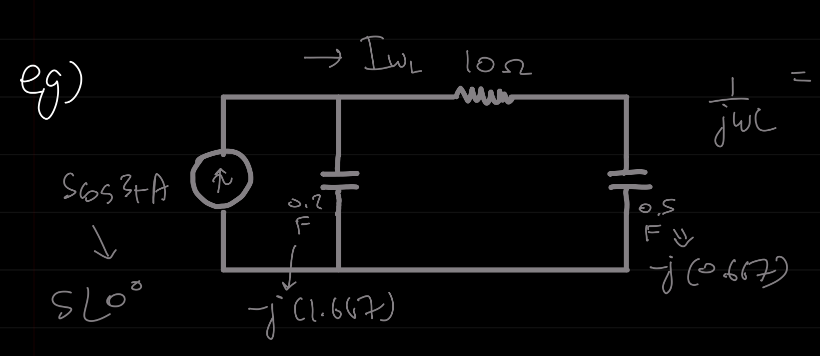
Find $i(t)$



Note: we have 2 sources operating at 2 different frequencies (ω)

① consider left src $\omega_L = 3rad/s$

eg)

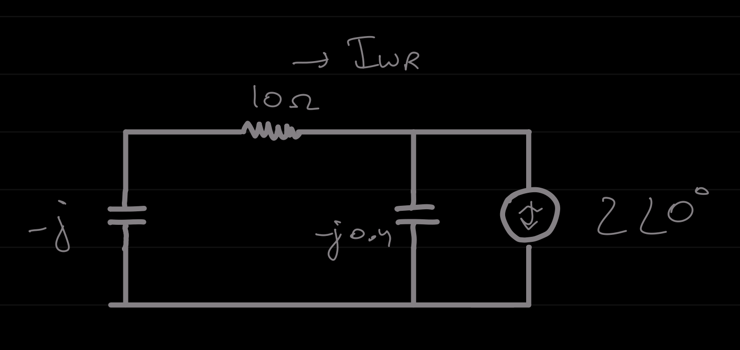


$$\frac{1}{j\omega C} = \frac{1}{j(3)0.2} = \frac{-j}{0.6}$$

$$I_{\omega_1} = 5\angle 0° \left(\frac{-j(1.667)}{-j(1.667)+10-j(0.667)}\right)$$

$\Rightarrow 811.7\angle -76.86°\,mA$

② consider right source $\omega_R = 5rad/s$



$$I_{\omega R} = 2\angle 0° \left(\frac{-j\cdot 0.4}{-j0.4+10+-j}\right)$$

$$\Rightarrow \frac{-j(0.8)}{-j(1.4)+10} \Rightarrow \frac{-j(0.8)[10+j(1.4)]}{100+1.96}$$

$$\frac{-j8+1.12}{101.96} \Rightarrow$$

$\Rightarrow 0.0792\angle -82.03°\,A = 79.2\angle -82.03\,mA$

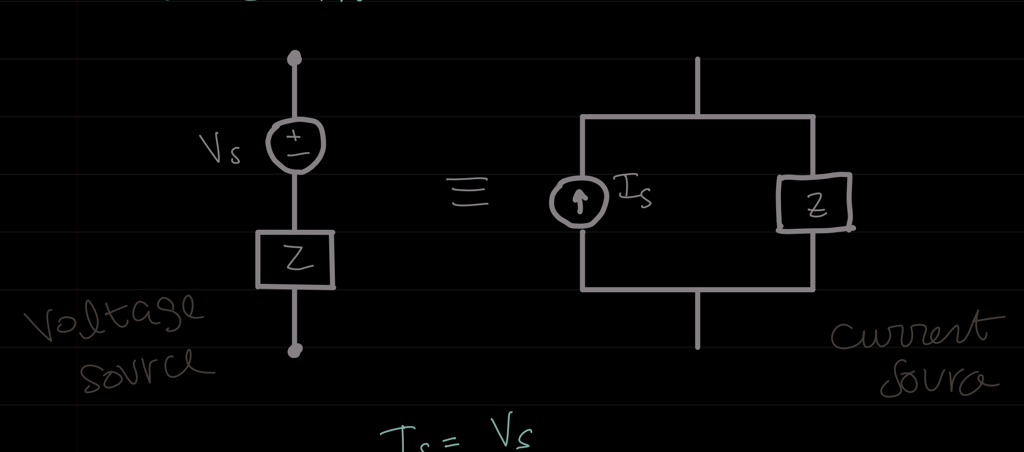
Applying superposition

$I = I_{\omega L} + I_{\omega R}$ ✗

Phasors can only be added when they correspond to same frequency

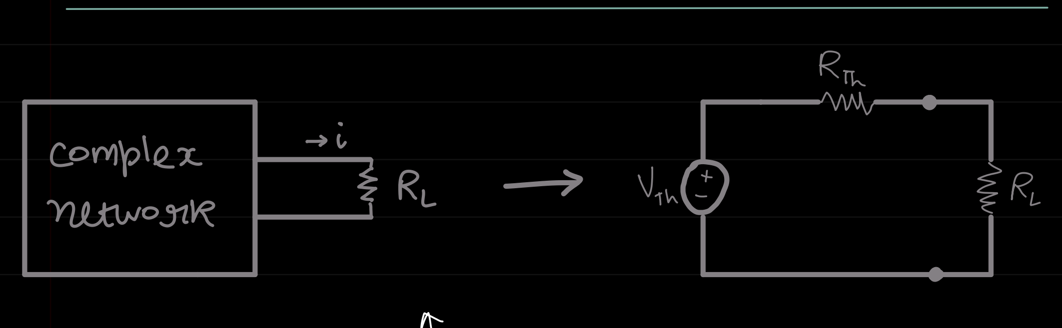
$i(t) = [811.7\cos(3t-76.86°) + 79.2\cos(5t-82.03)]\,mA$

* SOURCE TRANSFORMATION

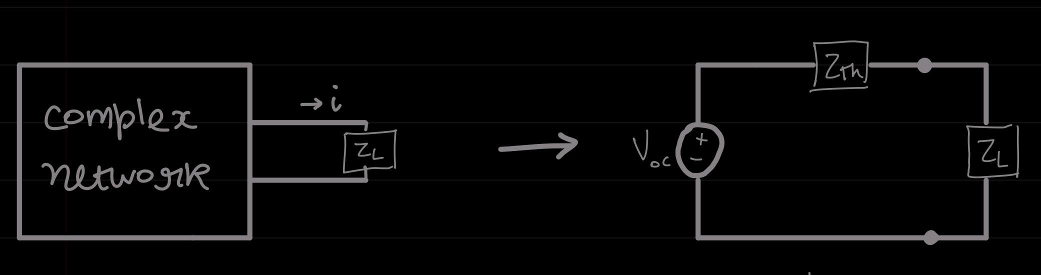


$I_s = \frac{V_s}{Z}$

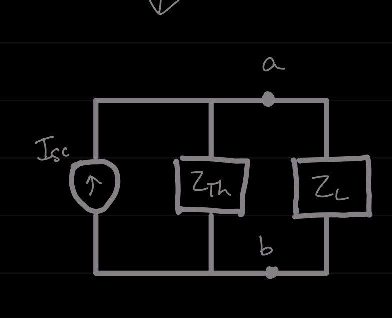
* THEVENIN & NORTON EQUIVALENT CKTS



Normally



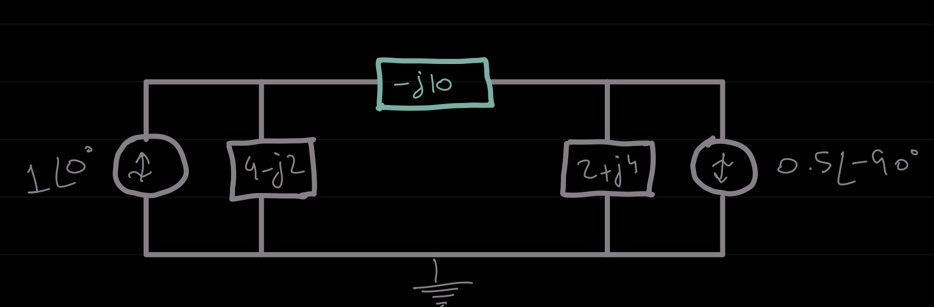
$V_{oc} = I_{sc} \cdot Z_{TH}$



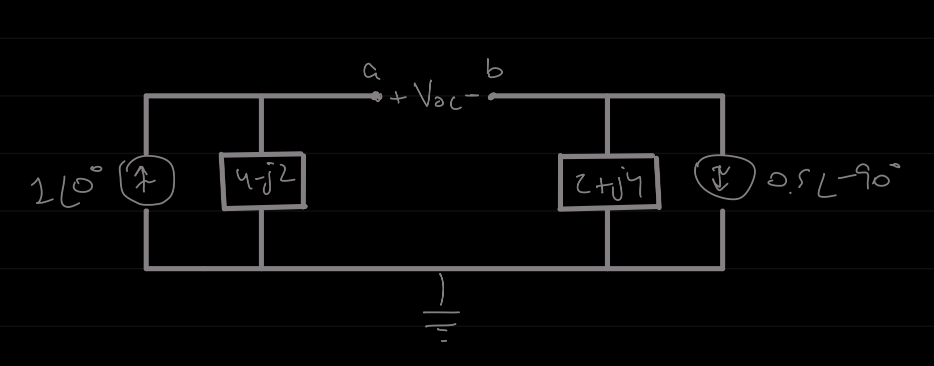
	V_{oc} (by open circuiting)	I_{sc} (by short circuiting)	Z_{TH} (ratio of V_{oc} & I_{sc})
① Independent sources	✓	✓	✓
② Independent + dependent sources	✓	✓	✓

③ Dependent sources only — we need to consider test sources
eg: 1A / 1V
and then find
$V_{oc} \Rightarrow I_{sc} \Rightarrow Z_{Th} = \frac{V_{oc}}{I_{sc}}$

eg) find Thevenin equivalent seen by $-j10\Omega$



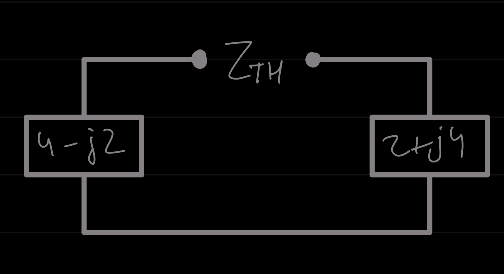
- : complex network
- : R_L



$$V_{oc} = V_a - V_b = 1\angle 0°(4-j2) - 0.5\angle -90°(2+j4)$$
$$= 4-j2+2-j1 = 6-j3\,V$$

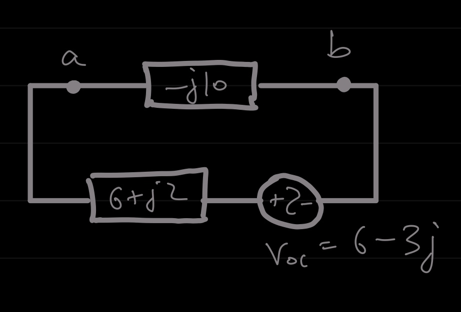
Find Z_{Th}

all volt src → short circuit
all curr src → open circuit



$Z_{TH} = 4-j2+2+j4 = 6+j2\,\Omega$

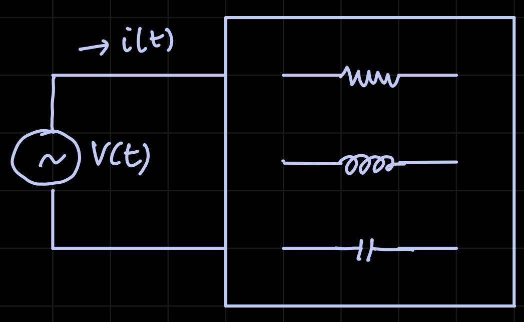
Equivalent circuit



$$I_{ab} = \frac{V_{oc}}{6+j2-j10} = \frac{V_{oc}}{6-j8}$$

* Lecture : 5 28/08/24

⇒ AC Power Analysis {Module : 3}



$\cos(A)\cos(B) = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

instantaneous power : $p(t) = v(t) \cdot i(t)$

$$v(t) = V_m \cdot \cos(\omega t + \theta)$$
$$i(t) = I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = V_m \cdot \cos(\omega t + \theta) \cdot I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = \underbrace{\frac{V_m I_m}{2} \cos(\theta - \phi)}_{\substack{\text{constant} \\ \text{(DC term)}}} + \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{\substack{\text{twice frequency} \\ \text{(harmonic)}}}$$

- Average Power

$$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2} \cos(\theta - \phi)\,dt + \underbrace{\frac{1}{T}\int_0^T \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{0}$$

$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)}$$

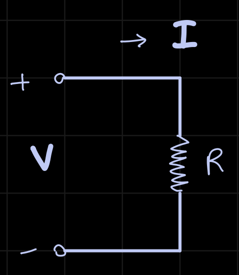
$$P_{avg} = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$$

$\mathbf{I} = I_m e^{j\phi}$
$\mathbf{I}^* = I_m e^{-j\phi}$
(conjugate)

real number ✓
phasor ✗

* Avg. Power absorbed

- by a purely resistive element

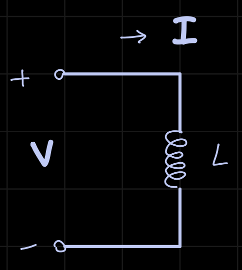


$$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$$

here, phase diff = 0°

$$P_{avg} = \frac{V_m \cdot I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

- by a purely inductive element



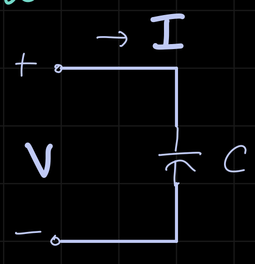
$$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$$

here, phase diff = 90°

So,

$P_{avg} = 0$ for inductor

- by a purely capacitive element



$$P_{avg} = \frac{V_m \cdot I_m}{2} \cdot \cos(-90°)$$

$P_{avg} = 0$ for capacitor

* Lecture: 6

$V(t) = V_m \cos(\omega t + \theta)$ ← Revision

$\downarrow$

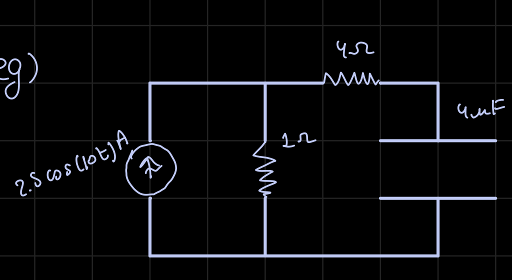
$V(t) = Re\{V_m e^{j\omega t} e^{j\theta}\}$

$\downarrow \; - e^{j\omega t}$

$V_m e^{j\theta} = V_m \angle \theta$

exponential polar

• eg)

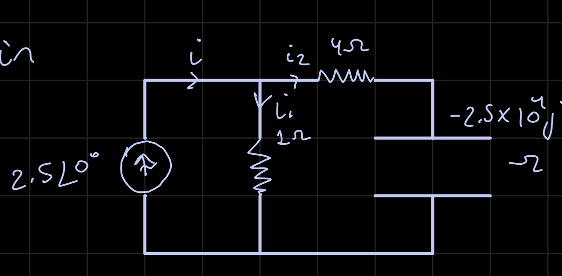


① find power delivered to each element at $t = 0, 10, 20ms$

② Find P_{avg} to each element

Ans) convert to freq. domain

$V_{1\Omega} = V_{4\Omega} + V_c$



① $P_{inst} = \frac{V_m I_m}{2} \cos(\theta - \phi) + \frac{V_m I_m}{2} \cos(2\omega t + \phi + \theta)$

imaginary part is way larger than real part

$I_1 = -2.5 \times \left(\frac{4 - j2.5 \times 10^4}{1 + 4 - j\,2.5 \times 10^4} \right) \simeq \underline{-2.5A}$

$\Rightarrow \frac{-10 + j625 \times 10^{8}}{5 - j\,25 \times 10^{3}} \Rightarrow \frac{-2 + j125}{1 - j50}$

$\Rightarrow \frac{-2 + j \cdot 125}{1 - j50} \times \frac{1 + j50}{1 + j50} \Rightarrow$

$I_{1\Omega}(t) = 2.5 \cos(10t)$

$V_{1\Omega} = (I_{1\Omega})(1\Omega) \Rightarrow V_{1\Omega}(t) = 2.5 \cos(10t)$

$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{4}$

$p_{1\Omega}(t) = 2.5 \cos(10t) \cdot 2.5 \cos(10t)$ W

$p_{1\Omega}(t=0) = 6.25W$

$p_{1\Omega}(t = 10ms) = 6.1877W$

$p_{1\Omega}(t = 20ms) = 6.033W$

$(P_{avg})_{1\Omega} = 3.125W$

⇒ P delivered to 4Ω

$P_{4\Omega}(t) = V_{4\Omega}(t) \times I_{4\Omega}(t)$

$I_{4\Omega} = I_s \times \frac{1}{5 - j2.5 \times 10^4} = 10^{-4} \angle 90° A$

$V_{4\Omega} = 4 \times I_{4\Omega} = 4 \times 10^{-4} \angle 90°\ V$

convert to time domain

$p_{4\Omega}(t) = 2 \times 10^{-8} + 2 \times 10^{-8} \cos(20t)W$

$t = 0: \; 4 \times 10^{-8}$ W

$t = 10ms = 3.96 \times 10^{-8}$ W

$t = 20ms = 3.87 \times 10^{-8}$ W

$P_{avg\,4\Omega} = 2 \times 10^{-8}$ W

⇒ P delivered to capacitor

$p_c(t) = V_c(t) \cdot I_c(t)$

$(p_{avg})_C = 0$ since phase diff = 90°

$I_c = I_2 = 10^{-4} \angle 90° A \Rightarrow I_c(t) = 10^{-4} \cos(10t + 90°)$

$V_c = (-j\,2.5 \times 10^{-4}) \times 10^{-4} \angle 90$

$-j \cdot 25 \times 10^{3} \times (+j10^{-4})$

$\underline{2.5\ V}$

$p_c(t) = 1.25 \times 10^{-4} \cos(20t + 90°)$ W

$t = 0 \Rightarrow p = 0W$

$t = 10ms \Rightarrow p = 2.48 \times 10^{-5}$ W

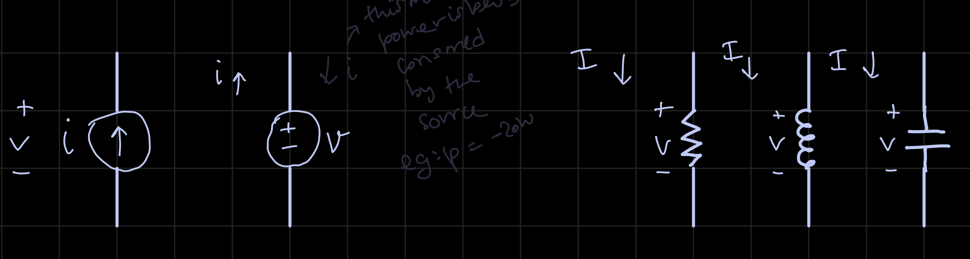
Note: we cannot multiply I_c and V_c in phasor form and then convert to time domain for getting P_c (power) because power does not has a phasor part. It is a real value.

* Correct or not?

depends on t ✗ ① $p_{1\Omega}(t) + p_{4\Omega}(t) + p_c(t) = \text{constant}\,1$

✓ ② $P_{avg\,1\Omega} + P_{avg\,4\Omega} + P_{avg\,c} = \text{constant}2 \equiv P_{avg\ source}$

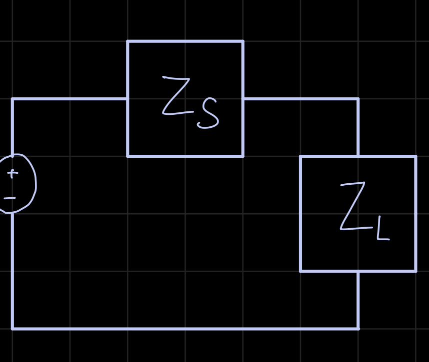
active sign convention passive sign convention



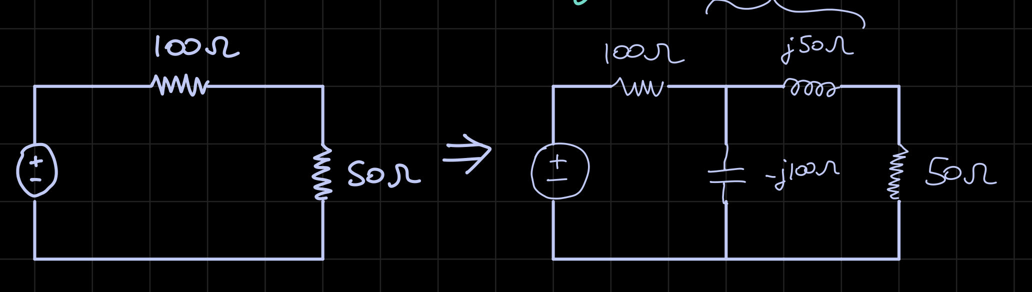
* Maximum Power Transfer Theorem

maximum power delivered to load when

$\boxed{Z_L = Z_S^*}$ complex conjugate of Z_S



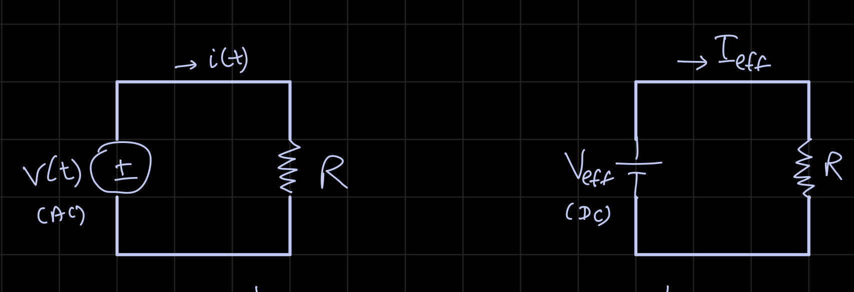
⇒ Impedance matching



$Z_L = Z_S^*$

* EFFECTIVE VALUE (Rms Value) of V and I

(voltage) (current)



if $P_{avg_1} = P_{avg_2}$

$\Rightarrow \frac{1}{T}\int_0^T i^2(t) R\, dt = I_{eff}^2 R$

$\Rightarrow \boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}} \equiv$ Root mean square

$i(t) \xrightarrow{\text{square}} i^2(t) \xrightarrow{\text{mean}} \frac{1}{T}\int_0^T i^2(t)\,dt \xrightarrow{\text{sqrt}} \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$

• Sinusoidally varying voltage

$\cos^2 x = \frac{\cos 2x + 1}{2}$

$i(t) = I_m \cos(\omega t + \phi)$

$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi)\,dt}$

$\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$ current rms value unit: A_{rms}

$\boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$ voltage rms value unit: V_{rms}

not a phasor; it is real number ≡ magnitude of current/voltage

eg: $I = 10 \angle 90° A \Rightarrow I = 5\sqrt{2} \angle 90° A$ rms

OR $V = 100\sqrt{2} \angle -60° V \Rightarrow V = 100 \angle -60° V_{rms}$

* Apparent Power

$P_{apparent} = I_{eff} V_{eff}$ unit: VA, not W (volt ampere)

* Power factor

$PF = \frac{\text{avg power}}{\text{apparent power}} = \frac{(V_m \cdot I_m)/2 \, \cos(\theta - \phi)}{V_{eff} \cdot I_{eff}}$

$PF = \cos(\underbrace{\theta - \phi}_{\text{power factor angle}})$

* Lecture : 7

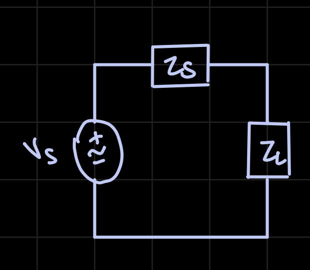
- Instantaneous Power : $p(t) = v(t)\, i(t)$
- Average Power : $P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$
- $P_{avg \cdot sources} = \Sigma P_{avg \cdot elements}$

* Note : $\overset{\text{impedence}}{Z} = \underset{\text{resistive}}{R} + j\overset{\text{reactive}}{X}$

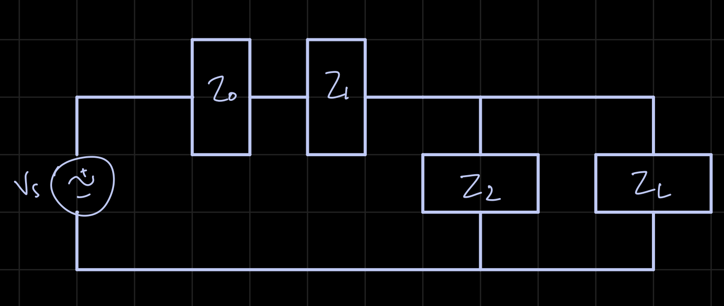
$P_{avg})_{resistive}$: $P \neq 0$

$P_{avg})_{reactive}$: $P = 0$

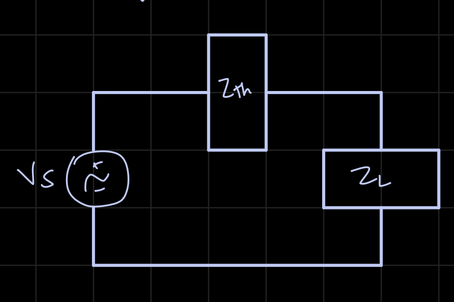
* Max Power Transfer :



Circuit has max. power when $Z_S = Z_L^*$ ← complex conjugate



⇓ apply thevenin's theorem



$Z_L = Z_{Th}^*$

* Power Factor

$$PF = \cos(\overset{\text{angle of voltage phasor}}{\theta} - \overset{\text{angle of current phasor}}{\phi}) = \frac{P_{average}}{P_{apparent}}$$

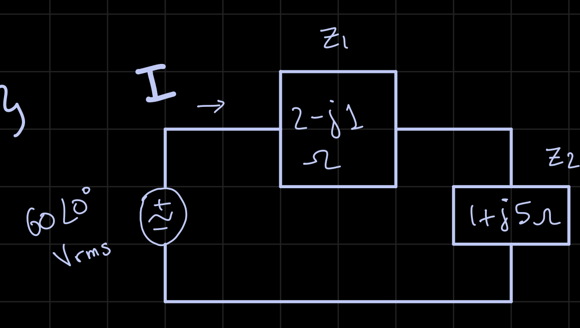
for purely resistive load : PF = 1 {Max} $\theta - \phi = \pm 90°$

for purely reactive load : PF = 0 {min}

Note → PF = 0.5 leading → capacitive $(\theta - \phi) < 0°$

PF = 0.5 lagging → inductive $(\theta - \phi) > 0°$

eg)



find

① Average power delivered to each load

② $P_{avg \cdot source}$ = ?

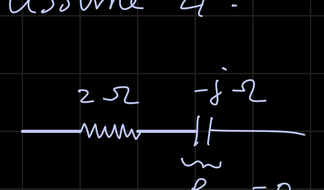
③ $P_{apparent \cdot src}$ = ?

④ PF of combined load = ?

Ans) $P_{avg\, Z_1} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$

voltage across Z_1, not the src

lets assume Z_1 :



So, $P_{avg\, Z_1}$ = Power delivered to 2Ω resistor

$= I_{eff}^2 R$

find $\mathbf{I}$ by KVL :

$$-60\angle 0° + \mathbf{I}(2 - j1 + 1 + 5) = 0$$

$$\mathbf{I} = \frac{60\angle 0°}{3 + j4} = \underbrace{12\angle -53.13°}_{I_{rms}} \text{ Arms}$$

$i(t) = 12\cos(\omega t - 53°)$ A rms

$\underset{(\text{Arms})}{I_{eff}} \rightarrow \underset{(A)}{I_m}$: $I_{eff} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = I_{eff}\sqrt{2}$

$I_{eff} = 12$ Arms $\Rightarrow I_m = 12\sqrt{2}$ A

$i(t) = 12\sqrt{2} \cos(\omega t - 53°)$ A

$P_{avg(Z_1)} = (12)^2 \times 2 = 288$ W

note: $P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$

Z_2 : I_{eff} same : series circuit

$I_{eff} = 12$

and $P_{avg})_{Z_2} = I_{eff}^2 R = (144)(1) = 144$ W

② $P_{avg})_{src} = V_{eff} I_{eff} \cos(\theta - \phi)$

$\mathbf{V}_{src} = 60\angle 0°$ Vrms $\Rightarrow V_{eff} = 60$ Vrms

$\mathbf{I}_{src} = 12\angle -53.13°$ Vrms $\Rightarrow I_{eff} = 12$ Arms

$\theta = 0°$, $\phi = -53.13°$

$P_{avg})_{src} = [60 \times 12] \cos(53.13°) = 432$ W

we can observe that $288 + 144 = 432$ and hence,

$P_{avg \cdot sources} = \Sigma P_{avg \cdot elements}$ holds

③ $P_{apparent} = V_{eff} \cdot I_{eff} = (60)(12) = 720$ W

④ PF of combined loads = PF of source

$PF = \cos(\theta - \phi) = \cos(\underbrace{0 + 53.13°}_{\theta - \phi > 0,\ \text{lagging}}) = 0.6$ lagging

* $P_{avg} = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$ $\mathbf{V} = V_m \angle\theta$, $\mathbf{I} = I_m \angle\phi$

$\mathbf{V}_{eff} = V_{eff}\angle\theta$

$\mathbf{I}_{eff} = I_{eff}\angle\phi$

$P_{avg} = Re\{\mathbf{V}_{eff}\mathbf{I}_{eff}\}$

* Complex Power

$\mathbf{S} = \mathbf{V}_{eff}\mathbf{I}_{eff}^*$ Volt Ampere {VA}

this is a complex number not a phasor of power

$\mathbf{S} = (V_{eff}\angle\theta)(I_{eff}\angle\phi)$

$= V_{eff} \cdot I_{eff}\, e^{j(\theta - \phi)}$

$$\Rightarrow \mathbf{S} = \underbrace{V_{eff} I_{eff} \cos(\theta - \phi)}_{\substack{\text{average power} \\ P \\ \text{(watt) W}}} + j \underbrace{V_{eff} I_{eff} \sin(\theta - \phi)}_{\substack{\text{reactive power} \\ Q \\ \text{(volt ampere reactance) VAR}}}$$

(volt ampere) VA

$$\boxed{S = P + jQ}$$

$|S| = V_{eff} I_{eff} = P_{apparent}$ (volt amp)

note: $Z = R + jX$

Purely resistive load : $Z = R$, $X = 0$

$S = \underset{V_{eff} I_{eff} \cos(0)}{P} + j\overset{V_{eff} I_{eff} \sin(\underbrace{\theta - \phi}_{0})}{Q}$

$P \neq 0$

$Q = 0$

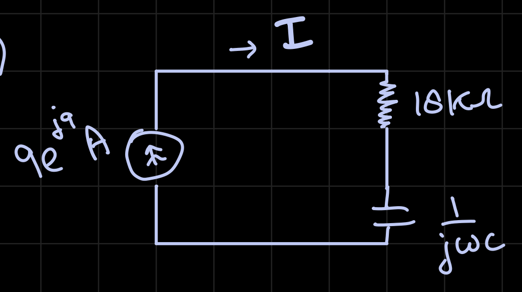
Purely reactive load

$S = \underset{V_{eff} I_{eff} \cos(90°)}{P} + j\overset{V_{eff} I_{eff} \sin(\overbrace{\theta - \phi}^{90°})}{Q}$

$P = 0$, $Q \neq 0$

Q signifies energy flow rate into or out of reactive component of load

eg)



assume $C = 1\mu F$, $\omega = 45$ rad/s

find:

① complex Power provided by src

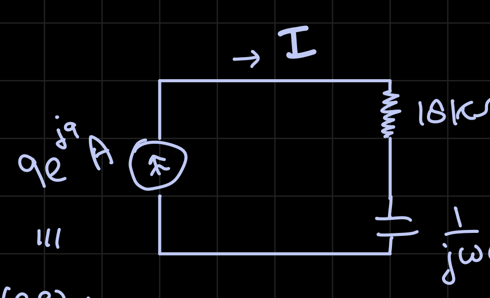
② Time average power absorbed by combined load

③ Reactive Power absorbed by combined load

④ $P_{apparent}$ absorbed " "

⑤ PF of " " "

Ans)



$\frac{1}{j\omega c} = \frac{1}{j \times 45 \times 1 \times 10^{-6}} = -j\frac{10^6}{45}$

$9e^{j9} \equiv 9\cos(9°) + j9\sin(9°)$

① $S_{source} = \overbrace{V_{eff}}^{\text{source}} \times I_{eff}$

$I_m = 9$ A

$I_{eff} = \frac{9}{\sqrt{2}}$ Arms

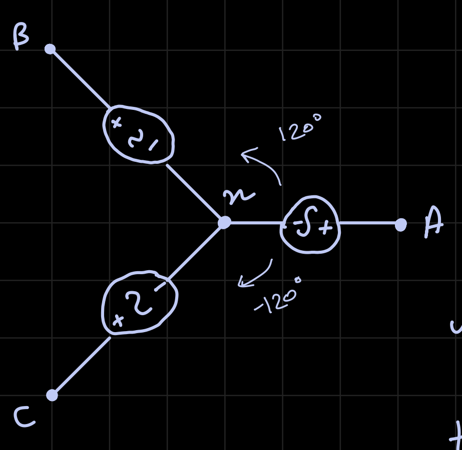
V_{eff} = ?, find $\mathbf{V}$

$$\mathbf{V} = \mathbf{I}(Z_{eq}) = 9\angle 9° \left(18 \times 10^3 - j\frac{10^6}{45}\right)$$

* Lecture : 8

- Polyphase Circuits (Chp 12)

⇒ Three Phase Source



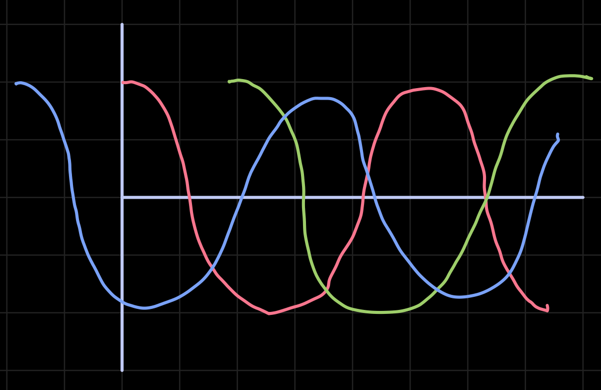
$V_{an} = 100\angle 0° \, V$
$V_{bn} = 100\angle 120° \, V$
$V_{cn} = 100\angle -120° \, V$

if $|V_{an}| = |V_{bn}| = |V_{cn}|$
& $V_{an} + V_{bn} + V_{cn} = 0$
then it is a

Balanced Source

total instantaneous power = constant

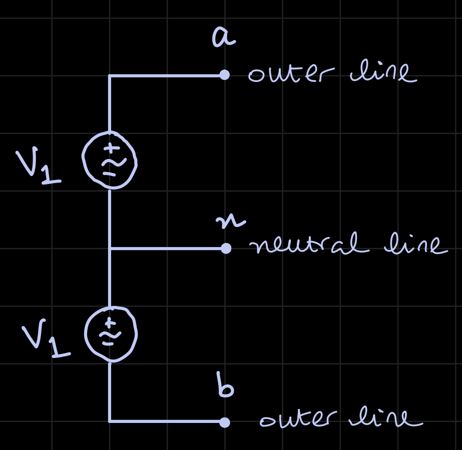
freq. domain	time domain
$100\angle 0°$	$100\cos(\omega t)$
$100\angle 120°$	$100\cos(\omega t + 120°)$
$100\angle -120°$	$100\cos(\omega t - 120°)$



total $p(t) \to$ constant

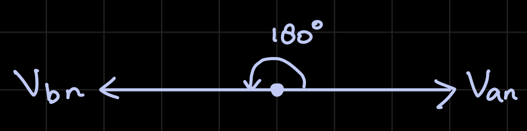
* Single Phase Three-wire Source

≡ Two phase source



$V_1 = V_m\angle 0°$

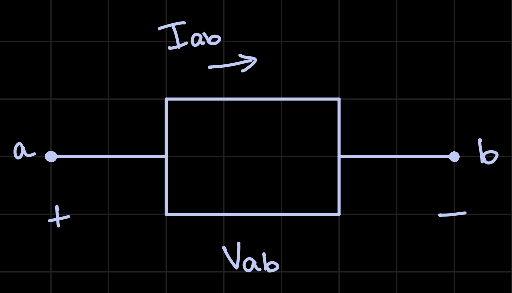
$V_{an} = V_1 = V_m\angle 0°$
$V_{bn} = -V_1 = V_m\angle 0° - 180°$



double script notation

$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$
$V_{bn} = -V_{nb}$

$V_{an} = V_A$ wrt V_n

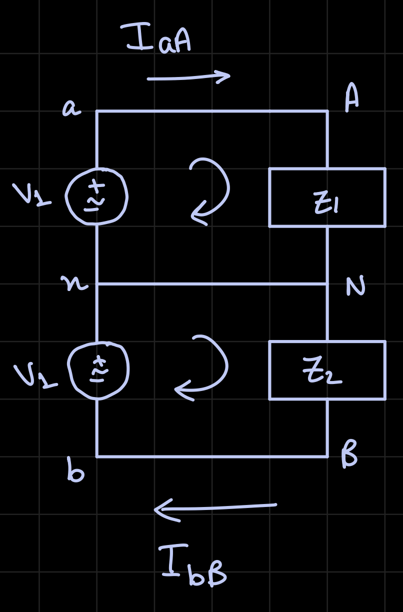


I_{ab}: current flowing from a to b

- Balanced single phase three wire source

Conditions :

$$\left.\begin{array}{l} |V_{an}| = |V_{bn}| \\ V_{an} + V_{bn} = 0 \end{array}\right\} \Rightarrow \text{Balanced Src}$$



$I_{aA} = \frac{V_1}{Z_1}$

$I_{bB} = \frac{V_1}{Z_2}$

$I_{nN} = -I_{aA} + I_{bB}$

$\Rightarrow \frac{V_1}{Z_2} - \frac{V_1}{Z_1}$

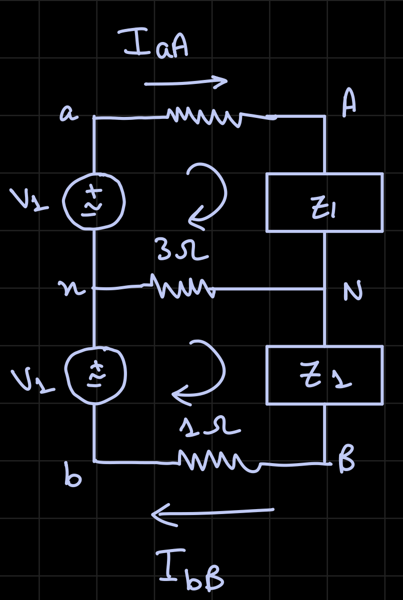
Assume $Z_1 = Z_2$

$I_{nN} = \frac{V_1}{Z_1} - \frac{V_2}{Z_1} = 0$

when both srcs and loads are equal

Balanced Load : current in neutral line is equal to zero.

all terms are phasors



even with resistance
$I_{nN} = 0 = I_{bB} - I_{aA}$
due to
① Balanced Load
≡ Symmetry

* Lecture : 9

09/09/24

time domain	freq. domain
$V_p\cos(\omega t+\theta^\circ)$	$V_p\angle\theta^\circ$
$V_p\cos(\omega t+\theta^\circ-180^\circ)$	$V_p\angle\theta-180^\circ$

we cannot directly say,

$$P_{avg} = VI \quad \times$$

but $$P_{avg} = \frac{1}{2}\mathrm{Re}\{VI^*\} \quad ✓$$

② a) $$I_{AB} = \frac{115\angle\theta^\circ + 115\angle\theta^\circ}{2(10+j2)} = \frac{230\angle\theta^\circ}{2(10+j2)}$$

$$= 11.27\angle(\theta^\circ - 11.3^\circ)A$$

$$\phi = \theta - 11.3^\circ$$

$$PF = \cos(\theta-\theta+11.3^\circ) = 0.98 \text{ lagging}$$

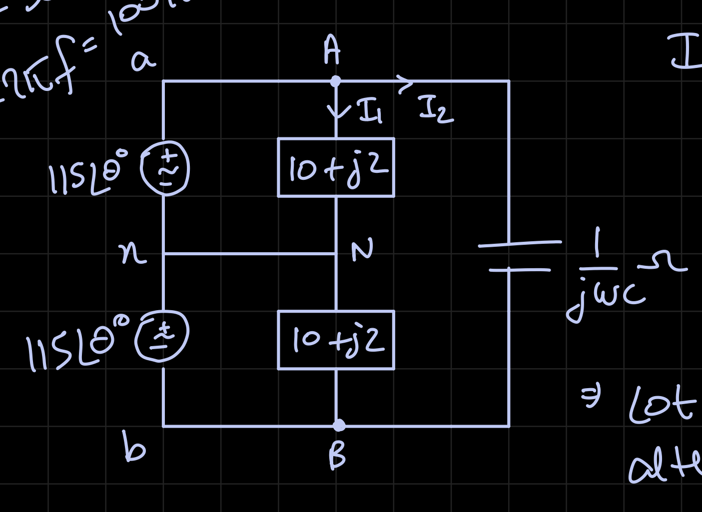
because PF angle is +ve

b) $PF = 1 \Rightarrow \cos(\theta-\phi) = 1 \Rightarrow \theta-\phi = 0$

$\Rightarrow \theta = \phi$

$f = 50h$

$\omega = 2\pi f = 100\pi$



$$I_{aA} = I_1 + I_2$$

$$\Rightarrow \frac{115\angle\theta^\circ}{10+j2} + \frac{230\angle\theta^\circ}{\left(\frac{1}{j\omega C}\right)}$$

$\Rightarrow$ lot of calculation

alternative method $\Rightarrow$

Note that this is still a balanced load, so we can remove nN since $I_{nN} = 0$

using complex power

$S \equiv$ complex power of total load

$$S = \frac{1}{2}VI^*$$

$$PF = \frac{\mathrm{Re}\{S\}}{|S|} = 1 \Rightarrow \mathrm{Re}\{S\} = |S|$$

so, $\mathrm{Im}\{S\} = 0$ here

$$S = \frac{1}{2}V_{an}I_{AN}^* + \frac{1}{2}V_{nb}I_{NB}^* + \frac{1}{2}V_{ab}I_{AB}^*$$

$$\Rightarrow \frac{1}{2}V_{an}\left(\frac{V_{an}}{Z_p}\right)^* + \frac{1}{2}V_{nb}\left(\frac{V_{nb}}{Z_p}\right)^* + \frac{1}{2}V_{ab}\left(\frac{V_{ab}}{Z_c}\right)^*$$

$$\Rightarrow \frac{1}{2}\frac{|V_{an}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{nb}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{ab}|^2}{|Z_c|^2}\times Z_c$$

$\mathrm{Re}\{115\angle0^\circ\}$

$\sqrt{10^2+2^2}$

$$\Rightarrow \frac{|V_{an}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{ab}|^2}{|Z_c|^2}\times Z_c$$

$\mathrm{Re}\{\frac{1}{j\omega C}\} = \frac{1}{j\omega C}\times\frac{-1}{j\omega C}$

$$\Rightarrow \frac{115^2}{10^4}(10+j2) + \frac{1}{2}\left(\frac{230^2}{\left(\frac{1}{\omega C}\right)^2}\right)\left(\frac{-j}{\omega C}\right)$$

$\frac{-1}{-1(\omega C)^2}$

$$\Rightarrow 115^2\left(\frac{\cancel{10}+j2}{10^4} - 2j\omega C\right)$$

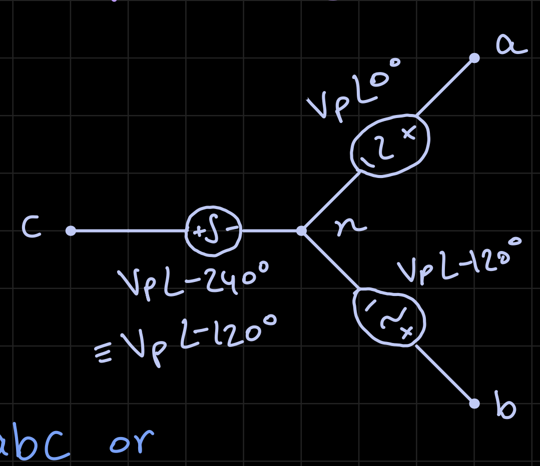
$$\Rightarrow 2\times115^2\left(\frac{j}{10^4} - j\omega C\right)$$

$$\Rightarrow 2\times115^2\left(\frac{1}{10^4} - \omega C\right) = 0$$

$$\frac{1}{10^4} - 100\pi C = 0$$

$$C = \frac{1}{10400\pi} = 30.60\,\mu F \quad ✓$$

* Three Phase Sources



$V_{an} = V_p\angle0^\circ$

$V_{bn} = V_p\angle-120^\circ$

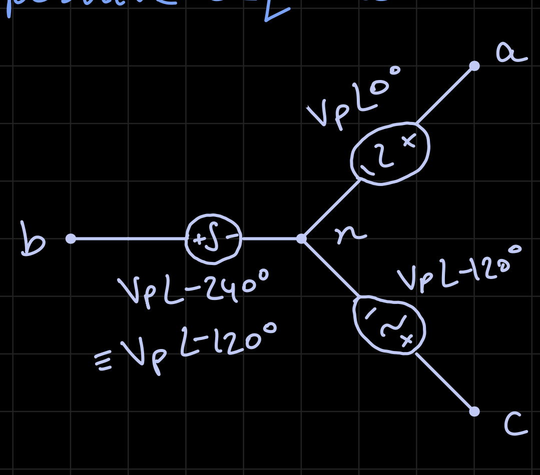
$V_{cn} = V_p\angle120^\circ)$

$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$

$V_{an}+V_{bn}+V_{cn} = 0$

↳ balanced source

abc or positive sequence



cba or negative sequence

just a naming thingy

Phase voltages V_{an}, V_{bn}, V_{cn}

Line-to-line Voltages OR line voltages V_{ab}, V_{bc}, V_{ac}

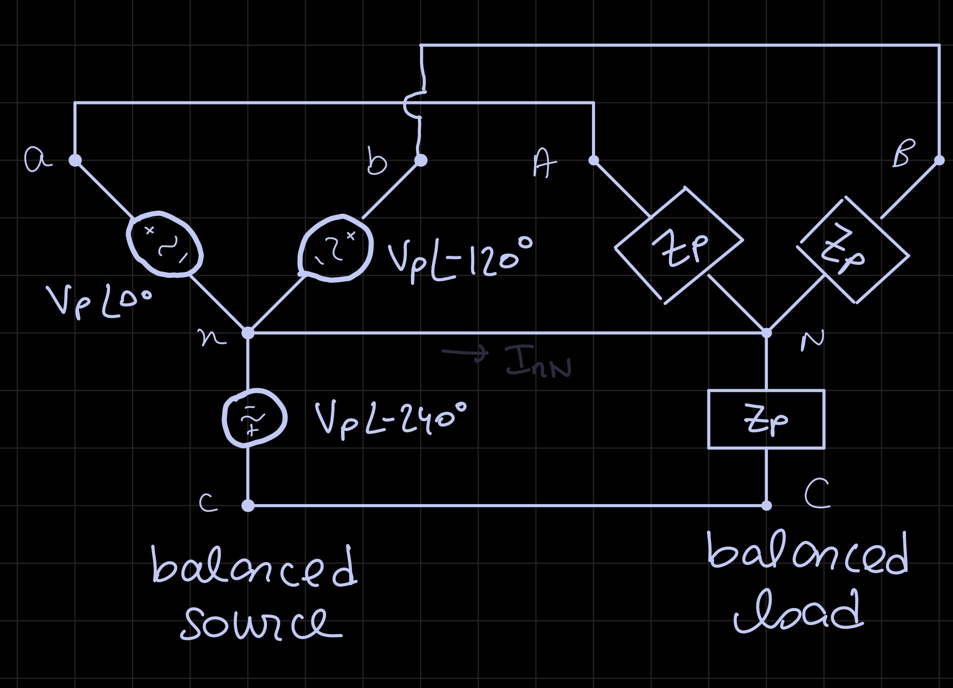
$$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$$

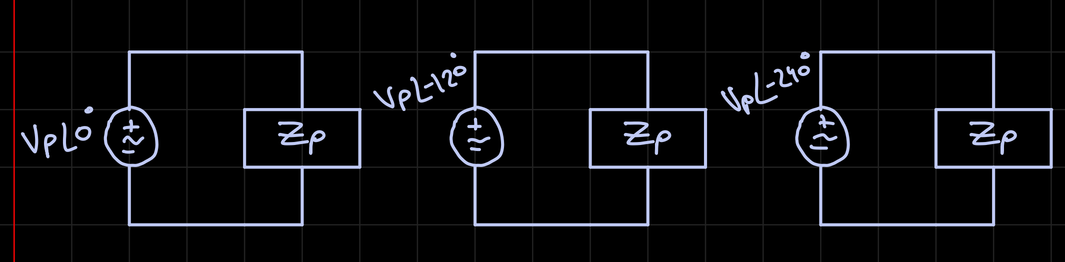
$$V_{ab} = \sqrt{3}V_p\angle30^\circ$$

$$V_{bc} = \sqrt{3}V_p\angle-90^\circ$$

$$V_{ca} = \sqrt{3}V_p\angle-210^\circ$$

* Y-Y connection





"Per-phase"

used for easier analysis

eg. to find total power by ckt, just add the three individual power obtained from above ckts.

going back to the Y-Y connection $\Rightarrow$

$$I_{aA} = \frac{V_{an}}{Z_p}$$

$$I_{bB} = \frac{V_{bn}}{Z_p} = \frac{V_{an}\angle-120^\circ}{Z_p} = I_{aA}\angle-120^\circ$$

$$I_{cC} = \frac{V_{cn}}{Z_p} = \frac{V_{an}\angle+120^\circ}{Z_p} = I_{aA}\angle+120^\circ$$

$$I_{nN} = I_{aA} + I_{bB} + I_{cC} = 0$$

eg} consider three phase balanced Y-Y connected system

$$V_{an} = 200\angle0^\circ\ V_{rms}$$
$$Z_p = 100\angle60^\circ\ \Omega$$

find phase and line voltage & current and total avg power delivered to load

Ans}

$V_{an} = 200\angle0^\circ\ V_{Rms}$

$V_{bn} = 200\angle-120^\circ\ V_{Rms}$

$V_{cn} = 200\angle+120^\circ\ V_{Rms}$

line voltage:

$$V_{ab} = \sqrt{3}V_p\angle30^\circ = 200\sqrt{3}\angle30^\circ\ V_{Rms}$$

$$V_{bc} = 200\sqrt{3}\angle-90^\circ\ V_{Rms}$$

$$V_{ca} = 200\sqrt{3}\angle-210^\circ = 200\sqrt{3}\angle150^\circ\ V_{rms}$$

line currents:

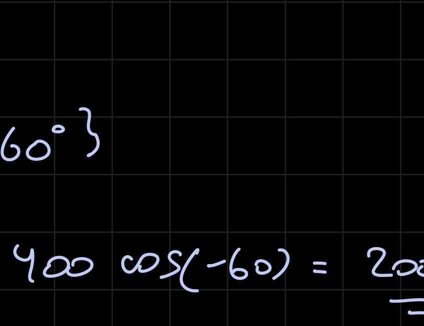
$$I_{aA} = \frac{V_{an}}{Z_p} = \frac{200\angle0^\circ}{100\angle60^\circ} = 2\angle-60^\circ\ A_{rms}$$

$$I_{bB} = 2\angle-180^\circ\ A_{rms}$$

$$I_{cC} = 2\angle-300^\circ\ A_{rms} = 2\angle60^\circ\ A_{rms}$$

total avg. power: $P_{avg} = \frac{1}{2}\mathrm{Re}\{VI^*\}$

in phase A :



$$P_{avg})_A = \frac{1}{2}\mathrm{Re}\{VI^*\}$$

$$= \mathrm{Re}\{V_{eff}I_{eff}\}$$

$$= \mathrm{Re}\{200\angle0^\circ \times 2\angle-60^\circ\}$$

$$\Rightarrow \mathrm{Re}\{400\angle-60^\circ\} \Rightarrow 400\cos(-60) = 200W$$

$P_{avg})_B = 200W$

$P_{avg})_C = 200W$

$\Big\}\ P_{avg})_{total} = 600W$