$p(t)$

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) \checkmark$$

$$P_{avg} = Re\left\{\frac{V I^*}{2}\right\}$$

$$v(t) = V_m \cos(\omega t + \theta)$$

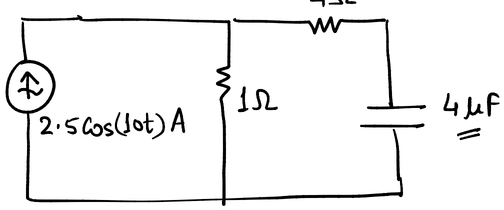
$$\downarrow$$

$$v(t) = Re\left\{ V_m e^{j\omega t + j\theta} \right\}$$

$$\downarrow \; e^{j\omega t}$$

$$V = V_m \angle \theta$$

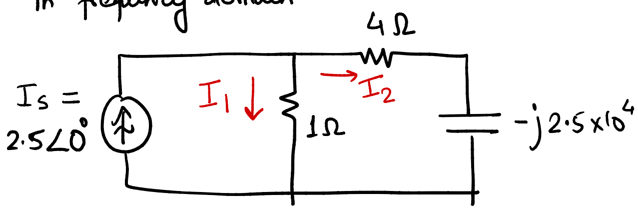
Example



① Find power delivered to each element at $t = 0, 10, 20$ ms

② Find average power delivered to each element.

In frequency domain



Power delivered to 1Ω

$$P_{1\Omega}(t) = v_{1\Omega}(t)\, i_{1\Omega}(t)$$

$$I_{1\Omega} = I_1 = I_s \frac{4 - j2.5\times10^4}{1 + 4 - j2.5\times10^4} \approx I_s \quad \Rightarrow i_{1\Omega}(t) = 2.5 \cos(10t) \text{ A}$$

$$V_{1\Omega} = (I_1)(1\Omega) \quad \Rightarrow \quad v_{1\Omega}(t) = 2.5 \cos(10t) \text{ V}$$

$$p_{1\Omega}(t) = 2.5\cos(10t) \cdot 2.5\cos(10t)$$

$$p_{1\Omega}(t) = 3.125 + 3.125\cos(20t) \text{ Watts}$$

$$p_{1\Omega}(t = 0) = 6.25 \text{ W}$$

$$p_{1\Omega}(t = 10\text{ms}) = 6.1877 \text{ W}$$

$$p_{1\Omega}(t = 20\text{ms}) = 6.0033 \text{ W}$$

Average power delivered to 1Ω

$$P_{avg\,1\Omega} = 3.125 \text{ W}$$

Power delivered to 4Ω :-

$$p_{4\Omega}(t) = v_{4\Omega}(t) \cdot i_{4\Omega}(t)$$

$$I_{4\Omega} = I_2 = I_s \frac{1}{5 - j2.5\times10^4} = 10^{-4}\angle 90^\circ \text{ A} \rightarrow i_{4\Omega}(t)$$

$$V_{4\Omega} = 4\,(I_{4\Omega}) = 4\times10^{-4}\angle 90^\circ \text{ V} \rightarrow v_{4\Omega}(t)$$

$$p_{4\Omega}(t) = 2\times10^{-8} + 2\times10^{-8}\cos(20t) \text{ W}$$

$$p_{4\Omega}(t=0) = 4\times10^{-8} \text{ W}$$

$$p_{4\Omega}(t=10\text{ms}) = 3.96\times10^{-8} \text{ W}$$

$$p_{4\Omega}(t=20\text{ms}) = 3.84\times10^{-8} \text{ W}$$

$$P_{avg\,4\Omega} = 2\times10^{-8} \text{ W}$$

Power delivered to capacitor :

$$p_c(t) =$$

$$P_{avg.c} =$$

$$I_c = I_2 = 10^{-4}\angle 90^\circ \text{ A} \longrightarrow i_c(t) \qquad \Rightarrow p(t) = i_c(t)\,v_c(t)$$

$$V_c = (-j2.5\times10^4)\,I_2 = 2.5 \text{ V} \longrightarrow v_c(t)$$

$$\cancel{\underbrace{\underline{I_c} \times \underline{V_c}}_{\text{Phasor of power}} \longrightarrow p(t)}$$

$$i_c(t) = 10^{-4}\cos(10t + 90^\circ)$$

$$v_c(t) = 2.5\cos(10t)$$

$$p_c(t) = 1.25\times10^{-4}\cos(20t + 90^\circ) \text{ W}$$

$$p_c(t=0) = 0 \text{ W}$$

$$p_c(t=10\text{ms}) = 2.48\times10^{-5} \text{ W}$$

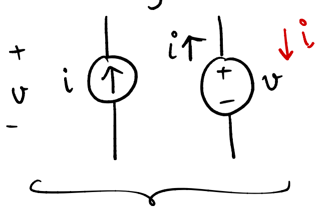
$$p_c(t=20\text{ms}) = 4.86\times10^{-5} \text{ W}$$

$$P_{avg.c} = 0$$

① $p_{1\Omega}(t) + p_{4\Omega}(t) + p_C(t) = \text{constant } 1$ ✗

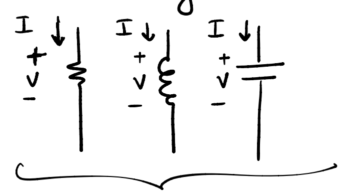
② $P_{avg,1\Omega} + P_{avg,4\Omega} + P_{avg,C} = \text{Constant } 2$ ✓ $= \underline{\underline{P_{avg,\,source}}}$

Active sign convention

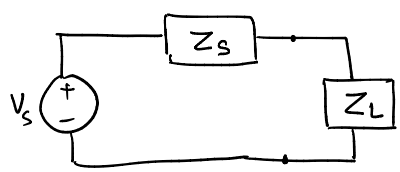


$v \cdot i = -20\,W$

Passive sign convention



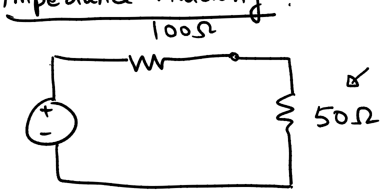
Maximum Power Transfer :



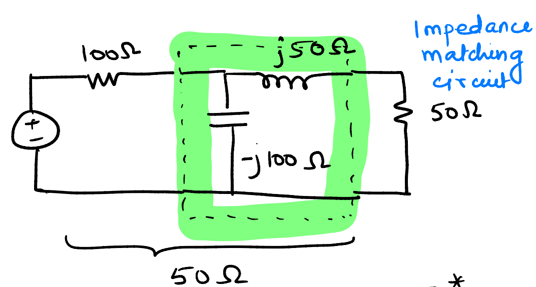
$R_s = R_L$

Max power delivered to load, when $\boxed{Z_L = Z_s^*}$ ✓

Impedance Matching :-

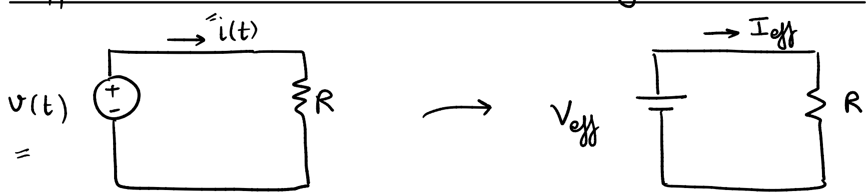


⇒



$Z_L = Z_s^*$

Effective value (RMS value) of voltage and current :—



v(t) ← P_{avg} = P_{avg}

$$\Rightarrow \frac{1}{T}\int_0^T i^2(t)\,R\,dt = I_{eff}^2 R = \frac{V_{eff}^2}{R}$$

$$\Rightarrow I_{eff} = \underbrace{\sqrt{\frac{1}{T}\int_0^T i^2(t)\,.\,dt}} =$$ Root mean square

Sinusoidally varying sources

$$i(t) = I_m \cos(\omega t + \phi)$$

$$2\cos^2 x = \cos 2x + 1$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi)\,dt}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$$ rms value

Suppose, $\mathbf{I} = 10\angle 90^\circ$ A

$$\Rightarrow \mathbf{I} = \frac{10}{\sqrt{2}}\angle 90^\circ \text{ A rms}$$

$$\boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$$ rms value

$$\mathbf{V}_1 = 100\angle -60^\circ \text{ V rms}$$

$$V_1 = 100\sqrt{2}\ \angle -60^\circ \text{ V}$$

Apparent Power

$$P_{apparent} = I_{eff} V_{eff}$$

Power Factor

$$PF = \frac{\text{average power}}{\text{apparent power}} = \frac{(V_m I_m / 2) \cos(\theta - \phi)}{V_{eff} I_{eff}} = \cos(\theta - \phi)$$

$$PF = \cos \underbrace{(\theta - \phi)}_{\text{Power factor angle}}$$