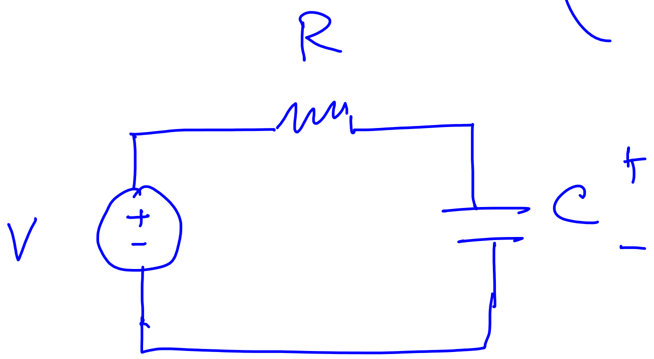
Quick Recap

RC circuit $\rightarrow$ charging, discharging

Unit step function

$$u(t) = \begin{cases} 0 & \forall\, t < 0 \\ 1 & \forall\, t \geq 0 \end{cases}$$



$v_c(0) = 0$

$$V_c(t) = V\left(1 - e^{-t/RC}\right) \quad \forall\, t \geq 0$$

time constant (τ)

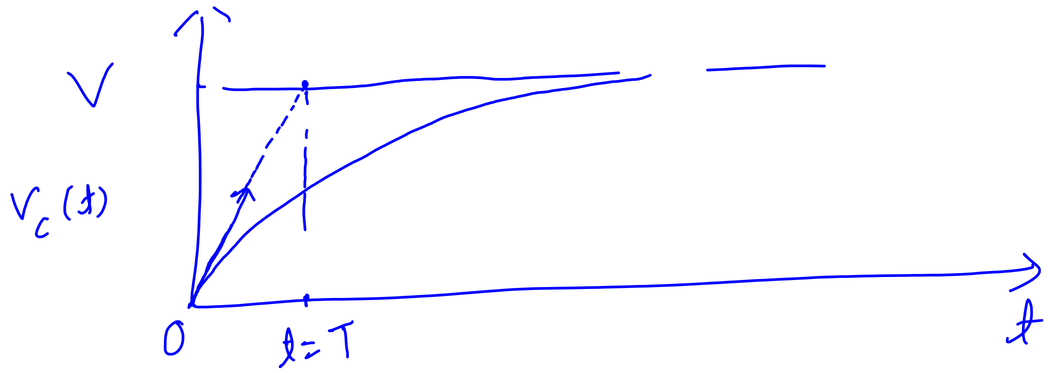
$\tau = RC$

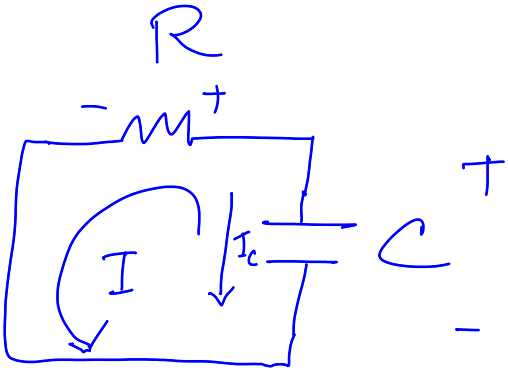
at $t = \tau$,

$$V_c(t) \approx 66\%\ V$$

$$\left.\frac{dV_c}{dt}\right|_{t=0} = \left.\frac{V}{RC}\, e^{-t/RC}\right|_{t=0} = \frac{V}{RC} = \frac{V}{T}$$

If the rate of change of V_c would have remained same as $\left.\frac{dV_c}{dt}\right|_{t=0}$ for $t>0$, the capacitor voltage would have reached V at $t = T$.

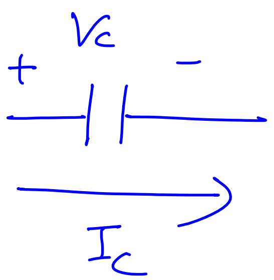




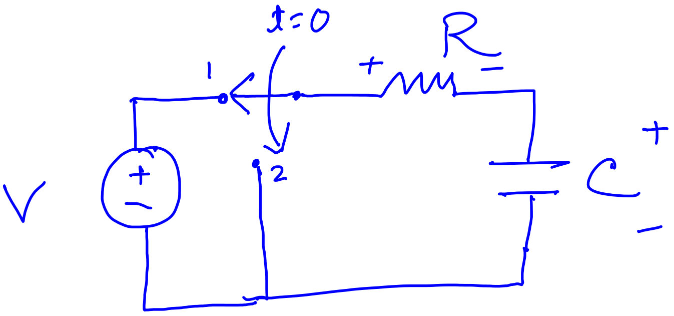
$v_c(0) = V$

$$V_c(t) = V e^{-t/RC}$$

$$I_c = C\frac{dV_c}{dt} = -I$$



$$I_c = C\frac{dV_c}{dt}$$



$\underline{\underline{V_C(0)}}$

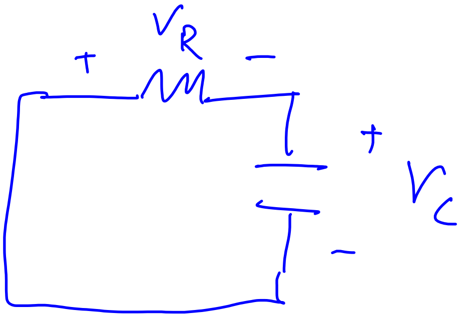
$$V_C(t) = \boxed{V}\, e^{-t/RC} \qquad \forall t \geq 0$$

$$\underline{V_C(0^-) = V \implies V_C(0^+) = V} \quad \text{if } I_C \neq \infty$$

$$\lim_{\varepsilon \to 0} V_C(0-\varepsilon) = \lim_{\varepsilon \to 0} V_C(0+\varepsilon) \quad \text{(continuity property)}$$

$$V_R(0^-) = 0 \qquad V_R(0^+) = -V \quad \text{(discontinuous)}$$

$\underline{\underline{t \geq 0}}$



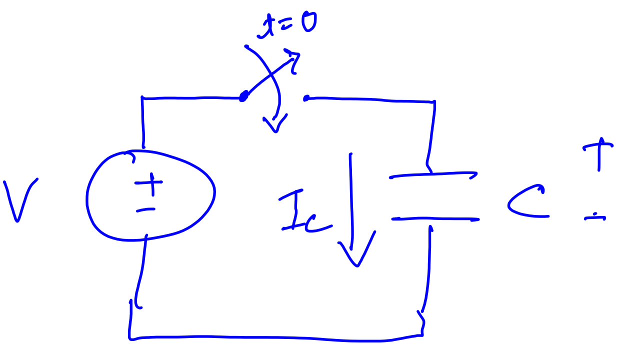
$$V_C(t) + V_R(t) = 0 \qquad \forall t \geq 0$$

$$V_C(0) + V_R(0) = 0$$

$$\implies V_R(0) = -V_C(0)$$

$$= -V$$

<u>Let's consider the following circuit</u>,

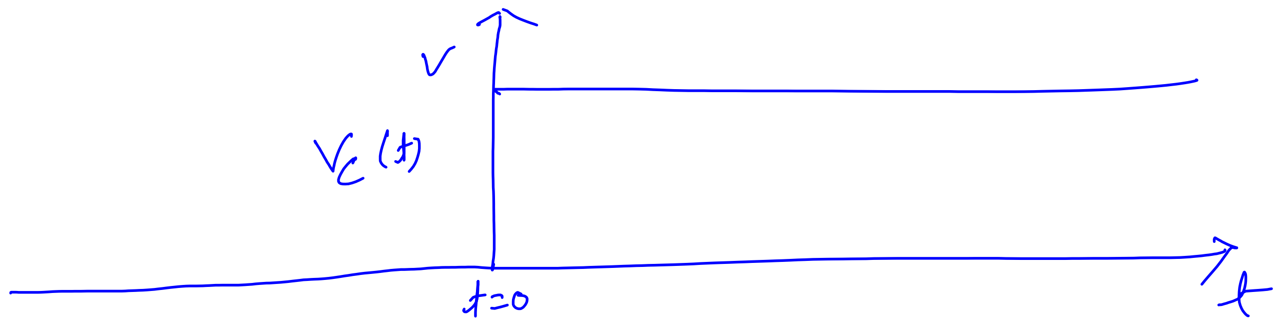


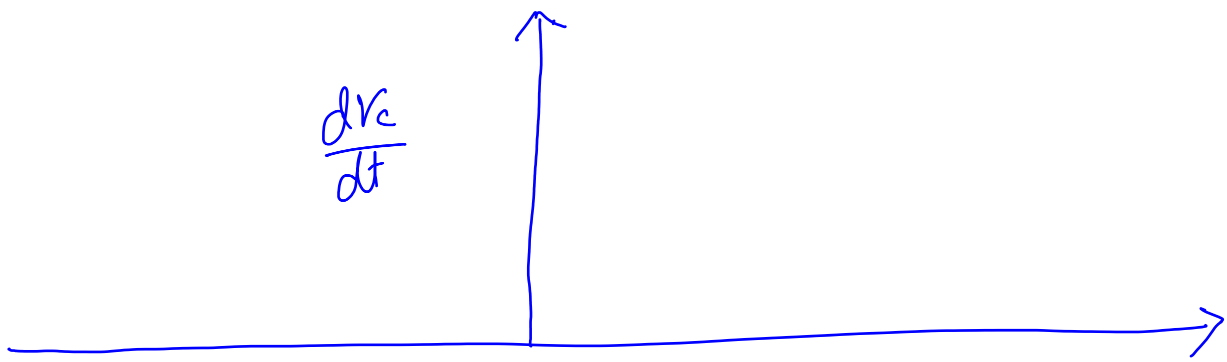
$$V_C(0^-) = 0$$

$V_c(0^+) = V$ (using KVL)

$$I_c = C \frac{dV_c}{dt}$$

$$V_c(t) = V\,u(t)$$



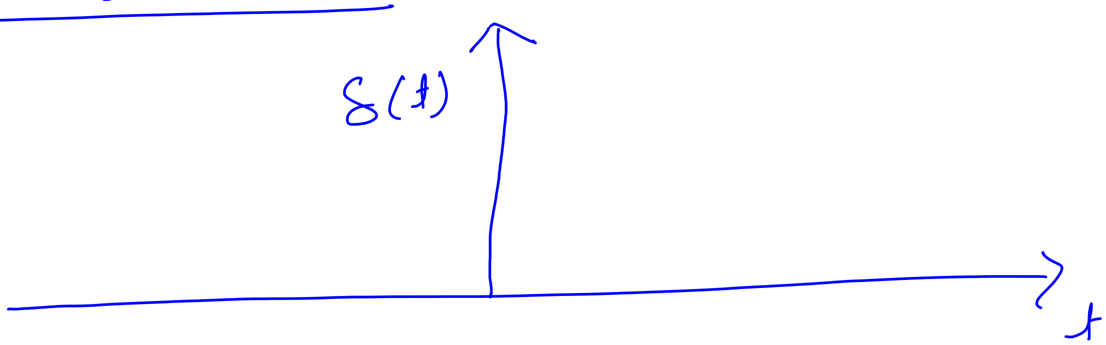


1) $\frac{dV_c}{dt} = 0 \quad \forall\, t \neq 0$

2) $\frac{dV_c}{dt}$ is not defined at $t=0$

3) $$\int_{-\infty}^{\infty} \frac{dV_c}{dt}\, dt = V$$

Dirac - delta function



1) $\delta(t) = 0 \quad \forall\, t \neq 0$

2) $\delta(t)$ is not defined at $t=0$

3) $\displaystyle\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

$$\Rightarrow \quad \frac{d\,u(t)}{dt} = \delta(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(t)\, dt$$

if $t < 0$,

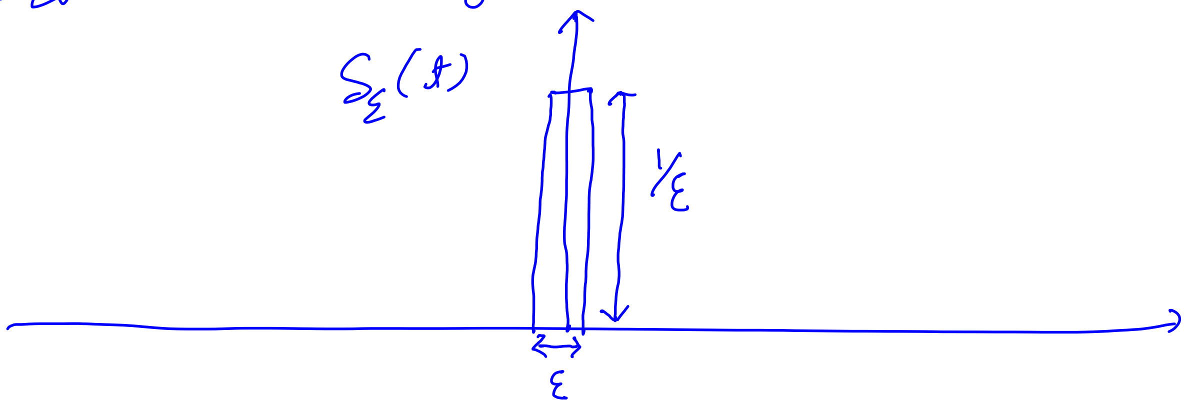
$$\boxed{u(t) = \int_{-\infty}^{t} \delta(t)\, dt = 0}$$

if $t \geq 0$,

$$u(t) = \int_{-\infty}^{t} \delta(t)\, dt = \int_{-\infty}^{\infty} \delta(t)\, dt - \int_{t}^{\infty} \delta(t)\, dt$$

$$= 1 - 0 = 1$$

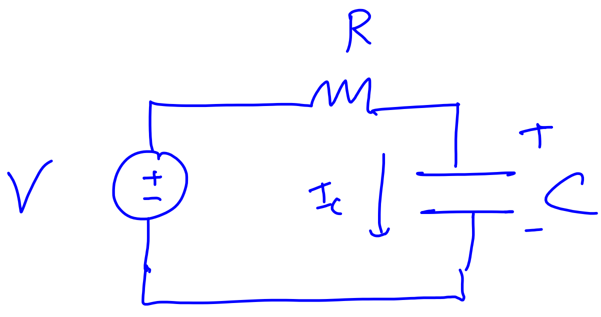
$$\int_{-\infty}^{+\infty} \delta(t)\, dt = \int_{0^-}^{0^+} \delta(t)\, dt = 1$$



$$\delta_\varepsilon(t) = 0 \quad \forall\, t > \varepsilon/2 \text{ and } \forall\, t < -\varepsilon/2$$

$$= 1/\varepsilon \quad -\varepsilon/2 \le t \le \varepsilon/2$$

$$\delta(t) = \underset{\varepsilon \to 0}{\mathrm{Lt}}\ \delta_\varepsilon(t)$$



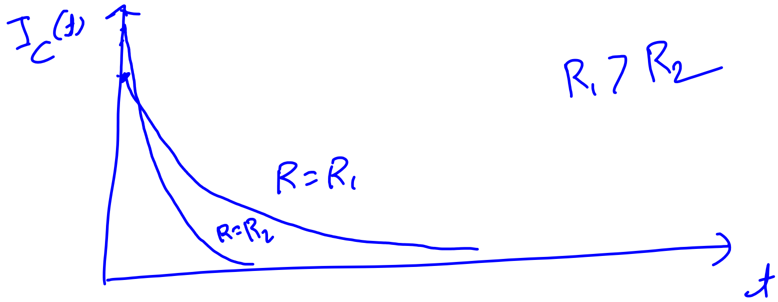
, $V_C(0^+) = 0$

$$V_C(t) = V\left(1 - e^{-t/RC}\right) \quad \forall\, t \ge 0$$

$$I_C(t) = C\,\frac{dV_C}{dt} = \frac{V}{R}\, e^{-t/RC}$$

$$\lim_{R \to 0} I_c(t) = \delta(t) \cdot C$$

(some constant) Try to derive the expression of C . . .



$R_1 > R_2$