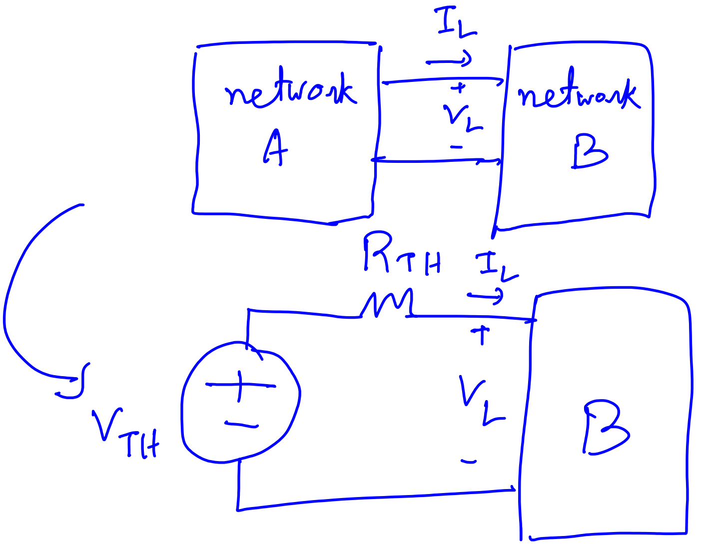
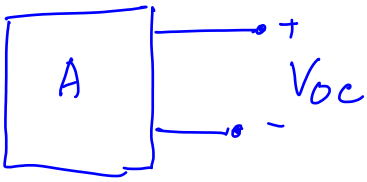
Quick Recap

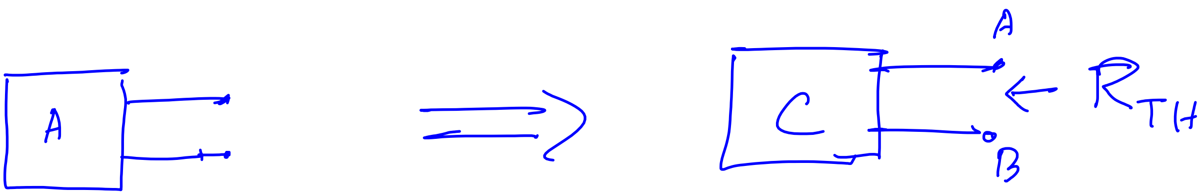
Thevnin's Theorem



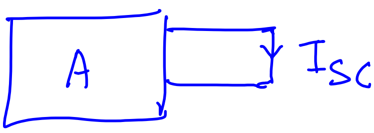
Thevnin equivalent circuit



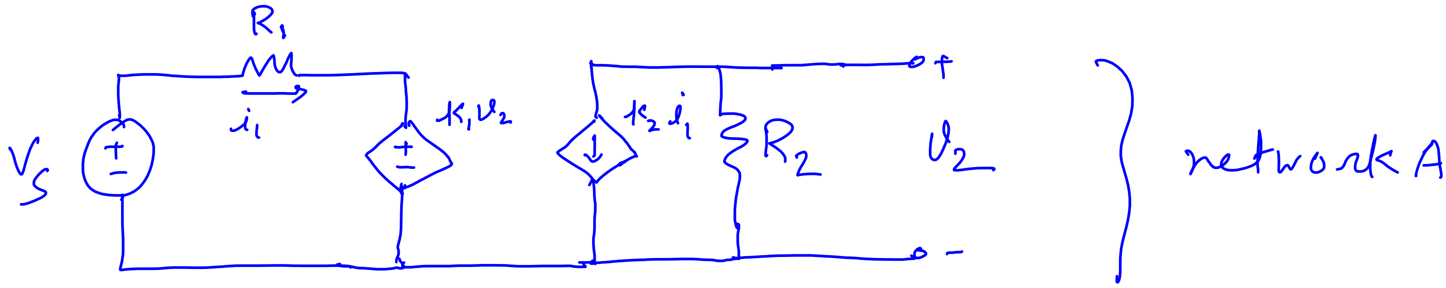
$V_{TH} = V_{OC}$

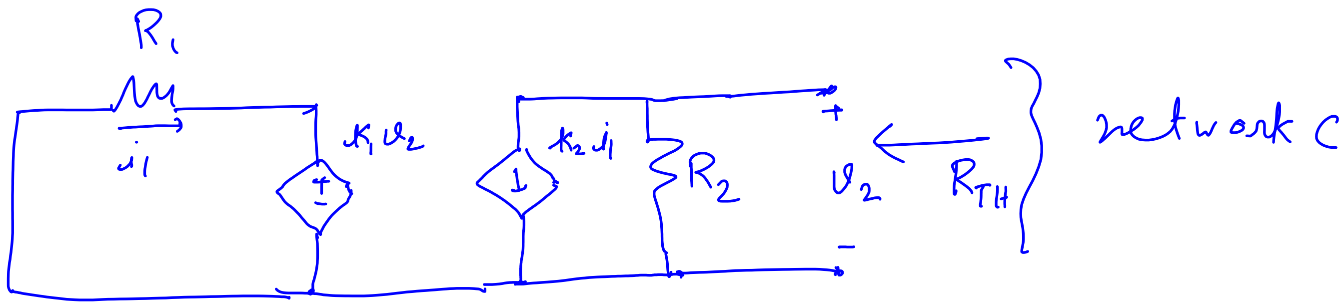


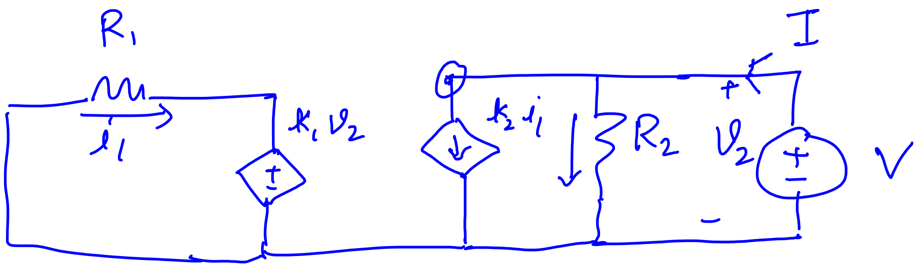
(switch off all the independent sources)



$I_N = I_{SC}$







$$i_1 R_1 + k_1 v_2 = 0 \quad \text{(KVL)}$$

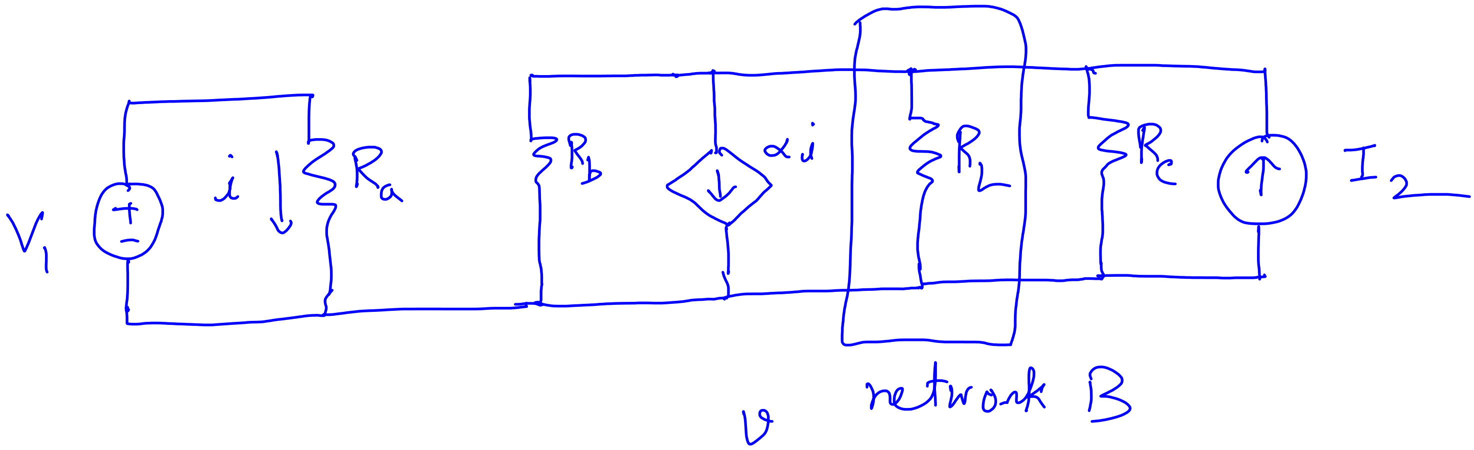
$$I = k_2 i_1 + \frac{v_2}{R_2} \quad \text{(KCL)} \qquad v_2 = V$$

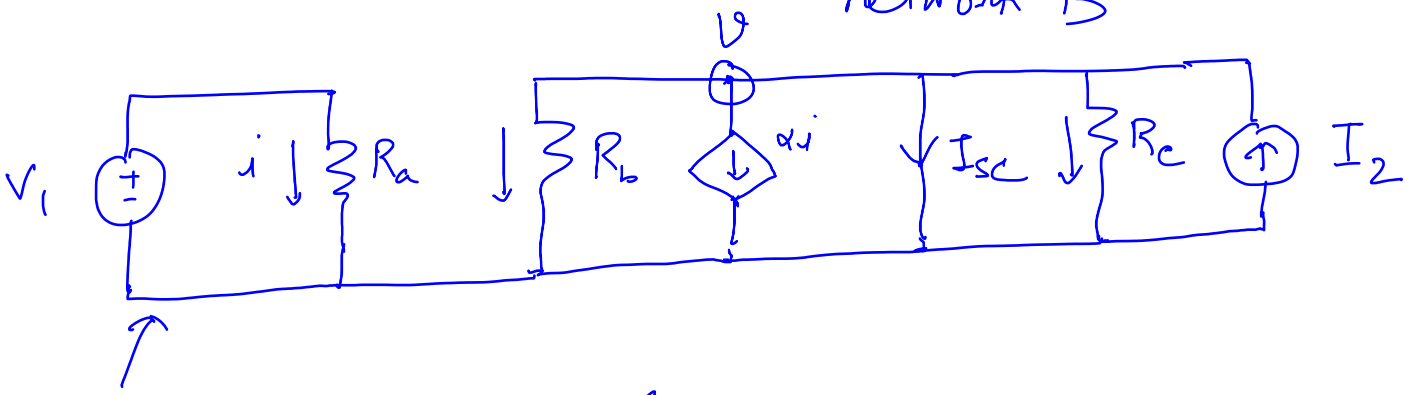
$$i_1 = -\frac{k_1 v_2}{R_1}$$

$$I = -\frac{k_1 k_2}{R_1} v_2 + \frac{v_2}{R_2}$$

$$= V\left(-\frac{k_1 k_2}{R_1} + \frac{1}{R_2}\right)$$

$$R_{TH} = \frac{V}{I} = \frac{1}{\left(-\frac{k_1 k_2}{R_1} + \frac{1}{R_2}\right)}$$



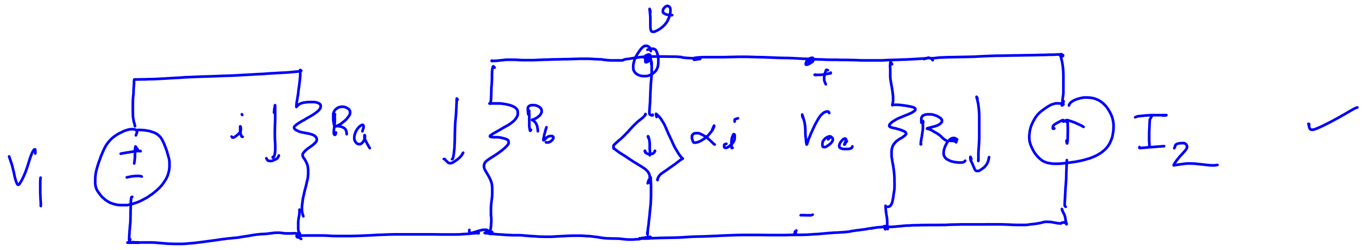


$V_1 = i R_a$ (KVL)

$I_2 = \frac{v}{R_b} + \frac{v}{R_c} + I_{sc} + \alpha i$ (KCL)

$I_{sc} = I_2 - \alpha i$

$I_N = I_{sc} = I_2 - \alpha \frac{V_1}{R_a}$



$v = V_{oc}$

$V_1 = i R_a$ (KVL)

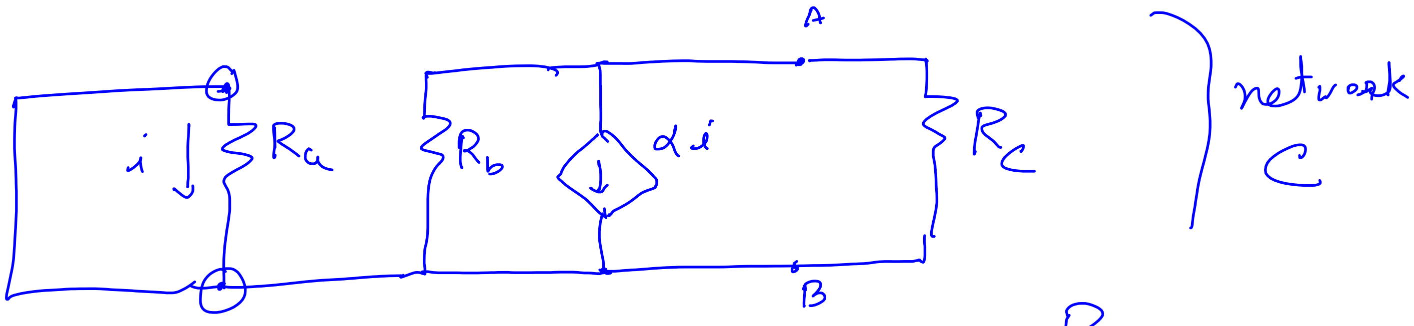
$I_2 = \frac{v}{R_b} + \frac{v}{R_c} + \alpha i$ (KCL)

$$I_2 = \frac{v}{R_b} + \frac{v}{R_c} + \alpha \frac{V_1}{R_a}$$

$$I_2 - \frac{\alpha V_1}{R_a} = v\left(\frac{1}{R_b} + \frac{1}{R_c}\right)$$

$$V_{TH} = V_{OC} = v = \frac{I_2 - \frac{\alpha V_1}{R_a}}{\left(\frac{1}{R_b} + \frac{1}{R_c}\right)} \qquad I_N$$

2) $$\frac{V_{TH}}{I_N} = \frac{1}{\left(\frac{1}{R_b} + \frac{1}{R_c}\right)} = R_{TH}$$



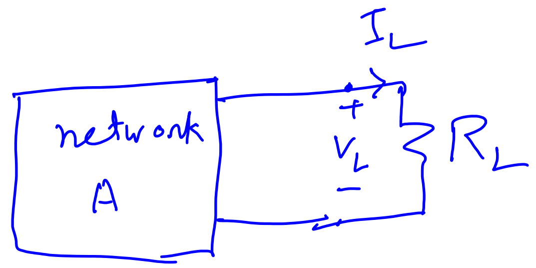
R_{AB}

$i = 0$

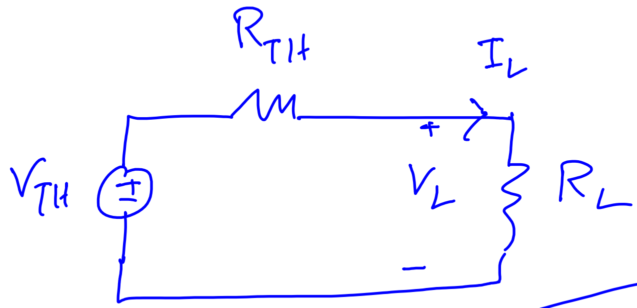
$$R_{AB}^{-1} = R_b^{-1} + R_c^{-1}$$

$$R_{TH} = R_{AB} = \frac{1}{\frac{1}{R_b} + \frac{1}{R_c}}$$

Maximum Power Transfer Theorem



$$P_L = V_L I_L$$



$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

$$P_L = I_L^2 R_L$$

$$P_L = \frac{V_{TH}^2}{(R_{TH} + R_L)^2} R_L$$

$$\frac{dP_L}{dR_L} = V_{TH}^2 \left[\frac{-2R_L}{(R_{TH} + R_L)^3} + \frac{1}{(R_{TH} + R_L)^2} \right]$$

$$= V_{TH}^2 \left[\frac{R_{TH} - R_L}{(R_{TH} + R_L)^3} \right]$$

$$\frac{dP_L}{dR_L} = 0 \Rightarrow R_L = R_{TH}$$

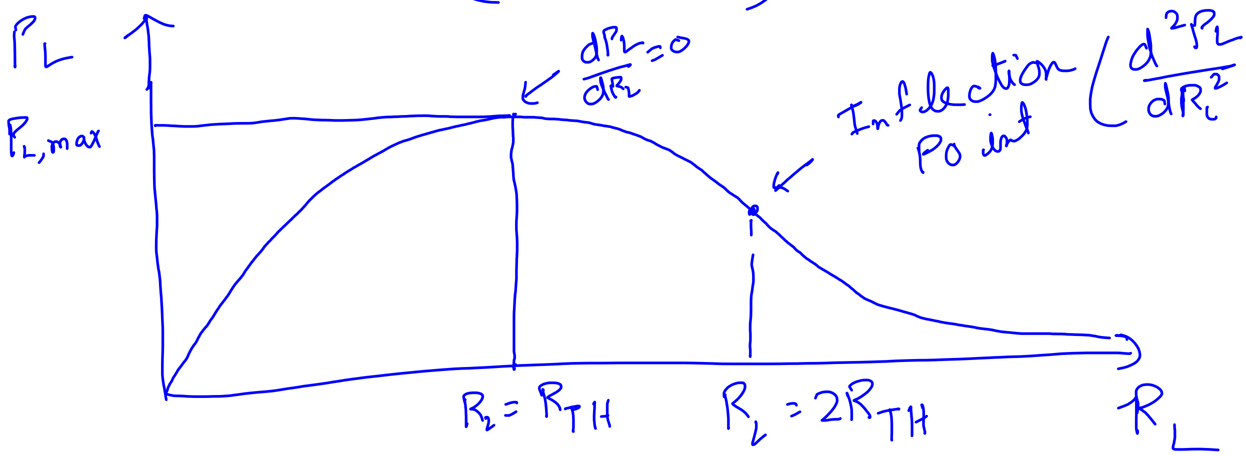
$$\frac{d^2 P_L}{dR_L^2} = V_{TH}^2 \left[\frac{-3(R_{TH} - R_L)}{(R_{TH}+R_L)^4} - \frac{1}{(R_{TH}+R_L)^3} \right]$$

$$= V_{TH}^2 \left[\frac{2R_L - 4R_{TH}}{(R_{TH}+R_L)^4} \right]$$

$$\left.\frac{d^2 P_L}{dR_L^2}\right|_{R_L = R_{TH}} < 0 \Rightarrow R_L = R_{TH}$$

is a maxima.

$$P_L = V_{TH}^2 \frac{R_L}{(R_{TH}+R_L)^2}$$



$$P_{L,max} = V_{TH}^2 \frac{R_{TH}}{(R_{TH}+R_{TH})^2} = \frac{V_{TH}^2}{4R_{TH}}$$