Review

Instantaneous Power : $p(t) = v(t)\, i(t)$

Average Power : $P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = \frac{1}{2} \mathrm{Re}\{ \mathbf{V}\mathbf{I}^* \}$

$$P_{avg,\,sources} = \sum P_{avg,\,elements}$$

P_{avg} for resistive load : $P_{avg} \neq 0$

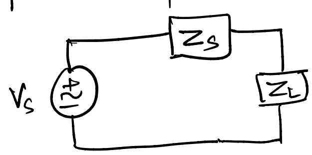
P_{avg} for reactive load : $P_{avg} = 0$

resistive

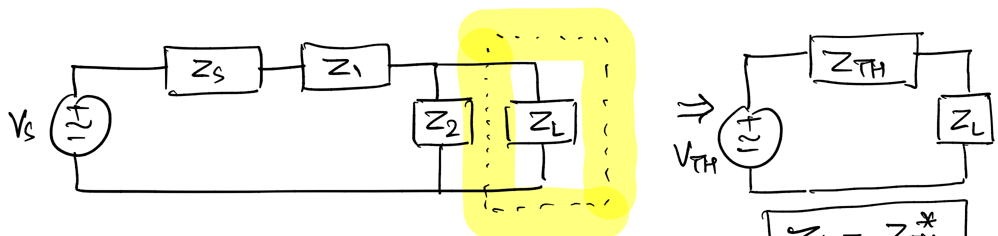
$Z = R + jX$

reactive

Max. power Transfer :



$Z_L = Z_s^*$



$$\boxed{Z_L = Z_{TH}^*}$$

Effective V and I values (RMS values)

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt}$$

$$i(t) = I_m \cos(\omega t + \phi)$$

$$I_{eff} = \frac{I_m}{\sqrt{2}} \quad , \quad V_{eff} = \frac{V_m}{\sqrt{2}}$$

$$\mathbf{I} = I_m \angle \phi = \sqrt{2}\, I_{eff} \angle \phi$$

I_{eff} A rms

V_{eff} V rms

Apparent Power

$P_{apparent} = I_{eff} \cdot V_{eff}$ VA (Volt-Ampere)

Power Factor

$$PF = \cos(\theta - \phi) = \frac{P_{average}}{P_{apparent}}$$

For purely resistive load: $PF = 1$ (max)

For purely reactive load: $PF = 0$ (min)

$\theta - \phi \quad 90^\circ$

$\theta - \phi \quad -90^\circ$

Assume PF of a load $= 0.5$

$$\cos(\theta - \phi) = 0.5$$

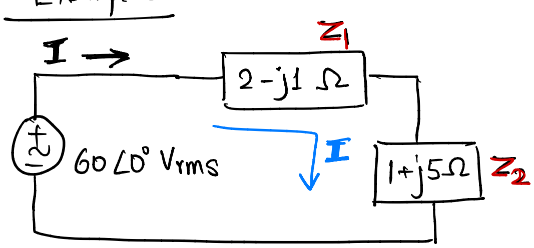
$$\Rightarrow (\theta - \phi) = \pm 60^\circ$$

$PF = 0.5$ leading $\rightarrow$ capacitive $(\theta - \phi) < 0$

$ 0.5$ lagging $\rightarrow$ inductive $(\theta - \phi) > 0$

$\boxed{\mathbf{I} \text{ w.r.t. } \mathbf{V}}$

Example

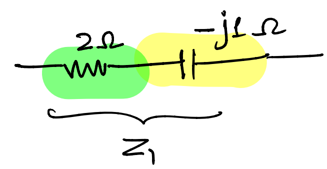


Find

1. Average power delivered to each loads
2. Average power supplied by source
3. Apparent power supplied by source
4. PF of combined load.

① $P_{avg, z_1} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} \text{Re}\{\mathbf{V}\mathbf{I}^*\}$

$$= \underline{I_{eff}^2 R}$$



$\mathbf{I} =$

Writing KVL;

$$-60\angle 0^\circ \text{ Vrms} + \mathbf{I}(2 - j1 + 1 + j5) = 0$$

$$\Rightarrow \mathbf{I} = \frac{60\angle 0^\circ \text{ Vrms}}{3 + j4\ \Omega} = \underbrace{12}_{I_{eff}}\angle -53.13^\circ \text{ A rms}$$

$$P_{avg, z_1} = (12)^2\, 2 = 288 \text{ W}$$

$$\mathbf{I} = 12 \angle -53^\circ \quad A\ rms$$

$$i(t) = \underbrace{12}_{I_{eff}} \cos(\omega t - 53^\circ) \quad \underline{A\ rms}$$

I_{eff} (A rms)	versus	I_m (A)

$$I_{eff} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = I_{eff}\sqrt{2}$$

$$I_{eff} = 12$$

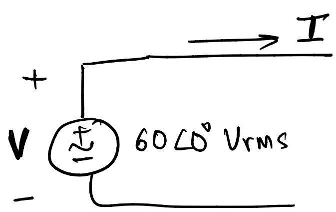
$$I_m = I_{eff}\sqrt{2} = \underline{12\sqrt{2}}$$

$$i(t) = \underbrace{12\sqrt{2}}_{I_m} \cos(\omega t - 53^\circ) \quad A$$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$$

$$P_{avg, Z_2} = I_{eff}^2 R = (12)^2 1 = 144 \quad W$$

2) $$P_{avg,\ source} = V_{eff} I_{eff} \cos(\theta - \phi)$$



$$\mathbf{V}_{source} = 60 \angle 0^\circ \ V\ rms$$

$$\mathbf{I}_{source} = 12 \angle -53.13^\circ \ A\ rms$$

$$P_{avg, source} = (60)(12) \cos(0 - (-53.13^\circ)) = 432\ W$$

$$P_{avg, Z_1} = 288\ W, \quad P_{avg, Z_2} = 144\ W$$

$$P_{avg, Z_1} + P_{avg, Z_2} = 432\ W$$

3) $$P_{apparent,\ source} = I_{eff} V_{eff} = (60)(12) = 720 \quad VA$$

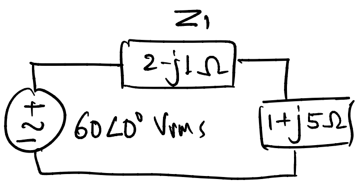
4) PF of combined loads = PF of source

$$PF = \cos(\theta - \phi) = \cos(\underbrace{0^\circ + 53.13^\circ}) = 0.6 \text{ lagging}$$

$$(\theta - \phi) > 0 \rightarrow \text{lagging}$$

$$(\theta - \phi) < 0 \rightarrow \text{leading}$$

$$P_{avg,Z_1} = I_{eff}^2 R$$



$$P_{avg,Z_1} = V_{eff} I_{eff} \cos(\theta - \phi)$$

$$\mathbf{V}_{Z_1} = 60\angle 0° \text{ Vrms} \times \frac{2-j1}{3+j4}$$

$$= \underbrace{26.83}\ \angle -79.7° \quad \text{Vrms}$$

$$\mathbf{I}_{Z_1} = \mathbf{I} = 12\angle -53.13° \text{ Arms}$$

$$P_{avg,Z_1} = 26.83 \times 12 \times \cos(-79.7° - (-53.13°)) = 288 \text{ W}$$

$$P_{average} = \frac{1}{2}\mathrm{Re}\{\mathbf{V}\mathbf{I}^*\} \quad ; \quad \mathbf{V} = V_m \angle\theta, \quad \mathbf{I} = I_m \angle\phi$$

$$\mathbf{V}_{eff} = V_{eff}\angle\theta = \frac{V_m}{\sqrt{2}}\angle\theta$$

$$\mathbf{I}_{eff} = I_{eff}\angle\phi = \frac{I_m}{\sqrt{2}}\phi$$

$$P_{average} = \mathrm{Re}\{\mathbf{V}_{eff}\mathbf{I}_{eff}^*\}$$

Complex Power

$$\mathbf{S} = \mathbf{V}_{eff}\mathbf{I}_{eff}^* \qquad \text{(unit : VA)}$$

Not a phasor of power.
This is a complex number.

$$\mathbf{S} = (V_{eff}\angle\theta°)(I_{eff}\angle -\phi) = V_{eff} \cdot I_{eff}\, e^{j(\theta-\phi)}$$

$$\Rightarrow \mathbf{S} = \underbrace{V_{eff} I_{eff}\cos(\theta-\phi)}_{\substack{P \\ \text{average power}}} + j\,\underbrace{V_{eff} I_{eff}\sin(\theta-\phi)}_{\substack{Q \\ \text{reactive power}}}$$

(VA) (W) (VAR)

$$\mathbf{S} = P + jQ$$

$$|\mathbf{S}| = V_{eff} I_{eff} = P_{apparant} \quad (VA)$$

$Z = R + jX$

Purely resistive load $Z = R$, $X = 0$

$$\mathbf{S} = P + jQ \longrightarrow V_{eff} I_{eff} \underbrace{\sin(\theta - \phi)}_{0}$$

$$P \downarrow \; V_{eff} I_{eff} \cos(\theta - \phi)$$

$P \neq 0$

$Q = 0$

Purely reactive load $Z = jX$, $R = 0$

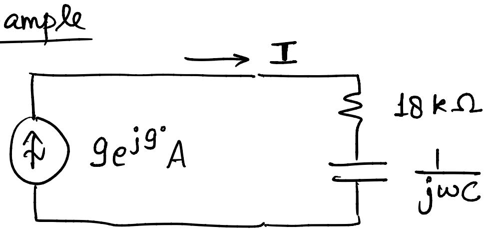
$\mathbf{S} = P + jQ$

$P = 0$

$Q \neq 0$

Q signifies energy flow rate into or out of reactive component of load.

Example



Assume $C = 1\,\mu f$, $\omega = 45$ rad/s

Find:

1. Complex power provided by source
2. Time-average power absorbed by combined load.

Find

3. Reactive power absorbed by combined load
4. Apparent power absorbed by " "
5. Power factor of " "

$S = 1.158 \times 10^6 \angle -50.99^\circ$ VA

$P_{avg} = 7.29 \times 10^5$ W

$Q = -9 \times 10^5$ VAR

$|S| = P_{apparent} = 1.158 \times 10^6$ VA

$pf = 0.629$ leading

$p(t) = 7.28 \times 10^5 + 1.158 \times 10^6 \cos(90t - 33^\circ)$ W