Quick Recap

Resistance (R)
Inductance (L)
Capacitance (C)

} Passive Elements

Volatage source
Current source

} Active Elements

Independent source
Dependent source

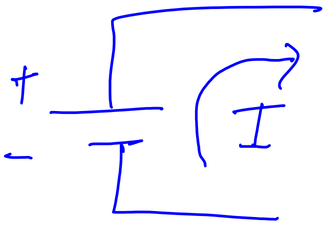
- Current-controlled
- Voltage controlled

$$R = \frac{\rho l}{A}$$ ✓

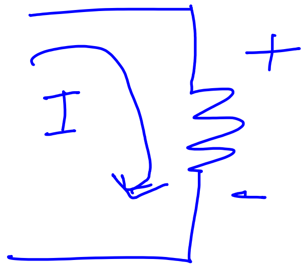
Algebraic Sum

KCL →

KVL →

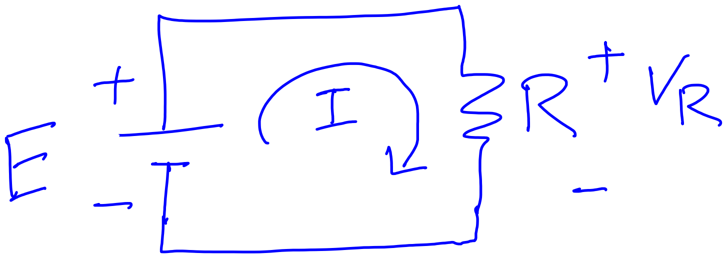


Power generated



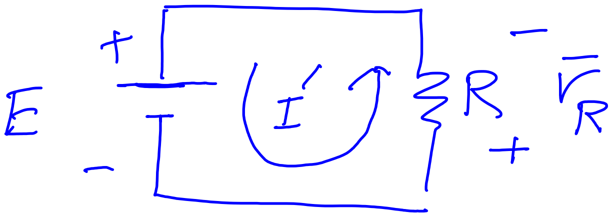
Power absorbed ✓

KCL and KVL are also applicable to circuits having C and L.



$$E - V_R = 0 \Rightarrow E = V_R = IR$$

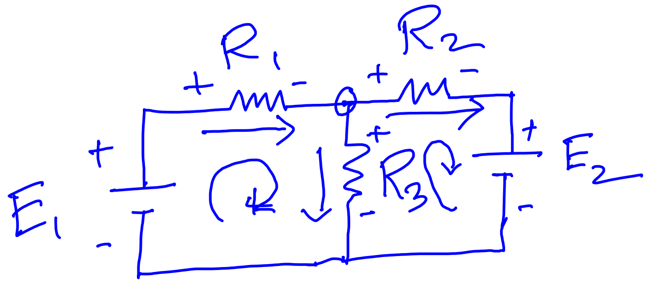
$$\underline{\underline{I}} = E/R$$



$$E + \bar{V}_R = 0 \Rightarrow I' = -E/R$$

Initial choice of current direction can be arbitrary ....

$I = -I'$ (in the above example)



I_1, I_2, I_3

$\downarrow$ $\downarrow$ $\downarrow$

R_1 R_2 R_3

$$\begin{cases} I_1 = I_2 + I_3 & \text{--- (1) (KCL)} \\ E_1 = I_1 R_1 + I_3 R_3 & \text{--- (2) (KVL)} \\ I_3 R_3 = I_2 R_2 + E_2 & \text{--- (3) (KVL)} \end{cases}$$

System of Linear Equations . .

$$\underbrace{A}_{3\times 3}\, x = \underbrace{b}_{3\times 1}$$

$$x = \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{3\times 1}$$

$$A = \begin{bmatrix} 1 & -1 & -1 \\ R_1 & 0 & R_3 \\ 0 & -R_2 & R_3 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ E_1 \\ E_2 \end{bmatrix}$$

Check for $\det(A) \neq 0$ for unique sol.

$$x = A^{-1} b$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{b}$$

C_1, C_2

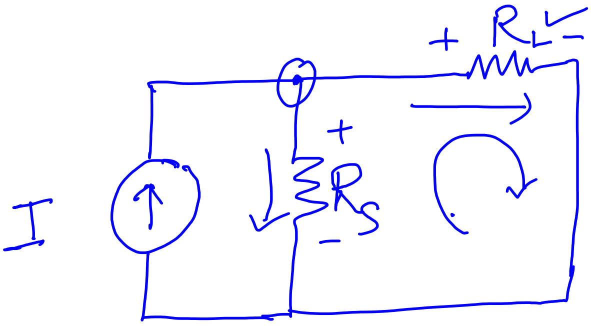
$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{b_1}{b_2}$ $\Rightarrow$ infinitely many solutions

$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}}$ $\longrightarrow$ no solution (parallel lines)

$\alpha_1 C_1 + \alpha_2 C_2 = 0 \Rightarrow \alpha_1 = 0$ and $\alpha_2 = 0$

C_1, C_2 are linearly independent.

Same concept can be extended to "n" equations . . .



I_s , I_L

$I = I_s + I_L$ (KCL)

$I_s R_s = I_L R_L$ (KVL)

$I_L = \frac{R_s}{R_L} I_s$

$I_L \left(1 + \frac{R_s}{R_L}\right) = I$

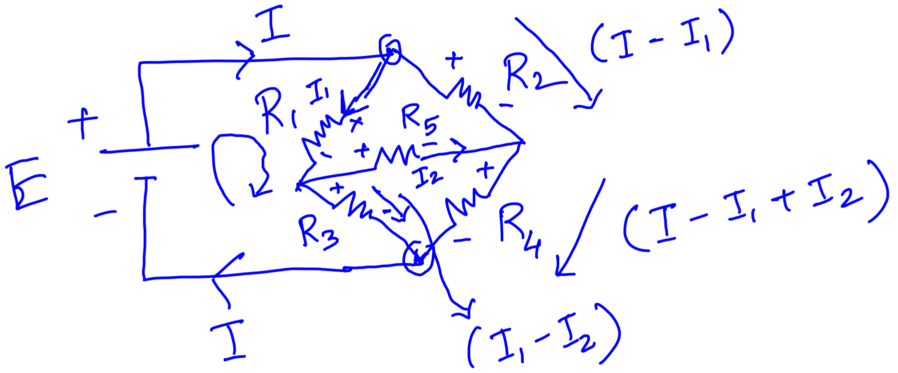
$I_L = \frac{R_L}{R_s + R_L} I$)

$A = \begin{bmatrix} 1 & 1 \\ R_s & -R_L \end{bmatrix}$

$b = \begin{bmatrix} I \\ 0 \end{bmatrix}$

$x = A^{-1} b$

$x = \begin{bmatrix} I_s \\ I_L \end{bmatrix}$



$$Ax = b$$

$$E = I_1 R_1 + (I_1 - I_2) R_3$$

$$I_1 R_1 + I_2 R_5 = (I - I_1) R_2$$

$$I_2 R_5 + (I - I_1 + I_2) R_4 = (I_1 - I_2) R_3$$

$$x = \begin{bmatrix} I \\ I_1 \\ I_2 \end{bmatrix}$$

$A \to 3 \times 3$

$b \to 3 \times 1$

A circuit having "b" no. of branches

How many minimum no. variables you need to compute "b" branch voltages and "b" branch currents?

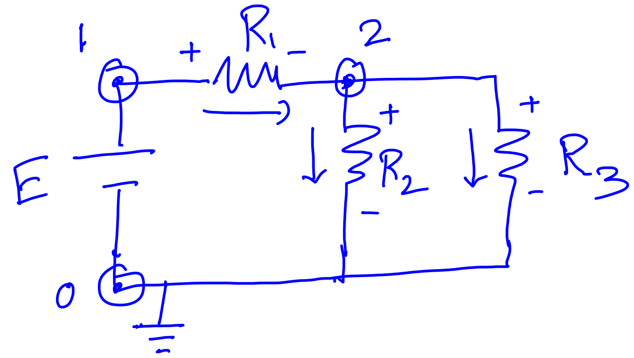
b no. of unknowns!

Let's use the following notations throughout ...

b = no. of branches/elements

n = no. of nodes

(in a circuit)



$n=3$ $\qquad b=4$

V_1 and V_2

$V_1 \rightarrow$ voltage of node 1 w.r.t node 0.

$V_2 \rightarrow$ .. ________ –

$\left.\begin{matrix} V_{R_1} \\ V_{R_2} \\ V_{R_3} \\ V_E \end{matrix}\right\}$ branch voltages $\rightarrow \left\{\begin{matrix} V_1 \\ V_2 \end{matrix}\right\}$ node-to-reference voltages $\rightarrow$ Independent variables

$V_1 = E$

$$\frac{V_1 - V_2}{R_1} = \frac{V_2}{R_2} + \frac{V_2}{R_3}$$

$\rightarrow$ KCL

Here, node "0" is considered as reference node.

$(n-1)$ voltage variables to solve for a linear circuit problem.

(node-to-reference voltages)
(datum)