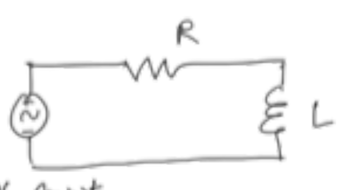
Sinusoidal steady state Analysis



$v_s = V_m \cos \omega t$

$v_s(t) = i(t) \cdot R + L\frac{di}{dt}$ ←

↗ $V_s = V_m(\cos \omega t + \theta)$

→ $i = I_1 \cos \omega t + I_2 \sin \omega t$ —

$I_1 = \frac{R V_m}{R^2 + \omega^2 L^2}$, $I_2 = \frac{\omega L V_m}{R^2 + \omega^2 L^2}$

$\Rightarrow i(t) = \frac{R V_m}{R^2 + \omega^2 L^2} \cos \omega t + \frac{\omega L V_m}{R^2 + \omega^2 L^2} \sin \omega t$ — ①

Can you write i(t) in compact form

$i(t) = A \cos(\omega t + \phi) = A(\cos \omega t \cos \phi - \sin \omega t \sin \phi)$ — ②

{ Use : $\cos(C+D) = \cos C \cos D - \sin C \sin D$ }

$A = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$

$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$

Comparing equ ① & ②

$A \cos \phi = R V_m / (R^2 + \omega^2 L^2)$

$A \sin \phi = -\omega L V_m / (R^2 + \omega^2 L^2)$

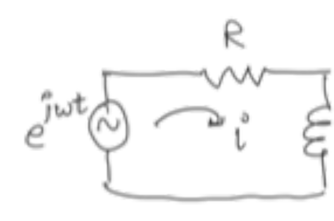
$A^2 \cos^2 \phi + A^2 \sin^2 \phi = A^2$

$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

Consider a complex source $e^{j\theta} = \cos\theta + j \sin \theta$

$V_m \cos \omega t + j V_m \sin \omega t = V_m e^{j\omega t}$

→ Real v source $= V_m \cos \omega t = Re\{V_m e^{j\omega t}\}$



$V_m e^{j\omega t} = iR + L\frac{di}{dt}$ —

Assume $i = I_m e^{j(\omega t + \phi)}$

V_m

$\Rightarrow V_m e^{j\omega t} = R \cdot I_m e^{j\omega t + j\phi} + L(j\omega) I_m e^{j\omega t + j\phi}$ —

$\Rightarrow \underbrace{I_m e^{j\omega t + j\phi}} = \frac{V_m e^{j\omega t}}{R + j\omega L}$ ←

eal $i(t) = Re\{I_m e^{j\omega t + j\phi}\} = Re\left\{\frac{V_m}{R + j\omega L} e^{j\omega t}\right\}$

→ $\frac{V_m}{R + j\omega L} \frac{(R - j\omega L)}{(R - j\omega L)} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} e^{-j \tan^{-1}\left(\frac{\omega L}{R}\right)}$

R

$x + jy \rightleftarrows r e^{j\theta} \rightarrow r\angle\theta$

$\boxed{x = r \cos\theta}$, $y = r \sin \theta$

$r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}(y/x)$

$i(t) = Re\left\{\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} e^{-j \tan^{-1}\left(\frac{\omega L}{R}\right)} \cdot e^{j\omega t}\right\}$

$= Re\left\{\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} e^{j\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)}\right\}$

$= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

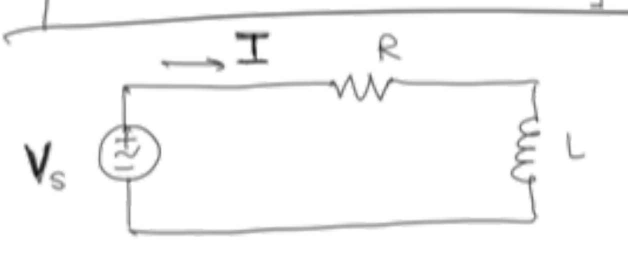
Phasor How we define phasor

$i(t) = I_m \cos(\omega t + \phi) = Re\{I_m e^{(j\omega t + j\phi)}\}$

⇓ remove $e^{j\omega t}$, $Re\{\ \}$

$\underbrace{I_m e^{j\phi} = I_m \angle\phi}_{\text{phasor}}$

time domain	Phasor representation
$i(t) = I_m \cos(\omega t + \phi)$	$I_m \angle\phi$
$V(t) = V_m \cos(\omega t + \theta)$	$V_m \angle\theta$
• real , function of t	• complex , constant
• notations : small letters	• Both capital letter



✓ $i(t) = I_m \cos(\omega t + \phi)$, $V(t) = V_m(\cos \omega t + \theta)$

✓ $i(t) = Re\{I_m e^{j\omega t + j\phi}\}$

✗ $i(t) = I_m \angle\phi$

✗ $i(t) = I_m e^{j(\omega t + \phi)}$

✗ $i(t) = I_m \angle \omega t + \phi$

✓ Phasor representation of $i(t) = I_m \angle\phi$

✓ Phasor representation of $v(t) = V_m \angle\theta$

from previous notes

$V_m e^{j\omega t + j\theta} = R\, I_m e^{j\omega t + j\phi} + L(j\omega) I_m e^{j\omega t + j\phi}$

$\Rightarrow \underbrace{V_m e^{j\theta}}_{\mathbf{V}} = (R + j\omega L) \underbrace{I_m e^{j\phi}}_{\mathbf{I}}$

→ $\boxed{\mathbf{V} = (R + j\omega L)\mathbf{I}}$

Phasor representation of relationship between i and v for R, L, C

Resistor

$\boxed{v = iR}$

Consider complex voltage & current

$v \to V_m e^{j\omega t + j\theta}$

$i \to I_m e^{j\omega t + j\phi}$

$\Rightarrow V_m \cancel{e^{j\omega t}} e^{j\theta} = R I_m \cancel{e^{j\omega t}} e^{j\phi}$

$\Rightarrow V_m e^{j\theta} = R\, I_m e^{j\phi}$

$\underbrace{\underbrace{V_m \angle\theta}}_{\mathbf{V}}$ $\underbrace{\underbrace{I_m \angle\phi}}_{\mathbf{I}}$

$\boxed{\mathbf{V} = R\mathbf{I}}$

Inductor

$v = L\frac{di}{dt}$

$v \to V_m e^{j\omega t + j\theta}$

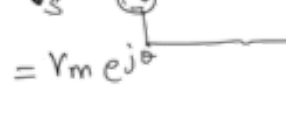
$i \to I_m e^{j\omega t + j\phi}$

$\Rightarrow V_m \cancel{e^{j\omega t}} e^{j\theta} = L \cdot I_m \cancel{e^{j\omega t}} e^{j\phi} (j\omega)$

$\Rightarrow \underbrace{V_m e^{j\theta}}_{\mathbf{V}} = j\omega L \cdot \underbrace{I_m e^{j\phi}}_{\mathbf{I}}$

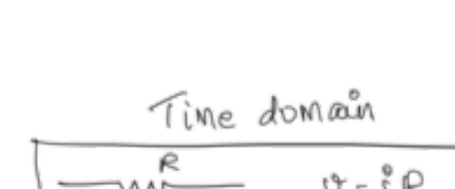
$\boxed{\mathbf{V} = j\omega L\mathbf{I}}$

Capacitor



$i = C\frac{dv}{dt}$

$\boxed{\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}}$



Phasor representation

Using KVL ,

$\mathbf{V}_S = \mathbf{V}_R + \mathbf{V}_L$

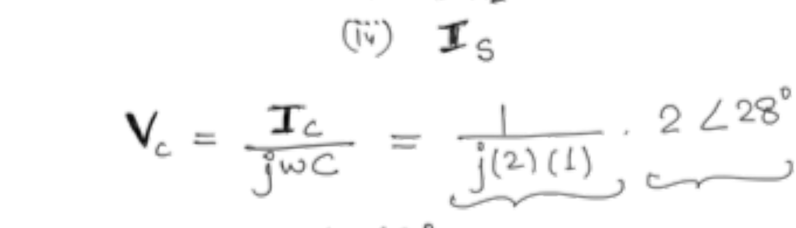
$= \mathbf{I}R + (j\omega L)\mathbf{I}$

$= \mathbf{I}(R + j\omega L)$

$\mathbf{I} = \frac{\mathbf{V}_S}{R + j\omega L} = \frac{V_m \angle\theta}{R + j\omega L}$

	Time domain	Phasor (frequency domain)
R	$v = iR$	$\mathbf{V} = \mathbf{I}R$
L	$v = L\frac{di}{dt}$	$\mathbf{V} = (j\omega L)\mathbf{I}$
C	$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$

Example



① Given : both sources $\mathbf{V}_s$ and $\mathbf{I}_s$ operate at $\omega = 2$ rad/s

② Given : $\mathbf{I}_c = 2\angle 28°$ A

find (i) $\mathbf{V}_c$ ←

(ii) $\mathbf{I}_{R_2}$

(iii) $\mathbf{I}_S$

$\mathbf{V}_c = \frac{\mathbf{I}_c}{j\omega C} = \underbrace{\frac{1}{j(2)(1)}} \cdot \underbrace{2\angle 28°} = (0.5\angle -90°)(2\angle 28°)$

$= 1\angle -62°$ V

$\mathbf{I}_{R_2} = \frac{\mathbf{V}_{R_2}}{2} = \frac{1\angle -62°}{2} = 0.5\angle -62°$ A

$\mathbf{I}_S = 0.5\angle -62° + 2\angle 28° = 2.06\angle 14°$ A