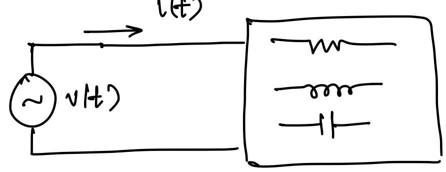
Module 3

AC Power Analysis



$\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

Instantaneous Power : $p(t) = v(t)\, i(t)$

$$v(t) = V_m \cos(\omega t + \theta)$$

$$i(t) = I_m \cos(\omega t + \phi)$$

$$p(t) = V_m \cos(\omega t + \theta) \cdot I_m \cos(\omega t + \phi)$$

$$= \underbrace{\frac{V_m I_m}{2} \cos(\theta - \phi)}_{\substack{\text{constant} \\ \text{(DC term)}}} + \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{\substack{\text{twice frequency} \\ \text{(harmonic)}}}$$

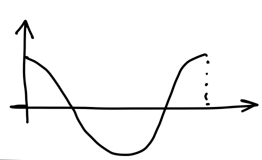
Average Power :

$$P_{avg} = \frac{1}{T} \int_0^T p(t)\, dt$$

$$= \underbrace{\frac{1}{T} \int_0^T \frac{V_m I_m}{2} \cos(\theta - \phi) \cdot dt}_{} + \underbrace{\frac{1}{T} \int_0^T \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)\, dt}_{0}$$

$$= \frac{V_m I_m}{2} \cos(\theta - \phi) + 0$$

Average of Cos



$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)}$$

$$P_{avg} = \frac{1}{2} \, Re\{ \mathbf{V} \mathbf{I}^* \}$$

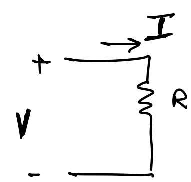
$$\mathbf{I} = I_m e^{j\phi}$$

$$\mathbf{I}^* = I_m e^{-j\phi}$$

$$\cancel{P_{avg} = \mathbf{V}\mathbf{I}^*}$$

No phasor for power !

Average power absorbed by a purely resistive element

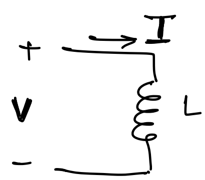


$$P_{avg} = \frac{V_m I_m}{2} \cos(\underbrace{\theta - \phi}_{0^\circ})$$

$\mathbf{V} = \mathbf{I}R$

$$P_{avg} = \frac{V_m I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

Purely inductive element



$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)$$

$$= \frac{V_m I_m}{2} \cos(90^\circ)$$

$$\boxed{P_{avg} = 0}$$

$\mathbf{V} = j\omega L \mathbf{I}$

$(\theta - \phi) =$

$V_m e^{j\theta} = j\omega L I_m e^{j\phi}$

$\Rightarrow \frac{V_m}{I_m} e^{j\theta - \phi} = j\omega L$

$\theta - \phi = 90^\circ$

Purely capacitive element

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)$$

$$\theta - \phi = -90^\circ$$

$$\boxed{P_{avg} = 0}$$