Quick Recap

Capacitors

$$C = \frac{\varepsilon A}{d}$$

$\varepsilon \rightarrow$ permitivity

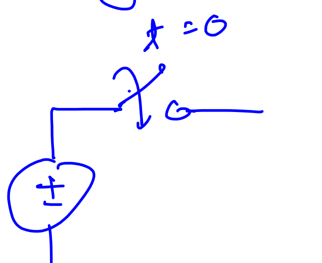
$\varepsilon_0 \rightarrow$ permitivity of free space (8.854 pF/m)

$\varepsilon = \varepsilon_r \varepsilon_0$

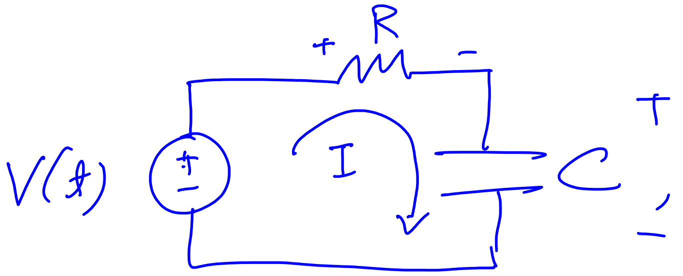
$\varepsilon_r \rightarrow$ Relative permitivity

RC circuits → charging
→ discharging

unit step function



Dirac-delta (Impulse function)



$V_c(0) = 0$

$$V(t) = V_R(t) + V_C(t)$$

$$= \underline{I(t)R} + \frac{1}{C}\int_0^t I(t)\,dt$$

$$\underline{V(t)} = RC\frac{dV_c}{dt} + V_c(t)$$

general structure of the problem,

$$\frac{dx}{dt} + \underline{P\,x = Q(t)}, \quad \forall\, t \geq 0, \quad \boxed{x(0) = x_0}$$

$$\underbrace{e^{Pt}\frac{dx}{dt} + P e^{Pt} x} = e^{Pt} Q(t)$$

$$x_d(t) = e^{Pt}\cdot x(t)$$

$$\frac{dx_d(t)}{dt} = \underbrace{e^{Pt}\frac{dx}{dt} + P e^{Pt} x(t)}$$

$$= e^{Pt} Q(t)$$

$$\frac{dx_d}{dt} = e^{Pt} Q(t)$$

$$x_d(t) = x_d(0) + \int_0^t e^{Pt} Q(t)\, dt$$

$$x(t) = x_d(0) e^{-Pt} + e^{-Pt} \int_0^t e^{P\tau} Q(\tau)\, d\tau$$

$$x_d(t) = e^{Pt} x(t)$$

$$x_d(0) = x(0)$$

natural response

forced response

✓ $$\frac{dx}{dt} + P x(t) = Q(t) \qquad x(0) = x_0$$

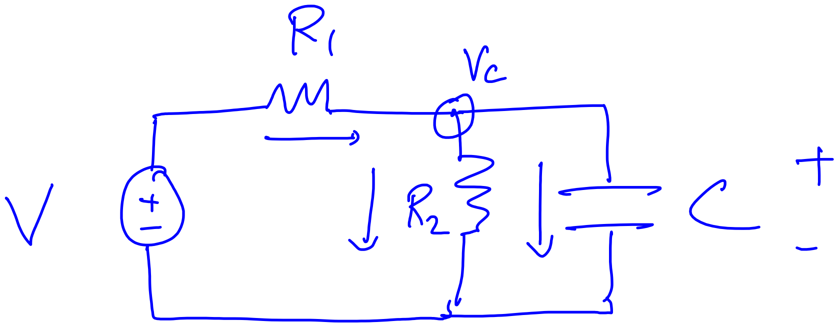
$$x(t) = x_n(t) + x_f(t)$$

natural response

forced response

$$\frac{dx_n}{dt} + P x_n(t) = 0 \qquad x_n(0) = x_0$$

$$\frac{dx_f}{dt} + P x_f(t) = Q(t) \qquad x_f(0) = 0$$



$$V_c(0) = 0$$

$$\frac{V - V_c}{R_1} = \frac{V_c}{R_2} + C\frac{dV_c}{dt}$$

$$\frac{V}{R_1} = V_c\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + C\frac{dV_c}{dt}$$

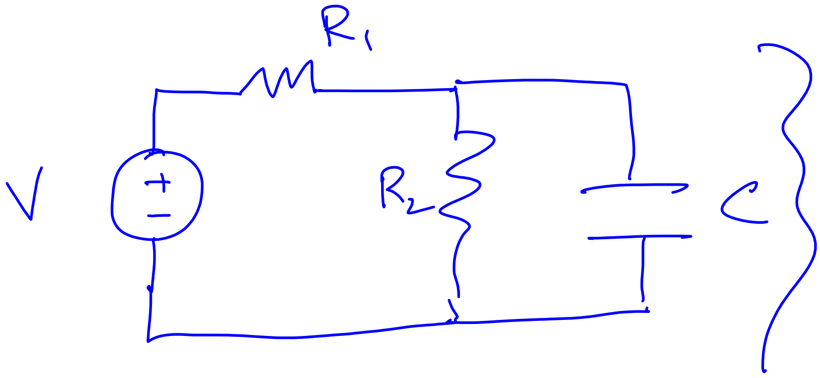
$$\frac{V}{R_1} = V_c \cdot \frac{1}{R_{eq}} + C\frac{dV_c}{dt}$$

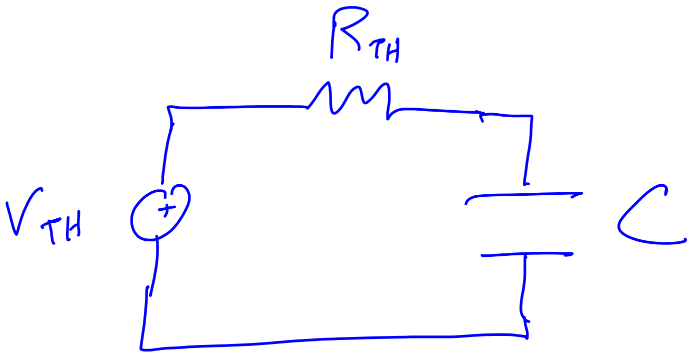
$$\frac{R_{eq} V}{R_1} = V_c + R_{eq} C \frac{dV_c}{dt}$$

$$V_c(t) = \frac{R_{eq} V}{R_1}\left(1 - e^{-t/R_{eq}C}\right)$$

$$= \frac{V}{R_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}\left(1 - e^{-t/R_{eq}C}\right)$$

$$= \underbrace{\frac{R_2 V}{R_1 + R_2}}_{V_{TH}}\left(1 - e^{-t/\underbrace{R_{eq}}_{R_{TH}}C}\right)$$

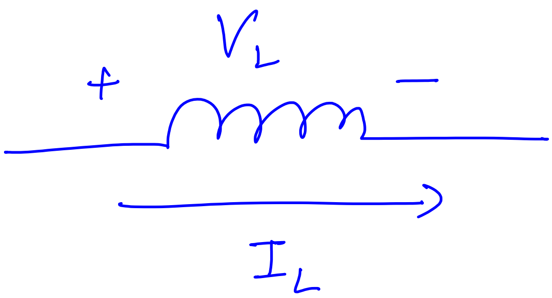




$$V_{TH} = \frac{R_2 V}{R_1 + R_2}$$

$$R_{TH} = (R_1 \parallel R_2) = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Inductor (L)



$$V_L = L \frac{dI_L}{dt}$$

Inductance (Henry)

Faraday's Law

$$V \propto \frac{d\psi}{dt}$$

$\psi \rightarrow$ magnetic flux linkage

$\Psi = N\phi$ $\longrightarrow$ magnetic flux (ϕ)

N $\longrightarrow$ no. of turns

$$\Psi = L\, i$$

$$L = \frac{N^2 \mu A}{l}$$ } Solenoid

(thin and very long coil)

$\mu \to$ permeability

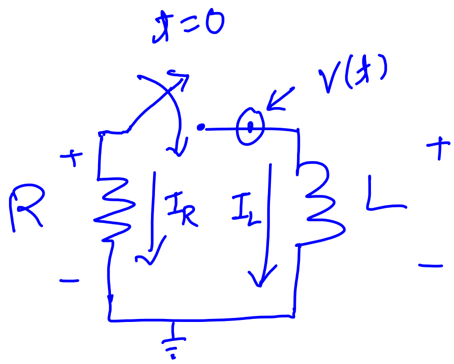
$A \to$ area of the core

$l \to$ length

$$\mu = \mu_r \mu_0$$

$\mu_0 \to 4\pi \times 10^{-7}$ H/m (free space)

Natural Response of a RL circuit



$$\boxed{V_L = V_R = V}$$

$$I_L(0^-) = I_0$$

$$V_C(t^-) = V_C(t^+) \quad \text{if} \quad I_C(t) \neq \infty$$

$$I_L(t^-) = I_L(t^+) \quad \text{if} \quad V_L(t) \neq \infty$$

(continuity of Inductor current)

Applying KVL,

$$I_L(t) + I_R(t) = 0 \qquad \forall\, t \geq 0$$

$$I_L(t) + \frac{V(t)}{R} = 0$$

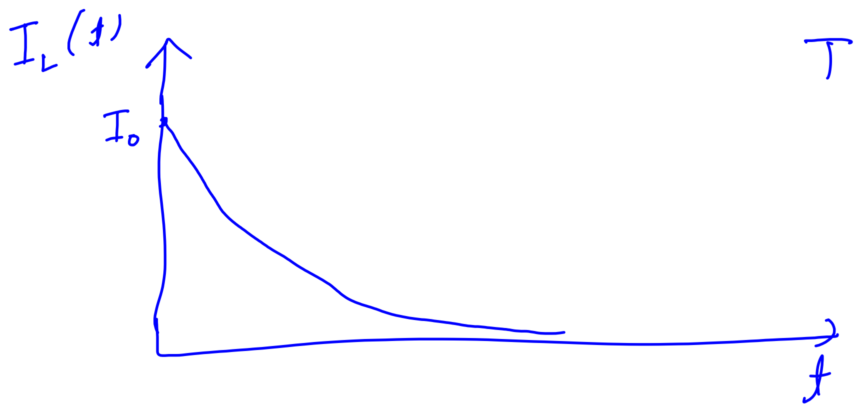
$$I_L(t) + \frac{L}{R}\frac{d I_L(t)}{dt} = 0$$

$$I_L(t) = I_L(0)\, e^{-\frac{R}{L}t}$$

$$= I_0\, e^{-\frac{R}{L}t}$$

$$= I_0\, e^{-t/T}$$

$$T = \frac{L}{R} \text{ (time constant)}$$



$$V_L(t) = L\frac{d i_L(t)}{dt} = -\frac{L I_0}{T} e^{-t/T}$$

$$V_L(t) = -(R I_0)\, e^{-t/T}$$

$I_L(0^-) = I_0 \qquad 2)\ I_L(0) = I_0$

$$I_L(t) + I_R(t) = 0 \qquad \forall\, t \geq 0$$

$$\Rightarrow \qquad I_R(0) = - I_L(0)$$

$$= - I_0$$

$$V_R(0) = R\, I_R(0) = - I_0 R$$

$$V_L(t) = V_R(t) \qquad \forall\, t \geq 0$$

$$\boxed{V_L(0) = - I_0 R}$$