tr5frbj 12/08/24

Circuit Theory and Devices
↗ tut as well

lab: LT Spice (2nd week onwards)

book: Engineering Circuit Analysis (9th Edition)

Relative grading
- Labs 20% — every 2 weeks (n−1)
- Quiz 20%
- Mid sem 30%
- End sem 30% — scientific calc

Course: 9 modules — chapter 10 onwards
{continuation of BE}

* Lecture : 1

⇒ Linear Circuits

x (input) $\longrightarrow \boxed{\text{system}} \longrightarrow y$ (output)

system is linear if

$Kx \rightarrow \boxed{\text{sys}} \rightarrow Ky$

where k is constant

"R" Resistor : linear element ✓
$V = iR$

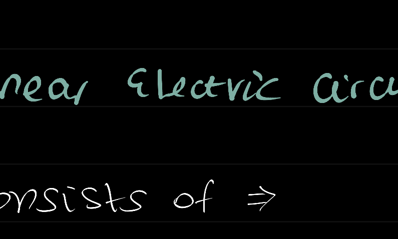
"L" Inductor : linear element ✓
$V = L\frac{di}{dt}$

"C" Capacitor : linear element ✓
$i = C\frac{dv}{dt}$

* Linear Electric circuits :

consists of ⇒

① R, L, C → linear elements
② Independent voltage & current sources
③ linear dependent sources



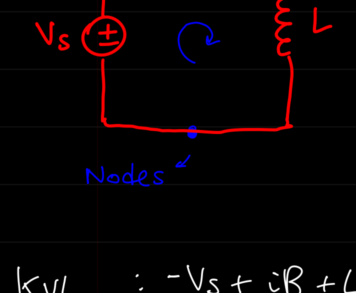
example
would not have been linear if $V_s = KV_x^2$

NOTE: diode and transistors are non-linear elements

* Response of a linear circuit

① full response = transient response + Steady State

transient response	Steady State
temporary	persists $t \to \infty$
due to sudden change	due to source
also called natural response	also called forced response
depends on linear elements R, L, C	depends on current/voltage source



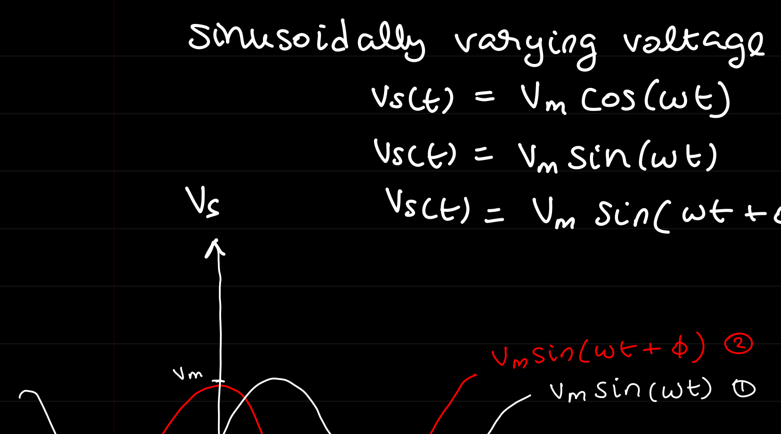
KVL : $-V_s + iR + L\frac{di}{dt} = 0$

$i_{final} = i_{transient} + i_{steady\ state}$

as $t \to \infty$, $i_{final} = i_{steady\ state}$

* Sinusoids & complex numbers

sinusoidally varying voltage source
$V_s(t) = V_m \cos(\omega t)$
$V_s(t) = V_m \sin(\omega t)$
$V_s(t) = V_m \sin(\omega t + \phi)$



② is leading ①
① is lagging ② } by angle ϕ

* TRIGO

$\sin(-\theta) = -\sin(\theta)$
$\cos(-\theta) = \cos(\theta)$
$\sin(A+B) = \sin A \cos B + \cos A \sin B$
$\cos(A+B) = \cos A \cos B - \sin A \sin B$

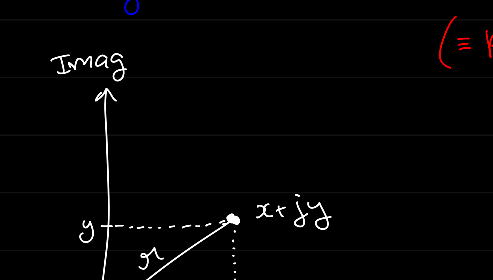
sin +ve	All +ve
tan +ve	cos +ve

* Complex Numbers

Real : x
complex : $\underbrace{x + iy}_{\text{rectangular form}}$ — real part, imaginary part, $i = \sqrt{-1}$ a.k.a. j

⇒ Euler's identity : $\underbrace{e^{j\theta}}_{\text{exponential form}} = \underbrace{\cos\theta + j\sin\theta}_{\text{rectangular form}}$

Complex Number
- rectangular: $x + iy$
- exponential: $re^{j\theta}$ (≡ polar form : $r\angle\theta$)



$r = \sqrt{x^2 + y^2}$
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$(r_1 e^{j\theta_1}) \times (r_2 e^{j\theta_2}) = (r_1 \cdot r_2)\, e^{j(\theta_1 + \theta_2)}$

AC	DC
• we can use transformers to vary voltage	• appliances need constant current
• transported easily over large distances	

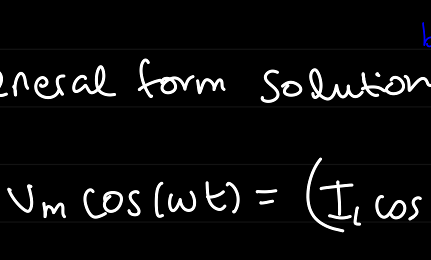
AC → DC : using rectifiers

* MODULE : 1 {CH = 10} 10:30 am

⇒ Sinusoidal steady state analysis

assuming transient has died out long back

Consider,



$V_s(t) = V_m \cos(\omega t)$
find $i(t)$

$V_s(t) = iR + L\frac{di}{dt}$

$\Rightarrow V_m \cos(\omega t) = iR + L\frac{di}{dt}$
both will take cos(ωt) form

General form Solution : $i = I_1 \cos \omega t + I_2 \sin \omega t$

$\Rightarrow \quad V_m \cos(\omega t) = (I_1 \cos \omega t + I_2 \sin \omega t)R + L\frac{di}{dt}$

$\frac{di}{dt} = \frac{d}{dt}(I_1 \cos \omega t + I_2 \sin \omega t)$
$\Rightarrow \omega(I_1(-\sin \omega t) + I_2 \cos \omega t)$
$\Rightarrow \omega(I_2 \cos \omega t - I_1 \sin \omega t)$

$V_m \cos(\omega t) = (I_1 \cos \omega t + I_2 \sin \omega t)R + (I_2 \cos \omega t - I_1 \sin \omega t)L\omega$

$\Rightarrow \cos \omega t \cdot (I_1 R + I_2 L) + \sin \omega t \cdot (I_2 R - I_1 L)$

$\Rightarrow (I_1 R + I_2 L - V_m)\cos(\omega t) + (I_2 R - I_1 L)\sin(\omega t) = 0$

* at $t = 0$,
$(I_1 R + I_2 L - V_m) \cdot 1 + (I_2 R - I_1 L) \cdot 0 = 0$

$\boxed{I_1 R + I_2 L = V_m}$

* at $t = \frac{\pi}{2\omega}$, $(I_1 R + I_2 L - V_m) \cdot 0 + (I_2 R - I_1 L) \cdot 1 = 0$

$I_1 = \frac{V_m L R}{R^2 + L}$ $\quad \boxed{I_1 = I_2 \cdot \frac{R}{L}}$

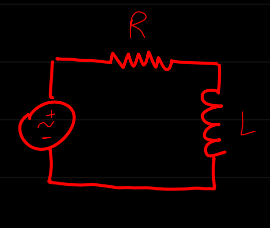
$I_1 R + I_2 L = V_m \Rightarrow I_2 \frac{R^2}{L} + I_2 L = V_m$
$\Rightarrow I_2\left(\frac{R^2 + L}{L}\right) = V_m \Rightarrow I_2 = \frac{V_m L^2}{R^2 + L}$

14/08/24 **Lecture : 2** 9:30 AM

Wednesday → Quiz : Chp 1, 2, 3

* Sinusoidal Steady state analysis

AC sources ↙ ↳ not considering any transient response



$V_s = V_m \cos(\omega t)$ ← angular freq

freq (hz) $= \dfrac{\omega}{2\pi}$

$$V_s(t) = i(t)R + L\frac{di(t)}{dt}$$

general soln: $i = I_1\cos(\omega t) + I_2 \sin(\omega t)$

put i from gen soln in above eqn

$$I_1 = \frac{RV_m}{R^2+\omega^2L^2}, \quad I_2 = \frac{\omega L V_m}{R^2+\omega^2L^2}$$

$$i(t) = \frac{RV_m}{R^2+\omega^2L^2}\cos(\omega t) + \frac{\omega L V_m}{R^2+\omega^2L^2}\sin(\omega t)$$

Note: use $\cos(A+B) = \cos A\cos B - \sin A \sin B$

⇒ So, $i(t) = A\cos(\omega t + \phi)$: general soln

becomes ⇒ $i(t) = A[\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)]$

$$\frac{RV_m}{R^2+\omega^2L^2}\cos(\omega t) + \frac{\omega L V_m}{R^2+\omega^2L^2}\sin(\omega t) = A\cos(\omega t)\cos(\phi) - A\sin(\omega t)\sin(\phi)$$

⇓

$$A\cos(\phi) = \frac{RV_m}{R^2+\omega^2L^2}, \quad A\sin(\phi) = \frac{-\omega L V_m}{R^2+\omega^2L^2}$$

$$A^2\cos^2(\phi) + A^2(\sin^2\phi) = A^2(1) = A^2 = \frac{R^2V_m^2 - \omega^2L^2V_m^2}{R^2+\omega^2L^2}$$

⇓

$$\frac{V_m^2(R^2-\omega^2L^2)}{R^2+\omega^2L^2}$$

$$A = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}$$

$$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

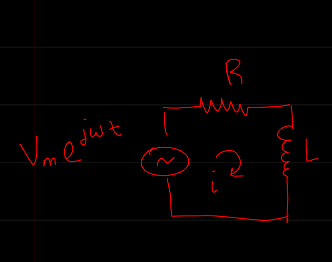
$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

⇒ consider a complex source

$$V_m\cos(\omega t) + jV_m\sin(\omega t) = V_m e^{j\omega t}$$

Real Voltage source $= V_m\cos(\omega t) = Re\{V_m e^{j\omega t}\}$

Note: Euler's formula → $e^{j\theta} = \cos\theta + j\sin\theta$



$$V_m e^{j\omega t} = iR + L\frac{di}{dt}$$

dont integrate ✗

general soln ✓

Assume $i = I_m e^{j(\omega t + \phi)}$

put eqn

$$V_m e^{j\omega t} = I_m e^{j\omega t + j\phi} R + L(j\omega) I_m e^{j\omega t + j\phi}$$

$$\Rightarrow I_m e^{j\omega t + j\phi} = \frac{V_m e^{j\omega t}}{R + j\omega L}$$

is this the real current? No

current cannot be complex

$$\text{Real } i(t) = Re\{I_m e^{j\omega t + j\phi}\} = Re\left\{\frac{V_m}{R+j\omega L} e^{j\omega t}\right\}$$

find the real part of this complex number by simplifying

{multiply by the conjugate ≡ factorize}

$$i(t) = \frac{V_m}{R+j\omega L}\left(\frac{R-j\omega L}{R-j\omega L}\right)e^{j\omega t} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} e^{-j\tan^{-1}\left(\frac{\omega L}{R}\right)} \cdot e^{j\omega t}$$

⇒ Note: rect form → polar form

$x + jy \rightleftarrows re^{j\theta}$

$x = r\cos\theta$, $y = r\sin\theta$

$r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$\text{Real } i(t) = Re\left\{\frac{V_m}{\sqrt{R^2+\omega^2L^2}} \cdot e^{j\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)}\right\}$$

$$* \quad i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

same answer as when we were not considering complex numbers

* PHASOR

$$i(t) = I_m\cos(\omega t + \phi) = Re\{I_m e^{j\omega t + j\phi}\}$$

⇓ remove $e^{j\omega t}$, $Re\{\ldots\}$

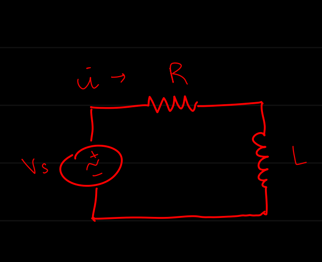
not changing with time

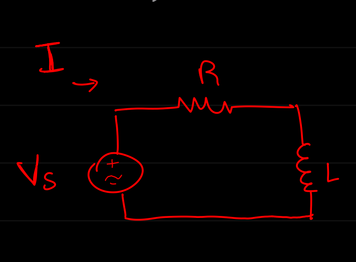
complex func ✗

complex number ✓

$$\underbrace{I_m e^{j\phi}}_{\text{phasor}} = I_m \angle \phi$$

time domain	phasor representation
$i(t) = I_m\cos(\omega t + \phi)$	$I_m\angle\phi$
$v(t) = V_m\cos(\omega t + \theta)$	$V_m\angle\theta$
⇒ real function of t	⇒ complex, constant
⇒ notations: Small letters	⇒ Bold, capital letters





$i(t) = I_m\cos(\omega t + \phi)$ ✓

$i(t) = Re\{I_m e^{j\omega t + j\phi}\}$ ✓

$i(t) = I_m\angle\phi$ ✗ complex constant

$i(t) = I_m e^{j(\omega t + \phi)}$ ✗ complex

$i(t) = I_m \angle \omega t + \phi$ ✗ complex phasor notation

phasor notation

but we can write ⇒ phasor notation of $i(t) = I_m\angle\phi = I_m\angle(\omega t + \phi)$

Note: $\angle\phi = e^{j\phi}$ or $\angle \omega t + \phi = e^{j\omega t + j\phi}$

$$\underbrace{V_m e^{j\theta}}_{V} = (R + j\omega L)\underbrace{I_m e^{j\phi}}_{I}$$

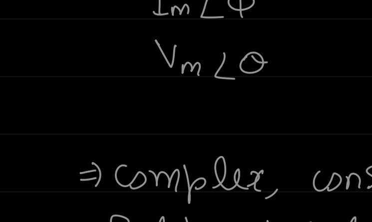
$$\boxed{V = (R + j\omega L)\, I}$$

↳ phasor notation

* phasor representation of relationship between i and v for R, L, C

① RESISTOR

time domain



$v = iR$

consider complex voltage & current

$v = V_m e^{j(\omega t + \theta)}$

$i = I_m e^{j(\omega t + \phi)}$

$$V_m e^{j\omega t + j\theta} = R \cdot I_m e^{j\omega t + j\phi}$$

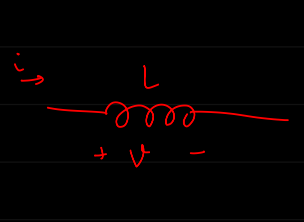
$$V_m \cancel{e^{j\omega t}} \cdot e^{j\theta} = R \cdot I_m \cancel{e^{j\omega t}} e^{j\phi}$$

$$V_m e^{j\theta} = R \cdot I_m e^{j\phi}$$

$$\underbrace{V_m\angle\theta}_{V} = \underbrace{I_m\angle\phi}_{I} \cdot R$$

$$\boxed{V = IR}$$

② INDUCTOR



$$v = L\frac{di}{dt}$$

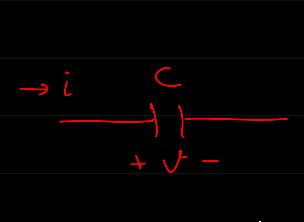
$v \to V_m e^{j(\omega t + \theta)}$ $\quad i \to I_m e^{j\omega t + j\phi}$

$$V_m e^{j\omega t} e^{j\theta} = L\, I_m e^{j\omega t} \cdot e^{j\phi}(j\omega)$$

$$\underbrace{V_m e^{j\theta}}_{V} = j\omega L \cdot \underbrace{I_m e^{j\phi}}_{I}$$

$$\boxed{V = j\omega L \cdot I}$$

③ CAPACITOR



$$i = C\frac{dv}{dt}$$

$i \to I_m e^{j(\omega t + \phi)}$

$v \to V_m e^{j(\omega t + \theta)}$

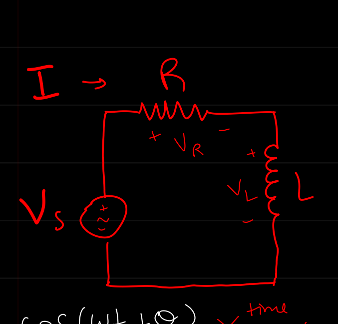
$$I_m \cancel{e^{j\omega t}} \cdot e^{j\phi} = C \cdot V_m \cancel{e^{j\omega t}} e^{j\theta} \cdot j\omega$$

$$\underbrace{I_m e^{j\phi}}_{I} = C \cdot j\omega \cdot \underbrace{V_m e^{j\theta}}_{V}$$

$$I = j\omega C \cdot V$$

$$\boxed{V = \frac{1}{j\omega C} \cdot I}$$

* PHASOR Representation using KVL



$$V_s = V_R + V_L$$

$$= IR + (j\omega L)I$$

$$= I(R + j\omega L)$$

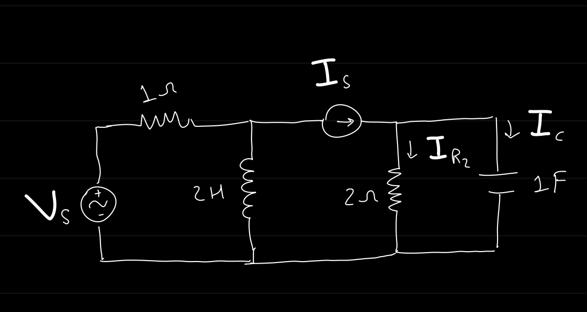
$V_s = V_m\cos(\omega t + \theta)$ ✗ time domain

$V_s = V_m e^{j(\omega t + \theta)}$ ✓ phasor notation

$$I = \frac{V_s}{R + j\omega L} = \frac{V_m\angle\theta}{R + j\omega L}$$

Time domain	Phasor (Frequency domain)
R: $v = iR$	$V = IR$
L: $v = L\frac{di}{dt}$	$V = j\omega L\, I$
C: $i = C\frac{dv}{dt}$	$V = \frac{1}{j\omega C}\, I$

⇒ Example



① Given: both AC sources, V_s & I_s operate at $\omega = 2$ rad/s

② Given: $I_c = 2\angle 28°$ A

Find (i) V_c

(ii) I_{R_2}

(iii) I_s

① $$V_c = \frac{I_c}{j\omega C} = \frac{1}{j(2)(1)} \cdot 2\angle 28°$$

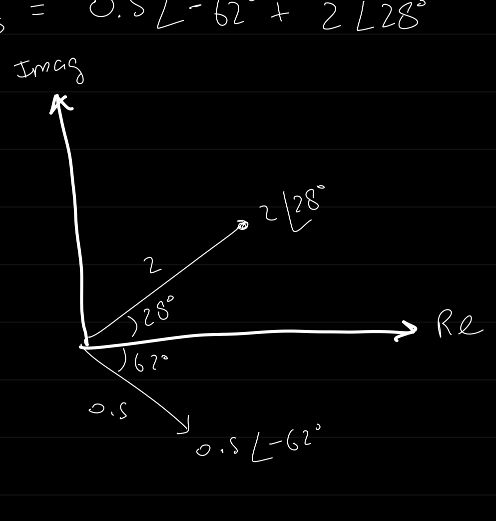
Note: $\frac{1}{j} = -j = -90°$

we add the angles directly which are being multiplied

$$V_c = (0.5\angle -90°)(2\angle 28°) = 1\angle -62°\ V$$

② $$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_{R_2}}{2} = \frac{1\angle -62°}{2} = 0.5\angle -62°\ A$$

③ $$I_s = 0.5\angle -62° + 2\angle 28°$$



$2.06\angle 14°$ A

* Lecture : 3 19/08/24 9:30AM

- Phasor : $i(t) = I_m \cos(\omega t + \phi) = Re\{I_m e^{j(\omega t+\phi)}\}$

Remove $e^{j\omega t}$ and $Re\{...\}$

$I_m e^{j\phi} = I_m \angle\phi$ } Phasor

exponential form — polar form

time domain ⇌ freq. domain

$V_R = IR$ $V_L = j\omega L I$ $V_c = \frac{1}{j\omega C} I$

* IMPEDANCE

$$Z = \frac{V}{I} = \frac{1}{R}$$

$Z_R = R$ — voltage and current in the same phase

$Z_L = j\omega L$ — voltage leads current by 90°

$Z_C = \frac{1}{j\omega C}$ — voltage lags current by 90°

$Z = x + jy$ (resistance, reactance)

* Admittance

$$Y = \frac{1}{Z}$$

$Y = x + jy$ (conductance, susceptance)

* Impedance combination

Series : $Z_{eq} = Z_1 + Z_2 + \dots Z_N$

Parallel : $Z_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots \frac{1}{Z_N}$

eg} Find impedance of series combination of an inductor L (L=5mH) and capacitor (C=100μF) $\omega = 10^4$ rad/s

5mH 100μF

⇒ $Z_{eq} = Z_L + Z_C = j\omega L + \frac{1}{j\omega C} = 10^4 \cdot j \cdot 5\times10^{-3} + \frac{10^{-4}\times10^{3}}{j \cdot 5\times 100 \times 10^{-6}}$

Note: $\frac{1}{j} = -j$ ⇒ $50j - j = 49j$

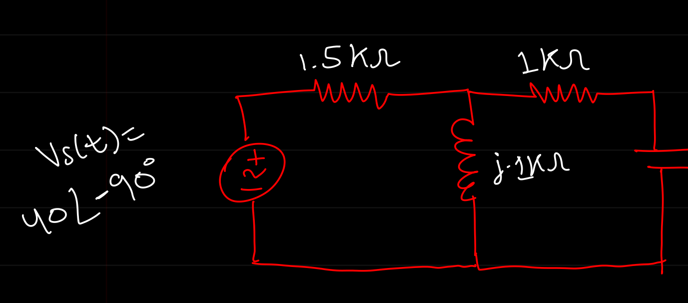
Note: $e^{j\theta} = \cos\theta + j\sin\theta = e\angle\theta$ ⇒ $0 + 49j = 49e^{j\frac{\pi}{2}}$

eg:

Find i(t)

$\frac{10^3(1-2j)(1+j)}{2}$, $500(1-2j)(1+j)$, $\frac{10^3(1-2j)}{1-j}$

step1} Convert to freq. domain (write in terms of phasor and Z)



$2000 - 2000j \parallel 2000j$

$\left(\frac{1}{1000-2000j} + \frac{1}{1000j}\right)^{-1}$

$-j \cdot 2k\Omega$ $\frac{1-j}{10^3 - 2\cdot10^3 j}$

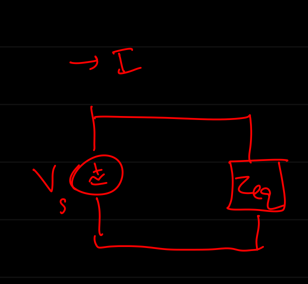
⇒ $\frac{1000j + 1000 - 2000j}{(10^3 - 2\cdot10^3 j)10^3 \cdot j}$

$\frac{1}{3}H \rightarrow Z_L = j\omega L = j \cdot 3000 \cdot \frac{1}{3} ⇒ j \cdot 1k$

$\frac{1}{6}\mu F \rightarrow Z_c = \frac{1}{j\omega C} = \frac{1}{j \cdot 3000 \times \frac{1}{6} \times 10^{-6}} ⇒ \frac{2\cdot10^3}{j}$

⇒ $-j \cdot 2k\Omega$

step2}



$I = \frac{V_s}{Z_{eq}}$

$Z_{eq} = 1.5k + [500(1+j)(1-2j)]$

⇒ $1500 + 500[1 - 2j + j + 2]$

⇒ $500[3 + 3 - j]$ ⇒ $500[6 - j]$

⇒ $3000 - 500j$

$\frac{j(1-2j)(1+j)}{2}$ ⇒ $\frac{(j+2)(1+j)}{2}$ ⇒ $\frac{0 + j^2 + 2 + 3j}{2}$ ⇒ $\frac{3j+1}{2}$

PPT} $Z_{eq} = 1.5 + \frac{(jL)(1-j2)}{1-j}$ kΩ

⇒ $\frac{4+3j}{2}$

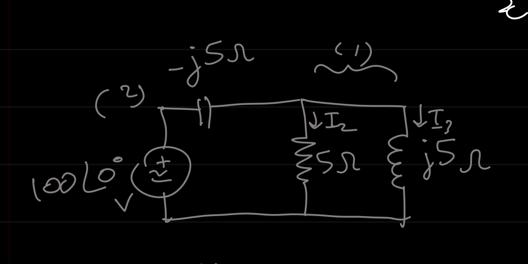
Note: complex number < $x+iy$, $r\angle\theta$

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$I_3 + I_2 = I_1$

eg}



find I_1, I_2, I_3

$Z_{eq}^{(1)} = \left(\frac{1}{5} + \frac{1}{5j}\right)^{-1} = \left(\frac{5j+5}{25j}\right)^{-1} = \frac{5j}{j+1}$

$Z_{eq}^{(2)} ⇒ -5j + \frac{5j}{j+1} ⇒ \frac{+5 - \cancel{5j} + \cancel{5j}}{j+1} = \frac{5}{j+1}$

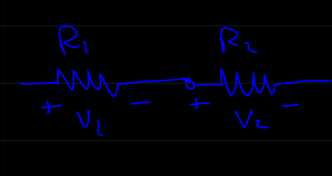
⇒ $\frac{5(j-1)}{-2}$ ⇒ $\frac{5-5j}{2}$

$\frac{5}{2} - \frac{5}{2}j$

$r = \sqrt{x^2 + y^2} = \sqrt{\frac{25}{4} + \frac{25}{4}} = \frac{\sqrt{50}}{2}$ $\frac{5\sqrt{2}}{2}$

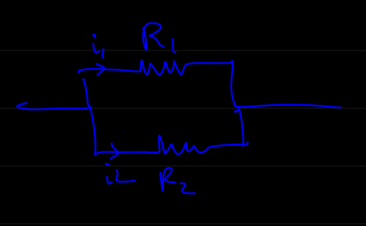
$\theta = \tan^{-1}(-1) ⇒ -45°$

Voltage division



$V_1 = V \cdot \frac{R_1}{R_1 + R_2}$

Current division



$i_1 = i \cdot \frac{R_2}{R_1 + R_2}$

$I_1 = \frac{V}{Z_{eq}} = \frac{100\angle0°}{\frac{5\sqrt{2}}{2}\angle-45°} ⇒ \frac{40\angle0°}{\sqrt{2}\angle-45°}$

$r\angle\theta = re^{j\theta} =$

$\frac{40(\cos 0 + j\sin 0)}{\sqrt{2}(\cos(45) - j\sin(45))}$

$\frac{40}{\sqrt{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)}$ ⇒ $\frac{40}{1-j}$ ⇒ $\frac{40(1+j)}{2} = 20(1+j)$

$20 + 20j$

$r = \sqrt{x^2+y^2} ⇒ \sqrt{400+400}$ ⇒ $20\sqrt{1+1} = 20\sqrt{2}$

$\theta = \tan^{-1}(1) = 45°$

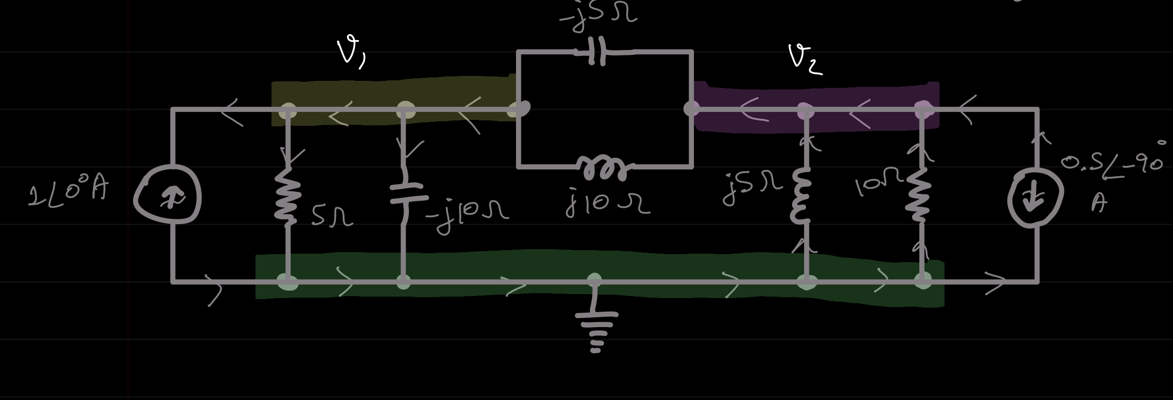
$I_1 = 20\sqrt{2}\angle45°$

$I_2 = I_1 \times \frac{j5}{5+j5}$ ⇒ $20\sqrt{2}\angle45° \times \frac{j(1-j)}{2}$

⇒ $10\sqrt{2}\angle45° \times (j+1)$

* NODAL ANALYSIS

Node ≡ Same voltage



at node 1:

$1\angle0° = \frac{V_1}{5} + \frac{jV_1}{10} + \frac{(V_2 - V_1)j}{10} - \frac{(V_2 - V_1)j}{5}$

$r\cos\theta + rj\sin\theta$

$r[1 + 0] ⇒ 1$

$10 = 2V_1 + \cancel{jV_1} + jV_2 - \cancel{jV_1} - 2V_2 j + 2V_1 j$

$10 = 2V_1 - jV_2 + 2jV_1$

$10 = 2V_1 + j(2V_1 - V_2)$ ①

at node 2:

$\frac{V_2 - V_1}{j10} + \frac{V_2 - V_1}{-j5} + \frac{V_2}{j5} + \frac{V_2}{10} = 0.5\angle-90°$

$0.5\cos 90° - 0.5j\sin 90°$ ⇒ $-0.5j$

$V_2 - V_1 + 2V_1 - \cancel{2V_2} + \cancel{2V_2} + V_2 j = 5$

$3V_2 - V_1 + V_2 j = 5$ ⇒ $V_1 = 3V_2 - 5 + V_2 j$

Put value of V_1 in ①

$10 = 6V_2 - 10 + 2V_2 j + j(6V_2 - 10 + 2V_2 j - V_2)$

$10 = 6V_2 - 10 + 2V_2 j + 6V_2 j - 10j - 2V_2 - V_2 j$

$20 = 4V_2 + 7V_2 j - 10j$

$20 + 10j = V_2(4 + 7j)$

$V_2 = \frac{(20+10j)(4-7j)}{16+49} = \frac{80 - 140j + 40j + 70}{65}$

$\frac{150 - 100j}{65}$ ⇒ $\frac{30 - 20j}{13}$

$V_2 = \frac{30}{13} - j\left(\frac{20}{13}\right)$

$r = \sqrt{\frac{900+400}{169}} = \frac{10}{\sqrt{13}}$

$\theta = \tan^{-1}\left(-\frac{2}{3}\right)$

$= -\tan\left(\frac{2}{3}\right) =$

$= -33.69$

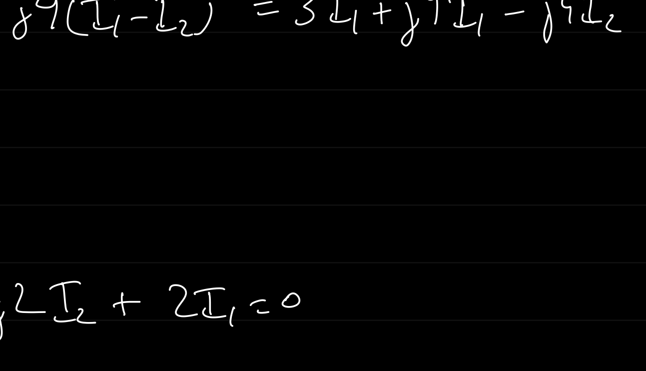
$V_2 = \frac{10\sqrt{13}}{13}\angle-33.69°$

$V_1 = 3V_2 - 5 + jV_2$

⇒ $V_2(3 + j) - 5 = \frac{30}{13} - \frac{20j}{13}(3+j) - 5$

⇒ $\frac{30 - 60j + 20 - 65}{13} = \frac{-60j - 15}{13}$ ⇒ $V_1 = \frac{15\sqrt{17}}{13}\angle-104°$

* Mesh Analysis



2 meshes

$10\angle0° = 3I_1 + j4(I_1 - I_2)$

$(I_2 - I_1)j4 - I_2(j2) + 2I_1 = 0$

$10\angle0° = 10 = 3I_1 + j4(I_1 - I_2) = 3I_1 + j4I_1 - j4I_2$

$j4I_2 + 10 = I_1(3$

$j4I_2 - j4I_1 - j2I_2 + 2I_1 = 0$

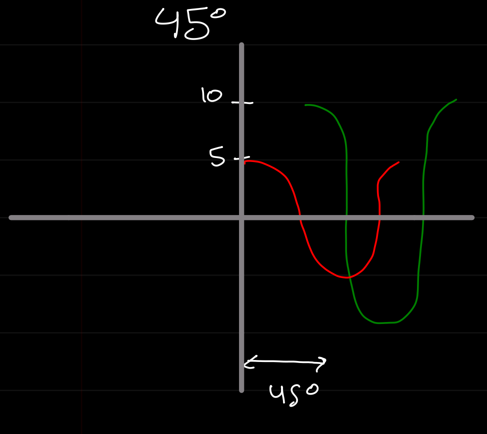
$j4I_2 - j4\Big(10 -$

* CTD Tutorial : 1

19/08/24 1:00 – 3:00

Q1) Determine the angle by which v_1 leads i_2 if $V_1 = 10\cos(10t - 45°)$ and i_1 is equal to

(1) $i_1 = 5\cos 10t$
45°



(2) $i_1 = 5\cos(10t - 40°)$
5°

(3) $i_1 = 5\cos(10t - 80°)$
35°

(4) $i_1 = 5\cos(10t + 40°)$
85°

(5) $5\sin(10t - 19°)$
$= 5\cos(10t - 19° - 90°)$
$= 5\cos(10t - 109°)$
Ans: 64°

Q2) Determine which waveform in each of the following pairs is lagging

(1) $\cos 4t$ leads $\sin 4t$ by $\frac{\pi}{2}$ {90°}

(2) $\cos(4t - 80°)$ lags $\cos(4t)$ by 80°

(3) $\cos(4t + 80°)$ leads $\cos(4t)$ by 80°

(4) $\sin(5t)$ lags $\cos(5t + 2°)$ by 92°

(5) $\sin 5t + \cos 5t$

Q3) Perform the indicated operation {express in rectangular & polar form}

(a) $\frac{2+j3}{1+8\angle 90°} - 4$

$8\angle 90°$
$\to r\cos\theta + j\sin\theta = 8 \cdot 0 + 8 \cdot j \cdot 1 = 8j$

$\Rightarrow \frac{2+j3}{1+8j} - 4 \Rightarrow \frac{2+j3-4-j32}{1+8j} \times \frac{1-8j}{1-8j}$

$\Rightarrow \frac{(-2+29j)(1-8j)}{8} = -2+16j+29j+(29\times 8)$

$\Rightarrow 230 + 45j$

$\Rightarrow 46 + 9j$

$\frac{2+j3}{1+8\angle 90°} - 4$

$8\angle 90° = 8\cdot\cos(90) + 8\cdot j\sin(90) = 8j$

$\frac{2+j3}{1+j8} \times \frac{1-8j}{1-8j} - 4$

$\Rightarrow \frac{2-16j+j3+24}{65} - 4 \Rightarrow \frac{26-13j}{65} - 4$

$\frac{-234-13j}{65} \Leftarrow \frac{26-13j-260}{65}$

x y $\boxed{\frac{-18}{5} - \frac{1}{5}\cdot j}$ rectangular form

$r = \sqrt{x^2+y^2} = \sqrt{\frac{18^2}{25} + \frac{1}{25}}$

$\Rightarrow \sqrt{\frac{325}{25}} = \sqrt{13}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{18}\right) = 3.179°$

polar form $\boxed{\sqrt{13}\angle 3.179°}$

(b) $\left(\frac{10\angle 25°}{5\angle -10°} + \frac{3\angle 15°}{3-5j}\right) j2$

Q4) Transform into Phasor form

(a) $11\sin 100t$

(b) $11\cos 100t$

(c) $11\cos(100t - 90°)$

(d) $3\cos 100t - 3\sin 100t$

Q5) Determine equivalent Admittance assuming $w = 1000$ rad/s

(a) 25Ω in series with 20mH

$Z_L = j\omega L = j \times 1000 \times 20 \times 10^{-3}$
$= j\cdot 20$

Admittance $(Y) = \frac{1}{Z_{eq}}$

$Z_{eq} = 25 + j20$

$Y = \frac{1}{25+j20} \times \frac{25-j20}{25-j20}$

$\boxed{\frac{1}{41} - \frac{4}{205}\cdot j} \Leftarrow \frac{5-j4}{205} \Leftarrow \frac{25-j\cdot 20}{625+400} \Rightarrow \frac{25-j\cdot 20}{1025}$

(b) 25Ω in parallel with 20mH

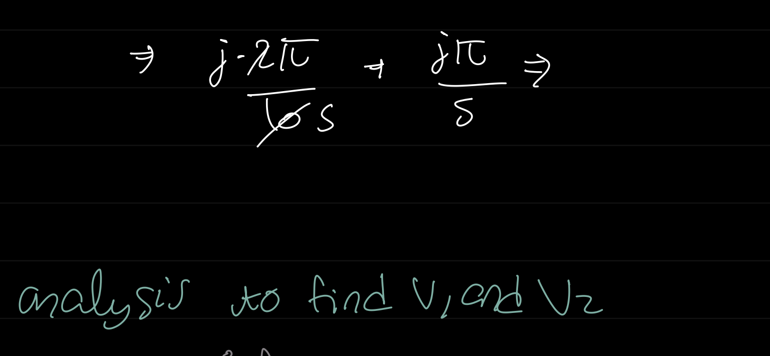
admittance $= \frac{1}{25} + \frac{1}{20j} = \frac{\overset{5}{\cancel{25}} + j\overset{4}{\cancel{20}}}{\underset{100}{\cancel{500}}j}$

$\frac{-j\cdot 5 - 4}{100} \Leftarrow \frac{-j(5+j\cdot 4)}{100} \Leftarrow \frac{5+j\cdot 4}{100j}$

$\frac{-1}{25} + \left(\frac{-j}{20}\right)$

Q6) Calculate the equivalent impedance as seen at the open terminals if the freq is ...

(a) 1Hz
(b) 1KHz
(c) 1MHz
(d) 1GHz



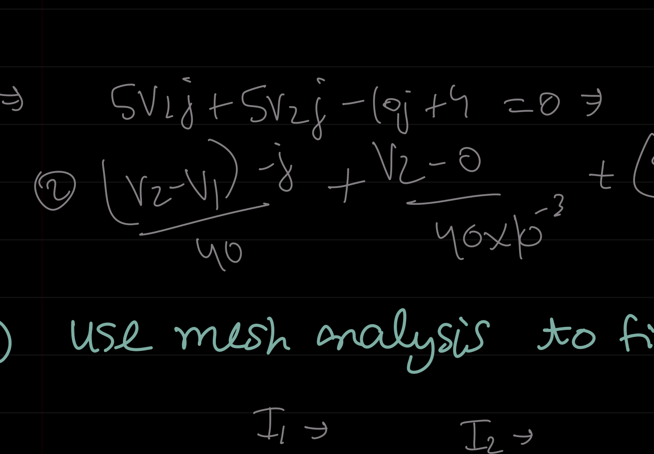
(a) $f = 1Hz$ $\omega = 2\pi f$ $= 2\pi$ radians/s

$Z_L = j\omega L = j\cdot 2\pi \cdot 100\times 10^{-3}$

$\Rightarrow \frac{j\cdot 2\pi}{\cancel{10}5} \Rightarrow \frac{j\pi}{5} \Rightarrow$

Q7) Use nodal analysis to find V_1 and V_2

$\frac{1}{-25\times j\times 10^{-3}}$



V1

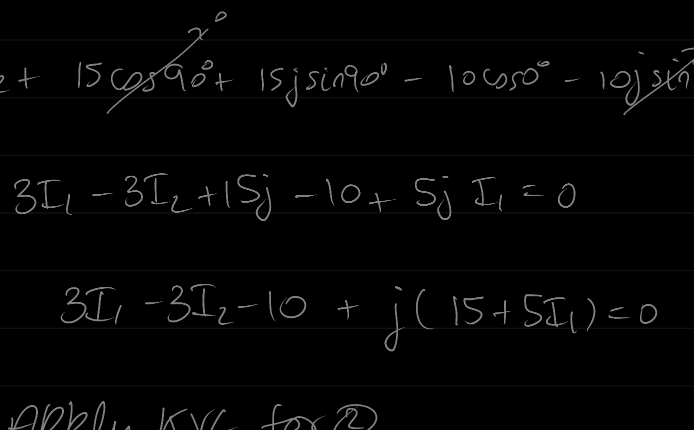
$\frac{V_1\cdot j}{20} - \frac{(V_1-V_2)j}{40} + (-50j\times 10^{-3}) + 20\times 10^{-3} = 0$

$\Rightarrow \frac{5(V_1 j + V_2 j)}{10} - \frac{10j}{10} + \frac{4}{10} = 0$

$\Rightarrow 5V_1 j + 5V_2 j - 10j + 4 = 0 \Rightarrow 4 + j(5V_1 + 5V_2 - 10) = 0$

② $\frac{(V_2 - V_1)\cdot j}{40} + \frac{V_2 - 0}{40\times 10^{-3}} + (50j\times 10^{-3}) = 0$

Q8) Use mesh analysis to find I_1 and I_2



Apply KVL for ①:

$3(I_1 - I_2) + 15\angle 90° - 10\angle 0° + I_1(j5) = 0$

$3I_1 - 3I_2 + 15\cancel{\cos 90°} + 15j\sin 90° - 10\cos 0° - 10j\cancel{\sin 0°} + I_1 5\cdot j = 0$

$3I_1 - 3I_2 + 15j - 10 + 5j I_1 = 0$

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

Apply KVL for ②

$-4I_2\cdot j + 20\angle 0° - 15\angle 90° + (I_2 - I_1)3 = 0$

$-4I_2\cdot j + 20 - 15j + 3I_2 - 3I_1 = 0$

$3I_2 - 3I_1 + 20 + j(-15 - 4I_2) = 0$

$3I_1 = 3I_2 + 20 + j(-15 - 4I_2)$

Put ② in ①

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

$3\cancel{I_2} + 20 + j(-15 - 4I_2) - 3\cancel{I_2} - 10 + j(15 + 5I_1) = 0$

$10 + j(-15 - 4I_2) + j\left(15 + 5\left[\frac{3I_2 + 20 - 15j - 4I_2\cdot j}{3}\right]\right) = 0$

$30 - 45j - 12I_2 j + j(45 + 15I_2 + 100 - 75j - 20I_2 j) = 0$

$30 - \cancel{45}j - 12I_2 j + \cancel{45}j + 15I_2 j + 100j + 75 + 20I_2 = 0$

$105 + 20I_2 + j(3I_2 + 100) = 0$

$105 + 20I_2 + j\cdot 3I_2 + j\cdot 100 = 0$

$I_2(20 + j\cdot 3) = -105 - j\cdot 100$

$I_2 = \frac{-105 - j\cdot 100}{20 + j\cdot 3} \times \frac{20 - j\cdot 3}{20 - j\cdot 3}$

$I_2 = \frac{-2100 + 315\cdot j - 2000j - 300}{409}$

$I_2 = \frac{-2400 - 1685j}{409}$

Homework:1 19/08/24

* Section 10.1

Q1} (a) $5\sin(5t - 9^\circ)$

$t = 0 \Rightarrow 5\sin(-9^\circ) = -5\sin(9^\circ) = -0.782$

$t = 0.01 \Rightarrow 5\sin(0.05 - 9^\circ) = -5\sin(8.95^\circ)$ ✗

$\Downarrow$ radians

$5\sin\left(\left[0.05 \times \frac{180}{\pi}\right) - 9^\circ\right) = 5\sin(2.86^\circ - 9^\circ)$

$\Rightarrow -5\sin(6.14) = -0.534$

$t = 0.1s \Rightarrow 5\sin(0.5 - 9^\circ) = 5\sin(28.64 - 9) = 5\sin(19.64)$

$\Rightarrow 1.6805$

(b) $4\cos 2t$

$t = 0 \Rightarrow 4\cos(0) = 4$

$t = 1 \Rightarrow 4\cos(2) = 3.997$

$t = 1.5 \Rightarrow 4\cos(3) = 3.994$

(c) $3.2\cos(6t + 15^\circ)$

$t = 0 \Rightarrow 3.2\cos(15^\circ) = 3.09$

$t = 0.01 \Rightarrow 3.2\cos(0.06 + 15^\circ) = 3.2\cos(3.43^\circ + 15^\circ)$

$\Rightarrow 3.2\cos(18.43^\circ)$

$\Rightarrow 3.035$

$t = 0.1 \Rightarrow 3.2\cos(0.6 + 15^\circ) = 3.2\cos(34.3^\circ + 15^\circ)$

$= 3.2\cos(49.3)$

$= 2.086$

Q2} (a) $300\sin(628t) \rightarrow 300 \cdot \cos(628t - 90^\circ)$

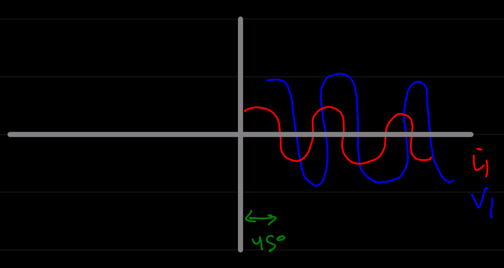
$4\sin(3\pi t + 30^\circ) \rightarrow 4\cos(3\pi t - 60^\circ)$

$14\sin(50t - 5^\circ) - 10\cos 50t \rightarrow$

Q3} $V_1 = 10\cos(10t - 45^\circ)$

(a) $i_1 = 5\cos 10t$

-45°

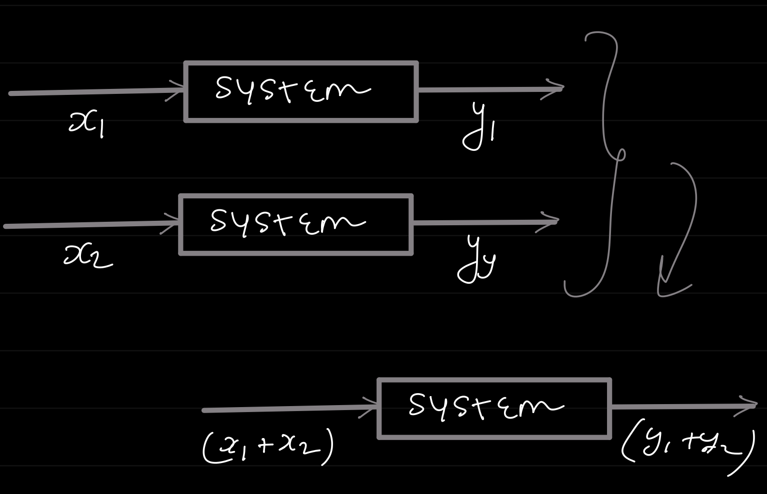


LECTURE: 4

9:30 – 11:00AM
21/08/24

* Superposition principle

active elements : provide energy eg: volt src / curr src
passive element : consume/ store energy
eg: R/L/C

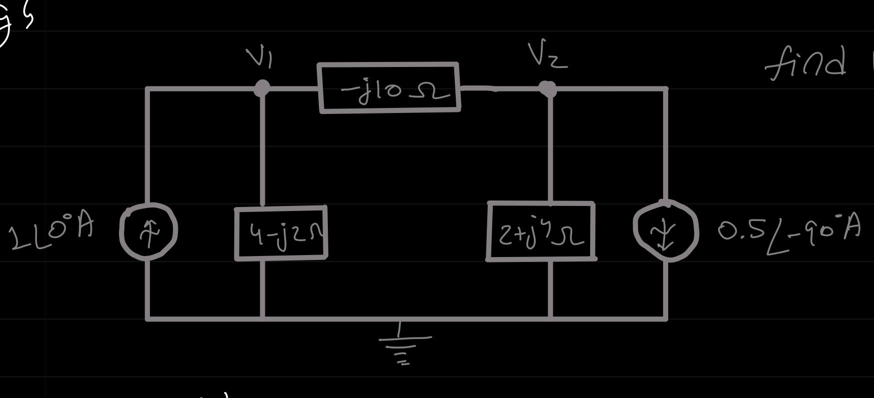


* Multiple independent sources

- consider separate independent sources "acting alone"
- add all responses

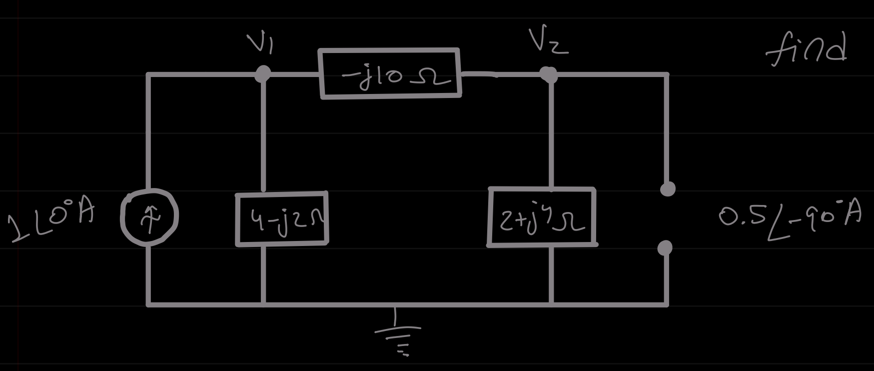
eg}

find V_2

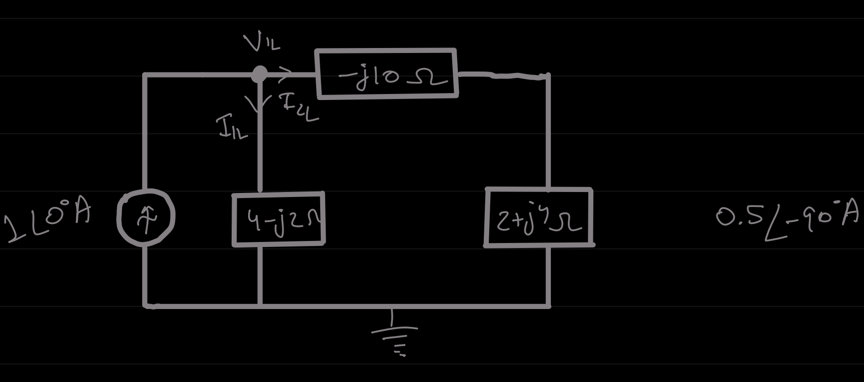


consider left side source
so we need to open the circuit at the right source

find V_2



⇓



$V_{1L} = I_{1L}(4-j2)$

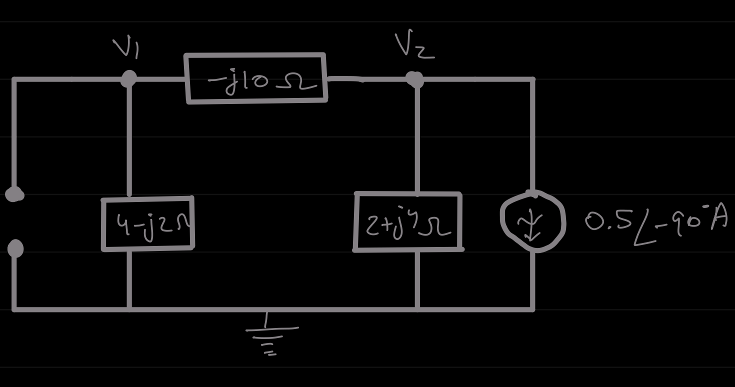
$\Rightarrow \dfrac{-j10+2+j4}{(4-j2)+(-j10+2+j4)}(4-j2)$

$\Rightarrow \dfrac{(2-j6)(4-j2)}{6-j8} \Rightarrow \dfrac{8-j4-24j-12}{6-8j}$

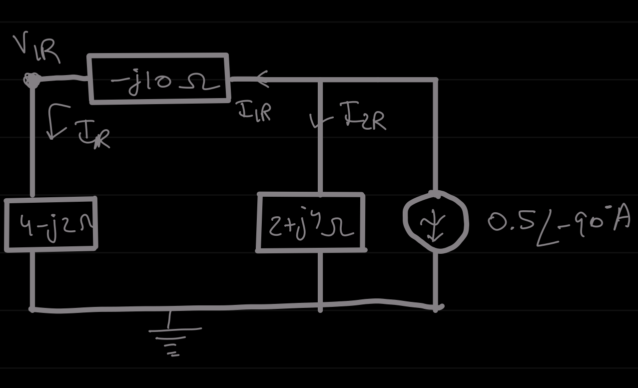
$\Rightarrow \dfrac{-4-28j}{6-8j} = 2-j2\,V$

$I_{1L}+I_{2L} = 1\angle 0^\circ = 1\cos 0^\circ + j1\sin 0^\circ = 1A$

consider right src



⇓



$V_{1R} = I_{1R}(4-j2)$

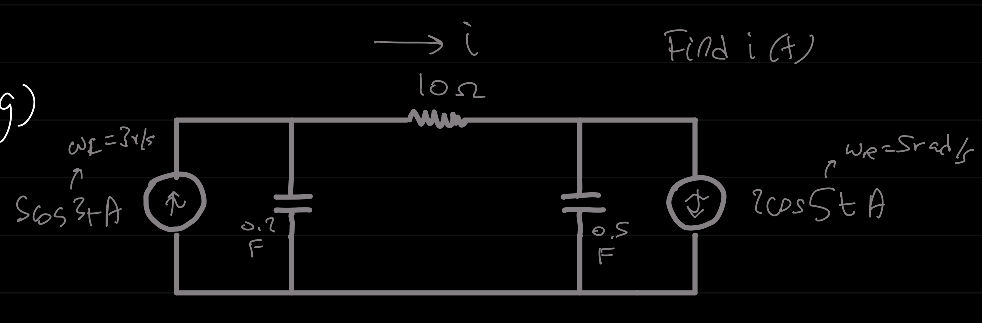
$I_{1R} = \dfrac{(-0.5\angle -90^\circ A)(2+j4)}{-j10+4-2j+2+j4}$

$V_{1R} = \dfrac{(j\cdot 0.5)(2+j4)(4-j2)}{6-j8}$

$\Rightarrow \dfrac{(j-2)(4-j2)}{6-j8} = \dfrac{j4+2-8+j4}{6-j8} = \dfrac{j8-6}{6-j8} = -1V$

$V_1 = V_{1L}+V_{1R} = 2-j2-1 = \boxed{1-j2\,V}$

eg) Find $i(t)$

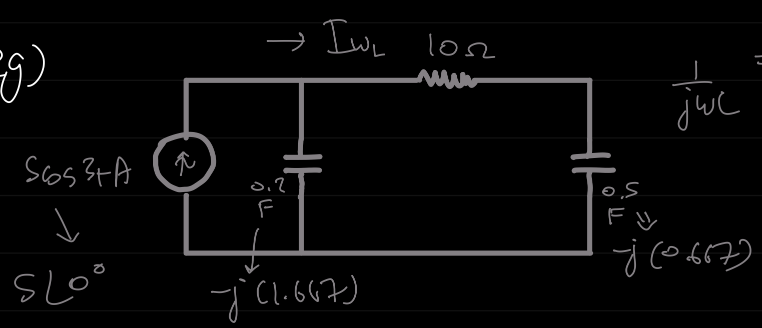


Note: we have 2 sources operating at 2 different frequencies (ω)

① consider left src $\omega_L = 3\text{rad/s}$

eg)

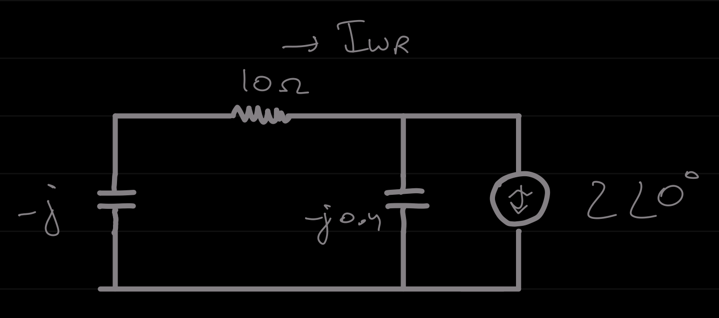
$\dfrac{1}{j\omega C} = \dfrac{1}{j(3)0.2} = \dfrac{-j}{0.6}$



$I_{wL} = 5\angle 0^\circ \left(\dfrac{-j(1.667)}{-j(1.667)+10-j0.667}\right)$

$\Rightarrow 811.7\angle -76.86^\circ\,mA$

② consider right source $\omega_R = 5\text{rad/s}$



$I_{wR} = 2\angle 0^\circ \left(\dfrac{-j\cdot 0.4}{-j0.4+10+-j}\right)$

$\Rightarrow \dfrac{-j(0.8)}{-j(1.4)+10} \Rightarrow \dfrac{-j(0.8)[10+j(1.4)]}{100+1.96}$

$\dfrac{-j8+1.12}{101.96} \Rightarrow$

$\Rightarrow 0.0792\angle -82.03^\circ\,A = 79.2\angle -82.03^\circ\,mA$

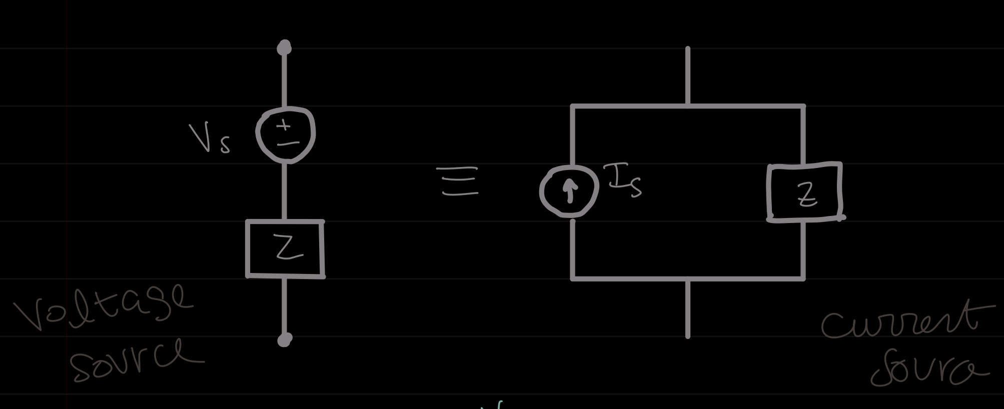
Applying superposition

$I = I_{wL}+I_{wR}$ ✗

Phasors can only be added when they correspond to same frequency

$i(t) = [811.7\cos(3t-76.86^\circ)+79.25\cos(5t-82.03^\circ)]\,mA$

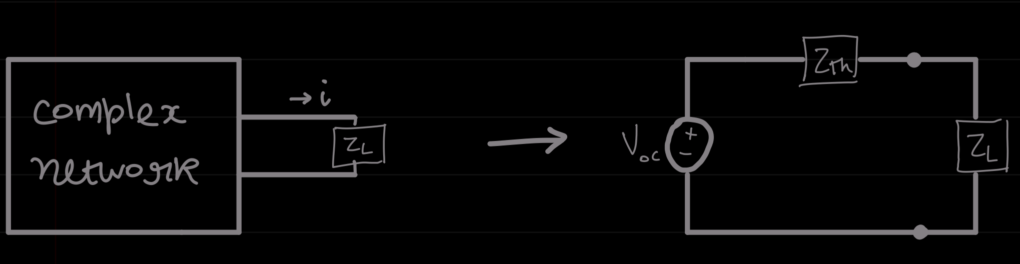
* SOURCE TRANSFORMATION



$I_s = \dfrac{V_s}{Z}$

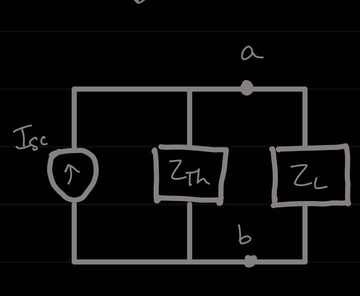
* THEVENIN & NORTON EQUIVALENT CKTS

Normally



↓

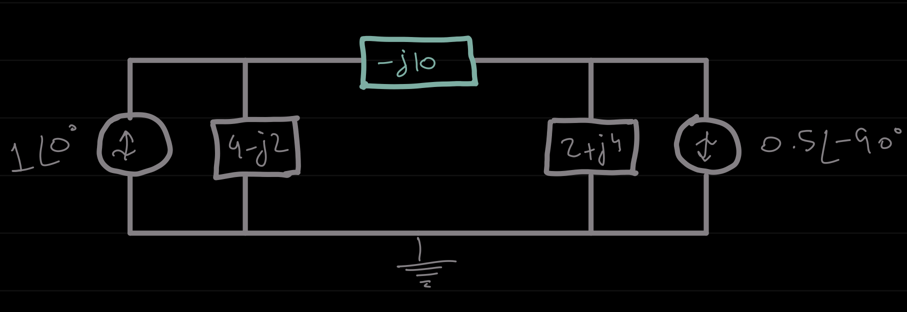
$V_{oc} = I_{sc}\cdot Z_{TH}$



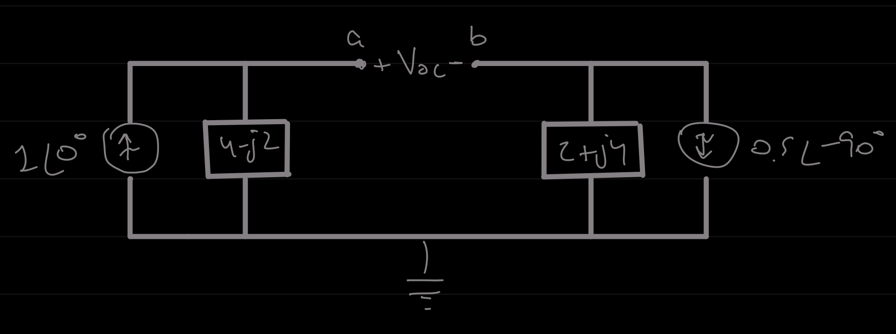
	V_{oc} (by open circuiting)	I_{sc} (by short circuiting)	Z_{TH} (ratio of V_{oc} & I_{sc})
① Independent sources	✓	✓	✓
② Independent + dependent sources	✓	✓	✓

③ Dependent sources only — we need to consider test sources eg: 1A / 1V and then find $V_{oc} \Rightarrow I_{sc} \Rightarrow Z_{Th} = \dfrac{V_{oc}}{I_{sc}}$

eg} find Thevenin equivalent seen by $-j10\Omega$



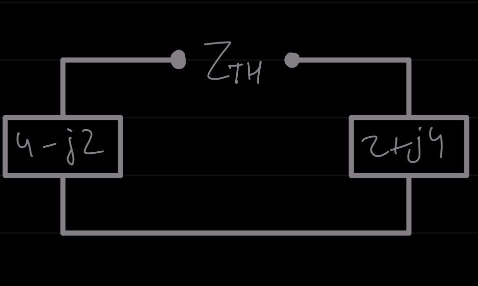
- : complex network
- : R_L



$V_{oc} = V_a - V_b = 1\angle 0^\circ(4-j2) - 0.5\angle -90^\circ(2+j4)$
$= 4-j2+2-j1 = 6-j3\,V$

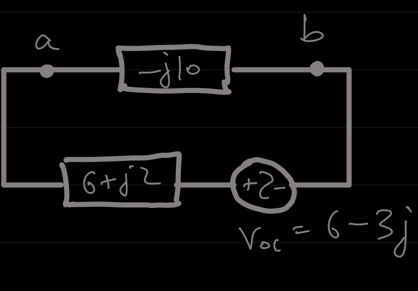
Find Z_{Th}

all volt src → short circuit
all curr src → open circuit



$Z_{TH} = 4-j2+2+j4 = 6+j2\,\Omega$

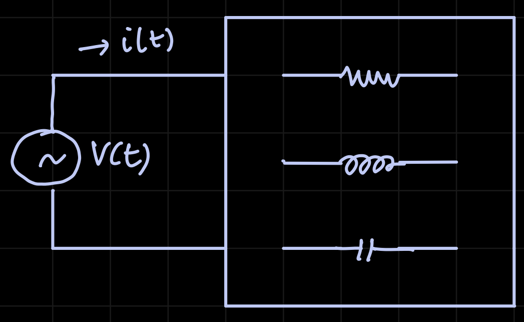
Equivalent circuit



$I_{ab} = \dfrac{V_{oc}}{6+j2-j10} = \dfrac{V_{oc}}{6-j8}$

* Lecture : 5 28/08/24

⇒ AC Power Analysis {Module : 3}



$\cos(A)\cos(B) = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

instantaneous power : $p(t) = v(t) \cdot i(t)$

$$v(t) = V_m \cdot \cos(\omega t + \theta)$$
$$i(t) = I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = V_m \cdot \cos(\omega t + \theta) \cdot I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = \underbrace{\frac{V_m I_m}{2} \cos(\theta - \phi)}_{\substack{\text{constant} \\ \text{(DC term)}}} + \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{\substack{\text{twice frequency} \\ \text{(harmonic)}}}$$

- Average Power

$$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2} \cos(\theta - \phi)\,dt + \underbrace{\frac{1}{T}\int_0^T \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{0}$$

$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)}$$

$P_{avg} = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$ $\mathbf{I} = I_m e^{j\phi}$

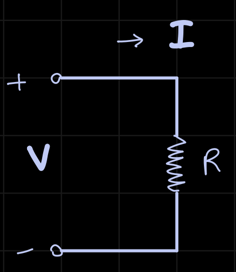
$\mathbf{I}^* = I_m e^{-j\phi}$ (conjugate)

real number ✓

phasor ✗

* Avg. Power absorbed

- by a purely resistive element

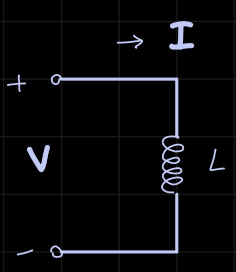


$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$

here, phase diff = 0°

$$P_{avg} = \frac{V_m \cdot I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

- by a purely inductive element

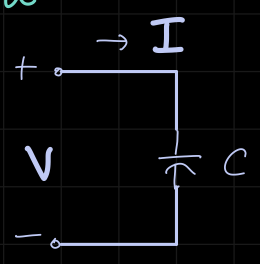


$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$

here, phase diff = 90°

So, $P_{avg} = 0$ for inductor

- by a purely capacitive element



$$P_{avg} = \frac{V_m \cdot I_m}{2} \cdot \cos(-90°)$$

$P_{avg} = 0$ for capacitor

* Lecture: 6

$V(t) = V_m \cos(\omega t + \theta)$ ← Revision

$\downarrow$

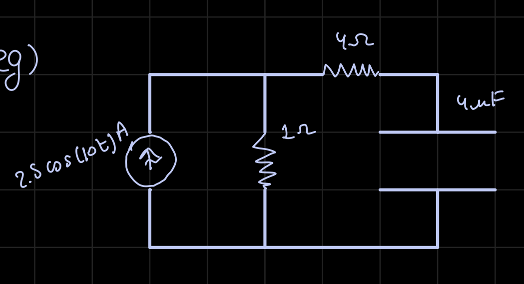
$V(t) = Re\{V_m e^{j\omega t} e^{j\theta}\}$

$\downarrow -e^{j\omega t}$

$V_m e^{j\theta} = V_m \angle\theta$

exponential   polar

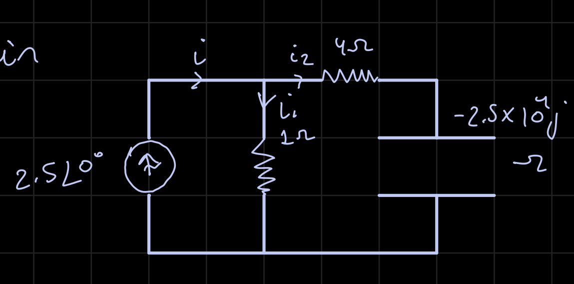
• eg)



① find power delivered to each element at $t = 0, 10, 20ms$

② Find P_{avg} to each element

Ans) convert to freq. domain



$V_{1\Omega} = V_{4\Omega} + V_C$

① $P_{inst} = \frac{V_m I_m}{2}\cos(\theta - \phi) + \frac{V_m I_m}{2}\cos(2\omega t + \phi + \theta)$

imaginary part is way larger than real part

$I_1 = -2.5 \times \left(\frac{4 - j2.5\times10^4}{1+4-j\,2.5\times10^4}\right) \simeq \underline{\underline{-2.5A}}$

$\Rightarrow \frac{-10 + j625\times10^{\cancel{00}}}{5 - j\,25\times10^3} \Rightarrow \frac{-2 + j125}{1 - j50}$

$\Rightarrow \frac{-2 + j\cdot125}{1 - j50} \times \frac{1 + j50}{1 + j50} \Rightarrow$

$I_{1\Omega}(t) = 2.5\cos(10t)$

$V_{1\Omega} = (I_{1\Omega})(1\Omega) \Rightarrow V_{1\Omega}(t) = 2.5\cos(10t)$

$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{4}$

$p_{1\Omega}(t) = 2.5\cos(10t) \cdot 2.5\cos(10t)$ W

$p_{1\Omega}(t=0) = 6.25W$

$p_{1\Omega}(t = 10ms) = 6.1877W$

$p_{1\Omega}(t = 20ms) = 6.033W$

$(P_{avg})_{1\Omega} = 3.125W$

$\Rightarrow$ P delivered to 4Ω

$P_{4\Omega}(t) = V_{4\Omega}(t) \times I_{4\Omega}(t)$

$I_{4\Omega} = I_S \times \frac{1}{5 - j2.5\times10^4} = 10^{-4}\angle 90°A$

$V_{4\Omega} = 4 \times I_{4\Omega} = 4\times10^{-4}\angle90°$ V

convert to time domain

$p_{4\Omega}(t) = 2\times10^{-8} + 2\times10^{-8}\cos(20t)W$

$t=0: 4\times10^{-8}$ W

$t = 10ms = 3.96\times10^{-8}$ W

$t = 20ms = 3.87\times10^{-8}$ W

$P_{avg\,4\Omega} = 2\times10^{-8}$ W

$\Rightarrow$ P delivered to capacitor

$p_C(t) = V_C(t) \cdot I_C(t)$

$(p_{avg})_C = 0$ since phase diff $= 90°$

$I_C = I_2 = 10^{-4}\angle90°A \Rightarrow I_C(t) = 10^{-4}\cos(10t + 90°)$

$V_C = (-j\,2.5\times10^{-4}) \times 10^{-4}\angle 90$

$-j \cdot 25\times10^3 \times (+j10^{-4})$

$\underline{\underline{2.5V}}$

$p_C(t) = 1.25\times10^{-4}\cos(20t + 90°)$ W

$t=0 \Rightarrow p = 0W$

$t = 10ms \Rightarrow p = 2.48\times10^{-5}W$

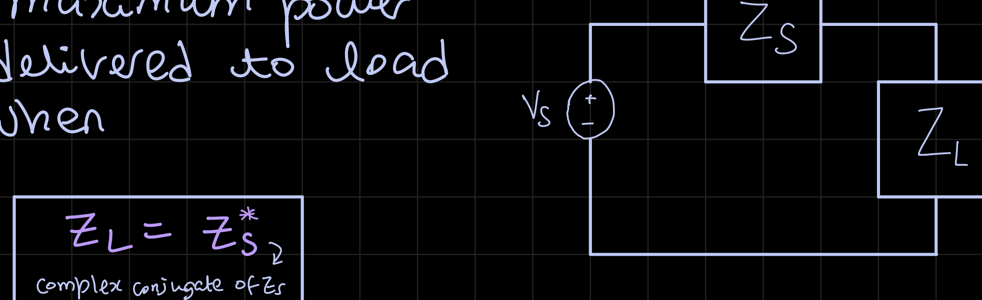
Note: we cannot multiply I_C and V_C in phasor form and then convert to time domain for getting P_C (power) because power does not has a phasor part. It is a real value.

* Correct or not?

depends on t ✗ ① $p_{1\Omega}(t) + p_{4\Omega}(t) + p_C(t) = \text{constant1}$

✓ ② $P_{avg\,1\Omega} + P_{avg\,4\Omega} + P_{avg\,C} = \text{constant2} \equiv P_{avg\ source}$

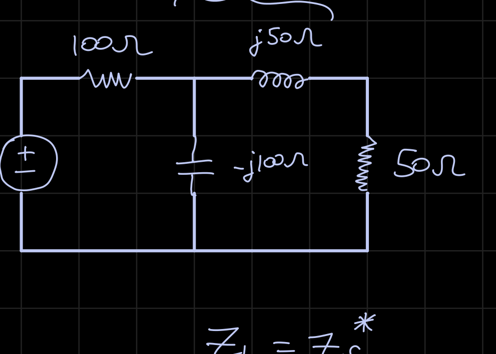
active sign convention   passive sign convention



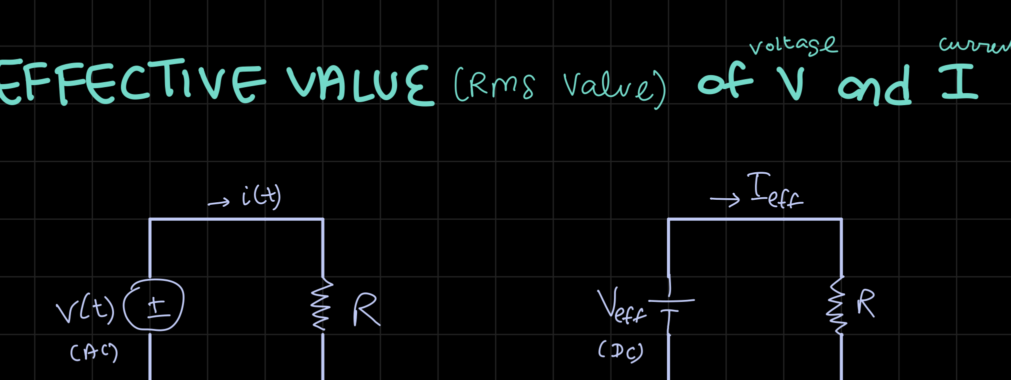
* Maximum Power Transfer Theorem

maximum power delivered to load when

$\boxed{Z_L = Z_S^*}$
complex conjugate of Z_S

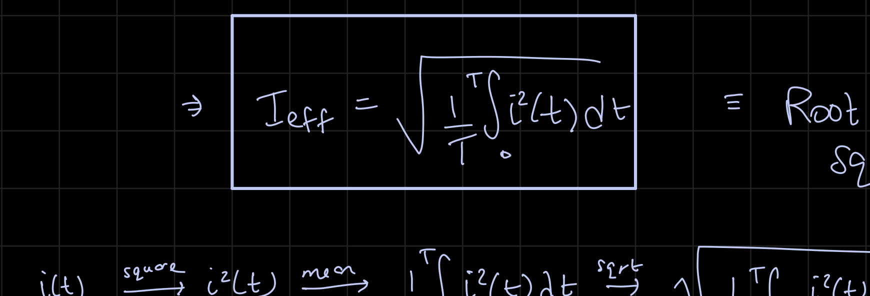


$\Rightarrow$ Impedance matching



$Z_L = Z_S^*$

* EFFECTIVE VALUE (Rms Value) of V (voltage) and I (current)



if $P_{avg_1} = P_{avg_2}$

$\Rightarrow \frac{1}{T}\int_0^T i^2(t)R\,dt = I_{eff}^2 R$

$\Rightarrow \boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}} \equiv$ Root mean square

$i(t) \xrightarrow{\text{square}} i^2(t) \xrightarrow{\text{mean}} \frac{1}{T}\int_0^T i^2(t)\,dt \xrightarrow{\text{sqrt}} \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$

• Sinusoidally varying voltage

$\cos^2 x = \cancel{\frac{2\cos x + 1}{2}}$

$i(t) = I_m\cos(\omega t + \phi)$

$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2\cos^2(\omega t + \phi)\,dt}$

not a phasor
it is real number
≡ magnitude of current/voltage

$\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$ current rms value   unit: A_{rms}

$\boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$ voltage rms value   unit: V_{rms}

eg: $I = 10\angle90°A \Rightarrow I = 5\sqrt{2}\angle90°A$ rms

OR $V = 100\sqrt{2}\angle-60°V \Rightarrow V = 100\angle-60°V_{rms}$

* Apparent Power

$P_{apparent} = I_{eff}V_{eff}$ unit: VA (volt ampere), not W

* Power factor

$PF = \frac{\text{avg power}}{\text{apparent power}} = \frac{(V_m \cdot I_m)/2\,\cos(\theta - \phi)}{V_{eff}\cdot I_{eff}}$

$PF = \cos(\underbrace{\theta - \phi}_{\text{power factor angle}})$

* Lecture : 7

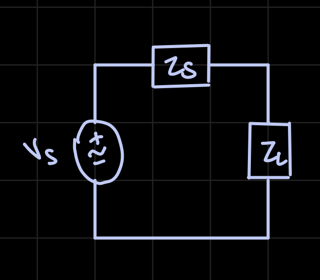
- Instantaneous Power : $p(t) = v(t)\, i(t)$
- Average Power : $P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = \frac{1}{2} Re\{\mathbf{V I^*}\}$
- $P_{avg.\,sources} = \Sigma P_{avg.\,elements}$

*** Note :** $\overset{\text{impedence}}{Z} = \underbrace{R}_{\text{resistive}} + j\overset{\text{reactive}}{X}$

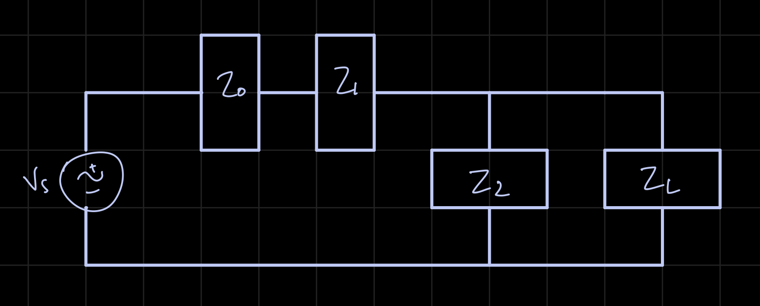
$P_{avg})_{resistive} : P \neq 0$

$P_{avg})_{reactive} : P = 0$

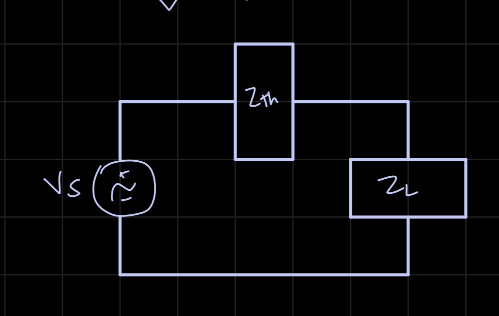
* Max Power Transfer :



Circuit has max. power when $Z_S = Z_L^*$ (complex conjugate)



⇓ apply thevenin's theorem



$Z_L = Z_{Th}^*$

* Power Factor

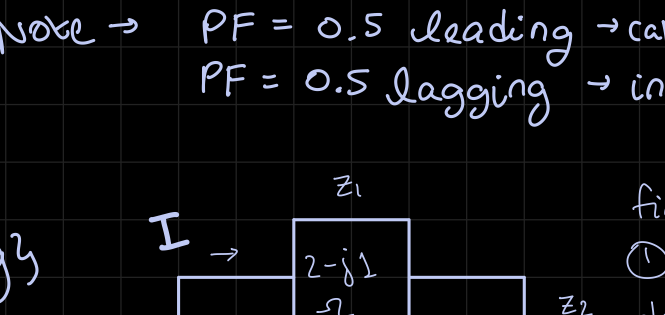
$$PF = \cos(\theta - \phi) = \frac{P_{average}}{P_{apparent}}$$

(θ: angle of voltage phasor, ϕ: angle of current phasor)

for purely resistive load : PF = 1 {Max} $\theta - \phi = \pm 90°$
for purely reactive load : PF = 0 {min}

Note → PF = 0.5 leading → capacitive $(\theta - \phi) < 0°$
PF = 0.5 lagging → inductive $(\theta - \phi) > 0°$

eg)

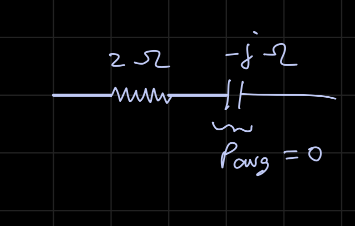


find
① Average power delivered to each load
② $P_{avg.\,source}$ = ?
③ $P_{apparent\,.\,src}$ = ?
④ PF of combined load = ?

Ans) $P_{avg\,Z_1} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} Re\{\mathbf{V I^*}\}$

$\mathbf{V}$: voltage across Z_1, not the src

lets assume Z_1 :



So, $P_{avg\,Z_1}$ = Power delivered to 2Ω resistor

$= I_{eff}^2 R$

find $\mathbf{I}$ by KVL :

$$-60\angle 0° + \mathbf{I}(2 - j1 + 1 + 5) = 0$$

$$\mathbf{I} = \frac{60\angle 0°}{3 + j4} = \underbrace{12\angle -53.13°}_{I_{rms}}\ A_{rms}$$

$i(t) = 12\cos(wt - 53°)\ A_{rms}$

$\underset{(A_{rms})}{I_{eff}} \rightarrow \underset{(A)}{I_m} : I_{eff} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = I_{eff}\sqrt{2}$

$I_{eff} = 12\ A_{rms} \Rightarrow I_m = 12\sqrt{2}\ A$

$i(t) = 12\sqrt{2}\cos(wt - 53°)\ A$

$P_{avg(Z_1)} = (12)^2 \times 2 = 288\ W$

note: $P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$

Z_2 : I_{eff} same : series circuit

$I_{eff} = 12$

and $P_{avg})_{Z_2} = I_{eff}^2 R = (144)(1) = 144\ W$

② $P_{avg})_{src} = V_{eff} I_{eff} \cos(\theta - \phi)$

$\mathbf{V}_{src} = 60\angle 0°\ V_{rms} \Rightarrow V_{eff} = 60\ V_{rms}$

$\mathbf{I}_{src} = 12\angle -53.13°\ V_{rms} \Rightarrow I_{eff} = 12\ A_{rms}$

$\theta = 0°,\ \phi = -53.13°$

$P_{avg})_{src} = [60 \times 12]\cos(53.13°) = 432\ W$

we can observe that $288 + 144 = 432$
and hence,

$P_{avg\,.\,sources} = \Sigma P_{avg.\,elements}$ holds

③ $P_{apparent} = V_{eff} \cdot I_{eff} = (60)(12) = 720\ W$

④ PF of combined loads = PF of source

$PF = \cos(\theta - \phi) = \cos(\underbrace{0 + 53.13°}_{\theta - \phi > 0,\ \text{lagging}}) = 0.6$ lagging

$$* \quad P_{avg} = \frac{1}{2} Re\{\mathbf{V I^*}\} \qquad \mathbf{V} = V_m\angle\theta,\quad \mathbf{I} = I_m\angle\phi$$

$\mathbf{V}_{eff} = V_{eff}\angle\theta$
$\mathbf{I}_{eff} = I_{eff}\angle\phi$

$P_{avg} = Re\{\mathbf{V}_{eff}\mathbf{I}_{eff}\}$

* Complex Power

$\mathbf{S} = \mathbf{V}_{eff}\mathbf{I}_{eff}^*$ Volt Ampere {VA}

this is a complex number not a phasor of power

$\mathbf{S} = (V_{eff}\angle\theta)(I_{eff}\angle\phi)$
$= V_{eff} \cdot I_{eff}\, e^{j(\theta - \phi)}$

$$\Rightarrow \mathbf{S} = \underbrace{V_{eff} I_{eff}\cos(\theta - \phi)}_{\substack{\text{average power} \\ P \\ \text{(watt) W}}} + j\underbrace{V_{eff} I_{eff}\sin(\theta - \phi)}_{\substack{\text{reactive power} \\ Q \\ \text{(volt ampere reactance) VAR}}}$$

(volt ampere) VA

$$\boxed{S = P + jQ}$$

$|S| = V_{eff} I_{eff} = P_{apparent}$ (volt amp)

note: $Z = R + jx$

Purely resistive load : $Z = R,\ X = 0$

$S = \underbrace{P}_{V_{eff} I_{eff} \cos(0)} + j\overset{V_{eff} I_{eff} \sin(\theta - \phi) = 0}{Q}$

$P \neq 0$
$Q = 0$

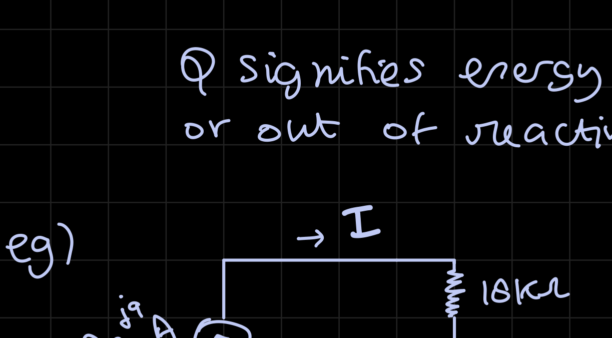
Purely reactive load

$S = \underbrace{P}_{V_{eff} I_{eff} \cos(90°)} + j\overset{V_{eff} I_{eff} \sin(\overbrace{\theta - \phi}^{90°})}{Q}$

$P = 0,\ Q \neq 0$

Q signifies energy flow rate into or out of reactive component of load

eg)

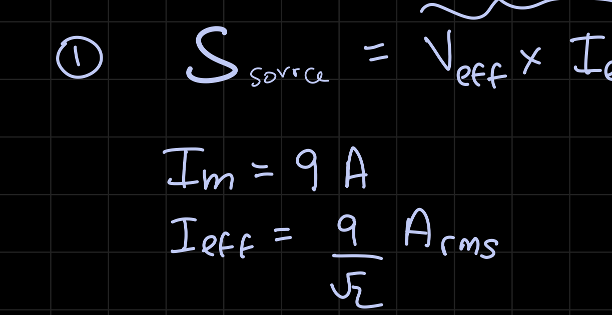


assume $C = 1\mu F$, $w = 45\ rad/s$

find:

① complex Power provided by src
② Time average power absorbed by combined load
③ Reactive Power absorbed by combined load
④ $P_{apparent}$ absorbed " "
⑤ PF of " " "

Ans)



$9e^{j9}$ A $\equiv 9\cos(9°) + j9\sin(9°)$

$\frac{1}{jwc} = \frac{1}{j \times 45 \times 1 \times 10^{-6}} = -j\frac{10^6}{45}$

① $S_{source} = \overbrace{V_{eff}}^{\text{source}} \times I_{eff}$

$I_m = 9\ A$
$I_{eff} = \frac{9}{\sqrt{2}}\ A_{rms}$

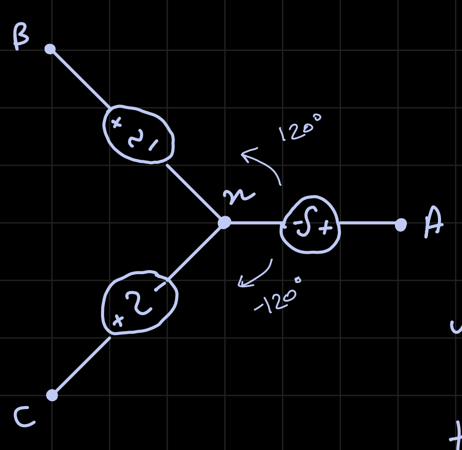
V_{eff} = ?, find $\mathbf{V}$

$$\mathbf{V} = \mathbf{I}(Z_{eq}) = 9\angle 9°\left(18 \times 10^3 - j\frac{10^6}{45}\right)$$

* Lecture : 8

- Polyphase Circuits (Chp 12)

⇒ Three Phase Source



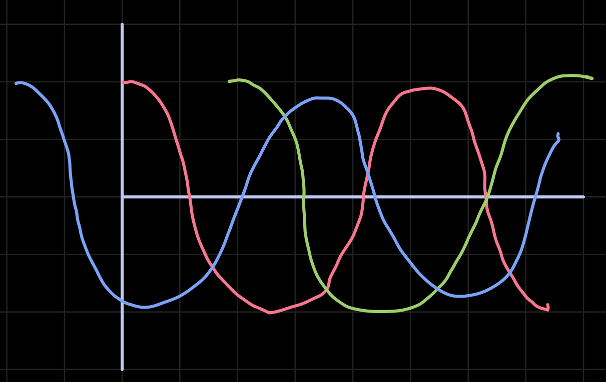
$V_{an} = 100\angle 0°\ V$
$V_{bn} = 100\angle 120°\ V$
$V_{cn} = 100\angle -120°\ V$

if $|V_{an}| = |V_{bn}| = |V_{cn}|$
& $V_{an} + V_{bn} + V_{cn} = 0$
then it is a

Balanced Source

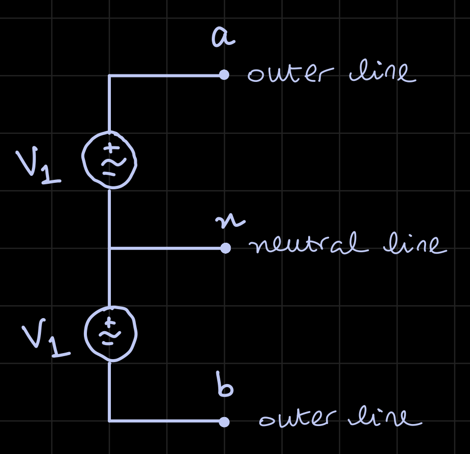
total instantaneous power = constant

freq. domain	time domain
$100\angle 0°$	$100\cos(\omega t)$
$100\angle 120°$	$100\cos(\omega t + 120°)$
$100\angle -120°$	$100\cos(\omega t - 120°)$



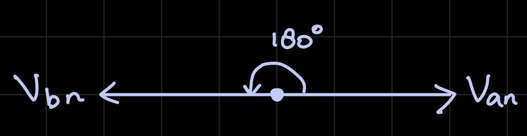
total $p(t) \to$ constant

* Single Phase Three-wire Source
≡ Two phase source



$V_1 = V_m\angle 0°$

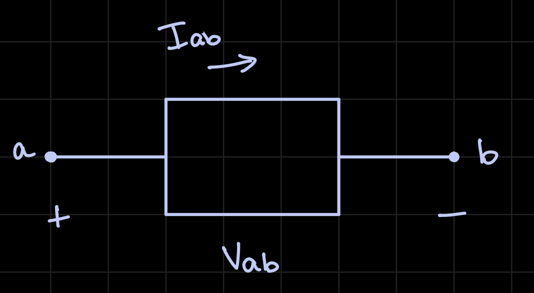
$V_{an} = V_1 = V_m\angle 0°$
$V_{bn} = -V_1 = V_m\angle 0° - 180°$



double script notation

$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$
$V_{bn} = -V_{nb}$

$V_{an} = V_A$ wrt V_n

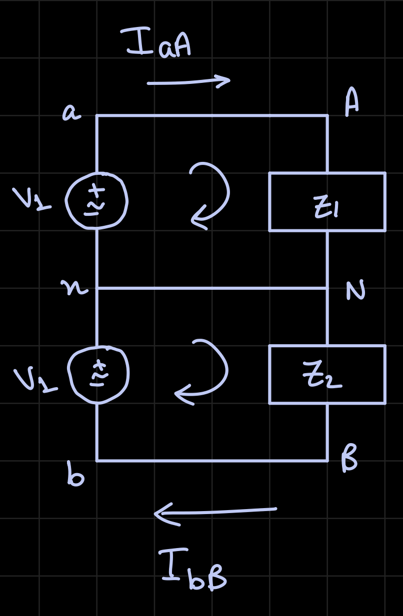


I_{ab}: current flowing from a to b

- Balanced single phase three wire source

Conditions:

$|V_{an}| = |V_{bn}|$
$V_{an} + V_{bn} = 0$
$\Big\}$ ⇒ Balanced Src



$I_{aA} = \frac{V_1}{Z_1}$

$I_{bB} = \frac{V_1}{Z_2}$

$I_{nN} = -I_{aA} + I_{bB}$

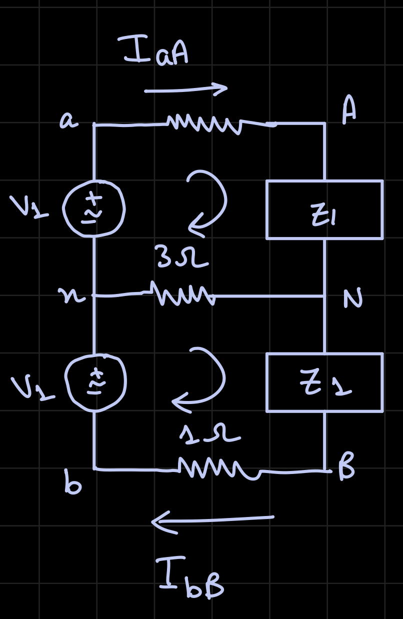
$\Rightarrow \frac{V_1}{Z_2} - \frac{V_1}{Z_1}$

Assume $Z_1 = Z_2$

$I_{nN} = \frac{V_1}{Z_1} - \frac{V_2}{Z_1} = 0$

when both srcs and loads are equal

Balanced Load : current in neutral line is equal to zero.



all terms are phasors

even with resistance
$I_{nN} = 0 = I_{bB} - I_{aA}$
due to
① Balanced Load
≡ Symmetry

* Lecture : 9

09/09/24

time domain | freq. domain
$V_p\cos(\omega t+\theta^\circ)$ | $V_p\angle\theta^\circ$
$V_p\cos(\omega t+\theta^\circ-180^\circ)$ | $V_p\angle\theta-180^\circ$

we cannot directly say,

$P_{avg} = VI$ ✗

but $P_{avg} = \frac{1}{2}Re\{VI^*\}$ ✓

② a) $I_{AB} = \frac{115\angle\theta^\circ + 115\angle\theta^\circ}{2(10+j2)} = \frac{230\angle\theta^\circ}{2(10+j2)}$

$= 11.27\angle(\theta^\circ - 11.3^\circ)A$

$\phi = \theta - 11.3^\circ$

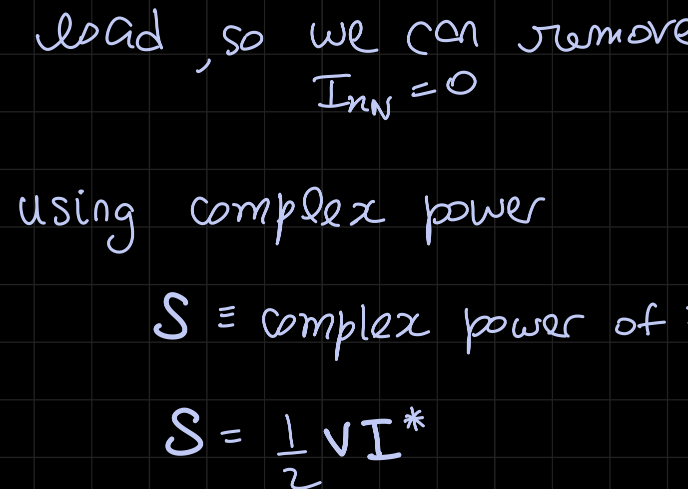
PF $= \cos(\theta - \theta + 11.3^\circ) = 0.98$ Lagging

because PF angle is +ve

b) $PF = 1 \Rightarrow \cos(\theta-\phi) = 1 \Rightarrow \theta - \phi = 0$

$\Rightarrow \theta = \phi$

$f = 50h$
$\omega = 2\pi f = 100\pi$



$I_{aA} = I_1 + I_2$

$\Rightarrow \frac{115\angle\theta^\circ}{10+j2} + \frac{230\angle\theta^\circ}{\left(\frac{1}{j\omega C}\right)}$

$\Rightarrow$ lot of calculation
alternative method $\Rightarrow$

Note that this is still a balanced load, so we can remove nN since $I_{nN} = 0$

using complex power

$S \equiv$ complex power of total load

$S = \frac{1}{2}VI^*$

$PF = \frac{Re\{S\}}{|S|} = 1 \Rightarrow Re\{S\} = |S|$

so, $Im\{S\} = 0$ here

$S = \frac{1}{2}V_{an}I_{AN}^* + \frac{1}{2}V_{nb}I_{NB}^* + \frac{1}{2}V_{ab}I_{AB}^*$

$\Rightarrow \frac{1}{2}V_{an}\left(\frac{V_{an}}{Z_p}\right)^* + \frac{1}{2}V_{nb}\left(\frac{V_{nb}}{Z_p}\right)^* + \frac{1}{2}V_{ab}\left(\frac{V_{ab}}{Z_c}\right)^*$

$\Rightarrow \frac{1}{2}\frac{|V_{an}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{nb}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{ab}|^2}{|Z_c|^2}\times Z_c$

$Re\{115\angle 0^\circ\}$

$\sqrt{10^2+2^2}$

$\Rightarrow \frac{|V_{an}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{ab}|^2}{|Z_c|^2}\times Z_c$

$Re\{\frac{1}{j\omega C}\} = \frac{1}{j\omega C}\times\frac{-1}{j\omega C}$

$\Rightarrow \frac{115^2}{104}(10+j2) + \frac{1}{2}\left(\frac{230^2}{\left(\frac{1}{\omega C}\right)^2}\right)\left(\frac{-j}{\omega C}\right)$ $\frac{-1}{-(\omega C)^2}$

$\Rightarrow 115^2\left(\frac{\cancel{10}+j2}{104} - 2j\omega C\right)$

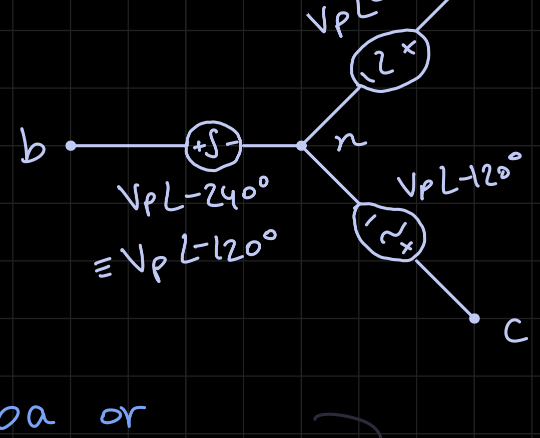
$\Rightarrow 2\times115^2\left(\frac{j}{104} - j\omega C\right)$

$\Rightarrow 2\times115^2\left(\frac{1}{104} - \omega C\right) = 0$

$\frac{1}{104} - 100\pi C = 0$

$C = \frac{1}{10400\pi} = 30.60\mu F$ ✓

* Three Phase Sources

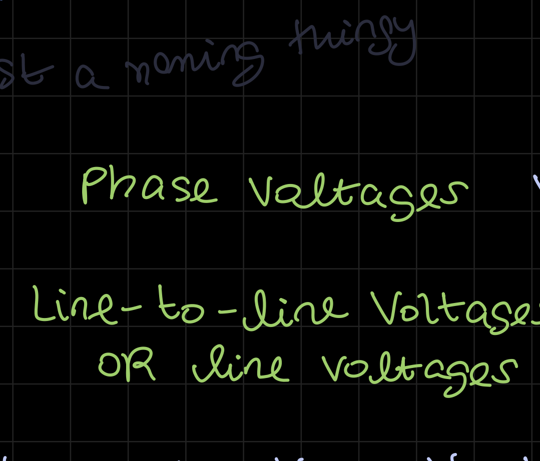


$V_{an} = V_p\angle 0^\circ$
$V_{bn} = V_p\angle -120^\circ$
$V_{cn} = V_p\angle 120^\circ$)

$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$
$V_{an} + V_{bn} + V_{cn} = 0$

↳ balanced source

abc or
positive sequence



cba or
negative sequence

just a naming thingy

Phase Voltages V_{an}, V_{bn}, V_{cn}

Line-to-line Voltages OR line voltages V_{ab}, V_{bc}, V_{ac}

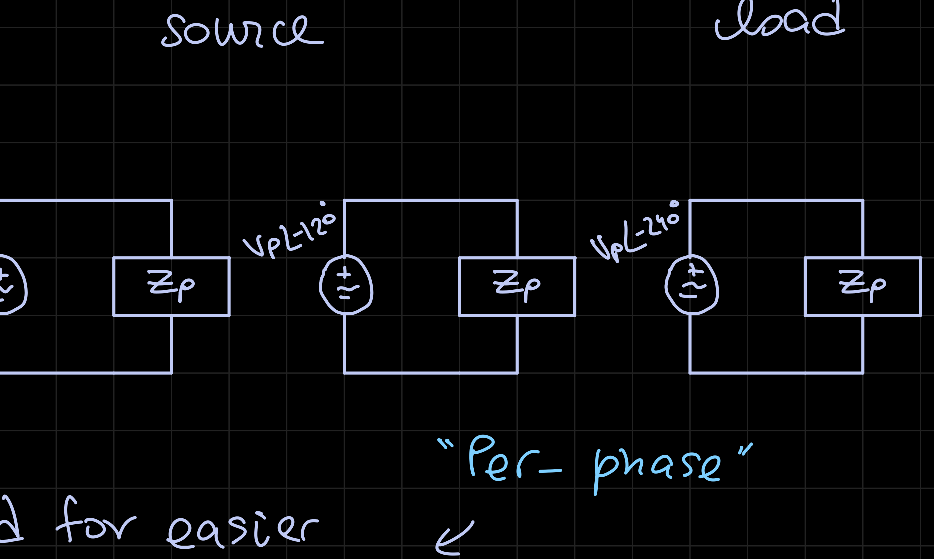
$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$

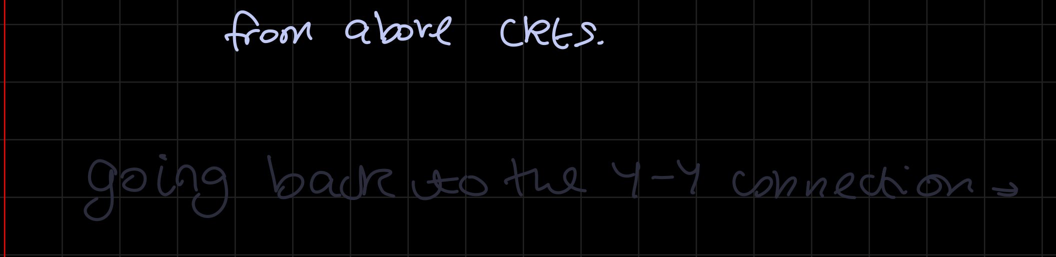
$V_{ab} = \sqrt{3}V_p\angle 30^\circ$

$V_{bc} = \sqrt{3}V_p\angle -90^\circ$

$V_{ca} = \sqrt{3}V_p\angle -210^\circ$

* Y-Y connection





"Per- phase"

used for easier analysis

eg. to find total power by ckt, just add the three individual power obtained from above ckts.

going back to the Y-Y connection $\Rightarrow$

$I_{aA} = \frac{V_{an}}{Z_p}$

$I_{bB} = \frac{V_{bn}}{Z_p} = \frac{V_{an}\angle -120^\circ}{Z_p} = I_{aA}\angle -120^\circ$

$I_{cC} = \frac{V_{cn}}{Z_p} = \frac{V_{an}\angle +120^\circ}{Z_p} = I_{aA}\angle +120^\circ$

$I_{nN} = I_{aA} + I_{bB} + I_{cC} = 0$

eg} consider three phase balanced Y-Y connected system

$V_{an} = 200\angle 0^\circ\ V_{rms}$
$Z_p = 100\angle 60^\circ\ \Omega$

find phase and line voltage & current and total avg power delivered to load

Ans}
$V_{an} = 200\angle 0^\circ\ V_{Rms}$
$V_{bn} = 200\angle -120^\circ\ V_{Rms}$
$V_{cn} = 200\angle +120^\circ\ V_{Rms}$

line voltage :

$V_{ab} = \sqrt{3}V_p\angle 30^\circ = 200\sqrt{3}\angle 30^\circ\ V_{Rms}$

$V_{bc} = 200\sqrt{3}\angle -90^\circ\ V_{Rms}$

$V_{ca} = 200\sqrt{3}\angle -210^\circ = 200\sqrt{3}\angle 150^\circ\ V_{rms}$

line currents:

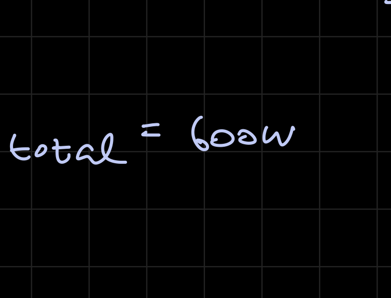
$I_{aA} = \frac{V_{an}}{Z_p} = \frac{200\angle 0^\circ}{100\angle 60^\circ} = 2\angle -60^\circ A_{rms}$

$I_{bB} = 2\angle -180^\circ\ A_{rms}$

$I_{cC} = 2\angle -300^\circ\ A_{rms} = 2\angle 60^\circ A_{rms}$

total avg. power: $P_{avg} = \frac{1}{2}Re\{VI^*\}$

in phase A :



$P_{avg})_A = \frac{1}{2}Re\{VI^*\}$

$= Re\{V_{eff}I_{eff}\}$

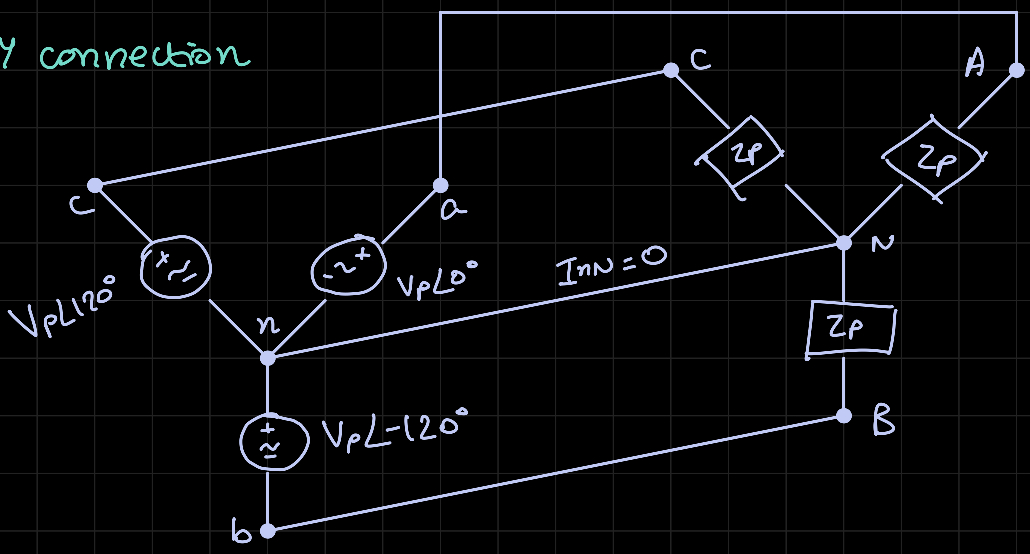
$= Re\{200\angle 0^\circ \times 2\angle -60^\circ\}$

$\Rightarrow Re\{400\angle -60^\circ\} \Rightarrow 400\cos(-60) = \underline{\underline{200W}}$

$P_{avg})_B = 200W$
$P_{avg})_C = 200W$
} $P_{avg})_{total} = 600W$

* lecture : 10 11/09/24

→ Y-Y connection



balanced load : all load are same

balanced src : all src magnitudes are equal

line : lines connecting load to src
aA cC
bB nN

Phase Voltages: $V_{AN} = V_{an}$, $V_{BN} = V_{bn}$, $V_{CN} = V_{cn}$

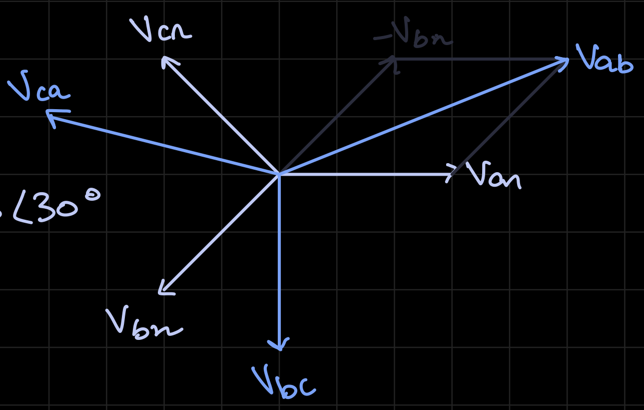
line voltages : V_{aA}, V_{bB}, V_{cC}

line currents : I_{aA}, I_{bB}, I_{cC}

phase currents : $I_{AN} = I_{aA}$, $I_{BN} = I_{bB}$, $I_{CN} = I_{cC}$

$V_{an} = V_p \angle 0°$
$V_{bn} = V_p \angle -120°$
$V_{cn} = V_p \angle -240°$

$V_{ab} = V_{an} - V_{bn}$



$$\underbrace{\begin{aligned} V_{ab} &= V_{an} - V_{bn} = \sqrt{3} V_p \angle 30° \\ V_{bc} &= \sqrt{3} V_p \angle -90° \\ V_{ca} &= \sqrt{3} V_p \angle -210° \end{aligned}}_{\text{line voltages}}$$

$$V_{line} = \sqrt{3} V_{phase}$$

line currents = phase currents

* Total Instantaneous Power

$$P_{total}(t) = p_A(t) + p_B(t) + p_C(t)$$

Classes & Attribs

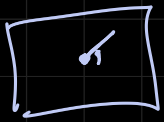
interface (entity)

- Bird
  - dmg (hits)
- Pig
  - health (hits)
- User
  - save stage
  - Score
  - level_status
- Level mng
  - max_score
  - no. of pigs
  - ↳ blocks
  - ↳ pigs
    - type (wood, glass, steel)

(entry point)
GAME
update score

Slingshot
angle
stretch

$V = 10$ W/A S,D



$V\cos\theta$, $V\sin\theta$

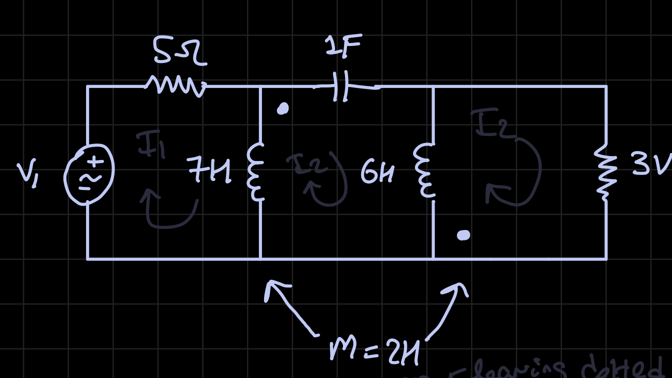
per second / rate

$-1 \to 0$

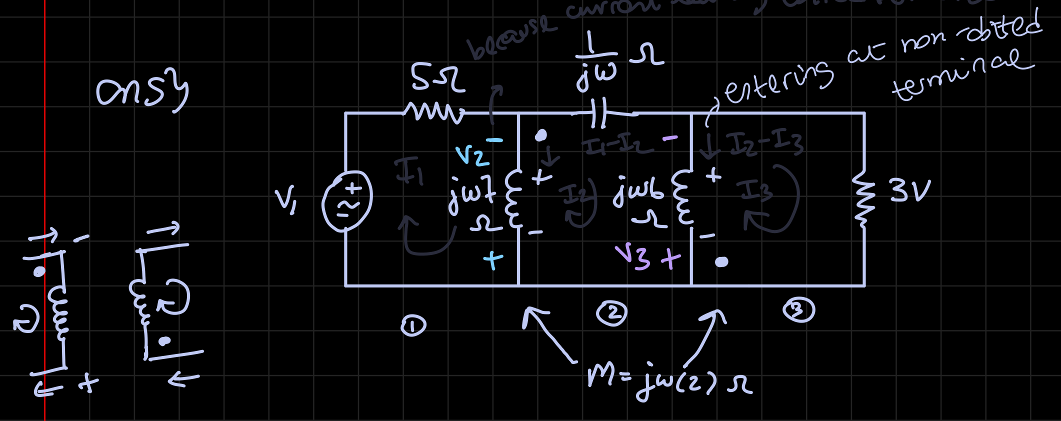
TODO : revise lecture 11 (missed due to SNS quiz)

* lecture 12:

eg3



ans}



① $-V_1 + 5I_1 - V_2 = 0$

$V_2 = (j\omega 7)(I_1 - I_2)(-1)$

because same sign of V_2 & V_3

$V_2 = -j\omega 7(I_1 - I_2) + j2\omega(I_2 - I_3)$

③ $3I_3 + V_3 = 0$

self + mutual inductance

same polarity

$V_3 = -j6\omega(I_2 - I_3) + j2\omega(I_1 - I_2)$

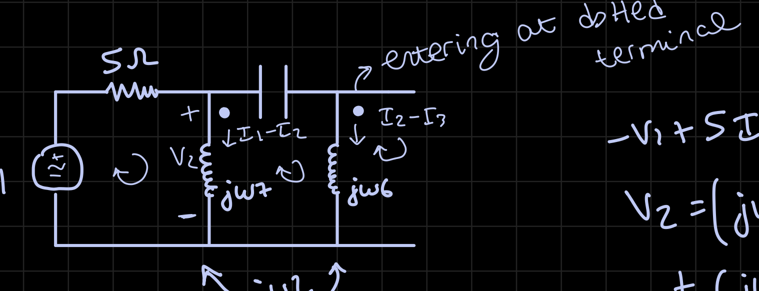
② $V_2 + \frac{1}{j\omega} I_2 - V_3 = 0$

$-j\omega 7(I_1 - I_2) + j2\omega(I_2 - I_3) + j6\omega(I_2 - I_3) - j2\omega(I_1 - I_2) + \frac{I_2}{j\omega} = 0$

$I_1(-j5\omega) + I_2(j17\omega) + I_3(-j8\omega) + \frac{I_2}{j\omega} = 0$

$I_1(-j5\omega) + I_2\left(j17\omega + \frac{1}{j\omega}\right) + I_3(-j8\omega) = 0$

eg3



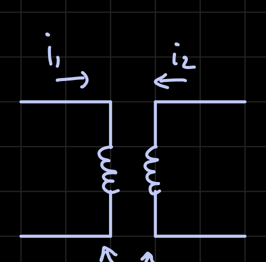
$-V_1 + 5I_1 + V_2 = 0$

$V_2 = (j\omega 7)(I_1 - I_2) + (j\omega 2)(I_2 - I_3)$

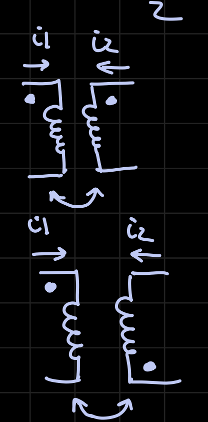
we donot need to care abt this sign if we use V_2, V_3 method.

* ENERGY STORED

$w(t) = \frac{1}{2} L(i(t))^2$ for only one inductor



$w(t) = \frac{1}{2} L(i_1)^2 + \frac{1}{2} L(i_2)^2 \pm M i_1(t) i_2(t)$



$\boxed{+ve}$ sign will occur iff both i_1 and i_2 are entering either dotted or undotted

$\boxed{-ve}$ sign iff both enter different (dotted / undotted)

• coupling coefficient (K)

$M \leq \sqrt{L_1 L_2}$

$K = \frac{M}{\sqrt{L_1 L_2}} \Rightarrow 0 \leq K \leq 1$

$\boxed{K \to 0}$ poor coupling or no coupling

$\boxed{K \to 1}$ strong coupling (very close to each other)

⇒ K depends on : distance btw coils ; size of coils ; ferrite core

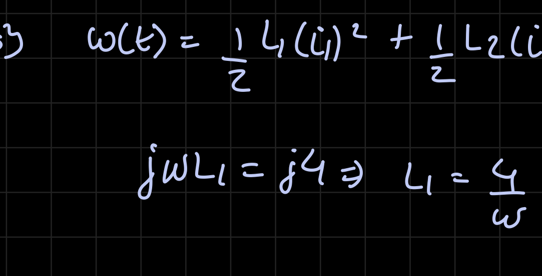
$y \propto \frac{1}{x}$ $\quad y \propto x$ $\quad y \propto x$

uses : wireless power transfer (inductively coupled)

- electric vehicle charging
- magsafe charging

eg3 let $i_s = 2\cos(10t)$ A. find total energy stored in network at $t = 0$ if $k = 0.6$ and

(a) xy terminals are open circuited
(b) xy are short circuited



Ans} $w(t) = \frac{1}{2} L_1(i_1)^2 + \frac{1}{2} L_2(i_2) \pm M i_1(t) i_2(t)$

$j\omega L_1 = j4 \Rightarrow L_1 = \frac{4}{\omega} = 0.4$ H

(a) if xy are open : $i_2 = 0$

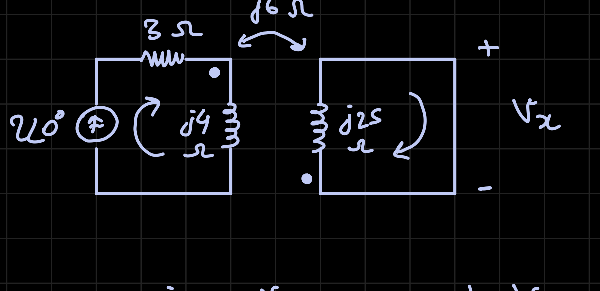
so, $w(t) = \frac{1}{2} \times 0.4 \times (2\cos 10t)^2$

$w(t) = 0.8\cos^2(10t)$

at $t = 0 \Rightarrow w(t) = 0.8$ J

(b) $w(t) = \frac{1}{2} L_1 \cdot i_1^2 + \frac{1}{2} L_2 i_2^2 - M \cdot i_1 \cdot i_2$

$i_1 = i_s = 2\angle 0°$



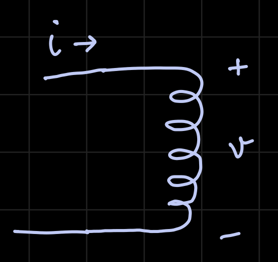
$i_2 = \frac{V_x}{j25}$ and $V_x = -j\omega M I_1$

$i_2 = \frac{-0.6(2\angle 0°)}{2.5} = -0.48$ A

$w(t=0) = \frac{1}{2}(0.4)(2^2) + \frac{1}{2}(\cdots$

* Lecture : 11 {Revision} [absent] 24/9/24

- **MODULE 5: Magnetically coupled circuits**



$$v = L\frac{di}{dt}$$

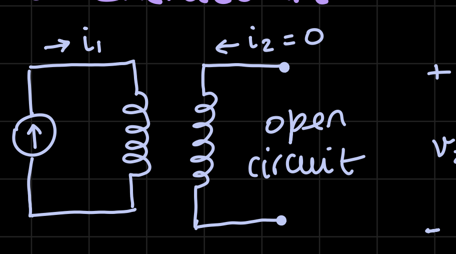
current → magnetic flux
$i \propto \phi$ — ①

magnetic flux → induced voltage
$v \propto \frac{d\phi}{dt}$ — ②

① and ② → $V = L\frac{di}{dt}$

for DC source : current is constant
hence $\frac{di}{dt} = 0$ and $\therefore v = 0$ (induced)

- **Mutual Inductance**

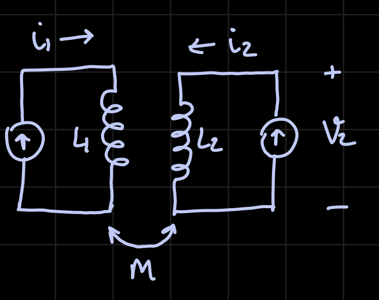


$$v_2 \propto \frac{di_1}{dt}$$

$$V_2 = \underset{\swarrow}{M}\frac{di_1}{dt}$$

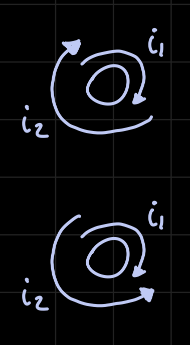
mutual inductance

L: self inductance



$$v_2 = \underbrace{L\frac{di_2}{dt}}_{\text{self inductance}} + \underbrace{M\frac{di_1}{dt}}_{\text{mutual inductance}}$$

* Additive /Substractive Property {two different coils}

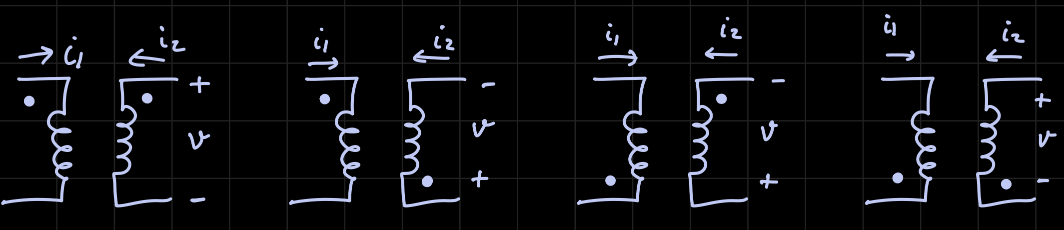


$$v_2 = L\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$v_2 = L\frac{di_2}{dt} - M\frac{di_1}{dt}$$

- **Dotted Notation**

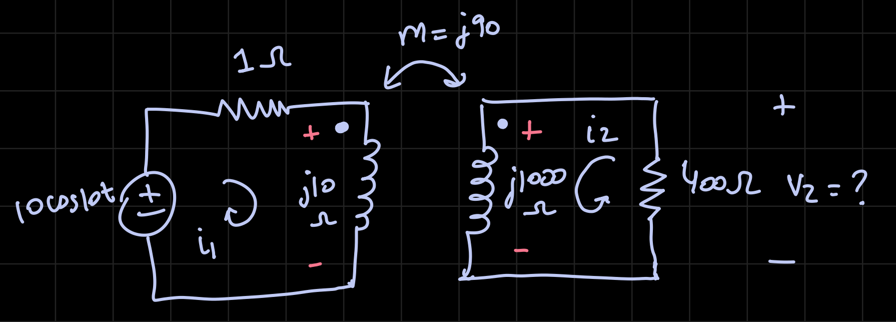
current entering $\underset{}{\_}$ terminal means (dotted/undotted)
+ve voltage reference at _



$$v = L_1\frac{di}{dt} + M\frac{di_2}{dt}$$ time domain

$$V = j\omega L_1 I_1 + j\omega M I_2$$ phasor domain

eg}



① $-10\angle 0° + I_1 + j10\, I_1 + j90\, I_2 = 0$

$I_1(1 + j10) + I_2(j90) = 10$

② $400 I_2 + j1000\, I_2 + j90\, I_1 = 0$

$I_1(j90) + I_2(400 + j1000) = 0$

CRAMMERS :

$$\Delta = \begin{vmatrix} 1+j10 & j1000 \\ j90 & 400+j1000 \end{vmatrix}$$

$= (1+j10)(400+j1000) + 90000$

$= 400 + j1000 + j4000 - 10.000 + 90.000$

$= j5000 + 80,400 = 80555\angle 3.550$

$$\Delta_1 = \begin{vmatrix} j1000 & 10 \\ 400+j1000 & 0 \end{vmatrix} = -4000 - j10.000$$
$= 10770\angle -111.8°$

$$\Delta_2 = \begin{vmatrix} 10 & 1+j10 \\ 0 & j90 \end{vmatrix} = j900 = 900\angle 90°$$

$$I_1 = \frac{\Delta}{\Delta_1} = -3.2 + 6.75j$$

full cramers part
wrong because I took
$M = L_2 = j1000$ above

$$D_0 = \begin{vmatrix} 1+j10 & j90 \\ j90 & 400+j1000 \end{vmatrix}$$

$= 400 + j1000 + j4000 - 10.000 + 8100$
$= j5000 - 1500 = 5220\angle 106.69°$

---

energy stored in the circuit

$$w(t) = \frac{1}{2}L_1 i_1(t)^2 + \frac{1}{2}L_2 i_2(t)^2$$
$$\pm M i_1(t) i_2(t)$$

+ : current entering same : • and • OR _ and _
- : current enteing different : • and _ OR _ and •

COUPLING COEFFICIENT (K)

$$K = \frac{M}{\sqrt{L_1 L_2}}$$: note ∌ $M \leq \sqrt{L_1 L_2}$

So, $0 \leq K \leq 1$