

tr5frbj 12/08/24

Circuit Theory and Devices

Lab: LT Spice (2nd week onwards)

book: Engineering Circuit Analysis (9th edition)

Relative grading

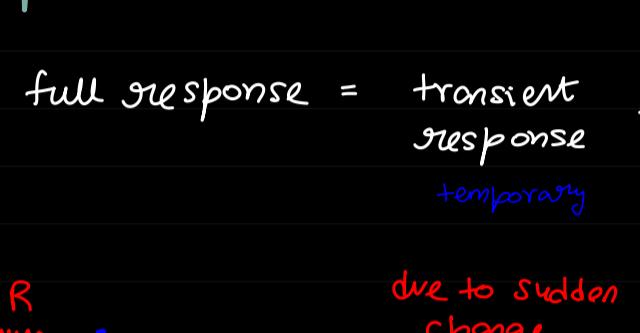
Labs	20%
Quiz	20%
Midsem	30%

Scientific calc

Course: 9 modules chapter 10 onwards
{continuation of BE3}

Lecture 1

⇒ Linear Circuits



System is linear if

$$Kx \rightarrow [\text{sys}] \rightarrow Ky$$

where K is constant

"R" Resistor: linear element ✓
 $V = iR$

"L" Inductor: linear element ✓
 $V = L \frac{di}{dt}$

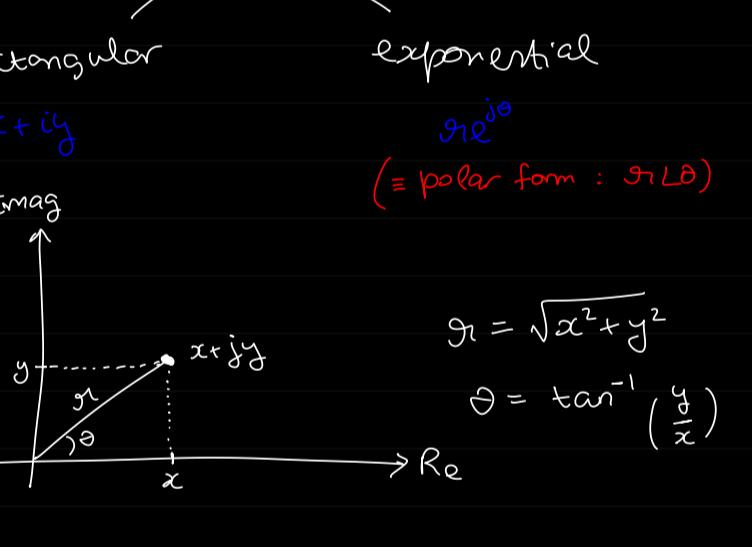
* Linear Electric Circuits:

consists of ⇒

① $R, L, C \rightarrow$ linear elements

② Independent voltage & current sources

③ Linear dependent sources



Note: diode and transistors are non-linear elements

* Response of a linear circuit

① full response = transient response + steady state

transient temporary persists $t \rightarrow \infty$

due to sudden change due to source also called natural response

depends on R, L, C depends on current/voltage source

$$KVL : -V_s + iR + L \frac{di}{dt} = 0$$

also called forced response

depends on current/voltage source

$$i_{\text{final}} = i_{\text{transient}} + i_{\text{steady state}}$$

$$\text{as } t \rightarrow \infty, i_{\text{final}} = i_{\text{steady state}}$$

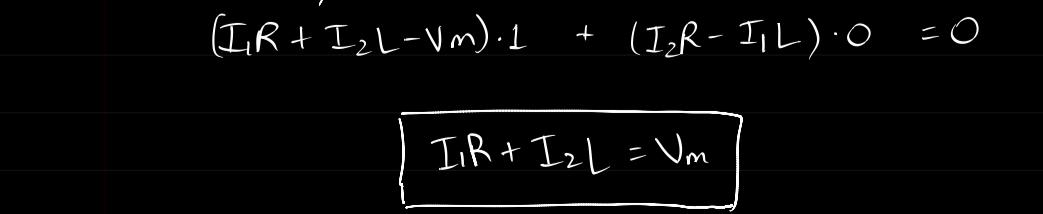
* Sinusoids & complex numbers

Sinusoidally varying voltage source

$$V_s(t) = V_m \cos(\omega t)$$

$$V_s(t) = V_m \sin(\omega t)$$

$$V_s(t) = V_m \sin(\omega t + \phi)$$



$$\Rightarrow V_m \cos(\omega t) = iR + L \frac{di}{dt}$$

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$$\Rightarrow V_m \cos(\omega t) = iR + L \frac{d}{dt}(I_1 \cos$$

* Section 10.1

(a) $Q_1 y \quad 5\sin(5t - 9^\circ)$

 $t=0 \Rightarrow 5\sin(-9^\circ) = -5\sin(9^\circ) = -0.782$

$t=0.01 \Rightarrow 5\sin(0.05 - 9^\circ) = -5\sin(8.95^\circ) \quad X$

\Downarrow
radians

$5\sin\left(\frac{0.05 \times 80}{\pi} - 9^\circ\right) = 5\sin(2.86^\circ - 9^\circ)$
 $\Rightarrow -5\sin(6.14^\circ) = -0.534$

$t=0.15 \Rightarrow 5\sin(0.5 - 9^\circ) = 5\sin(28.64^\circ - 9^\circ) = 5\sin(19.64^\circ)$
 $\Rightarrow 1.6805$

(b) $4\cos 2t$

$t=0 \Rightarrow 4\cos(0) = 4$

$t=1 \Rightarrow 4\cos(2) = 3.997$

$t=1.5 \Rightarrow 4\cos(3) = 3.994$

(c) $3.2 \cos(6t + 15^\circ)$

$t=0 \Rightarrow 3.2 \cos(15^\circ) = 3.09$

$t=0.01 \Rightarrow 3.2 \cos(0.06 + 15^\circ) = 3.2 \cos(3.43^\circ + 15^\circ)$
 $\Rightarrow 3.2 \cos(18.43^\circ)$
 $\Rightarrow 3.035$

$t=0.1 \Rightarrow 3.2 \cos(0.6 + 15^\circ) = 3.2 \cos(34.3^\circ + 15^\circ)$
 $= 3.2 \cos(49.3^\circ)$
 $= 2.086$

Q2) (a) $300\sin(628t) \rightarrow 300 \cdot \cos(628t - 90^\circ)$

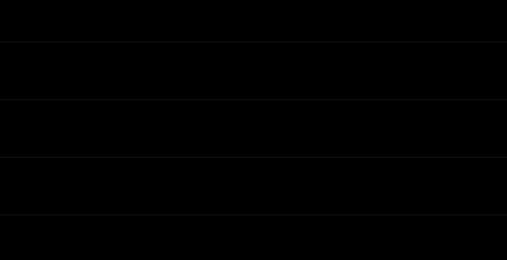
$4\sin(3\pi t + 30^\circ) \rightarrow 4\cos(3\pi t - 60^\circ)$

$14\sin(50t - 8^\circ) - 10\cos 50t \rightarrow$

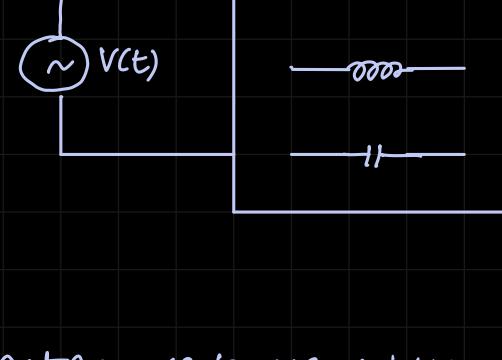
Q3) $V_L = 10\cos(10t - 45^\circ)$

(a) $i_L = 5\cos 10t$

-45°



⇒ AC Power Analysis {Module: 3}



$$\cos(A) \cos(B) =$$

$$\frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

Instantaneous power: $p(t) = v(t) \cdot i(t)$

$$v(t) = V_m \cos(\omega t + \theta)$$

$$i(t) = I_m \cos(\omega t + \phi)$$

$$p(t) = V_m \cos(\omega t + \theta) \cdot I_m \cos(\omega t + \phi)$$

$$p(t) = \underbrace{\frac{V_m I_m}{2} \cos(\theta - \phi)}_{\text{constant}} + \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{\text{twice frequency (harmonic)}}$$

• Average Power

$$P_{avg} = \frac{1}{T} \int_0^T p(t) dt$$

$$= \frac{1}{T} \int_0^T \frac{V_m I_m}{2} \cos(\theta - \phi) dt + \underbrace{\frac{1}{T} \int_0^T \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi) dt}_{0}$$

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)$$

* Avg. Power absorbed

• by a purely resistive element

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)$$

here, phase diff = 0°

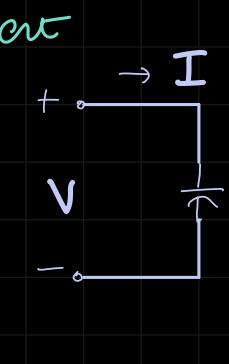


$$P_{avg} = \frac{V_m \cdot I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

• by a purely inductive element

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)$$

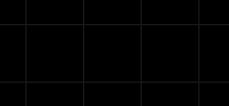
here, phase diff = 90°



$$P_{avg} = 0 \text{ for inductor}$$

• by a purely capacitive element

$$P_{avg} = \frac{V_m \cdot I_m}{2} \cos(-90^\circ)$$



* Lecture: 6

$$v(t) = V_m \cos(\omega t + \phi) \quad \leftarrow \text{Revision}$$

$$\downarrow$$

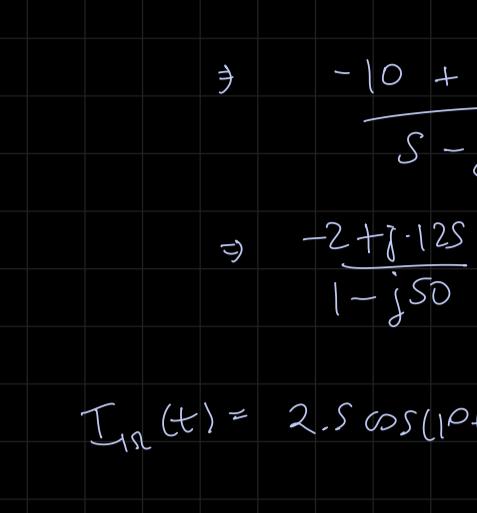
$$v(t) = R \Re \{ V_m e^{j\omega t} e^{j\phi} \}$$

$$\downarrow -e^{j\omega t}$$

$$V_m e^{j\phi} = V_m \angle \phi$$

exponential polar

• eq)

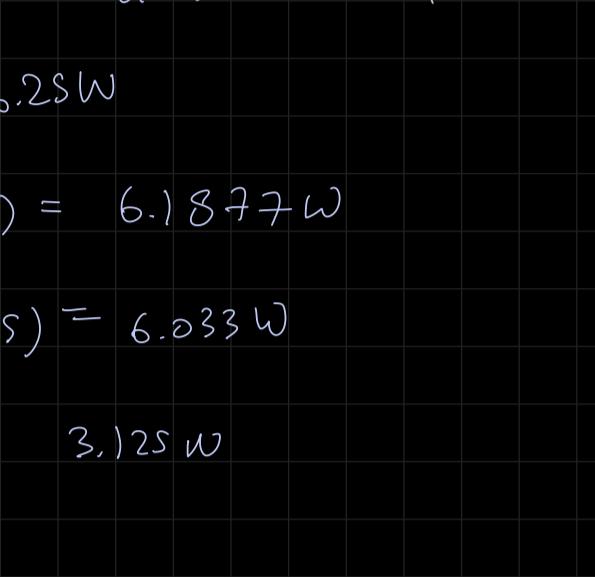


① find power delivered to each element at $t = 0, 10, 20 \text{ ms}$

② find P_{avg} to each element

(ans) convert to freq. domain

$$V_{1\text{R}} = V_{u\text{R}} + V_c$$



$$\textcircled{1} \quad P_{inst} = \frac{V_m I_m \cos(\theta - \phi)}{2} + \frac{V_m I_m}{2} \cos(2\omega t + \phi + \theta)$$

$$\textcircled{1} \quad I_1 = -2.5 \times \left(\frac{4 - j2.5 \times 10^4}{1 + j - j2.5 \times 10^4} \right) \approx -2.5 \text{ A}$$

$$\Rightarrow \frac{-10 + j62.5 \times 10^4}{5 - j2.5 \times 10^4} \rightarrow \frac{-2 + j12.5}{1 - j50}$$

$$\Rightarrow \frac{-2 + j12.5}{1 - j50} \times \frac{1 + j50}{1 + j50} =$$

$$I_{1\text{R}}(t) = 2.5 \cos(10t)$$

$$V_{1\text{R}} = (I_{1\text{R}})(2 \Omega) \Rightarrow V_{1\text{R}}(t) = 2.5 \cos(10t)$$

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$P_{1\text{R}}(t) = 2.5 \cos(10t) \cdot 2.5 \cos(10t) \text{ W}$$

$$P_{1\text{R}}(t=0) = 6.25 \text{ W}$$

$$P_{1\text{R}}(t=20 \text{ ms}) = 6.033 \text{ W}$$

$$(P_{avg})_{1\text{R}} = 3.125 \text{ W}$$

\Rightarrow P delivered to 4 ohm

$$P_{4\text{R}}(t) = V_{u\text{R}}(t) \times I_{u\text{R}}(t)$$

$$\textcircled{2} \quad I_{4\text{R}} = I_s \times \frac{1}{5 - j2.5 \times 10^4} = 10^{-4} \angle 90^\circ \text{ A}$$

$$V_{u\text{R}} = 4 \times I_{4\text{R}} = 4 \times 10^{-4} \angle 90^\circ \text{ V}$$

convert to time domain

$$P_{4\text{R}}(t) = 2 \times 10^{-8} + 2 \times 10^{-8} \cos(20t) \text{ W}$$

$$t=0^- \quad 4 \times 10^{-8} \text{ W}$$

$$t=10 \text{ ms} = 3.96 \times 10^{-8} \text{ W}$$

$$P_{4\text{R}}(t=20 \text{ ms}) = 3.87 \times 10^{-8} \text{ W}$$

$$(P_{avg})_{4\text{R}} = 2 \times 10^{-8} \text{ W}$$

\Rightarrow P delivered to capacitor

$$P_c(t) = V_c(t) \cdot I_c(t)$$

$$\textcircled{3} \quad (P_{avg})_c = 0 \quad \text{since phase diff} = 90^\circ$$

$$I_c = I_2 = 10^{-4} \angle 90^\circ \text{ A} \Rightarrow I_c(t) = 10^{-4} \cos(10t + 90^\circ)$$

$$V_c = (-j2.5 \times 10^{-4}) \times 10^{-4} \angle 90^\circ$$

$$-j2.5 \times 10^{-4} \times (+j10^{-4}) = 2.5 \times 10^{-8} \text{ V}$$

$$P_c(t) = 1.25 \times 10^{-8} \cos(20t + 90^\circ) \text{ W}$$

$$t=0 \Rightarrow P=0 \text{ W}$$

$$t=10 \text{ ms} \Rightarrow P=2.48 \times 10^{-8} \text{ W}$$

$$(P_{avg})_c = 1.25 \times 10^{-8} \text{ W}$$

$$\text{Note: we cannot multiply } I_c \text{ and } V_c \text{ in phasor form and then convert to time domain for getting } P_c \text{ (power) because power does not have a phasor part. It is a real value.}$$

* Correct or not?

$$\text{depends on } \times \quad \textcircled{1} \quad P_{1\text{R}}(t) + P_{4\text{R}}(t) + P_c(t) = \text{constant 1}$$

$$\checkmark \quad \textcircled{2} \quad P_{avg\ 1\text{R}} + P_{avg\ 4\text{R}} + P_{avg\ c} = \text{constant 2} \quad \text{= } P_{avg\ \text{source}}$$

active sign convention

passive sign convention

* Maximum Power Transfer Theorem

maximum power delivered to load when

$$Z_L = Z_S^*$$

complex conjugate of Z_S



\Rightarrow Impedance Matching

$$\text{impedance matching circuit}$$

$$\text{original source } \xrightarrow{\sim} \text{load } \xrightarrow{\sim} \text{impedance matching circuit}$$

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$$\text{original source }$$

* Lecture: 7

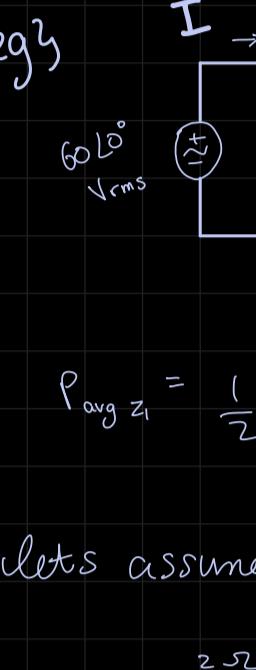
- Instantaneous Power: $p(t) = v(t)i(t)$
- Average Power: $P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = \frac{1}{2} R_{eff} \{VI^*\}$
- $P_{avg, sources} = \sum P_{avg, elements}$

* Note: $Z = R + jX$
 \downarrow resistive reactive

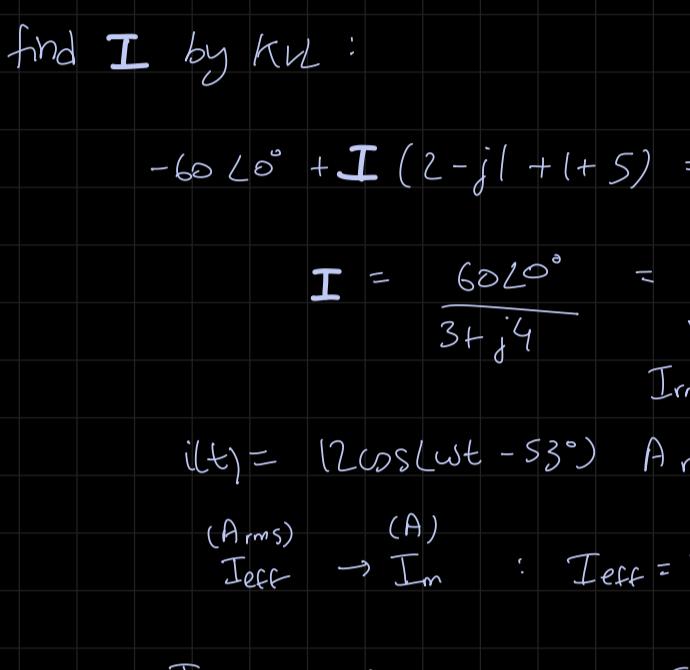
$P_{avg, resistive} : P \neq 0$

$P_{avg, reactive} : P = 0$

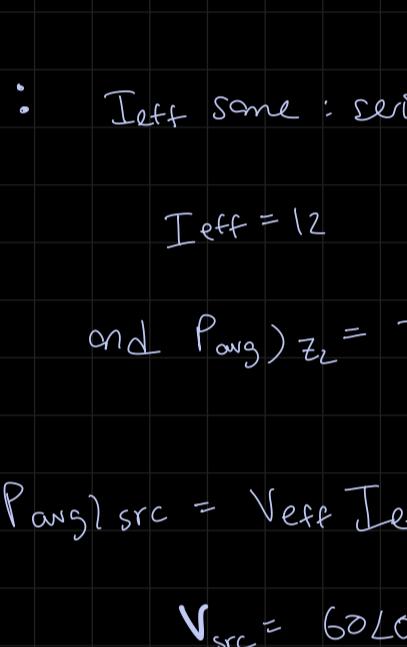
* Max Power Transfer:



Circuit has max. power when $Z_L = Z_s^*$ complex conjugate



↓ apply Thevenin's theorem



$$Z_L = Z_{th}$$

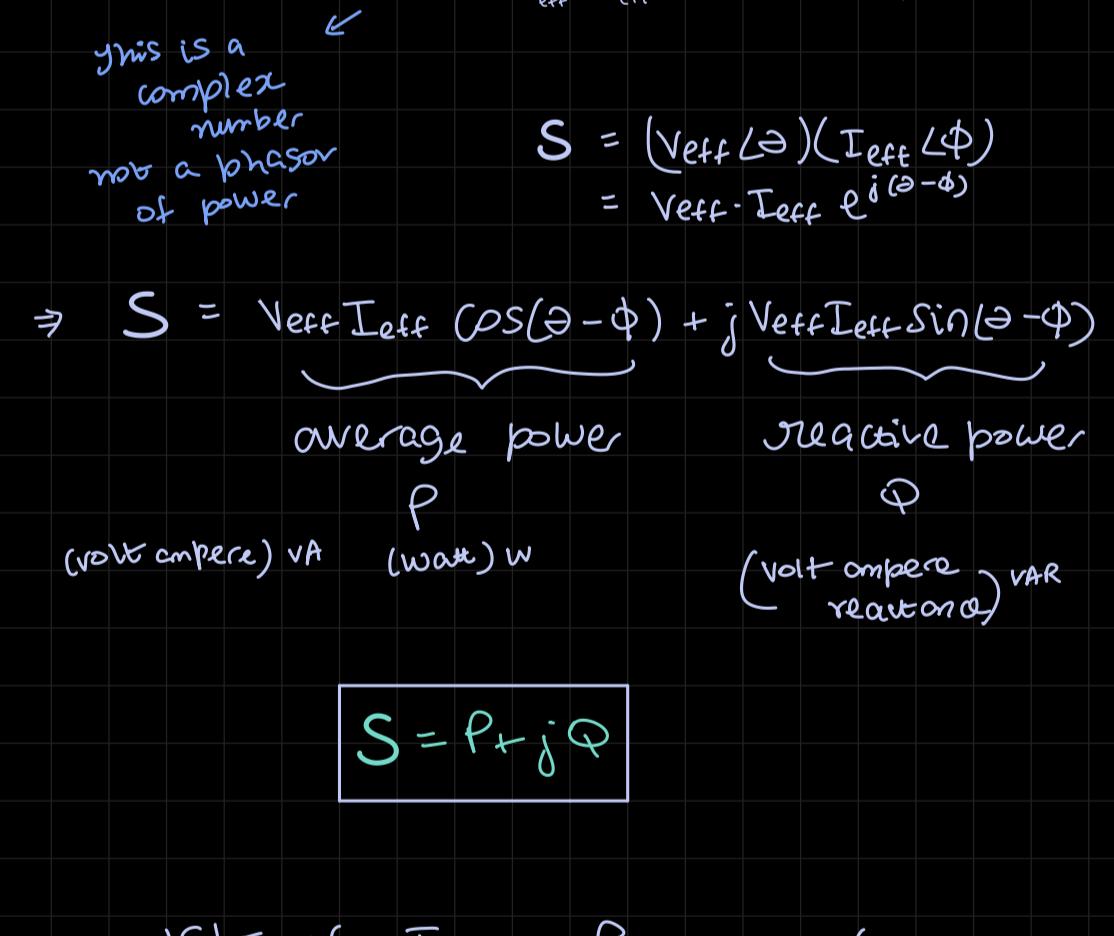
* Power Factor

$$PF = \cos(\theta - \phi) = \frac{\text{Power active}}{\text{Power apparent}}$$

angle of voltage phasor
↑ angle of current phasor

for purely resistive load: $PF = 1$ {Max} $\theta - \phi = 0^\circ$
 for purely reactive load: $PF = 0$ {min}

Note $\rightarrow PF = 0.5$ leading \rightarrow capacitive $(\theta - \phi) < 0^\circ$
 $PF = 0.5$ lagging \rightarrow inductive $(\theta - \phi) > 0^\circ$



(Ans) $P_{avg, Z_1} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} R_{eff} \{VI^*\}$

voltage across Z_1 , not the src

lets assume Z_1 :

$$\frac{2-j1}{2+j1} = \frac{-j2}{1+j4} \quad \text{so, } P_{avg, Z_1} = \text{Power delivered to } 2\Omega \text{ resistor}$$

$$= I_{eff}^2 R$$

find I by KVL:

$$-60 \angle 0^\circ + I(2-j1 + 1+j5) = 0$$

$$I = \frac{60 \angle 0^\circ}{3+j4} = 12 \angle -53.13^\circ \text{ Arms}$$

$$i(t) = 12 \cos(\omega t - 53.13^\circ) \text{ A rms}$$

$$\frac{(A_{rms})}{I_{eff}} \rightarrow \frac{(A)}{I_m} : I_{eff} = I_m \Rightarrow I_m = I_{eff} \sqrt{2}$$

$$I_{eff} = 12 \text{ Arms} \Rightarrow I_m = 12\sqrt{2} \text{ A}$$

$$i(t) = 12\sqrt{2} \cos(\omega t - 53^\circ) \text{ A}$$

$$P_{avg, Z_1} = (12)^2 \times 2 = 288 \text{ W}$$

$$\text{note: } P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$$

$$S = V_{eff} I_{eff} \cos(\theta - \phi) \quad \text{volt ampere (VA)}$$

$$S = (V_{eff} \angle \theta)(I_{eff} \angle \phi) \quad \text{volt ampere (VA)}$$

$$= V_{eff} \cdot I_{eff} e^{j(\theta-\phi)}$$

$$S = P + jQ \quad \text{volt ampere reactive (VAR)}$$

$$|S| = \sqrt{P^2 + Q^2} = P_{apparent} \quad \text{volt amp}$$

$$P_{apparent} = V_{eff} \cdot I_{eff} = (60)(12) = 720 \text{ W}$$

$$Q_{apparent} = V_{eff} I_{eff} \sin(\theta - \phi) = 720 \sin(53.13^\circ) = 480 \text{ VAR}$$

$$P_{avg, source} = V_{eff} I_{eff} \cos(\theta - \phi)$$

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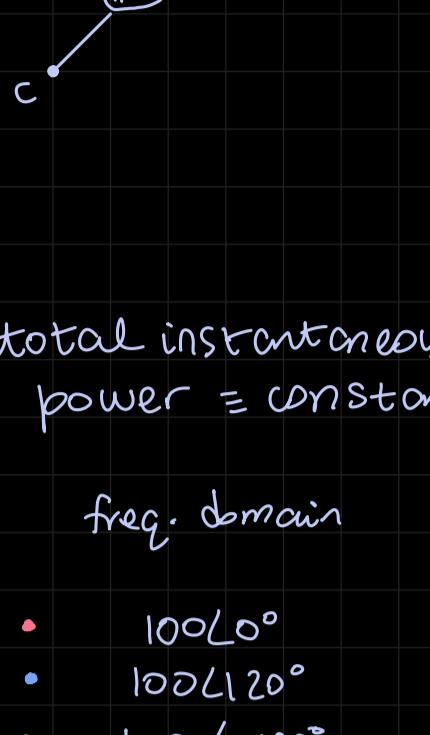
$$|S| = \sqrt{P^2 + Q^2} = P_{apparent} \quad \text{volt amp}$$

$$P_{apparent} = V_{eff} \cdot I_{eff} = (60$$

* Lecture - 8

• Polyphase Circuits (Chp 12)

⇒ Three Phase Source



$$V_{an} = 100\angle 0^\circ \text{ V}$$

$$V_{bn} = 100\angle 120^\circ \text{ V}$$

$$V_{cn} = 100\angle -120^\circ \text{ V}$$

if $|V_{an}| = |V_{bn}| = |V_{cn}|$
 & $V_{an} + V_{bn} + V_{cn} = 0$
 then it is a

Balanced Source

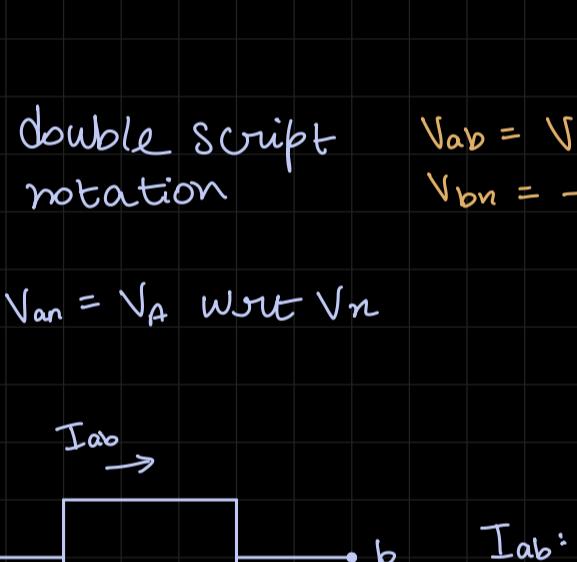
total instantaneous power = constant

freq. domain

- $100\angle 0^\circ$
- $100\angle 120^\circ$
- $100\angle -120^\circ$

time domain

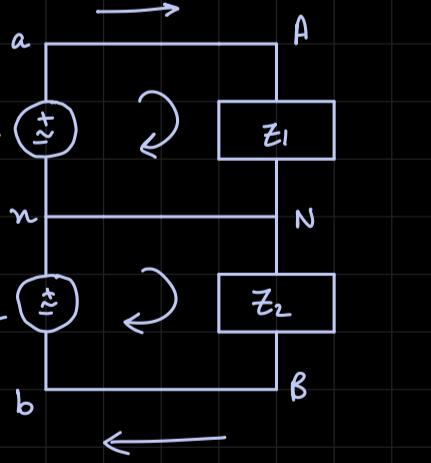
- $100 \cos(\omega t)$
- $100 \cos(\omega t + 120^\circ)$
- $100 \cos(\omega t - 120^\circ)$



total $p(t) \rightarrow \text{constant}$

* Single Phase Three-wire Source

≡ Two phase source



$$V_1 = V_m \angle 0^\circ$$

$$V_{an} = V_1 = V_m \angle 0^\circ$$

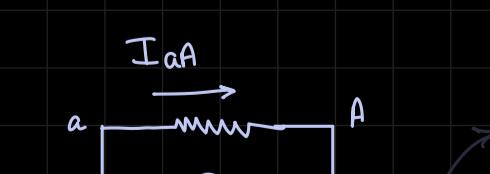
$$V_{bn} = -V_1 = V_m \angle 0^\circ - 180^\circ$$

$$V_{bn} \leftarrow \xrightarrow{180^\circ} V_{an}$$

double script notation

$$\begin{aligned} V_{ab} &= V_{an} - V_{bn} = V_{an} + V_{nb} \\ V_{bn} &= -V_{nb} \end{aligned}$$

$$V_{an} = V_A \text{ wrt } V_n$$



$$I_{aa} = \frac{V_1}{Z_1}$$

$$I_{bb} = \frac{V_1}{Z_2}$$

$$I_{nn} = -I_{aa} + I_{bb}$$

$$\Rightarrow \frac{V_1}{Z_1} - \frac{V_1}{Z_2}$$

Assume $Z_1 = Z_2$

$$I_{nn} = \frac{V_1}{Z_1} - \frac{V_1}{Z_1} = 0$$

when both srcs &
and loads are equal

Balanced Load : current in neutral line is equal to zero.

all terms are phasors

even with resistance,

$$I_{nn} = 0 = I_{bb} - I_{aa}$$

due to

① Balanced Load

≡ Symmetry

* Lecture: 9

09/09/29

time domain freq. domain

$$\begin{array}{ll} \sqrt{V_p} \cos(\omega t + \theta^\circ) & \sqrt{V_p} L^{\theta^\circ} \\ \sqrt{V_p} \cos(\omega t + \theta^\circ - 180^\circ) & \sqrt{V_p} L^{\theta^\circ - 180^\circ} \end{array}$$

We cannot directly say,

$$P_{avg} = \sqrt{I} \quad \times$$

$$\text{but } P_{avg} = \frac{1}{2} \operatorname{Re}\{ \sqrt{I} I^* \} \quad \checkmark$$

$$\begin{aligned} \textcircled{2} \quad a) \quad I_{AB} &= \frac{115 L^{\theta^\circ} + 115 L^{\theta^\circ}}{2(10 + j^2)} = \frac{230 L^{\theta^\circ}}{2(10 + j^2)} \\ &= 11.27 / (\theta^\circ - 11.3^\circ) A \end{aligned}$$

$$\phi = \theta - 11.3^\circ$$

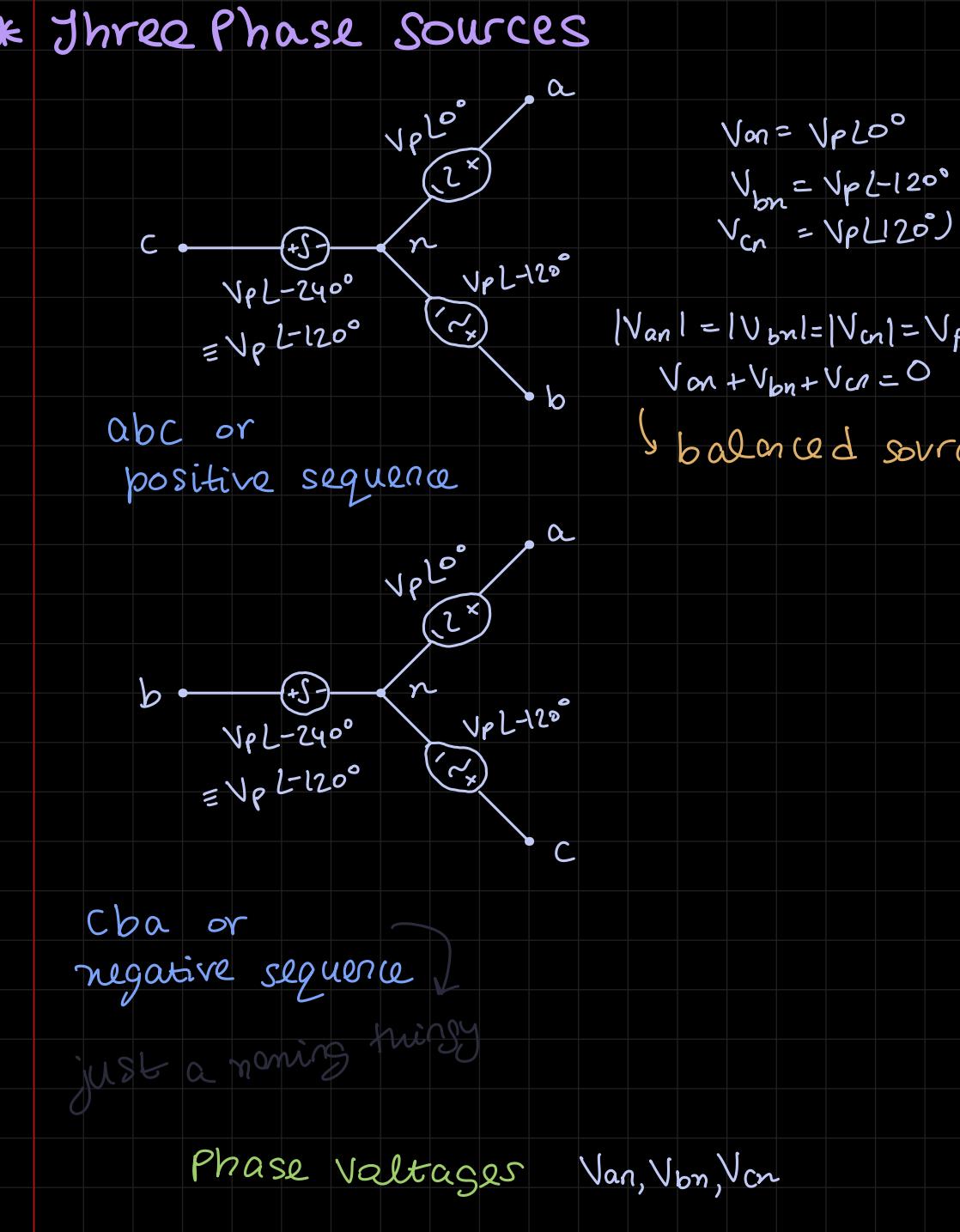
$$PF = \cos(\theta - \phi + 11.3^\circ) = 0.98 \text{ lagging}$$

because PF angle is true

$$b) \quad PF = 1 \Rightarrow \cos(\theta - \phi) = 1 \Rightarrow \theta - \phi = 0$$

$$\Rightarrow \theta = \phi$$

$$f = 50 \text{ Hz}$$



Note that this is still a balanced load, so we can remove MN since $I_{MN} = 0$

using complex power

$S = \text{complex power of total load}$

$$S = \frac{1}{2} \sqrt{I} I^*$$

$$PF = \frac{\operatorname{Re}\{ S \}}{|S|} = 1 \Rightarrow \operatorname{Re}\{ S \} = |S|$$

so, $\operatorname{Im}\{ S \} = 0$ here

$$S = \frac{1}{2} V_{an} I_{an}^* + \frac{1}{2} V_{nb} I_{nb}^* + \frac{1}{2} V_{ab} I_{ab}^*$$

$$\Rightarrow \frac{1}{2} V_{an} \left(\frac{V_{an}}{Z_p} \right)^* + \frac{1}{2} V_{nb} \left(\frac{V_{nb}}{Z_p} \right)^* + \frac{1}{2} V_{ab} \left(\frac{V_{ab}}{Z_p} \right)^*$$

$$\Rightarrow \frac{1}{2} \frac{|V_{an}|^2}{|Z_p|^2} Z_p + \frac{1}{2} \frac{|V_{nb}|^2}{|Z_p|^2} Z_p + \frac{1}{2} \frac{|V_{ab}|^2}{|Z_p|^2} Z_p$$

$$\Rightarrow \frac{115^2}{10^4} (10 + j^2) + \frac{1}{2} \left(\frac{230^2}{(j\omega C)^2} \right) \left(\frac{-j}{\omega C} \right) = \frac{1}{j\omega C} \times \frac{-1}{j\omega C}$$

$$\Rightarrow 115^2 \left(\frac{10 + j^2}{10^4} - \frac{1}{j\omega C} \right)$$

$$\Rightarrow 2 \times 115^2 \left(\frac{j}{10^4} - \frac{1}{j\omega C} \right)$$

$$\Rightarrow 2 \times 115^2 \left(\frac{1}{10^4} - \omega C \right) = 0$$

$$\frac{1}{10^4} - 100\pi C = 0$$

$$C = \frac{1}{10400\pi} = 30.6 \mu F \quad \checkmark$$

* Three Phase Sources



$$V_{an} = V_p L^{\theta^\circ}$$

$$V_{bn} = V_p L^{-120^\circ}$$

$$V_{cn} = V_p L^{120^\circ}$$

$$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$$

$$V_{an} + V_{bn} + V_{cn} = 0$$

balanced source

abc or positive sequence

cba or negative sequence

just a naming thingy

Phase Voltages V_{an}, V_{bn}, V_{cn}

Line-to-line Voltages V_{ab}, V_{bc}, V_{ca} OR line voltages

$$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$$

$$V_{ab} = \sqrt{3} V_p L^{30^\circ}$$

$$V_{bc} = \sqrt{3} V_p L^{-90^\circ}$$

$$V_{ca} = \sqrt{3} V_p L^{-210^\circ}$$

$$I_{aa} = \frac{V_{an}}{Z_p} = \frac{200 L^{\theta^\circ}}{100 L^{60^\circ}} = 2 L^{-60^\circ} A_{rms}$$

$$I_{bb} = 2 L^{-180^\circ} A_{rms}$$

$$I_{cc} = 2 L^{-300^\circ} A_{rms} = 2 L^{60^\circ} A_{rms}$$

$$\text{total avg. power: } P_{avg} = \frac{1}{2} \operatorname{Re}\{ \sqrt{I} I^* \}$$

in phase A:

$$P_{avg,A} = \frac{1}{2} \operatorname{Re}\{ \sqrt{I} I^* \}$$

$$= \operatorname{Re}\{ \sqrt{I_{eff}} I_{eff}^* \}$$

$$= \operatorname{Re}\{ 200 L^{\theta^\circ} \times 2 L^{-60^\circ} \}$$

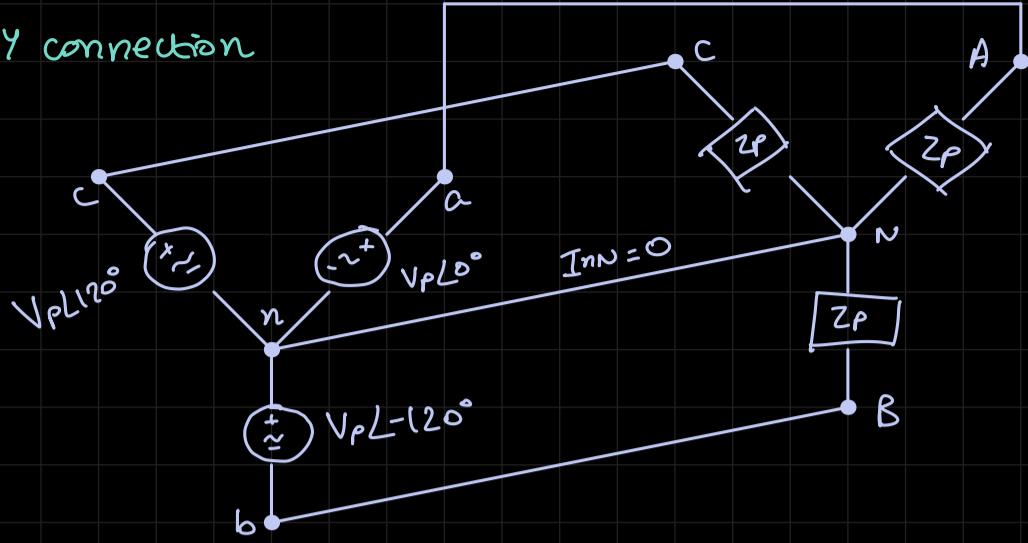
$$\Rightarrow \operatorname{Re}\{ 400 L^{-60^\circ} \} \Rightarrow 400 \cos(-60^\circ) = 200 W$$

$$P_{avg,B} = 200 W$$

$$P_{avg,C} = 200 W$$

$$P_{avg,\text{total}} = 600 W$$

→ Y-Y connection



balanced load: all load are same

balanced src: all src magnitudes are equal

line: lines connecting load to src

aa cC

bb nn

Phase Voltages: $V_{AN} = V_{an}$, $V_{BN} = V_{bn}$, $V_{CN} = V_{cn}$ line voltages: V_{ab} , V_{bc} , V_{ca} line currents: I_{aA} , I_{bB} , I_{cC} phase currents: $I_{AN} = I_{aA}$, $I_{BN} = I_{bB}$, $I_{CN} = I_{cC}$

$$V_{an} = V_p L 0^\circ$$

$$V_{bn} = V_p L -120^\circ$$

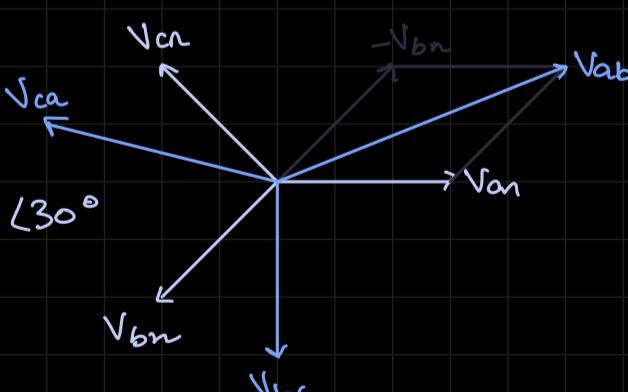
$$V_{cn} = V_p L -240^\circ$$

$$V_{ab} = V_{an} - V_{bn}$$

$$V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_p L 30^\circ$$

$$V_{bc} = \sqrt{3} V_p L -90^\circ$$

$$V_{ca} = \sqrt{3} V_p L -210^\circ$$



line voltages

$$V_{\text{line}} = \sqrt{3} V_{\text{phase}}$$

line currents = phase currents

* Total Instantaneous Power

$$P_{\text{total}}(t) = P_A(t) + P_B(t) + P_C(t)$$

Classes & Attribs

interface (entity)

~~ ~~~

Bird

↓

dmg
(chits)

Pig

↓

health
(units)

User

↓

save
stage

level mg

blocks

pigs

type

→ max-score

→ no. of pigs

center point

Game

update score

(wood

glass

steel)

Score

level-status

Slingshot

angle

stretch

↙

b°



$$v = 10 \text{ m/s}$$

0°

90°

S, P

$$\overline{\sqrt{v \cos \theta}, \sqrt{v \sin \theta}}$$

per second

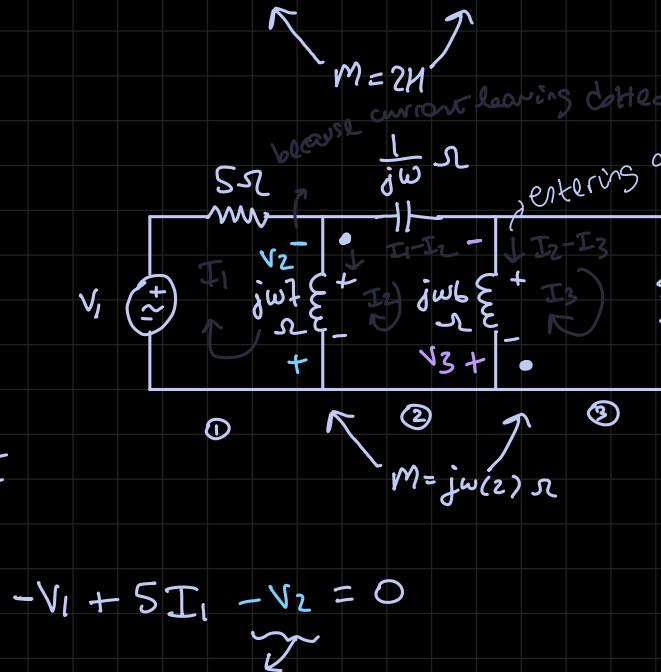
/ rate

-l → 0

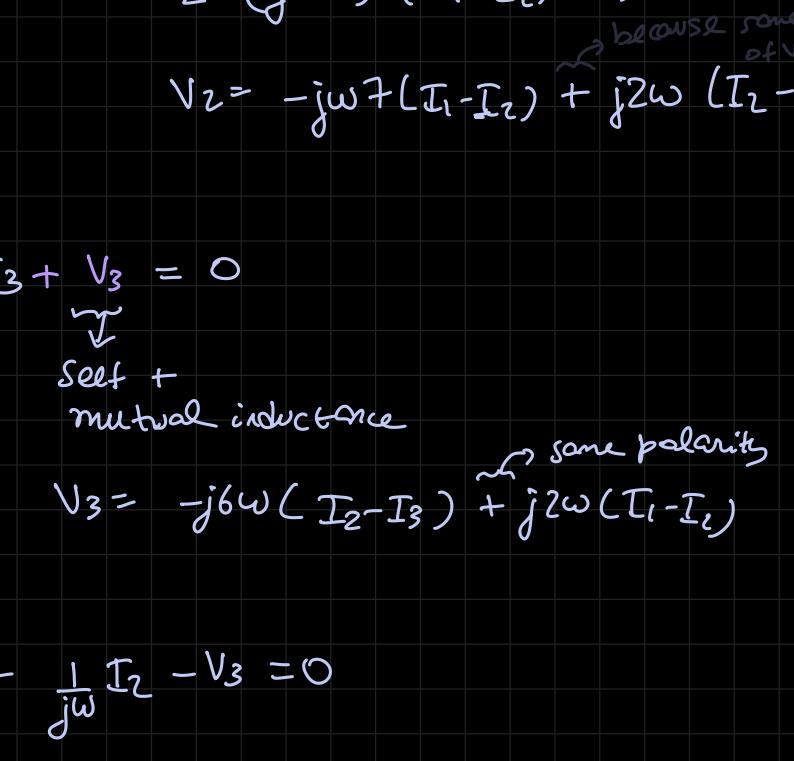
* TODO: revise lecture 11 (missed due to SNS quiz)

* Lecture 12:

eg 3



ans 3



$$\textcircled{1} \quad -V_1 + 5I_1 - V_2 = 0$$

$$V_2 = (j\omega 7)(I_1 - I_2)(-1)$$

$$V_2 = -j\omega 7(I_1 - I_2) + j2\omega(I_2 - I_3)$$

$$\textcircled{3} \quad 3I_3 + V_3 = 0$$

Self + mutual inductance

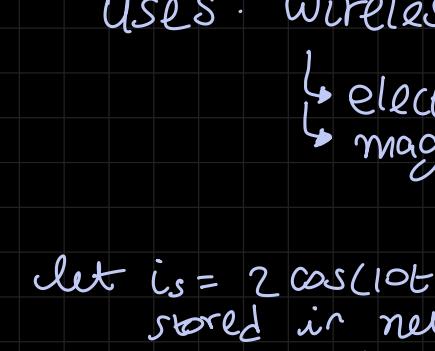
$$V_3 = -j6\omega(I_2 - I_3) + j2\omega(I_1 - I_2) \quad \text{some polarity}$$

$$\textcircled{2} \quad V_2 + \frac{1}{j\omega} I_2 - V_3 = 0$$

$$-j\omega 7(I_1 - I_2) + j2\omega(I_2 - I_3) + j6\omega(I_2 - I_3) - j2\omega(I_1 - I_2) + \frac{I_2}{j\omega} = 0$$

$$I_1(-j5\omega) + I_2(j17\omega) + I_3(-j8\omega) + \frac{I_2}{j\omega} = 0$$

eg 3



$$-V_1 + 5I_1 + V_2 = 0$$

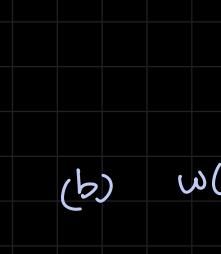
$$V_2 = (j\omega 7)(I_1 - I_2)$$

+ (j\omega 2)(I_2 - I_3)

We don't need to care about this sign if we use V_2, V_3 method.

* ENERGY STORED

$$W(t) = \frac{1}{2} L(i(t))^2 \quad \text{for only one inductor}$$



$$W(t) = \frac{1}{2} L(i_1)^2 + \frac{1}{2} L(i_2)^2 \pm M i_1(t) i_2(t)$$

[+ve] sign with occur iff both i_1 and i_2 are entering either dotted or undotted

[-ve] sign iff both enter different (dotted/undotted)

- Coupling coefficient (K)

$$M \leq \sqrt{L_1 L_2}$$

$$K = \frac{M}{\sqrt{L_1 L_2}} \Rightarrow 0 \leq K \leq 1$$

$K \rightarrow 0$

poor coupling or no coupling

$K \rightarrow 1$

Strong coupling (very close to each other)

$\Rightarrow K$ depends on: distance; size; ferrite b/w coils; of coils core

$$y \propto \frac{1}{x}$$

$$y \propto x$$

$$y \propto x$$

uses: wireless power transfer \leftarrow (inductively coupled)

↳ electric vehicle charging

↳ magSafe charging

eg 3 let $i_s = 2 \cos(10t)$ A. find total energy stored in network at $t=0$ if $K=0.6$ and

(a) if x_y terminals are open circuited

(b) if x_y are short circuited

$$\text{Ans 3} \quad W(t) = \frac{1}{2} L_1(i_1)^2 + \frac{1}{2} L_2(i_2)^2 \pm M i_1(t) i_2(t)$$

$$j\omega L_1 = j^4 \Rightarrow L_1 = \frac{4}{\omega} = 0.4 \text{ H}$$

(a) if x_y are open: $i_2 = 0$

$$\therefore W(t) = \frac{1}{2} \times 0.4 \times (2 \cos 10t)^2$$

$$W(t) = 0.8 \cos^2(10t)$$

$$\text{at } t=0 \Rightarrow W(t) = 0.8 \text{ J}$$

$$(b) \quad W(t) = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M i_1 i_2$$

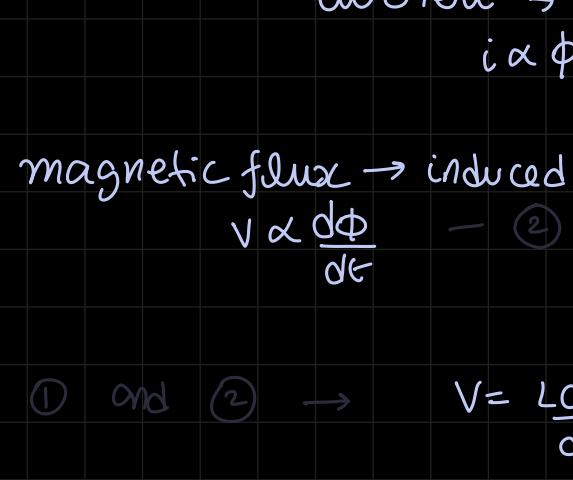
$$i_1 = i_s = 2 \angle 0^\circ$$

$$i_2 = \frac{V_x}{j2S} \quad \text{and} \quad V_x = -j\omega M I_1$$

$$i_2 = -0.6 \frac{(2 \angle 0^\circ)}{2.5} = -0.48A$$

$$W(t=0) = \frac{1}{2} (0.4) (2^2) + \frac{1}{2} (-0.48)^2$$

• MODULE 5: Magnetically coupled circuits



current \rightarrow magnetic flux
 $i \propto \phi$ — ①

magnetic flux \rightarrow induced voltage

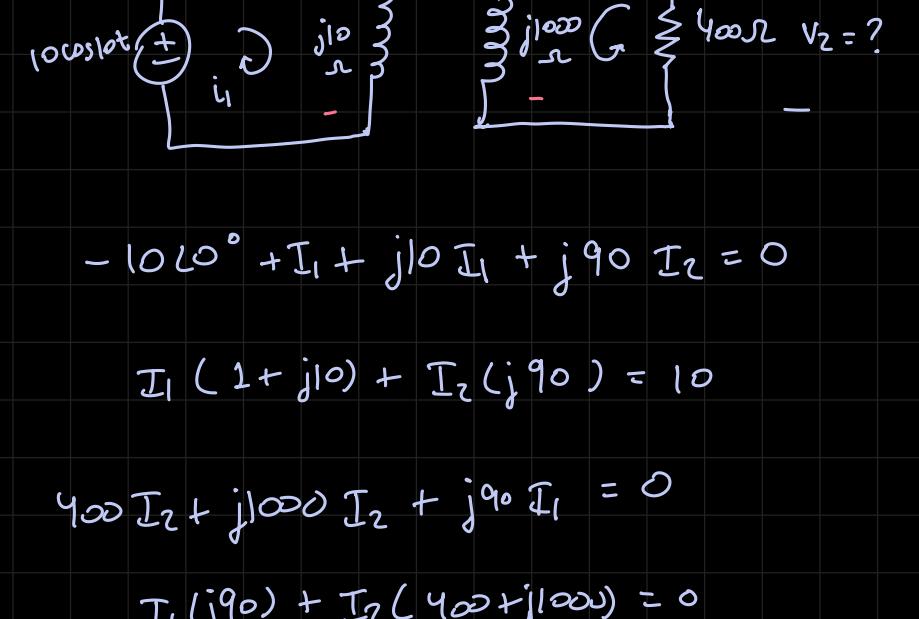
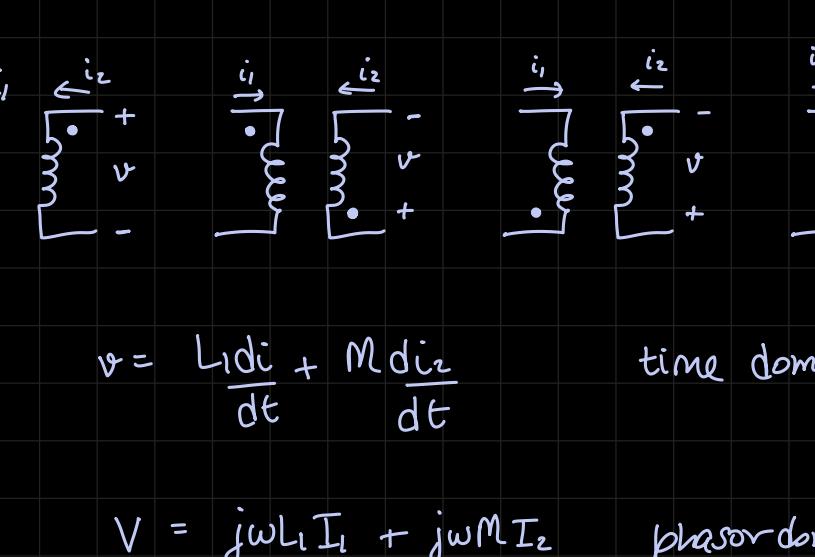
$$V \propto \frac{d\phi}{dt} \quad \text{— ②}$$

$$\textcircled{1} \text{ and } \textcircled{2} \rightarrow V = L \frac{di}{dt}$$

for DC source: current is constant

hence $\frac{di}{dt} = 0$ and $\therefore V = 0$
 (induced)

• Mutual Inductance



* Additive/Subtractive Property {two different coils?}

$$i_1 \text{ (clockwise)} \quad i_2 \text{ (counter-clockwise)} \quad v_2 = L \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$i_1 \text{ (clockwise)} \quad i_2 \text{ (clockwise)} \quad v_2 = L \frac{di_2}{dt} - M \frac{di_1}{dt}$$

• Dotted Notation

Current entering + terminal means
 +ve voltage reference at -

$$v = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad \text{time domain}$$

$$V = j\omega L_1 I_1 + j\omega M I_2 \quad \text{phasor domain}$$

$$\text{eg: } \begin{array}{c} 1 \Omega \\ \text{10cos}(\omega t) \end{array} \quad \begin{array}{c} i_1 \\ \text{+} \\ \text{-} \end{array} \quad \begin{array}{c} i_2 \\ \text{+} \\ \text{-} \end{array} \quad \begin{array}{c} M=j90 \\ \text{+} \\ \text{-} \end{array} \quad \begin{array}{c} 4 \Omega \\ \text{+} \\ \text{-} \end{array} \quad \begin{array}{c} i_2 \\ \text{+} \\ \text{-} \end{array} \quad \begin{array}{c} 4 \Omega \\ \text{+} \\ \text{-} \end{array} \quad \begin{array}{c} i_2 \\ \text{+} \\ \text{-} \end{array}$$

$$\textcircled{1} \quad -10\cos^2 \theta + I_1 + j10 I_1 + j90 I_2 = 0$$

$$I_1(1+j10) + I_2(j90) = 10$$

$$\textcircled{2} \quad 4\Omega I_2 + j1000 I_2 + j90 I_1 = 0$$

$$I_1(j90) + I_2(4\Omega + j1000) = 0$$

CURRENTS:

$$\Delta = \begin{vmatrix} 1+j10 & j90 \\ j90 & 4\Omega + j1000 \end{vmatrix}$$

$$= (1+j10)(4\Omega + j1000) + 90000$$

$$= 400 + j1000 + j4000 - 10.000 + 90.000$$

$$= j5000 + 80,400 = 80555 \angle 3.55^\circ$$

$$\Delta_1 = \begin{vmatrix} j1000 & 10 \\ 4\Omega + j1000 & 0 \end{vmatrix} = -4000 - j10.000 = 10770 \angle -111.8^\circ$$

$$\Delta_2 = \begin{vmatrix} 10 & 1+j10 \\ 0 & j90 \end{vmatrix} = j900 = 900 \angle 90^\circ$$

$$I_1 = \frac{\Delta}{\Delta_1} = -3.2 + 6.75j$$

full current part

wrong because I took $M = L_2 = j1000$ above

$$\Delta_0 = \begin{vmatrix} 1+j10 & j90 \\ j90 & 4\Omega + j1000 \end{vmatrix}$$

$$= 400 + j1000 + j4000 - 10.000 + 8100$$

$$= j5000 - 1500 = 5220 \angle 106.69^\circ$$

ENERGY STORED IN THE CIRCUIT

$$w(t) = \frac{1}{2} L_1 i_1(t)^2 + \frac{1}{2} L_2 i_2(t)^2$$

$$\pm M i_1(t) i_2(t)$$

+: current entering same: • and • OR - and -

-: current entering different: • and - OR - and •

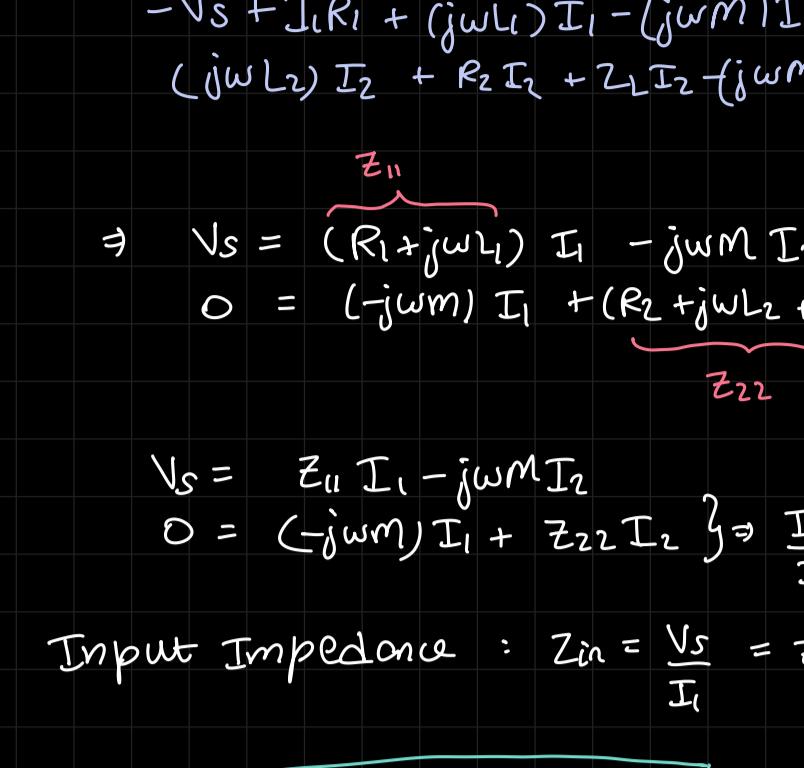
COUPLING COEFFICIENT (k)

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad ; \quad \text{note } \Rightarrow M \leq \sqrt{L_1 L_2}$$

$$\therefore 0 \leq k \leq 1$$

• LINEAR TRANSFORMER

When $V \propto \frac{di}{dt} \Rightarrow$ no magnetic core



$$-Vs + I_1 R_1 + (j\omega L_1) I_1 - (j\omega M) I_2 = 0$$

$$(j\omega L_2) I_2 + R_2 I_2 + Z_L I_2 - (j\omega M) I_1 = 0$$

$$\Rightarrow V_s = (R_1 + j\omega L_1) I_1 - j\omega M I_2$$

$$0 = (-j\omega M) I_1 + (R_2 + j\omega L_2 + Z_L) I_2$$

$$V_s = Z_{11} I_1 - j\omega M I_2$$

$$0 = (-j\omega M) I_1 + Z_{22} I_2 \Rightarrow \frac{I_2}{I_1} = \frac{j\omega M}{Z_{22}}$$

$$\text{Input Impedance : } Z_{in} = \frac{V_s}{I_1} = Z_{11} - j\omega M \frac{I_2}{I_1}$$

$$Z_{in} = Z_{11} + \underbrace{\frac{\omega^2 M^2}{Z_{22}}}_{\text{Reflective Impedance}}$$

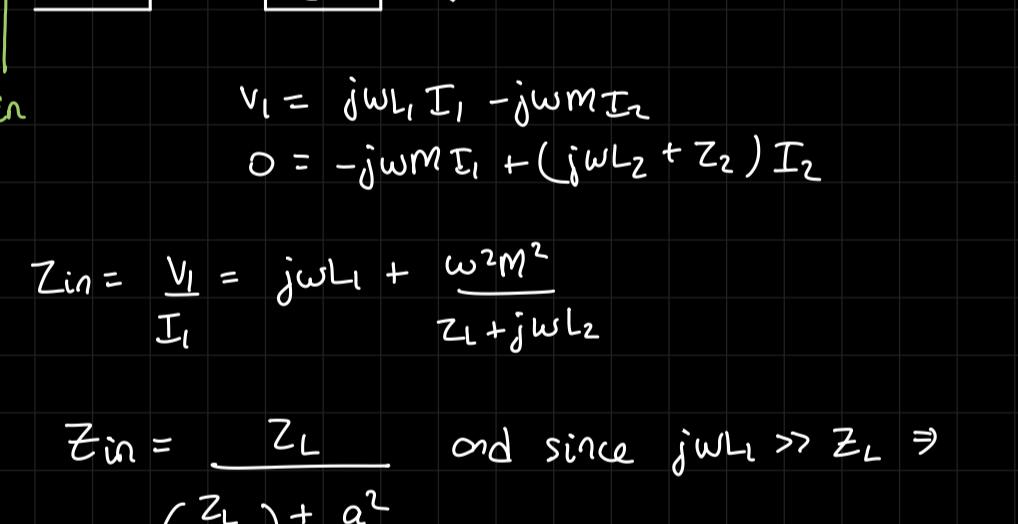
where

$$Z_{11} = R_1 + j\omega L_1, \quad Z_{22} = R_2 + j\omega L_2 + Z_L$$

- if $\kappa = \frac{M}{\sqrt{L_1 L_2}} = 0 \Rightarrow Z_{in} = Z_{11}$ i.e. no coupling

• T EQUIVALENT NETWORK

(no mutual coupling)



$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Note: no mutual coupling since there is no arrow for it

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Note: if dots are on

opposite sides, replace M with $-M$

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

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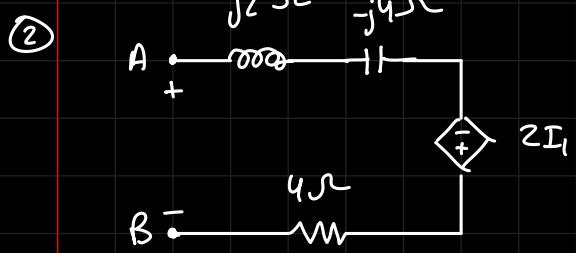
$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

* CTD Practice

① Quiz: 1 (set A)

mesh analysis: current
nodal analysis: voltage

• midsem : 2023



mesh analysis voltage:

* Lecture 15

13/10/24

• Phasor vs S-domain

Assumptions:

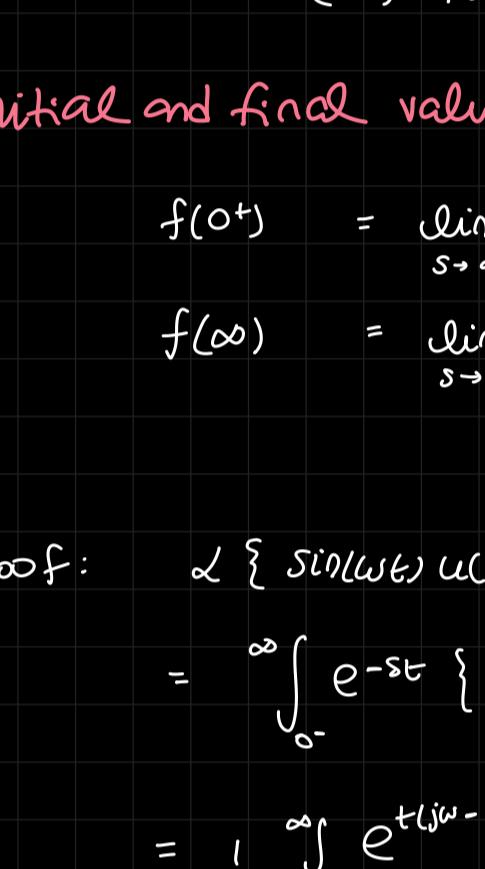
zero initial conditions

sinusoidally varying source

"steady state"

<u>time domain</u>	<u>freq. domain</u>
$v(t) = V_m \cos(\omega t + \phi)$ $= \operatorname{Re} \{ V_m e^{j\omega t} e^{j\phi} \}$	$V = V_m e^{j\phi}$
$v(t) = V_m e^{\sigma t} \cos(\omega t + \phi)$ $= \operatorname{Re} \{ V_m e^{\sigma t} e^{j\phi} e^{j\omega t} \}$	$V = V_m e^{j\phi}$ where $s = \sigma + j\omega$ complex frequency
R	R
L	$L(s) = sL$
C	$\frac{1}{(s + j\omega)C} = \frac{1}{sC}$

e.g.



$$V_{in}(s) = 5e^{-2t} \cos(3t + 45^\circ)$$

$$s = \sigma + j\omega$$

$$s = -2 + j3$$

$$V_{in} = 5e^{-2t} = 5/45^\circ V$$

Voltage divider
for finding V_{out}

$$V_{out} = \frac{5/45^\circ \times (-0.02)(-2+j3) \times (-0.01)(-2+j3)}{10 + (-0.02)(-2+j3) + (-0.01)(-2+j3)}$$

$$= 5.86 L-24.4$$

$$= 5.86 e^{-2t} \cos(3t - 24.4) V$$

for zero initial condition, we can use phasors
for $e^{\sigma t} \cos(\omega t + \phi)$

Can we still use phasors for $\delta(t)$, $u(t)$, $\sin(\omega t)$, $\cos(\omega t)$, e^{-at} ?

NO

but Laplace transform works ✓

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) dt \Rightarrow F(s)$$

variable ↴

$$\begin{array}{lll} \mathcal{S}(t) & \downarrow & f_1(t) \pm f_2(t) \\ u(t) & \frac{1}{s} & k f(t) \\ tu(t) & \frac{1}{s^2} & \frac{d}{dt} f(t) \\ e^{-at} u(t) & \frac{1}{s+a} & s F(s) - f(0^+) \\ te^{-at} u(t) & \frac{1}{(s+a)^2} & \int_0^t f(t) dt \\ \sin(\omega t) u(t) & \frac{\omega}{s^2 + \omega^2} & f_1(t) * f_2(t) \\ \cos(\omega t) u(t) & \frac{s}{s^2 + \omega^2} & F_1(t) \cdot F_2(t) \\ e^{-at} \sin(\omega t) u(t) & \frac{\omega}{(s+a)^2 + \omega^2} & f(t) e^{-at} \\ e^{-at} \cos(\omega t) u(t) & \frac{s}{(s+a)^2 + \omega^2} & F(s+a) \end{array}$$

$$e^{-at} \cos(\omega t) u(t) = \frac{s}{(s+a)^2 + \omega^2}$$

use L'Hopital's rule

$$\lim_{s \rightarrow \infty} \frac{f(x)}{g(x)} = \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}$$

$$\Rightarrow f(0^+) = \lim_{s \rightarrow \infty} \frac{4}{2e^{2s}} = \frac{1}{\infty} = 0$$

$$\text{final value : } f(\infty) = \lim_{s \rightarrow 0} 4e^{-2s} (s+50) = 4(50) = 200$$

$$\Rightarrow f(0^+) = \lim_{s \rightarrow \infty} s F(s) = \frac{5s^2 + 10}{2s(s^2 + 6s + 10)} = \frac{5s^2 + 10}{2s^3 + 16s^2 + 10s}$$

$$\dim_{s \rightarrow \infty} \frac{10s}{4s^2 + 6s} \Rightarrow \lim_{s \rightarrow \infty} \frac{10}{s^2} = \frac{10}{4} = \frac{5}{2}$$

$$f(\infty) = \lim_{s \rightarrow 0} \frac{5s^2 + 10}{2s^3 + 16s^2 + 10s} = 1$$

$$\text{Q: complex : find ILT of } F(s) = \frac{s^2 + 6}{s^2 + 2s + 7}$$

$$F(s) = 1 - \frac{1}{s^2 + 2s + 7} = 1 - \frac{\sqrt{6}}{\sqrt{6}(s^2 + (\sqrt{6})^2)}$$

$$f(t) = \delta(t) - \frac{1}{\sqrt{6}} \sin(\sqrt{6}t) u(t) \quad \times$$

$$\Rightarrow \frac{(s+1) + 1}{(s+1)^2 + (\sqrt{3})^2} \Rightarrow \frac{s+1}{(s+1)^2 + (\sqrt{3})^2} + \frac{1}{s^2 + s + 7} \cos(\sqrt{3}t) u(t)$$

$$\frac{1}{s^2 + 2s + 7}$$

$$\downarrow$$

$$s = -1 \pm j\sqrt{3}$$

$$\text{Impedances in S-domain}$$

$$\text{Resistor : } \frac{v(t)}{R} \rightarrow i(t) \quad v(t) = i(t) R \quad V(s) = I(s) R$$

$$\frac{v(s)}{R} \rightarrow I(s) \quad V(s) = I(s) R$$

$$\text{Inductor : } \frac{v(t)}{L} \rightarrow i(t) \quad v(t) = L \frac{di(t)}{dt} \quad V(s) = sI(s) - i(0^-)$$

$$\frac{v(s)}{L} \rightarrow I(s) \quad V(s) = sI(s) - i(0^-)$$

$$\frac{1}{Cs} \rightarrow I(s) \quad V(s) = \frac{1}{Cs} I(s) + \frac{v(0^+)}{s}$$

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$$\frac{1}{Cs} \rightarrow I(s) \quad V(s) = \frac{1}{Cs} I(s) + \frac{v(0^+)}{s}$$

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$$\frac{1}{Cs} \rightarrow I(s) \quad V(s) = \frac{1}{Cs} I(s) + \frac{v(0^+)}{s}$$

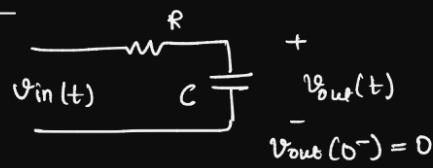
$$\frac{1}{Cs} \rightarrow I(s) \quad V(s) = \frac{1}{Cs} I(s) + \frac{v(0^+)}{s}$$

$$\frac{1}{Cs} \rightarrow I(s) \quad V(s$$

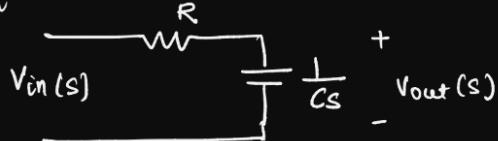
* Lecture 1b {missed}

Transfer function :

lets consider :



Freq. domain



$$V_{out}(s) = \frac{1/Cs}{R + 1/Cs} V_{in}(s)$$

$$\Rightarrow \underbrace{\frac{V_{out}(s)}{V_{in}(s)}}_{\text{transfer function. } H(s)} = \frac{1}{sRC+1}$$

transfer function. $H(s)$

$$V_{out}(t) = \mathcal{L}^{-1} \left\{ V_{out}(s) \right\} = \mathcal{L}^{-1} \left\{ H(s) \cdot V_{in}(s) \right\}$$

defines the system.

• Stability

Stable: bounded o/p for bounded i/p

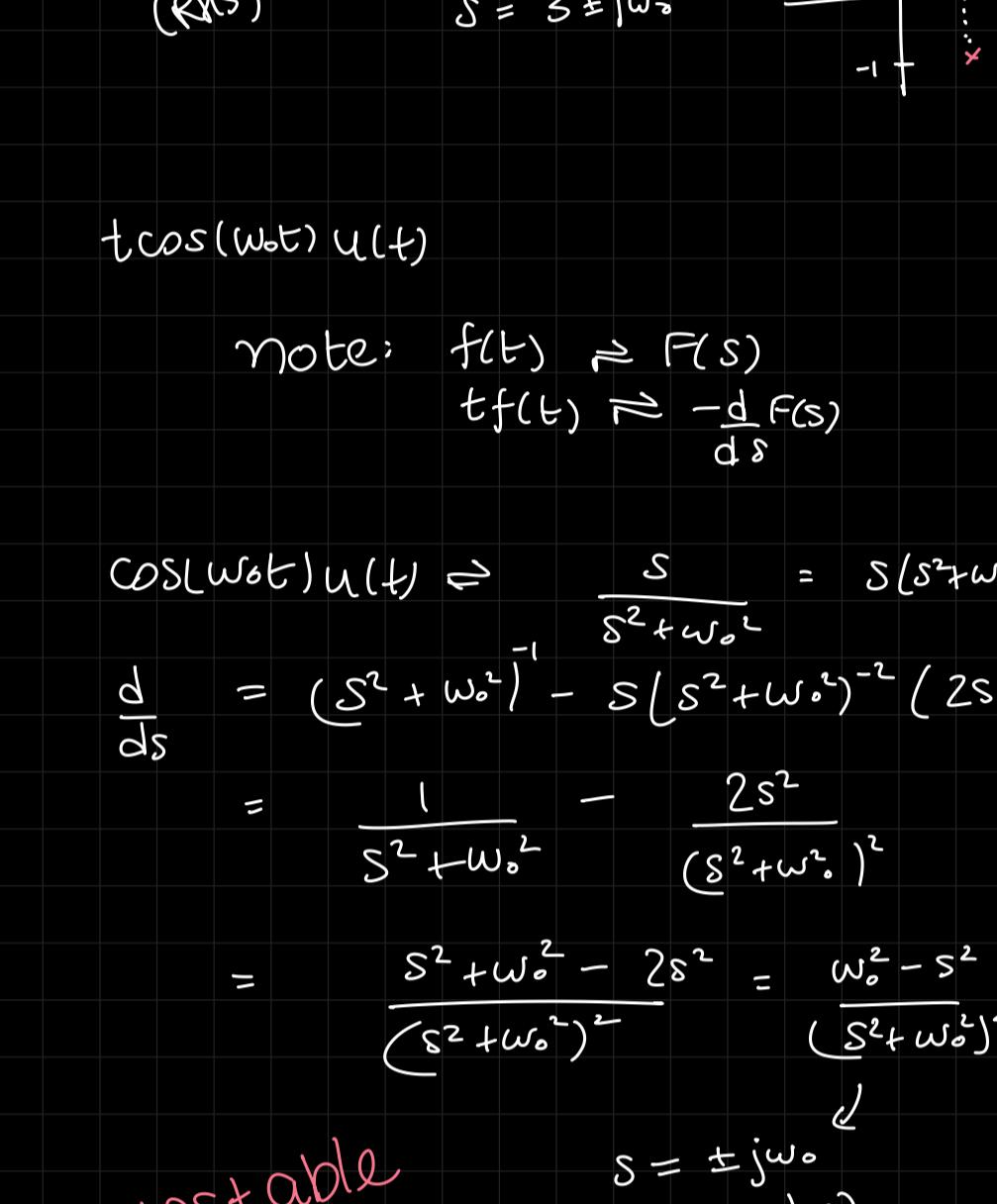
Unstable: unbounded o/p for bounded i/p

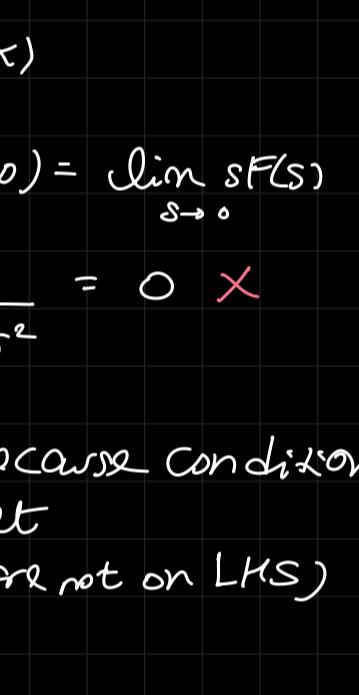
marginally stable: oscillating OR bounded offset

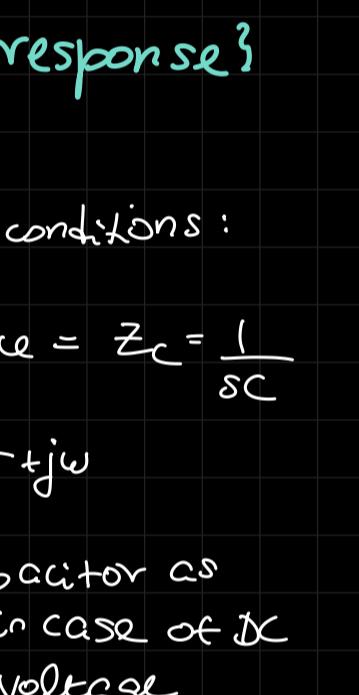
let: $e^{\sigma t} \cos(\omega_0 t) u(t)$ ① $\sigma > 0$: unstable② $\sigma < 0$: stable③ $\sigma = 0$: marginally stable

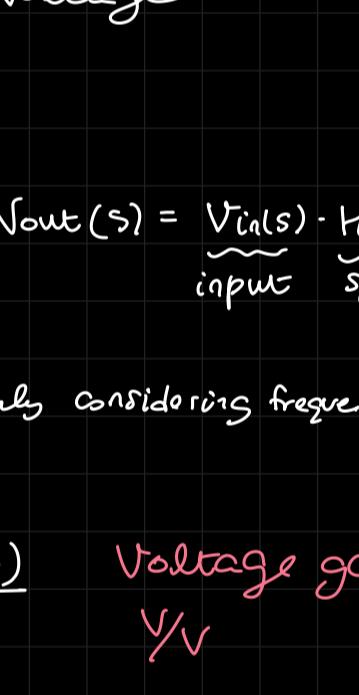
* Stability Criterion

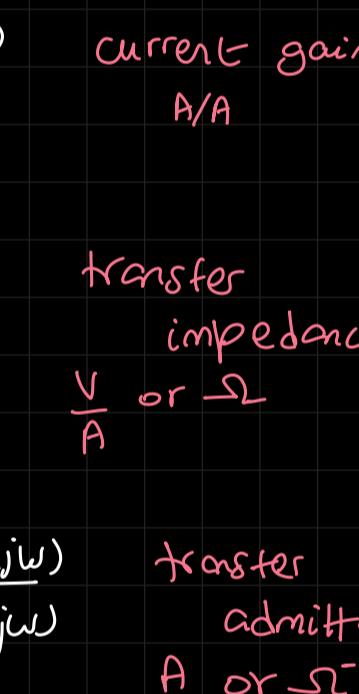
System transfer fn $H(s) = k \frac{(s-z_1)(s-z_2)(s-z_3) \dots (s-z_n)}{(s-p_1)(s-p_2)(s-p_3) \dots (s-p_n)}$

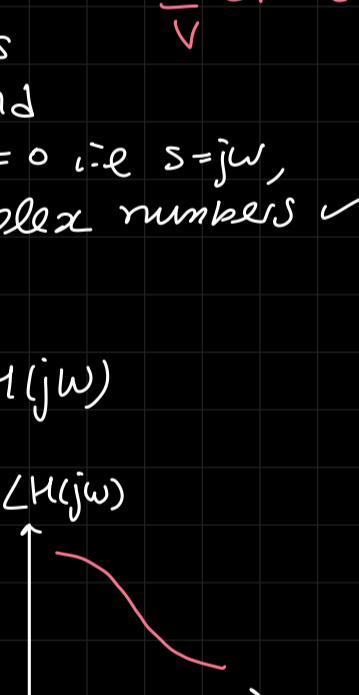
• system is stable if all poles of $SH(s)$ lie on LHS of complex plane• Marginally stable if 1st order pole at $s=0$ OR 1st order complex conjugate pole pair• Unstable system if All other cases
→ 1st poles on RHS OR
→ 2nd or higher order pole at $s=0$ OR
→ 2nd or higher order conjugate pole pair at jw-axisorder of poles = order of s'' at jw axis:
could be unstable
or marginally stablef(t) $\delta F(s)$

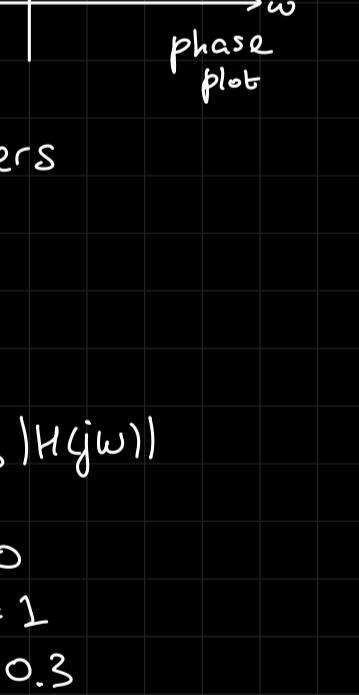
① $e^{-2t} u(t) \quad \frac{s+1}{s+2}$ 

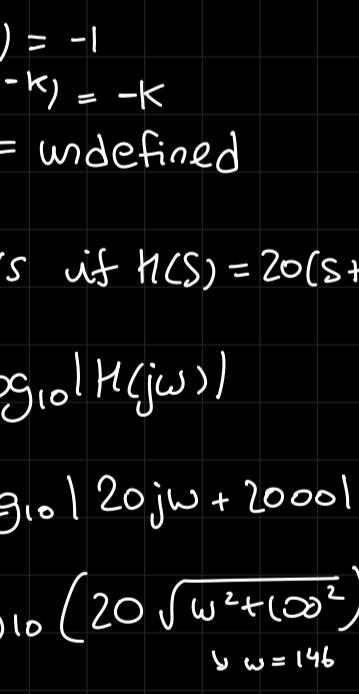
② $e^{4t} u(t) \quad \frac{1}{s-4}$ 

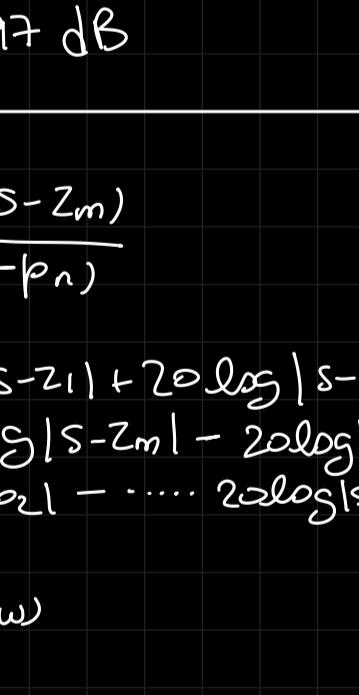
③ $u(t) \quad \frac{1}{s}$ 

④ $e^{-3t} u(t) + e^{+5t} u(t) \quad s \left\{ \frac{1}{s+3} + \frac{1}{s-5} \right\}$ 

⑤ $\frac{s}{(s+1)(s+2)(s+3)}$ 

⑥ $\frac{s+14}{(s+1-\sqrt{2})(s+1+\sqrt{2})}$ 

⑦ $t u(t) \quad \frac{1}{s^2}$ 

⑧ $e^{-3t} \sin(\omega_0 t) u(t) \quad \frac{s - \omega_0}{(s+3)^2 + \omega_0^2}$ 

note: $f(t) \approx F(s)$
 $t f(t) \approx -\frac{d}{ds} F(s)$

$\cos(\omega_0 t) u(t) \Rightarrow \frac{s}{s^2 + \omega_0^2} = s(s^2 + \omega_0^2)^{-1}$

$\frac{d}{ds} \frac{s}{s^2 + \omega_0^2} = \frac{1}{(s^2 + \omega_0^2)^2} - s \frac{2s}{(s^2 + \omega_0^2)^2}$

$= \frac{s^2 + \omega_0^2 - 2s^2}{(s^2 + \omega_0^2)^2} = \frac{\omega_0^2 - s^2}{(s^2 + \omega_0^2)^2}$

$H(j\omega) = \frac{1}{s^2 + \omega_0^2} \quad s = \pm j\omega_0 \quad (\text{2nd order})$

$H(j\omega) = |H(j\omega)| \angle H(j\omega)$

$H(j\omega) = 20 \log_{10} |H(j\omega)| + 20 \log_{10} |\angle H(j\omega)|$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 10000000000000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100000000000000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000000000000000000000000000000000000000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 1000\omega_0$

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$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0} \right) \quad \text{if } \omega = 100\omega_0$

$H_{dB} = 20 \log_{10} \left(\frac{\omega}{\omega_0}$

- Lecture 18 {Bode Plots} Missed
23/10/24

Lecture 19

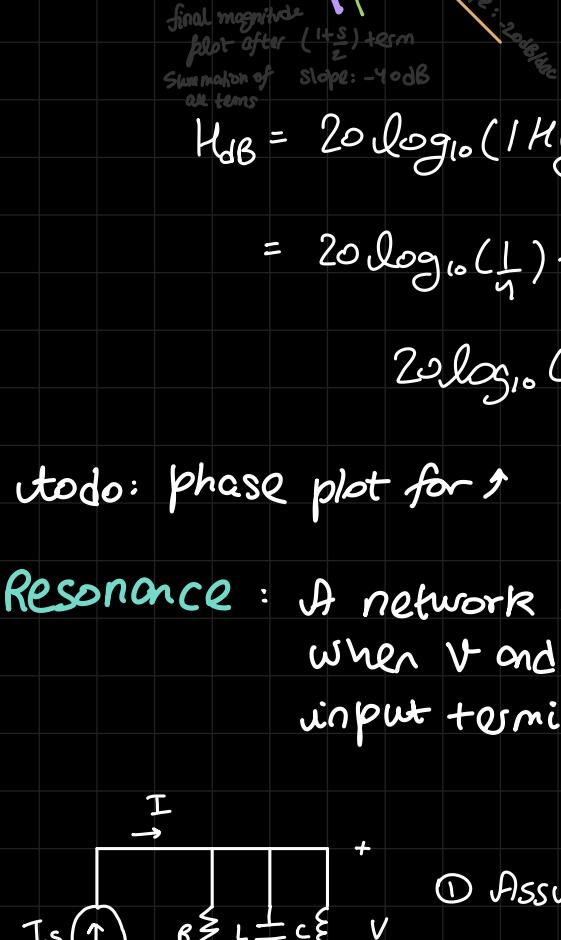
04/11/24

eg) draw bode plots of $H(s) = \frac{s+1}{s(s+2)^2}$

zeroes: 1 $\rightarrow s = -1$

poles: 3 $\rightarrow s = 0, -2, -2$

$$H(s) = \frac{(1+s/1)}{s(1+s/2)^2} = \frac{1}{4} \cdot \frac{1}{s} \cdot \left(\frac{1+s/1}{1+s/2}\right) \cdot \left(\frac{1+s/2}{1+s/2}\right)$$



Simple pole case

so, we have corner frequency at $\omega = \alpha$ i.e.

at $\omega = 1$ & $\omega = 2$ respectively

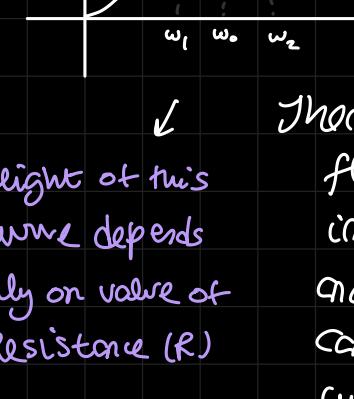
$$H_{dB} = 20 \log_{10}(|H(j\omega)|)$$

$$= 20 \log_{10}\left(\frac{1}{\sqrt{2}}\right) + 20 \log_{10}\left(\frac{1}{j\omega}\right)$$

$$20 \log_{10}(1 + \frac{1}{\omega}) + 40 \log_{10}(1 + \frac{1}{\omega^2})$$

todo: phase plot for \rightarrow

- Resonance: A network is in resonance when V and I at network input terminals are in phase



① Assume: $I_S = I_0 \cos(\omega t + \phi)$

$$V(j\omega) = \frac{1}{Z(j\omega)} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

for resonance \Rightarrow V and I are in phase
(purely resistive circuits)

$$\text{i.e. } \omega C - \frac{1}{\omega L} = 0$$

$$\Rightarrow \omega = \frac{1}{\sqrt{LC}} = \omega_0 \text{ "resonant frequency"}$$

- ② Assume $I_S = I_0 e^{-\sigma t} \cos(\omega t + \phi)$

$$Y(s) = \frac{1}{Z(s)} = \frac{1}{R} + sC + \frac{1}{sL}$$

$$= \frac{s^2 + s(1/RC) + (1/LC)}{(1/C)s}$$

$V_{out} = V \Rightarrow$ transfer function?

$I_{in} = I_S$

↓

$$\frac{V(s)}{I_S(s)} = Z(s) = \frac{1}{Y(s)}$$

transfer impedance

ω_1, ω_2 : half power frequency

height of this curve depends only on value of Resistance (R)
depends on L & C as well

there will be some current flowing in capacitor & inductor but it is equal and opposite so it cancels out and net total current flows only through the resistor

width of the curve depends on L & C as well

At resonance:

$$I_S = I_R$$

$$I_C = -I_L \Rightarrow I_C + I_L = 0$$

quality factor

$$Q = 2\pi \frac{\text{max. energy stored}}{\text{total energy lost per period}}$$

$$= 2\pi \frac{[w_L(t) + w_C(t)]_{\text{max}}}{P_R \cdot T}$$

P_R : avg power lost in resistor

$$= \frac{1}{2} |I|^2 R \text{ WATTS}$$

T: time period (sec)

$$w_L(t) : \frac{1}{2} L i_L^2(t) \text{ (J)}$$

$$w_C(t) : \frac{1}{2} C v_C^2(t) \text{ (J)}$$

$$Q = \omega_0 R C = R \sqrt{\frac{C}{L}}$$

$$\alpha = \text{damping factor} = \frac{1}{2RC} = \frac{\omega_0}{2Q}$$

$$\omega_d : \text{damped frequency} = \sqrt{\omega_0^2 - \alpha^2}$$

$$= \omega_0 \sqrt{1 - \left(\frac{1}{2Q}\right)^2}$$

HALF POWER BANDWIDTH (BW)

$$BW = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

$$\text{where } \omega_c = \sqrt{\omega_1 \omega_2}$$

note: $BW \propto \frac{1}{Q}$.

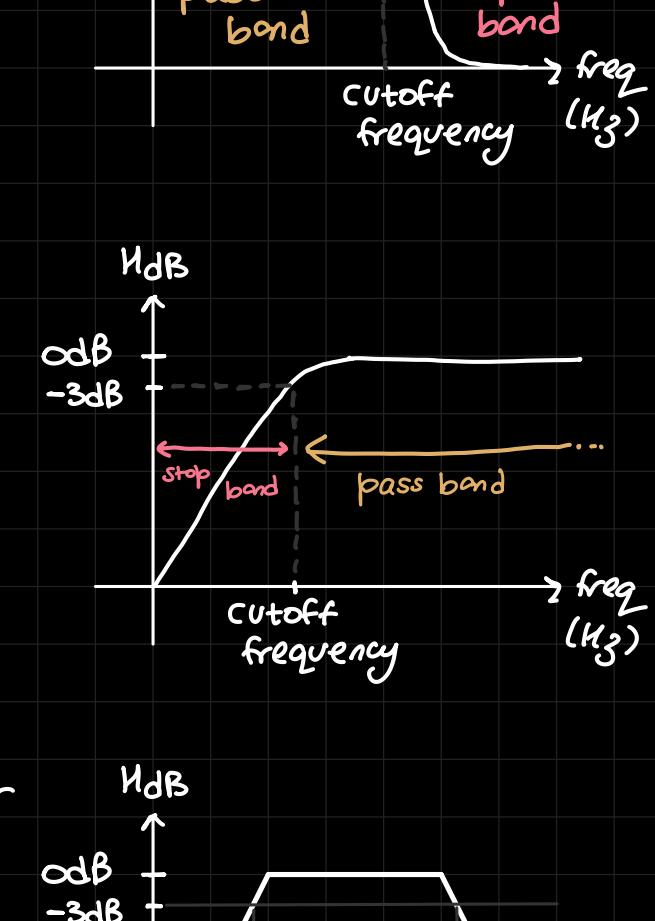
note: if $Q > 5$: high-Q circuit

Lecture 20

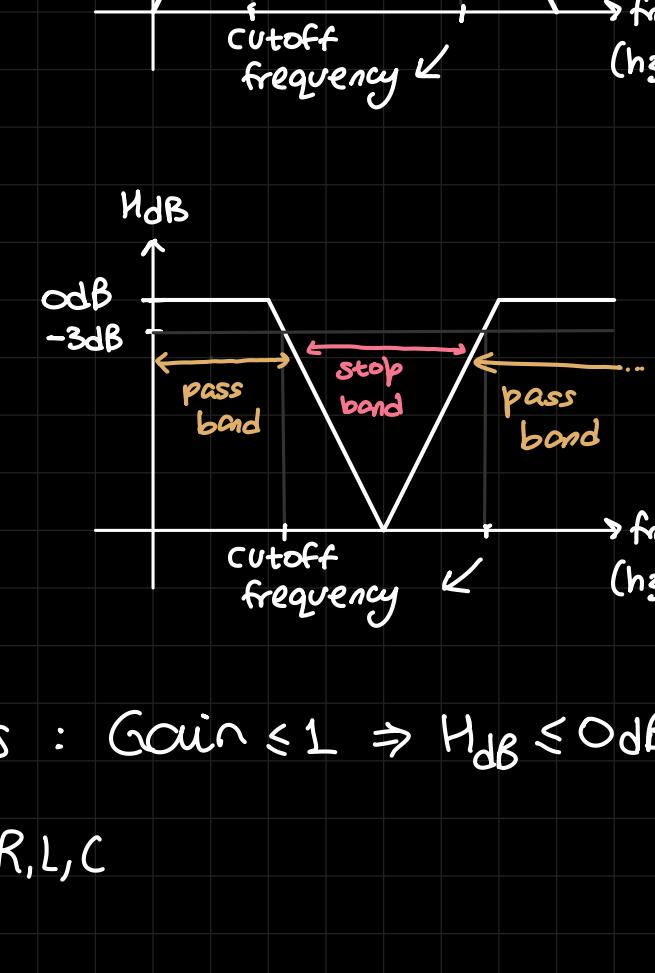
06/11/24

FILTER DESIGN

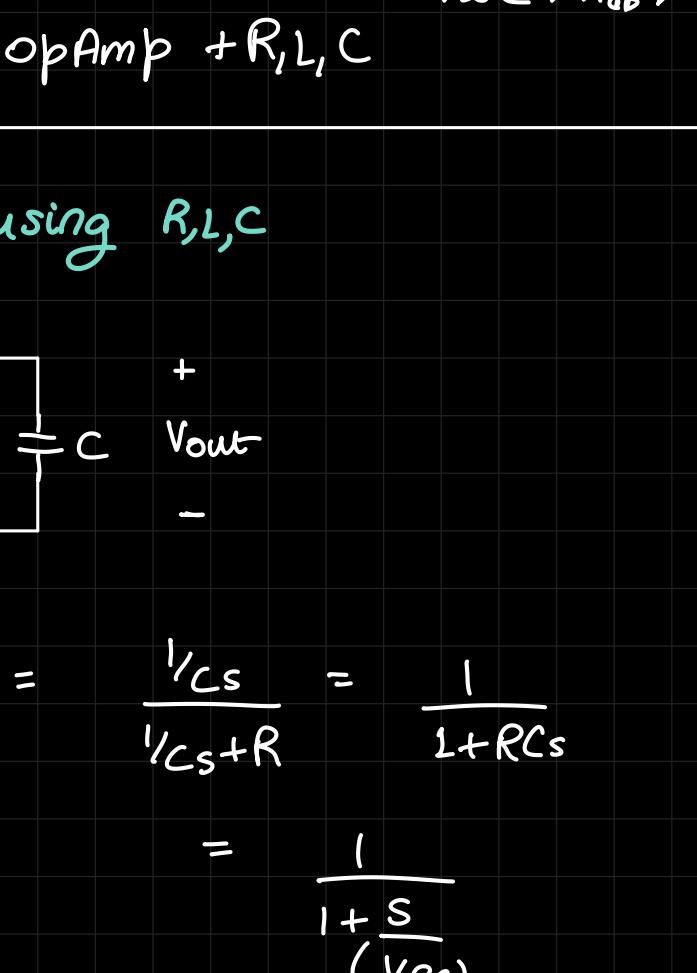
① Low Pass filter



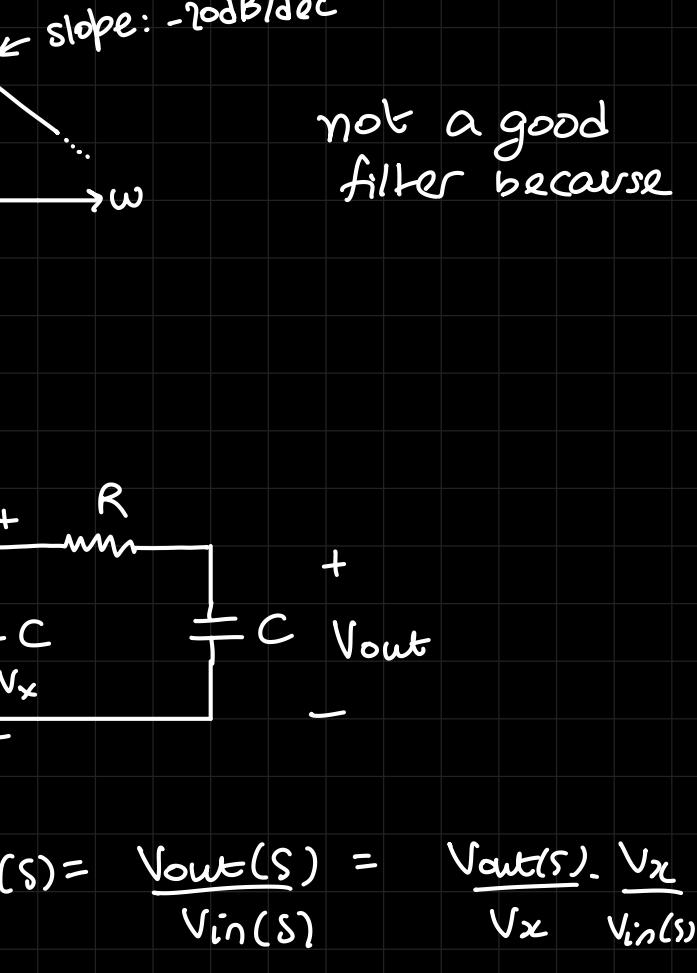
② High Pass filter



③ Band Pass Filter



④ Band Stop Filter

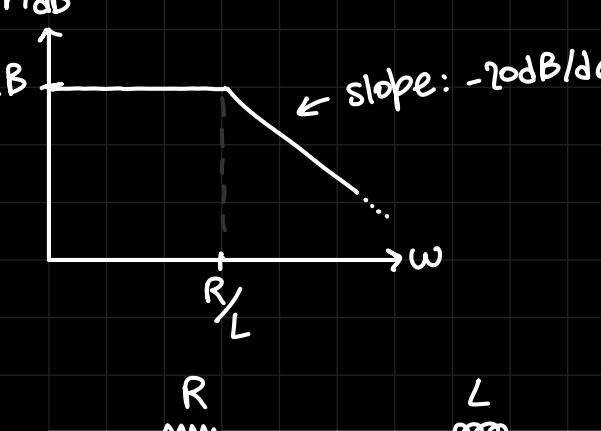


- Passive filters : Gain $\leq 1 \Rightarrow H_{dB} \leq 0dB$

using R, L, C

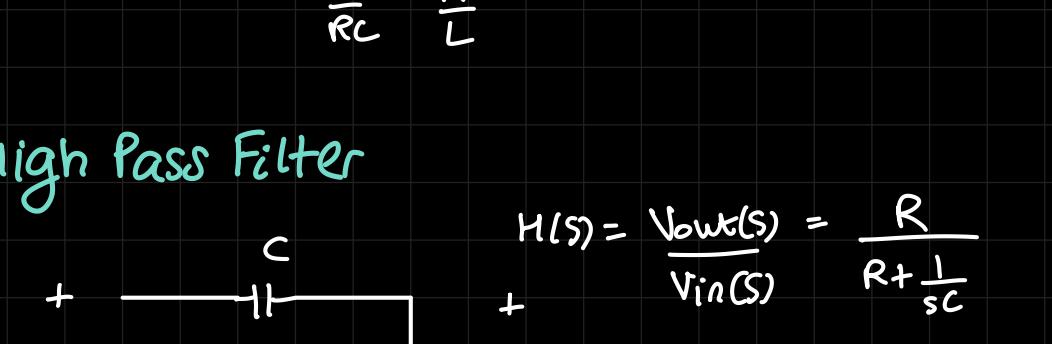
- Active filters : Gain can be more than 1

here $\Rightarrow H_{dB} \geq 0$

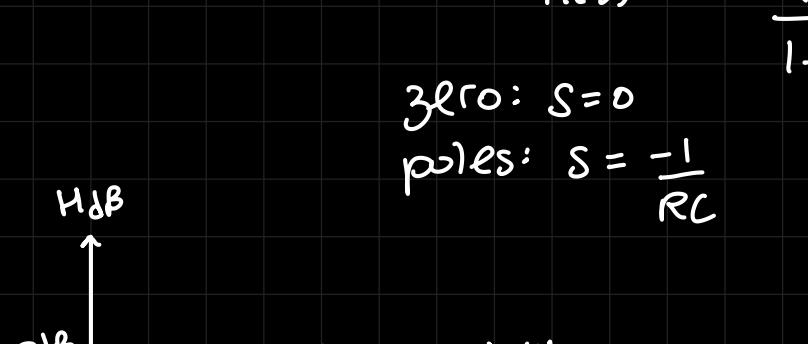


$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + R} = \frac{1}{1 + RCs}$$

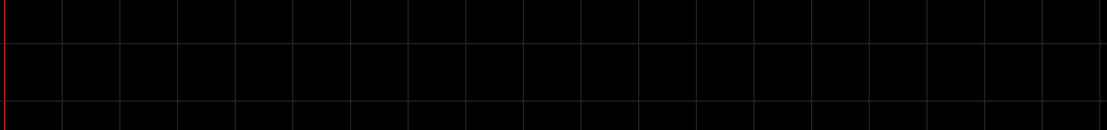
$$\omega_c = \frac{1}{RC}$$



- Another 1st order LPF :



$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + sL}$$



$$poles: s = 0$$

$$zeros: s = -\frac{1}{RL}$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 + \frac{s}{(1/RC)} \cdot \frac{1}{1 + \frac{s}{(RL)}}}$$

- High Pass Filter

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{sC}}$$

$$= \frac{RCs}{RCs + 1}$$

$$= \frac{s}{(1/RC)}$$

$$zeros: s = 0$$

$$poles: s = -\frac{1}{RC}$$

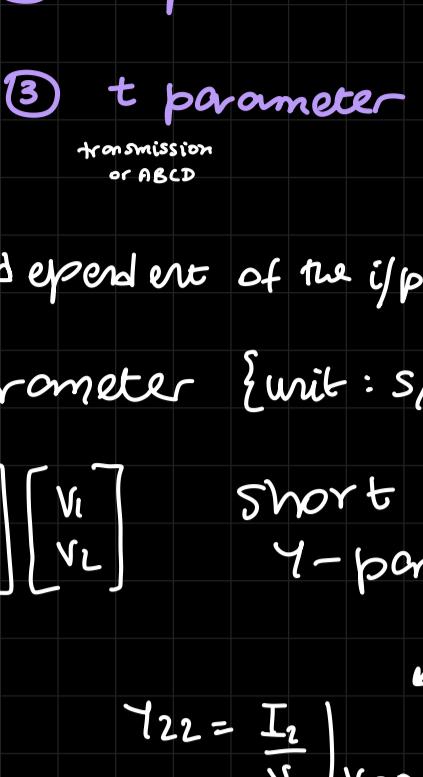
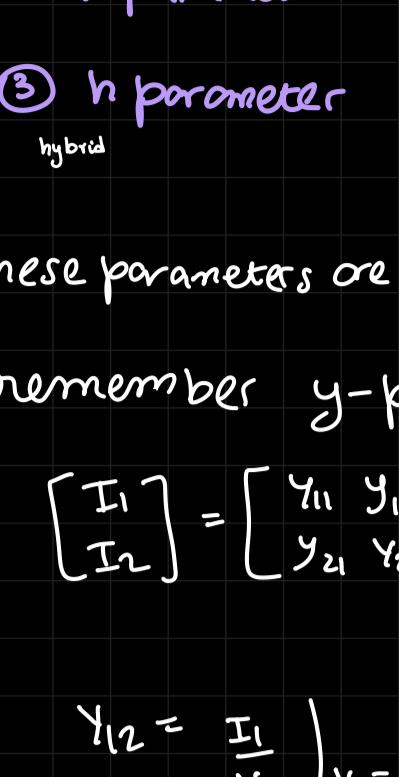
$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1 + \frac{s}{(1/RC)} \cdot \frac{1}{1 + \frac{s}{(RL)}}}$$

Lecture 22

13/11/24

port: a pair of terminals where a signal can enter or exit

Port: A pair of terminals at which a signal can enter or leave a network.



The network ("box") can have only 1st order elements and dependent sources. (R, L, C)

Parameters

① Y parameter

② Z parameter

③ h parameter

④ t parameter
transmission or ABCD

These parameters are independent of the i/p or o/p

remember y-parameter {unit: S/Ω^{-1} }

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{short circuit } Y\text{-parameter}$$

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0} \quad Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} \quad Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}$$

Z parameter {unit: Ω }

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \quad Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = 125 \Omega$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = 25 \Omega$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = 25 \Omega$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = 75 \Omega$$

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} = 25 \Omega$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = 125 \Omega$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} = 125 \Omega$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0} = 25 \Omega$$

HYBRID PARAMETER (h)

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 5 \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -2$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 0.1 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 5 \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -2$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 0.1 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 25 \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -125 \Omega$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 25 \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -125 \Omega$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 25 \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -125 \Omega$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 25 \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -125 \Omega$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 25 \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -125 \Omega$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

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$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 25 \Omega$$

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$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -125 \Omega$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

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$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

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$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 25 \Omega$$

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$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 25 \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -125 \Omega$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 25 \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -125 \Omega$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = 25 \Omega$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -125 \Omega$$

$$h_{21} = \left. \frac{I_2}{V_1} \right|_{I_1=0} = 125 \Omega$$

$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$

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$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = -125 \Omega$$

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$$h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = 25 \Omega$$