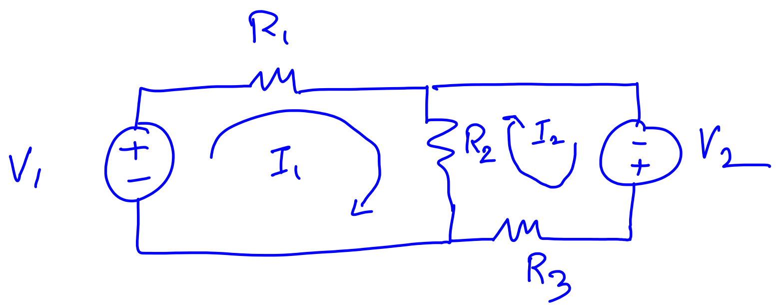
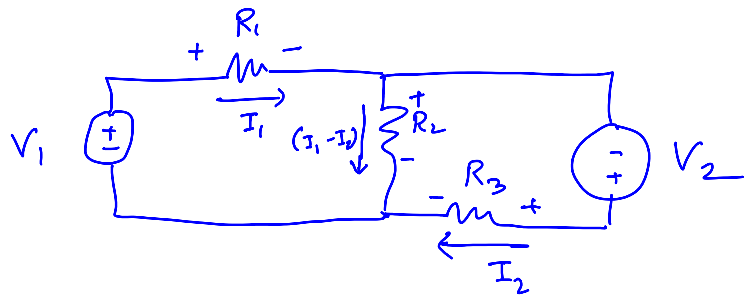
Quick Recap



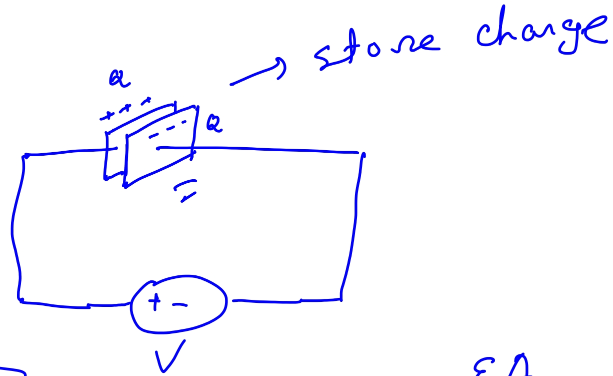


$$V_1 - V_{R_1} - V_{R_2} = 0$$

$$V_2 - V_{R_3} + V_{R_2} = 0$$

$$\left.\begin{aligned} V_{R_1} &= I_1 R_1 \\ V_{R_2} &= (I_1 - I_2) R_2 \\ V_{R_3} &= I_2 R_3 \end{aligned}\right\}$$

Capacitors



$Q = \underbrace{C}_{\text{capacitance}} V$

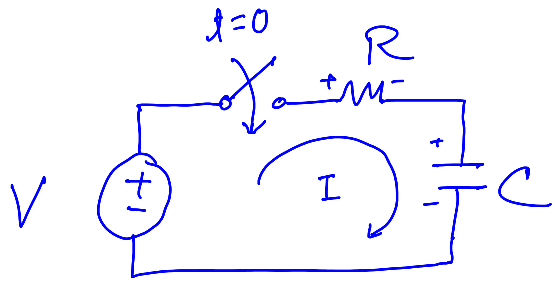
$$\boxed{I = \frac{dQ}{dt} = C\frac{dV}{dt}}$$

$$C = \frac{\varepsilon A}{d}$$

$\varepsilon \to$ permitivity
$A \to$ area of the plate
$d \to$ distance between the plates

unit of capacitance Farad $= \frac{\text{Coulomb}}{\text{volt}}$ }

Capacitor Charging →



$V_R = IR$

$V = V_R + V_c$

$I = C \frac{dV_c}{dt}$

$V = IR + V_c$

$= CR \frac{dV_c}{dt} + V_c$

$V = RC \frac{dV_c}{dt} + V_c$

$$\Rightarrow \int_0^{V_c} \frac{dV_c}{V - V_c} = \int_0^t \frac{1}{RC}\, dt$$

$$\Rightarrow -\ln\left(\frac{V - V_c}{V}\right) = \frac{t}{RC}$$

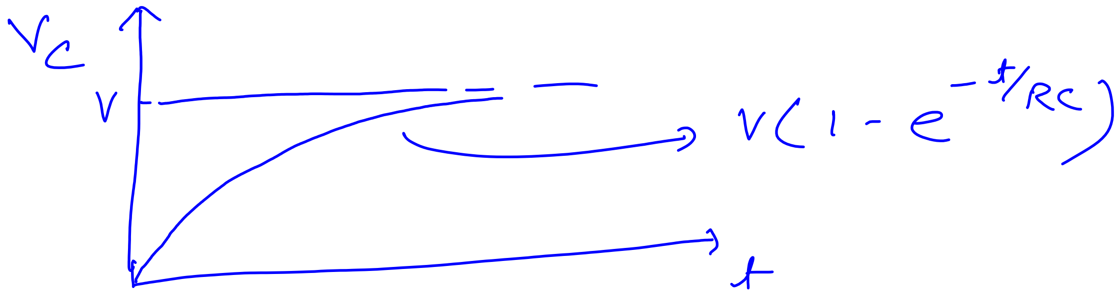
$$\Rightarrow \frac{V - V_c}{V} = e^{-t/RC}$$

$$\Rightarrow V_c = V - V e^{-t/RC}$$

$$V_c = V\left(1 - e^{-t/RC}\right) \quad \forall t \geq 0$$

at $t = 0$ $V_c = 0$

at $t \to \infty$ $V_c \to V$ at $t \to \infty$ the capacitor is fully charged

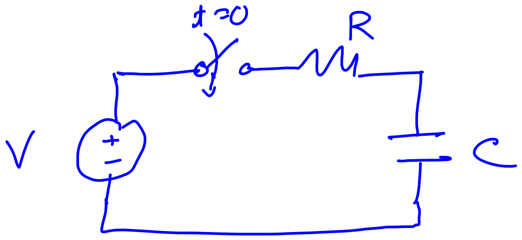


$$I = C\frac{dV_c}{dt} = \frac{CV}{RC}e^{-t/RC} = \frac{V}{R}e^{-t/RC}$$

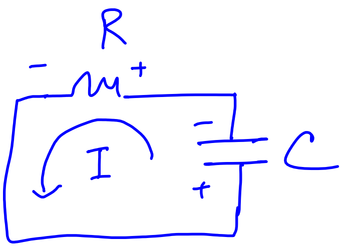
at $t=0$ the discharged capacitor acts as a short circuit

at $t \to \infty$ the capacitor acts as an open circuit.

Capacitor discharging



After a very long time remove the voltage source and connect the R and C together.



$$V_R + V_C = 0$$

$$V_R = IR \quad , \quad I = C\frac{dV_c}{dt}$$

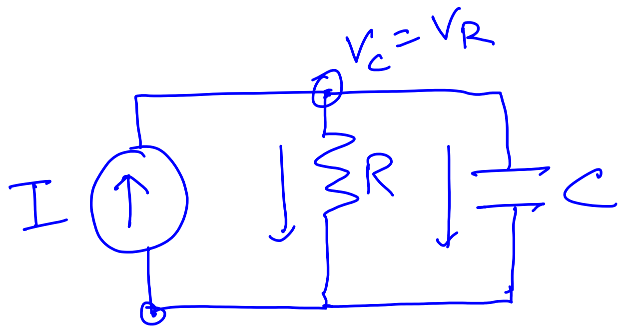
$$IR + V_c = 0$$

$$RC\frac{dV_c}{dt} + V_c = 0$$

$$\int_{V_C(0)=V}^{V_C} \frac{dV_C}{V_C} = \int_0^t -\frac{1}{RC}\,dt$$

$$\ln \frac{V_C}{V} = -\frac{t}{RC}$$

$$\Rightarrow \quad V_C = V e^{-t/RC} \quad (\text{discharging})$$



$$I = \frac{V_C}{R} + C\frac{dV_C}{dt}$$

$$\Rightarrow \int_0^{V_C} \frac{dV_C}{IR - V_C} = \int_0^t \frac{1}{RC}\,dt$$

$$\Rightarrow \quad -\ln \frac{IR - V_C}{IR} = \frac{t}{RC}$$

$$\Rightarrow \quad IR - V_C = IR\, e^{-t/RC}$$

$$\Rightarrow \quad V_C = IR\left(1 - e^{-t/RC}\right)$$

$$I_C = C\frac{dV_C}{dt} = \frac{IRC}{RC} e^{-t/RC} = I e^{-t/RC}$$