tr5frbj 12/08/24

Circuit Theory and Devices

Lab: LT Spice (2nd week onwards) — tut as well

book: Engineering circuit Analysis (9th Edition)

Relative grading

Labs	20%
Quiz (every 2 weeks, (n−1))	20%
Mid sem	30%
End sem	30%

Mid sem, End sem ⇐ scientific calc

Course: 9 modules — chapter 10 onwards
{continuation of BE}

* Lecture : 1

⇒ Linear Circuits

$$\underset{\text{input}}{x} \longrightarrow \boxed{\text{system}} \longrightarrow \underset{\text{output}}{y}$$

system is linear if

$$Kx \rightarrow \boxed{\text{sys}} \rightarrow Ky$$

where k is constant

"R" Resistor : linear element ✓
$V = iR$

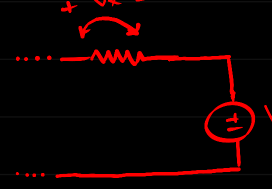
"L" Inductor : linear element ✓
$V = L\frac{di}{dt}$

"C" Capacitor : linear element ✓
$i = C\frac{dv}{dt}$

* Linear Electric circuits :

consists of ⇒

① R, L, C → linear elements
② Independent voltage & current sources
③ linear dependent sources



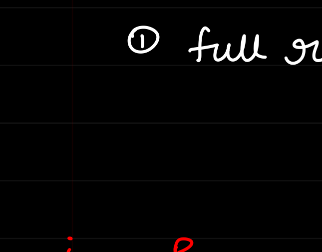
example — would not have been linear if $V_s = KV_x^2$

NOTE: diode and transistors are non-linear elements

* Response of a linear circuit

① full response = transient response + steady state

transient response	steady state
temporary	persists $t \to \infty$
due to sudden change	due to source
also called natural response	also called forced response
depends on linear elements R, L, C	depends on current/voltage source



KVL : $-V_s + iR + L\frac{di}{dt} = 0$

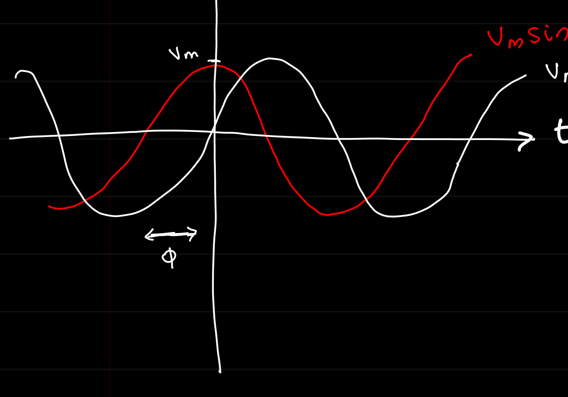
$$i_{final} = i_{transient} + i_{steady\ state}$$

as $t \to \infty$, $i_{final} = i_{steady\ state}$

* Sinusoids & complex numbers

sinusoidally varying voltage source

$$V_s(t) = V_m \cos(\omega t)$$
$$V_s(t) = V_m \sin(\omega t)$$
$$V_s(t) = V_m \sin(\omega t + \phi)$$



② is leading ①
① is lagging ②
} by angle ϕ

* TRIGO

$$\sin(-\theta) = -\sin(\theta)$$
$$\cos(-\theta) = \cos(\theta)$$
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

sin +ve	All +ve
tan +ve	cos +ve

* Complex Numbers

Real : x

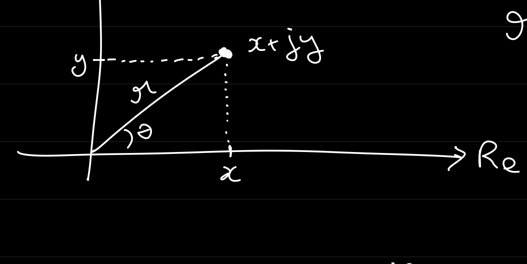
complex : $\underbrace{x + iy}_{\text{rectangular form}}$

real part: x, imaginary part: y, $i = \sqrt{-1}$ a.k.a. j

⇒ Euler's identity : $\underbrace{e^{j\theta}}_{\text{exponential form}} = \underbrace{\cos\theta + j\sin\theta}_{\text{rectangular form}}$

Complex Number

- rectangular: $x + iy$
- exponential: $re^{j\theta}$ ($\equiv$ polar form : $r\angle\theta$)



$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$(r_1 e^{j\theta_1}) \times (r_2 e^{j\theta_2}) = (r_1 \cdot r_2)\, e^{j(\theta_1 + \theta_2)}$$

AC	DC
• we can use transformers to vary voltage	• appliances need constant current
• transported easily over large distances	

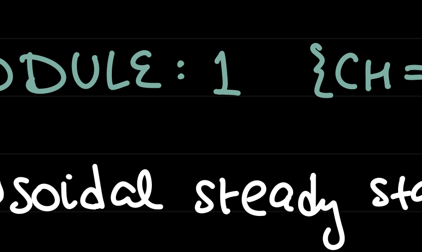
AC → DC : using rectifiers

* MODULE : 1 {CH = 10} 10:30 am

⇒ Sinusoidal steady state analysis

assuming transient has died out long back

Consider,



$V_s(t) = V_m \cos(\omega t)$
find $i(t)$

$$V_s(t) = iR + L\frac{di}{dt}$$

$$\Rightarrow V_m \cos(\omega t) = iR + L\frac{di}{dt}$$

both will take cos(ωt) form

General form Solution : $i = I_1 \cos \omega t + I_2 \sin \omega t$

$$\Rightarrow V_m \cos(\omega t) = (I_1 \cos \omega t + I_2 \sin \omega t)R + L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{d}{dt}(I_1 \cos \omega t + I_2 \sin \omega t)$$
$$\Rightarrow \omega(I_1(-\sin \omega t) + I_2 \cos \omega t)$$
$$\Rightarrow \omega(I_2 \cos \omega t - I_1 \sin \omega t)$$

$$V_m \cos(\omega t) = (I_1 \cos \omega t + I_2 \sin \omega t)R + (I_2 \cos \omega t - I_1 \sin \omega t)L\omega$$

$$\Rightarrow \cos \omega t \cdot (I_1 R + I_2 L) + \sin \omega t \cdot (I_2 R - I_1 L)$$

$$\Rightarrow (I_1 R + I_2 L - V_m)\cos(\omega t) + (I_2 R - I_1 L)\sin(\omega t) = 0$$

* at $t = 0$,
$$(I_1 R + I_2 L - V_m) \cdot 1 + (I_2 R - I_1 L) \cdot 0 = 0$$

$$\boxed{I_1 R + I_2 L = V_m}$$

* at $t = \frac{\pi}{2\omega}$, $(I_1 R + I_2 L - V_m) \cdot 0 + (I_2 R - I_1 L) \cdot 1 = 0$

$$\boxed{I_1 = I_2 \cdot \frac{R}{L}}$$

$I_1 R + I_2 L = V_m \Rightarrow I_2 \frac{R^2}{L} + I_2 L = V_m$
$\Rightarrow I_2\left(\frac{R^2 + L}{L}\right) = V_m \Rightarrow I_2 = \frac{V_m L^2}{R^2 + L}$

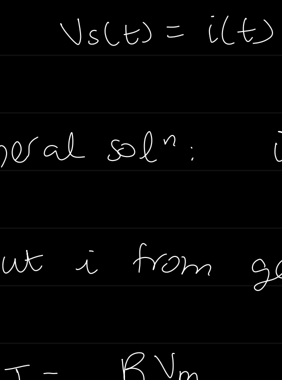
$$I_1 = \frac{V_m L R}{R^2 + L}$$

14/08/24 **Lecture : 2** 9:30 AM

Wednesday → Quiz : Chp 1, 2, 3

* Sinusoidal Steady state analysis

AC sources ↙ ↳ not considering any transient response



angular freq

$V_s = V_m \cos(\omega t)$

$freq\ (hz) = \frac{\omega}{2\pi}$

$V_s(t) = i(t)R + L\frac{di(t)}{dt}$

general soln: $i = I_1\cos(\omega t) + I_2\sin(\omega t)$

put i from gen soln in above eqn

$I_1 = \frac{RV_m}{R^2+\omega^2L^2}$, $I_2 = \frac{\omega LV_m}{R^2+\omega^2L^2}$

$i(t) = \frac{RV_m}{R^2+\omega^2L^2}\cos(\omega t) + \frac{\omega LV_m}{R^2+\omega^2L^2}\sin(\omega t)$

Note: use $\cos(A+B) = \cos A\cos B - \sin A\sin B$

⇒ So, $i(t) = A\cos(\omega t+\phi)$: general soln
becomes ⇒ $i(t) = A[\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)]$

$\frac{RV_m}{R^2+\omega^2L^2}\cos(\omega t) + \frac{\omega LV_m}{R^2+\omega^2L^2}\sin(\omega t) = A\cos(\omega t)\cos(\phi) - A\sin(\omega t)\sin(\phi)$

⇓

$A\cos(\phi) = \frac{RV_m}{R^2+\omega^2L^2}$, $A\sin(\phi) = \frac{-\omega LV_m}{R^2+\omega^2L^2}$

$A^2\cos^2(\phi) + A^2(\sin^2\phi) = A^2(1) = A^2 = \frac{R^2V_m^2 - \omega^2L^2V_m^2}{R^2+\omega^2L^2}$

⇓

$\frac{V_m^2(R^2-\omega^2L^2)}{R^2+\omega^2L^2}$

$A = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}$

$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$

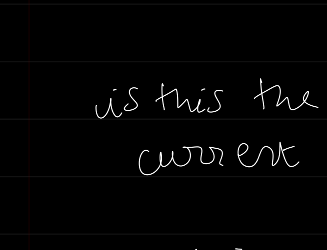
$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

⇒ Consider a complex source

$V_m\cos(\omega t) + jV_m\sin(\omega t) = V_me^{j\omega t}$

Real Voltage source $= V_m\cos(\omega t) = Re\{V_me^{j\omega t}\}$

Note: Euler's formula → $e^{j\theta} = \cos\theta + j\sin\theta$



$V_me^{j\omega t} = iR + L\frac{di}{dt}$

dont integrate ✗
general soln ✓

Assume $i = I_me^{j(\omega t+\phi)}$
put eqn

$V_me^{j\omega t} = I_me^{j\omega t+j\phi}R + L(j\omega)I_me^{j\omega t+j\phi}$

⇒ $I_me^{j\omega t+j\phi} = \frac{V_me^{j\omega t}}{R+j\omega L}$

is this the real current? No
current cannot be complex

Real $i(t) = Re\{I_me^{j\omega t+j\phi}\} = Re\left\{\frac{V_m}{R+j\omega L}e^{j\omega t}\right\}$

find the real part of this
complex number by simplifying
{multiply by the conjugate ≡ factorize}

$i(t) = \frac{V_m}{R+j\omega L}\left(\frac{R-j\omega L}{R-j\omega L}\right)e^{j\omega t} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}e^{-j\tan^{-1}\left(\frac{\omega L}{R}\right)}\cdot e^{j\omega t}$

⇒ Note: rect form → polar form

$x+jy \rightleftarrows re^{j\theta}$

$x = r\cos\theta$, $y = r\sin\theta$

$r = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Real $i(t) = Re\left\{\frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cdot e^{j\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)}\right\}$

* $i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

Same answer as when we were
not considering complex numbers

* PHASOR

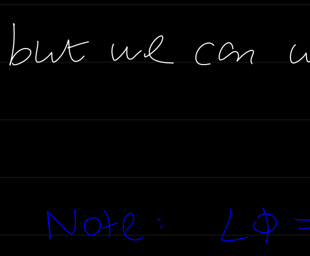
$i(t) = I_m\cos(\omega t+\phi) = Re\{I_me^{j\omega t+j\phi}\}$

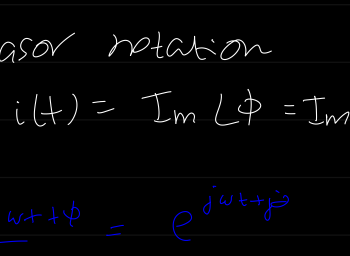
⇓ remove $e^{j\omega t}$, $Re\{\ldots\}$

not changing with time
complex func ✗
complex number ✓

$\underbrace{I_me^{j\phi}}_{phasor} = I_m\angle\phi$

time domain	phasor representation
$i(t) = I_m\cos(\omega t+\phi)$	$I_m\angle\phi$
$v(t) = V_m\cos(\omega t+\theta)$	$V_m\angle\theta$
⇒ real function of t	⇒ complex, constant
⇒ notations: small letters	⇒ Bold, capital letters





$i(t) = I_m\cos(\omega t+\phi)$ ✓
$i(t) = Re\{I_me^{j\omega t+j\phi}\}$ ✓
$i(t) = I_m\angle\phi$ ✗ complex constant
$i(t) = I_me^{j(\omega t+\phi)}$ ✗ complex
$i(t) = I_m\angle\omega t+\phi$ ✗ complex phasor notation

phasor notation

but we can write ⇒ phasor notation
of $i(t) = I_m\angle\phi = I_m\angle(\omega t+\phi)$

Note: $\angle\phi = e^{j\phi}$ or $\angle\omega t+\phi = e^{j\omega t+j\phi}$

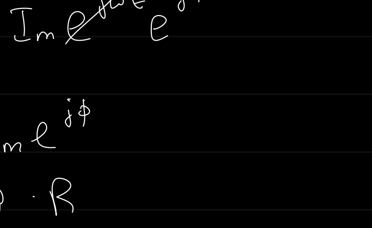
$\underbrace{V_me^{j\theta}}_{V} = (R+j\omega L)\underbrace{I_me^{j\phi}}_{I}$

$\boxed{V = (R+j\omega L)I}$

↳ phasor notation

* phasor representation of relationship between i and v for R, L, C

① RESISTOR



$v = iR$

consider complex voltage & current

$v = V_me^{j(\omega t+\theta)}$
$i = I_me^{j(\omega t+\phi)}$

$V_me^{j\omega t+j\theta} = R\cdot I_me^{j\omega t+j\phi}$

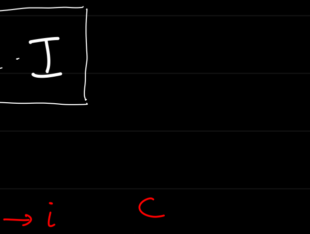
$V_m\cancel{e^{j\omega t}}\cdot e^{j\theta} = R\cdot I_m\cancel{e^{j\omega t}}e^{j\phi}$

$V_me^{j\theta} = R\cdot I_me^{j\phi}$

$\underbrace{V_m\angle\theta}_{V} = \underbrace{I_m\angle\phi}_{I}\cdot R$

$\boxed{V = IR}$

② INDUCTOR



$v = L\frac{di}{dt}$

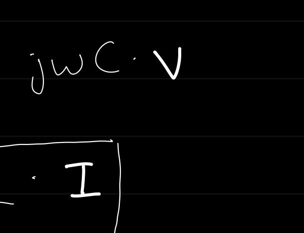
$v \to V_me^{j(\omega t+\theta)}$, $i \to I_me^{j\omega t+j\phi}$

$V_me^{j\omega t}e^{j\theta} = L\,I_me^{j\omega t}\cdot e^{j\phi}(j\omega)$

$\underbrace{V_me^{j\theta}}_{V} = j\omega L\cdot\underbrace{I_me^{j\phi}}_{I}$

$\boxed{V = j\omega L\cdot I}$

③ CAPACITOR



$i = C\frac{dv}{dt}$

$i \to I_me^{j(\omega t+\phi)}$
$v \to V_me^{j(\omega t+\theta)}$

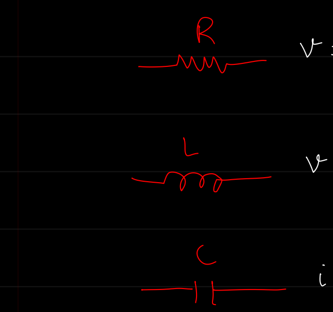
$I_m\cancel{e^{j\omega t}}\cdot e^{j\phi} = C\cdot V_m\cancel{e^{j\omega t}}e^{j\theta}\cdot j\omega$

$\underbrace{I_me^{j\phi}}_{I} = C\cdot j\omega\cdot\underbrace{V_me^{j\theta}}_{V}$

$I = j\omega C\cdot V$

$\boxed{V = \frac{1}{j\omega C}\cdot I}$

* PHASOR Representation using KVL

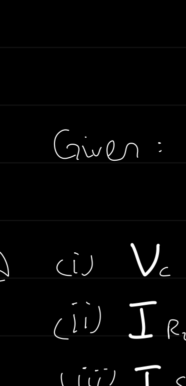


$V_s = V_R + V_L$
$= IR + (j\omega L)I$
$= I(R+j\omega L)$

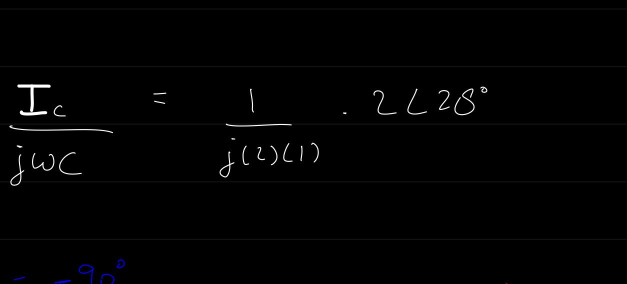
$V_s = V_m\cos(\omega t+\theta)$ ✗ time domain
$V_s = V_me^{j(\omega t+\theta)}$ ✓ phasor notation

$I = \frac{V_s}{R+j\omega L} = \frac{V_m\angle\theta}{R+j\omega L}$

Time domain		Phasor (Frequency domain)
R	$v = iR$	$V = IR$
L	$v = L\frac{di}{dt}$	$V = j\omega L\,I$
C	$i = C\frac{dv}{dt}$	$V = \frac{1}{j\omega C}I$



⇒ Example



① Given: both AC sources, V_s & I_s operate at $\omega = 2\ rad/s$

② Given: $I_c = 2\angle 28°\ A$

Find (i) V_c
(ii) I_{R_2}
(iii) I_s

① $V_c = \frac{I_c}{j\omega C} = \frac{1}{j(2)(1)}\cdot 2\angle 28°$

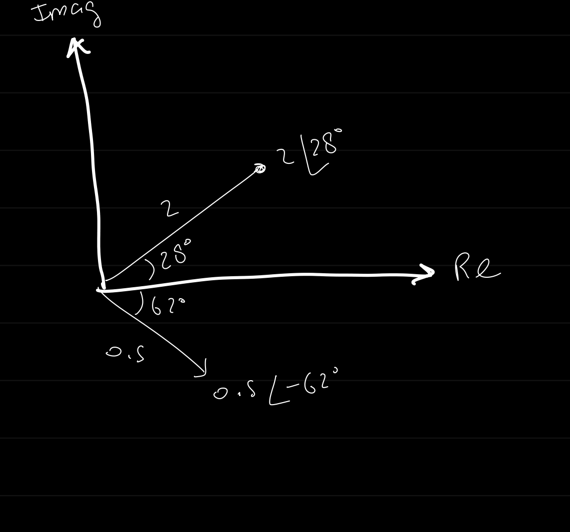
Note: $\frac{1}{j} = -j = -90°$

we add the angles directly which are being multiplied

$V_c = (0.5\angle -90°)(2\angle 28°) = 1\angle -62°\ V$

② $I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_{R_2}}{2} = \frac{1\angle -62°}{2} = 0.5\angle -62°\ A$

③ $I_s = 0.5\angle -62° + 2\angle 28°$



$2.06\angle 14°\ A$

* Lecture: 3

19/08/24 9:30AM

- Phasor: $i(t) = I_m \cos(\omega t + \phi) = Re\{I_m e^{j(\omega t+\phi)}\}$

Remove $e^{j\omega t}$ and $Re\{...\}$

$$I_m e^{j\phi} = I_m \angle \phi \quad \} \text{ Phasor}$$

exponential form — polar form

time domain ⇄ freq. domain

$V_R = IR \qquad V_L = j\omega L I \qquad V_c = \frac{1}{j\omega C} I$

* IMPEDANCE

$$Z = \frac{V}{I} = \frac{1}{R}$$

$Z_R = R$ — voltage and current in the same phase

$Z_L = j\omega L$ — voltage leads current by 90°

$Z_C = \frac{1}{j\omega C}$ — voltage lags current by 90°

$$Z = x + jy$$

resistance ↙ ↘ reactance

* Admittance

$$Y = \frac{1}{Z}$$

$$Y = x + jy$$

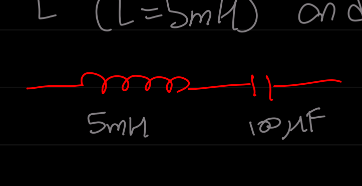
conductance ↙ ↘ susceptance

* Impedance combination

Series : $Z_{eq} = Z_1 + Z_2 + \dots Z_N$

Parallel : $Z_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots \frac{1}{Z_N}$

eg) Find impedance of series combination of an inductor L (L=5mH) and capacitor (C=100μF) $\omega = 10^4$ rad/s



$\Rightarrow Z_{eq} = Z_L + Z_C = j\omega L + \frac{1}{j\omega C} = 10^4 \cdot j \cdot 5\times10^{-3} + \frac{10^{-4}\times10^3}{j\cdot 5\times 100 \times 10^{-6}}$

Note: $\frac{1}{j} = -j$ $\qquad \Rightarrow 50j - j = 49j$

Note: $e^{j\theta} = \cos\theta + j\sin\theta = e\angle\theta$ $\qquad \Rightarrow 0 + 49j = 49e^{j\cdot\frac{\pi}{2}}$

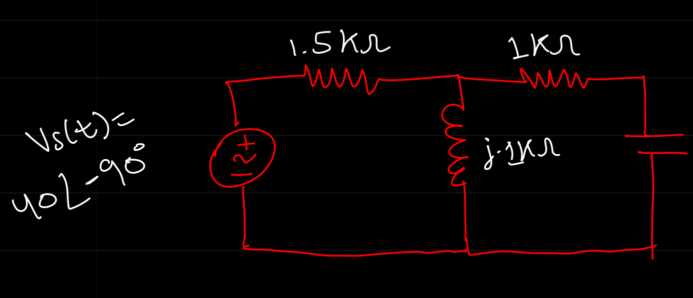
eg:

Find i(t)

$\frac{10^3(1-2j)(1+j)}{2}$

$500(1-2j)(1+j) \quad 10^3\frac{(1-2j)}{1-j}$

step1) Convert to freq. domain (write in terms of phasor and Z)



$1000 - 2000j \parallel 1000j$

$\left(\frac{1}{1000-2000j} + \frac{1}{1000j}\right)^{-1}$

$\frac{1-j}{10^3 - 2\cdot10^3 j}$

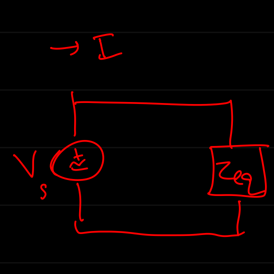
$\Rightarrow \frac{1000j + 1000 - 2000j}{(10^3 - 2\cdot10^3 j)10^3 j}$

$\frac{1}{3}H \rightarrow Z_L = j\omega L = j\cdot 3000 \cdot \frac{1}{3} \Rightarrow j\cdot 1k$

$\frac{1}{6}\mu F \rightarrow Z_c = \frac{1}{j\omega C} = \frac{1}{j\cdot 3000 \times \frac{1}{6} \times 10^{-6}} \Rightarrow \frac{2\cdot10^3}{j}$

$\Rightarrow -j\cdot 2K\Omega$

step2)



$$I = \frac{V_s}{Z_{eq}}$$

$Z_{eq} = 1.5^k + [500(1+j)(1-2j)]$

$\Rightarrow 1500 + 500[1 - 2j + j + 2)$

$\Rightarrow 500[3 + 3 - j] \Rightarrow 500[6 - j]$

$\Rightarrow 3000 - 500j$

$\frac{j(1-2j)(1+j)}{2} \Rightarrow \frac{(j+2)(1+j)}{2} \Rightarrow \frac{j + j^2 + 2 + 2j}{2}$

PPT) $Z_{eq} = 1.5 + \frac{(j1)(1-j2)}{1-j}$ KΩ $\qquad \frac{3j+1}{2}$

$\Rightarrow \frac{4+3j}{2}$

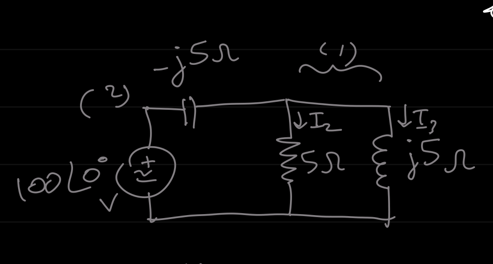
Note: complex number $\begin{cases} x+iy \\ r\angle\theta \end{cases}$

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$I_3 + I_2 = I_1$

eg)



find I_1, I_2, I_3

$Z_{eq}^{(1)} = \left(\frac{1}{5} + \frac{1}{5j}\right)^{-1} = \left(\frac{5j+5}{25j}\right)^{-1} = \frac{5j}{j+1}$

$Z_{eq} \Rightarrow^{(2)} -5j + \frac{5j}{j+1} \Rightarrow \frac{+5 - 5j + 5j}{j+1} = \frac{5}{j+1}$

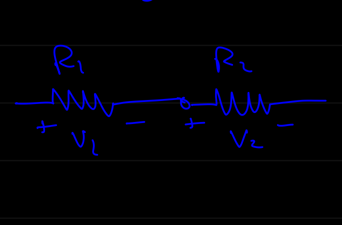
$\Rightarrow \frac{5(j-1)}{-2} \Rightarrow \frac{5-5j}{2}$

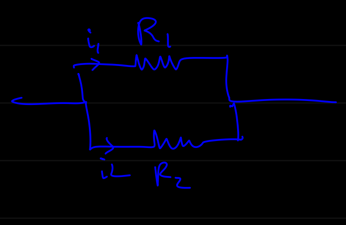
$\frac{5}{2} - \frac{5}{2}j$

$r = \sqrt{x^2+y^2} = \sqrt{\frac{25}{4} + \frac{25}{4}} = \frac{\sqrt{50}}{2}$ $\quad \frac{5\sqrt{2}}{2}$

$\theta = \tan^{-1}(-1) \Rightarrow -45°$

Voltage division — Current division





$V_1 = V\cdot\frac{R_1}{R_1+R_2} \qquad i_1 = i\cdot\frac{R_2}{R_1+R_2}$

$I_1 = \frac{V}{Z_{eq}} = \frac{\cancel{100}^{20}\angle 0°}{\frac{5\sqrt{2}}{2}\angle -45°} \Rightarrow \frac{40\angle 0°}{\sqrt{2}\angle -45°}$

$\frac{40(\cos 0 + j\sin 0)}{\sqrt{2}(\cos(45) - j\sin(45))} \qquad r\angle\theta = re^{j\theta} =$

$\frac{40}{\sqrt{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)} \Rightarrow \frac{40}{1-j} \Rightarrow \frac{40(1+j)}{2} = 20(1+j)$

$20 + 20j$

$r = \sqrt{x^2+y^2} \Rightarrow \sqrt{400+400} \Rightarrow 20\sqrt{1+1} = 20\sqrt{2}$

$\theta = \tan^{-1}(1) = 45°$

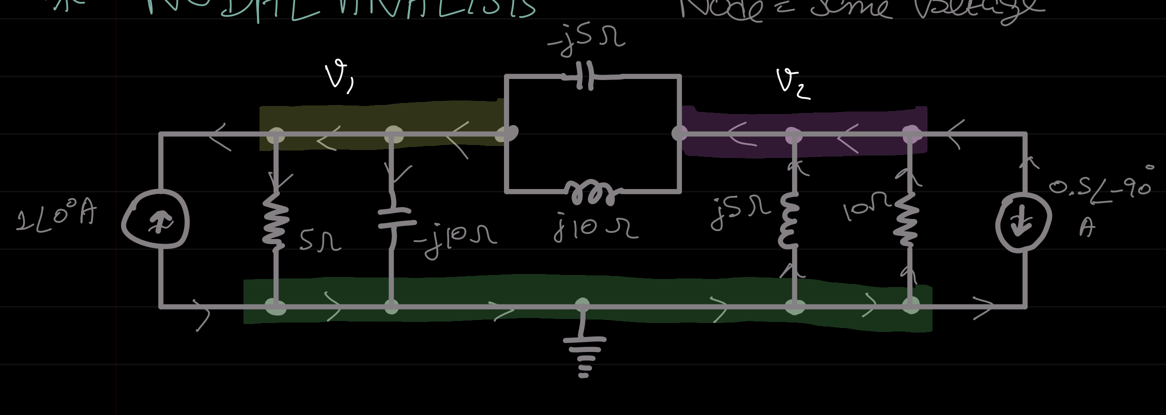
$I_1 = 20\sqrt{2}\angle 45°$

$I_2 = I_1 \times \frac{j5}{5+j5} \Rightarrow \cancel{20}^{10}\sqrt{2}\angle 45° \times \frac{j(1-j)}{\cancel{2}}$

$\Rightarrow 10\sqrt{2}\angle 45° \times (j+1)$

* NODAL ANALYSIS

Node ≡ Same voltage



at node 1:

$1\angle 0° = \frac{V_1}{5} + j\frac{V_1}{10} + \frac{(V_2 - V_1)j}{10} - \frac{(V_2 - V_1)j}{5}$

$r\cos\theta + rj\sin\theta$

$r[1 + 0) \Rightarrow 1$

$10 = 2V_1 + \cancel{jV_1} + jV_2 - \cancel{jV_1} - 2V_2 j + 2V_1 j$

$10 = 2V_1 - jV_2 + 2jV_1$

$10 = 2V_1 + j(2V_1 - V_2)$ ①

at node 2:

$\frac{V_2 - V_1}{j10} + \frac{V_2 - V_1}{-j5} + \frac{V_2}{j5} + \frac{V_2}{10} = 0.5\angle -90°$

$0.5\cos 90° - 0.5j\sin 90°$

$\Rightarrow -0.5j$

$V_2 - V_1 + 2V_1 - \cancel{2V_2} + \cancel{2V_2} + V_2 j = 5$

$3V_2 - V_1 + V_2 j = 5 \Rightarrow V_1 = 3V_2 - 5 + V_2 j$

Put value of V_1 in ①

$10 = 6V_2 - 10 + 2V_2 j + j(6V_2 - 10 + 2V_2 j - V_2)$

$10 = 6V_2 - 10 + 2V_2 j + 6V_2 j - 10j - 2V_2 - V_2 j$

$20 = 4V_2 + 7V_2 j - 10j$

$20 + 10j = V_2(4 + 7j)$

$V_2 = \frac{(20+10j)(4-7j)}{16+49} = \frac{80 - 140j + 40j + 70}{65}$

$\frac{150 - 100j}{65} \Rightarrow \frac{30-20j}{13}$

$V_2 = \frac{30}{13} - j\left(\frac{20}{13}\right)$

$r = \sqrt{\frac{900+400}{169}} = \frac{10}{\sqrt{13}}$

$\theta = \tan^{-1}\left(-\frac{2}{3}\right)$

$= -\tan\left(\frac{2}{3}\right) =$

$= -33.69$

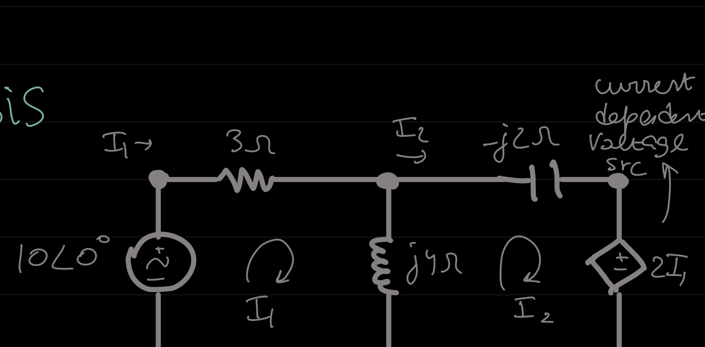
$V_2 = \frac{10\sqrt{13}}{13}\angle -33.69°$

$V_1 = 3V_2 - 5 + jV_2$

$\Rightarrow V_2(3+j) - 5 = \frac{30}{13} - \frac{20j}{13}(3+j) - 5$

$\Rightarrow \frac{30 - 60j + 20 - 65}{13} = \frac{-60j - 15}{13} \Rightarrow V_1 = \frac{15\sqrt{17}}{13}\angle -104°$

* Mesh Analysis



2 meshes

$10\angle 0° = 3I_1 + j4(I_1 - I_2)$

$(I_2 - I_1)j4 - I_2(j2) + 2I_1 = 0$

$10\angle 0° = 10 = 3I_1 + j4(I_1 - I_2) = 3I_1 + j4I_1 - j4I_2$

$j4I_2 + 10 = I_1(3$

$j4I_2 - j4I_1 - j2I_2 + 2I_1 = 0$

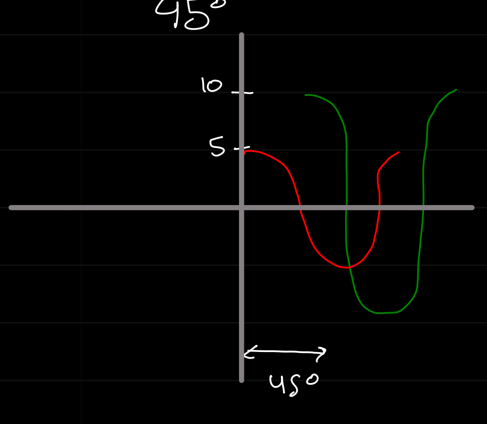
$j4I_2 - j4(10 -$

CTD Tutorial : 1

19/08/24 1:00 – 3:00

Q1) Determine the angle by which v_1 leads i_2 if $v_1 = 10\cos(10t - 45^\circ)$ and i_1 is equal to

(1) $i_1 = 5\cos 10t$
45°



(3) $i_1 = 5\cos(10t - 40^\circ)$
5°

(2) $i_1 = 5\cos(10t - 80^\circ)$
35°

(4) $i_1 = 5\cos(10t + 40^\circ)$
85°

(5) $5\sin(10t - 19^\circ)$
$= 5\cos(10t - 19^\circ - 90^\circ)$
$= 5\cos(10t - 109^\circ)$
Ans : 64°

Q2) Determine which waveform in each of the following pairs is lagging

(1) $\cos 4t$ leads $\sin 4t$ by $\frac{\pi}{2}$ $\{90^\circ\}$

(2) $\cos(4t - 80^\circ)$ lags $\cos(4t)$ by 80°

(3) $\cos(4t + 80^\circ)$ leads $\cos(4t)$ by 80°

(4) $\sin(5t)$ lags $\cos(5t + 2^\circ)$ by 92°

(5) $\sin 5t + \cos 5t$

Q3) Perform the indicated operation {express in rectangular & polar form}

(a) $\frac{2 + j3}{1 + 8\angle 90^\circ} - 4$

$8\angle 90^\circ$
$\hookrightarrow j\cos\theta + j\sin\theta = 8 \cdot 0 + 8 \cdot j \cdot 1 = 8j$

$\Rightarrow \frac{2 + j3}{1 + 8j} - 4 \Rightarrow \frac{2 + j3 - 4 - j32}{1 + 8j} \times \frac{1 - 8j}{1 - 8j}$

$\Rightarrow \frac{(-2 + 29j)(1 - 8j)}{8} = -2 + 16j + 29j + (29 \times 8)$

$\Rightarrow 230 + 45j$

$\Rightarrow 46 + 9j$

$\frac{2 + j3}{1 + 8\angle 90^\circ} - 4$
$\Downarrow$
$8\angle 90^\circ = 8 \cdot \cos(90) + 8 \cdot j\sin(90) = 8j$

$\frac{2 + j3}{1 + j8} \times \frac{1 - 8j}{1 - 8j} - 4$

$\Rightarrow \frac{2 - 16j + j3 + 24}{65} - 4 \Rightarrow \frac{26 - 13j - 4}{65}$

$\Downarrow$

$\frac{-234 - 13j}{65} \Leftarrow \frac{26 - 13j - 260}{65}$

$x \quad y$ $\boxed{\frac{-18}{5} - \frac{1}{5} \cdot j}$ rectangular form

$\Downarrow$

$r = \sqrt{x^2 + y^2} = \sqrt{\frac{18^2}{25} + \frac{1}{25}}$

$\Rightarrow \sqrt{\frac{325}{25}} = \sqrt{13}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{18}\right) = 3.179^\circ$

polar form $\boxed{\sqrt{13}\angle 3.179^\circ}$

(b) $\left(\frac{10\angle 25^\circ}{5\angle -10^\circ} + \frac{3\angle 15^\circ}{3 - 5j}\right) j2$

Q4) Transform into Phasor form

(a) $11\sin 100t$

(b) $11\cos 100t$

(c) $11\cos(100t - 90^\circ)$

(d) $3\cos 100t - 3\sin 100t$

Q5) Determine equivalent Admittance assuming $w = 1000$ rad/s

(a) 25Ω in series with $20mH$

$\downarrow$

$Z_L = jwL = j \times 1000 \times 20 \times 10^{-3}$
$= j \cdot 20$

Admittance $(Y) = \frac{1}{Z_{eq}}$

$Z_{eq} = 25 + j20 \nearrow Y = \frac{1}{25 + j20} \times \frac{25 - j20}{25 - j20}$

$\boxed{\frac{1}{41} - \frac{4}{205} \cdot j} \Leftarrow \frac{5 - j4}{205} \Leftarrow \frac{25 - j \cdot 20}{625 + 400} \Rightarrow \frac{25 - j \cdot 20}{1025}$

(b) 25Ω in parallel with $20mH$

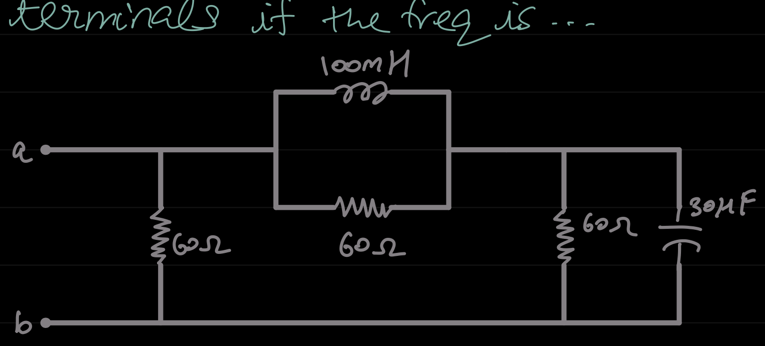
admittance $= \frac{1}{25} + \frac{1}{20j} = \frac{\overset{5}{\cancel{25}} + j\overset{4}{\cancel{20}}}{\underset{100}{\cancel{500}}j}$

$\frac{-j \cdot 5 - 4}{100} \Leftarrow \frac{-j(5 + j \cdot 4)}{100} \Leftarrow \frac{5 + j \cdot 4}{100j}$

$\hookrightarrow \frac{-1}{25} + \left(\frac{-j}{20}\right)$

Q6) Calculate the equivalent impedance as seen at the open terminals if the freq is...

(a) 1Hz
(b) 1KHz
(c) 1MHz
(d) 1GHz



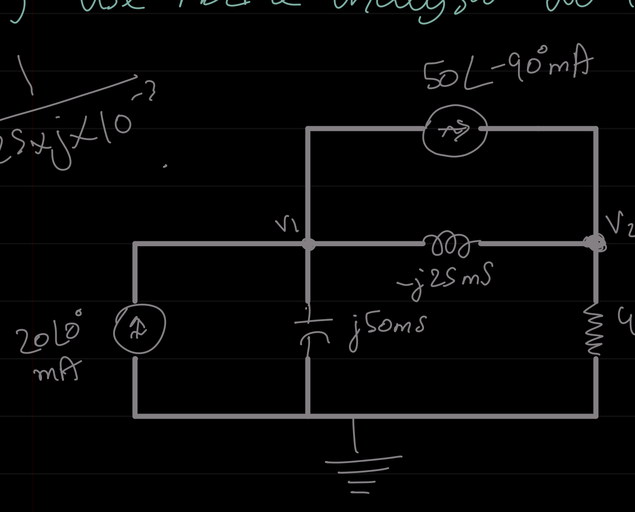
(a) $f = 1Hz$ $\quad w = 2\pi f = 2\pi$ radians/s

$Z_L = jwL = j \cdot 2\pi \cdot 100 \times 10^{-3}$

$\Rightarrow \frac{j \cdot 2\pi}{\cancel{10}5} \Rightarrow \frac{j\pi}{5} \Rightarrow$

Q7) Use nodal analysis to find V_1 and V_2

$\frac{1}{-25 \times j \times 10^{-3}}$



$\boxed{V1}$

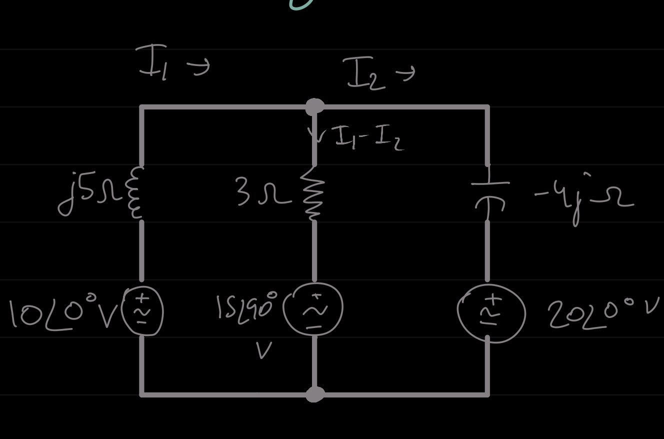
$\frac{V_1 \cdot j}{20} - \frac{(V_1 - V_2)j}{40} + (-50j \times 10^{-3}) + 20 \times 10^{-3} = 0$

$\Rightarrow \frac{5(V_1 j + V_2 j)}{10} - \frac{10j}{10} + \frac{4}{10} = 0$

$\Rightarrow 5V_1 j + 5V_2 j - 10j + 4 = 0 \Rightarrow 4 + j(5V_1 + 5V_2 - 10) = 0$

② $\frac{(V_2 - V_1) \cdot j}{40} + \frac{V_2 - 0}{40 \times 10^{-3}} + (50j \times 10^{-3}) = 0$

Q8) Use mesh analysis to find I_1 and I_2



Apply KVL for ①:

$3(I_1 - I_2) + 15\angle 90^\circ - 10\angle 0^\circ + I_1(j5) = 0$

$3I_1 - 3I_2 + 15\cancel{\cos 90^\circ} + 15j\sin 90^\circ - 10\cos 0^\circ - 10j\cancel{\sin 0^\circ} + I_1 5 \cdot j = 0$

$3I_1 - 3I_2 + 15j - 10 + 5j I_1 = 0$

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

Apply KVL for ②

$-4I_2 \cdot j + 20\angle 0^\circ - 15\angle 90^\circ + (I_2 - I_1)3 = 0$

$-4I_2 \cdot j + 20 - 15j + 3I_2 - 3I_1 = 0$

$3I_2 - 3I_1 + 20 + j(-15 - 4I_2) = 0$

$3I_1 = 3I_2 + 20 + j(-15 - 4I_2)$

Put ② in ①

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

$3\cancel{I_2} + 20 + j(-15 - 4I_2) - 3\cancel{I_2} - 10 + j(15 + 5I_1) = 0$

$10 + j(-15 - 4I_2) + j\left(15 + 5\left[\frac{3I_2 + 20 - 15j - 4I_2 \cdot j}{3}\right]\right) = 0$

$30 - 45j - 12I_2 j + j(45 + 15I_2 + 100 - 75j - 20I_2 j) = 0$

$30 - \cancel{45}j - 12I_2 j + \cancel{45}j + 15I_2 j + 100j + 75 + 20I_2 = 0$

$105 + 20I_2 + j(3I_2 + 100) = 0$

$105 + 20I_2 + j \cdot 3I_2 + j \cdot 100 = 0$

$I_2(20 + j \cdot 3) = -105 - j \cdot 100$

$I_2 = \frac{-105 - j \cdot 100}{20 + j \cdot 3} \times \frac{20 - j \cdot 3}{20 - j \cdot 3}$

$I_2 = \frac{-2100 + 315 \cdot j - 2000j - 300}{409}$

$I_2 = \frac{-2400 - 1685j}{409}$

Home work:1 19/08/24

* Section 10.1

Q1) (a) $5\sin(5t-9^\circ)$

$t=0 \Rightarrow 5\sin(-9^\circ) = -5\sin(9^\circ) = -0.782$

$t=0.01 \Rightarrow 5\sin(0.05-9^\circ) = -5\sin(8.95^\circ)$ X

$\Downarrow$ radians

$5\sin\left(\left(0.05\times\frac{180}{\pi}\right)-9^\circ\right) = 5\sin(2.86^\circ-9^\circ)$

$\Rightarrow -5\sin(6.14) = -0.534$

$t=0.1s \Rightarrow 5\sin(0.5-9^\circ) = 5\sin(28.64-9) = 5\sin(19.64)$

$\Rightarrow 1.6805$

(b) $4\cos 2t$

$t=0 \Rightarrow 4\cos(0) = 4$

$t=1 \Rightarrow 4\cos(2) = 3.997$

$t=1.5 \Rightarrow 4\cos(3) = 3.994$

(c) $3.2\cos(6t+15^\circ)$

$t=0 \Rightarrow 3.2\cos(15^\circ) = 3.09$

$t=0.01 \Rightarrow 3.2\cos(0.06+15^\circ) = 3.2\cos(3.43^\circ+15^\circ)$

$\Rightarrow 3.2\cos(18.43^\circ)$

$\Rightarrow 3.035$

$t=0.1 \Rightarrow 3.2\cos(0.6+15^\circ) = 3.2\cos(34.3^\circ+15^\circ)$

$= 3.2\cos(49.3)$

$= 2.086$

Q2) (a) $300\sin(628t) \rightarrow 300\cdot\cos(628t-90^\circ)$

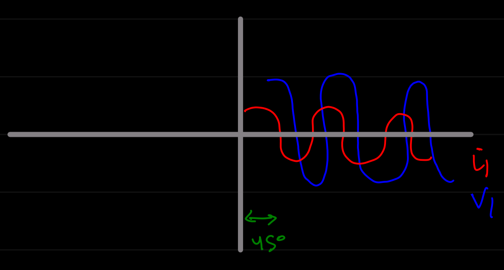
$4\sin(3\pi t+30^\circ) \rightarrow 4\cos(3\pi t-60^\circ)$

$14\sin(50t-5^\circ) - 10\cos 50t \rightarrow$

Q3) $V_1 = 10\cos(10t-45^\circ)$

(a) $i_1 = 5\cos 10t$

-45°

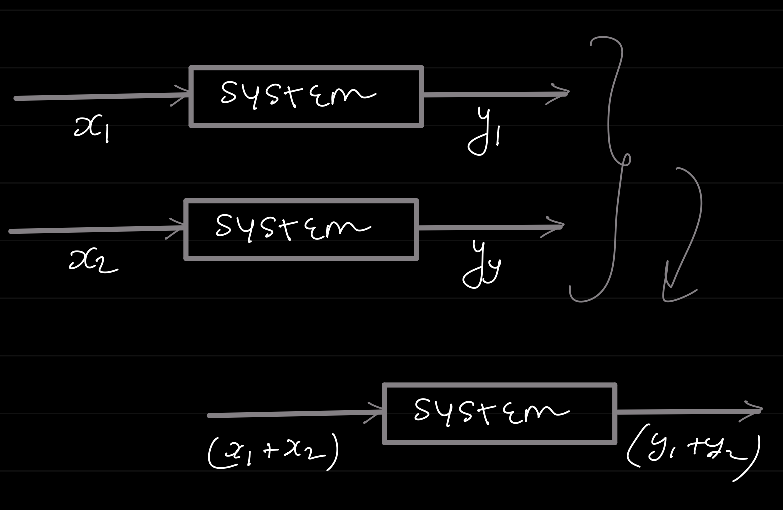


9:30 – 11:00AM
21/08/24

LECTURE: 4

* Superposition principle

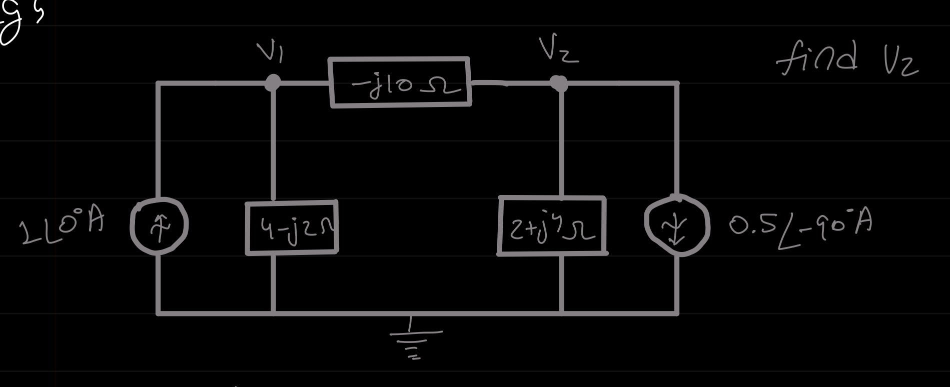
active elements : provide energy eg: volt src / curr src
passive element : consume/ store energy eg: R/L/C



* Multiple independent sources

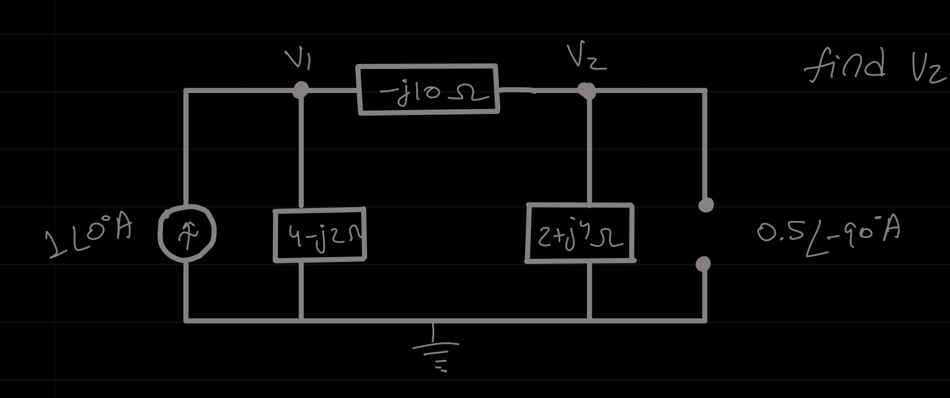
- consider separate independent sources "acting alone"
- add all responses

eg:



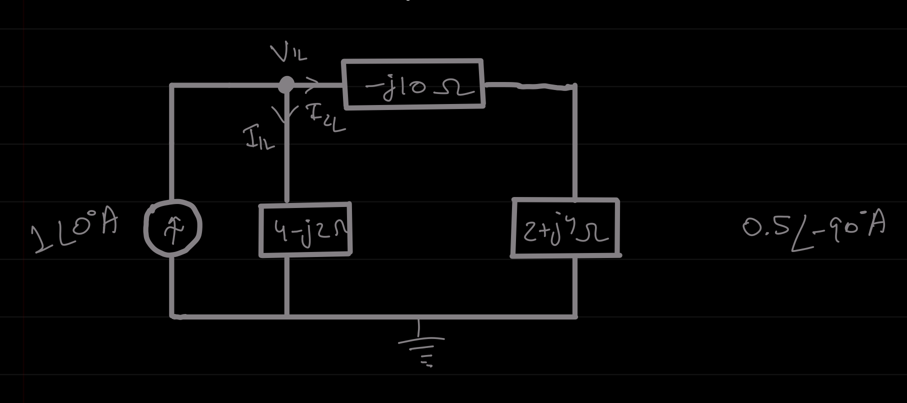
find V_2

consider left side source
so we need to open the circuit at the right source



find V_2

⇓



$V_{1L} = I_{1L}(4-j2)$

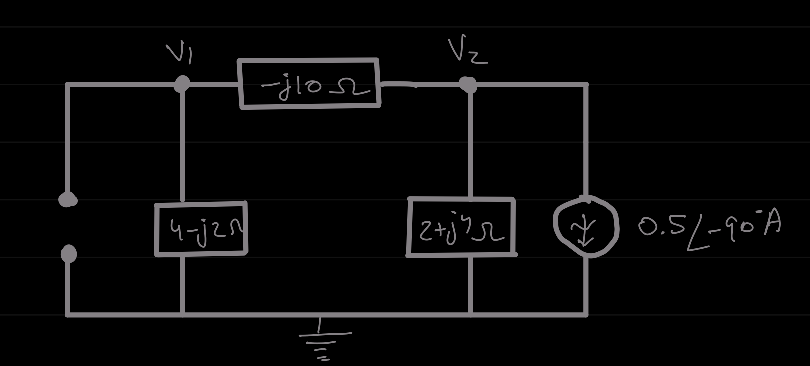
$\Rightarrow \dfrac{-j10+2+j4}{(4-j2)+(-j10+2+j4)}(4-j2)$

$\Rightarrow \dfrac{(2-j6)(4-j2)}{6-j8} \Rightarrow \dfrac{8-j4-24j-12}{6-8j}$

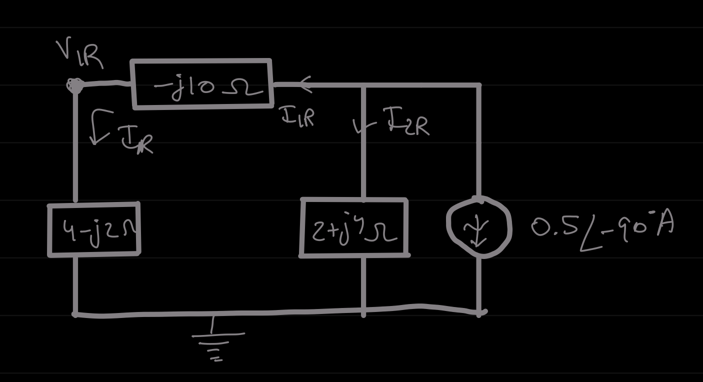
$\Rightarrow \dfrac{-4-28j}{6-8j} = 2-j2V$

$I_{1L}+I_{2L} = 1\angle 0° = 1\cos 0° + j1\sin 0° = 1A$

consider right src



⇓



$V_{1R} = I_{1R}(4-j2)$

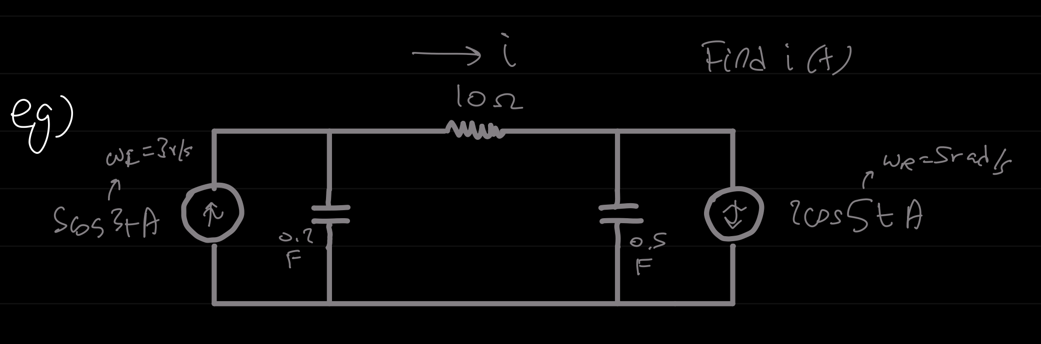
$I_{1R} = \dfrac{(-0.5\angle -90° A)(2+j4)}{-j10+4-2j+2+j4}$

$V_{1R} = \dfrac{(j\cdot 0.5)(2+j4)(4-j2)}{6-j8}$

$\Rightarrow \dfrac{(j-2)(4-j2)}{6-j8} = \dfrac{j4+2-8+j4}{6-j8} = \dfrac{j8-6}{6-j8}$
$= -1V$

$V_1 = V_{1L}+V_{1R} = 2-j2-1 = \boxed{1-j2V}$

eg)

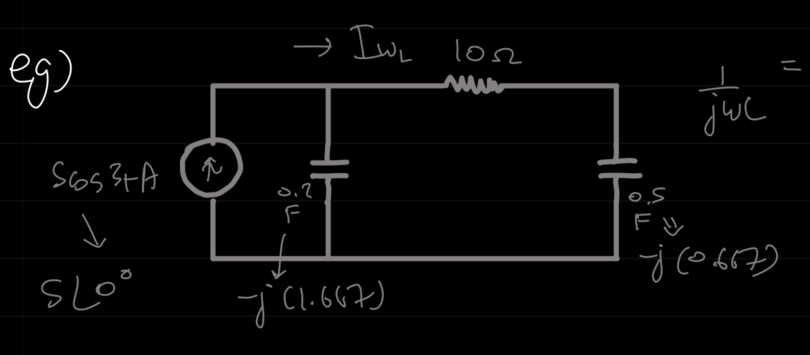


Find i(t)

Note: we have 2 sources operating at 2 different frequencies (ω)

① consider left src $\omega_L = 3rad/s$

eg)

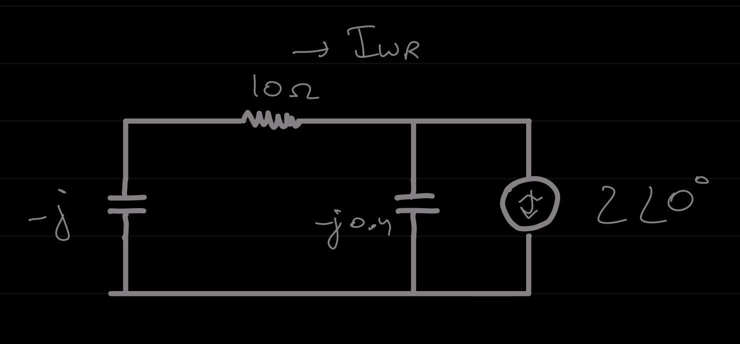


$\dfrac{1}{j\omega C} = \dfrac{1}{j(3)0.2} = \dfrac{-j}{0.6}$

$I_{\omega_1} = 5\angle 0° \left(\dfrac{-j(1.667)}{-j(1.667)+10-j(0.667)}\right)$

$\Rightarrow 811.7\angle -76.86° mA$

② consider right source $\omega_R = 5rad/s$



$I_{\omega_R} = 2\angle 0° \left(\dfrac{-j\cdot 0.4}{-j0.4+10+-j}\right)$

$\Rightarrow \dfrac{-j(0.8)}{-j(1.4)+10} \Rightarrow \dfrac{-j(0.8)[10+j(1.4)]}{100+1.96}$

$\dfrac{-j8+1.12}{101.96} \Rightarrow$

$\Rightarrow 0.0792\angle -82.03° A = 79.2\angle -82.03 mA$

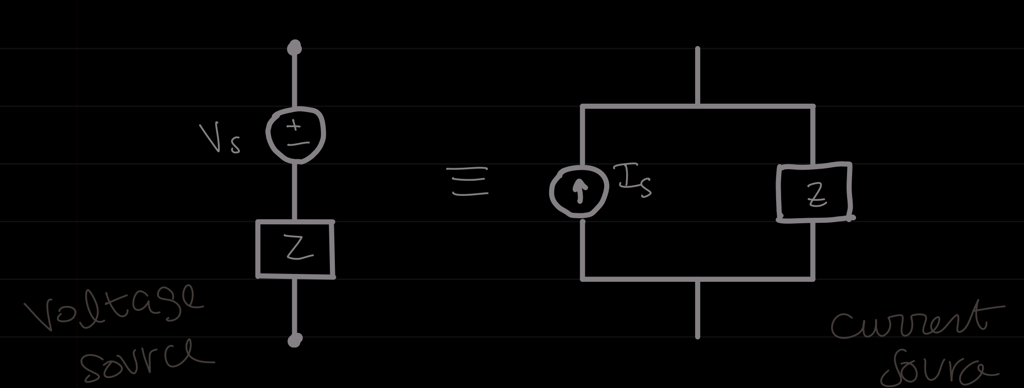
Applying superposition

$I = I_{\omega_L} + I_{\omega_R}$ ✗

Phasors can only be added when they correspond to same frequency

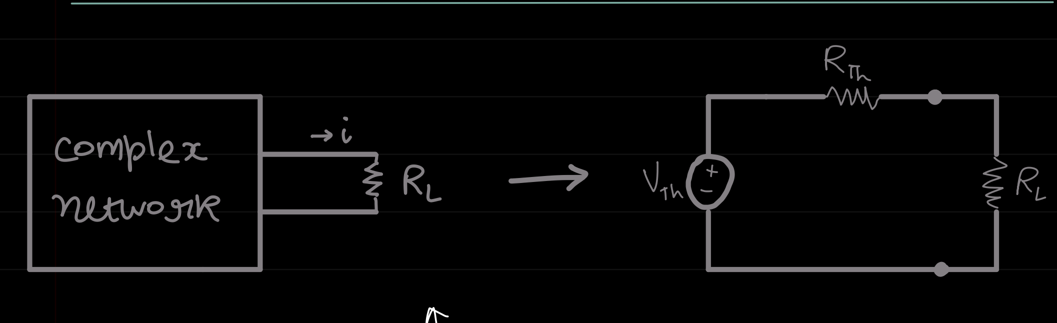
$i(t) = [811.7\cos(3t-76.86°) + 79.2\cos(5t-82.03)]$ mA

* SOURCE TRANSFORMATION

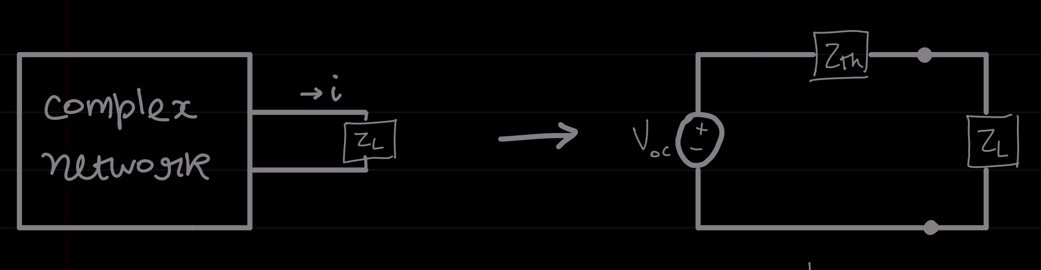


$I_s = \dfrac{V_s}{Z}$

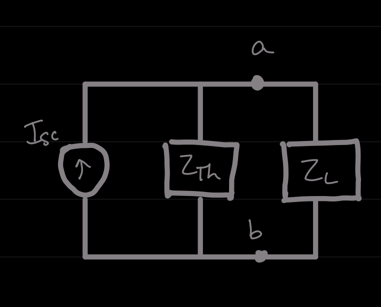
* THEVENIN & NORTON EQUIVALENT CKTS



Normally



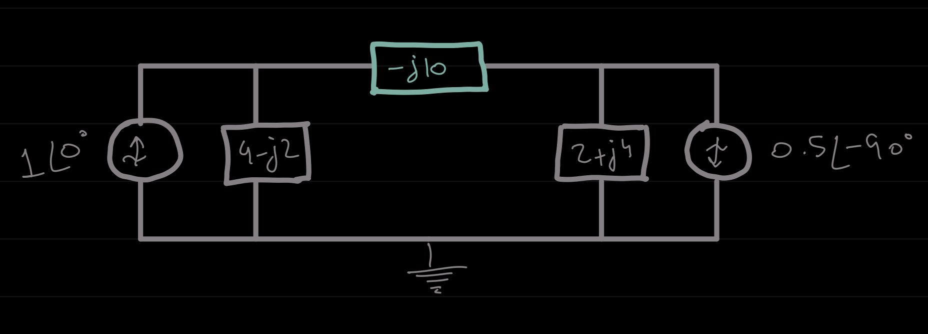
$V_{oc} = I_{sc}\cdot Z_{TH}$



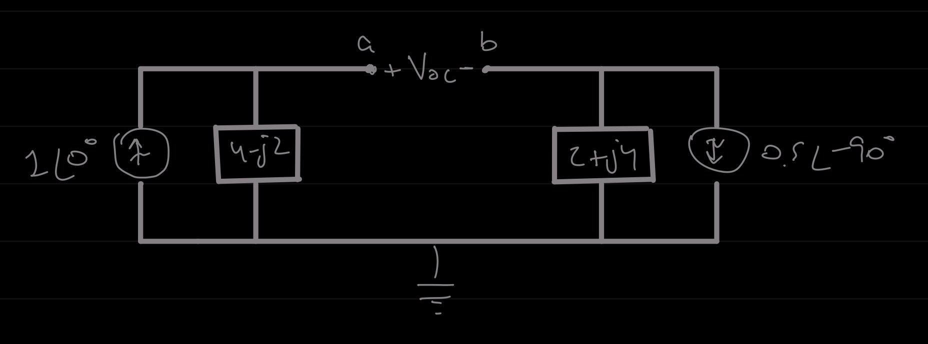
	V_{oc} (by open circuiting)	I_{sc} (by short circuiting)	Z_{TH} (ratio of V_{oc} & I_{sc})
① Independent sources	✓	✓	✓
② Independent + dependent sources	✓	✓	✓

③ Dependent sources only — we need to consider test sources eg: 1A / 1V and then find $V_{oc} \Rightarrow I_{sc} \Rightarrow Z_{Th} = \dfrac{V_{oc}}{I_{sc}}$

eg: find Thevenin equivalent seen by $-j10\Omega$



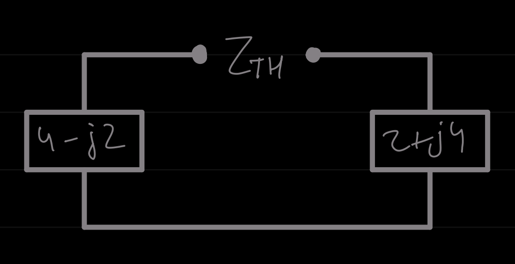
- : complex network
- : R_L



$V_{oc} = V_a - V_b = 1\angle 0°(4-j2) - 0.5\angle -90°(2+j4)$
$= 4-j2+2-j1 = 6-j3\ V$

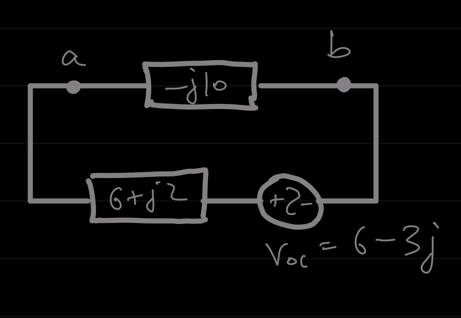
Find Z_{Th}

all volt src → short circuit
all curr src → open circuit



$Z_{TH} = 4-j2+2+j4 = 6+j2\ \Omega$

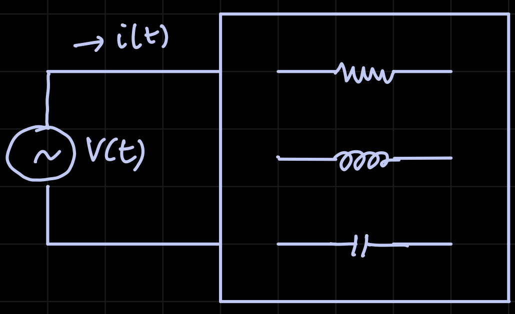
Equivalent circuit



$I_{ab} = \dfrac{V_{oc}}{6+j2-j10} = \dfrac{V_{oc}}{6-j8}$

* Lecture : 5 28/08/24

⇒ AC Power Analysis {Module : 3}



$\cos(A)\cos(B) = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

instantaneous power : $p(t) = v(t) \cdot i(t)$

$$v(t) = V_m \cdot \cos(\omega t + \theta)$$
$$i(t) = I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = V_m \cdot \cos(\omega t + \theta) \cdot I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = \underbrace{\frac{V_m I_m}{2} \cos(\theta - \phi)}_{\text{constant (DC term)}} + \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{\text{twice frequency (harmonic)}}$$

- Average Power

$$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2} \cos(\theta - \phi)\,dt + \underbrace{\frac{1}{T}\int_0^T \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{0}$$

$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)}$$

$$P_{avg} = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$$

$\mathbf{I} = I_m e^{j\phi}$

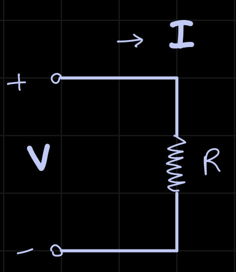
$\mathbf{I}^* = I_m e^{-j\phi}$ (conjugate)

real number ✓

phasor ✗

* Avg. Power absorbed

- by a purely resistive element

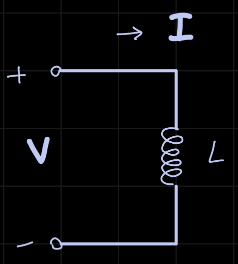


$$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$$

here, phase diff = 0°

$$P_{avg} = \frac{V_m \cdot I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

- by a purely inductive element



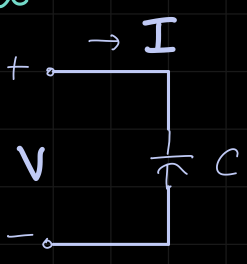
$$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$$

here, phase diff = 90°

So,

$P_{avg} = 0$ for inductor

- by a purely capacitive element



$$P_{avg} = \frac{V_m \cdot I_m}{2} \cdot \cos(-90°)$$

$P_{avg} = 0$ for capacitor

* Lecture: 6

$V(t) = V_m \cos(\omega t + \theta)$ ← Revision

$\downarrow$

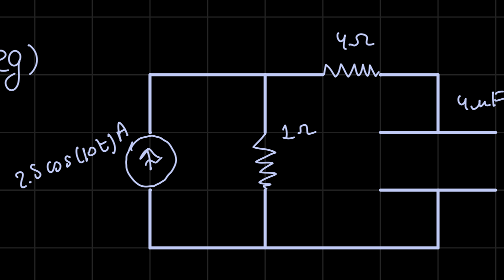
$V(t) = Re\{V_m e^{j\omega t} e^{j\theta}\}$

$\downarrow$ $- e^{j\omega t}$

$V_m e^{j\theta} = V_m \angle \theta$

exponential — polar

• eg)

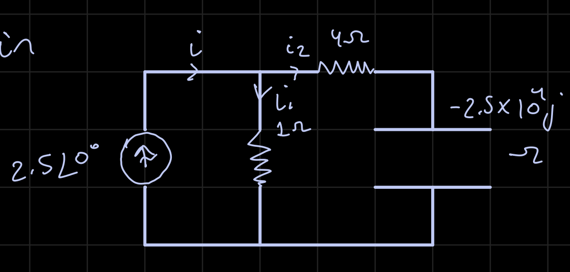


① find power delivered to each element at $t = 0, 10, 20ms$

② Find P_{avg} to each element

Ans) convert to freq. domain

$V_{1\Omega} = V_{4\Omega} + V_c$



① $P_{inst} = \frac{V_m I_m}{2} \cos(\theta - \phi) + \frac{V_m I_m}{2} \cos(2\omega t + \phi + \theta)$

imaginary part is way larger than real part

$I_1 = -2.5 \times \left(\frac{4 - j2.5 \times 10^4}{1 + 4 - j\, 2.5 \times 10^4} \right) \simeq \underline{\underline{-2.5A}}$

$\Rightarrow \frac{-10 + j625 \times 10^{\cancel{00}}}{5 - j\, 25 \times 10^3} \Rightarrow \frac{-2 + j125}{1 - j50}$

$\Rightarrow \frac{-2 + j \cdot 125}{1 - j50} \times \frac{1 + j50}{1 + j50} \Rightarrow$

$I_{1\Omega}(t) = 2.5 \cos(10t)$

$V_{1\Omega} = (I_{1\Omega})(1\,\Omega) \Rightarrow V_{1\Omega}(t) = 2.5 \cos(10t)$

$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{4}$

$p_{1\Omega}(t) = 2.5 \cos(10t) \cdot 2.5 \cos(10t)$ W

$p_{1\Omega}(t = 0) = 6.25W$

$p_{1\Omega}(t = 10ms) = 6.1877W$

$p_{1\Omega}(t = 20ms) = 6.033W$

$(P_{avg})_{1\Omega} = 3.125W$

⇒ P delivered to 4Ω

$P_{4\Omega}(t) = V_{4\Omega}(t) \times I_{4\Omega}(t)$

$I_{4\Omega} = I_s \times \frac{1}{5 - j2.5 \times 10^4} = 10^{-4} \angle 90° A$

$V_{4\Omega} = 4 \times I_{4\Omega} = 4 \times 10^{-4} \angle 90°$ V

convert to time domain

$p_{4\Omega}(t) = 2 \times 10^{-8} + 2 \times 10^{-8} \cos(20t)$ W

$t = 0$: 4×10^{-8} W

$t = 10ms = 3.96 \times 10^{-8}$ W

$t = 20ms = 3.87 \times 10^{-8}$ W

$P_{avg\,4\Omega} = 2 \times 10^{-8}$ W

⇒ P delivered to capacitor

$p_c(t) = V_c(t) \cdot I_c(t)$

$(p_{avg})_C = 0$ since phase diff $= 90°$

$I_c = I_2 = 10^{-4} \angle 90° A \Rightarrow I_c(t) = 10^{-4} \cos(10t + 90°)$

$V_c = (-j\, 2.5 \times 10^{-4}) \times 10^{-4} \angle 90$

$-j \cdot 25 \times 10^3 \times (+j10^{-4})$

$\underline{\underline{2.5\,V}}$

$p_c(t) = 1.25 \times 10^{-4} \cos(20t + 90°)$ W

$t = 0 \Rightarrow p = 0W$

$t = 10ms \Rightarrow p = 2.48 \times 10^{-5}$ W

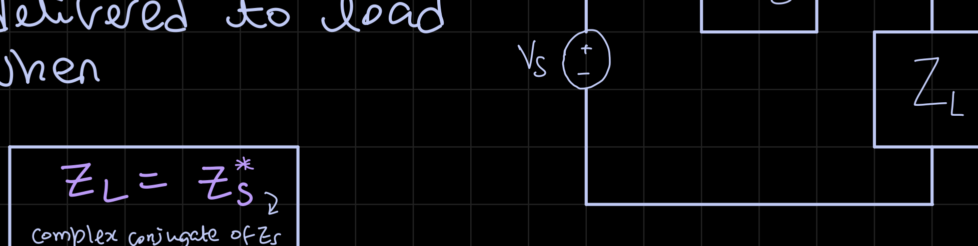
Note: we cannot multiply I_c and V_c in phasor form and then convert to time domain for getting P_c (power) because power does not has a phasor part. It is a real value.

* Correct or not ?

depends on t ✗ ① $p_{1\Omega}(t) + p_{4\Omega}(t) + p_c(t) = $ constant1

✓ ② $p_{avg\,1\Omega} + p_{avg\,4\Omega} + p_{avg\,c} = $ constant2 $\equiv P_{avg}$ source

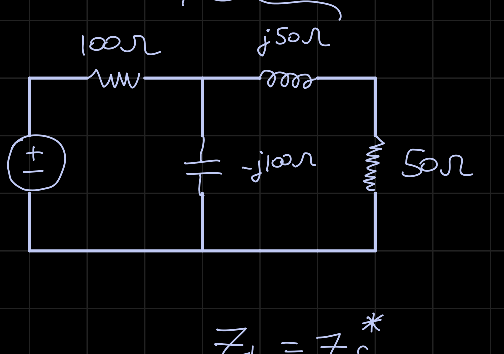
active sign convention — passive sign convention



* Maximum Power Transfer Theorem

maximum power delivered to load when

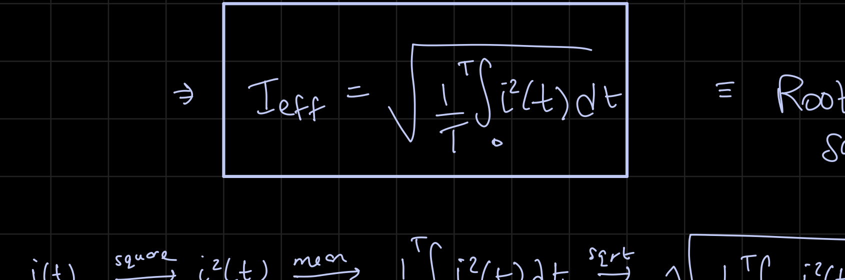
$\boxed{Z_L = Z_S^*}$ complex conjugate of Z_s



⇒ Impedance matching

$Z_L = Z_S^*$

* EFFECTIVE VALUE (Rms Value) of V (voltage) and I (current)



if $P_{avg_1} = P_{avg_2}$

$\Rightarrow \frac{1}{T}\int_0^T i^2(t) R\, dt = I_{eff}^2 R$

$\Rightarrow \boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt}} \equiv$ Root mean square

$i(t) \xrightarrow{\text{square}} i^2(t) \xrightarrow{\text{mean}} \frac{1}{T}\int_0^T i^2(t)\, dt \xrightarrow{\text{sqrt}} \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt}$

• Sinusoidally varying voltage

$i(t) = I_m \cos(\omega t + \phi)$

$\cos^2 x = \frac{2\cos x + 1}{2}$

$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi)\, dt}$

$\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$ current rms value

$\boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$ voltage rms value

eg: $I = 10\angle 90° A \Rightarrow I = 5\sqrt{2} \angle 90° A$ rms

OR $V = 100\sqrt{2} \angle -60° V \Rightarrow V = 100 \angle -60° V_{rms}$

* Apparent Power

$P_{apparent} = I_{eff} V_{eff}$

* Power factor

$PF = \frac{\text{avg power}}{\text{apparent power}} = \frac{(V_m \cdot I_m)/2 \cos(\theta - \phi)}{V_{eff} \cdot I_{eff}}$

$PF = \cos(\underbrace{\theta - \phi}_{\text{power factor angle}})$