tr5frbj 12/08/24

Circuit Theory and Devices

tut as well

Lab: LT Spice (2nd week onwards)

book: Engineering Circuit Analysis (9th Edition)

Relative grading

Labs	20%
Quiz (every 2 weeks (n-1))	20%
Mid sem	30%
End sem	30%

scientific calc ⇐ { Mid sem, End sem }

Course: 9 modules chapter 10 onwards
{continuation of BE}

* Lecture : 1

⇒ Linear Circuits

x (input) $\longrightarrow$ $\boxed{\text{system}}$ $\longrightarrow$ y (output)

system is linear if

$$Kx \rightarrow \boxed{\text{sys}} \rightarrow Ky$$

where k is constant

"R" Resistor: linear element ✓
$V = iR$

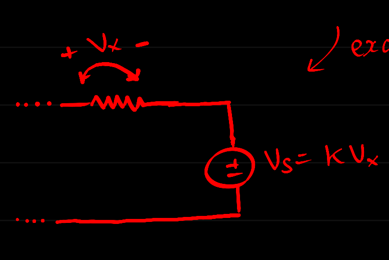
"L" Inductor: linear element ✓
$V = L\dfrac{di}{dt}$

"C" Capacitor: linear element ✓
$i = C\dfrac{dv}{dt}$

* Linear Electric Circuits:

consists of ⇒

1. R, L, C → linear elements
2. Independent voltage & current sources
3. linear dependent sources



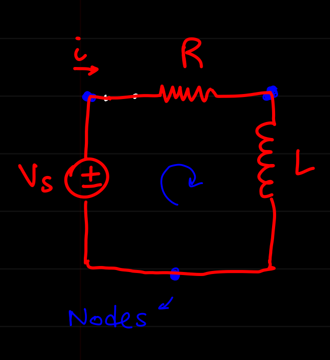
example
would not have been linear if $V_s = KV_x^2$

NOTE: diode and transistors are non-linear elements

* Response of a linear circuit

① full response = transient response + Steady State

transient response	Steady State
temporary	persists $t \to \infty$
due to sudden change	due to source
also called natural response	also called forced response
depends on linear elements R, L, C	depends on current/voltage source



KVL : $-V_s + iR + L\dfrac{di}{dt} = 0$

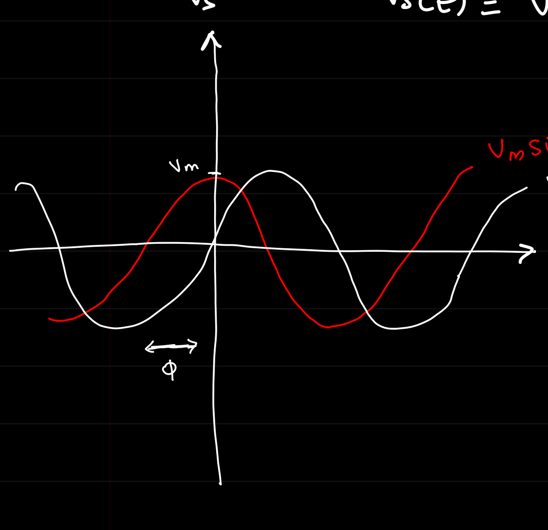
$$i_{final} = i_{transient} + i_{steady\ state}$$

as $t \to \infty$, $i_{final} = i_{steady\ state}$

* Sinusoids & complex numbers

sinusoidally varying voltage source

$$V_s(t) = V_m \cos(\omega t)$$
$$V_s(t) = V_m \sin(\omega t)$$
$$V_s(t) = V_m \sin(\omega t + \phi)$$



② is leading ①
① is lagging ②
} by angle ϕ

* TRIGO

$$\sin(-\theta) = -\sin(\theta)$$
$$\cos(-\theta) = \cos(\theta)$$
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

sin +ve	All +ve
tan +ve	cos +ve

* Complex Numbers

Real : x

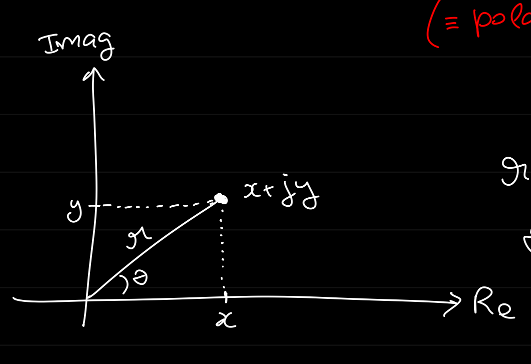
complex : $\underbrace{x + iy}_{\text{rectangular form}}$

x: real part; y: imaginary part; $i = \sqrt{-1}$ a.k.a. j

⇒ Euler's identity: $\underbrace{e^{j\theta}}_{\text{exponential form}} = \underbrace{\cos\theta + j\sin\theta}_{\text{rectangular form}}$

Complex Number

- rectangular: $x + iy$
- exponential: $re^{j\theta}$ ($\equiv$ polar form : $r\angle\theta$)



$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$(r_1 e^{j\theta_1}) \times (r_2 e^{j\theta_2}) = (r_1 \cdot r_2)\, e^{j(\theta_1 + \theta_2)}$$

AC	DC
• we can use transformers to vary voltage	• appliances need constant current
• transported easily over large distances	

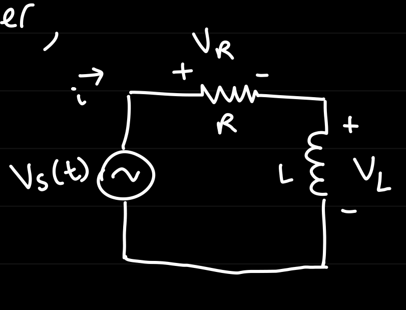
AC → DC : using rectifiers

* MODULE: 1 {CH = 10} 10:30 am

⇒ Sinusoidal steady state analysis

assuming transient has died out long back

Consider,



$V_s(t) = V_m \cos(\omega t)$
find $i(t)$

$$V_s(t) = iR + L\frac{di}{dt}$$

$$\Rightarrow V_m \cos(\omega t) = iR + L\frac{di}{dt}$$

both will take $\cos(\omega t)$ form

General form solution : $i = I_1 \cos \omega t + I_2 \sin \omega t$

$$\Rightarrow V_m \cos(\omega t) = (I_1 \cos \omega t + I_2 \sin \omega t)R + L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{d}{dt}(I_1 \cos \omega t + I_2 \sin \omega t)$$
$$\Rightarrow \omega(I_1(-\sin \omega t) + I_2 \cos \omega t)$$
$$\Rightarrow \omega(I_2 \cos \omega t - I_1 \sin \omega t)$$

$$V_m \cos(\omega t) = (I_1 \cos \omega t + I_2 \sin \omega t)R + (I_2 \cos \omega t - I_1 \sin \omega t)L\omega$$

$$\Rightarrow \cos \omega t \cdot (I_1 R + I_2 L) + \sin \omega t \cdot (I_2 R - I_1 L)$$

$$\Rightarrow (I_1 R + I_2 L - V_m)\cos(\omega t) + (I_2 R - I_1 L)\sin(\omega t) = 0$$

* at $t = 0$,
$$(I_1 R + I_2 L - V_m) \cdot 1 + (I_2 R - I_1 L) \cdot 0 = 0$$

$$\boxed{I_1 R + I_2 L = V_m}$$

* at $t = \dfrac{\pi}{2\omega}$, $(I_1 R + I_2 L - V_m) \cdot 0 + (I_2 R - I_1 L) \cdot 1 = 0$

$$\boxed{I_1 = I_2 \cdot \frac{R}{L}}$$

$I_1 R + I_2 L = V_m \Rightarrow I_2 \dfrac{R^2}{L} + I_2 L = V_m \Rightarrow I_2\left(\dfrac{R^2 + L}{L^2}\right) = V_m \Rightarrow I_2 = \dfrac{V_m L^2}{R^2 + L}$

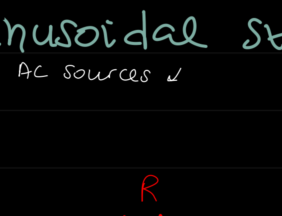
$I_1 = \dfrac{V_m L R}{R^2 + L}$

14/08/24 # Lecture : 2 9:30 AM

Wednesday → Quiz : Chp 1, 2, 3

* Sinusoidal Steady state analysis

AC sources ↙ ↳ not considering any transient response



angular freq

$V_s = V_m \cos(\omega t)$

$freq\ (hz) = \frac{\omega}{2\pi}$

$V_s(t) = i(t) R + L \frac{di(t)}{dt}$

general solⁿ: $i = I_1 \cos(\omega t) + I_2 \sin(\omega t)$

put i from gen solⁿ in above eqⁿ

$I_1 = \frac{R V_m}{R^2 + \omega^2 L^2}$, $I_2 = \frac{\omega L V_m}{R^2 + \omega^2 L^2}$

$i(t) = \frac{R V_m}{R^2 + \omega^2 L^2} \cos(\omega t) + \frac{\omega L V_m}{R^2 + \omega^2 L^2} \sin(\omega t)$

Note: use $\cos(A+B) = \cos A \cos B - \sin A \sin B$

⇒ So, $i(t) = A\cos(\omega t + \phi)$: general solⁿ

becomes ⇒ $i(t) = A[\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)]$

$\frac{R V_m}{R^2 + \omega^2 L^2} \cos(\omega t) + \frac{\omega L V_m}{R^2 + \omega^2 L^2} \sin(\omega t) = A\cos(\omega t)\cos(\phi) - A\sin(\omega t)\sin(\phi)$

⇓

$A\cos(\phi) = \frac{R V_m}{R^2 + \omega^2 L^2}$, $A\sin(\phi) = \frac{-\omega L V_m}{R^2 + \omega^2 L^2}$

$A^2\cos^2(\phi) + A^2(\sin^2\phi) = A^2(1) = A^2 = \frac{R^2 V_m^2 - \omega^2 L^2 V_m^2}{R^2 + \omega^2 L^2}$

⇓

$\frac{V_m^2 (R^2 - \omega^2 L^2)}{R^2 + \omega^2 L^2}$

$A = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$

$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$

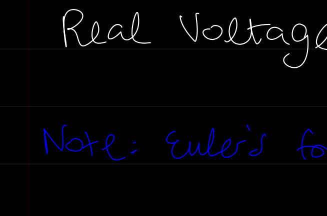
$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

⇒ Consider a complex source

$V_m \cos(\omega t) + j V_m \sin(\omega t) = V_m e^{j\omega t}$

Real Voltage source $= V_m \cos(\omega t) = Re\{V_m e^{j\omega t}\}$

Note: Euler's formula → $e^{j\theta} = \cos\theta + j\sin\theta$



$V_m e^{j\omega t} = iR + L\frac{di}{dt}$

dont integrate ✗

general solⁿ ✓

Assume $i = I_m e^{j(\omega t + \phi)}$

put eqⁿ

$V_m e^{j\omega t} = I_m e^{j\omega t + j\phi} R + L(j\omega) I_m e^{j\omega t + j\phi}$

⇒ $I_m e^{j\omega t + j\phi} = \frac{V_m e^{j\omega t}}{R + j\omega L}$

is this the real current? No

current cannot be complex

Real $i(t) = Re\{I_m e^{j\omega t + j\phi}\} = Re\left\{\frac{V_m}{R + j\omega L} e^{j\omega t}\right\}$

find the real part of this complex number by simplifying

{multiply by the conjugate ≡ factorize}

$i(t) = \frac{V_m}{R + j\omega L} \cdot \left(\frac{R - j\omega L}{R - j\omega L}\right) e^{j\omega t} = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} e^{-j\tan^{-1}\left(\frac{\omega L}{R}\right)} \cdot e^{j\omega t}$

⇒ Note: rect form → polar form

$x + jy \rightleftharpoons r e^{j\theta}$

$x = r\cos\theta$, $y = r\sin\theta$

$r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Real $i(t) = Re\left\{\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cdot e^{j\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)}\right\}$

* $i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$

Same answer as when we were not considering complex numbers

* PHASOR

$i(t) = I_m \cos(\omega t + \phi) = Re\{I_m e^{j\omega t + j\phi}\}$

⇓ remove $e^{j\omega t}$, $Re\{\ldots\}$

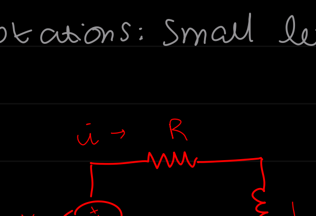
not changing with time

complex func ✗

complex number ✓

$\underbrace{I_m e^{j\phi}}_{\text{phasor}} = I_m \angle \phi$

time domain	phasor representation
$i(t) = I_m \cos(\omega t + \phi)$	$I_m \angle \phi$
$v(t) = V_m \cos(\omega t + \theta)$	$V_m \angle \theta$
⇒ real function of t	⇒ complex, constant
⇒ notations: small letters	⇒ Bold, capital letters



$i(t) = I_m \cos(\omega t + \phi)$ ✓

$i(t) = Re\{I_m e^{j\omega t + j\phi}\}$ ✓

$i(t) = I_m \angle \phi$ ✗ complex constant

$i(t) = I_m e^{j(\omega t + \phi)}$ ✗ complex

$i(t) = I_m \angle \omega t + \phi$ ✗ complex phasor notation

phasor notation

but we can write ⇒ phasor notation of $i(t) = I_m \angle \phi = I_m \angle(\omega t + \phi)$

Note: $\angle \phi = e^{j\phi}$ or $\angle \omega t + \phi = e^{j\omega t + j\phi}$

$\underbrace{V_m e^{j\theta}}_{V} = (R + j\omega L) \underbrace{I_m e^{j\phi}}_{I}$

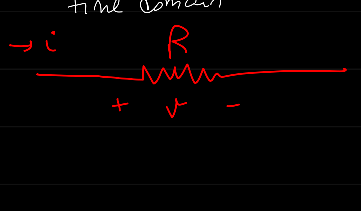
$\boxed{V = (R + j\omega L) I}$

↳ phasor notation

* phasor representation of relationship between i and v for R, L, C

① RESISTOR

time domain



$v = iR$

consider complex voltage & current

$v = V_m e^{j(\omega t + \theta)}$

$i = I_m e^{j(\omega t + \phi)}$

$V_m e^{j\omega t + j\theta} = R \cdot I_m e^{j\omega t + j\phi}$

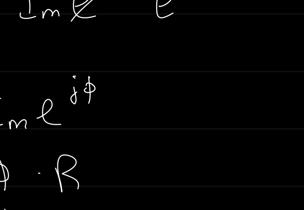
$V_m \cancel{e^{j\omega t}} \cdot e^{j\theta} = R \cdot I_m \cancel{e^{j\omega t}} e^{j\phi}$

$V_m e^{j\theta} = R \cdot I_m e^{j\phi}$

$\underbrace{V_m \angle \theta}_{V} = \underbrace{I_m \angle \phi}_{I} \cdot R$

$\boxed{V = IR}$

② INDUCTOR



$v = L\frac{di}{dt}$

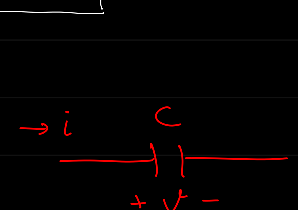
$v \to V_m e^{j(\omega t + \theta)}$ $i \to I_m e^{j\omega t + j\phi}$

$V_m e^{j\omega t} e^{j\theta} = L\, I_m e^{j\omega t} \cdot e^{j\phi} (j\omega)$

$\underbrace{V_m e^{j\theta}}_{V} = j\omega L \cdot \underbrace{I_m e^{j\phi}}_{I}$

$\boxed{V = j\omega L \cdot I}$

③ CAPACITOR



$i = C\frac{dv}{dt}$

$i \to I_m e^{j(\omega t + \phi)}$

$v \to V_m e^{j(\omega t + \theta)}$

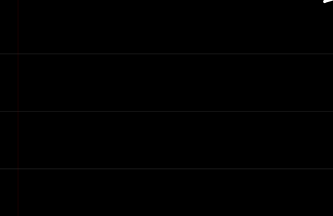
$I_m \cancel{e^{j\omega t}} \cdot e^{j\phi} = C \cdot V_m \cancel{e^{j\omega t}} e^{j\theta} \cdot j\omega$

$\underbrace{I_m e^{j\phi}}_{I} = C \cdot j\omega \cdot \underbrace{V_m e^{j\theta}}_{V}$

$I = j\omega C \cdot V$

$\boxed{V = \frac{1}{j\omega C} \cdot I}$

* PHASOR Representation using KVL



$V_s = V_R + V_L$

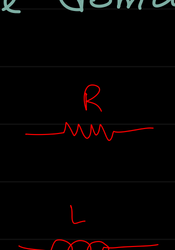
$= IR + (j\omega L) I$

$= I(R + j\omega L)$

$V_s = V_m \cos(\omega t + \theta)$ ✗ time domain

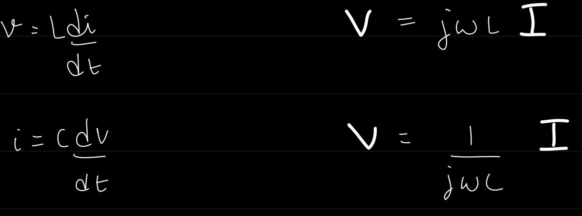
$V_s = V_m e^{j(\omega t + \theta)}$ ✓ phasor notation

$I = \frac{V_s}{R + j\omega L} = \frac{V_m \angle \theta}{R + j\omega L}$



Time domain	Phasor (≡ Frequency domain)
$v = iR$	$V = IR$
$v = L\frac{di}{dt}$	$V = j\omega L\, I$
$i = C\frac{dv}{dt}$	$V = \frac{1}{j\omega C} I$

⇒ Example



① Given: both AC sources, V_s & I_s operate at $\omega = 2\ rad/s$

② Given: $I_c = 2\angle 28°\ A$

Find (i) V_c

(ii) I_{R_2}

(iii) I_s

① $V_c = \frac{I_c}{j\omega C} = \frac{1}{j(2)(1)} \cdot 2\angle 28°$

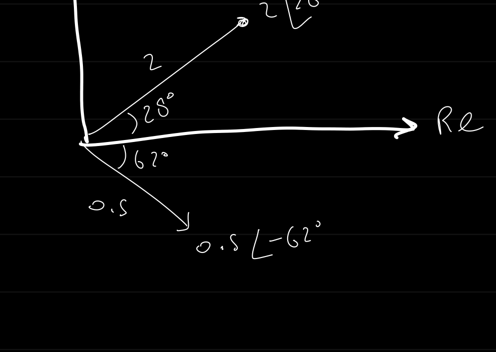
Note: $\frac{1}{j} = -j = -90°$

we add the angles directly which are being multiplied

$V_c = (0.5\angle -90°)(2\angle 28°) = 1\angle -62°\ V$

② $I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_{R_2}}{2} = \frac{1\angle -62°}{2} = 0.5\angle -62°\ A$

③ $I_s = 0.5\angle -62° + 2\angle 28°$



$2.06\angle 14°\ A$

- Phasor: $i(t) = I_m \cos(\omega t + \phi) = Re\{I_m e^{j(\omega t+\phi)}\}$

Remove $e^{j\omega t}$ and $Re\{...\}$

$$I_m e^{j\phi} = I_m \angle \phi \quad \} \text{ Phasor}$$

exponential form   polar form

time domain ⇄ freq. domain

$V_R = IR \qquad V_L = j\omega L I \qquad V_c = \frac{1}{j\omega C} I$

* IMPEDANCE

$$Z = \frac{V}{I} = \frac{1}{R}$$

$Z_R = R$ — voltage and current in the same phase

$Z_L = j\omega L$ — voltage leads current by 90°

$Z_c = \frac{1}{j\omega C}$ — voltage lags current by 90°

$$Z = x + jy$$

resistance ↙ ↘ reactance

* Admittance

$$Y = \frac{1}{Z}$$

$$Y = x + jy$$

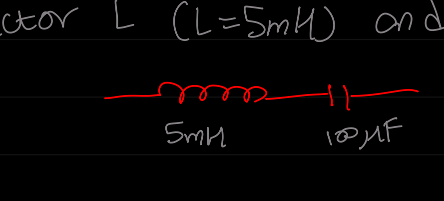
conductance ↙ ↘ susceptance

* Impedance combination

Series : $Z_{eq} = Z_1 + Z_2 + \dots Z_N$

Parallel : $Z_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots \frac{1}{Z_N}$

eg} Find impedance of series combination of an inductor L (L=5mH) and capacitor (C=100μF) $\omega = 10^4$ rad/s



$\Rightarrow Z_{eq} = Z_L + Z_C = j\omega L + \frac{1}{j\omega C} = 10^4 \cdot j \cdot 5\times10^{-3} + \frac{10^{-4}\times10^{3}}{j\cdot 5\times 100 \times 10^{-6}}$

Note: $\frac{1}{j} = -j$ $\qquad \Rightarrow 50j - j = 49j$

Note: $e^{j\theta} = \cos\theta + j\sin\theta = e\angle\theta$ $\qquad \Rightarrow 0 + 49j = 49e^{j\cdot\frac{\pi}{2}}$

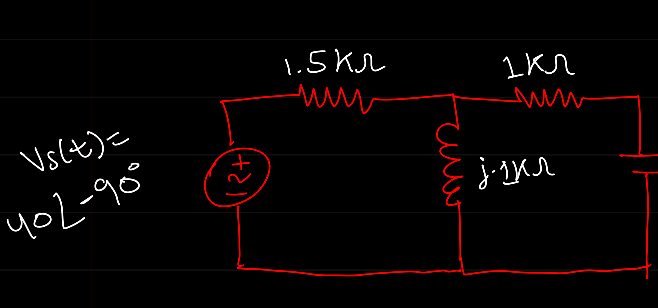
eg:

Find i(t)

$\frac{10^3(1-2j)(1+j)}{2}$

$500(1-2j)(1+j)$ $\quad 10^3\frac{(1-2j)}{1-j}$

step1} Convert to freq. domain (write in terms of phasor and Z)



$2000 - 2000j \parallel 1000j$

$\left(\frac{1}{1000-2000j} + \frac{1}{1000j}\right)^{-1}$

$\frac{1-j}{10^3 - 2\cdot10^3 j}$

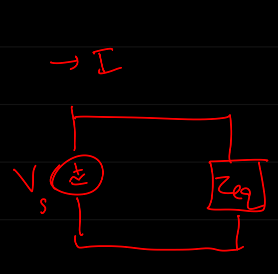
$\Rightarrow \frac{1000j + 1000 - 2000j}{(10^3 - 2\cdot10^3 j)10^3 j}$

$\frac{1}{3}H \rightarrow Z_L = j\omega L = j\cdot 3000 \cdot \frac{1}{3} \Rightarrow j\,1K$

$\frac{1}{6}\mu F \rightarrow Z_c = \frac{1}{j\omega C} = \frac{1}{j\cdot 3000 \times \frac{1}{6} \times 10^{-6}} \Rightarrow \frac{2\cdot10^3}{j}$

$\Rightarrow -j\cdot 2K\Omega$

step2}



$$I = \frac{V_s}{Z_{eq}}$$

$Z_{eq} = 1.5K + [500(1+j)(1-2j)]$

$\Rightarrow 1500 + 500[1 - 2j + j + 2]$

$\Rightarrow 500[3 + 3 - j] \Rightarrow 500[6 - j]$

$\Rightarrow 3000 - 500j$

$\frac{j(1-2j)(1+j)}{2} \Rightarrow \frac{(j+2)(1+j)}{2} \Rightarrow \frac{j+j^2+2+2j}{2}$

PPT} $\qquad Z_{eq} = 1.5 + \frac{(jL)(1-j2)}{1-j}$ KΩ $\qquad \frac{3j+1}{2}$

$\Rightarrow \frac{4+3j}{2}$

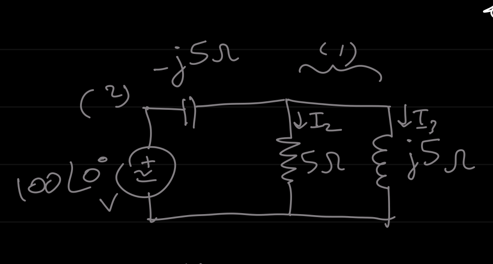
Note: complex number $<$ $x+iy$ / $r\angle\theta$

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$I_3 + I_2 = I_1$

eg}



find I_1, I_2, I_3

$Z_{eq}^{(1)} = \left(\frac{1}{5} + \frac{1}{5j}\right)^{-1} = \left(\frac{5j+5}{25j}\right)^{-1} = \frac{5j}{j+1}$

$Z_{eq} \Rightarrow^{(2)} -5j + \frac{5j}{j+1} \Rightarrow \frac{+5 - 5j + 5j}{j+1} = \frac{5}{j+1}$

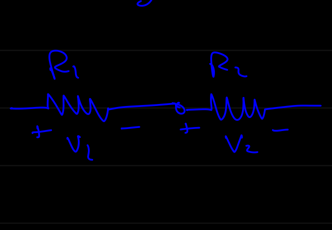
$\Rightarrow \frac{5(j-1)}{-2} \Rightarrow \frac{5-5j}{2}$

$\frac{5}{2} - \frac{5}{2}j$ $\qquad \frac{5\sqrt{2}}{2}$

$r = \sqrt{x^2+y^2} = \sqrt{\frac{25}{4} + \frac{25}{4}} = \frac{\sqrt{50}}{2}$

$\theta = \tan^{-1}(-1) \Rightarrow -45°$

Voltage division



$V_1 = V\cdot\frac{R_1}{R_1+R_2}$

Current division

$i_1 = i\cdot\frac{R_2}{R_1+R_2}$

$I_1 = \frac{V}{Z_{eq}} = \frac{\cancel{100}^{20}\angle 0°}{\frac{5\sqrt{2}}{2}\angle -45°} \Rightarrow \frac{40\angle 0°}{\sqrt{2}\angle -45°}$

$\frac{40(\cos 0 + j\sin 0)}{\sqrt{2}(\cos(45) - j\sin(45))}$ $\qquad r\angle\theta = re^{j\theta} =$

$\frac{40}{\sqrt{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)} \Rightarrow \frac{40}{1-j} \Rightarrow \frac{40(1+j)}{2} = 20(1+j)$

$20 + 20j$

$r = \sqrt{x^2+y^2} \Rightarrow \sqrt{400+400}$

$\Rightarrow 20\sqrt{1+1} = 20\sqrt{2}$

$\theta = \tan^{-1}(1) = 45°$

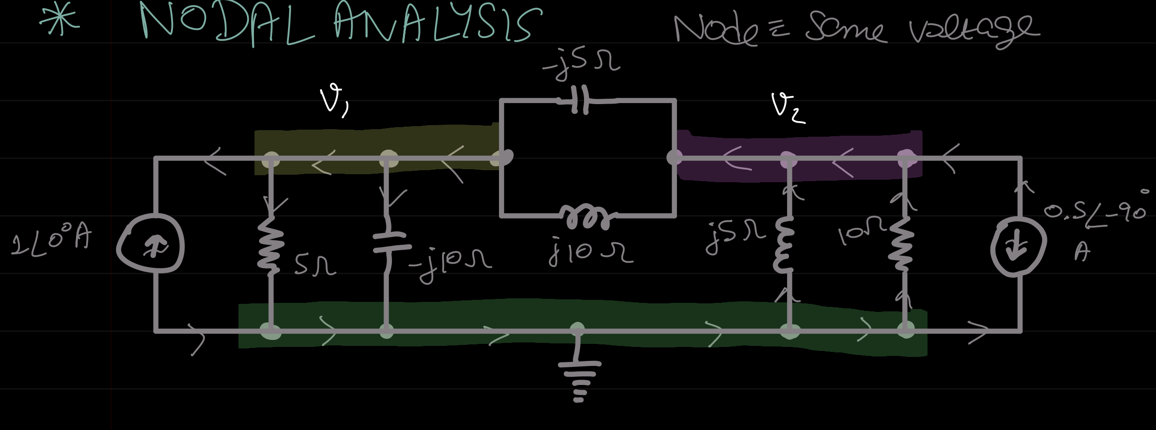
$I_1 = 20\sqrt{2}\angle 45°$

$I_2 = I_1 \times \frac{j5}{5+j5} \Rightarrow \cancel{20}^{10}\sqrt{2}\angle 45° \times \frac{j(1-j)}{\cancel{2}}$

$\Rightarrow 10\sqrt{2}\angle 45° \times (j+1)$

* NODAL ANALYSIS

Node ≡ Same voltage



at node 1:

$1\angle 0° = \frac{V_1}{5} + j\frac{V_1}{10} + \frac{(V_2 - V_1)j}{10} - \frac{(V_2-V_1)j}{5}$

$r\cos\theta + rj\sin\theta$

$r[1 + 0] \Rightarrow 1$

$10 = 2V_1 + \cancel{jV_1} + jV_2 - \cancel{jV_1} - 2V_2 j + 2V_1 j$

$10 = 2V_1 - jV_2 + 2jV_1$

$10 = 2V_1 + j(2V_1 - V_2)$ ①

at node 2:

$\frac{V_2 - V_1}{j10} + \frac{V_2 - V_1}{-j5} + \frac{V_2}{j5} + \frac{V_2}{10} = 0.5\angle -90°$

$0.5\cancel{\cos 90°} - 0.5j\sin 90°$

$\Rightarrow -0.5j$

$V_2 - V_1 + 2V_1 - \cancel{2V_2} + \cancel{2V_2} + V_2 j = 5$

$3V_2 - V_1 + V_2 j = 5 \Rightarrow V_1 = 3V_2 - 5 + V_2 j$

Put value of V_1 in ①

$10 = 6V_2 - 10 + 2V_2 j + j(6V_2 - 10 + 2V_2 j - V_2)$

$10 = 6V_2 - 10 + 2V_2 j + 6V_2 j - 10j - 2V_2 - V_2 j$

$20 = 4V_2 + 7V_2 j - 10j$ $\qquad \frac{30-20j}{13}$

$20 + 10j = V_2(4 + 7j)$ $\qquad \frac{150-100j}{65}$

$V_2 = \frac{(20+10j)(4-7j)}{16+49} = \frac{80 - 140j + 40j + 70}{65}$

$V_2 = \frac{30}{13} - j\left(\frac{20}{13}\right)$ $\qquad r = \sqrt{\frac{900+400}{169}} = \frac{10}{\sqrt{13}}$

$\theta = \tan^{-1}\left(\frac{-2}{3}\right)$

$V_2 = \frac{10\sqrt{13}}{13}\angle -33.69°$

$= -\tan\left(\frac{2}{3}\right) =$

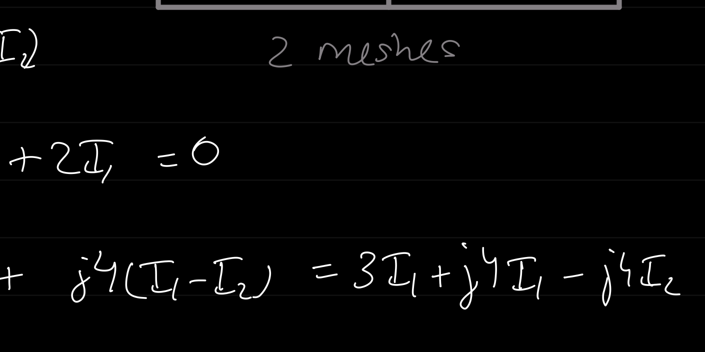
$= -33.69$

$V_1 = 3V_2 - 5 + jV_2$

$\Rightarrow V_2(3+j) - 5 = \frac{30}{13} - \frac{20j}{13}(3+j) - 5$

$\Rightarrow \frac{30 - 60j + 20 - 65}{13} = \frac{-60j - 15}{13} \Rightarrow V_1 = \frac{15\sqrt{17}}{13}\angle -104°$

* mesh Analysis



2 meshes

$10\angle 0° = 3I_1 + j4(I_1 - I_2)$

$(I_2 - I_1)j4 - I_2(j2) + 2I_1 = 0$

$10\angle 0° = 10 = 3I_1 + j4(I_1 - I_2) = 3I_1 + j4I_1 - j4I_2$

$j4I_2 + 10 = I_1(3$

$j4I_2 - j4I_1 - j2I_2 + 2I_1 = 0$

$j4I_2 - j4(10 -$

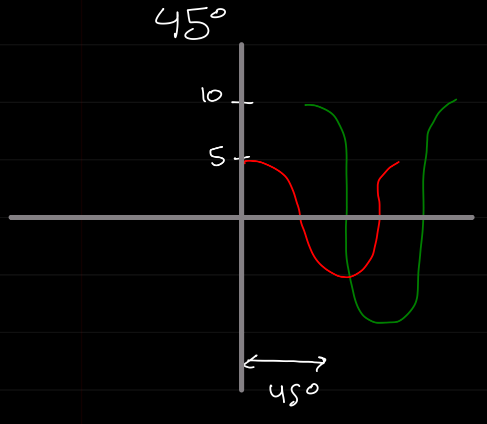
* CTD Tutorial : 1

19/08/24 1:00 – 3:00

Q1) Determine the angle by which v_1 leads i_2 if $V_1 = 10\cos(10t - 45°)$ and i_1 is equal to

(1) $i_1 = 5\cos 10t$

$45°$



(3) $i_1 = 5\cos(10t - 40°)$

$5°$

(2) $i_1 = 5\cos(10t - 80°)$

$35°$

(4) $i_1 = 5\cos(10t + 40°)$

$85°$

(5) $5\sin(10t - 19°)$

$= 5\cos(10t - 19° - 90°)$

$= 5\cos(10t - 109°)$

Ans : $64°$

Q2) Determine which waveform in each of the following pairs is lagging

(1) $\cos 4t$ leads $\sin 4t$ by $\frac{\pi}{2}$ {90°}

(2) $\cos(4t - 80°)$ lags $\cos(4t)$ by $80°$

(3) $\cos(4t + 80°)$ leads $\cos(4t)$ by $80°$

(4) $\sin(5t)$ lags $\cos(5t + 2°)$ by $92°$

(5) $\sin 5t + \cos 5t$

Q3) Perform the indicated operation {express in rectangular & polar form}

(a) $\frac{2+j3}{1+8\angle 90°} - 4$

$8\angle 90°$

$\hookrightarrow r\cos\theta + jr\sin\theta = 8.0 + 8.j.1 = 8j$

$\Rightarrow \frac{2+j3}{1+8j} - 4 \Rightarrow \frac{2+j3-4-j32}{1+8j} \times \frac{1-8j}{1-8j}$

$\Rightarrow \frac{(-2+29j)(1-8j)}{8} = -2+16j+29j+(29\times8)$

$\Rightarrow 230 + 45j$

$\Rightarrow 46 + 9j$

$\frac{2+j3}{1+8\angle 90°} - 4$

$\Downarrow$

$8\angle 90° = 8\cdot\cos(90) + 8.j\sin(90) = 8j$

$\frac{2+j3}{1+j8} \times \frac{1-8j}{1-8j} - 4$

$\Rightarrow \frac{2-16j+j3+24}{65} - 4 \Rightarrow \frac{26-13j}{65} - 4$

$\Downarrow$

$\frac{-234-13j}{65} \Leftarrow \frac{26-13j-260}{65}$

$x \quad y \quad \boxed{\frac{-18}{5} - \frac{1}{5}.j}$ rectangular form

$\Downarrow$

$r = \sqrt{x^2+y^2} = \sqrt{\frac{18^2}{25} + \frac{1}{25}}$

$\Rightarrow \sqrt{\frac{325}{25}} = \sqrt{13}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{18}\right) = 3.179°$

polar form $\boxed{\sqrt{13}\angle 3.179°}$

(b) $\left(\frac{10\angle 25°}{5\angle -10°} + \frac{3\angle 15°}{3-5j}\right) j2$

Q4) Transform into Phasor form

(a) $11\sin 100t$

(b) $11\cos 100t$

(c) $11\cos(100t - 90°)$

(d) $3\cos 100t - 3\sin 100t$

Q5) Determine equivalent Admittance assuming $w = 1000$ rad/s

(a) 25Ω in series with $20mH$

$\Downarrow$

$Z_L = jwL = j \times 1000 \times 20 \times 10^{-3}$

$= j \cdot 20$

Admittance $(Y) = \frac{1}{Z_{eq}}$

$Z_{eq} = 25 + j20 \nearrow \quad Y = \frac{1}{25+j20} \times \frac{25-j20}{25-j20}$

$\boxed{\frac{1}{41} - \frac{4}{205}.j} \Leftarrow \frac{5-j4}{205} \Leftarrow \frac{25-j\cdot20}{625+400} \Rightarrow \frac{25-j\cdot20}{1025}$

(b) 25Ω in parallel with $20mH$

$\text{admittance} = \frac{1}{25} + \frac{1}{20j} = \frac{\overset{5}{\cancel{25}} + j\overset{4}{\cancel{20}}}{\underset{100}{\cancel{500}}j}$

$\frac{-j.5-4}{100} \Leftarrow \frac{-j(5+j.4)}{100} \Leftarrow \frac{5+j.4}{100j}$

$\hookrightarrow \frac{-1}{25} + \left(\frac{-j}{20}\right)$

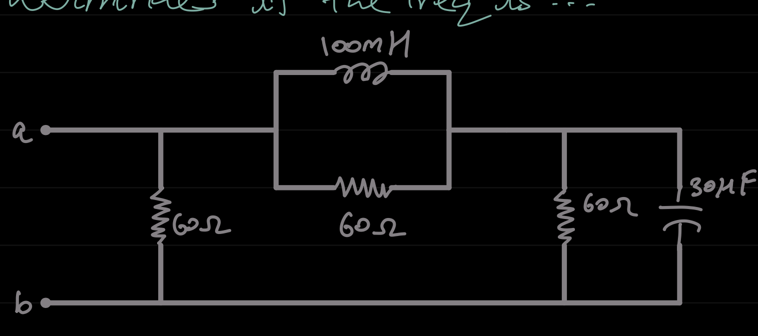
Q6) Calculate the equivalent impedance as seen at the open terminals if the freq is ...

(a) 1Hz

(b) 1KHz

(c) 1MHz

(d) 1GHz



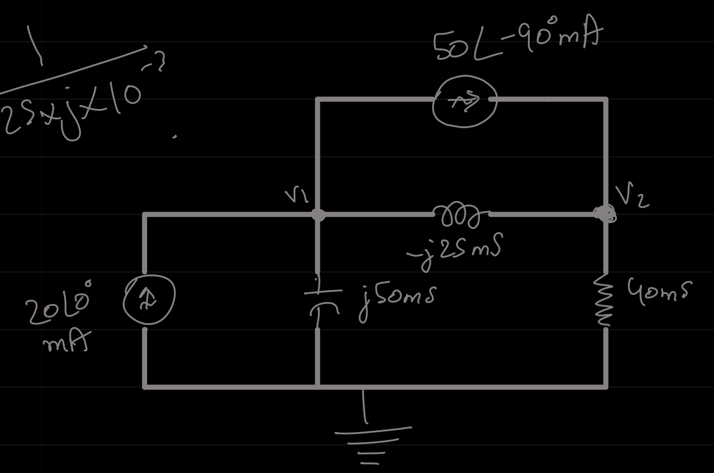
(a) $f = 1Hz \quad w = 2\pi f = 2\pi$ radians/s

$Z_L = jwL = j\cdot 2\pi \cdot 100 \times 10^{-3}$

$\Rightarrow \frac{j\cdot 2\pi}{\cancel{10}5} \Rightarrow \frac{j\pi}{5} \Rightarrow$

Q7) Use nodal analysis to find V_1 and V_2

$\frac{1}{-25\times j\times 10^{-3}}$



$\boxed{V1}$

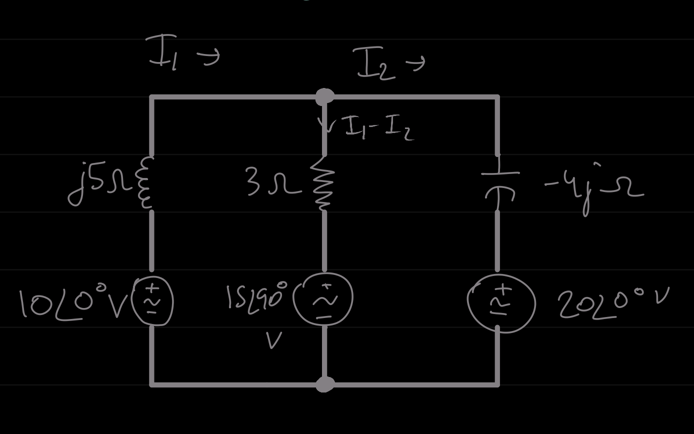
$\frac{V_1\cdot j}{20} - \frac{(V_1-V_2)j}{40} + (-50j \times 10^{-3}) + 20\times10^{-3} = 0$

$\Rightarrow \frac{5(V_1j + V_2j)}{10} - \frac{10j}{10} + \frac{4}{10} = 0$

$\Rightarrow 5V_1j + 5V_2j - 10j + 4 = 0 \Rightarrow 4 + j(5V_1 + 5V_2 - 10) = 0$

② $\frac{(V_2 - V_1)}{40}\cdot -j + \frac{V_2 - 0}{40\times10^{-3}} + (50j\times10^{-3}) = 0$

Q8) Use mesh analysis to find I_1 and I_2



Apply KVL for ①:

$3(I_1 - I_2) + 15\angle 90° - 10\angle 0° + I_1(j5) = 0$

$3I_1 - 3I_2 + 15\cancel{\cos 90°} + 15j\sin 90° - 10\cos 0° - 10j\cancel{\sin 0°} + I_1 5.j = 0$

$3I_1 - 3I_2 + 15j - 10 + 5j\, I_1 = 0$

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

Apply KVL for ②

$-4I_2\cdot j + 20\angle 0° - 15\angle 90° + (I_2 - I_1)3 = 0$

$-4I_2\cdot j + 20 - 15j + 3I_2 - 3I_1 = 0$

$3I_2 - 3I_1 + 20 + j(-15 - 4I_2) = 0$

$3I_1 = 3I_2 + 20 + j(-15 - 4I_2)$

Put ② in ①

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

$3\cancel{I_2} + 20 + j(-15 - 4I_2) - 3\cancel{I_2} - 10 + j(15 + 5I_1) = 0$

$10 + j(-15 - 4I_2) + j\left(15 + 5\left[\frac{3I_2 + 20 - 15j - 4I_2\cdot j}{3}\right]\right) = 0$

$30 - 45j - 12I_2j + j(45 + 15I_2 + 100 - 75j - 20I_2j) = 0$

$30 - \cancel{45}j - 12I_2j + \cancel{45}j + 15I_2j + 100j + 75 + 20I_2 = 0$

$105 + 20I_2 + j(3I_2 + 100) = 0$

$105 + 20I_2 + j\cdot 3I_2 + j\cdot 100 = 0$

$I_2(20 + j\cdot 3) = -105 - j\cdot 100$

$I_2 = \frac{-105 - j\cdot 100}{20 + j\cdot 3} \times \frac{20 - j\cdot 3}{20 - j\cdot 3}$

$I_2 = \frac{-2100 + 315\cdot j - 2000j - 300}{409}$

$I_2 = \frac{-2400 - 1685j}{409}$

Home work:1 19/08/24

* Section 10.1

Q1} (a) $5\sin(5t-9^\circ)$

$t=0 \Rightarrow 5\sin(-9^\circ) = -5\sin(9^\circ) = -0.782$

$t=0.01 \Rightarrow 5\sin(0.05-9^\circ) = -5\sin(8.95^\circ)$ X

$\Downarrow$ radians

$5\sin\left(\left[0.05\times\frac{180}{\pi}\right]-9^\circ\right) = 5\sin(2.86^\circ-9^\circ)$

$\Rightarrow -5\sin(6.14) = -0.534$

$t=0.1s \Rightarrow 5\sin(0.5-9^\circ) = 5\sin(28.64-9) = 5\sin(19.64)$

$\Rightarrow 1.6805$

(b) $4\cos 2t$

$t=0 \Rightarrow 4\cos(0) = 4$

$t=1 \Rightarrow 4\cos(2) = 3.997$

$t=1.5 \Rightarrow 4\cos(3) = 3.994$

(c) $3.2\cos(6t+15^\circ)$

$t=0 \Rightarrow 3.2\cos(15^\circ) = 3.09$

$t=0.01 \Rightarrow 3.2\cos(0.06+15^\circ) = 3.2\cos(3.43^\circ+15^\circ)$

$\Rightarrow 3.2\cos(18.43^\circ)$

$\Rightarrow 3.035$

$t=0.1 \Rightarrow 3.2\cos(0.6+15^\circ) = 3.2\cos(34.3^\circ+15^\circ)$

$= 3.2\cos(49.3)$

$= 2.086$

Q2} (a) $300\sin(628t) \rightarrow 300\cdot\cos(628t-90^\circ)$

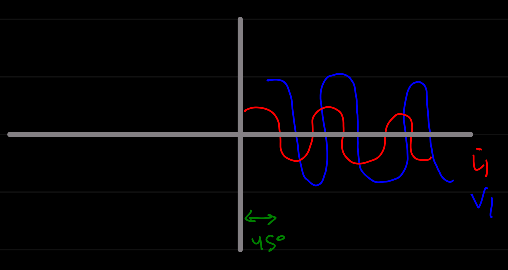
$4\sin(3\pi t+30^\circ) \rightarrow 4\cos(3\pi t-60^\circ)$

$14\sin(50t-5^\circ) - 10\cos 50t \rightarrow$

Q3} $v_1 = 10\cos(10t-45^\circ)$

(a) $i_1 = 5\cos 10t$

-45°

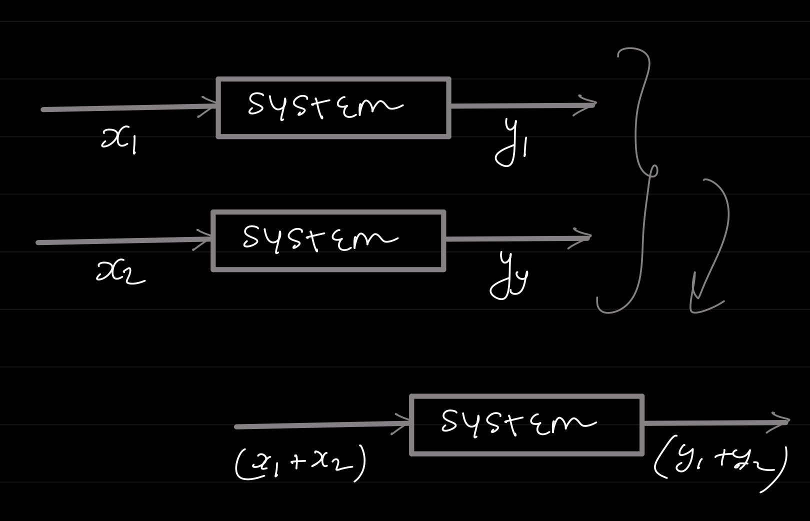


9:30 – 11:00AM
21/08/24

LECTURE: 4

* Superposition principle

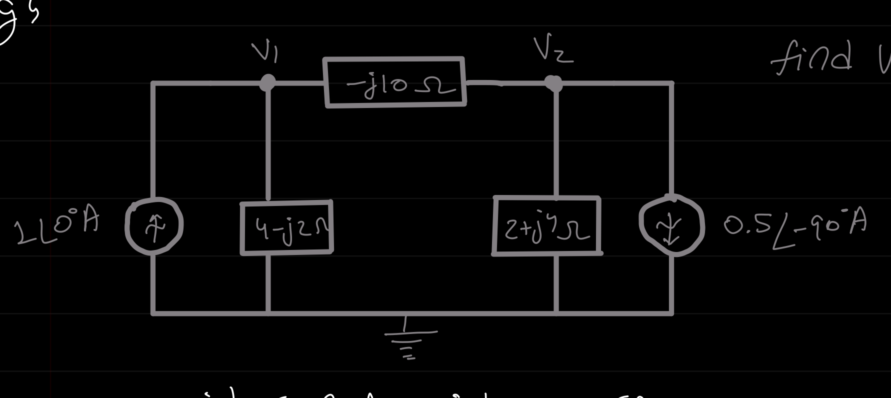
active elements : provide energy eg: volt src / curr src
passive element : consume/ store energy eg: R/L/C



* Multiple independent sources

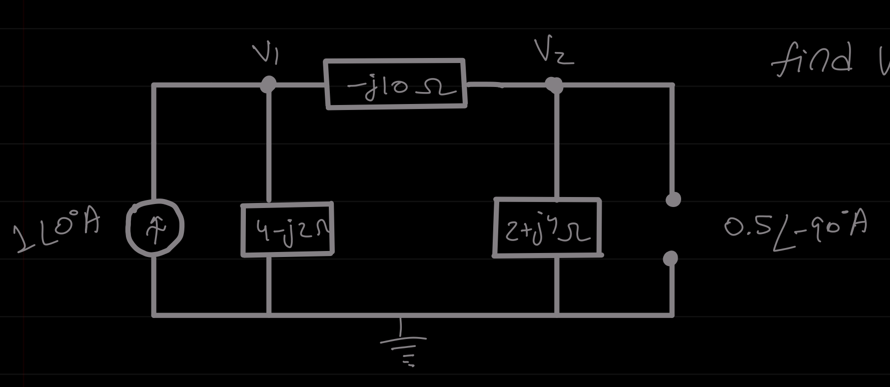
- consider separate independent sources "acting alone"
- add all responses

eg?



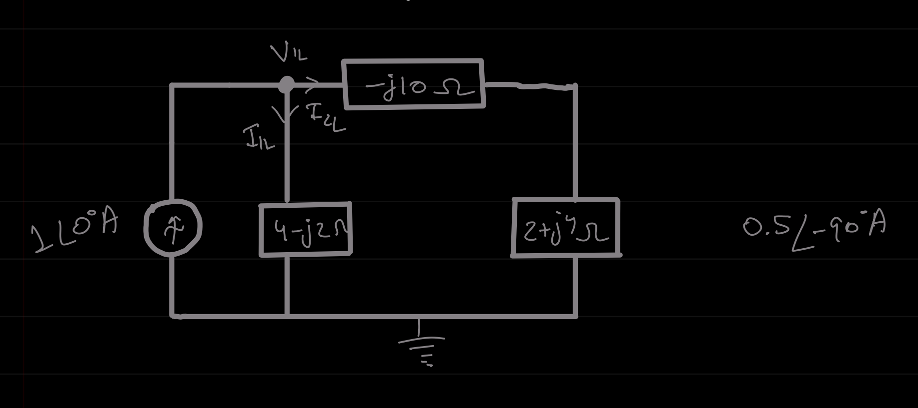
find V_2

consider left side source
so we need to open the circuit at the right source



find V_2

⇓



$V_{1L} = I_{1L}(4-j2)$

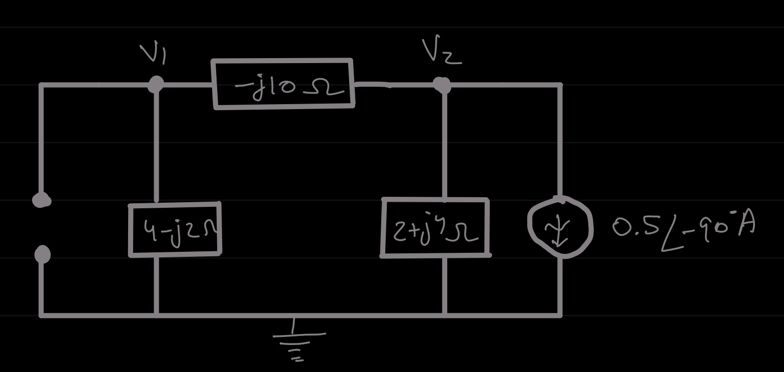
$$\Rightarrow \frac{-j10+2+j4}{(4-j2)+(-j10+2+j4)}(4-j2)$$

$$\Rightarrow \frac{(2-j6)(4-j2)}{6-j8} \Rightarrow \frac{8-j4-24j-12}{6-8j}$$

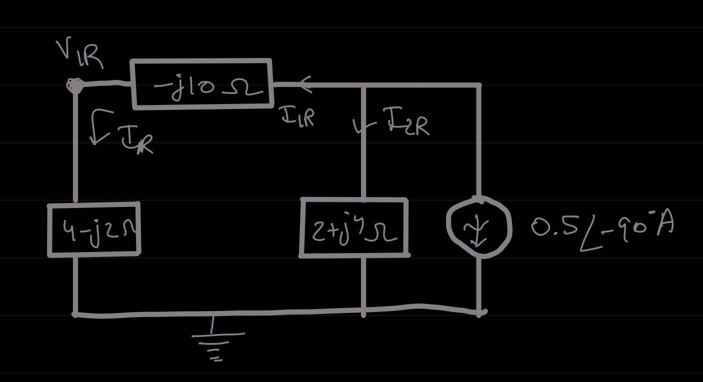
$$\Rightarrow \frac{-4-28j}{6-8j} = 2-j2\,V$$

$I_{1L}+I_{2L} = 1\angle 0^\circ = 1\cos 0^\circ + j1\sin 0^\circ = 1A$

consider right src



⇓



$V_{1R} = I_{1R}(4-j2)$

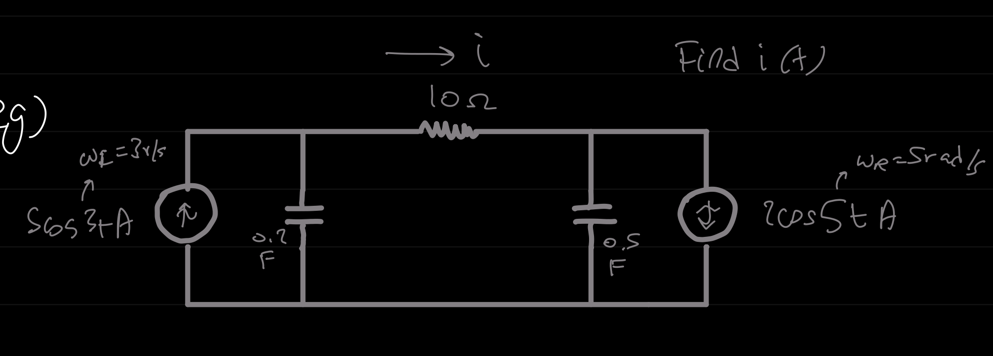
$$I_{1R} = \frac{(-0.5\angle -90^\circ A)(2+j4)}{-j10+4-2j+2+j4}$$

$$V_{1R} = \frac{(j\cdot 0.5)(2+j4)(4-j2)}{6-j8}$$

$$\Rightarrow \frac{(j-2)(4-j2)}{6-j8} = \frac{j4+2-8+j4}{6-j8} = \frac{j8-6}{6-j8} = -1V$$

$V_1 = V_{1L}+V_{1R} = 2-j2-1 = \boxed{1-j2V}$

eg)

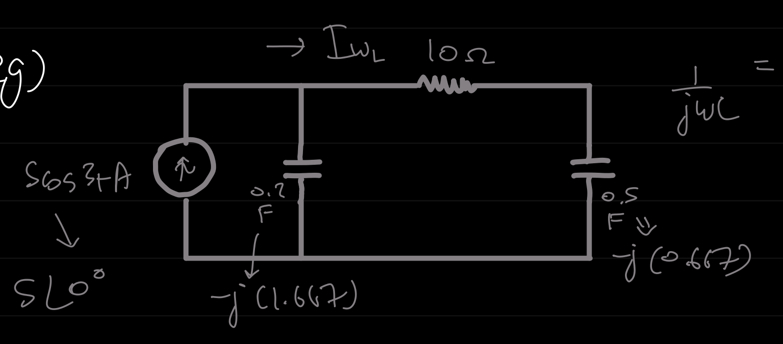


Find $i(t)$

Note: we have 2 sources operating at 2 different frequencies (ω)

① consider left src $\omega_L = 3rad/s$

eg)

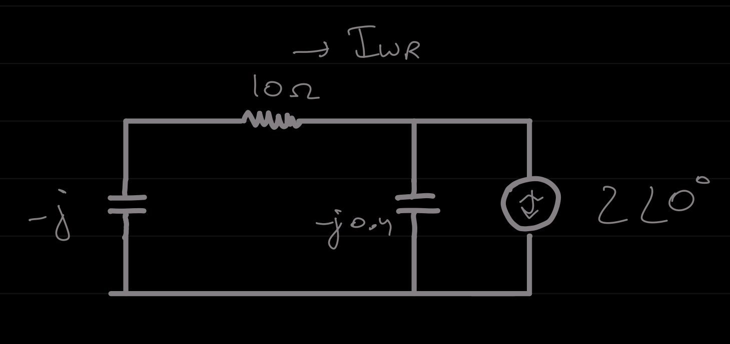


$$\frac{1}{j\omega C} = \frac{1}{j(3)0.2} = \frac{-j}{0.6}$$

$$I_{\omega_1} = 5\angle 0^\circ\left(\frac{-j(1.667)}{-j(1.667)+10-j(0.667)}\right)$$

$\Rightarrow 811.7\angle -76.86^\circ mA$

② consider right source $\omega_R = 5rad/s$



$$I_{\omega R} = 2\angle 0^\circ\left(\frac{-j\cdot 0.4}{-j0.4+10+-j}\right)$$

$$\Rightarrow \frac{-j(0.8)}{-j(1.4)+10} \Rightarrow \frac{-j(0.8)[10+j(1.4)]}{100+1.96}$$

$$\Rightarrow \frac{-j8+1.12}{101.96}$$

$\Rightarrow 0.0792\angle -82.03^\circ A = 79.2\angle -82.03\,mA$

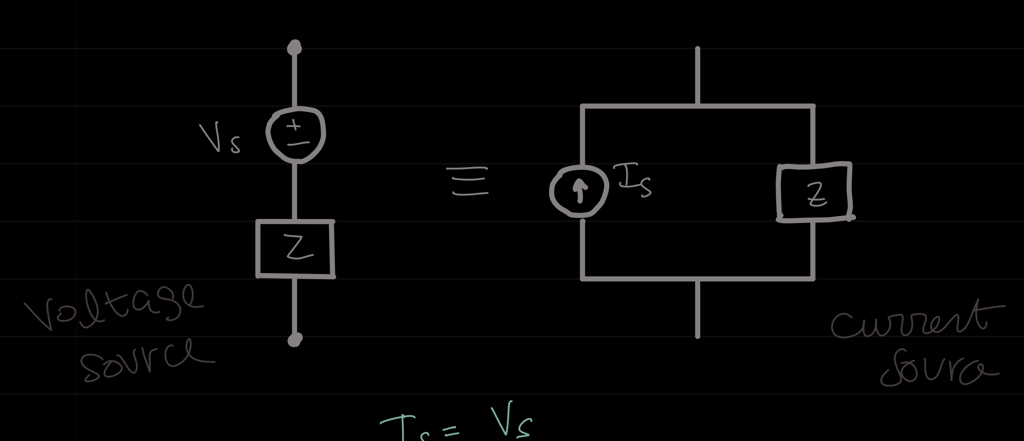
Applying superposition

$I = I_{\omega L} + I_{\omega R}$ ✗

Phasors can only be added when they correspond to same frequency

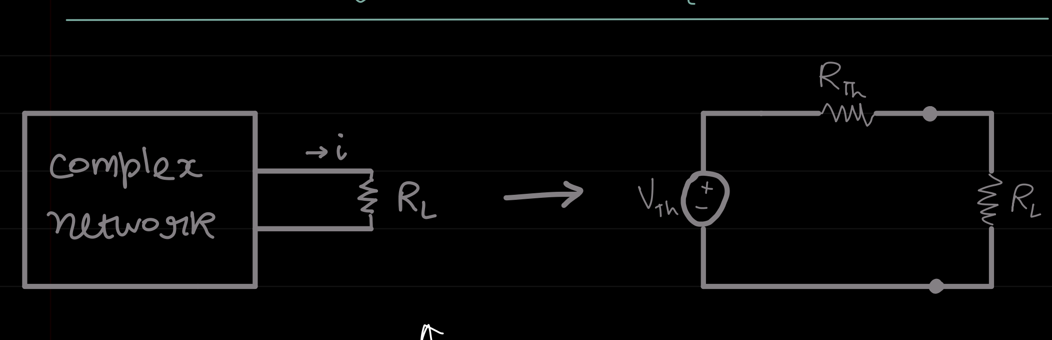
$i(t) = [811.7\cos(3t-76.86^\circ) + 79.2\cos(5t-82.03^\circ)]\,mA$

* SOURCE TRANSFORMATION

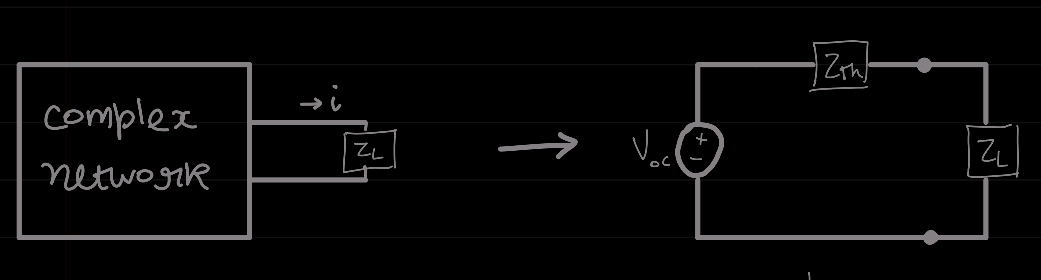


$$I_s = \frac{V_s}{Z}$$

* THEVENIN & NORTON EQUIVALENT CKTS

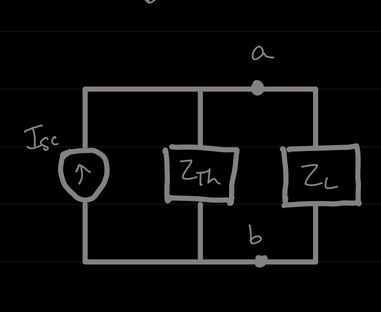


Normally



↓

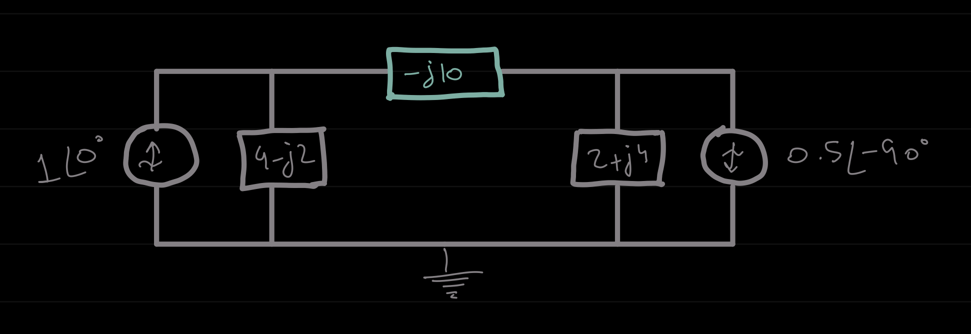
$V_{oc} = I_{sc}\cdot Z_{TH}$



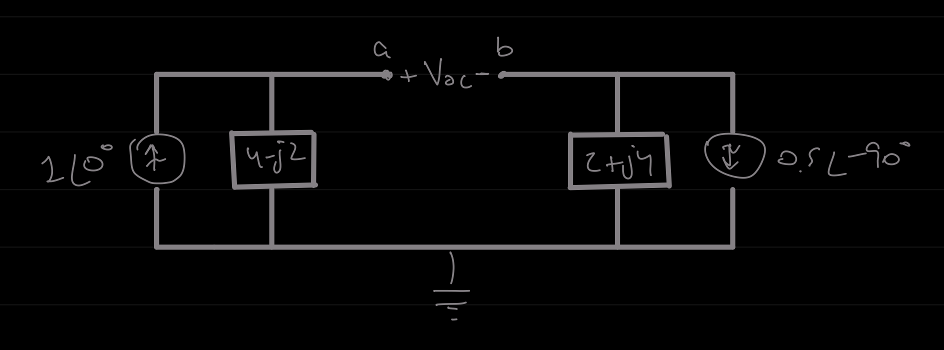
	V_{oc} (by open circuiting)	I_{sc} (by short circuiting)	Z_{TH} (ratio of V_{oc} & I_{sc})
① Independent sources	✓	✓	✓
② Independent + dependent sources	✓	✓	✓

③ Dependent sources only — we need to consider test sources eg: 1A / 1V and then find $V_{oc} \Rightarrow I_{sc} \Rightarrow Z_{Th} = \frac{V_{oc}}{I_{sc}}$

eg? find Thevenin equivalent seen by $-j10\,\Omega$



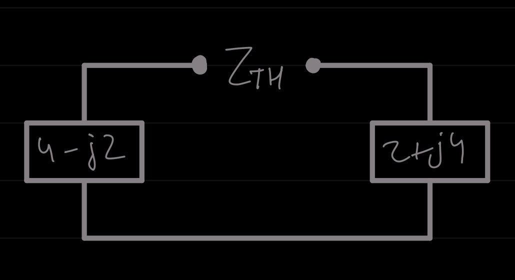
- : complex network
- : R_L



$$V_{oc} = V_a - V_b = 1\angle 0^\circ(4-j2) - 0.5\angle -90^\circ(2+j4)$$
$$= 4-j2+2-j1 = 6-j3\,V$$

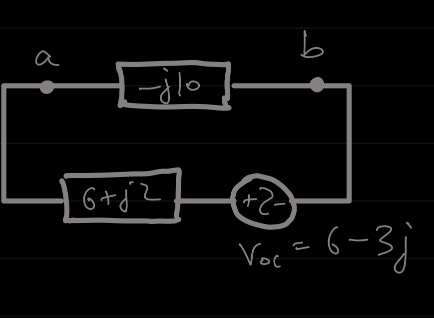
Find Z_{Th}

all volt src → short circuit
all curr src → open circuit



$Z_{TH} = 4-j2+2+j4 = 6+j2\ \Omega$

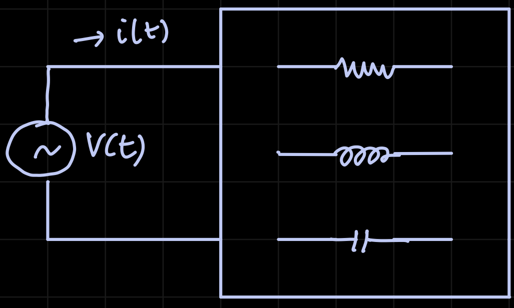
Equivalent circuit



$$I_{ab} = \frac{V_{oc}}{6+j2-j10} = \frac{V_{oc}}{6-j8}$$

* Lecture : 5 28/08/24

⇒ AC Power Analysis {Module : 3}



$\cos(A)\cos(B) = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

instantaneous power : $p(t) = v(t) \cdot i(t)$

$$v(t) = V_m \cdot \cos(\omega t + \theta)$$
$$i(t) = I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = V_m \cdot \cos(\omega t + \theta) \cdot I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = \underbrace{\frac{V_m I_m}{2} \cos(\theta - \phi)}_{\substack{\text{constant} \\ \text{(DC term)}}} + \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{\substack{\text{twice frequency} \\ \text{(harmonic)}}}$$

- Average Power

$$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2} \cos(\theta - \phi)\,dt + \underbrace{\frac{1}{T}\int_0^T \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{0}$$

$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)}$$

$P_{avg} = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$

real number ✓
phasor ✗

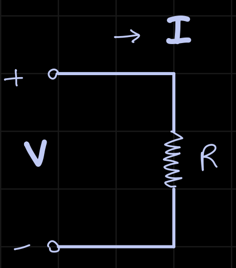
$\mathbf{I} = I_m e^{j\phi}$
$\mathbf{I}^* = I_m e^{-j\phi}$
(conjugate)

* Avg. Power absorbed

- by a purely resistive element

$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$

here, phase diff = 0°

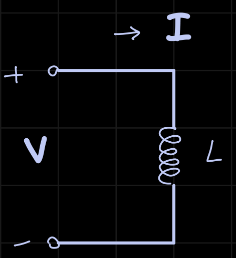


$$P_{avg} = \frac{V_m \cdot I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

- by a purely inductive element

$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$

here, phase diff = 90°

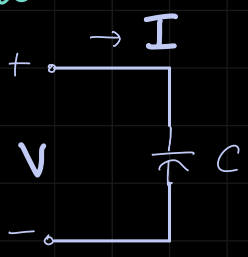


So,

$P_{avg} = 0$ for inductor

- by a purely capacitive element

$P_{avg} = \frac{V_m \cdot I_m}{2} \cdot \cos(\pm 90°)$



$P_{avg} = 0$ for capacitor

* Lecture : 6

$v(t) = V_m \cos(\omega t + \theta)$ ← Revision

$\downarrow$

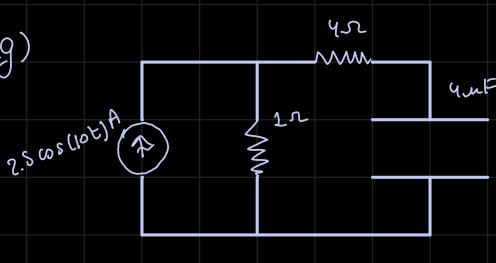
$v(t) = Re\{V_m e^{j\omega t} e^{j\theta}\}$

$\downarrow$ $-e^{j\omega t}$

$V_m e^{j\theta} = V_m \angle \theta$

exponential polar

- eg)

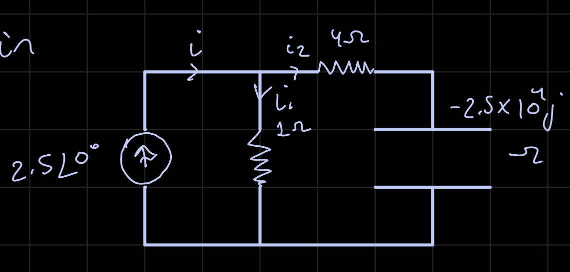


① find power delivered to each element at $t = 0, 10, 20ms$

② Find P_{avg} to each element

Ans) convert to freq. domain

$V_{1\Omega} = V_{4\Omega} + V_c$



① $P_{inst} = \frac{V_m I_m}{2} \cos(\theta - \phi) + \frac{V_m I_m}{2} \cos(2\omega t + \phi + \theta)$

imaginary part is way larger than real part

$I_1 = -2.5 \times \left(\frac{4 - j2.5 \times 10^4}{1 + 4 - j\,25 \times 10^4} \right) \simeq \underline{\underline{-2.5A}}$

$\Rightarrow \frac{-10 + j625 \times 1\cancel{00}}{5 - j\,25 \times 10^3} \Rightarrow \frac{-2 + j125}{1 - j50}$

$\Rightarrow \frac{-2 + j \cdot 125}{1 - j50} \times \frac{1 + j50}{1 + j50} \Rightarrow$

$I_{1\Omega}(t) = 2.5 \cos(10t)$

$V_{1\Omega} = (I_{1\Omega})(1\Omega) \Rightarrow V_{1\Omega}(t) = 2.5 \cos(10t)$

$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{4}$

$p_{1\Omega}(t) = 2.5 \cos(10t) \cdot 2.5 \cos(10t)$ W

$p_{1\Omega}(t = 0) = 6.25W$

$p_{1\Omega}(t = 10ms) = 6.1877W$

$p_{1\Omega}(t = 20ms) = 6.033W$

$(P_{avg})_{1\Omega} = 3.125W$

⇒ P delivered to 4Ω

$P_{4\Omega}(t) = V_{4\Omega}(t) \times I_{4\Omega}(t)$

$I_{4\Omega} = I_S \times \frac{1}{5 - j2.5 \times 10^4} = 10^{-4} \angle 90° A$

$V_{4\Omega} = 4 \times I_{4\Omega} = 4 \times 10^{-4} \angle 90°$ V

convert to time domain

$p_{4\Omega}(t) = 2 \times 10^{-8} + 2 \times 10^{-8} \cos(20t)$ W

$t = 0 : 4 \times 10^{-8}$ W

$t = 10ms = 3.96 \times 10^{-8}$ W

$t = 20ms = 3.84 \times 10^{-8}$ W

$P_{avg\,4\Omega} = 2 \times 10^{-8}$ W

⇒ P delivered to capacitor

$p_c(t) = V_c(t) \cdot I_c(t)$

$(p_{avg})_C = 0$ since phase diff $= 90°$

$I_c = I_2 = 10^{-4} \angle 90° A \Rightarrow I_c(t) = 10^{-4} \cos(10t + 90°)$

$V_c = (-j\,2.5 \times 10^{-4}) \times 10^{-4} \angle 90$

$-j \cdot 25 \times 10^3 \times (+j10^{-4})$

$\underline{\underline{2.5\,V}}$

$p_c(t) = 1.25 \times 10^{-4} \cos(20t + 90°)$ W

$t = 0 \Rightarrow p = 0W$

$t = 10ms \Rightarrow p = 2.48 \times 10^{-5}$ W

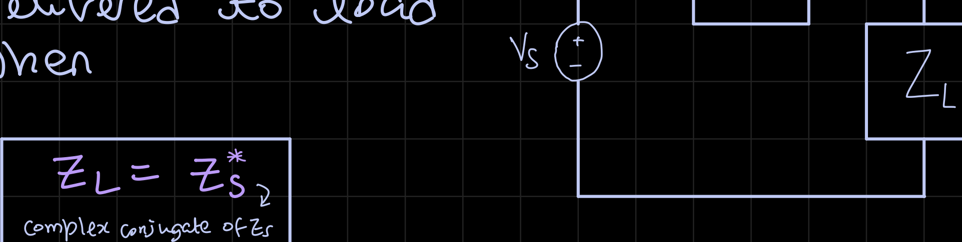
Note: we cannot multiply I_c and V_c in phasor form and then convert to time domain for getting P_c (power) because power does not has a phasor part. It is a real value.

* Correct or not ?

depends on t ✗ ① $p_{1\Omega}(t) + p_{4\Omega}(t) + p_c(t) = $ constant1

✓ ② $P_{avg\,1\Omega} + p_{avg\,4\Omega} + P_{avg\,c} = $ constant2 $\equiv P_{avg}$ source

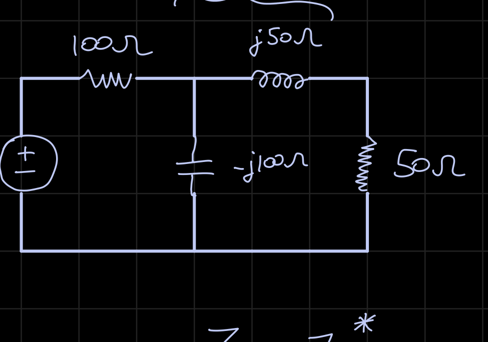
active sign convention | passive sign convention



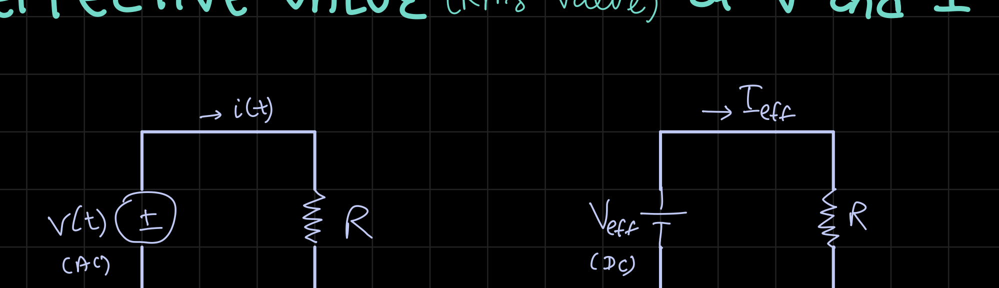
* Maximum Power Transfer Theorem

maximum power delivered to load when

$\boxed{Z_L = Z_S^*}$
complex conjugate of Z_S



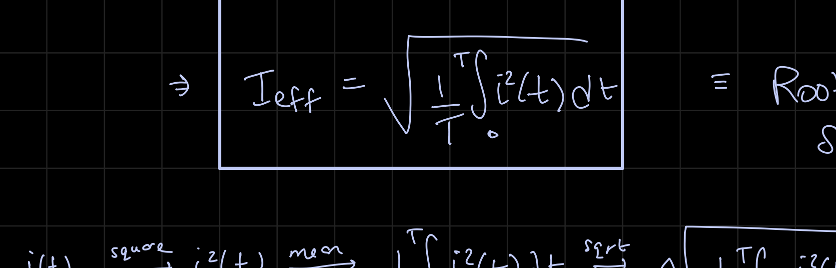
⇒ Impedance matching



$Z_L = Z_S^*$

* EFFECTIVE VALUE (RMS Value) of V and I

voltage current



if $P_{avg_1} = P_{avg_2}$

$\Rightarrow \frac{1}{T}\int_0^T i^2(t) R\, dt = I_{eff}^2 R$

$\Rightarrow \boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}}$ ≡ Root mean square

$i(t) \xrightarrow{square} i^2(t) \xrightarrow{mean} \frac{1}{T}\int_0^T i^2(t)\,dt \xrightarrow{sqrt} \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$

- Sinusoidally varying voltage

$\cos^2 x = \frac{2\cos x + 1}{2}$

$i(t) = I_m \cos(\omega t + \phi)$

$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi)\,dt}$

$\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$ current rms value unit: A_{rms}

$\boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$ voltage rms value unit: V_{rms}

not a phasor
it is real number
≡ magnitude of current / voltage

eg: $I = 10\angle 90° A \Rightarrow I = 5\sqrt{2}\angle 90° A$ rms

OR $V = 100\sqrt{2}\angle -60° V \Rightarrow V = 100\angle -60° V_{rms}$

* Apparent Power

$P_{apparent} = I_{eff} V_{eff}$ unit: VA, not W

volt ampere

* Power factor

$PF = \frac{\text{avg power}}{\text{apparent power}} = \frac{(V_m \cdot I_m)/2 \cos(\theta - \phi)}{V_{eff} \cdot I_{eff}}$

$PF = \cos(\theta - \phi)$

power factor angle

* Lecture : 7

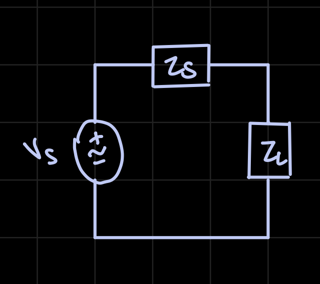
- Instantaneous Power : $p(t) = v(t)\, i(t)$
- Average Power : $P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = \frac{1}{2} Re\{V I^*\}$
- $P_{avg.\,sources} = \sum P_{avg.\,elements}$

* Note : $Z = R + jX$ — impedence; R: resistive; X: reactive

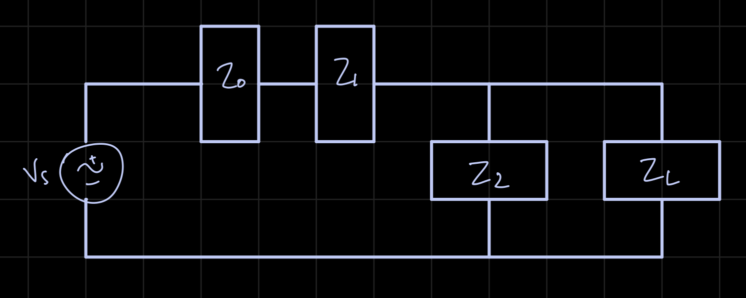
$P_{avg})_{resistive}$: $P \neq 0$

$P_{avg})_{reactive}$: $P = 0$

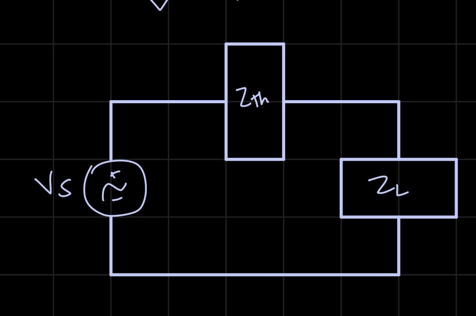
* Max Power Transfer :



Circuit has max. power when $Z_S = Z_L^*$ (complex conjugate)



⇓ apply thevenin's theorem



$Z_L = Z_{Th}^*$

* Power Factor

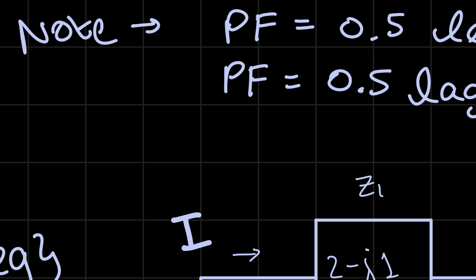
$$PF = \cos(\theta - \phi) = \frac{P_{average}}{P_{apparent}}$$

θ: angle of voltage phasor; φ: angle of current phasor

for purely resistive load : PF = 1 {Max} $\theta - \phi = \pm 90°$
for purely reactive load : PF = 0 {min}

Note → PF = 0.5 leading → capacitive $(\theta - \phi) < 0°$
PF = 0.5 lagging → inductive $(\theta - \phi) > 0°$

eg)

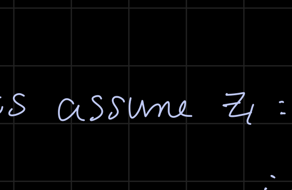


find
1. Average power delivered to each load
2. $P_{avg.\,source} = ?$
3. $P_{apparent.\,src} = ?$
4. PF of combined load = ?

Ans) $P_{avg\, Z_1} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} Re\{V I^*\}$

V: voltage across Z_1, not the src

lets assume Z_1 :



So, $P_{avg\, Z_1}$ = Power delivered to 2Ω resistor

$= I_{eff}^2 R$

find I by KVL :

$$-60\angle 0° + I(2 - j1 + 1 + 5) = 0$$

$$I = \frac{60\angle 0°}{3 + j4} = \underbrace{12\angle -53.13°}_{I_{rms}} \; A_{rms}$$

$i(t) = 12\cos(wt - 53°)\ A_{rms}$

$\underset{I_{eff}}{(A_{rms})} \rightarrow \underset{I_m}{(A)}$: $I_{eff} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = I_{eff}\sqrt{2}$

$I_{eff} = 12\, A_{rms} \Rightarrow I_m = 12\sqrt{2}\, A$

$i(t) = 12\sqrt{2} \cos(wt - 53°)\ A$

$P_{avg\,(Z_1)} = (12)^2 \times 2 = 288\, W$

note: $P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$

Z_2 : I_{eff} same : series circuit

$I_{eff} = 12$

and $P_{avg})_{Z_2} = I_{eff}^2 R = (144)(1) = 144\, W$

2. $P_{avg})_{src} = V_{eff} I_{eff} \cos(\theta - \phi)$

$V_{src} = 60\angle 0°\ V_{rms} \Rightarrow V_{eff} = 60\ V_{rms}$

$I_{src} = 12\angle -53.13°\ V_{rms} \Rightarrow I_{eff} = 12\, A_{rms}$

$\theta = 0°, \quad \phi = -53.13°$

$P_{avg})_{src} = [60 \times 12] \cos(53.13°) = 432\, W$

we can observe that $288 + 144 = 432$ and hence,

$P_{avg\,.\,sources} = \sum P_{avg.\,elements}$ holds

3. $P_{apparent} = V_{eff} \cdot I_{eff} = (60)(12) = 720\, W$

4. PF of combined loads = PF of source

$PF = \cos(\theta - \phi) = \cos(\underbrace{0 + 53.13°}_{\theta - \phi > 0,\ lagging}) = 0.6$ lagging

$$* \quad P_{avg} = \frac{1}{2} Re\{V I^*\}$$

$V = V_m \angle\theta$
$I = I_m \angle\phi$

$V_{eff} = V_{eff} \angle\theta$
$I_{eff} = I_{eff} \angle\phi$

$P_{avg} = Re\{V_{eff} I_{eff}\}$

* Complex Power

$S = V_{eff} I_{eff}^*$ Volt Ampere {VA}

this is a complex number not a phasor of power

$S = (V_{eff}\angle\theta)(I_{eff}\angle\phi)$
$= V_{eff} \cdot I_{eff}\, e^{j(\theta - \phi)}$

$$\Rightarrow S = \underbrace{V_{eff} I_{eff} \cos(\theta - \phi)}_{\substack{\text{average power} \\ P \\ \text{(watt) W}}} + j\underbrace{V_{eff} I_{eff} \sin(\theta - \phi)}_{\substack{\text{reactive power} \\ Q \\ \text{(volt ampere reactance) VAR}}}$$

(volt ampere) VA

$$\boxed{S = P + jQ}$$

$|S| = V_{eff} I_{eff} = P_{apparent}$ (volt amp)

note: $Z = R + jX$

Purely resistive load : $Z = R,\ X = 0$

$S = P + jQ$, with $P = V_{eff} I_{eff} \cos(0)$ and $Q = V_{eff} I_{eff} \sin(\underbrace{\theta - \phi}_{0}) = 0$

$P \neq 0$
$Q = 0$

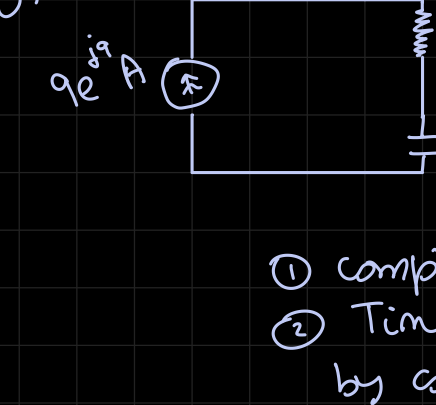
Purely reactive load

$S = P + jQ$, with $P = V_{eff} I_{eff} \cos(90°)$ and $Q = V_{eff} I_{eff} \sin(\overbrace{\theta - \phi}^{90°})$

$P = 0, \quad Q \neq 0$

Q signifies energy flow rate into or out of reactive component of load

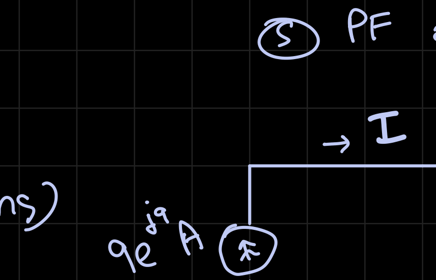
eg)



assume $C = 1\mu F$, $w = 45\, rad/s$

find:
1. complex Power provided by src
2. Time average power absorbed by combined load
3. Reactive Power absorbed by combined load
4. $P_{apparent}$ absorbed " "
5. PF of " " "

Ans)



$9e^{j9}$ ≡ $9\cos(9°) + j9\sin(9°)$

$\frac{1}{jwc} = \frac{1}{j \times 45 \times 1 \times 10^{-6}} = -j\frac{10^6}{45}$

1. $S_{source} = \underbrace{V_{eff} \times I_{eff}}_{source}$

$I_m = 9\, A$
$I_{eff} = \frac{9}{\sqrt{2}}\, A_{rms}$

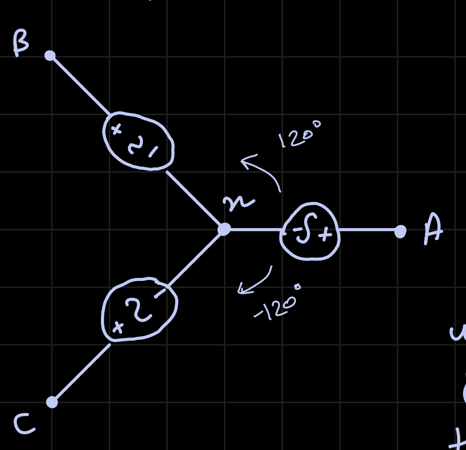
$V_{eff} = ?$, find V

$$V = I(Z_{eq}) = 9\angle 9° \left(18 \times 10^3 - j\frac{10^6}{45}\right)$$

* Lecture : 8

- Polyphase Circuits (Chp 12)

⇒ Three Phase Source



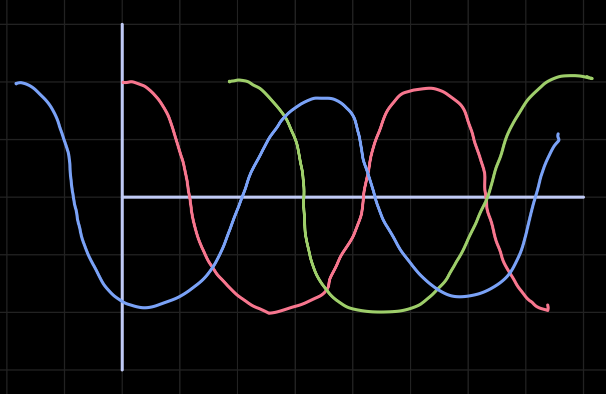
$V_{an} = 100\angle 0^\circ$ V
$V_{bn} = 100\angle 120^\circ$ V
$V_{cn} = 100\angle -120^\circ$ V

if $|V_{an}| = |V_{bn}| = |V_{cn}|$
& $V_{an} + V_{bn} + V_{cn} = 0$
then it is a

Balanced Source

total instantaneous power = constant

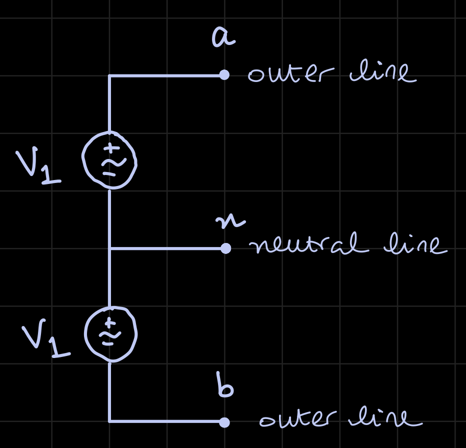
freq. domain	time domain
$100\angle 0^\circ$	$100\cos(\omega t)$
$100\angle 120^\circ$	$100\cos(\omega t + 120^\circ)$
$100\angle -120^\circ$	$100\cos(\omega t - 120^\circ)$



total $p(t) \rightarrow$ constant

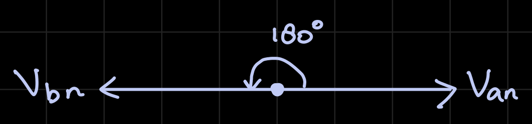
* Single Phase Three-wire Source

≡ Two phase source



$V_1 = V_m\angle 0^\circ$

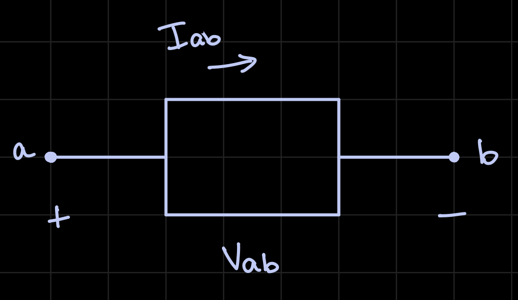
$V_{an} = V_1 = V_m\angle 0^\circ$
$V_{bn} = -V_1 = V_m\angle 0^\circ - 180^\circ$



double script notation

$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$
$V_{bn} = -V_{nb}$

$V_{an} = V_A$ wrt V_n

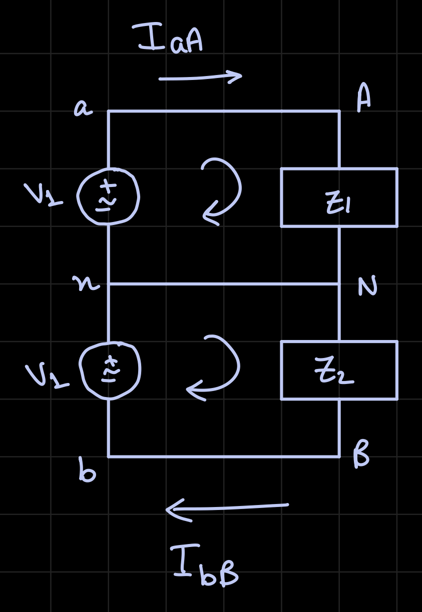


I_{ab}: current flowing from a to b

- Balanced single phase three wire source

Conditions:

$|V_{an}| = |V_{bn}|$
$V_{an} + V_{bn} = 0$
⇒ Balanced Src



$I_{aA} = \frac{V_1}{Z_1}$

$I_{bB} = \frac{V_1}{Z_2}$

$I_{nN} = -I_{aA} + I_{bB}$

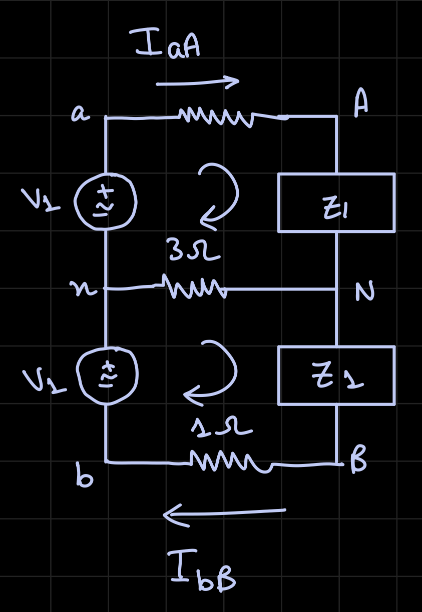
$\Rightarrow \frac{V_1}{Z_2} - \frac{V_1}{Z_1}$

Assume $Z_1 = Z_2$

$I_{nN} = \frac{V_1}{Z_1} - \frac{V_2}{Z_1} = 0$

when both srcs and loads are equal

Balanced Load : current in neutral line is equal to zero.



all terms are phasors

even with resistance
$I_{nN} = 0 = I_{bB} - I_{aA}$
due to
① Balanced Load
≡ Symmetry

* Lecture : 9

09/09/24

time domain	freq. domain
$V_p \cos(\omega t + \theta°)$	$V_p \angle \theta°$
$V_p \cos(\omega t + \theta° - 180°)$	$V_p \angle \theta - 180°$

we cannot directly say,

$$P_{avg} = VI \quad \times$$

but $\quad P_{avg} = \frac{1}{2} Re\{VI^*\} \quad ✓$

② a) $$I_{AB} = \frac{115\angle\theta° + 115\angle\theta°}{2(10+j2)} = \frac{230\angle\theta°}{2(10+j2)}$$

$$= 11.27\angle(\theta° - 11.3°)A$$

$$\phi = \theta - 11.3°$$

$$PF = \cos(\theta - \theta + 11.3°) = 0.98 \text{ lagging}$$

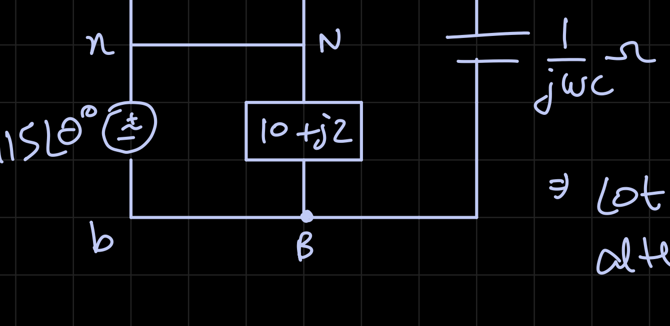
because PF angle is +ve

b) $PF = 1 \Rightarrow \cos(\theta - \phi) = 1 \Rightarrow \theta - \phi = 0$

$$\Rightarrow \theta = \phi$$

$f = 50h$

$\omega = 2\pi f = 100\pi$



$$I_{aA} = I_1 + I_2$$

$$\Rightarrow \frac{115\angle 0°}{10+j2} + \frac{230\angle 0°}{\left(\frac{1}{j\omega C}\right)}$$

$\Rightarrow$ lot of calculation
alternative method $\Rightarrow$

Note that this is still a balanced load, so we can remove nN since $I_{nN} = 0$

using complex power

$S \equiv$ complex power of total load

$$S = \frac{1}{2} V I^*$$

$$PF = \frac{Re\{S\}}{|S|} = 1 \Rightarrow Re\{S\} = |S|$$

so, $Im\{S\} = 0$ here

$$S = \frac{1}{2} V_{an} I_{AN}^* + \frac{1}{2} V_{nb} I_{NB}^* + \frac{1}{2} V_{ab} I_{AB}^*$$

$$\Rightarrow \frac{1}{2} V_{an} \left(\frac{V_{an}}{Z_p}\right)^* + \frac{1}{2} V_{nb} \left(\frac{V_{nb}}{Z_p}\right)^* + \frac{1}{2} V_{ab} \left(\frac{V_{ab}}{Z_c}\right)^*$$

$$\Rightarrow \frac{1}{2} \frac{|V_{an}|^2}{|Z_p|^2} \times Z_p + \frac{1}{2} \frac{|V_{nb}|^2}{|Z_p|^2} \times Z_p + \frac{1}{2} \frac{|V_{ab}|^2}{|Z_c|^2} \times Z_c$$

$Re\{115\angle 0°\}$

$$\Rightarrow \frac{|V_{an}|^2}{|Z_p|^2} \times Z_p + \frac{1}{2} \frac{|V_{ab}|^2}{|Z_c|^2} \times Z_c$$

$\sqrt{10^2 + 2^2}$

$\rightarrow Re\{\frac{1}{j\omega C}\} = \frac{1}{j\omega C} \times \frac{-1}{j\omega C}$

$$\Rightarrow \frac{115^2}{104}(10 + j2) + \frac{1}{2}\left(\frac{230^2}{\left(\frac{1}{\omega C}\right)^2}\right)\left(\frac{-j}{\omega C}\right)$$

$\frac{-1}{-1(\omega C)^2}$

$$\Rightarrow 115^2\left(\frac{\cancel{10} + j2}{104} - 2j\omega C\right)$$

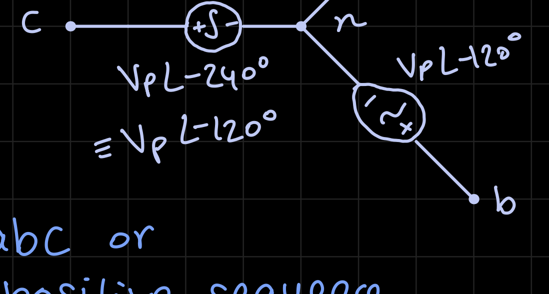
$$\Rightarrow 2 \times 115^2 \left(\frac{j}{104} - j\omega C\right)$$

$$\Rightarrow 2 \times 115^2 \left(\frac{1}{104} - \omega C\right) = 0$$

$$\frac{1}{104} - 100\pi C = 0$$

$$C = \frac{1}{10400\pi} = 30.60\,\mu F \quad ✓$$

* Three Phase Sources



$V_{an} = V_p\angle 0°$
$V_{bn} = V_p\angle -120°$
$V_{cn} = V_p\angle 120°$

$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$
$V_{an} + V_{bn} + V_{cn} = 0$

↳ balanced source

abc or
positive sequence

cba or
negative sequence

just a naming thingy

Phase Voltages $\quad V_{an}, V_{bn}, V_{cn}$

Line-to-line Voltages OR line voltages $\quad V_{ab}, V_{bc}, V_{ac}$

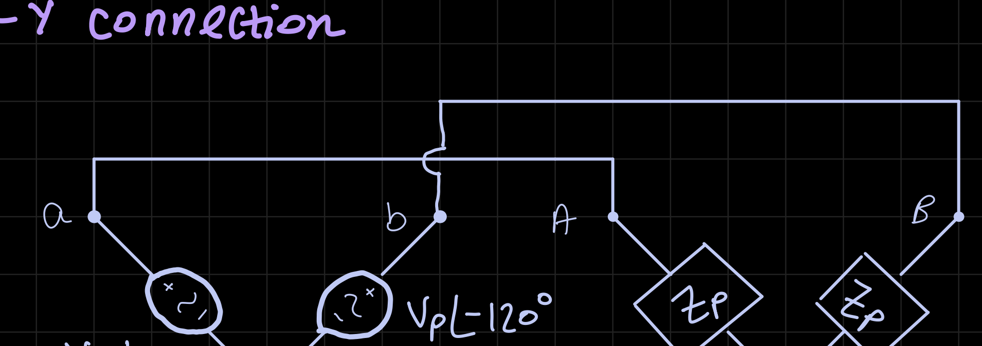
$$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$$

$$V_{ab} = \sqrt{3}\, V_p \angle 30°$$

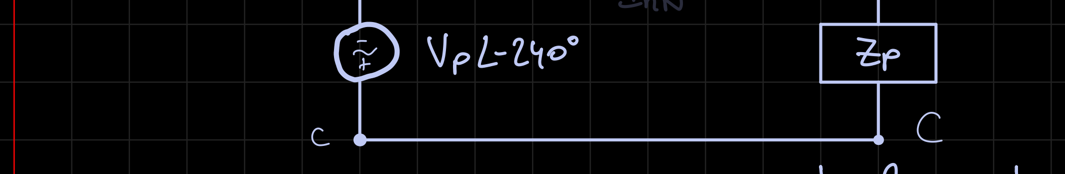
$$V_{bc} = \sqrt{3}\, V_p \angle -90°$$

$$V_{ca} = \sqrt{3}\, V_p \angle -210°$$

* Y-Y connection



balanced source $\qquad$ balanced load



"Per- phase"

used for easier analysis

eg. to find total power by ckt, just add the three individual power obtained from above ckts.

going back to the Y-Y connection $\Rightarrow$

$$I_{aA} = \frac{V_{an}}{Z_p}$$

$$I_{bB} = \frac{V_{bn}}{Z_p} = \frac{V_{an}\angle -120°}{Z_p} = I_{aA}\angle -120°$$

$$I_{cC} = \frac{V_{cn}}{Z_p} = \frac{V_{an}\angle +120°}{Z_p} = I_{aA}\angle +120°$$

$$I_{nN} = I_{aA} + I_{bB} + I_{cC} = 0$$

eg} consider three phase balanced Y-Y connected system

$$V_{an} = 200\angle 0° \ V_{rms}$$
$$Z_p = 100\angle 60° \ \Omega$$

find phase and line voltage & current and total avg power delivered to load

Ans}

$V_{an} = 200\angle 0° \ V_{Rms}$
$V_{bn} = 200\angle -120° \ V_{Rms}$
$V_{cn} = 200\angle +120° \ V_{Rms}$

line voltage :

$$V_{ab} = \sqrt{3}\, V_p \angle 30° = 200\sqrt{3}\angle 30° \ V_{Rms}$$

$$V_{bc} = 200\sqrt{3}\angle -90° \ V_{Rms}$$

$$V_{ca} = 200\sqrt{3}\angle -210° = 200\sqrt{3}\angle 150° \ V_{rms}$$

line currents:

$$I_{aA} = \frac{V_{an}}{Z_p} = \frac{200\angle 0°}{100\angle 60°} = 2\angle -60° \ A_{rms}$$

$$I_{bB} = 2\angle -180° \ A_{rms}$$

$$I_{cC} = 2\angle -300° \ A_{rms} = 2\angle 60° \ A_{rms}$$

total avg. power: $P_{avg} = \frac{1}{2} Re\{V I^*\}$

in phase A :

$$P_{avg})_A = \frac{1}{2} Re\{VI^*\}$$

$$= Re\{V_{eff} I_{eff}\}$$

$$= Re\{200\angle 0° \times 2\angle -60°\}$$

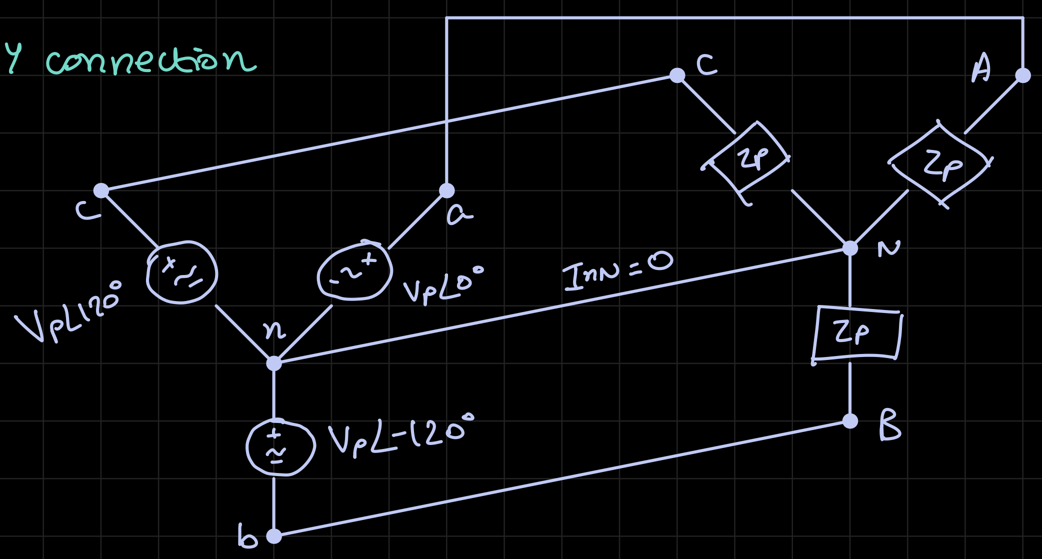
$$\Rightarrow Re\{400\angle -60°\} \Rightarrow 400\cos(-60) = 200W$$

$P_{avg})_B = 200W$
$P_{avg})_C = 200W$

$\Big\} \ P_{avg})_{total} = 600W$

* lecture : 10 11/09/24

→ Y-Y connection



balanced load : all load are same

balanced src : all src magnitudes are equal

line : lines connecting load to src
aA cC
bB nN

Phase Voltages: $V_{AN} = V_{an}$, $V_{BN} = V_{bn}$, $V_{CN} = V_{cn}$

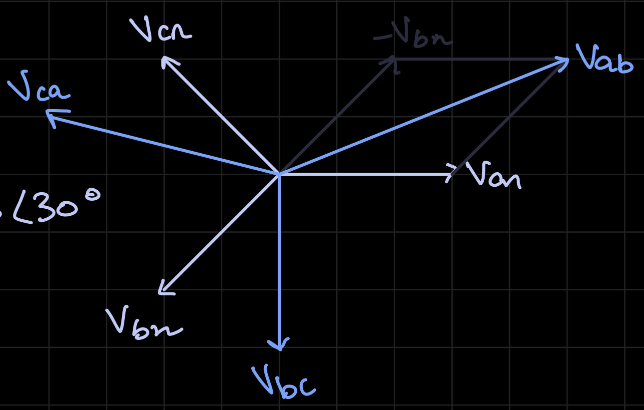
line voltages : V_{aA}, V_{bB}, V_{cC}

line currents : I_{aA}, I_{bB}, I_{cC}

phase currents : $I_{AN} = I_{aA}$, $I_{BN} = I_{bB}$, $I_{CN} = I_{cC}$

$V_{an} = V_p \angle 0°$
$V_{bn} = V_p \angle -120°$
$V_{cn} = V_p \angle -240°$

$V_{ab} = V_{an} - V_{bn}$



$V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_p \angle 30°$
$V_{bc} = \sqrt{3} V_p \angle -90°$
$V_{ca} = \sqrt{3} V_p \angle -210°$

line voltages

$V_{line} = \sqrt{3} V_{phase}$

line currents = phase currents

* Total Instantaneous Power

$$P_{total}(t) = p_A(t) + p_B(t) + p_C(t)$$

Classes & Attribs

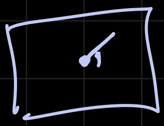
interface (entity)

- Bird → dmg (hits)
- Pig → health (hits)
- User → save stage → Score, level_status
- Level mng → max_score, no. of pigs
  - ↳ blocks
  - ↳ pigs → type (wood glass steel)

GAME (entry point)

update score

Slingshot
- angle
- stretch

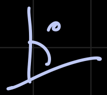


$V = 10$ W/A S, D

$V\cos\theta, V\sin\theta$

per second / rate

$-1 \to 0$

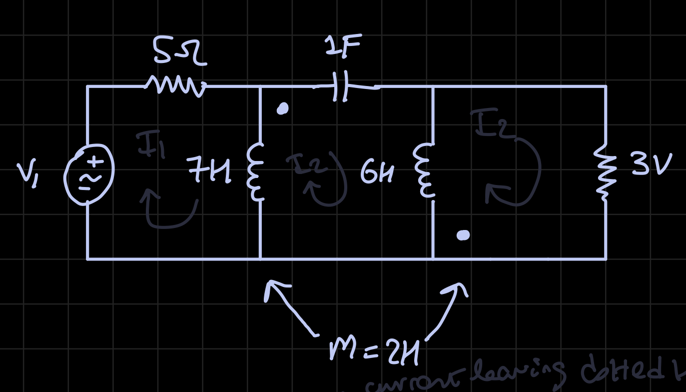


V

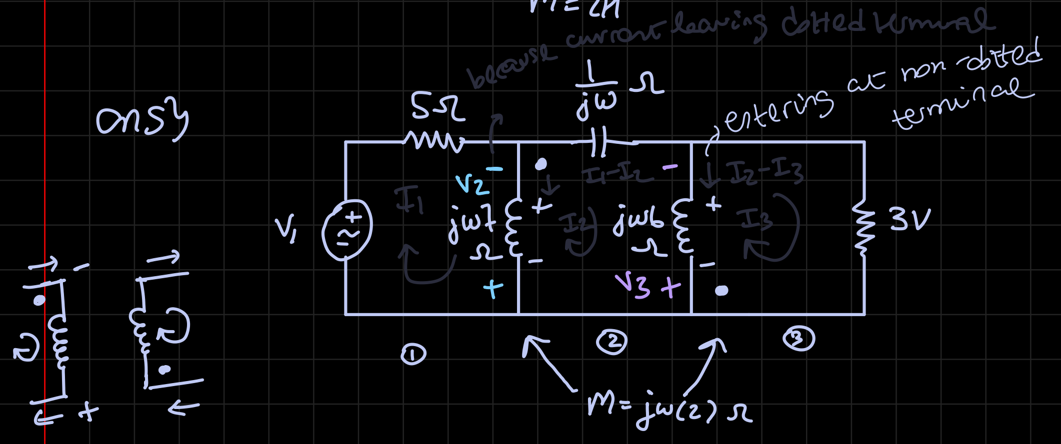
TODO : revise lecture 11 (missed due to SNS quiz)

* lecture 12:

eg 3



ans}



① $-V_1 + 5I_1 - V_2 = 0$

$$V_2 = (j\omega 7)(I_1 - I_2)(-1)$$

because same sign of V_2 & V_3

$$V_2 = -j\omega 7(I_1 - I_2) + j2\omega(I_2 - I_3)$$

③ $3I_3 + V_3 = 0$

self + mutual inductance

same polarity

$$V_3 = -j6\omega(I_2 - I_3) + j2\omega(I_1 - I_2)$$

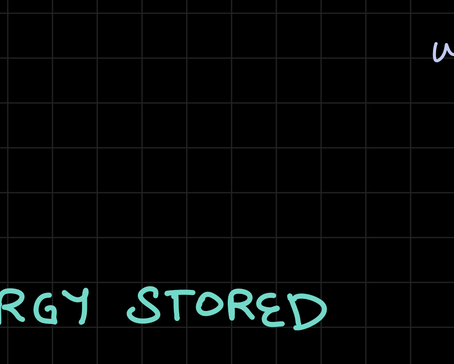
② $V_2 + \frac{1}{j\omega} I_2 - V_3 = 0$

$$-j\omega 7(I_1 - I_2) + j2\omega(I_2 - I_3) + j6\omega(I_2 - I_3) - j2\omega(I_1 - I_2) + \frac{I_2}{j\omega} = 0$$

$$I_1(-j5\omega) + I_2(j17\omega) + I_3(-j8\omega) + \frac{I_2}{j\omega} = 0$$

$$I_1(-j5\omega) + I_2\left(j17\omega + \frac{1}{j\omega}\right) + I_3(-j8\omega) = 0$$

eg3



entering at dotted terminal

$-V_1 + 5I_1 + V_2 = 0$

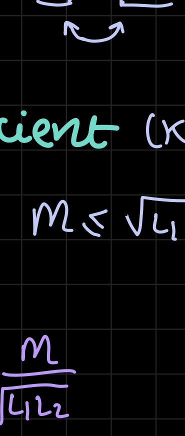
$V_2 = (j\omega 7)(I_1 - I_2) + (j\omega 2)(I_2 - I_3)$

we donot need to care abt this sign if we use V_2, V_3 method.

* ENERGY STORED

$w(t) = \frac{1}{2} L(i(t))^2$ for only one inductor

$$w(t) = \frac{1}{2} L(i_1)^2 + \frac{1}{2} L(i_2)^2 \pm M i_1(t) i_2(t)$$



$\boxed{+ve}$ sign will occur iff both i_1 and i_2 are entering either dotted or undotted

$\boxed{-ve}$ sign iff both enter different (dotted / undotted)

• coupling coefficient (K)

$$M \leq \sqrt{L_1 L_2}$$

$$K = \frac{M}{\sqrt{L_1 L_2}} \Rightarrow 0 \leq K \leq 1$$

$\boxed{K \to 0}$ poor coupling or no coupling

$\boxed{K \to 1}$ strong coupling (very close to each other)

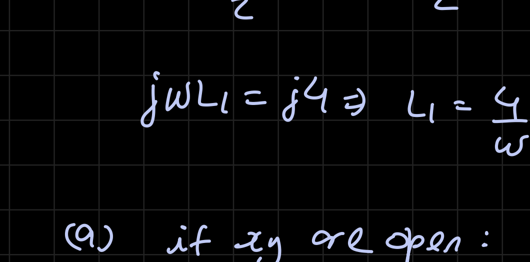
⇒ K depends on : distance btw coils ; size of coils ; ferrite core

$y \propto \frac{1}{x}$ $y \propto x$ $y \propto x$

uses : wireless power transfer (inductively coupled)
- electric vehicle charging
- magsafe charging

eg3 let $i_s = 2\cos(10t)$ A. find total energy stored in network at $t = 0$ if $k = 0.6$ and

(a) xy terminals are open circuited
(b) xy are short circuited



Ans} $w(t) = \frac{1}{2} L_1(i_1)^2 + \frac{1}{2} L_2(i_2) \pm M i_1(t) i_2(t)$

$$j\omega L_1 = j4 \Rightarrow L_1 = \frac{4}{\omega} = 0.4\ H$$

(a) if xy are open : $i_2 = 0$

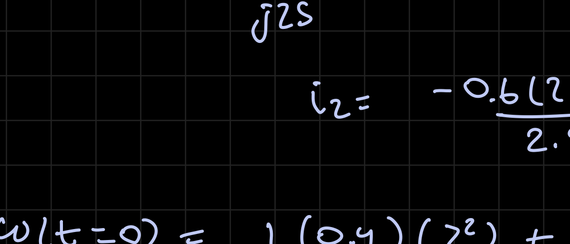
$$\text{so, } w(t) = \frac{1}{2} \times 0.4 \times (2\cos 10t)^2$$

$$w(t) = 0.8\cos^2(10t)$$

$$\text{at } t = 0 \Rightarrow w(t) = 0.8\ J$$

(b) $w(t) = \frac{1}{2} L_1 \cdot i_1^2 + \frac{1}{2} L_2 i_2^2 - M \cdot i_1 \cdot i_2$

$$i_1 = i_s = 2\angle 0°$$



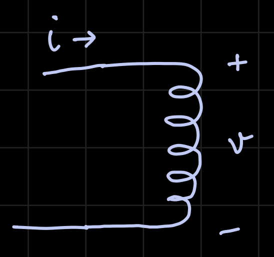
$i_2 = \frac{V_x}{j25}$ and $V_x = -j\omega M I_1$

$$i_2 = \frac{-0.6(2\angle 0°)}{2.5} = -0.48A$$

$$w(t=0) = \frac{1}{2}(0.4)(2^2) + \frac{1}{2}(\cdots$$

* Lecture : 11 {Revision} [absent] 24/9/24

- **MODULE 5: Magnetically coupled circuits**



$v = L\frac{di}{dt}$

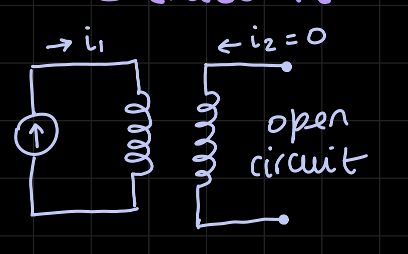
current → magnetic flux
$i \propto \phi$ — ①

magnetic flux → induced voltage
$v \propto \frac{d\phi}{dt}$ — ②

① and ② → $v = L\frac{di}{dt}$

for DC source : current is constant
hence $\frac{di}{dt} = 0$ and $\therefore v = 0$ (induced)

- **Mutual Inductance**

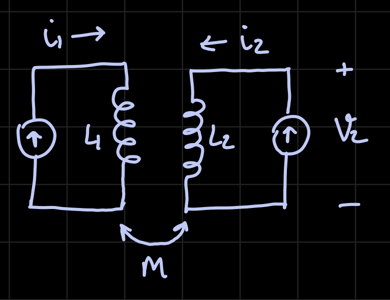


$v_2 \propto \frac{di_1}{dt}$

$v_2 = M\frac{di_1}{dt}$

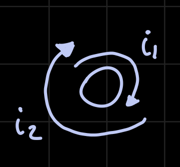
L : self inductance

mutual inductance

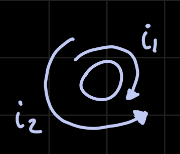


$v_2 = \underbrace{L\frac{di_2}{dt}}_{\text{self inductance}} + \underbrace{M\frac{di_1}{dt}}_{\text{mutual inductance}}$

* Additive / Substractive Property {two different coils}



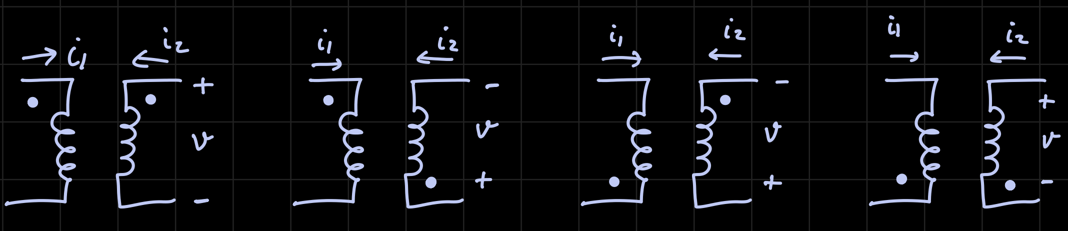
$v_2 = L\frac{di_2}{dt} + M\frac{di_1}{dt}$



$v_2 = L\frac{di_2}{dt} - M\frac{di_1}{dt}$

- **Dotted Notation**

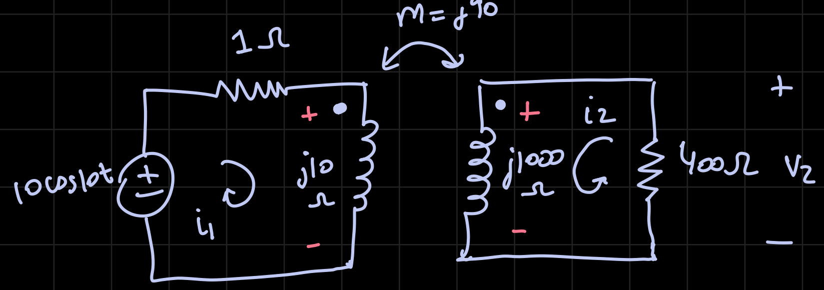
current entering __ terminal means
dotted/undotted
+ve voltage reference at __



$v = L_1\frac{di}{dt} + M\frac{di_2}{dt}$ time domain

$V = j\omega L_1 I_1 + j\omega M I_2$ phasor domain

eg3



① $-10\angle 0^\circ + I_1 + j10\,I_1 + j90\,I_2 = 0$

$I_1(1 + j10) + I_2(j90) = 10$

② $400\,I_2 + j1000\,I_2 + j90\,I_1 = 0$

$I_1(j90) + I_2(400 + j1000) = 0$

CRAMMERS :

$$\Delta = \begin{vmatrix} 1+j10 & j1000 \\ j90 & 400+j1000 \end{vmatrix}$$

$= (1+j10)(400+j1000) + 90000$

$= 400 + j1000 + j4000 - 10.000 + 90.000$

$= j5000 + 80{,}400 = 80555\angle 3.55^\circ$

$$\Delta_1 = \begin{vmatrix} j1000 & 10 \\ 400+j1000 & 0 \end{vmatrix} = -4000 - j10.000$$
$= 10770\angle -111.8^\circ$

$$\Delta_2 = \begin{vmatrix} 10 & 1+j10 \\ 0 & j90 \end{vmatrix} = j900 = 900\angle 90^\circ$$

$I_1 = \frac{\Delta}{\Delta_1} = -3.2 + 6.75j$

full cramers part
wrong because I took
M = L2 = j1000 above

$$D_0 = \begin{vmatrix} 1+j10 & j90 \\ j90 & 400+j1000 \end{vmatrix}$$

$= 400 + j1000 + j4000 - 10.000 + 8100$

$= j5000 - 1500 = 5220\angle 106.69^\circ$

$D_1 =$