14/08/24 # Signals And Systems 11:00Am

Evaluation : (relative grading) yooooooooooo :fire emoji:		
	Attendance	10%
	Quiz	16%
	Assignments	15%
	mid sem	24%
	end sem	35%

Book: Signals & System 2nd edition by Alan V. oppenheimer

Lecture : 1 14/08/24 : 11:00AM

* Linear Algebra Revision

⇒ GROUP : consists of ....

- A set G
- A rule / binary operation "$*$"
  - a. associative
    $$x*(y*z) = (x*y)*z \quad \forall x,y,z \in G$$
  - b. There exists an element "e" called the identity of group G such that
    $$e*x = x*e = x \quad \forall x \in G$$
  - c. $\forall x \in G, \exists\ x^{-1}$ such that
    $$x*x^{-1} = e = x^{-1}*x \quad \forall x \in G$$
  - d. if $x*y = y*x \quad \forall x,y \in G$, this group is called

Commutative / Abelian Group

Note: we only need (1), (2.a), (2.b), (2.c), for a set to be a group.

eg: A set of invertible $n \times n$ matrices with binary operation = matrix multiplicati
But it is not an Abellion Group

eg: set of continuous time periodic signals with time period T with "$*$" = "$+$"

⇒ FIELD : consists of the following

- A set F
- Two binary operations "$+$" and "$\cdot$" such that ....
  - $(F,+)$ is an abelian group
  - define $F^* = F - \{0\} \Rightarrow (F^*, \cdot)$ is an abelian group
  - multiplication operation distributes over addition
    - left distributive
      $$x\cdot(y+z) = xy + xz \quad \forall x,y,z \in F$$
    - Right distributive
      $$(x+y)\cdot z = xz + yz \quad \forall x,y,z \in F$$

eg: F = Real Numbers $\mathbb{R}$

* Vector Space : A set V with a map...

- '$+$' : $V \times V \to V$
  $(v_1, v_2) \to (v_1 + v_2)$ called vector addition
- '$\cdot$' : $F \times V \to V$
  $(a, v_1) \to (a \times v_1)$ called scalar multiplication

... is called a F-vector space or vector space over the field F if the following are satisfied :

- $(V,+)$ is an abelian group
- $a\cdot(v_1+v_2) = av_1 + av_2$
- $(a_1+a_2)\cdot v = a_1 v + a_2 v$
- $(\alpha\beta)v = \alpha(\beta v) = \beta(\alpha v)$
- $1 \cdot v = v$
- $a \cdot 0 = 0$
- $0 \cdot v = 0$
- if $v \neq 0$, then $a \cdot v = 0$ implies $a = 0$
- if V is a vector space over field F, then any linear combination of vectors lying in V (with scalars from F) would again lie in V

* METRIC SPACE : metric is a map

$$d : X \times X \to R$$

satisfies the following : $(\forall x,y,z \in X)$

- $d(x,y) \geq 0$ and $d(x,y) = 0$ if $x = y$
- $d(x,y) = d(y,x)$
- $d(x,y) \leq d(x,z) + d(z,y)$

This map is called a metric and a set equipped with this map is called a metric space and is denoted by (X, d)

eg: Euclidean distance

Note: metric is merely a generalization of the notion of distance

* NORM : let V be a F-vector space.

A map $\|\cdot\| : V \to R$ is called a norm if it satisfies ...

- $\|\bar{v}\| \geq 0$ and $\|\bar{v}\| = 0$ if $\bar{v} = 0$
- $\|a\bar{v}\| = |a|\ \|v\|$
- $\|v_1 + v_2\| \leq \|v_1\| + \|v_2\|$

A vector space equipped with a norm is called a normed vector space

eg: let V be a F-vector space with a nor

prove that $d(v_1, v_2) = \|v_1 - v_2\|$ is a proper metric

Lecture : 2

16/08/24 : 9:30Am

* Inner Product :

let V be a F-vector space
A map,

$$\langle , \rangle : V \times V \to F$$

is called an inner product if ...

- $\langle v,v \rangle \geq 0$ and $\langle v,v \rangle = 0$ iff $v = 0$
- $\langle v_1, v_2 \rangle = \overline{\langle v_2, v_1 \rangle}$ " $\overline{}$ " : complex conjugate operation
- it is linear in the first co-ordinate
  $$\langle a_1 v_1 + a_2 v_2, w \rangle = a_1 \langle v_1, w \rangle + a_2 \langle v_2, w \rangle$$
  $\forall\ v_1, v_2, w \in V$ and $a_1, a_2 \in F$
- it is conjugate linear in the second co-ordinate
  $$\langle v, a_1 w_1 + a_2 w_2 \rangle = \bar{a}_1 \langle v, w_1 \rangle + \bar{a}_2 \langle v, w_2 \rangle$$
  $\forall\ w_1, w_2, v \in V$ and $a_1, a_2 \in F$

measures cosine similarity
" norm(x) . norm(y) . cosθ

eg: dot product
$$\langle x, y \rangle = x_1 y_1 + x_2 y_2 + \dots x_n y_n$$

complex inner Product

complex conjugate

$$\langle x, y \rangle = \sum_{i=1}^{n} x_i \cdot \bar{y}_i$$

* Linear Independence of Vectors

A set of vectors $v_1, v_2, \dots v_n$ in $V_n(F)$ is called linearly independent (LI) if

$$\sum_{i=1}^{n} a_i v_i = 0 \qquad \text{implies } a_i = 0\ \forall i$$

Note: In a 2d space, we represent all non-colinear vectors using 2 vectors

Definition : The number of maximal LI vectors in a vector space V is called the DIMENSION of the vector space and the maximal LI vectors is called a BASIS for V.

weighted linear combination of vector V

if $\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$ is a basis for $V_n(F)$, then $\forall\ v_i \in V$ & $a_i \in F$

$$V = \sum_{i=1}^{n} a_i e_i$$

$V = a_1 e_1 + a_2 e_2 + \dots a_n e_n$

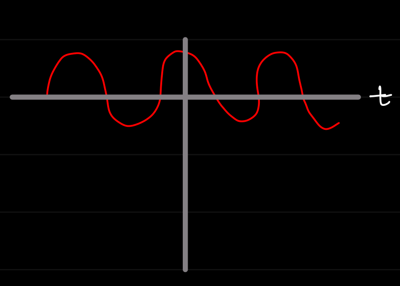
* ORTHOGONAL & ORTHONORMAL Basis

A set of basis vectors $(v_1, v_2 \dots v_n)$ spanning an inner product space V if :

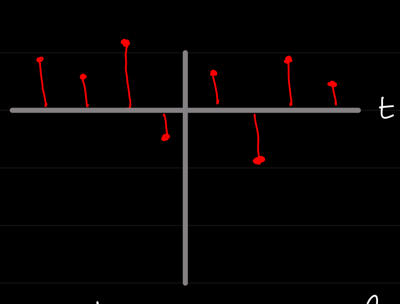
$$v_i \neq 0 \quad \forall\ i$$
$$\langle v_i, v_j \rangle \neq 0 \quad \forall\ i \,\&\, j$$

SIGNALS (CTS & DTS)

- Continuous time signal



- Discrete time signal



- Even Signal : symmetric about vertical axis
  $$x(t) = x(-t) \quad \forall\ t$$
  eg: cosine function
- Odd Signal : non-symmetric about vertical axis
  $$x(-t) = -x(t) \quad \forall\ t$$
  eg: sine function

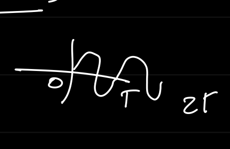
signal notation: $x(t)$ continuous time signal
$x[n]$ discrete time signal

images are two dimensional signal
videos are 3d
videos with audio : 4d

* Even/odd component of a signal:

$$\text{even}\{x(t)\} = \frac{x(t) + x(-t)}{2}$$

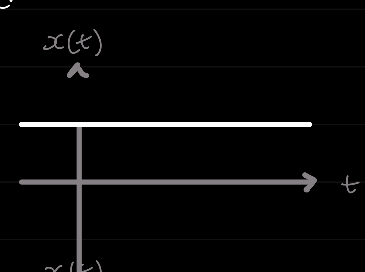
$$\text{odd}\{x(t)\} = \frac{x(t) - x(-t)}{2}$$



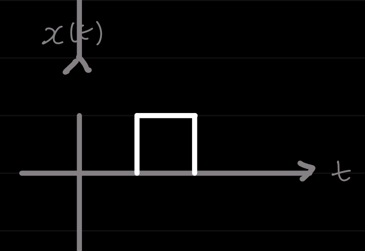
* PERIODIC : if $x(t) = x(t+T)\ \forall\ t$
then T is the time period

The smallest time period T defined for this property is called the fundamental time period

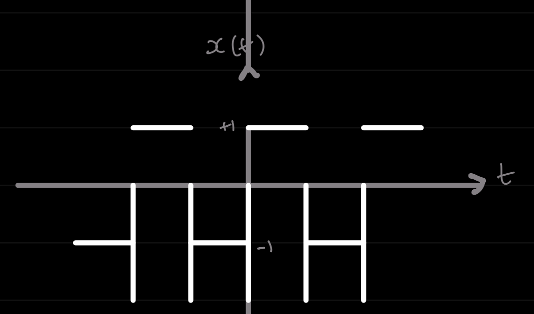
eg:



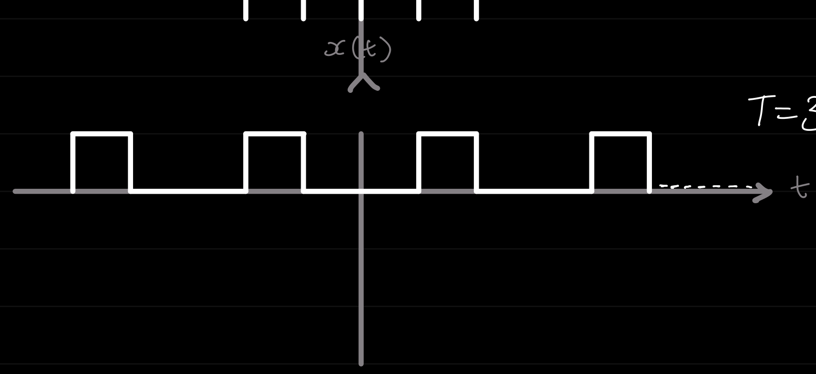
Not periodic



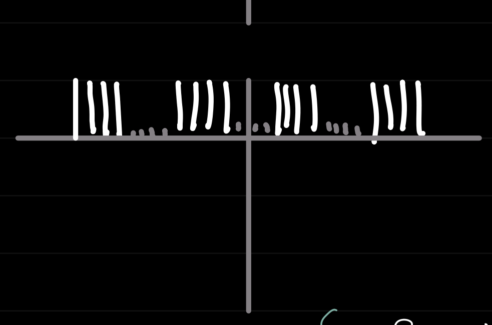
$T \to \infty$
Aperiodic



Periodic with T=2



T=3



T = 8

classification $\begin{cases} \text{Probabilistic} \\ \text{Dataministic} \end{cases}$

* ENERGY OF SIGNAL

Continuous Time signal:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Discrete Time signal

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

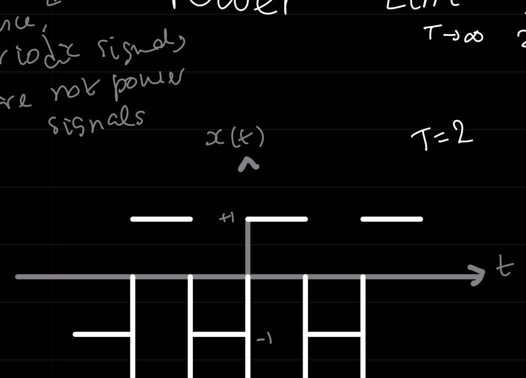
Note: Periodic Signals are power signals ✓
Aperiodic signals are not power signals ✗

T=∞
so, power = not defined
hence, Aperiodic signals are not power signals

avg. energy in
" a time duration

$$\text{Power} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

T=2



compute power?

$$P = \lim_{T \to \infty} \frac{1}{2T} \int$$

$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x[n]|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{x=-N}^{N} |x[n]|^2$$