tr5frbj 12/08/24

Circuit Theory and Devices

tut as well

Lab: LT Spice (2nd week onwards)

book: Engineering Circuit Analysis (9th Edition)

Relative grading

Labs	20%
Quiz (every 2 weeks (n-1))	20%
Mid sem	30%
End sem	30%

Mid sem, End sem ⇐ scientific calc

Course: 9 modules — chapter 10 onwards
{continuation of BE}

* Lecture : 1

⇒ Linear Circuits

x (input) → [system] → y (output)

system is linear if

$Kx \rightarrow \boxed{sys} \rightarrow Ky$

where k is constant

"R" Resistor: linear element ✓
$V = iR$

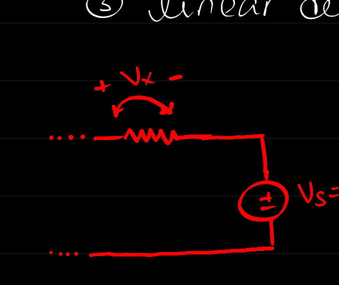
"L" Inductor: linear element ✓
$V = L\frac{di}{dt}$

"C" Capacitor: linear element ✓
$i = C\frac{dv}{dt}$

* Linear Electric Circuits:

consists of ⇒

1. R, L, C → linear elements
2. Independent voltage & current sources
3. linear dependent sources



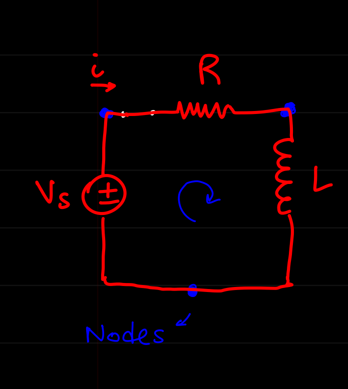
example
would not have been linear if $V_s = KV_x^2$

NOTE: diode and transistors are non-linear elements

* Response of a linear circuit

① full response = transient response + Steady State

transient response	Steady State
temporary	persists $t \to \infty$
due to sudden change	due to source
also called natural response	also called forced response
depends on linear elements R, L, C	depends on current/voltage source



KVL : $-V_s + iR + L\frac{di}{dt} = 0$

$i_{final} = i_{transient} + i_{steady\ state}$

as $t \to \infty$, $i_{final} = i_{steady\ state}$

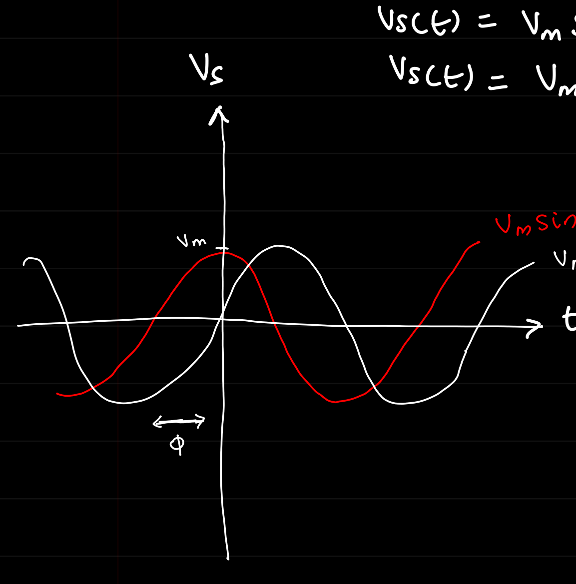
* Sinusoids & complex numbers

sinusoidally varying voltage source

$V_s(t) = V_m \cos(\omega t)$

$V_s(t) = V_m \sin(\omega t)$

$V_s(t) = V_m \sin(\omega t + \phi)$



② is leading ①
① is lagging ②
by angle φ

* TRIGO

$\sin(-\theta) = -\sin(\theta)$

$\cos(-\theta) = \cos(\theta)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

sin +ve	All +ve
tan +ve	cos +ve

* Complex Numbers

Real : x

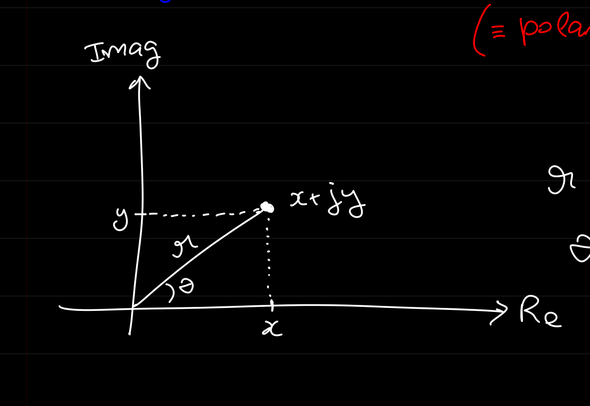
complex : $\underbrace{x + iy}_{\text{rectangular form}}$

x: real part; y: imaginary part; $i = \sqrt{-1}$ a.k.a. j

⇒ **Euler's identity** : $\underbrace{e^{j\theta}}_{\text{exponential form}} = \underbrace{\cos\theta + j\sin\theta}_{\text{rectangular form}}$

Complex Number

- rectangular: $x + iy$
- exponential: $re^{j\theta}$ (≡ polar form : $r\angle\theta$)



$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$(r_1 e^{j\theta_1}) \times (r_2 e^{j\theta_2}) = (r_1 \cdot r_2)\, e^{j(\theta_1 + \theta_2)}$

AC	DC
• we can use transformers to vary voltage	• appliances need constant current
• transported easily over large distances	

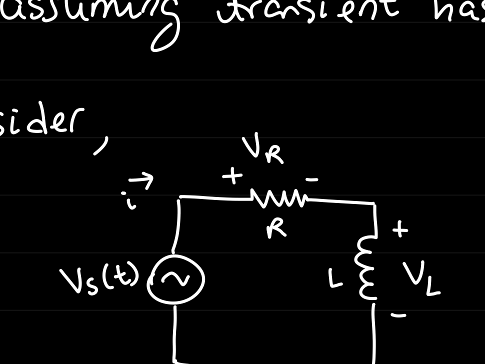
AC → DC : using rectifiers

* MODULE: 1 {CH = 10} 10:30 am

⇒ Sinusoidal steady state analysis

assuming transient has died out long back

Consider,



$V_s(t) = V_m \cos(\omega t)$
find $i(t)$

$V_s(t) = iR + L\frac{di}{dt}$

$\Rightarrow V_m \cos(\omega t) = iR + L\frac{di}{dt}$

both will take cos(ωt) form

General form Solution : $i = I_1 \cos\omega t + I_2 \sin\omega t$

$\Rightarrow V_m \cos(\omega t) = (I_1 \cos\omega t + I_2 \sin\omega t)R + L\frac{di}{dt}$

$\frac{di}{dt} = \frac{d}{dt}(I_1 \cos\omega t + I_2 \sin\omega t)$

$\Rightarrow \omega(I_1(-\sin\omega t) + I_2 \cos\omega t)$

$\Rightarrow \omega(I_2 \cos\omega t - I_1 \sin\omega t)$

$V_m \cos(\omega t) = (I_1 \cos\omega t + I_2 \sin\omega t)R + (I_2 \cos\omega t - I_1 \sin\omega t)L\omega$

$\Rightarrow \cos\omega t \cdot (I_1 R + I_2 L) + \sin\omega t \cdot (I_2 R - I_1 L)$

$\Rightarrow (I_1 R + I_2 L - V_m)\cos(\omega t) + (I_2 R - I_1 L)\sin(\omega t) = 0$

* at $t = 0$,
$(I_1 R + I_2 L - V_m) \cdot 1 + (I_2 R - I_1 L) \cdot 0 = 0$

$$\boxed{I_1 R + I_2 L = V_m}$$

* at $t = \frac{\pi}{2\omega}$, $(I_1 R + I_2 L - V_m) \cdot 0 + (I_2 R - I_1 L) \cdot 1 = 0$

$$\boxed{I_1 = I_2 \cdot \frac{R}{L}}$$

$I_1 R + I_2 L = V_m \Rightarrow I_2 \frac{R^2}{L} + I_2 L = V_m$
$\Rightarrow I_2\left(\frac{R^2 + L^2}{L}\right) = V_m \Rightarrow I_2 = \frac{V_m L^2}{R^2 + L}$

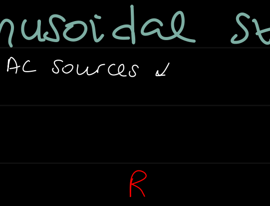
$I_1 = \frac{V_m L R}{R^2 + L}$

14/08/24 # Lecture : 2 9:30 AM

Wednesday → Quiz : Chp 1, 2, 3

* Sinusoidal Steady state analysis

AC sources ↙ ↳ not considering any transient response



$V_s = V_m \cos(\omega t)$ (angular freq)

$freq\,(hz) = \frac{\omega}{2\pi}$

$$V_s(t) = i(t)R + L\frac{di(t)}{dt}$$

general solⁿ: $i = I_1\cos(\omega t) + I_2\sin(\omega t)$

put i from gen solⁿ in above eqⁿ

$$I_1 = \frac{RV_m}{R^2+\omega^2L^2}, \quad I_2 = \frac{\omega LV_m}{R^2+\omega^2L^2}$$

$$i(t) = \frac{RV_m}{R^2+\omega^2L^2}\cos(\omega t) + \frac{\omega LV_m}{R^2+\omega^2L^2}\sin(\omega t)$$

Note: use $\cos(A+B) = \cos A\cos B - \sin A\sin B$

⇒ So, $i(t) = A\cos(\omega t+\phi)$: general solⁿ

becomes ⇒ $i(t) = A[\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)]$

$$\frac{RV_m}{R^2+\omega^2L^2}\cos(\omega t) + \frac{\omega LV_m}{R^2+\omega^2L^2}\sin(\omega t) = A\cos(\omega t)\cos(\phi) - A\sin(\omega t)\sin(\phi)$$

⇓

$$A\cos(\phi) = \frac{RV_m}{R^2+\omega^2L^2}, \quad A\sin(\phi) = \frac{-\omega LV_m}{R^2+\omega^2L^2}$$

$$A^2\cos^2(\phi) + A^2(\sin^2\phi) = A^2(1) = A^2 = \frac{R^2V_m^2 - \omega^2L^2V_m^2}{R^2+\omega^2L^2}$$

⇓

$$\frac{V_m^2(R^2-\omega^2L^2)}{R^2+\omega^2L^2}$$

$$A = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}$$

$$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

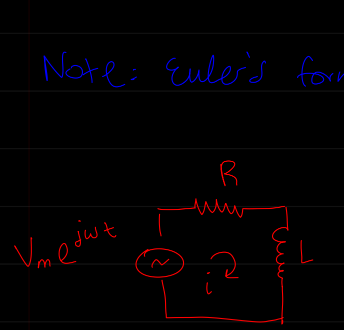
$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

⇒ consider a complex source

$$V_m\cos(\omega t) + jV_m\sin(\omega t) = V_m e^{j\omega t}$$

Real Voltage source $= V_m\cos(\omega t) = Re\{V_m e^{j\omega t}\}$

Note: Euler's formula → $e^{j\theta} = \cos\theta + j\sin\theta$



$$V_m e^{j\omega t} = iR + L\frac{di}{dt}$$

dont integrate ✗

general solⁿ ✓

Assume $i = I_m e^{j(\omega t+\phi)}$

put eqⁿ

$$V_m e^{j\omega t} = I_m e^{j\omega t+j\phi}R + L(j\omega)I_m e^{j\omega t+j\phi}$$

$$\Rightarrow I_m e^{j\omega t+j\phi} = \frac{V_m e^{j\omega t}}{R+j\omega L}$$

is this the real current? No

current cannot be complex

$$\text{Real } i(t) = Re\{I_m e^{j\omega t+j\phi}\} = Re\left\{\frac{V_m}{R+j\omega L}e^{j\omega t}\right\}$$

find the real part of this complex number by simplifying

{multiply by the conjugate = factorize}

$$i(t) = \frac{V_m}{R+j\omega L}\left(\frac{R-j\omega L}{R-j\omega L}\right)e^{j\omega t} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}e^{-j\tan^{-1}\left(\frac{\omega L}{R}\right)}\cdot e^{j\omega t}$$

⇒ Note: rect form → polar form

$$x+jy \rightleftarrows re^{j\theta}$$

$$x = r\cos\theta, \quad y = r\sin\theta$$

$$r = \sqrt{x^2+y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\text{Real } i(t) = Re\left\{\frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cdot e^{j\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)}\right\}$$

$$* \quad i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

same answer as when we were not considering complex numbers

* PHASOR

$$i(t) = I_m\cos(\omega t+\phi) = Re\{I_m e^{j\omega t+j\phi}\}$$

⇓ remove $e^{j\omega t}$, $Re\{\ldots\}$

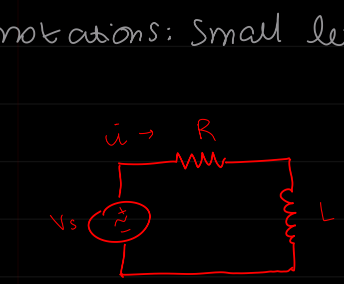
not changing with time

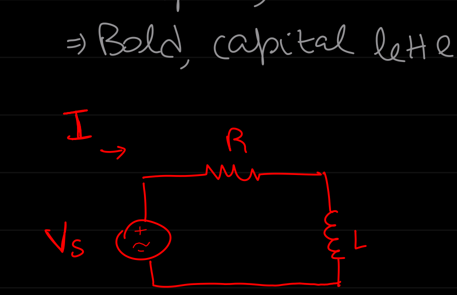
complex func ✗

complex number ✓

$$\underbrace{I_m e^{j\phi}}_{\text{phasor}} = I_m\angle\phi$$

time domain	phasor representation
$i(t) = I_m\cos(\omega t+\phi)$	$I_m\angle\phi$
$v(t) = V_m\cos(\omega t+\theta)$	$V_m\angle\theta$
⇒ real function of t	⇒ complex, constant
⇒ notations: small letters	⇒ Bold, capital letters





$i(t) = I_m\cos(\omega t+\phi)$ ✓

$i(t) = Re\{I_m e^{j\omega t+j\phi}\}$ ✓

$i(t) = I_m\angle\phi$ ✗ complex constant

$i(t) = I_m e^{j(\omega t+\phi)}$ ✗ complex

$i(t) = I_m\angle\omega t+\phi$ ✗ complex phasor notation

phasor notation

but we can write ⇒ phasor notation of $i(t) = I_m\angle\phi = I_m\angle(\omega t+\phi)$

Note: $\angle\phi = e^{j\phi}$ or $\angle\omega t+\phi = e^{j\omega t+j\phi}$

$$\underbrace{V_m e^{j\theta}}_{V} = (R+j\omega L)\underbrace{I_m e^{j\phi}}_{I}$$

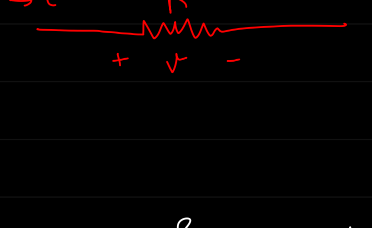
$$\boxed{V = (R+j\omega L)I}$$

↳ phasor notation

* phasor representation of relationship between i and v for R, L, C

① RESISTOR

time domain



$v = iR$

consider complex voltage & current

$v = V_m e^{j(\omega t+\theta)}$

$i = I_m e^{j(\omega t+\phi)}$

$$V_m e^{j\omega t+j\theta} = R\cdot I_m e^{j\omega t+j\phi}$$

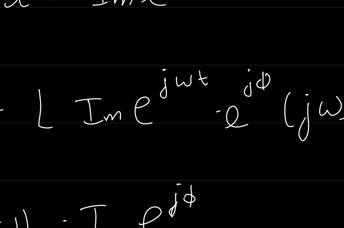
$$V_m \cancel{e^{j\omega t}}\cdot e^{j\theta} = R\cdot I_m \cancel{e^{j\omega t}}e^{j\phi}$$

$$V_m e^{j\theta} = R\cdot I_m e^{j\phi}$$

$$\underbrace{V_m\angle\theta}_{V} = \underbrace{I_m\angle\phi}_{I}\cdot R$$

$$\boxed{V = IR}$$

② INDUCTOR



$v = L\frac{di}{dt}$

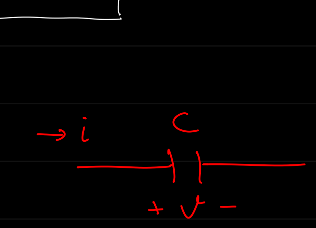
$v \to V_m e^{j(\omega t+\theta)}$ $\quad i \to I_m e^{j\omega t+j\phi}$

$$V_m e^{j\omega t}e^{j\theta} = L\,I_m e^{j\omega t}\cdot e^{j\phi}(j\omega)$$

$$\underbrace{V_m e^{j\theta}}_{V} = j\omega L\cdot\underbrace{I_m e^{j\phi}}_{I}$$

$$\boxed{V = j\omega L\cdot I}$$

③ CAPACITOR



$i = C\frac{dv}{dt}$

$i \to I_m e^{j(\omega t+\phi)}$

$v \to V_m e^{j(\omega t+\theta)}$

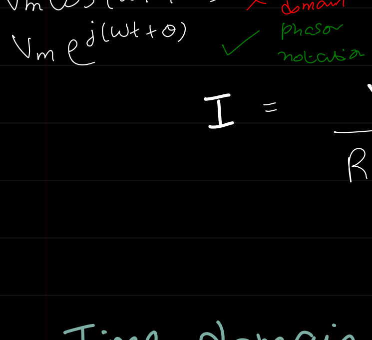
$$I_m\cancel{e^{j\omega t}}\cdot e^{j\phi} = C\cdot V_m\cancel{e^{j\omega t}}e^{j\theta}\cdot j\omega$$

$$\underbrace{I_m e^{j\phi}}_{I} = C\cdot j\omega\cdot\underbrace{V_m e^{j\theta}}_{V}$$

$$I = j\omega C\cdot V$$

$$\boxed{V = \frac{1}{j\omega C}\cdot I}$$

* PHASOR Representation using KVL



$$V_s = V_R + V_L$$

$$= IR + (j\omega L)I$$

$$= I(R+j\omega L)$$

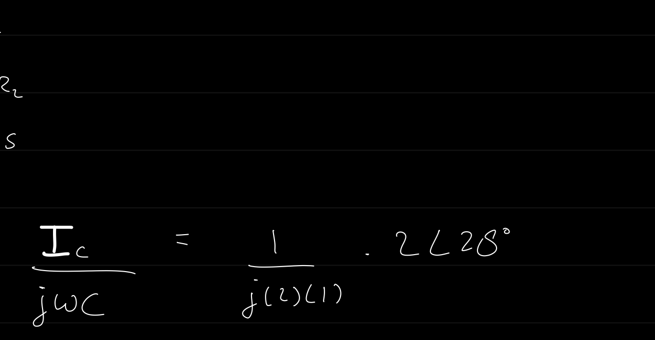
$V_s = V_m\cos(\omega t+\theta)$ ✗ time domain

$V_s = V_m e^{j(\omega t+\theta)}$ ✓ phasor notation

$$I = \frac{V_s}{R+j\omega L} = \frac{V_m\angle\theta}{R+j\omega L}$$

Time domain	Phasor (Frequency domain)
R: $v = iR$	$V = IR$
L: $v = L\frac{di}{dt}$	$V = j\omega L\,I$
C: $i = C\frac{dv}{dt}$	$V = \frac{1}{j\omega C}\,I$

⇒ Example



① Given: both AC sources, V_s & I_s operate at $\omega = 2\,rad/s$

② Given: $I_c = 2\angle 28^\circ A$

Find (i) V_c

(ii) I_{R_2}

(iii) I_s

① $$V_c = \frac{I_c}{j\omega C} = \frac{1}{j(2)(1)}\cdot 2\angle 28^\circ$$

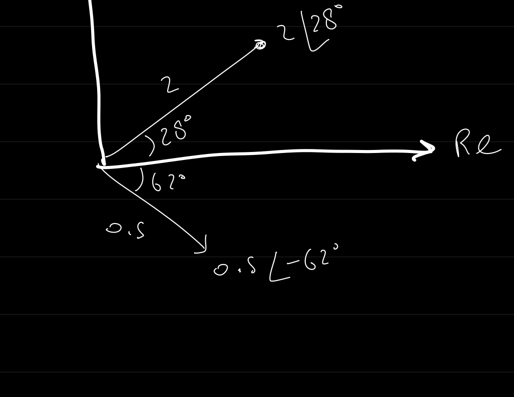
Note: $\frac{1}{j} = -j = -90^\circ$

we add the angles directly which are being multiplied

$$V_c = (0.5\angle -90^\circ)(2\angle 28^\circ) = 1\angle -62^\circ V$$

② $$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_{R_2}}{2} = \frac{1\angle -62^\circ}{2} = 0.5\angle -62^\circ A$$

③ $$I_s = 0.5\angle -62^\circ + 2\angle 28^\circ$$



$2.06\angle 14^\circ A$

Lecture: 3

19/08/24 9:30AM

- Phasor: $i(t) = I_m \cos(\omega t + \phi) = Re\{I_m e^{j(\omega t+\phi)}\}$

Remove $e^{j\omega t}$ and $Re\{...\}$

$$\underbrace{I_m e^{j\phi}}_{\text{exponential form}} = \underbrace{I_m \angle \phi}_{\text{polar form}} \quad \Big\} \text{ Phasor}$$

time domain $\rightleftarrows$ freq. domain

$V_R = IR \qquad V_L = j\omega L\, I \qquad V_c = \dfrac{1}{j\omega C} I$

* IMPEDANCE

$$Z = \frac{V}{I} = \frac{1}{R}$$

$Z_R = R$	$Z_L = j\omega L$	$Z_c = \dfrac{1}{j\omega C}$
voltage and current in the same phase	voltage leads current by 90°	voltage lags current by 90°

$$Z = \underbrace{x}_{\text{resistance}} + j\underbrace{y}_{\text{reactance}}$$

* Admittance

$$Y = \frac{1}{Z}$$

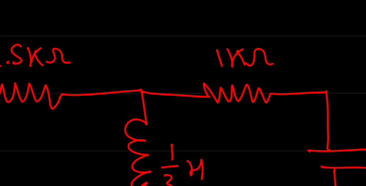
$$Y = \underbrace{x}_{\text{conductance}} + j\underbrace{y}_{\text{susceptance}}$$

* Impedance combination

Series : $Z_{eq} = Z_1 + Z_2 + \dots Z_N$

Parallel : $Z_{eq} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \dots \dfrac{1}{Z_N}$

eg) Find impedance of series combination of an inductor L (L=5mH) and capacitor (C=100μF) $\omega = 10^4$ rad/s

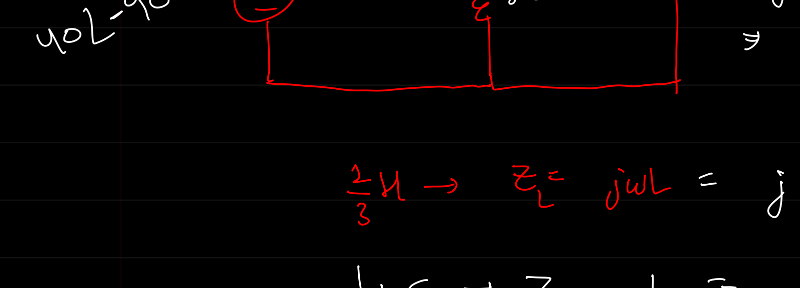


$\Rightarrow Z_{eq} = Z_L + Z_C = j\omega L + \dfrac{1}{j\omega C} = 10^4 \cdot j \cdot 5\times10^{-3} + \dfrac{10^{-4}\times10^3}{j\cdot 5\times 100 \times 10^{-6}}$

Note: $\frac{1}{j} = -j$ $\qquad \Rightarrow 50j - j = 49j$

Note: $e^{j\theta} = \cos\theta + j\sin\theta = e\angle\theta$ $\qquad \Rightarrow 0 + 49j = 49e^{j\cdot\frac{\pi}{2}}$

eg:

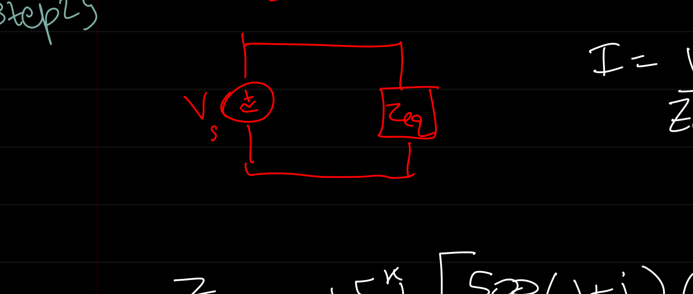


Find i(t)

$\dfrac{10^3(1-2j)(1+j)}{2}$

$500(1-2j)(1+j) \Leftarrow \dfrac{10^3(1-2j)}{1-j}$

step1) Convert to freq. domain (write in terms of phasor and Z)



$2000 - 2000j \parallel 2000j$

$\left(\dfrac{1}{1000 - 2000j} + \dfrac{1}{1000j}\right)^{-1}$

$-j\cdot 2K\Omega \qquad \dfrac{1-j}{10^3 - 2\cdot10^3 j}$

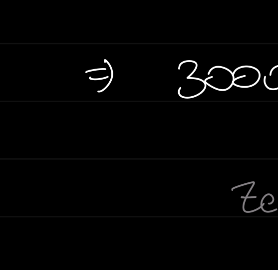
$\Rightarrow \dfrac{1000j + 1000 - 2000j}{(10^3 - 2\cdot10^3 j)10^3 j}$

$\frac{1}{3}H \to Z_L = j\omega L = j\cdot 3000 \cdot \frac{1}{3} \Rightarrow j\cdot 1k$

$\frac{1}{6}\mu F \to Z_c = \dfrac{1}{j\omega C} = \dfrac{1}{j\cdot 3000 \times \frac{1}{6} \times 10^{-6}} \Rightarrow \dfrac{2\cdot10^3}{j}$

$\Rightarrow -j\cdot 2K\Omega$

step2)



$I = \dfrac{V_s}{Z_{eq}}$

$Z_{eq} = 1.5K + [500(1+j)(1-2j)]$

$\Rightarrow 1500 + 500[1 - 2j + j + 2]$

$\Rightarrow 500[3 + 3 - j] \Rightarrow 500[6 - j]$

$\Rightarrow 3000 - 500j$

$\dfrac{j(1-2j)(1+j)}{2} \Rightarrow \dfrac{(j+2)(1+j)}{2} \Rightarrow \dfrac{j + j^2 + 2 + 2j}{2} \Rightarrow \dfrac{3j+1}{2}$

PPT)

$Z_{eq} = 1.5 + \dfrac{(j1)(1-j2)}{1-j} K\Omega$

$\Rightarrow \dfrac{4+3j}{2}$

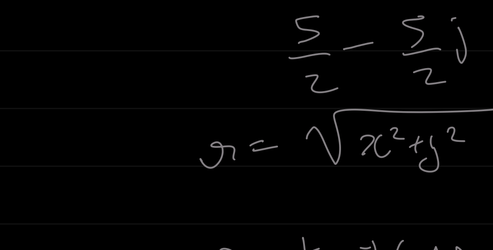
Note: complex number $\begin{cases} x + iy \\ r\angle\theta \end{cases}$

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$I_3 + I_2 = I_1$

eg)



find I_1, I_2, I_3

$Z_{eq}^{(1)} = \left(\dfrac{1}{5} + \dfrac{1}{5j}\right)^{-1} = \left(\dfrac{5j+5}{25j}\right)^{-1} = \dfrac{5j}{j+1}$

$Z_{eq} \Rightarrow^{(2)} -5j + \dfrac{5j}{j+1} \Rightarrow \dfrac{+5 - 5j + 5j}{j+1} = \dfrac{5}{j+1}$

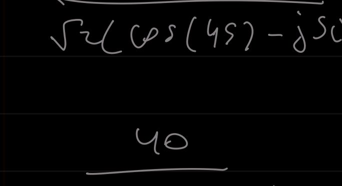
$\Rightarrow \dfrac{5(j-1)}{-2} \Rightarrow \dfrac{5-5j}{2}$

$\dfrac{5}{2} - \dfrac{5}{2}j$

$r = \sqrt{x^2 + y^2} = \sqrt{\dfrac{25}{4} + \dfrac{25}{4}} = \dfrac{\sqrt{50}}{2} \Rightarrow \dfrac{5\sqrt{2}}{2}$

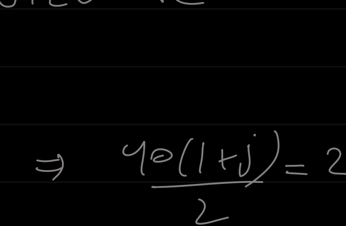
$\theta = \tan^{-1}(-1) \Rightarrow -45°$

Voltage division



$V_1 = V \cdot \dfrac{R_1}{R_1 + R_2}$

Current division



$i_1 = i \cdot \dfrac{R_2}{R_1 + R_2}$

$I_1 = \dfrac{V}{Z_{eq}} = \dfrac{\cancel{100}^{20}\angle 0°}{\frac{5\sqrt{2}}{2}\angle -45°} \Rightarrow \dfrac{40\angle 0°}{\sqrt{2}\angle -45°}$

$r\angle\theta = re^{j\theta} =$

$\dfrac{40(\cos 0 + j\sin 0)}{\sqrt{2}(\cos(45) - j\sin(45))}$

$\dfrac{40}{\sqrt{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)} \Rightarrow \dfrac{40}{1-j} \Rightarrow \dfrac{40(1+j)}{2} = 20(1+j)$

$20 + 20j$

$r = \sqrt{x^2+y^2} \Rightarrow \sqrt{400+400}$

$\Rightarrow 20\sqrt{1+1} = 20\sqrt{2}$

$\theta = \tan^{-1}(1) = 45°$

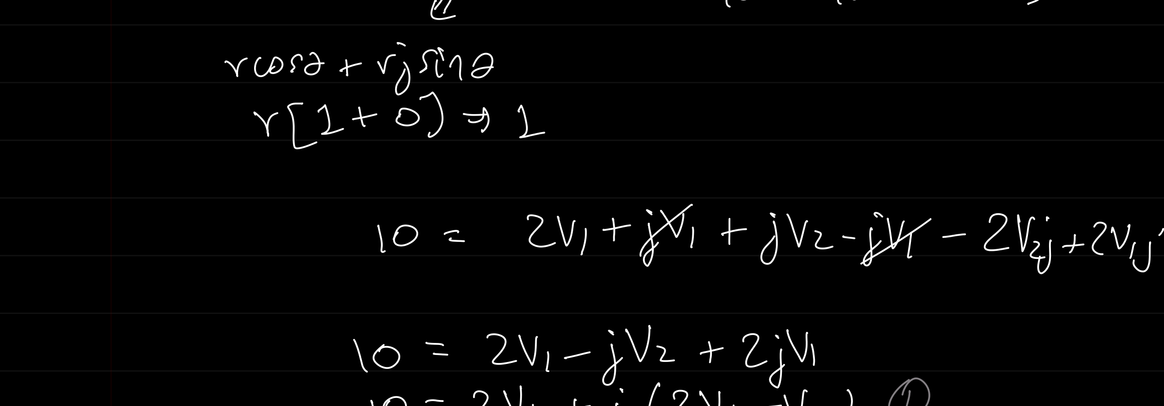
$I_1 = 20\sqrt{2}\angle 45°$

$I_2 = I_1 \times \dfrac{j5}{5+j5} \Rightarrow \cancel{20}^{10}\sqrt{2}\angle 45° \times \dfrac{j(1-j)}{\cancel{2}}$

$\Rightarrow 10\sqrt{2}\angle 45° \times (j+1)$

* NODAL ANALYSIS

Node ≡ Same voltage



at node 1:

$1\angle 0° = \dfrac{V_1}{5} + j\dfrac{V_1}{10} + \dfrac{(V_2 - V_1)j}{10} - \dfrac{(V_2 - V_1)j}{5}$

$r\cos\theta + rj\sin\theta$

$r[1 + 0] \Rightarrow 1$

$10 = 2V_1 + \cancel{jV_1} + jV_2 - \cancel{jV_1} - 2V_2 j + 2V_1 j$

$10 = 2V_1 - jV_2 + 2jV_1$

$10 = 2V_1 + j(2V_1 - V_2)$ ①

at node 2:

$\dfrac{V_2 - V_1}{j10} + \dfrac{V_2 - V_1}{-j5} + \dfrac{V_2}{j5} + \dfrac{V_2}{10} = 0.5\angle -90°$

$0.5\cos 90° - 0.5j\sin 90°$

$\Rightarrow -0.5j$

$V_2 - V_1 + 2V_1 - \cancel{2V_2} + \cancel{2V_2} + V_2 j = 5$

$3V_2 - V_1 + V_2 j = 5 \Rightarrow V_1 = 3V_2 - 5 + V_2 j$

Put value of V_1 in ①

$10 = 6V_2 - 10 + 2V_2 j + j(6V_2 - 10 + 2V_2 j - V_2)$

$10 = 6V_2 - 10 + 2V_2 j + 6V_2 j - 10j - 2V_2 - V_2 j$

$20 = 4V_2 + 7V_2 j - 10j$

$20 + 10j = V_2(4 + 7j)$

$V_2 = \dfrac{(20+10j)(4-7j)}{16+49} = \dfrac{80 - 140j + 40j + 70}{65}$

$\dfrac{150 - 100j}{65} \Rightarrow \dfrac{30 - 20j}{13}$

$V_2 = \dfrac{30}{13} - j\left(\dfrac{20}{13}\right)$

$r = \sqrt{\dfrac{900 + 400}{169}} = \dfrac{10}{\sqrt{13}}$

$\theta = \tan^{-1}\left(-\frac{2}{3}\right)$

$= -\tan\left(\frac{2}{3}\right) =$

$= -33.69$

$V_2 = \dfrac{10\sqrt{13}}{13}\angle -33.69°$

$V_1 = 3V_2 - 5 + jV_2$

$\Rightarrow V_2(3 + j) - 5 = \dfrac{30}{13} - \dfrac{20j}{13}(3+j) - 5$

$\Rightarrow \dfrac{30 - 60j + 20 - 65}{13} = \dfrac{-60j - 15}{13} \Rightarrow V_1 = \dfrac{15\sqrt{17}}{13}\angle -104°$

* mesh Analysis

2 meshes

$10\angle 0° = 3I_1 + j4(I_1 - I_2)$

$(I_2 - I_1)j4 - I_2(j2) + 2I_1 = 0$

$10\angle 0° = 10 = 3I_1 + j4(I_1 - I_2) = 3I_1 + j4I_1 - j4I_2$

$j4I_2 + 10 = I_1(3$

$j4I_2 - j4I_1 - j2I_2 + 2I_1 = 0$

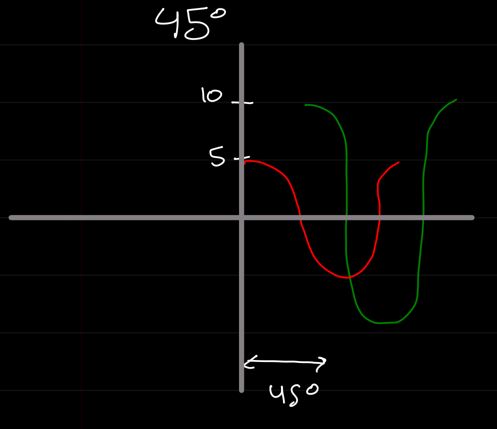
$j4I_2 - j4(10 -$

✱ CTD Tutorial : 1

19/08/24   1:00 – 3:00

Q1) Determine the angle by which v_1 leads i_2 if $V_1 = 10\cos(10t - 45°)$ and i_1 is equal to

(1) $i_1 = 5\cos 10t$
$45°$



(3) $i_1 = 5\cos(10t - 40°)$
$5°$

(2) $i_1 = 5\cos(10t - 80°)$
$35°$

(4) $i_1 = 5\cos(10t + 40°)$
$85°$

(5) $5\sin(10t - 19°)$
$= 5\cos(10t - 19° - 90°)$
$= 5\cos(10t - 109°)$
Ans : $64°$

Q2) Determine which waveform in each of the following pairs is lagging

(1) $\cos 4t$ leads $\sin 4t$ by $\frac{\pi}{2}$ {90°}

(2) $\cos(4t - 80°)$ lags $\cos(4t)$ by $80°$

(3) $\cos(4t + 80°)$ leads $\cos(4t)$ by $80°$

(4) $\sin(5t)$ lags $\cos(5t + 2°)$ by $92°$

(5) $\sin 5t + \cos 5t$

Q3) Perform the indicated operation {express in rectangular & polar form}

(a) $\frac{2 + j3}{1 + 8\angle 90°} - 4$

$8\angle 90°$
$\hookrightarrow r\cos\theta + j\sin\theta = 8.0 + 8.j.1 = 8j$

$\Rightarrow \frac{2 + j3}{1 + 8j} - 4 \Rightarrow \frac{2 + j3 - 4 - j32}{1 + 8j} \times \frac{1 - 8j}{1 - 8j}$

$\Rightarrow \frac{(-2 + 29j)(1 - 8j)}{8} = -2 + 16j + 29j + (29 \times 8)$

$\Rightarrow 230 + 45j$

$\Rightarrow 46 + 9j$

$\frac{2 + j3}{1 + 8\angle 90°} - 4$
$\Downarrow$
$8\angle 90° = 8 \cdot \cos(90) + 8.j\sin(90) = 8j$

$\frac{2 + j3}{1 + j8} \times \frac{1 - 8j}{1 - 8j} - 4$

$\Rightarrow \frac{2 - 16j + j3 + 24}{65} - 4 \Rightarrow \frac{26 - 13j}{65} - 4$

$\Downarrow$

$\frac{-234 - 13j}{65} \Leftarrow \frac{26 - 13j - 260}{65}$

$x \quad y \quad \boxed{\frac{-18}{5} - \frac{1}{5} \cdot j}$ rectangular form

$\Downarrow$

$r = \sqrt{x^2 + y^2} = \sqrt{\frac{18^2}{25} + \frac{1}{25}}$

$\Rightarrow \sqrt{\frac{325}{25}} = \sqrt{13}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{18}\right) = 3.179°$

polar form $\boxed{\sqrt{13}\angle 3.179°}$

(b) $\left(\frac{10\angle 25°}{5\angle -10°} + \frac{3\angle 15°}{3 - 5j}\right) j2$

Q4) Transform into Phasor form

(a) $11\sin 100t$

(b) $11\cos 100t$

(c) $11\cos(100t - 90°)$

(d) $3\cos 100t - 3\sin 100t$

Q5) Determine equivalent Admittance assuming $w = 1000$ rad/s

(a) 25Ω in series with $20mH$

$\Downarrow$

$Z_L = j\omega L = j \times 1000 \times 20 \times 10^{-3}$
$= j \cdot 20$

Admittance $(Y) = \frac{1}{Z_{eq}}$

$Z_{eq} = 25 + j20 \nearrow \quad Y = \frac{1}{25 + j20} \times \frac{25 - j20}{25 - j20}$

$\boxed{\frac{1}{41} - \frac{4}{205} \cdot j} \Leftarrow \frac{5 - j4}{205} \Leftarrow \frac{25 - j \cdot 20}{625 + 400} \Rightarrow \frac{25 - j \cdot 20}{1025}$

(b) 25Ω in parallel with $20mH$

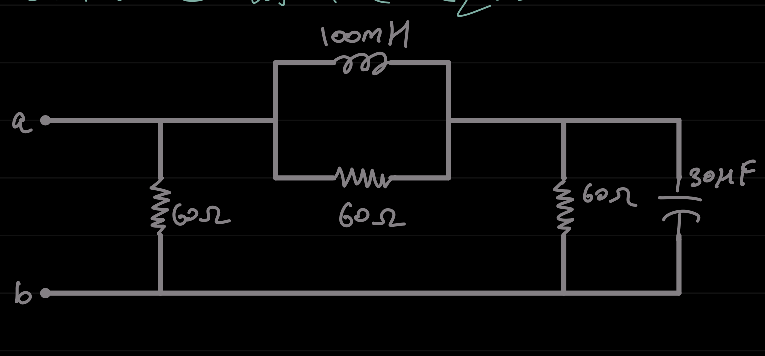
admittance $= \frac{1}{25} + \frac{1}{20j} = \frac{\overset{5}{\cancel{25}} + j\overset{4}{\cancel{20}}}{\underset{100}{\cancel{500}}j}$

$\frac{-j \cdot 5 - 4}{100} \Leftarrow \frac{-j(5 + j \cdot 4)}{100} \Leftarrow \frac{5 + j \cdot 4}{100j}$

$\hookrightarrow \frac{-1}{25} + \left(\frac{-j}{20}\right)$

Q6) Calculate the equivalent impedance as seen at the open terminals if the freq is ...

(a) 1Hz
(b) 1KHz
(c) 1MHz
(d) 1GHz



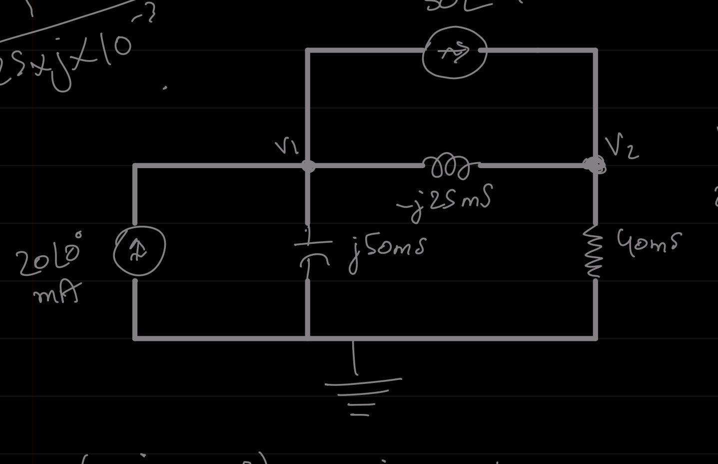
(a) $f = 1Hz \quad w = 2\pi f = 2\pi$ radians/s

$Z_L = j\omega L = j \cdot 2\pi \cdot 100 \times 10^{-3}$

$\Rightarrow \frac{j \cdot 2\pi}{\cancel{10}5} \Rightarrow \frac{j\pi}{5} \Rightarrow$

Q7) Use nodal analysis to find V_1 and V_2

$\frac{1}{-25 \times j \times 10^{-3}}$



$\boxed{V1}$

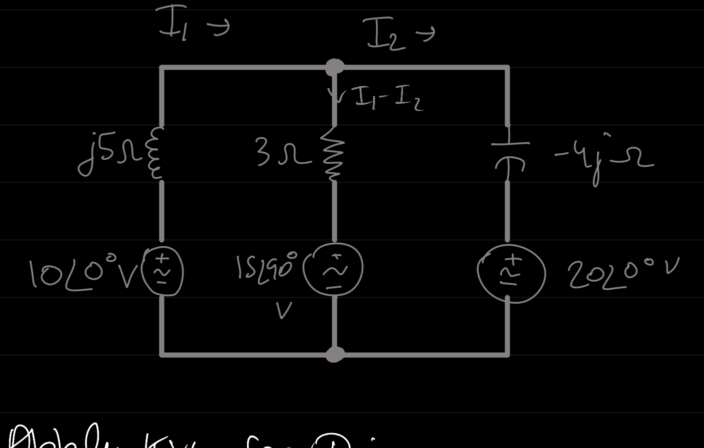
$\frac{V_1 \cdot j}{20} - \frac{(V_1 - V_2)j}{40} + (-50j \times 10^{-3}) + 20 \times 10^{-3} = 0$

$\Rightarrow \frac{5(V_1 j + V_2 j)}{10} - \frac{10j}{10} + \frac{4}{10} = 0$

$\Rightarrow 5V_1 j + 5V_2 j - 10j + 4 = 0 \Rightarrow 4 + j(5V_1 + 5V_2 - 10) = 0$

② $\frac{(V_2 - V_1) \cdot -j}{40} + \frac{V_2 - 0}{40 \times 10^{-3}} + (50j \times 10^{-3}) = 0$

Q8) Use mesh analysis to find I_1 and I_2



Apply KVL for ①:

$3(I_1 - I_2) + 15\angle 90° - 10\angle 0° + I_1(j5) = 0$

$3I_1 - 3I_2 + 15\overset{0}{\cancel{\cos 90°}} + 15j\sin 90° - 10\cos 0° - 10j\overset{0}{\cancel{\sin 0°}} + I_1 5 \cdot j = 0$

$3I_1 - 3I_2 + 15j - 10 + 5j\,I_1 = 0$

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

Apply KVL for ②

$-4I_2 \cdot j + 20\angle 0° - 15\angle 90° + (I_2 - I_1)3 = 0$

$-4I_2 \cdot j + 20 - 15j + 3I_2 - 3I_1 = 0$

$3I_2 - 3I_1 + 20 + j(-15 - 4I_2) = 0$

$3I_1 = 3I_2 + 20 + j(-15 - 4I_2)$

Put ② in ①

$3I_1 - 3I_2 - 10 + j(15 + 5I_1) = 0$

$\cancel{3I_2} + 20 + j(-15 - 4I_2) - \cancel{3I_2} - 10 + j(15 + 5I_1) = 0$

$10 + j(-15 - 4I_2) + j\left(15 + 5\left[\frac{3I_2 + 20 - 15j - 4I_2 \cdot j}{3}\right]\right) = 0$

$30 - 45j - 12I_2 j + j(45 + 15I_2 + 100 - 75j - 20I_2 j) = 0$

$30 - \cancel{45}j - 12I_2 j + \cancel{45}j + 15I_2 j + 100j + 75 + 20I_2 = 0$

$105 + 20I_2 + j(3I_2 + 100) = 0$

$105 + 20I_2 + j \cdot 3I_2 + j \cdot 100 = 0$

$I_2(20 + j \cdot 3) = -105 - j \cdot 100$

$I_2 = \frac{-105 - j \cdot 100}{20 + j \cdot 3} \times \frac{20 - j \cdot 3}{20 - j \cdot 3}$

$I_2 = \frac{-2100 + 315 \cdot j - 2000j - 300}{409}$

$I_2 = \frac{-2400 - 1685j}{409}$

Home work:1 19/08/24

* Section 10.1

Q1} (a) $5\sin(5t-9^\circ)$

$t=0 \Rightarrow 5\sin(-9^\circ) = -5\sin(9^\circ) = -0.782$

$t=0.01 \Rightarrow 5\sin(0.05-9^\circ) = -5\sin(8.95^\circ)$ X

$\Downarrow$ radians

$5\sin\left(\left[0.05\times\frac{180}{\pi}\right)-9^\circ\right) = 5\sin(2.86^\circ-9^\circ)$

$\Rightarrow -5\sin(6.14) = -0.534$

$t=0.1s \Rightarrow 5\sin(0.5-9^\circ) = 5\sin(28.64-9) = 5\sin(19.64)$

$\Rightarrow 1.6805$

(b) $4\cos 2t$

$t=0 \Rightarrow 4\cos(0) = 4$

$t=1 \Rightarrow 4\cos(2) = 3.997$

$t=1.5 \Rightarrow 4\cos(3) = 3.994$

(c) $3.2\cos(6t+15^\circ)$

$t=0 \Rightarrow 3.2\cos(15^\circ) = 3.09$

$t=0.01 \Rightarrow 3.2\cos(0.06+15^\circ) = 3.2\cos(3.43^\circ+15^\circ)$

$\Rightarrow 3.2\cos(18.43^\circ)$

$\Rightarrow 3.035$

$t=0.1 \Rightarrow 3.2\cos(0.6+15^\circ) = 3.2\cos(34.3^\circ+15^\circ)$

$= 3.2\cos(49.3)$

$= 2.086$

Q2} (a) $300\sin(628t) \rightarrow 300\cdot\cos(628t-90^\circ)$

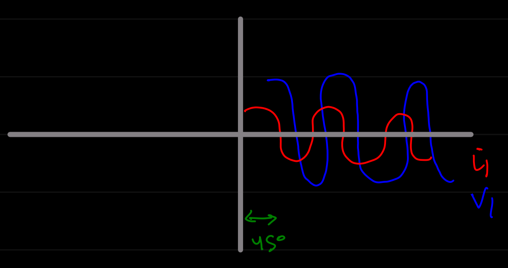
$4\sin(3\pi t+30^\circ) \rightarrow 4\cos(3\pi t-60^\circ)$

$14\sin(50t-5^\circ) - 10\cos 50t \rightarrow$

Q3} $v_1 = 10\cos(10t-45^\circ)$

(a) $i_1 = 5\cos 10t$

-45°

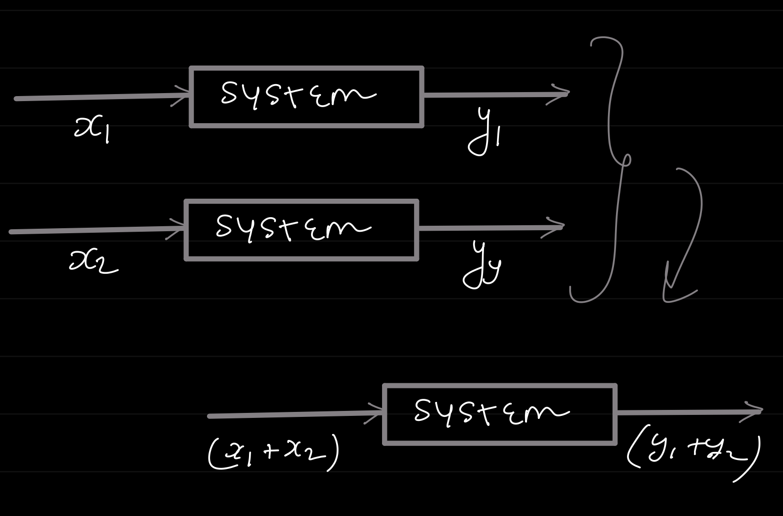


9:30 – 11:00AM
21/08/24

LECTURE: 4

* Superposition principle

active elements : provide energy eg: volt src / curr src
passive element : consume / store energy
eg: R / L / C

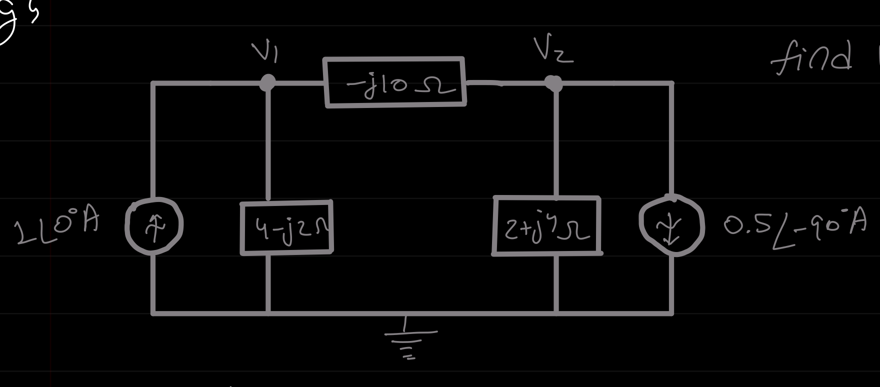


* Multiple independent sources

- consider separate independent sources "acting alone"
- add all responses

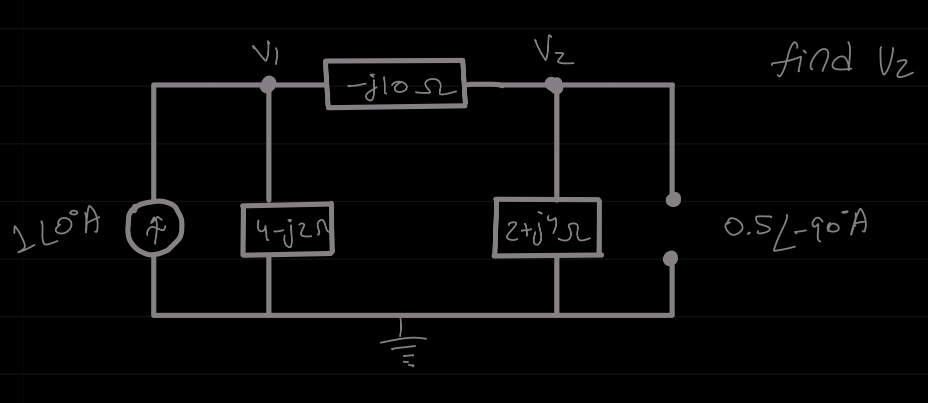
eg)

find V_2

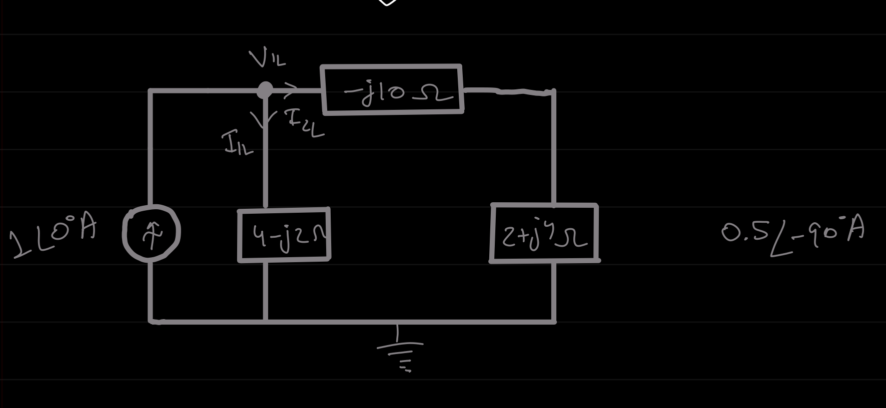


consider left side source
so we need to open the circuit at the right source

find V_2



⇓



$V_{1L} = I_{1L}(4-j2)$

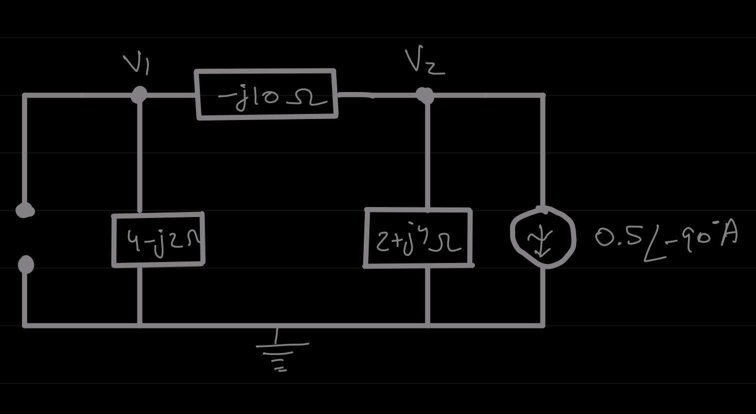
$\Rightarrow \dfrac{-j10+2+j4}{(4-j2)+(-j10+2+j4)}(4-j2)$

$\Rightarrow \dfrac{(2-j6)(4-j2)}{6-j8} \Rightarrow \dfrac{8-j4-24j-12}{6-8j}$

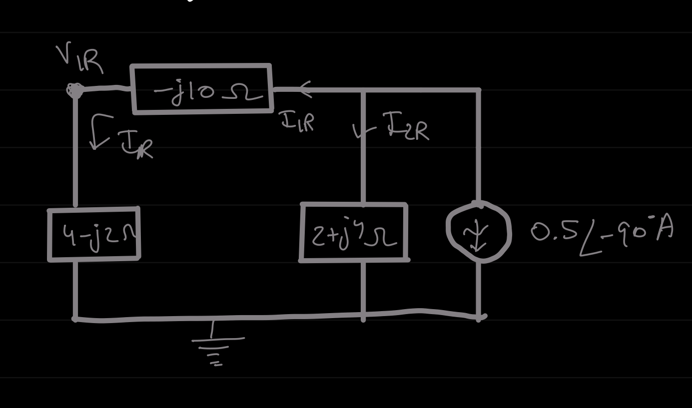
$\Rightarrow \dfrac{-4-28j}{6-8j} = 2-j2\,V$

$I_{1L}+I_{2L} = 1\angle 0° = 1\cos 0° + j\,1\sin 0° = 1A$

consider right src



⇓



$V_{1R} = I_{1R}(4-j2)$

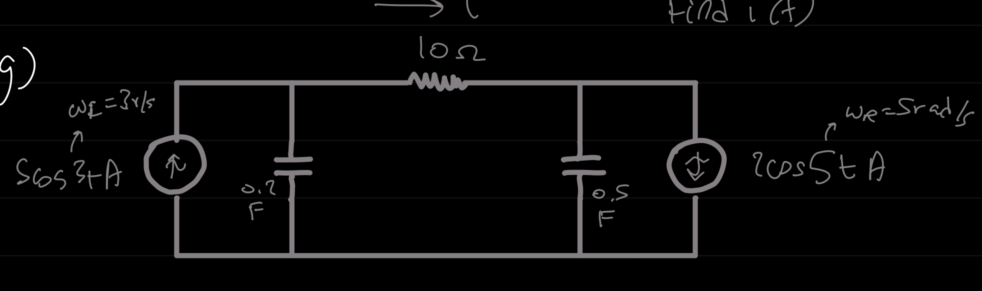
$I_{1R} = \dfrac{(-0.5\angle -90° A)(2+j4)}{-j10+4-2j+2+j4}$

$V_{1R} = \dfrac{(j\cdot 0.5)(2+j4)(4-j2)}{6-j8}$

$\Rightarrow \dfrac{(j-2)(4-j2)}{6-j8} = \dfrac{j4+2-8+j4}{6-j8} = \dfrac{j8-6}{6-j8} = -1V$

$V_1 = V_{1L}+V_{1R} = 2-j2-1 = \boxed{1-j2\,V}$

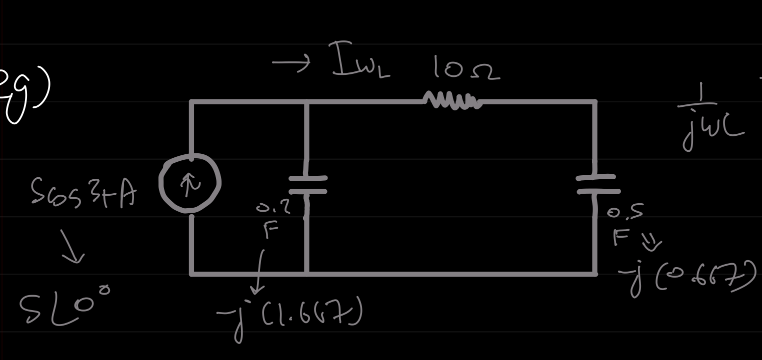
eg) Find i(t)



Note: we have 2 sources operating at 2 different frequencies (ω)

① consider left src $\omega_L = 3\,rad/s$

eg)

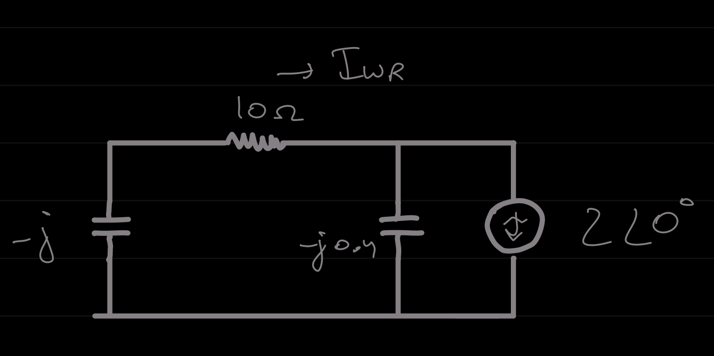


$\dfrac{1}{j\omega C} = \dfrac{1}{j(3)0.2} = \dfrac{-j}{0.6}$

$I_{\omega_1} = 5\angle 0°\left(\dfrac{-j(1.667)}{-j(1.667)+10-j(0.667)}\right)$

$\Rightarrow 811.7\angle -76.86°\,mA$

② consider right source $\omega_R = 5\,rad/s$



$I_{\omega R} = 2\angle 0°\left(\dfrac{-j\cdot 0.4}{-j0.4+10+-j}\right)$

$\Rightarrow \dfrac{-j(0.8)}{-j(1.4)+10} \Rightarrow \dfrac{-j(0.8)[10+j(1.4)]}{100+1.96}$

$\Downarrow$

$\dfrac{-j8+1.12}{101.96} \Rightarrow$

$\Rightarrow 0.0792\angle -82.03°\,A = 79.2\angle -82.03\,mA$

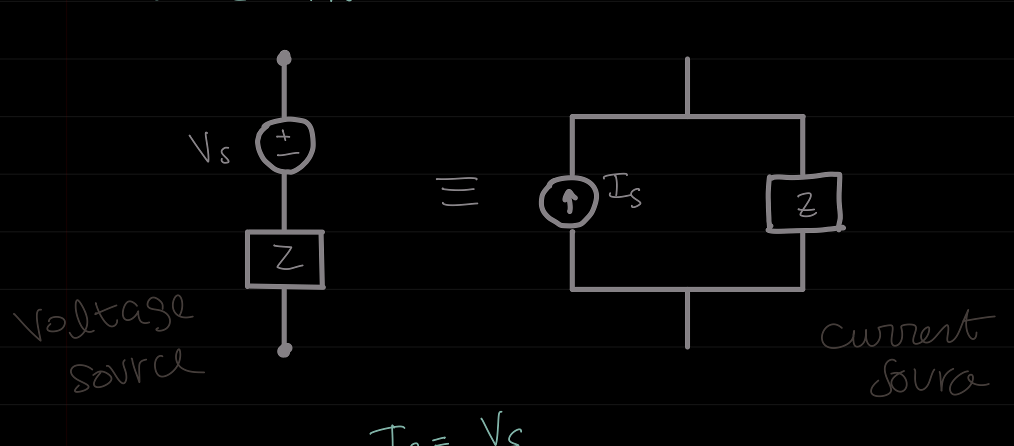
Applying superposition

$I = I_{\omega L} + I_{\omega R}$ ✗

Phasors can only be added when they correspond to same frequency

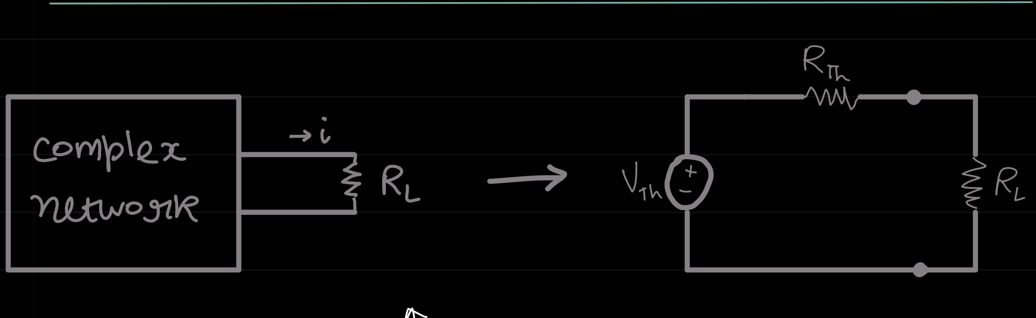
$i(t) = [811.7\cos(3t-76.86°) + 79.25\cos(5t-82.05)]\,mA$

* SOURCE TRANSFORMATION

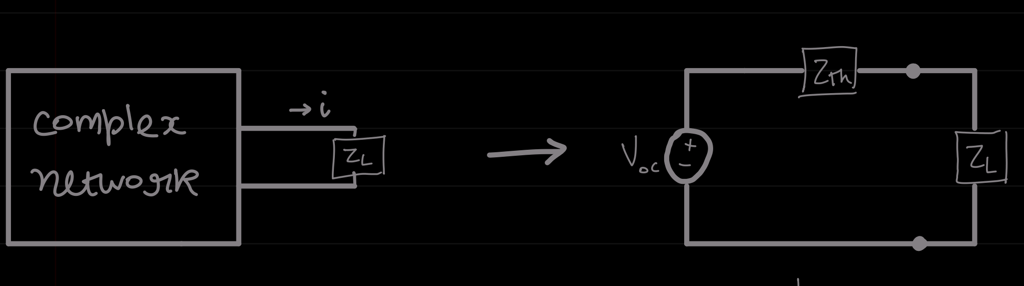


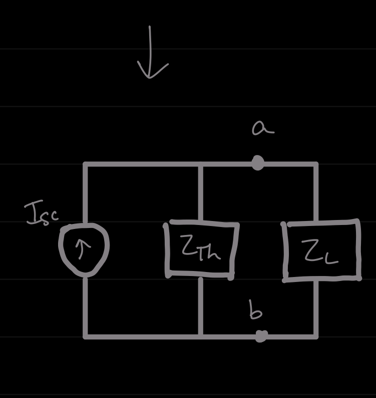
$I_s = \dfrac{V_s}{Z}$

* THEVENIN & NORTON EQUIVALENT CKTS



Normally



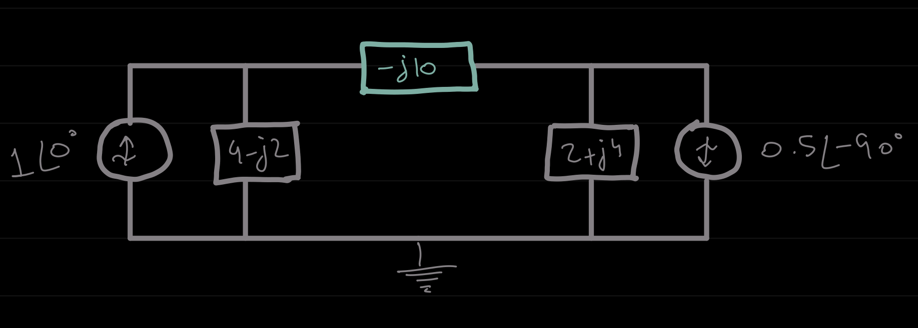


$V_{oc} = I_{sc} \cdot Z_{TH}$

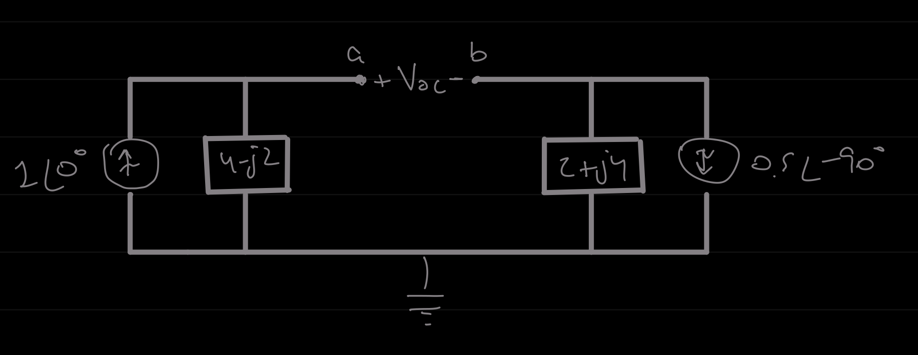
	by open circuiting V_{oc}	by short circuiting I_{sc}	Z_{TH} ratio of V_{oc} & I_{sc}
① Independent sources	✓	✓	✓
② Independent + dependent sources	✓	✓	✓

③ Dependent sources only — we need to consider test sources eg: 1A / 1V and then find $V_{oc} \Rightarrow I_{sc} \Rightarrow Z_{Th} = \dfrac{V_{oc}}{I_{sc}}$

eg) find Thevenin equivalent seen by $-j10\Omega$



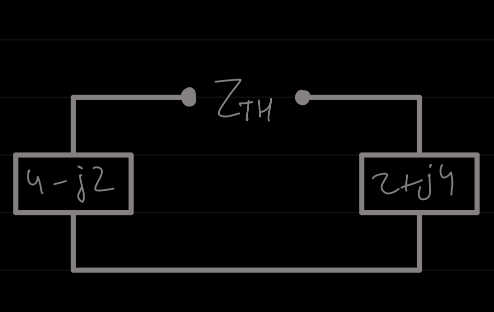
- : complex network
- : R_L



$V_{oc} = V_a - V_b = 1\angle 0°(4-j2) - 0.5\angle -90°(2+j4)$
$= 4-j2+2-j1 = 6-j3\,V$

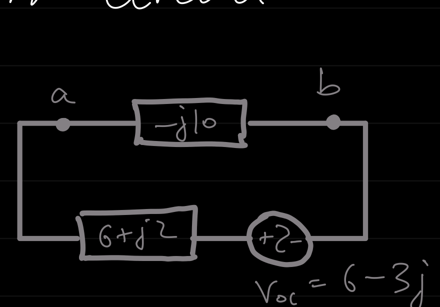
Find Z_{Th}

all volt src → short circuit
all curr src → open circuit



$Z_{TH} = 4-j2+2+j4 = 6+j2\,\Omega$

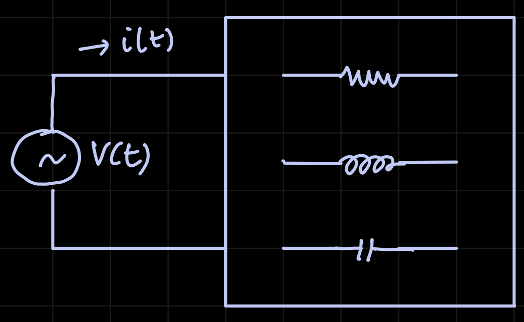
Equivalent circuit



$I_{ab} = \dfrac{V_{oc}}{6+j2-j10} = \dfrac{V_{oc}}{6-j8}$

* Lecture : 5 28/08/24

⇒ AC Power Analysis {Module : 3}



$\cos(A)\cos(B) = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

instantaneous power : $p(t) = v(t) \cdot i(t)$

$$v(t) = V_m \cdot \cos(\omega t + \theta)$$
$$i(t) = I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = V_m \cdot \cos(\omega t + \theta) \cdot I_m \cdot \cos(\omega t + \phi)$$

$$p(t) = \underbrace{\frac{V_m I_m}{2} \cos(\theta - \phi)}_{\substack{\text{constant} \\ \text{(DC term)}}} + \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{\substack{\text{twice frequency} \\ \text{(harmonic)}}}$$

- Average Power

$$P_{avg} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2} \cos(\theta - \phi)\,dt + \underbrace{\frac{1}{T}\int_0^T \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{0}$$

$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)}$$

$$P_{avg} = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$$

real number ✓
phasor ✗

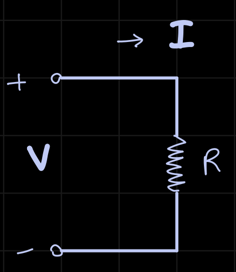
$\mathbf{I} = I_m e^{j\phi}$
$\mathbf{I}^* = I_m e^{-j\phi}$
(conjugate)

* Avg. Power absorbed

- by a purely resistive element

$$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$$

here, phase diff = 0°

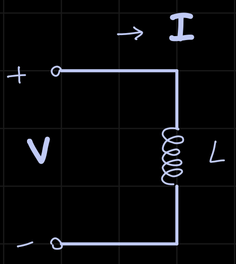


$$P_{avg} = \frac{V_m \cdot I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

- by a purely inductive element

$$P_{avg} = \frac{V_m I_m}{2} \cdot \cos(\theta - \phi)$$

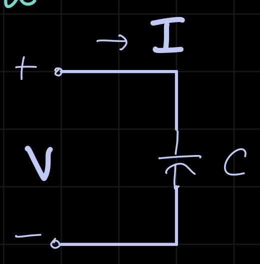
here, phase diff = 90°



So,
$P_{avg} = 0$ for inductor

- by a purely capacitive element

$$P_{avg} = \frac{V_m \cdot I_m}{2} \cdot \cos(-90°)$$



$P_{avg} = 0$ for capacitor

* Lecture : 6

$v(t) = V_m \cos(\omega t + \theta)$ ← Revision

↓

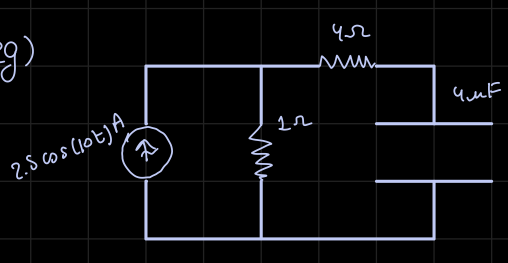
$v(t) = Re\{V_m e^{j\omega t} e^{j\theta}\}$

↓ $-e^{j\omega t}$

$V_m e^{j\theta} = V_m \angle \theta$

exponential polar

• eg)

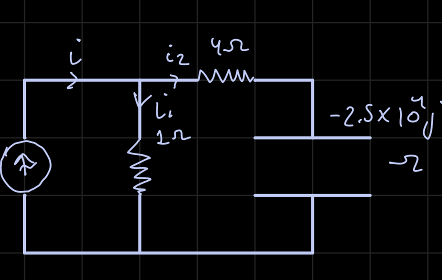


① find power delivered to each element at $t = 0, 10, 20ms$

② Find P_{avg} to each element

Ans) convert to freq. domain

$V_{1\Omega} = V_{4\Omega} + V_c$



① $P_{inst} = \frac{V_m I_m}{2} \cos(\theta - \phi) + \frac{V_m I_m}{2} \cos(2\omega t + \phi + \theta)$

imaginary part is way larger than real part

$I_1 = -2.5 \times \left(\frac{4 - j2.5 \times 10^4}{1 + 4 - j\,2.5 \times 10^4} \right) \simeq \underline{\underline{-2.5A}}$

$\Rightarrow \frac{-10 + j625 \times 10^{2}}{5 - j\,25 \times 10^3} \Rightarrow \frac{-2 + j125}{1 - j50}$

$\Rightarrow \frac{-2 + j \cdot 125}{1 - j50} \times \frac{1 + j50}{1 + j50} \Rightarrow$

$I_{1\Omega}(t) = 2.5 \cos(10t)$

$V_{1\Omega} = (I_{1\Omega})(1\Omega) \Rightarrow V_{1\Omega}(t) = 2.5 \cos(10t)$

$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{4}$

$p_{1\Omega}(t) = 2.5 \cos(10t) \cdot 2.5 \cos(10t)$ W

$p_{1\Omega}(t=0) = 6.25W$

$p_{1\Omega}(t = 10ms) = 6.1877W$

$p_{1\Omega}(t = 20ms) = 6.033W$

$(P_{avg})_{1\Omega} = 3.125W$

⇒ P delivered to 4Ω

$P_{4\Omega}(t) = V_{4\Omega}(t) \times I_{4\Omega}(t)$

$I_{4\Omega} = I_s \times \frac{1}{5 - j2.5 \times 10^4} = 10^{-4} \angle 90° A$

$V_{4\Omega} = 4 \times I_{4\Omega} = 4 \times 10^{-4} \angle 90°$ V

convert to time domain

$p_{4\Omega}(t) = 2 \times 10^{-8} + 2 \times 10^{-8} \cos(20t)$ W

$t = 0: \; 4 \times 10^{-8}$ W

$t = 10ms = 3.96 \times 10^{-8}$ W

$t = 20ms = 3.87 \times 10^{-8}$ W

$P_{avg\,4\Omega} = 2 \times 10^{-8}$ W

⇒ P delivered to capacitor

$p_c(t) = V_c(t) \cdot I_c(t)$

$(P_{avg})_C = 0$ since phase diff = 90°

$I_c = I_2 = 10^{-4} \angle 90° A \Rightarrow I_c(t) = 10^{-4} \cos(10t + 90°)$

$V_c = (-j\,2.5 \times 10^{4}) \times 10^{-4} \angle 90$

$-j \cdot 25 \times 10^3 \times (+j10^{-4})$

$\underline{\underline{2.5\,V}}$

$p_c(t) = 1.25 \times 10^{-4} \cos(20t + 90°)$ W

$t = 0 \Rightarrow p = 0W$

$t = 10ms \Rightarrow p = 2.48 \times 10^{-5}$ W

Note: we cannot multiply I_c and V_c in phasor form and then convert to time domain for getting P_c (power) because power does not has a phasor part. It is a real value.

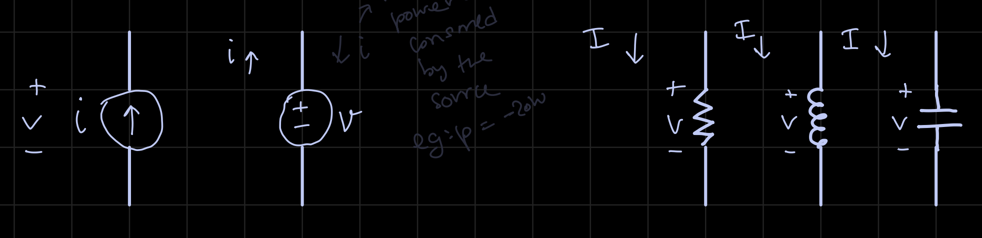
* Correct or not ?

depends on t ✗ ① $p_{1\Omega}(t) + p_{4\Omega}(t) + p_c(t) = \text{constant}1$

✓ ② $P_{avg\,1\Omega} + P_{avg\,4\Omega} + P_{avg\,c} = \text{constant}2 \equiv P_{avg\ source}$

active sign convention

passive sign convention

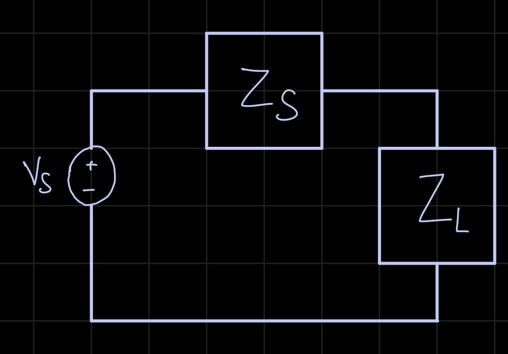


this means power is being consumed by the source eg: p = -20W

* Maximum Power Transfer Theorem

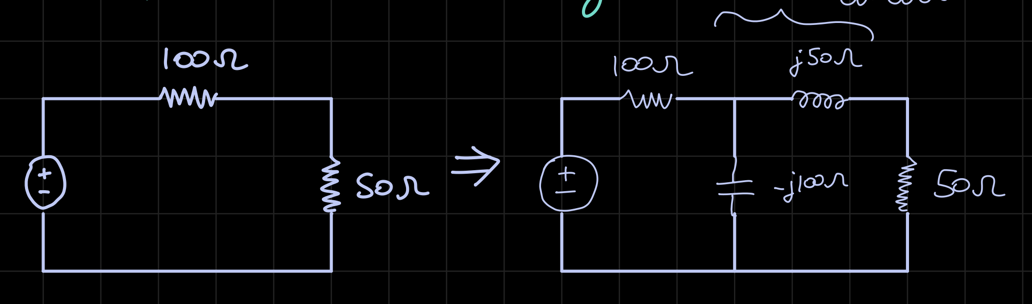
maximum power delivered to load when

$\boxed{Z_L = Z_s^*}$ complex conjugate of Z_s



⇒ Impedance matching

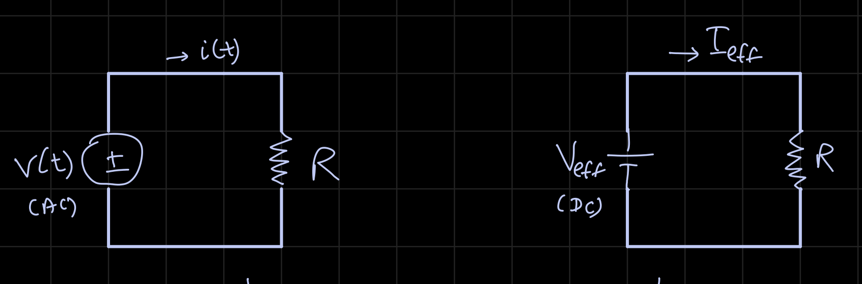
impedance matching circuit



$Z_L = Z_s^*$

* EFFECTIVE VALUE (Rms Value) of V and I

(voltage) (current)



if $P_{avg_1} = P_{avg_2}$

$\Rightarrow \frac{1}{T}\int_0^T i^2(t) R \, dt = I_{eff}^2 R$

$\Rightarrow \boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}} \equiv$ Root mean square

$i(t) \xrightarrow{square} i^2(t) \xrightarrow{mean} \frac{1}{T}\int_0^T i^2(t)\,dt \xrightarrow{sqrt} \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$

• Sinusoidally varying voltage

$i(t) = I_m \cos(\omega t + \phi)$

$\cos^2 x = \frac{2\cos x + 1}{2}$

$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi)\,dt}$

$\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$ current rms value unit: A_{rms}

$\boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$ voltage rms value unit: V_{rms}

not a phasor it is real number = magnitude of current | voltage

eg: $I = 10\angle 90° A \Rightarrow I = 5\sqrt{2}\angle 90° A$ rms

OR $V = 100\sqrt{2}\angle -60° V \Rightarrow V = 100\angle -60° V_{rms}$

* Apparent Power

$P_{apparent} = I_{eff} V_{eff}$ unit: VA, not W (volt ampere)

* Power factor

$PF = \frac{\text{avg power}}{\text{apparent power}} = \frac{(V_m \cdot I_m)/2 \cos(\theta - \phi)}{V_{eff} \cdot I_{eff}}$

$PF = \cos(\underbrace{\theta - \phi}_{\text{power factor angle}})$

* Lecture : 7

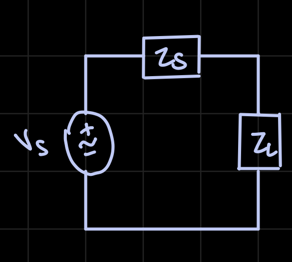
- Instantaneous Power : $p(t) = v(t)\, i(t)$
- Average Power : $P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = \frac{1}{2} Re\{V I^*\}$
- $P_{avg.\,sources} = \Sigma P_{avg.\,elements}$

*** Note :** $\overset{\text{impedence}}{Z} = \underset{\text{resistive}}{R} + j\overset{\text{reactive}}{X}$

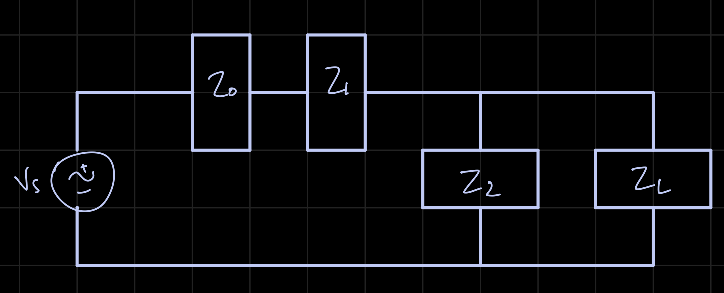
$P_{avg})_{resistive} : P \neq 0$

$P_{avg})_{reactive} : P = 0$

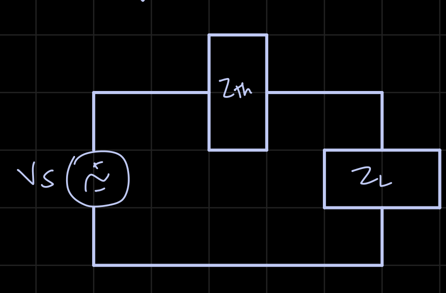
* Max Power Transfer :



Circuit has max. power when $Z_S = Z_L^*$ (complex conjugate)



⇓ apply thevenin's theorem



$Z_L = Z_{Th}^*$

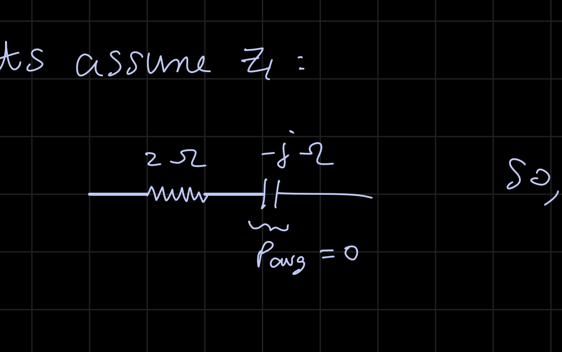
* Power Factor

$PF = \cos(\overset{\text{angle of voltage phasor}}{\theta} - \overset{\text{angle of current phasor}}{\phi}) = \frac{P_{average}}{P_{apparent}}$

for purely resistive load : PF = 1 {max} $\theta - \phi = \pm 90°$
for purely reactive load : PF = 0 {min}

Note → PF = 0.5 leading → capacitive $(\theta - \phi) < 0°$
PF = 0.5 lagging → inductive $(\theta - \phi) > 0°$

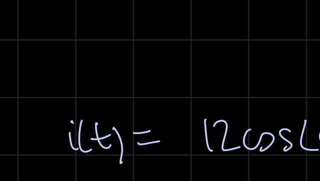
eg)



find
1. Average power delivered to each load
2. $P_{avg.\,source} = ?$
3. $P_{apparent\,.\,src} = ?$
4. PF of combined load = ?

Ans) $P_{avg\, Z_1} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} Re\{V I^*\}$ — V: voltage across Z_1, not the src

lets assume Z_1 :



So, $P_{avg\, Z_1}$ = Power delivered to 2Ω resistor

$= I_{eff}^2 R$

find **I** by KVL :

$-60\angle 0° + I(2 - j1 + 1 + 5) = 0$

$I = \frac{60\angle 0°}{3 + j4} = \underbrace{12\angle -53.13°}_{I_{rms}}$ Arms

$i(t) = 12\cos(\omega t - 53°)$ A rms

$\underset{(A_{rms})}{I_{eff}} \rightarrow \underset{(A)}{I_m}$: $I_{eff} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = I_{eff}\sqrt{2}$

$I_{eff} = 12\, A_{rms} \Rightarrow I_m = 12\sqrt{2}$ A

$i(t) = 12\sqrt{2}\cos(\omega t - 53°)$ A

$P_{avg\,(Z_1)} = (12)^2 \times 2 = 288$ W

note: $P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$

Z_2 : I_{eff} same : series circuit

$I_{eff} = 12$

and $P_{avg})_{Z_2} = I_{eff}^2 R = (144)(1) = 144$ W

2. $P_{avg})_{src} = V_{eff} I_{eff} \cos(\theta - \phi)$

$V_{src} = 60\angle 0°$ Vrms ⇒ $V_{eff} = 60$ Vrms

$I_{src} = 12\angle -53.13°$ Vrms ⇒ $I_{eff} = 12$ Arms

$\theta = 0°$, $\phi = -53.13°$

$P_{avg})_{src} = [60 \times 12]\cos(53.13°) = 432$ W

we can observe that 288 + 144 = 432
and hence,

$P_{avg\,.\,sources} = \Sigma P_{avg.\,elements}$ holds

3. $P_{apparent} = V_{eff} \cdot I_{eff} = (60)(12) = 720$ W

4. PF of combined loads = PF of source

$PF = \cos(\theta - \phi) = \cos(\underbrace{0 + 53.13°}_{\theta - \phi > 0,\ \text{lagging}}) = 0.6$ lagging

$* \quad P_{avg} = \frac{1}{2} Re\{V I^*\}$ $\quad V = V_m\angle\theta$, $I = I_m\angle\phi$

$V_{eff} = V_{eff}\angle\theta$
$I_{eff} = I_{eff}\angle\phi$

$P_{avg} = Re\{V_{eff} I_{eff}\}$

* Complex Power

$S = V_{eff} I_{eff}^*$ Volt Ampere {VA}

this is a complex number not a phasor of power

$S = (V_{eff}\angle\theta)(I_{eff}\angle\phi)$
$= V_{eff} \cdot I_{eff}\, e^{j(\theta - \phi)}$

$\Rightarrow S = \underbrace{V_{eff} I_{eff} \cos(\theta - \phi)}_{\text{average power } P} + j\underbrace{V_{eff} I_{eff} \sin(\theta - \phi)}_{\text{reactive power } Q}$

(volt ampere) VA $\quad$ (watt) W $\quad$ (volt ampere reactance) VAR

$\boxed{S = P + jQ}$

$|S| = V_{eff} I_{eff} = P_{apparent}$ (volt amp)

note: $Z = R + jX$

Purely resistive load : $Z = R$, $X = 0$

$S = \underset{V_{eff} I_{eff} \cos(0)}{P} + j\overset{V_{eff} I_{eff} \sin(\underbrace{\theta - \phi}_{0}) = 0}{Q}$

$P \neq 0$
$Q = 0$

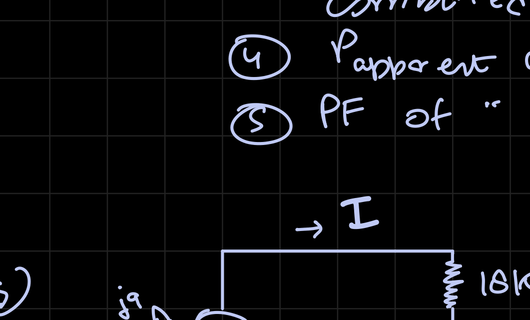
Purely reactive load

$S = \underset{V_{eff} I_{eff} \cos(90°)}{P} + j\overset{V_{eff} I_{eff} \sin(\overbrace{\theta - \phi}^{90°})}{Q}$

$P = 0$, $Q \neq 0$

Q signifies energy flow rate into or out of reactive component of load

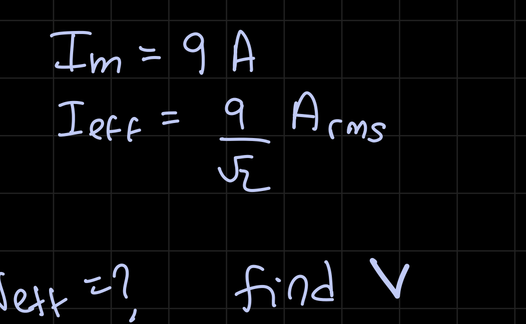
eg)



assume $C = 1\mu F$, $\omega = 45$ rad/s

find:
1. complex Power provided by src
2. Time average power absorbed by combined load
3. Reactive Power absorbed by combined load
4. $P_{apparent}$ absorbed " "
5. PF of " " "

Ans)



$9e^{j9}$ A $\equiv 9\cos(9°) + j9\sin(9°)$

$\frac{1}{j\omega C} = \frac{1}{j \times 45 \times 1 \times 10^{-6}} = -j\frac{10^6}{45}$

1. $S_{source} = \overbrace{V_{eff}}^{\text{source}} \times I_{eff}$

$I_m = 9$ A
$I_{eff} = \frac{9}{\sqrt{2}}$ Arms

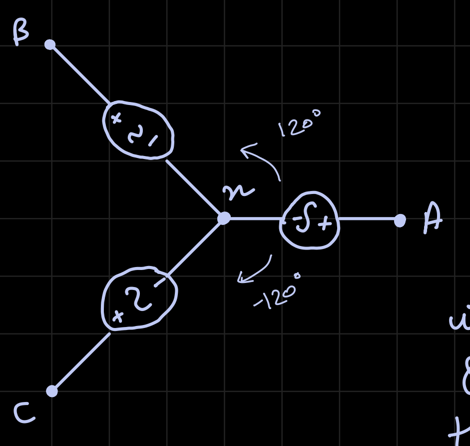
$V_{eff} = ?$, find V

$V = I(Z_{eq}) = 9\angle 9°\left(18 \times 10^3 - j\frac{10^6}{45}\right)$

* Lecture : 8

- Polyphase Circuits (Chp 12)

⇒ Three Phase Source



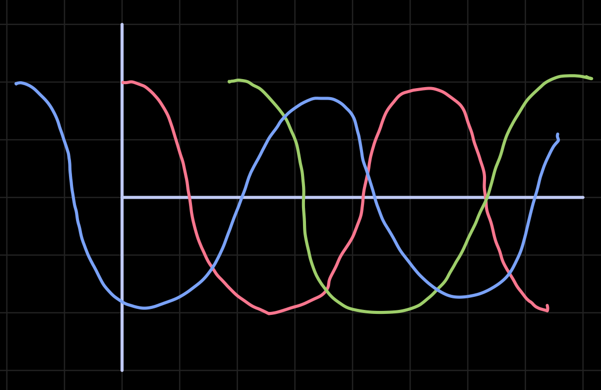
$V_{an} = 100\angle 0° \, V$
$V_{bn} = 100\angle 120° \, V$
$V_{cn} = 100\angle -120° \, V$

if $|V_{an}| = |V_{bn}| = |V_{cn}|$
& $V_{an} + V_{bn} + V_{cn} = 0$
then it is a

Balanced Source

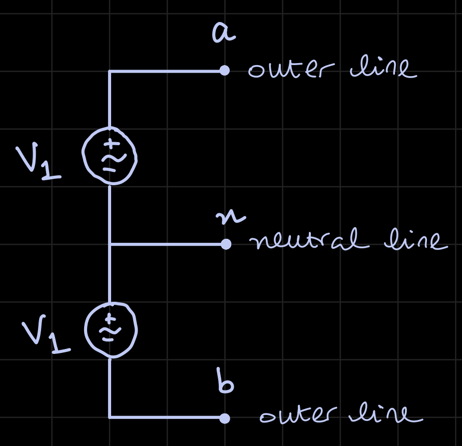
total instantaneous power = constant

freq. domain	time domain
$100\angle 0°$	$100 \cos(\omega t)$
$100\angle 120°$	$100 \cos(\omega t + 120°)$
$100\angle -120°$	$100 \cos(\omega t - 120°)$



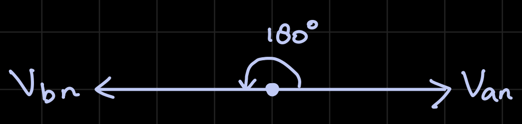
total $p(t) \to$ constant

* Single Phase Three-wire Source
≡ Two phase source



$V_1 = V_m \angle 0°$

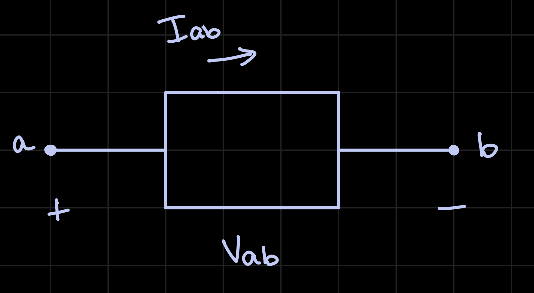
$V_{an} = V_1 = V_m \angle 0°$
$V_{bn} = -V_1 = V_m \angle 0° - 180°$



double script notation

$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$
$V_{bn} = -V_{nb}$

$V_{an} = V_A$ wrt V_n

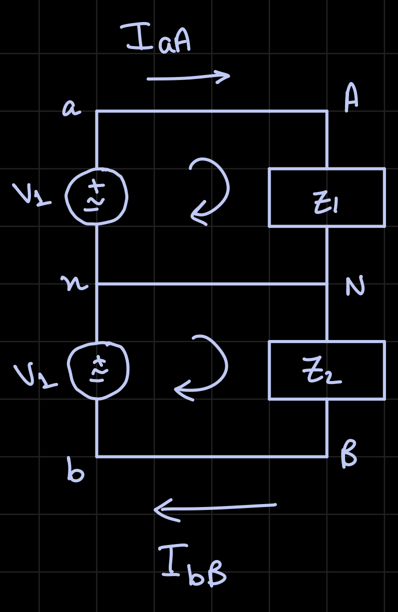


I_{ab}: current flowing from a to b

- Balanced single phase three wire source

Conditions:

$|V_{an}| = |V_{bn}|$
$V_{an} + V_{bn} = 0$
⇒ Balanced Src



$I_{aA} = \frac{V_1}{Z_1}$

$I_{bB} = \frac{V_1}{Z_2}$

$I_{nN} = -I_{aA} + I_{bB}$

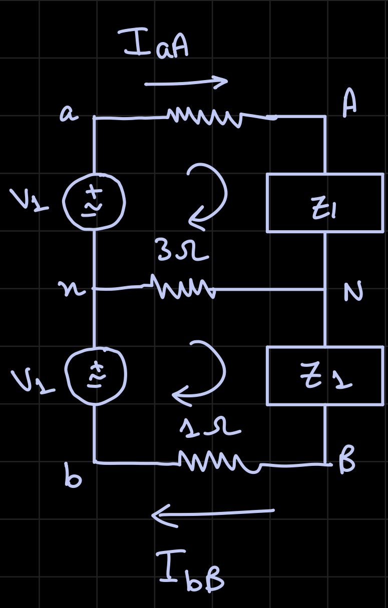
$\Rightarrow \frac{V_1}{Z_2} - \frac{V_1}{Z_1}$

Assume $Z_1 = Z_2$

$I_{nN} = \frac{V_1}{Z_1} - \frac{V_2}{Z_1} = 0$

when both srcs and loads are equal

Balanced Load : current in neutral line is equal to zero.



all terms are phasors

even with resistance
$I_{nN} = 0 = I_{bB} - I_{aA}$
due to
① Balanced Load
≡ Symmetry

* Lecture : 9 09/09/24

time domain	freq. domain
$V_p\cos(\omega t+\theta^\circ)$	$V_p\angle\theta^\circ$
$V_p\cos(\omega t+\theta^\circ-180^\circ)$	$V_p\angle\theta-180^\circ$

we cannot directly say,

$$P_{avg} = VI \quad \times$$

but $$P_{avg} = \frac{1}{2}\mathrm{Re}\{VI^*\} \quad ✓$$

② a) $$I_{AB} = \frac{115\angle\theta^\circ + 115\angle\theta^\circ}{2(10+j2)} = \frac{230\angle\theta^\circ}{2(10+j2)}$$

$$= 11.27\angle(\theta^\circ - 11.3^\circ)A$$

$$\phi = \theta - 11.3^\circ$$

$$PF = \cos(\theta - \theta + 11.3^\circ) = 0.98 \text{ lagging}$$

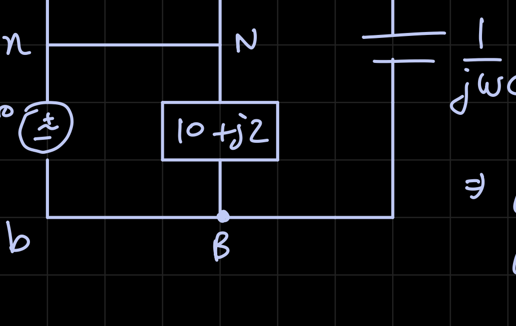
because PF angle is +ve

b) $PF = 1 \Rightarrow \cos(\theta-\phi) = 1 \Rightarrow \theta - \phi = 0$

$\Rightarrow \theta = \phi$

$f = 50h$

$\omega = 2\pi f = 100\pi$



$$I_{aA} = I_1 + I_2$$

$$\Rightarrow \frac{115\angle\theta^\circ}{10+j2} + \frac{230\angle\theta^\circ}{\left(\frac{1}{j\omega c}\right)}$$

$\Rightarrow$ Lot of calculation
alternative method $\Rightarrow$

Note that this is still a balanced load, so we can remove nN since $I_{nN} = 0$

using complex power

$S \equiv$ complex power of total load

$$S = \frac{1}{2}VI^*$$

$$PF = \frac{\mathrm{Re}\{S\}}{|S|} = 1 \Rightarrow \mathrm{Re}\{S\} = |S|$$

so, $\mathrm{Im}\{S\} = 0$ here

$$S = \frac{1}{2}V_{an}I_{AN}^* + \frac{1}{2}V_{nb}I_{NB}^* + \frac{1}{2}V_{ab}I_{AB}^*$$

$$\Rightarrow \frac{1}{2}V_{an}\left(\frac{V_{an}}{Z_p}\right)^* + \frac{1}{2}V_{nb}\left(\frac{V_{nb}}{Z_p}\right)^* + \frac{1}{2}V_{ab}\left(\frac{V_{ab}}{Z_c}\right)^*$$

$$\Rightarrow \frac{1}{2}\frac{|V_{an}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{nb}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{ab}|^2}{|Z_c|^2}\times Z_c$$

$\mathrm{Re}\{115\angle 0^\circ\}$

$\sqrt{10^2+2^2}$

$$\Rightarrow \frac{|V_{an}|^2}{|Z_p|^2}\times Z_p + \frac{1}{2}\frac{|V_{ab}|^2}{|Z_c|^2}\times Z_c$$

$\mathrm{Re}\{\frac{1}{j\omega c}\} = \frac{1}{j\omega c}\times\frac{-1}{j\omega c}$

$$\Rightarrow \frac{115^2}{104}(10+j2) + \frac{1}{2}\left(\frac{230^2}{\left(\frac{1}{\omega c}\right)^2}\right)\left(\frac{-j}{\omega c}\right)$$

$\frac{-1}{-1(\omega c)^2}$

$$\Rightarrow 115^2\left(\frac{\cancel{10}+j2}{104} - 2j\omega c\right)$$

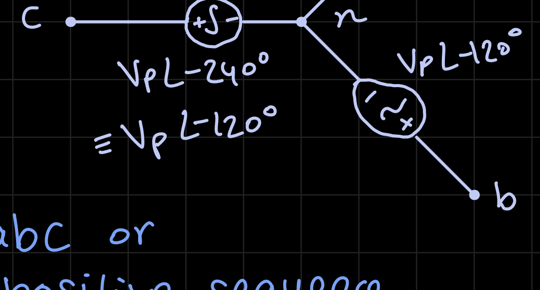
$$\Rightarrow 2\times 115^2\left(\frac{j}{104} - j\omega c\right)$$

$$\Rightarrow 2\times 115^2\left(\frac{1}{104} - \omega c\right) = 0$$

$$\frac{1}{104} - 100\pi C = 0$$

$$C = \frac{1}{10400\pi} = 30.60\,\mu F \quad ✓$$

* Three Phase Sources



$V_{an} = V_p\angle 0^\circ$

$V_{bn} = V_p\angle -120^\circ$

$V_{cn} = V_p\angle 120^\circ)$

$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$

$V_{an} + V_{bn} + V_{cn} = 0$

↳ balanced source

abc or
positive sequence

cba or
negative sequence

just a naming thingy

Phase voltages V_{an}, V_{bn}, V_{cn}

Line-to-line Voltages OR line voltages V_{ab}, V_{bc}, V_{ac}

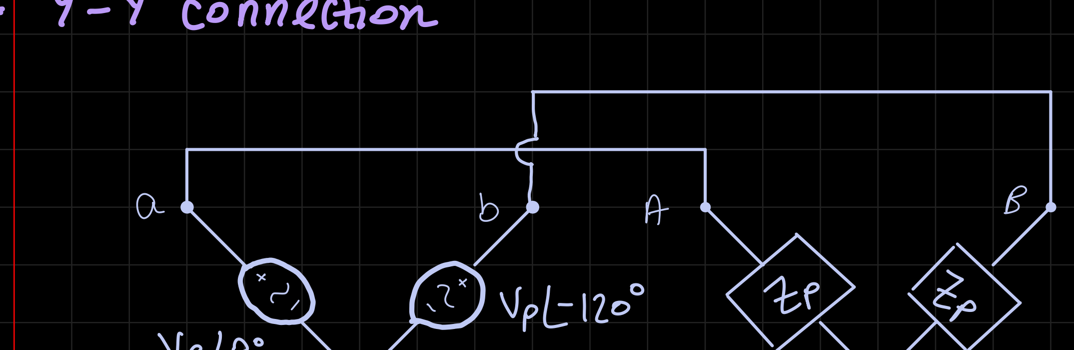
$$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$$

$$V_{ab} = \sqrt{3}V_p\angle 30^\circ$$

$$V_{bc} = \sqrt{3}V_p\angle -90^\circ$$

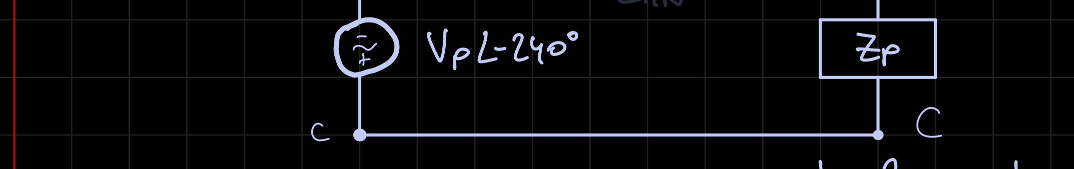
$$V_{ca} = \sqrt{3}V_p\angle -210^\circ$$

* Y-Y connection



balanced source

balanced load



"Per- phase"

used for easier analysis

eg. to find total power by ckt, just add the three individual power obtained from above ckts.

going back to the Y-Y connection $\Rightarrow$

$$I_{aA} = \frac{V_{an}}{Z_p}$$

$$I_{bB} = \frac{V_{bn}}{Z_p} = \frac{V_{an}\angle -120^\circ}{Z_p} = I_{aA}\angle -120^\circ$$

$$I_{cC} = \frac{V_{cn}}{Z_p} = \frac{V_{an}\angle +120^\circ}{Z_p} = I_{aA}\angle +120^\circ$$

$$I_{nN} = I_{aA} + I_{bB} + I_{cC} = 0$$

eg} consider three phase balanced Y-Y connected system

$V_{an} = 200\angle 0^\circ\ V_{rms}$

$Z_p = 100\angle 60^\circ\ \Omega$

find phase and line voltage & current and total avg power delivered to load

Ans}

$V_{an} = 200\angle 0^\circ\ V_{RMS}$

$V_{bn} = 200\angle -120^\circ\ V_{RMS}$

$V_{cn} = 200\angle +120^\circ\ V_{RMS}$

line voltage :

$$V_{ab} = \sqrt{3}V_p\angle 30^\circ = 200\sqrt{3}\angle 30^\circ\ V_{RMS}$$

$$V_{bc} = 200\sqrt{3}\angle -90^\circ\ V_{RMS}$$

$$V_{ca} = 200\sqrt{3}\angle -210^\circ = 200\sqrt{3}\angle 150^\circ\ V_{rms}$$

line currents:

$$I_{aA} = \frac{V_{an}}{Z_p} = \frac{200\angle 0^\circ}{100\angle 60^\circ} = 2\angle -60^\circ\ A_{rms}$$

$$I_{bB} = 2\angle -180^\circ\ A_{rms}$$

$$I_{cC} = 2\angle -300^\circ\ A_{rms} = 2\angle 60^\circ\ A_{rms}$$

total avg. power: $P_{avg} = \frac{1}{2}\mathrm{Re}\{VI^*\}$

in phase A :

$$P_{avg})_A = \frac{1}{2}\mathrm{Re}\{VI^*\}$$

$$= \mathrm{Re}\{V_{eff}I_{eff}\}$$

$$= \mathrm{Re}\{200\angle 0^\circ \times 2\angle -60^\circ\}$$

$$\Rightarrow \mathrm{Re}\{400\angle -60^\circ\} \Rightarrow 400\cos(-60) = 200W$$

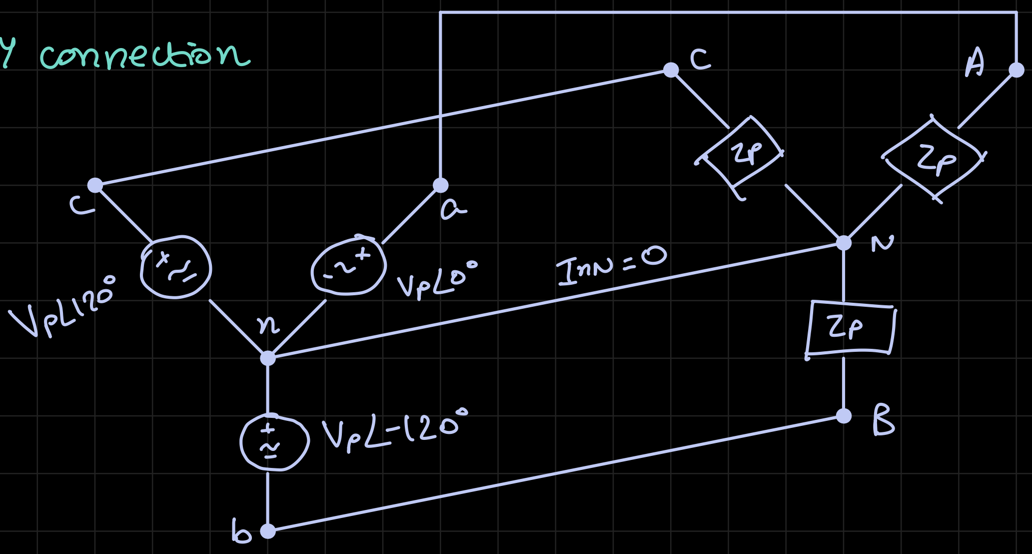
$P_{avg})_B = 200W$

$P_{avg})_C = 200W$

$\}\ P_{avg})_{total} = 600W$

* lecture : 10 11/09/24

→ Y-Y connection



balanced load : all load are same

balanced src : all src magnitudes are equal

line : lines connecting load to src
aA cC
bB nN

Phase Voltages: $V_{AN} = V_{an}$, $V_{BN} = V_{bn}$, $V_{CN} = V_{cn}$

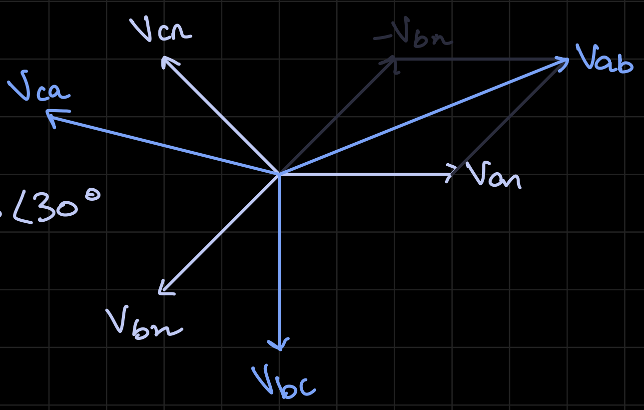
line voltages : V_{aA}, V_{bB}, V_{cC}

line currents : I_{aA}, I_{bB}, I_{cC}

phase currents : $I_{AN} = I_{aA}$, $I_{BN} = I_{bB}$, $I_{CN} = I_{cC}$

$V_{an} = V_p\angle 0^\circ$
$V_{bn} = V_p\angle -120^\circ$
$V_{cn} = V_p\angle -240^\circ$

$V_{ab} = V_{an} - V_{bn}$



$$\underbrace{\begin{aligned} V_{ab} &= V_{an} - V_{bn} = \sqrt{3} V_p \angle 30^\circ \\ V_{bc} &= \sqrt{3} V_p \angle -90^\circ \\ V_{ca} &= \sqrt{3} V_p \angle -210^\circ \end{aligned}}_{\text{line voltages}}$$

$V_{line} = \sqrt{3} V_{phase}$

line currents = phase currents

* Total Instantaneous Power

$$P_{total}(t) = p_A(t) + p_B(t) + p_C(t)$$

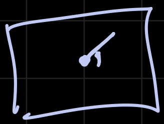
Classes & Attribs

interface (entity)

- Bird → dmg (hits)
- Pig → health (hits)
- User → save stage → Score, level_status
- Level mng → max_score, no. of pigs
  - ↳ blocks → type (wood, glass, steel)
  - ↳ pigs

(entry point) GAME
update score

Slingshot
angle
stretch



V=10

W/A

S,D

$V\cos\theta,\ V\sin\theta$

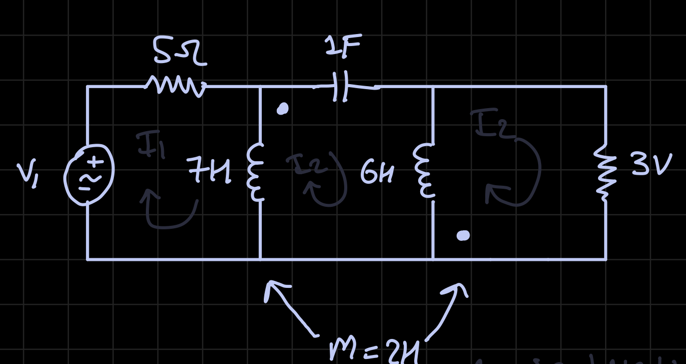
per second / rate

$-1 \to 0$

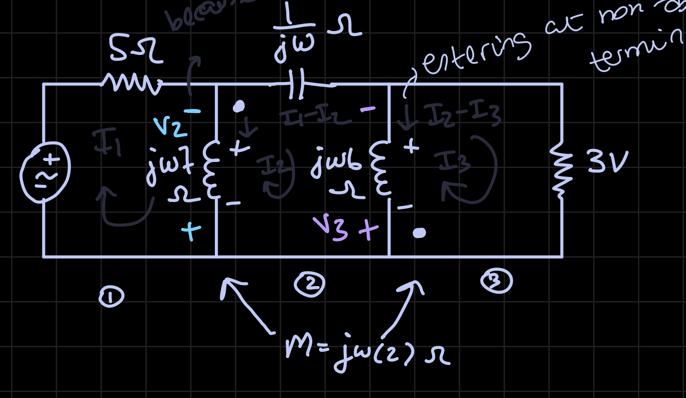
TODO : revise lecture 11 (missed due to SNS quiz)

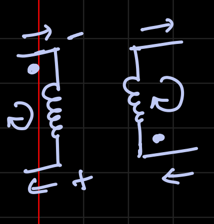
* lecture 12:

eg 3



ans}





① $-V_1 + 5I_1 - V_2 = 0$

$$V_2 = (j\omega 7)(I_1 - I_2)(-1)$$

because same sign of V_2 & V_3

$$V_2 = -j\omega 7(I_1 - I_2) + j2\omega(I_2 - I_3)$$

③ $3I_3 + V_3 = 0$

self + mutual inductance

same polarity

$$V_3 = -j6\omega(I_2 - I_3) + j2\omega(I_1 - I_2)$$

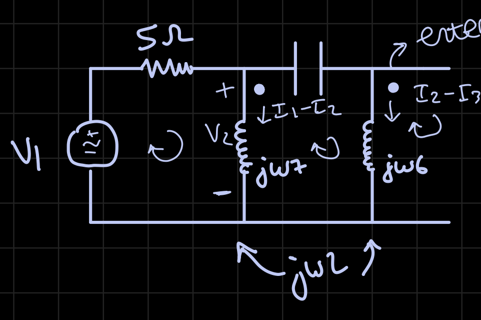
② $V_2 + \frac{1}{j\omega}I_2 - V_3 = 0$

$$-j\omega 7(I_1 - I_2) + j2\omega(I_2 - I_3) + j6\omega(I_2 - I_3) - j2\omega(I_1 - I_2) + \frac{I_2}{j\omega} = 0$$

$$I_1(-j5\omega) + I_2(j17\omega) + I_3(-j8\omega) + \frac{I_2}{j\omega} = 0$$

$$I_1(-j5\omega) + I_2\left(j17\omega + \frac{1}{j\omega}\right) + I_3(-j8\omega) = 0$$

eg3



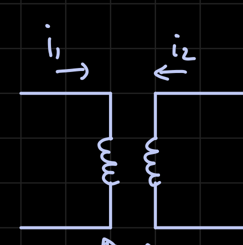
$-V_1 + 5I_1 + V_2 = 0$

$V_2 = (j\omega 7)(I_1 - I_2) + (j\omega 2)(I_2 - I_3)$

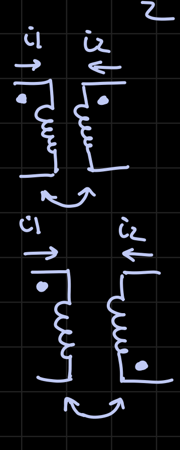
we donot need to care abt this sign if we use V_2, V_3 method.

* ENERGY STORED

$$w(t) = \frac{1}{2}L(i(t))^2$$ for only one inductor



$$w(t) = \frac{1}{2}L(i_1)^2 + \frac{1}{2}L(i_2)^2 \pm M i_1(t) i_2(t)$$



$\boxed{+ve}$ sign will occur iff both i_1 and i_2 are entering either dotted or undotted

$\boxed{-ve}$ sign iff both enter different (dotted / undotted)

- **coupling coefficient** (K)

$$M \leq \sqrt{L_1 L_2}$$

$$K = \frac{M}{\sqrt{L_1 L_2}} \Rightarrow 0 \leq K \leq 1$$

$\boxed{K \to 0}$ poor coupling or no coupling

$\boxed{K \to 1}$ strong coupling (very close to each other)

⇒ K depends on: distance btw coils ; size of coils ; ferrite core

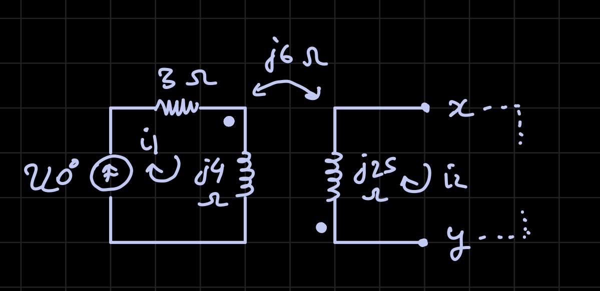
$y \propto \frac{1}{x}$ $\quad y \propto x$ $\quad y \propto x$

uses: wireless power transfer (inductively coupled)
- electric vehicle charging
- magsafe charging

eg3 let $i_s = 2\cos(10t)$ A. find total energy stored in network at $t = 0$ if $k = 0.6$ and

(a) xy terminals are open circuited

(b) xy are short circuited



Ans} $w(t) = \frac{1}{2}L_1(i_1)^2 + \frac{1}{2}L_2(i_2) \pm M i_1(t) i_2(t)$

$$j\omega L_1 = j4 \Rightarrow L_1 = \frac{4}{\omega} = 0.4\,H$$

(a) if xy are open: $i_2 = 0$

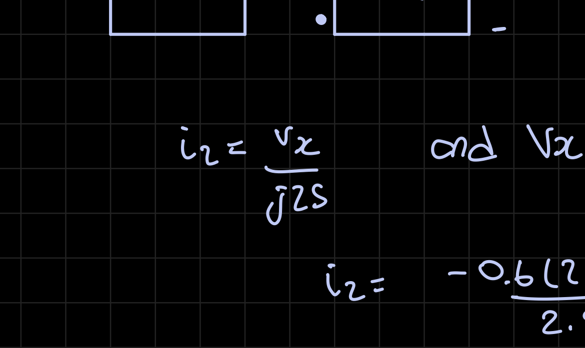
$$\text{so, } w(t) = \frac{1}{2} \times 0.4 \times (2\cos 10t)^2$$

$$w(t) = 0.8\cos^2(10t)$$

$$\text{at } t = 0 \Rightarrow w(t) = 0.8\ J$$

(b) $w(t) = \frac{1}{2}L_1 \cdot i_1^2 + \frac{1}{2}L_2 i_2^2 - M \cdot i_1 \cdot i_2$

$$i_1 = i_s = 2\angle 0°$$



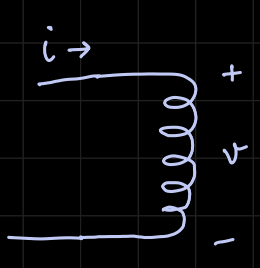
$i_2 = \frac{V_x}{j25}$ and $V_x = -j\omega M I_1$

$$i_2 = \frac{-0.6(2\angle 0°)}{2.5} = -0.48A$$

$$w(t=0) = \frac{1}{2}(0.4)(2^2) + \frac{1}{2}(\cdots$$

* Lecture : 11 {Revision} [absent] 24/9/24

• MODULE 5: Magnetically coupled circuits



$v = L\dfrac{di}{dt}$

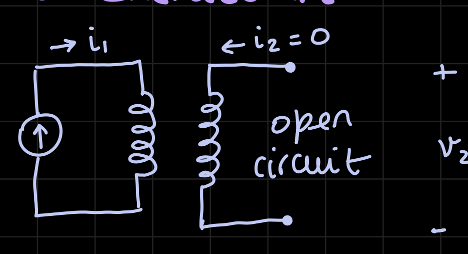
current → magnetic flux
$i \propto \phi$ — ①

magnetic flux → induced voltage
$v \propto \dfrac{d\Phi}{dt}$ — ②

① and ② → $V = L\dfrac{di}{dt}$

for DC source : current is constant
hence $\dfrac{di}{dt} = 0$ and $\therefore v = 0$ (induced)

• Mutual Inductance

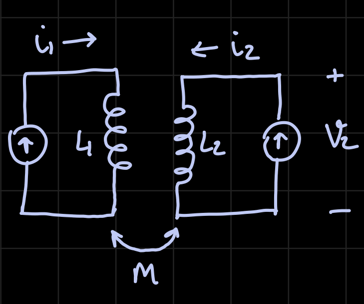


$v_2 \propto \dfrac{di_1}{dt}$

$V_2 = M\dfrac{di_1}{dt}$

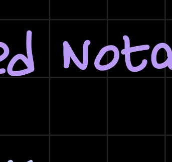
L : self inductance

$\underbrace{M}_{\text{mutual inductance}}$

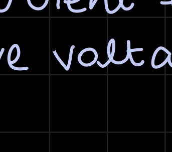


$v_2 = \underbrace{L\dfrac{di_2}{dt}}_{\text{self inductance}} + \underbrace{M\dfrac{di_1}{dt}}_{\text{mutual inductance}}$

* Additive / Substractive Property {two different coils}



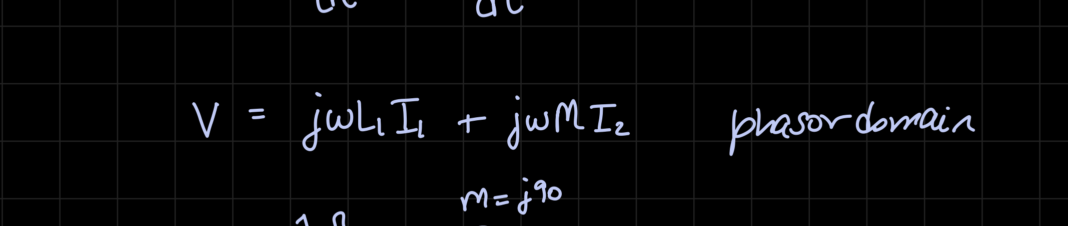
$v_2 = L\dfrac{di_2}{dt} + M\dfrac{di_1}{dt}$



$v_2 = L\dfrac{di_2}{dt} - M\dfrac{di_1}{dt}$

• Dotted Notation

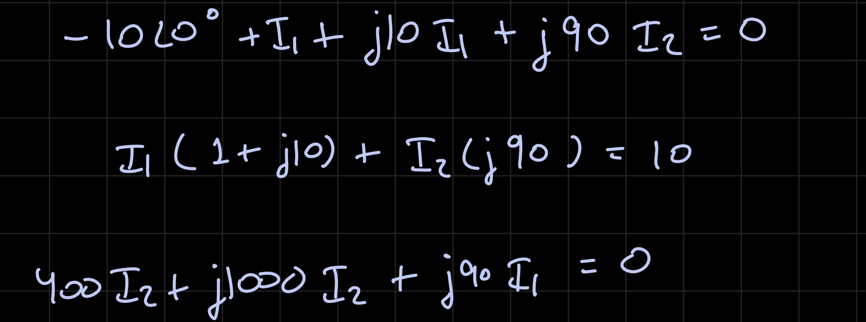
current entering _ terminal means
+ve voltage reference at _
dotted/undotted



$v = L_1\dfrac{di}{dt} + M\dfrac{di_2}{dt}$ time domain

$V = j\omega L_1 I_1 + j\omega M I_2$ phasor domain

eg}



① $-10\angle 0° + I_1 + j10\, I_1 + j90\, I_2 = 0$

$I_1(1 + j10) + I_2(j90) = 10$

② $400 I_2 + j1000\, I_2 + j90\, I_1 = 0$

$I_1(j90) + I_2(400 + j1000) = 0$

CRAMMERS :

$\Delta = \begin{vmatrix} 1+j10 & j1000 \\ j90 & 400+j1000 \end{vmatrix}$

$= (1+j10)(400+j1000) + 90000$

$= 400 + j1000 + j4000 - 10.000 + 90.000$

$= j5000 + 80,400 = 80555\angle 3.55°$

$\Delta_1 = \begin{vmatrix} j1000 & 10 \\ 400+j1000 & 0 \end{vmatrix} = -4000 - j10.000$
$= 10770\angle -111.8°$

$\Delta_2 = \begin{vmatrix} 10 & 1+j10 \\ 0 & j90 \end{vmatrix} = j900 = 900\angle 90°$

$I_1 = \dfrac{\Delta}{\Delta_1} = -3.2 + 6.75j$

full cramers part
wrong because I took
M = L2 = j1000 above

$D_0 = \begin{vmatrix} 1+j10 & j90 \\ j90 & 400+j1000 \end{vmatrix}$

$= 400 + j1000 + j4000 - 10.000 + 8100$
$= j5000 - 1500 = 5220\angle 106.69°$

---

energy stored in the circuit

$w(t) = \dfrac{1}{2}L_1 i_1(t)^2 + \dfrac{1}{2}L_2 i_2(t)^2$
$\pm M i_1(t) i_2(t)$

+ : current entering same : • and • OR _ and _
− : current entering different : • and _ OR _ and •

COUPLING COEFFICIENT (K)

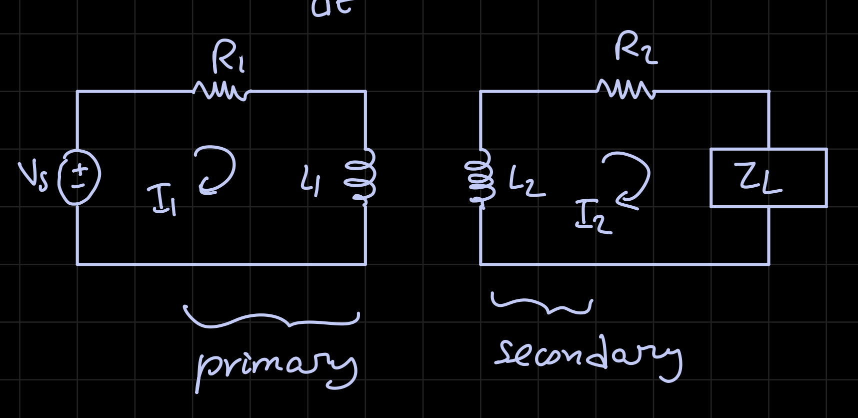
$K = \dfrac{M}{\sqrt{L_1 L_2}}$: note ∋ $M \leq \sqrt{L_1 L_2}$

So, $0 \leq K \leq 1$

* lecture 13 25/09/24

- **LINEAR TRANSFORMER**

when $v \propto \frac{di}{dt}$ ⇒ no magnetic core



$$-V_s + I_1R_1 + (j\omega L_1)I_1 - (j\omega M)I_2 = 0$$
$$(j\omega L_2)I_2 + R_2I_2 + Z_LI_2 - (j\omega M)I_1 = 0$$

$$\Rightarrow \quad V_s = \overbrace{(R_1 + j\omega L_1)}^{Z_{11}} I_1 - j\omega M I_2$$
$$0 = (-j\omega M) I_1 + \underbrace{(R_2 + j\omega L_2 + Z_L)}_{Z_{22}} I_2$$

$$\left.\begin{aligned} V_s &= Z_{11} I_1 - j\omega M I_2 \\ 0 &= (-j\omega M) I_1 + Z_{22} I_2 \end{aligned}\right\} \Rightarrow \frac{I_2}{I_1} = \frac{j\omega M}{Z_{22}}$$

Input Impedance : $Z_{in} = \frac{V_s}{I_1} = Z_{11} - j\omega M \frac{I_2}{I_1}$

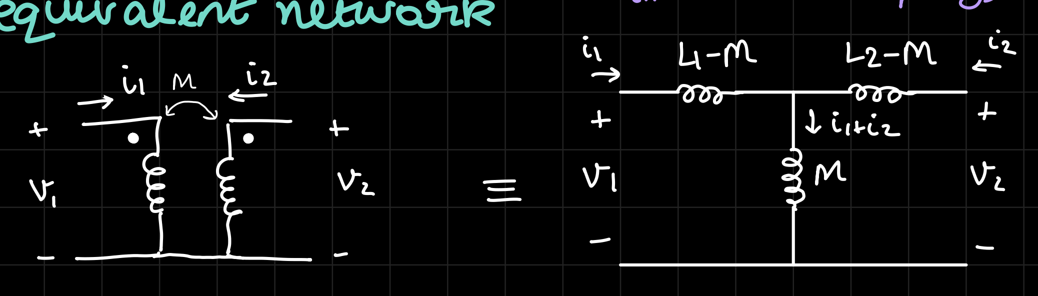
$$\boxed{Z_{in} = Z_{11} + \underbrace{\frac{\omega^2 M^2}{Z_{22}}}_{\text{Reflective Impedence}}}$$

where

$$Z_{11} = R_1 + j\omega L_1 \quad , \quad Z_{22} = R_2 + j\omega L_2 + Z_L$$

- if $K = \frac{M}{\sqrt{L_1L_2}} = 0 \Rightarrow Z_{in} = Z_{11}$ i.e. no coupling

- **T equivalent network** (no mutual coupling)



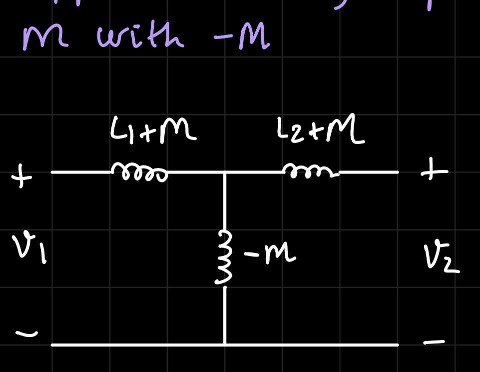
$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$
$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

Note: no mutual coupling since there is no arrow for it

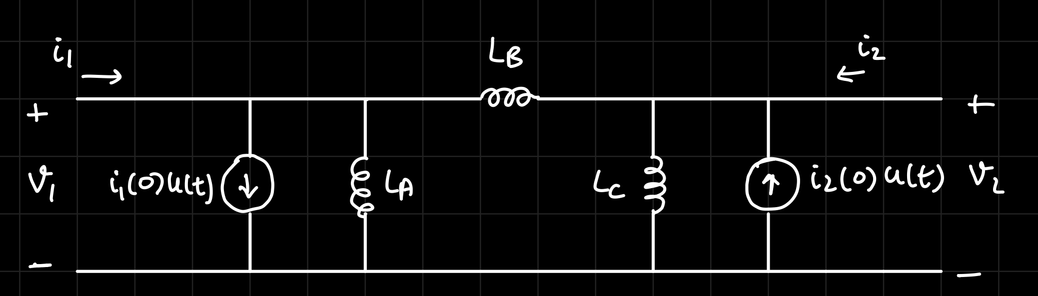
$$V_1 = (L_1 - M)\frac{di_1}{dt} + M\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right)$$
$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

similarly, we get V_2 same as we got in the left circuit. Hence, both circuits are equivalent.

Note: if dots are on opposite sides, replace M with -M



- **π EQUIVALENT NETWORK**



$$L_A = \frac{L_1L_2 - M^2}{L_2 - M}$$

$$L_B = \frac{L_1L_2 - M^2}{M}$$

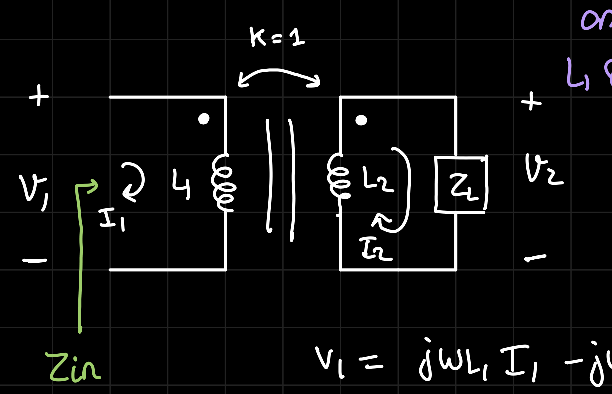
$$L_C = \frac{L_1L_2 - M^2}{L_1 - M}$$

use nodal analysis to prove this

$$v = L\frac{di}{dt} \Rightarrow i = \frac{1}{L}\int_{-\infty}^{t} v\,dt$$
$$i = i(0)u(t) + \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

if dots are on opposite side : $M \to -M$

- **Ideal Transformer** $K = 1$ ⇒ achieved only with ferrite core (iron) and $L_1 \,\&\, L_2 >> Z_L$



$$V_1 = j\omega L_1 I_1 - j\omega M I_2$$
$$0 = -j\omega M I_1 + (j\omega L_2 + Z_2) I_2$$

$$Z_{in} = \frac{V_1}{I_1} = j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2}$$

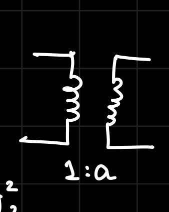
$$Z_{in} = \frac{Z_L}{\left(\frac{Z_L}{j\omega L_1}\right) + a^2}$$ and since $j\omega L_1 >> Z_L$ ⇒

$$Z_{in} = \frac{Z_L}{a^2}$$

$$\left.\begin{aligned} &\text{Current ratio} : \frac{I_2}{I_1} = \frac{1}{a} \\ &\text{Voltage ratio} : \frac{V_2}{V_1} = a \end{aligned}\right\}$$ replace a by -a if dots are on opp. side

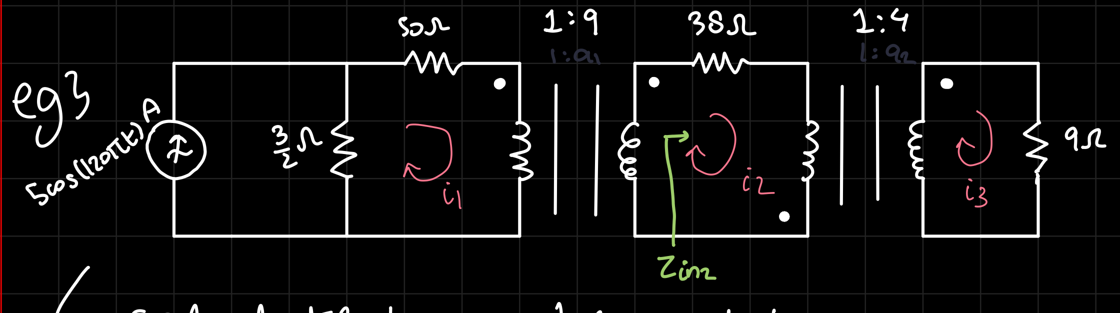
- **TURN RATIO**

let it be 1:a
then



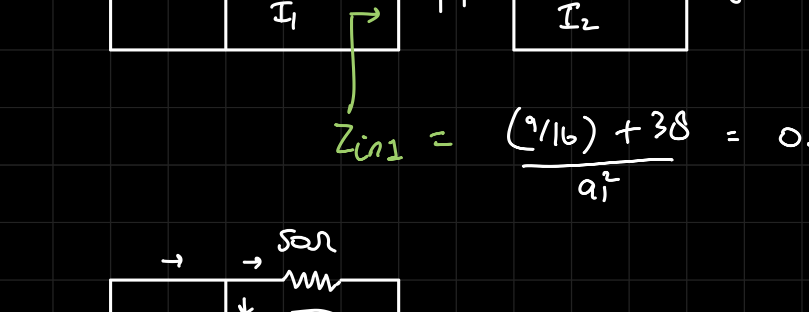
$$a = \frac{L_2}{L_1} = \frac{N_2^2}{N_1^2}$$

eg3

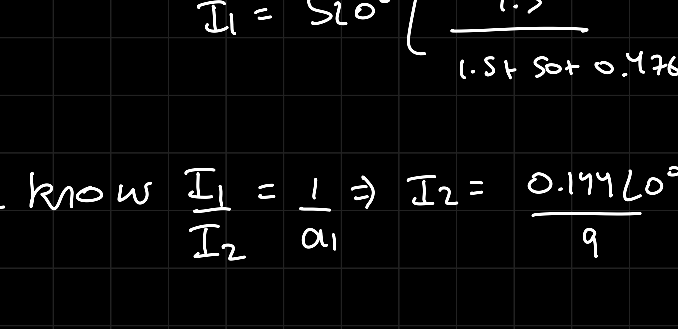


calculate power delivered to each resistor

$$Z_{in2} = \frac{Z_L}{a^2} = \frac{9}{16}\,\Omega$$



$$Z_{in1} = \frac{(9/16) + 38}{a_1^2} = 0.476\,\Omega$$



$$I_1 = 5\angle 0° \left(\frac{1.5}{1.5 + 50 + 0.476}\right) = 0.144\angle 0° A$$

We know $\frac{I_1}{I_2} = \frac{1}{a_1} \Rightarrow I_2 = \frac{0.144\angle 0°}{9} = 0.016\angle 0° A$

$$\frac{I_3}{I_2} = \frac{-1}{a_2} \Rightarrow I_3 = \frac{-0.016\angle 0°}{4} = -4\angle 0° mA$$

$$P_{avg\ 1.5\Omega} = \frac{1}{2}(5 - 0.144)^2 \times 1.5 = 17.67W$$

$$P_{avg\ 50\Omega} = \frac{1}{2}(0.144)^2 \times 50 = 0.518W$$

$$P_{avg\ 38\Omega} = \frac{1}{2}(0.016)^2 \times 38 = 0.0048W$$

$$P_{avg\ 9\Omega} = \frac{1}{2}(-4)^2 \times 10^{-6} \times 9 = 72 \times 10^{-6} W$$

* lecture : 14

• module : 6 → Circuit Analysis in S-domain
(textbook Chp 14)

$$V_m \cos(\omega t + \theta) = Re\{V_m e^{j\omega t + \theta}\}$$

$\Downarrow$

to phasor how?

↓ Remove $Re\{\}$, $e^{j\omega t}$

$$V_m e^{j\theta} = V_m \angle \theta$$

in phasor form, we can eliminate the differential equations and converted to algebraic equations

we cannot use phasors if not sinusoidal signals ??

let $v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$

$V(t)$ is not sinusoidal because extra term but we can write in phasor form using laplace transform

put $\sigma = 0, \omega = 0 \Rightarrow v(t) = V_m$ DC source

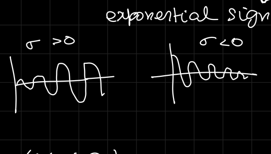
$\sigma = 0, \omega \neq 0 \Rightarrow v(t) = V_m \cos(\omega t + \theta)$ AC source

sinusoidally varying ✓ ↲

$\sigma \neq 0, \omega = 0 \Rightarrow v(t) = V_m e^{\sigma t} \cos(\theta) = V_1 e^{\sigma t}$ ↲

exponential signal

$\sigma \neq 0, \omega \neq 0 \Rightarrow$



$$v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$$

$$= V_m e^{\sigma t}\left[\frac{e^{j(\omega t + \theta)} + e^{-j(\omega t + \theta)}}{2}\right]$$

$$= \frac{V_m e^{j\theta}}{2}\left[e^{\overbrace{(\sigma + j\omega)}^{S}t} + e^{\overbrace{(\sigma - j\omega)}^{S^*}t}\right]$$

$S \equiv$ complex frequency

$S = \sigma + j\omega$

σ = Neper Frequency (neper/sec)

ω = Radian frequency $= 2\pi f$ (rad/s)

• Laplace Transform

$$f(t) \longrightarrow \boxed{\text{Laplace Transform}} \longrightarrow F(S)$$

LT : invertible ✓

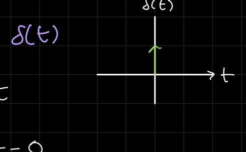
$$F(S) \xrightarrow[\text{Transform}]{\text{Laplace}} f(t)$$

one sided laplace transform $F(S) = \mathcal{L}(f(t)) = \int_{0^-}^{\infty} e^{-st} f(t)\,dt$

$$f(t) = \frac{1}{2\pi j}\int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{st} F(S)\,ds$$

$$f(t) \Longleftrightarrow F(S)$$

• Unit Step Function : u(t)



$$\mathcal{L}(u(t)) = \int_{0^-}^{\infty} e^{-st} u(t)\,dt$$

$$= \int_0^{\infty} e^{-st} dt = \left[\frac{e^{-st}}{-s}\right]_0^{\infty} = -\frac{e^{-\infty}}{s} + \frac{e^{0}}{s}$$

$$= \frac{1}{s} \checkmark$$

$$\boxed{u(t) \rightleftharpoons \frac{1}{s}}$$

note : $\frac{1}{s+5} \Leftrightarrow e^{-5t} u(t)$

* Unit Impulse function $\delta(t)$

$$\mathcal{L}(\delta(t)) = \int_{0^-}^{\infty} e^{-st} \delta(t)\,dt$$

$\delta(t) = 1$ at $t = 0$

$$\boxed{\delta(t) \rightleftharpoons 1}$$

* Exponential

$$e^{-\alpha t} u(t) \Longleftrightarrow \frac{1}{s + \alpha}$$

$$t e^{-\alpha t} u(t) \Longleftrightarrow \frac{1}{(s+\alpha)^2}$$

• Laplace Transform Theorems

① linearity theorems

$$\mathcal{L}\{f_1(t) + f_2(t)\} = \mathcal{L}\{f_1(t)\} + \mathcal{L}\{f_2(t)\}$$

summation in time domain ≡ summation in phasor domain

$$\mathcal{L}\{k f(t)\} = k\,\mathcal{L}\{f(t)\}$$

k is a constant

multiplication in time ≡ multiplication in phasor

eg} $V(S) = \frac{7}{s} - \frac{31}{s+17}$ find $v(t) = ?$

$$v(t) = \mathcal{L}^{-1}\{V(S)\} = 7\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 31\,\mathcal{L}^{-1}\left\{\frac{1}{s+17}\right\}$$

$$v(t) = 7u(t) - 31 e^{-17t} u(t) = (7 - 31 e^{-17t})\,u(t)$$

• ZEROES & POLES

for some functions : $V(S) = \frac{N(S)}{D(S)}$

zeroes : Values of S that lead to numerator, $N(S) = 0$

Poles : Values of S that lead to denominator, $D(S) = 0$

eg: $V(S) = \frac{s-3}{s(s+6)}$

Zeroes : $s = 3$

Poles : $s = 0, -6$

eg} $V(S) = \frac{1}{(s+\alpha)(s+\beta)}$ find $v(t)$

convert to : $V(S) = \frac{A}{s+\alpha} + \frac{B}{s+\beta}$

↳ method of residue

$$\frac{1}{(s+\alpha)(s+\beta)} = \frac{A}{s+\alpha} + \frac{B}{s+\beta}$$

$$1 = A(s+\beta) + B(s+\alpha)$$

when $s = -\alpha \Rightarrow 1 = A(\beta - \alpha)$

$$A = \frac{1}{\beta - \alpha}$$

when $s = -\beta \Rightarrow 1 = B(\alpha - \beta)$

$$B = \frac{-1}{\beta - \alpha}$$

$$V(S) = \left(\frac{\frac{1}{\beta-\alpha}}{s+\alpha}\right) + \left(\frac{\frac{-1}{\beta-\alpha}}{s+\beta}\right)$$

$$v(t) = \frac{1}{(\beta-\alpha)} e^{-\alpha t} u(t) - \frac{1}{(\beta-\alpha)} e^{-\beta t} u(t)$$

$$= \frac{1}{(\beta-\alpha)}\left(e^{-\alpha t} - e^{-\beta t}\right) u(t)$$

eg} $I(S) = \frac{7s+5}{s^2+s}$ $i(t) = ?$

zeroes : $\frac{-5}{7}$

poles : $0, -1$

$$I(S) = \frac{7s+5}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$$A = \frac{1}{\beta - \alpha} = \frac{1}{1}, \quad B = \frac{-1}{\beta - \alpha} = -1$$

$$7s + 5 = A(s+1) + B(s)$$

$$7s + 5 = (A+B)s + A$$

$A + B = 7 \Rightarrow B = 2$

$A = 5$

$$I(S) = \frac{5}{s} + \frac{2}{s+1}$$

$$\mathcal{L}^{-1}(I(S)) = 5u(t) + 2e^{-t}u(t)$$

⇒ Repeated Poles

$$F(S) = \frac{2}{s^3 + 12s^2 + 36s} = \frac{2}{s(s^2 + 12s + 36)}$$

$$F(S) = \frac{2}{s(s+2)^2}$$

$$F(S) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+2}$$

if we try to solve like earlier, we get inconsistent eqns like $2 = 0$

for repeated poles :

$$V(S) = \frac{N(S)}{(s-p)^n}$$

$$V(S) = \frac{a_n}{(s-p)^n} + \frac{a_{n-1}}{(s-p)^{n-1}} + \frac{a_{n-2}}{(s-p)^{n-2}} + \dots \frac{a_1}{(s-p)}$$

$$F(S) = \frac{A}{s} + \frac{B}{(s+6)^2} + \frac{C}{(s+6)}$$

$$2 = A(s+6)^2 + B(s) + C(s)(s+6)$$

when $s = 0 \Rightarrow 2 = 36A \Rightarrow A = \frac{1}{18}$

when $s = -6 \Rightarrow 2 = -6B \Rightarrow B = -\frac{1}{3}$

differentiate to find C

$$2 = A(s+6)^2 + Bs + Cs(s+6)$$

$\frac{d(2)}{ds} \Rightarrow 0 = 2A(s+6) + B + C(s+6) + Cs$

$\Rightarrow s = -6 \Rightarrow 0 = B + C(-6)$

$$C = \frac{B}{6} = \frac{-1}{18}$$

So, $A = \frac{1}{18}, B = \frac{-1}{3}, C = \frac{-1}{18}$

$$F(t) = \frac{1}{18}u(t) - \frac{1}{3}\left(t e^{-6t} u(t)\right) - \frac{1}{18} e^{-6t} u(t)$$

• THEOREMS :

① Time differentiation theorem

$$f(t) \rightleftharpoons F(S)$$

$$\boxed{\frac{d}{dt} f(t) \rightleftharpoons sF(S) - f(0^-)}$$

$L\frac{di}{dt} \rightarrow j\omega L I = sLI$

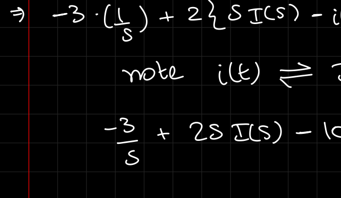
↳ when $\sigma = 0$

② Time Integration theorem

$$f(t) \rightleftharpoons F(S)$$

$$\int_{0^-}^{t} f(t)\,dt \rightleftharpoons \frac{F(S)}{s}$$

eg}



$i(t) = ?$

KVL? $\Rightarrow -3u(t) + 2\frac{di}{dt} + 4i(t) = 0$

then integrate

but better way → Laplace

we cant use phasors because source ≠ sinusoidal and we have a transient from $i(0^-) = 5A$

$$\Rightarrow -3 \cdot \left(\frac{1}{s}\right) + 2\{sI(S) - i(0^-)\} + 4I(S) = 0$$

note $i(t) \rightleftharpoons I(S)$

$$-\frac{3}{s} + 2sI(S) - 10 + 4I(S) = 0$$

$$I(S) = \frac{\frac{3}{s} + 10}{2s + 4} = \frac{10s + 3}{2s^2 + 4s}$$

$$= \frac{A}{2s} + \frac{B}{s+2}$$

$$10s + 3 = As + 2A + 2Bs$$

$$10s + 3 = s(A + 2B) + 2A$$

$A = \frac{3}{2}$, $A + 2B = 10$

$B = \frac{17}{4}$

$$\Rightarrow I(S) = \frac{3/2}{2s} + \frac{17/4}{s+2} \Rightarrow i(t) = ?$$

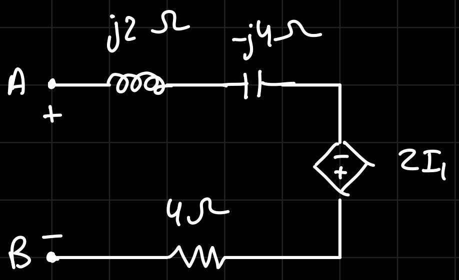
$$i(t) = \frac{3}{4}u(t) + \frac{17}{4}e^{-2t}u(t) \checkmark$$

* CTD Practice

① Quiz : 1 (set A)

mesh analysis: current
nodal analysis: voltage

- midsem : 2023

② 

thevenin's voltage :

* lecture 15

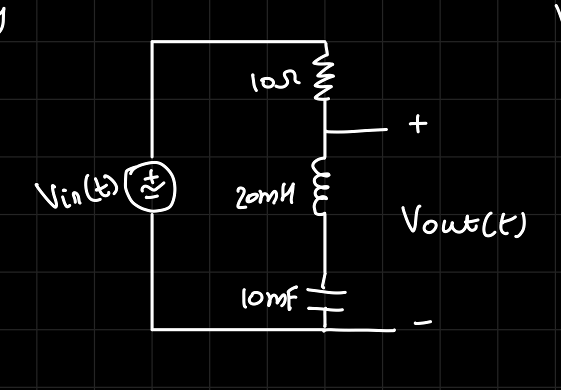
13/10/24

• Phasor vs S-domain

Assumptions:
zero initial conditions
sinusoidally varying source
"steady state"

time domain	freq. domain
$v(t) = V_m \cos(\omega t + \theta) = Re\{V_m e^{j\omega t} e^{j\theta}\}$	$V = V_m e^{j\theta}$
$v(t) = V_m e^{\sigma t}\cos(\omega t+\theta) = Re\{V_m e^{\sigma t} e^{j\theta} e^{j\omega t}\}$	$V = V_m e^{j\theta}$ where $s = \sigma + j\omega$ complex frequency
R	R
L	$L(\sigma + j\omega) = sL$
C	$\frac{1}{(\sigma+j\omega)C} = \frac{1}{sC}$

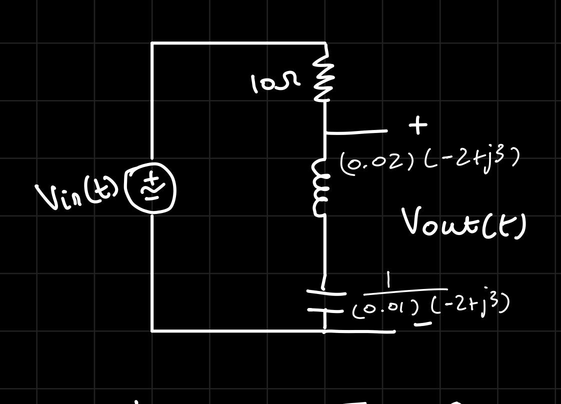
eg}



$V_{in}(t) = 5e^{-2t}\cos(3t+45°)$

$S = \sigma + j\omega$
$S = -2 + j3$

$V_{in} = 5e^{j45} = 5\angle 45° \text{ V}$



Voltage divider for finding Vout

$$V_{out} = \frac{5\angle 45° \times (0.02)(-2+j3) \times \frac{1}{(0.01)(-2+j3)}}{10 + (0.02)(-2+j3) + \frac{1}{(0.01)(-2+j3)}}$$
$$= 5.86 \angle -24.4$$

$$V_{out}(t) = Re\{5.86 e^{j(-24.4)} e^{(\sigma+j\omega)t}\}$$
$$= 5.86 e^{-2t}\cos(3t - 24.4) \text{ V}$$

for zero initial condition, we can use phasors for $e^{\sigma t}\cos(\omega t + \theta)$

Can we still use phasors for $\delta(t)$, $u(t)$, $\sin(\omega t)u(t)$
NO
but laplace transform works ✓

$$\mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t)\,dt \Rightarrow F(s)$$
variable ✓

$\delta(t)$	1	$f_1(t) \pm f_2(t)$	$F_1(t) \pm F_2(t)$
$u(t)$	$1/s$	$kf(t)$	$KF(t)$
$tu(t)$	$1/s^2$	$\frac{d}{dt} f(t)$	$sF(s) - f(0^-)$
$e^{-at}u(t)$	$1/(s+a)$		
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\int_{0^-}^{t} f(t)dt$	$F(s)/s$
$\sin(\omega t)u(t)$	$\frac{\omega}{s^2+\omega^2}$	$f_1(t) * f_2(t)$	$F_1(t)\cdot F_2(t)$
$\cos(\omega t)u(t)$	$\frac{s}{s^2+\omega^2}$	$f(t-a)u(t-a)$	$e^{-as}F(s)$
$e^{-at}\sin(\omega t)u(t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$f(t)e^{-at}$	$F(s+a)$

$$e^{-at}\cos(\omega t)u(t) = \frac{s}{(s+a)^2+\omega^2}$$

Initial and final value theorems

$$f(0^+) = \lim_{s\to\infty} sF(s)$$
$$f(\infty) = \lim_{s\to 0} sF(s)$$
all poles of sF(s) lie in left half plane

proof: $\mathcal{L}\{\sin(\omega t)u(t)\}$

$$= \int_{0^-}^{\infty} e^{-st}\left\{\frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)u(t)\right\}dt$$

$$= \frac{1}{2j}\int_{0^-}^{\infty} e^{t(j\omega - \sigma - j\omega)}u(t) - e^{-t(j\omega - \sigma - j\omega)}\,dt$$

$$= \frac{1}{2j}\int_{0^-}^{\infty} e^{-\sigma t}u(t) - e^{\sigma t}u(t)\,dt$$

$$= \frac{1}{2j}\mathcal{L}\{e^{j\omega t}\} - \mathcal{L}\{e^{-j\omega t}\}$$

$$= \frac{1}{2j}\left[\left(\frac{1}{s-j\omega}\right) - \left(\frac{1}{s+j\omega}\right)\right]$$

$$= \frac{1}{2j}\frac{2j\omega}{s^2+\omega^2} = \frac{\omega}{s^2+\omega^2}$$

example

① find $f(0^+)$ and $f(\infty)$ for $F(s) = \frac{4e^{-2s}(s+50)}{s}$

$$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{4e^{-2s}(s+50)\cancel{s}}{\cancel{s}}$$

$$= \lim_{s\to\infty}\frac{4(s+50)}{e^{2s}}$$
$\frac{\infty}{\infty}$ form

use L'Hôpital's rule

$$\lim_{x\to\infty}\frac{f(x)}{g(x)} = \lim_{x\to\infty}\frac{f'(x)}{g'(x)}$$

$$\Rightarrow f(0^+) = \lim_{s\to\infty}\frac{4}{2e^{2s}} = \frac{1}{\infty}\text{ form} = \underline{\underline{0}}$$

final value: $f(\infty) = \lim_{s\to 0} 4e^{-2s}(s+50)$
$= 4(50) = \underline{\underline{200}}$

② find $f(0^+)$ for $F(s) = \frac{5s^2+10}{2s(s^2+3s+5)}$

$$\Rightarrow f(0^+) = \lim_{s\to\infty} sF(s) = \frac{5s^2+10}{2s^2+6s+10}$$

$$\lim_{s\to\infty}\frac{10s}{4s+6} \Rightarrow \lim_{s\to\infty} = \frac{10}{4} = \frac{5}{2}$$

$$f(\infty) = \lim_{s\to 0}\frac{5s^2+10}{2(s^2+3s+5)} = 1$$

example: find ILT of $F(s) = \frac{s^2+6}{s^2+7}$

$$F(s) = 1 - \frac{1}{s^2+7} = 1 - \frac{\sqrt{7}}{\sqrt{7}(s^2+(\sqrt{7})^2)}$$

$$f(t) = \delta(t) - \frac{1}{\sqrt{7}}\sin(\sqrt{7}t)u(t)$$

eg} find ILT of $\frac{s+2}{s^2+2s+4}$

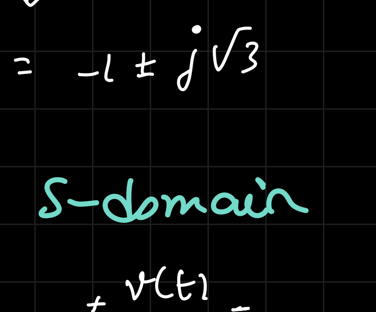
$$\Rightarrow \frac{1}{s+2} \Rightarrow f(t) = e^{-2t}u(t) \quad \times$$

$$\Rightarrow \frac{(s+1)+1}{(s+1)^2 + (\sqrt{3})^2} \Rightarrow \frac{s+1}{(s+1)^2+(\sqrt{3})^2} + \frac{1}{s^2+s+5}$$
$\Downarrow$
$\cos(\sqrt{3}t)u(t)$

$s^2 + 2s + 4$
$\Downarrow$
$s = -1 \pm j\sqrt{3}$

Impedances in S-domain

Resistor:



$v(t) = iR$

$V(s) = I(s)R$

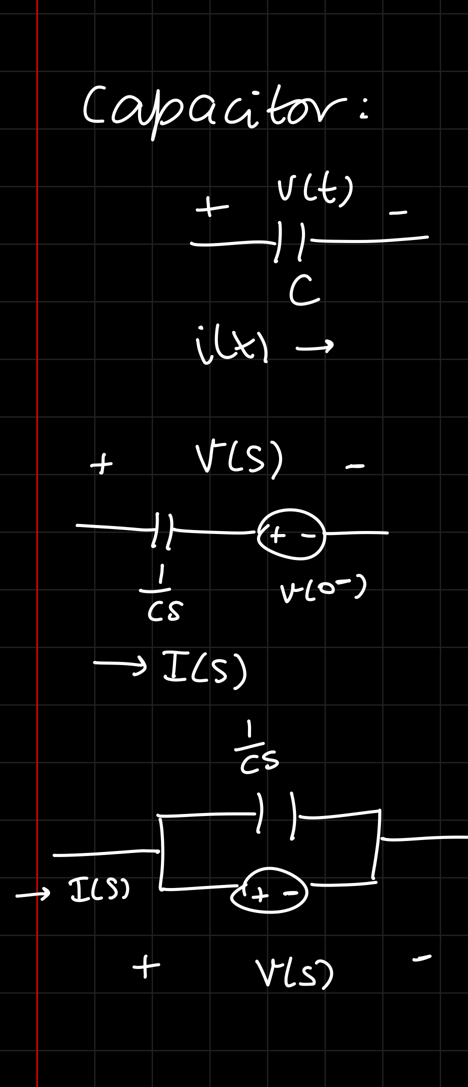
Inductor:

$$v(t) = L\frac{di(t)}{dt}$$

$$V(s) = L(sI(s) - i(0^-))$$

$$V(s) = sL\,I(s) - L\,i(0^-)$$

Capacitor:



$$i(t) = C\frac{dv}{dt}$$

$$I(s) = C[sV(s) - v(0^-)]$$

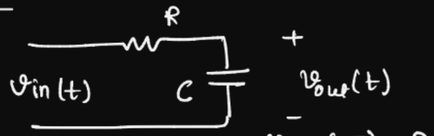
$$I(s) = CsV(s) - Cv(0^-)$$

$$V(s) = \frac{1}{Cs}I(s) + \frac{v(0^-)}{s}$$

* Lecture 16 {missed}

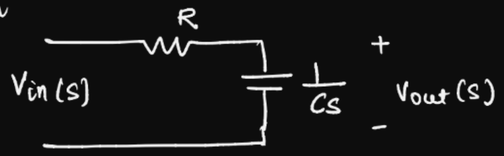
Transfer function :

Lets consider :



$v_{out}(0^-) = 0$

freq. domain



$$V_{out}(s) = \frac{1/Cs}{R + 1/Cs} V_{in}(s)$$

$$\Rightarrow \underbrace{\frac{V_{out}(s)}{V_{in}(s)}}_{\text{transfer function } H(s)} = \frac{1}{sRC+1}$$

$$v_{out}(t) = \mathcal{L}^{-1}\{V_{out}(s)\} = \mathcal{L}^{-1}\{\underbrace{H(s)}_{\text{defines the system.}} \cdot V_{in}(s)\}$$

* lecture: 17

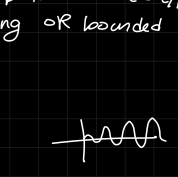
21/10/24

• Stability

Stable: bounded o/p for bounded i/p
unstable: unbounded o/p for bounded i/p
marginally stable: oscillating OR bounded offset

let: $e^{\sigma t}\cos(w_0 t)\,u(t)$

① $\sigma > 0$: unstable
② $\sigma < 0$: stable
③ $\sigma = 0$: marginally stable



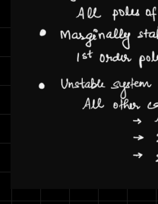
*
Stability Criterion

system transfer fn $H(s) = k\dfrac{(s-z_1)(s-z_2)(s-z_3)\cdots(s-z_m)}{(s-p_1)(s-p_2)(s-p_3)\cdots(s-p_n)}$

- System is stable if
  All poles of $sH(s)$ lie on LHS of complex plane
- Marginally stable if
  1st order pole at $s=0$ OR 1st order complex conjugate pole pair at $j\omega$-axis
- Unstable system if
  All other cases
  → Any poles on RHS OR
  → 2nd or higher order pole at $s=0$ OR
  → 2nd or higher order conjugate pole pair at $j\omega$-axis

$s = \sigma + j\omega$

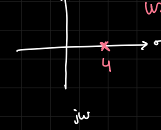
order of poles ≡ order of s

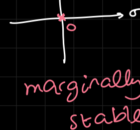
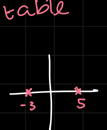
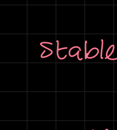


stable | unstable

at $j\omega$ axis:
could be unstable
or marginally stable

	$f(t)$	$sF(s)$	
①	$e^{-2t}u(t)$	$s\cdot\frac{1}{s+2}$	Stable
②	$e^{4t}u(t)$	$s\,\frac{1}{s-4}$	unstable
③	$u(t)$	$s\cdot\frac{1}{s}$ don't cancel we have one zero and one pole	marginally stable
④	$e^{-3t}u(t) + e^{+5t}u(t)$	$s\left\{\frac{1}{s+3} + \frac{1}{s-5}\right\}$	unstable
⑤		$s\cdot\frac{8}{(s+1)(s+2)(s+3)}$	stable
⑥		$s\cdot\frac{14}{(s+1-\sqrt{2})(s-1+\sqrt{2})}$	unstable
⑦	$tu(t)$	$\frac{1}{s^2}$ ↳ poles = 2, 2nd order pole at $s=0$	unstable

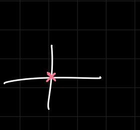


⑧ $e^{-3t}\sin(w_0 t)\,u(t)$ $\quad s\,\dfrac{w_0}{(s+3)^2+w_0^2}$

stable

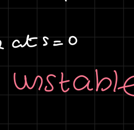
poles:
$(s+3)^2 + w_0^2 = 0$
$(s+3)^2 = -w_0^2$
$s+3 = \pm\sqrt{-1}\,w_0$
$s = -3 \pm jw_0$



⑨ $\sin(w_0 t)\,u(t)$ $\quad s\cdot\dfrac{w_0}{s^2+w_0^2}$

marginally stable

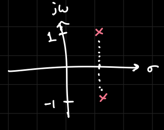
$s^2 = -w_0^2$
$s = \pm jw_0$



⑩ $e^{3t}\sin(w_0 t)\,u(t)$ $\quad s\cdot\dfrac{w_0}{(s-3)^2+w_0^2}$

unstable (RHS)

$s-3 = \pm jw_0$
$s = 3 \pm jw_0$



⑪ $t\cos(w_0 t)\,u(t)$

note: $f(t) \rightleftharpoons F(s)$
$tf(t) \rightleftharpoons -\dfrac{d}{ds}F(s)$

$\cos(w_0 t)u(t) \Rightarrow \dfrac{s}{s^2+w_0^2} = s(s^2+w_0^2)^{-1}$

$$\frac{d}{ds} = (s^2+w_0^2)^{-1} - s(s^2+w_0^2)^{-2}(2s)$$
$$= \frac{1}{s^2+w_0^2} - \frac{2s^2}{(s^2+w_0^2)^2}$$
$$= \frac{s^2+w_0^2-2s^2}{(s^2+w_0^2)^2} = \frac{w_0^2-s^2}{(s^2+w_0^2)^2}$$

$s = \pm jw_0$ (2nd order)

unstable

• Final Value Theorem

$$f(t\to\infty) = \lim_{s\to 0} sF(s)$$
↳ when all poles are on LHS (condition)

• Initial Value theorem

$$f(0) = \lim_{s\to\infty} sF(s) \quad \text{(no condition)}$$

eg} $f(t) = \sin(w_0 t)\,u(t)$

final value: $f(\infty) = \lim_{s\to 0} sF(s)$

$= \lim_{s\to 0} s\,\dfrac{w_0}{w_0^2+s^2} = 0$ ✗

indeterminant because condition not met
(all poles are not on LHS)

• Chapter 15 {frequency response}

assuming zero initial conditions:

capicitor impedance $= Z_C = \dfrac{1}{sC}$

where $s = \sigma + j\omega$

we represent capacitor as open circuit in case of DC input voltage

Transfer function:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} \Rightarrow V_{out}(s) = \underbrace{V_{in}(s)}_{input}\cdot\underbrace{H(s)}_{system}$$

let $\sigma = 0$ since we are only considering frequency
$\Rightarrow s = j\omega$

$H(j\omega) = \dfrac{V_{out}(j\omega)}{V_{in}(j\omega)}$ Voltage gain V/V

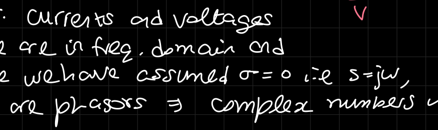
$H(j\omega) = \dfrac{I_{out}(j\omega)}{I_{in}(j\omega)}$ current gain A/A

$H(j\omega) = \dfrac{V_{out}(j\omega)}{I_{in}(j\omega)}$ transfer impedance $\frac{V}{A}$ or Ω

$H(j\omega) = \dfrac{I_{out}(j\omega)}{V_{in}(j\omega)}$ transfer admittance $\frac{A}{V}$ or Ω^{-1}

note: currents and voltages here are in freq. domain and since we have assumed $\sigma = 0$ i.e $s = j\omega$, they are phasors ⇒ complex numbers ✓

$$H(j\omega) = |H(j\omega)|\,\angle H(j\omega)$$



here these are low pass filters

• DECIBELS SCALE (dB)

$$H_{dB} = 20\log_{10}|H(j\omega)|$$

note:
$\log_{10}1 = 0$
$\log_{10}10 = 1$
$\log_{10}2 = 0.3$
$\log_{10}(0.1) = -1$
$\log_{10}(10^{-k}) = -k$
$\log_{10}(0) =$ undefined

find H_{dB} at $w = 146$ rad/s if $H(s) = 20(s+100)$

⇒ $H_{dB} = 20\log_{10}|H(j\omega)|$
$= 20\log_{10}|20j\omega + 2000|$
$= 20\log_{10}\left(20\sqrt{\omega^2+100^2}\right)$ ↳ $\omega = 146$
$= 70.97$ dB

---

let ....
$$H(s) = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

$H_{dB} = 20\log|K| + 20\log|s-z_1| + 20\log|s-z_2| + \cdots 20\log|s-z_m| - 20\log|s-p_1| - 20\log|s-p_2| - \cdots 20\log|s-p_n|$

let $H(j\omega) = H_1(j\omega)\cdot H_2(j\omega)$

$20\log_{10}|H(j\omega)| = 20\log_{10}|H_1(j\omega)| + 20\log|H_2(j\omega)|$
$\angle H(j\omega) = \angle H_1(j\omega) + \angle H_2(j\omega)$

⇒ consider: $H(s) = 1 + \dfrac{s}{a}$ → simple zero

$H(j\omega) = 1 + \dfrac{j\omega}{a}$

$H_{dB} = 20\log_{10}\left|1 + j\left(\frac{\omega}{a}\right)\right|$
$= 20\log_{10}\left(\sqrt{1^2 + \frac{\omega^2}{a^2}}\right)$

① let $\omega \lll a$
⇒ $\frac{\omega}{a}$ → negligible
$H_{dB} = 20\log_{10}(1) = \underline{0\text{ dB}}$

② if $\omega \ggg a$
⇒ 1^2 term becomes negligible
$H_{dB} = \underline{20\log_{10}\left(\frac{\omega}{a}\right)\text{ dB}}$

③ if $\omega = 10a$
$H_{dB} = 20\log(10) = 20$ dB

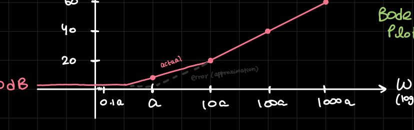
④ $\omega = 100a$
$H_{dB} = 20\log(100) = 40$ dB

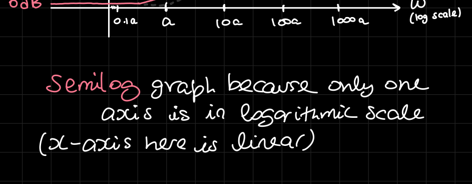
⑤ $\omega = 1000a$
$H_{dB} = 60$ dB

⑥ $\omega = 10000a$
$H_{dB} = 80$ dB

Observation: $\omega \times 10 \rightarrow H_{dB} + 20$ dB

Using log scale instead of ω directly





Semilog graph because only one axis is in logarithmic scale
(x-axis here is linear)

now let $\omega = a$ ⇒ $H_{dB} = 20\log_{10}(\sqrt{2})$
$= 20 \times \frac{1}{2}\log_{10}(2)$
$= 3$ dB

corner frequency / break freq at $\omega = a$

- Lecture 18 {Bode Plots} Missed

23/10/24

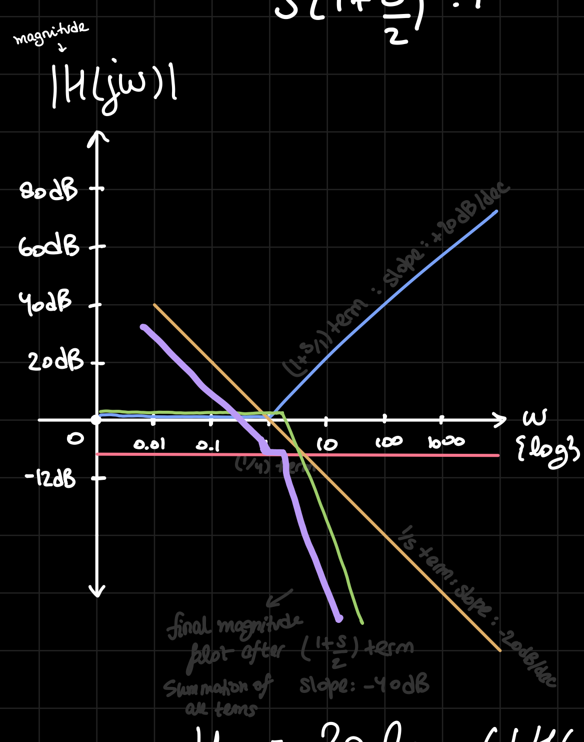
• lecture 19 04/11/24

eg 3 draw bode plots of $H(s) = \frac{s+1}{s(s+2)^2}$

zeroes : 1 → $s = -1$
poles : 3 → $s = 0, -2, -2$

$$H(s) = \frac{(1+s/1)}{s(1+\frac{s}{2})^2 \cdot 4} = \frac{1}{4}\cdot\frac{1}{s}\cdot\left(1+\frac{s}{1}\right)\cdot\frac{1}{(1+\frac{s}{2})^2}$$

simple pole case so, we have corner frequency at $w = a$ i.e. at $w = 1$ & $w = 2$ respectively



$$H_{dB} = 20\log_{10}(|H(jw)|)$$

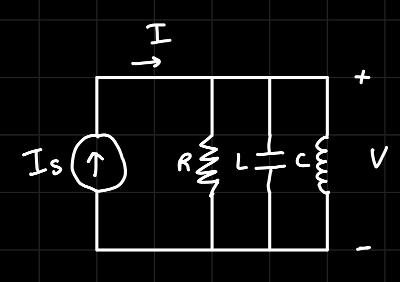
$$= 20\log_{10}\left(\frac{1}{4}\right) + 20\log_{10}\left(\left|\frac{1}{jw}\right|\right)$$

-20 log(w) straight line no break frequency

$$20\log_{10}\left(1+\frac{s}{1}\right) + 40\log_{10}\left(1+\frac{s}{2}\right)$$

todo: phase plot for ↑

• **Resonance** : A network is in resonance when V and I at network input terminals are in phase



① Assume : $I_s = I_0 \cos(wt + \phi)$

$$Y(jw) = \frac{1}{Z(jw)} = \frac{1}{R} + jwC + \frac{1}{jwL}$$

$$= \frac{1}{R} + j\left(wC - \frac{1}{wL}\right)$$

for resonance ⇒ V and I are in phase (purely resistive circuits)

i.e. $wC - \frac{1}{wL} = 0$

⇒ $w = \frac{1}{\sqrt{LC}} = w_0$ "resonant frequency"

② Assume $I_s = I_0 e^{\sigma t}\cos(wt + \phi)$

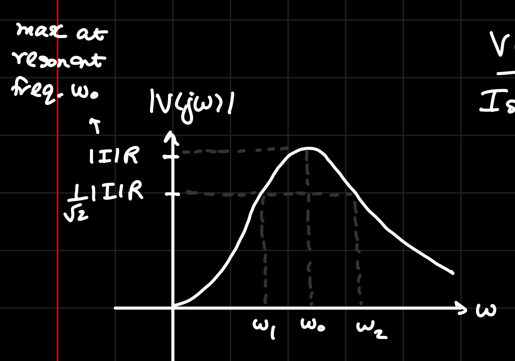
$$Y(s) = \frac{1}{Z(s)} = \frac{1}{R} + sC + \frac{1}{sL}$$

$$= \frac{s^2 + s(1/RC) + (1/LC)}{(1/C)\,s}$$

$V_{out} = V$
$I_{in} = I_s$ ⇒ transfer function?

$$\frac{V(s)}{I_s(s)} = Z(s) = \frac{1}{Y(s)}$$

transfer impedance



w_1, w_2 : half power frequency

height of this curve depends only on value of Resistance (R)

Width of the curve depends on L & C as well

There will be some current flowing in capacitor & inductor but it is equal and opposite so it cancels out and net total current flows only through the resistor

At resonance :

$I_s = I_R$
$I_C = -I_L$ ⇒ $I_C + I_L = 0$

Quality factor

$$Q = 2\pi \frac{\text{max. energy stored}}{\text{total energy lost per time period}}$$

$$= 2\pi \frac{[w_L(t) + w_C(t)]_{max}}{P_R \cdot T}$$

P_R : avg power lost in resistor
$= \frac{1}{2}|I|^2 R$ WATTS

T : time period (sec)
$w_L(t)$: $\frac{1}{2}Li_L^2(t)$ (J)
$w_C(t)$: $\frac{1}{2}Cv_C^2(t)$ (J)

$$Q = w_0 RC = R\sqrt{\frac{C}{L}}$$

$$\alpha = \text{damping factor} = \frac{1}{2RC} = \frac{w_0}{2Q}$$

$$w_d : \text{damped frequency} = \sqrt{w_0^2 - \alpha^2}$$

$$= w_0\sqrt{1 - \left(\frac{1}{2Q}\right)^2}$$

• HALF POWER BANDWIDTH (BW)

$$BW = w_2 - w_1 = \frac{w_0}{Q}$$

where $w_0 = \sqrt{w_1 w_2}$

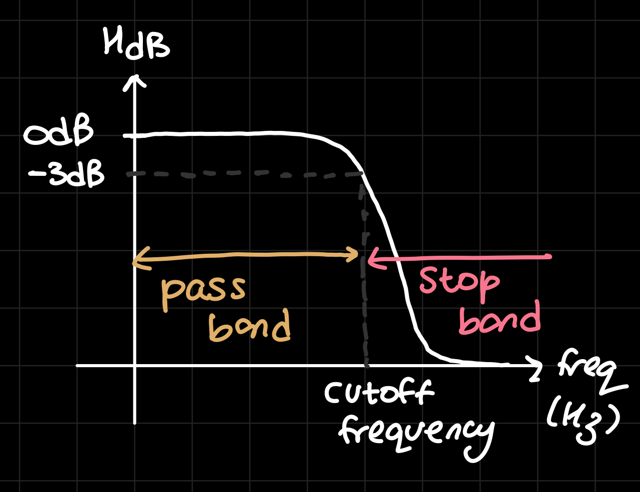
note: $BW \propto \frac{1}{Q}$

note: if $Q > 5$: high-Q circuit

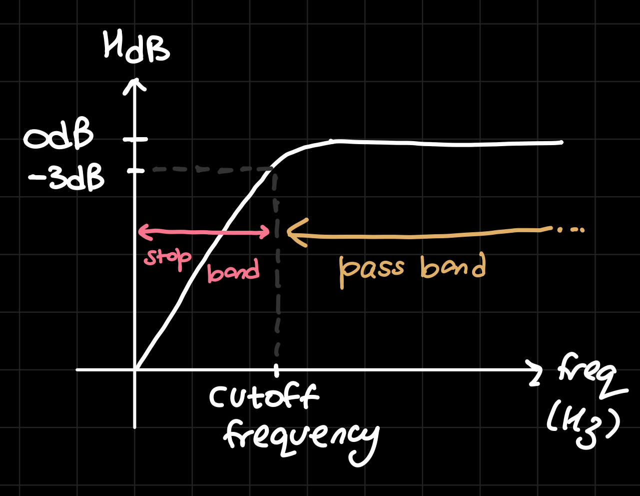
- Lecture 20 06/11/24

→ **FILTER DESIGN**

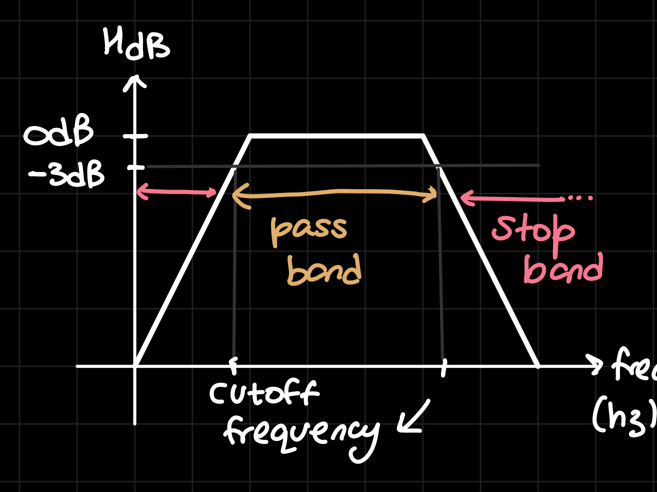
① Low Pass filter



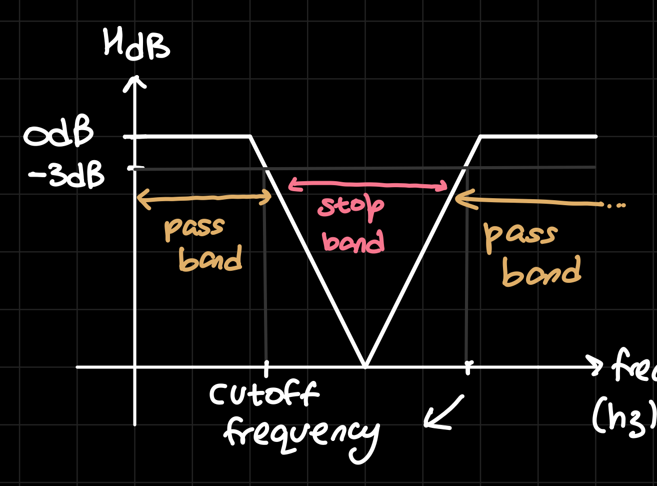
② High Pass filter



③ Band Pass Filter



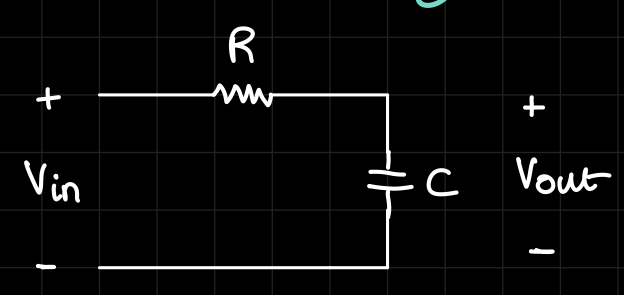
④ Band Pass Filter



- Passive filters : Gain $\leq 1 \Rightarrow H_{dB} \leq 0dB$
  ↳ using R, L, C

- Active filters : Gain can be more than 1
  here ⇒ $H_{dB} \geq 0$
  ↳ using opAmp + R, L, C

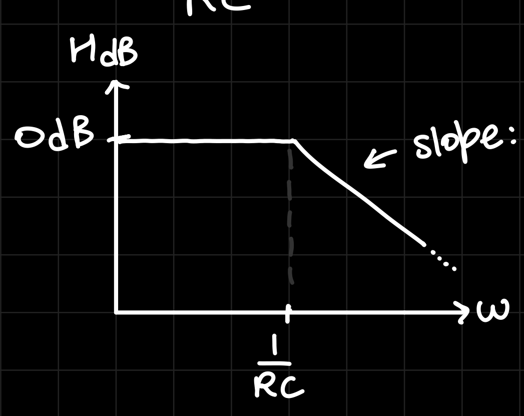
- 1st order LPF using R,L,C



$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1/Cs}{1/Cs + R} = \frac{1}{1+RCs}$$

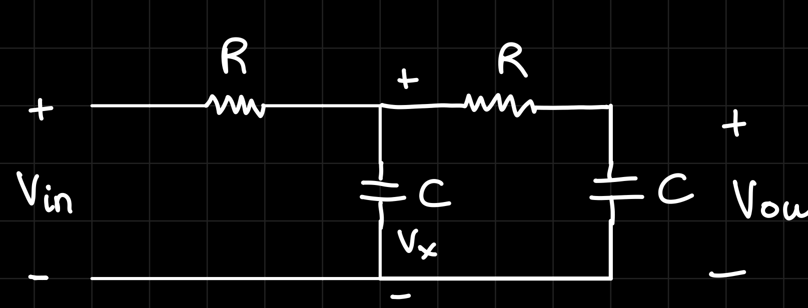
$$= \frac{1}{1+\frac{s}{(1/RC)}}$$

$$\omega_c = \frac{1}{RC}$$



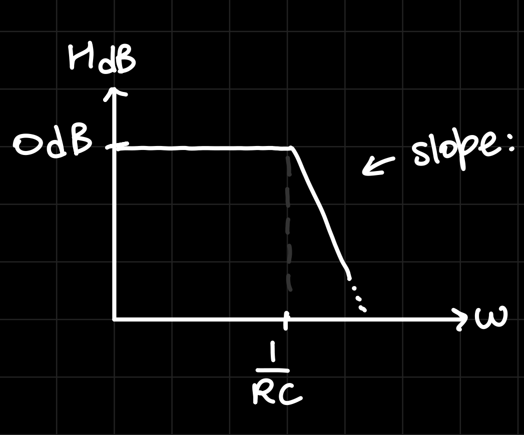
not a good filter because

- 2nd order LPF :



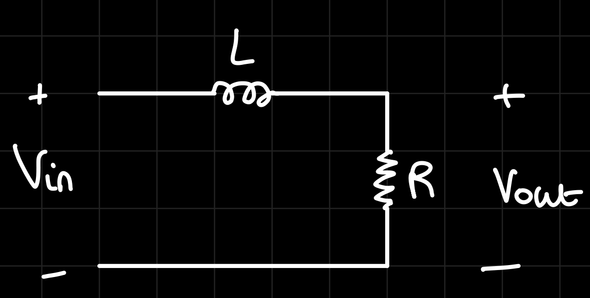
$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{V_{out}(s)}{V_x} \cdot \frac{V_x}{V_{in}(s)}$$

$$H(s) = \frac{1}{1+\frac{s}{\left(\frac{1}{RC}\right)}} \cdot \frac{1}{1+\frac{s}{\left(\frac{1}{RC}\right)}}$$



cascading
↓
better filter ✓

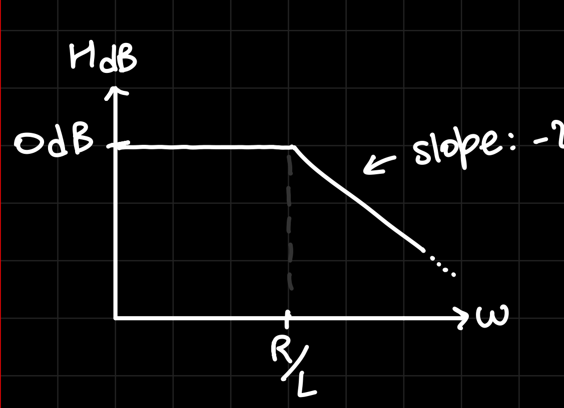
- Another 1st order LPF

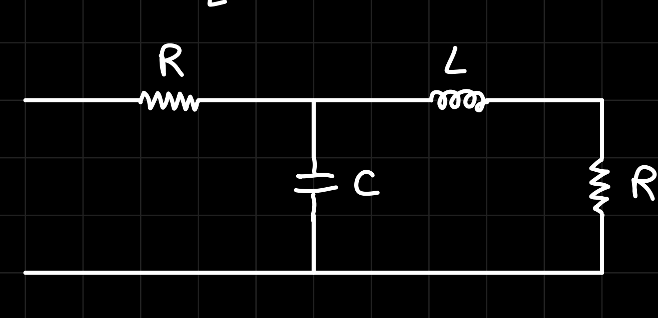


$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

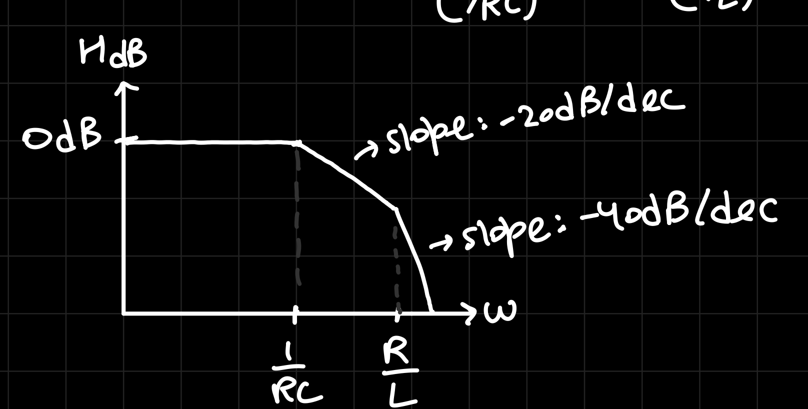
$$= \frac{R}{R+sL}$$

$$= \frac{1}{1+\frac{s}{(R/L)}}$$

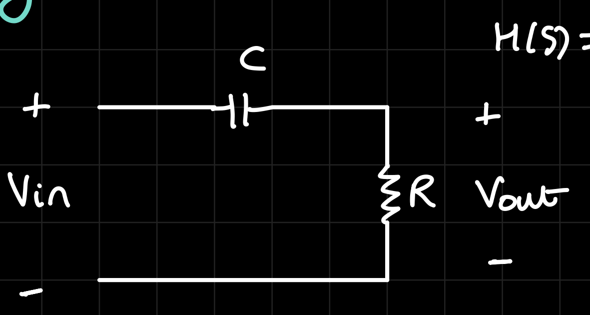




$$H(s) = \frac{1}{1+\frac{s}{(1/RC)}} \cdot \frac{1}{1+\frac{s}{(R/L)}}$$



- High Pass Filter

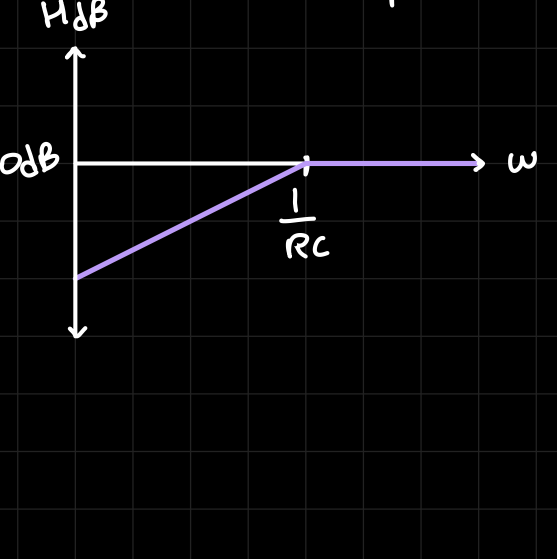


$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R+\frac{1}{sC}}$$

$$= \frac{RCs}{RCs+1}$$

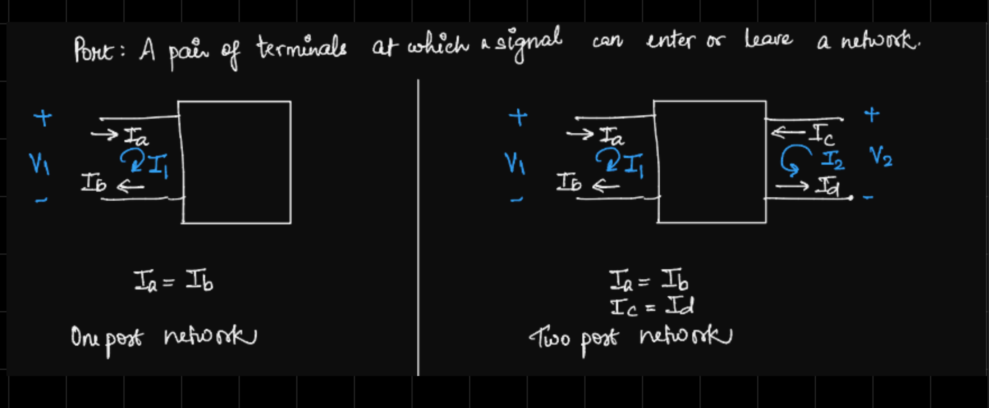
$$H(s) = \frac{\frac{s}{(1/RC)}}{1+\frac{s}{(1/RC)}}$$

zero: $s = 0$
poles: $s = \frac{-1}{RC}$



lecture 22 13/11/24

port : a pair of terminals where a signal can enter or exit



The network ("box") can have only 1st order elements and dependent sources. (R,L,C)

Parameters

① Y parameter ② Z parameter

③ h parameter (hybrid) ③ t parameter (transmission or ABCD)

These parameters are independent of the i/p or o/p

remember y-parameter {unit : S/Ω^{-1}}

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

short circuit Y-parameter

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} \qquad Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} \qquad Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$

Z parameter {unit : Ω}

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

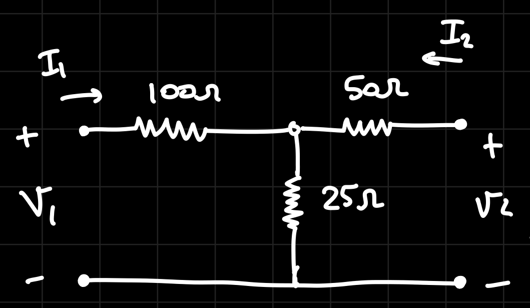
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

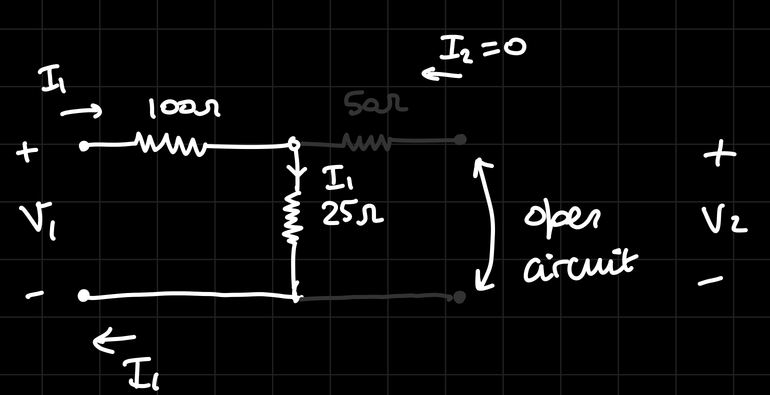
open circuit Z parameters

eg}



find z parameter

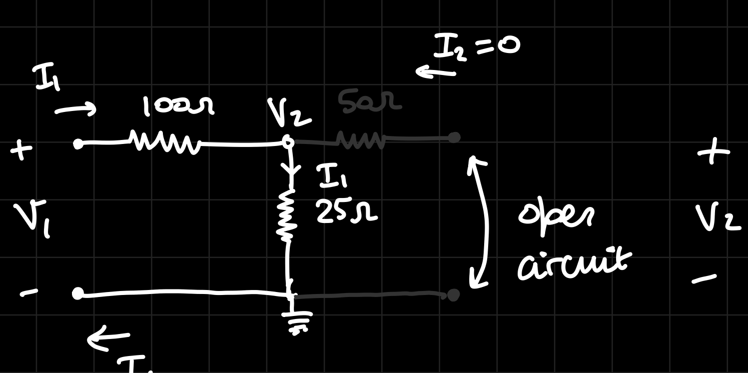
$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$



$V_1 = I_1(125)$

$Z_{11} = 125$

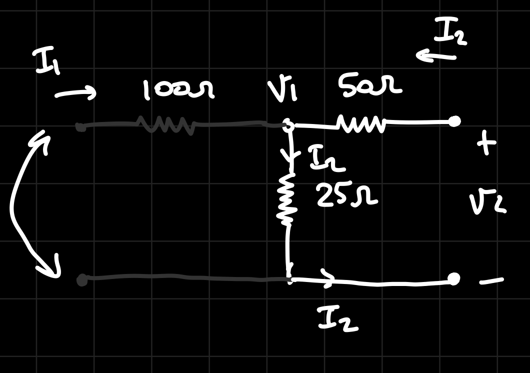
$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$



$$Z_{21} = \frac{V_2}{I_1} = 25\Omega$$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

OC



$Z_{22} = 75\Omega$

$Z_{12} = 25\Omega$

observation: $Z_{21} = Z_{12}$

Reciprocal / Bilateral network

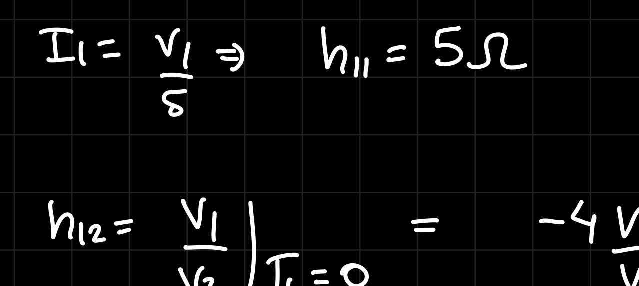
HYBRID PARAMETER (h)

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0}\ \Omega \ (\text{SC}) \qquad h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}\ V/V \ (\text{OC})$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0}\ \frac{V}{V} \ (\text{OC}) \qquad h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}\ \Omega \ (\text{SC})$$

eg}



$$I_1 = 0.8V_2 + \frac{V_1}{5} \quad ; \quad I_2 = \frac{V_2 - 0.1V_1}{2}$$

① $h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$

$I_1 = \frac{V_1}{5} \Rightarrow h_{11} = 5\Omega$

② $h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = -4\frac{V}{V}$

③ $h_{21} = \frac{V_2}{V_1}\Big|_{I_2=0} = 0.1\frac{V}{V}$

④ $h_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} \Rightarrow 4V_2 + V_1 = 0$

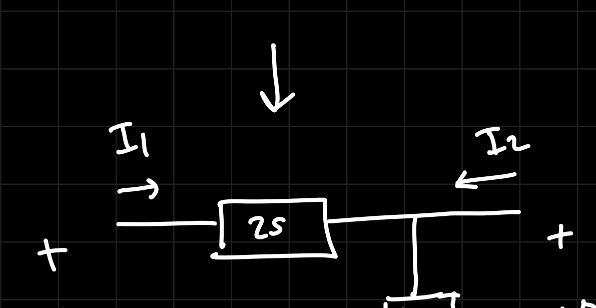
$V_1 = -4V_2$

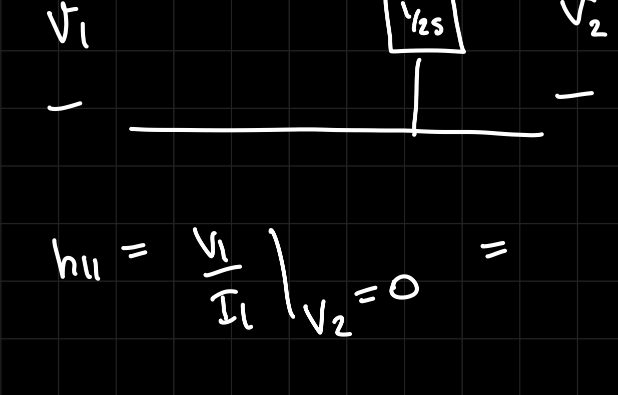
$h_{22} = 3.33\ \Omega$

$I_2 = \frac{V_2 - 0.4V_2}{2}$

$\Rightarrow \frac{V_2}{I_2} = \frac{2}{0.6} = \frac{10}{3}$

eg}





$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} =$

$h_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \frac{1}{2s}$

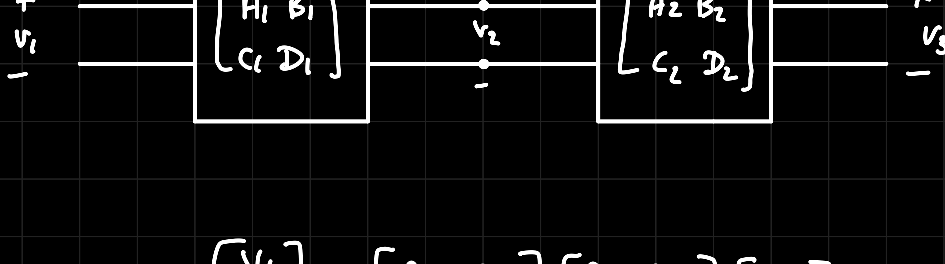
$h_{12} = \frac{V_1}{I_2}$

$$[h] = \begin{bmatrix} 2s\Omega & 1\ V/V \\ 1\ A/A & 2s\Omega \end{bmatrix}$$

Transmission parameter (ABCD)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Cascading :



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}\begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$$

eg)

(admittance)
- Y Parameters {unit: S / Ω^{-1}}

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$I_1 = Y_{11} V_1 + Y_{12} V_2$
$I_2 = Y_{21} V_1 + Y_{22} V_2$

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ $Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$ $Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

(impedance)
- Z parameter {unit: Ω}

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$V_1 = Z_{11} I_1 + Z_{12} I_2$
$V_2 = Z_{21} I_1 + Z_{22} I_2$

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$

$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$ $Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$

if $Z_{12} = Z_{21}$ then network is called bilateral/reciprocal network

- H Parameter
(H ≡ hybrid)

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$

$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ Ω $h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$ V/V

$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ A/A $h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$ Ω^{-1}

- T Parameter (ABCD)
(T ≡ transmission)

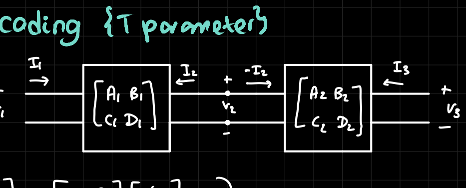
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$V_1 = AV_2 - BI_2$
$I_1 = CV_2 - DI_2$

$A = \frac{V_1}{V_2}\Big|_{I_2=0}$ $B = \frac{-V_1}{I_2}\Big|_{V_2=0}$

$C = \frac{I_1}{V_2}\Big|_{I_2=0}$ $D = \frac{-I_1}{I_2}\Big|_{V_2=0}$

- cascoding {T parameter}

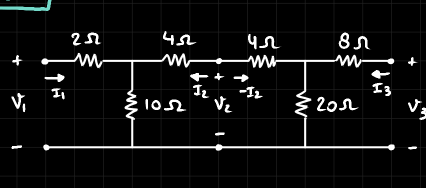


$$\left.\begin{aligned} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} &= \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \\ \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} &= \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix} \end{aligned}\right\} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$$

The coefficients are multiplied (matrix multiplication)

- Examples

16.10

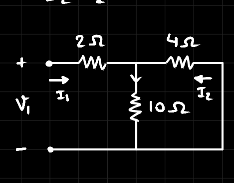


$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$A_1 = \frac{V_1}{V_2}\Big|_{I_2=0} \Rightarrow \frac{V_1}{I_1} = 12$ & $\frac{V_2}{I_1} = 10$

$\frac{V_1}{V_2} = 1.2$ V/V

$B_1 = \frac{-V_1}{I_2}\Big|_{V_2=0}$

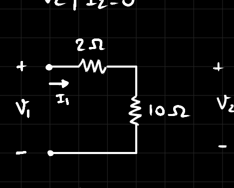


$V_1 = 2I_1 + 10I_1 + 10I_2$

$V_1 = 12I_1 + 10I_2$
$V_1 = 2I_1 - 4I_2$

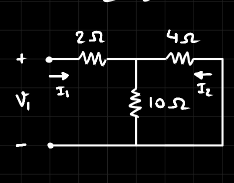
$5V_1 = -34 I_2$
$\frac{-V_1}{I_2} = \frac{34}{5} = 6.8\Omega$

$C_1 = \frac{I_1}{V_2}\Big|_{I_2=0} = 0.1\ \Omega^{-1}$



$V_1 = I_1(12)$
$V_2 = I_1(10)$
$\frac{I_1}{V_2} = \frac{1}{10}$

$D_1 = \frac{-I_1}{I_2}\Big|_{V_2=0} = 1.4$ A/A



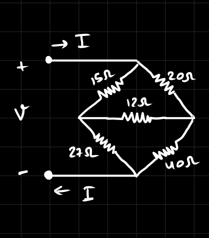
$V_1 = 12I_1 + 10I_2$
$V_1 = 2I_1 - 4I_2$
$I_1 - 2I_2 = 6I_1 + 5I_2$
$5I_1 = -7I_2$
$\frac{-I_1}{I_2} = \frac{7}{5}$

- Questions

16.3 (21)

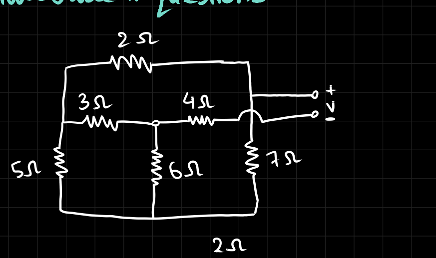
Zin?

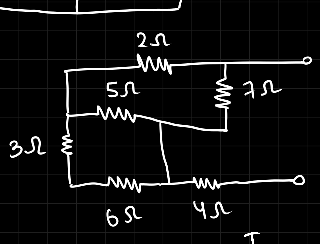
$Z_{in} = \frac{V}{I}$

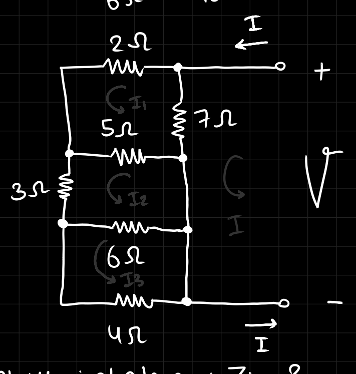


- Tutorial 11 questions

①







input impedance: Zin = ?

$Z_{in} = \frac{\Delta_Z}{\Delta_{11}}$

$-V + 7(I - I_1) = 0$
① $7I - 7I_1 = V$

$2I_1 + 5I_1 - 5I_2 + 7I_1 - 7I = 0$
② $-7I + 14I_1 - 5I_2 = 0$

$3I_2 + 6I_2 - 6I_3 + 5I_2 - 5I_1 = 0$
③ $-5I_1 + 14I_2 - 6I_3 = 0$

$4I_3 + 6I_3 - 6I_2 = 0$
④ $-6I_2 + 10I_3 = 0$

conclusion:

$$\begin{bmatrix} V \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 & -7 & 0 & 0 \\ -7 & 14 & -5 & 0 \\ 0 & -5 & 14 & -6 \\ 0 & 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I \\ I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$Z_{in} = \frac{\Delta_Z}{\Delta_{11}} = \frac{\begin{vmatrix} 7 & -7 & 0 & 0 \\ -7 & 14 & -5 & 0 \\ 0 & -5 & 14 & -6 \\ 0 & 0 & -6 & 10 \end{vmatrix}}{\begin{vmatrix} 14 & -5 & 0 \\ -5 & 14 & -6 \\ 0 & -6 & 10 \end{vmatrix}}$$

$$= \frac{7\begin{vmatrix} 14 & -5 & 0 \\ -5 & 14 & -6 \\ 0 & -6 & 10 \end{vmatrix} - 7\begin{vmatrix} -7 & -5 & 0 \\ 0 & 14 & -6 \\ 0 & -6 & 10 \end{vmatrix}}{\begin{vmatrix} 14 & -5 & 0 \\ -5 & 14 & -6 \\ 0 & -6 & 10 \end{vmatrix}}$$

$\Rightarrow 14(140 - 36) + 5(-50) = 14(104) - 250 = 1456 - 250 = 1206$

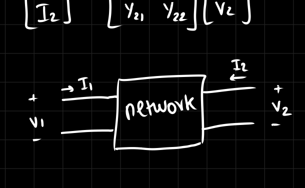
$\Rightarrow -7(140 - 36) = -7(104) = -728$

$Z_{in} = \frac{7(1206) + 7(728)}{1206} = \frac{7(1934)}{1206}$

$\approx 11.22\ \Omega$

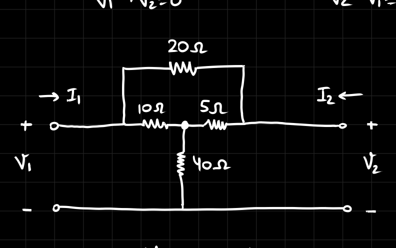
②

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

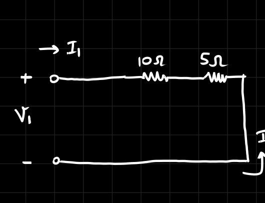


$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ $Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$ $Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$



① $V_2 = 0 \Rightarrow Y_{11}$ and Y_{21}

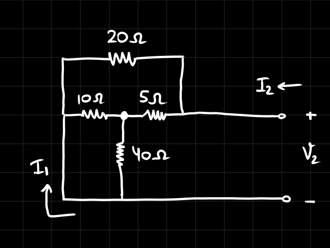


$I_2 = -I_1$

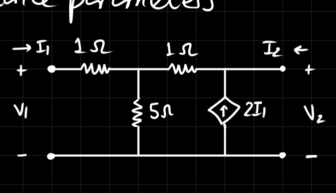
$V_1 = I_1(15)$
$\frac{I_1}{V_1} = \frac{1}{15}$
$\frac{I_2}{V_1} = \frac{-1}{15}$

$Y_{11} = \frac{1}{15}$, $Y_{21} = \frac{-1}{15}$

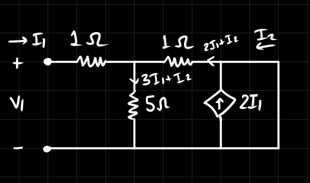
② $V_1 = 0$



③ admittance parameters

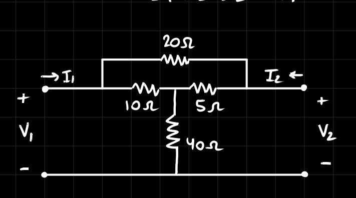


① $V_2 = 0$ $Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ $Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$

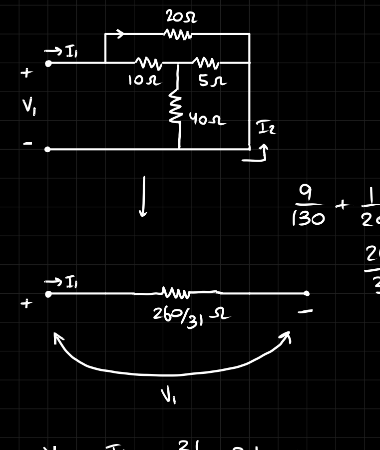


$-V_1 + I_1 + 10I_1 + 5I_2 = 0$
$9I_1 + 5I_2 = V_1$

②



$V_2 = 0$

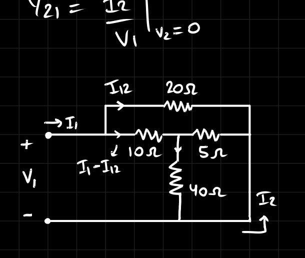


$\frac{9}{130} + \frac{1}{20}$

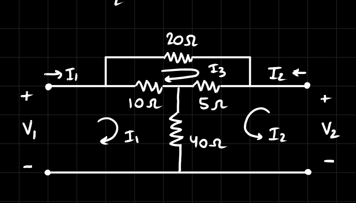
$\frac{260}{31}$

$Y_{11} = \frac{I_1}{V_1} = \frac{31}{260}\ \Omega^{-1}$ ✓

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$



how about finding parameters using KVL equations



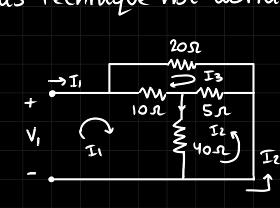
$-V_1 + 10I_1 - 10I_3 + 40I_1 + 40I_2 = 0$
$50I_1 + 40I_2 - 10I_3 = V_1$

$-V_2 + 5I_2 + 5I_3 + 40I_2 + 40I_1 = 0$
$40I_1 + 45I_2 + 5I_3 = V_2$

$20I_3 + 5I_3 + 5I_2 + 10I_3 - 10I_1 = 0$
$-10I_1 + 5I_2 + 35I_3 = 0$

$$\begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 50 & 40 & -10 \\ 40 & 45 & 5 \\ -10 & 5 & 35 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

this technique not working



$-V_1 + 10I_1 - 10I_3 + 40I_1 + 40I_2 = 0$
$30I_1 + 40I_2 - 10I_3 = V_1$

$20I_3 + 5I_3 + 5I_2 + 10I_3 - 10I_1 = 0$
$-10I_1 + 5I_2 + 35I_3 = 0$

$5I_2 + 5I_3 + 40I_2 + 40I_1 = 0$
$40I_1 + 45I_2 + 5I_3 = 0$

$$\begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 30 & 40 & -10 \\ -10 & 5 & 35 \\ 40 & 45 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$\Delta =$

$\Delta_1 = \begin{vmatrix} 40 & -10 & V_1 \\ 5 & 35 & 0 \\ 45 & 5 & 0 \end{vmatrix} = V_1(25 - 45 \cdot 35)$

$\Delta = 30(25 - 1575) - 40(-50 - 1400) - 10(-450 - 800)$

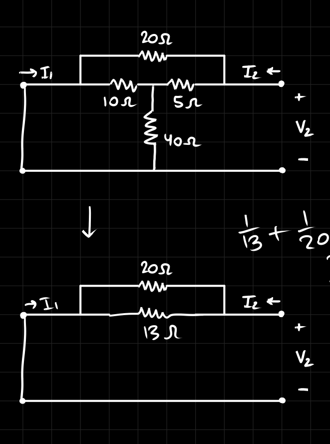
$= 30(-1550) + 40(1450) + 10(1250)$
$= -46500 + 58000 + 12500$
$= 58000 - 34000 = 24000$

$\Delta_2 = \begin{vmatrix} 30 & -10 & V_1 \\ -10 & 35 & 0 \\ 40 & 5 & 0 \end{vmatrix} = V_1(-50 - 1400)$
$= V_1(-1450)$

$I_2 = \frac{\Delta_2}{\Delta} = \frac{V_1(-1450)}{24000}$

$\frac{I_2}{V_1} = \frac{-145}{2400} = \frac{-29}{480}$

$V_1 = 0$ }

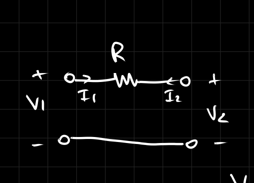


$\frac{1}{13} + \frac{1}{20}$ $\frac{260}{33}$

$\frac{33}{260}$

$Y_{22} = \frac{33}{260} = 0.1269\ \Omega^{-1}$

④ (a)



$I_2 = -I_1$

$\frac{V_2 - V_1}{R} = -I_1$

$V_2 - V_1 + I_1 R = 0$
$V_2 - V_1 - I_2 R = 0$
$V_1 + I_2 R = 0$

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{1}{R}$

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{-1}{R}$

$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} = \frac{-1}{R}$

$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{1}{R}$

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} = \infty$

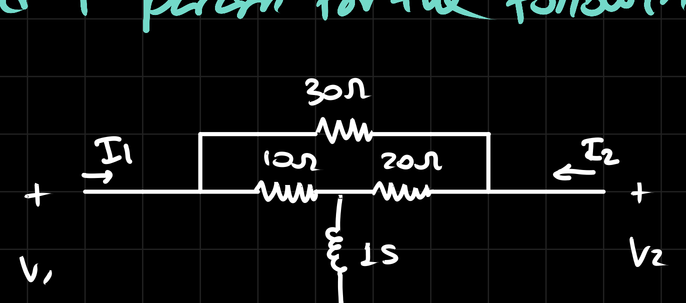
$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} = \infty$

$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = \infty$

$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0} = \infty$

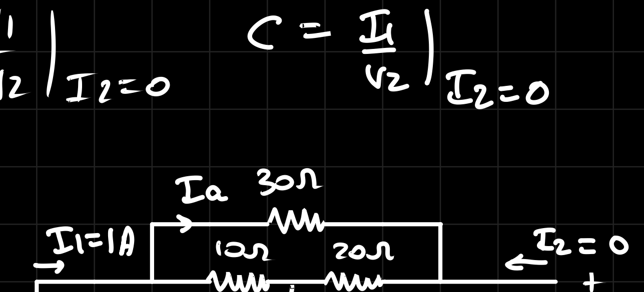
• lecture 23

Q) find T param for the following



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ $\quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$

let $I_1 = 1A$



$$-V_1 + 10 - 10I_a + 1s(1) + 0.1V_1 = 0$$

$$s + 10 - 0.9V_1 = 10I_a$$

$$-V_2 + 20I_a + s + 0.1V_1 = 0$$

$$30I_a + 20I_a + 10I_a - 10 = 0$$
$$60I_a = 10$$
$$I_a = \frac{1}{6}$$

① $-0.9V_1 = \frac{10}{6} - 10 - s$

$$0.9V_1 = \frac{50}{6} + s$$

$$V_1 = \frac{\frac{50}{6} + s}{0.9}$$

$$V_2 = \frac{20}{6} + s + 0.1V_1$$

$$V_2 = \frac{20}{6} + s + \frac{10.4}{6} = \frac{30.4}{6} + s$$

$$= s + 5.0\bar{6}$$

we know $I_1 = 1A$

so, $A = \frac{V_1}{V_2}$ $\quad C = \frac{I_1}{V_2} = \frac{1}{s+5.07}$

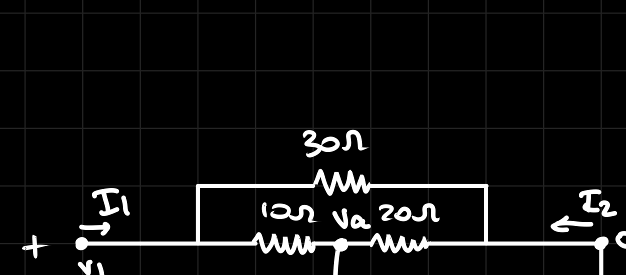
finding B and D

$$V_1 = -BI_2 \big|_{V_2=0}$$

$$I_1 = -DI_2 \big|_{V_2=0}$$

$B = \frac{-V_1}{I_2}$, $D = \frac{-I_1}{I_2}$

$\boxed{V_2 = 0}$



$$\frac{V_a - V_1}{10} + \frac{V_a - 0.1}{s} + \frac{V_a}{20} = 0$$

$$I_1 = \frac{V_1 - 0}{30} + \frac{V_1 - V_a}{10}$$

$$I_2 = \frac{-V_a}{20} + \frac{-V_1}{30}$$

let $V_1 = 1V$

• conversion between parameters

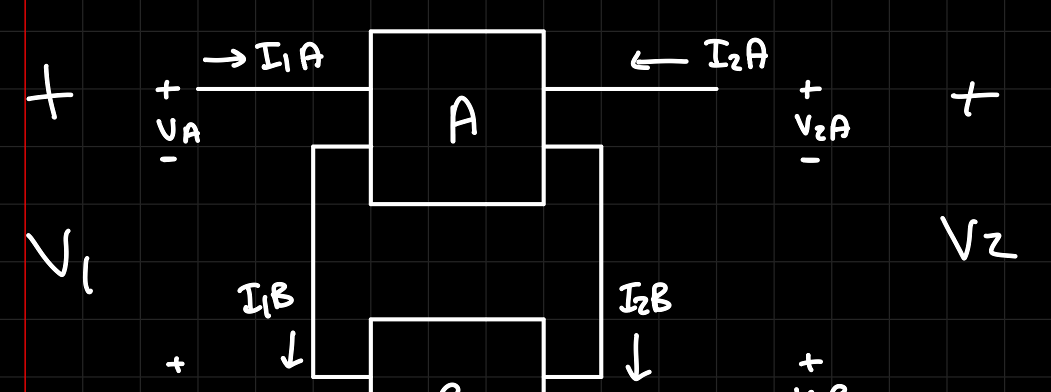
① $Z \rightarrow Y$

$Y_{11} = \frac{Z_{22}}{\Delta z}$ $\quad Y_{12} = \frac{-Z_{12}}{\Delta z}$

$Y_{21} = \frac{-Z_{21}}{\Delta z}$ $\quad Y_{22} = \frac{Z_{11}}{\Delta z}$

• Connection btw 2 port networks

① series

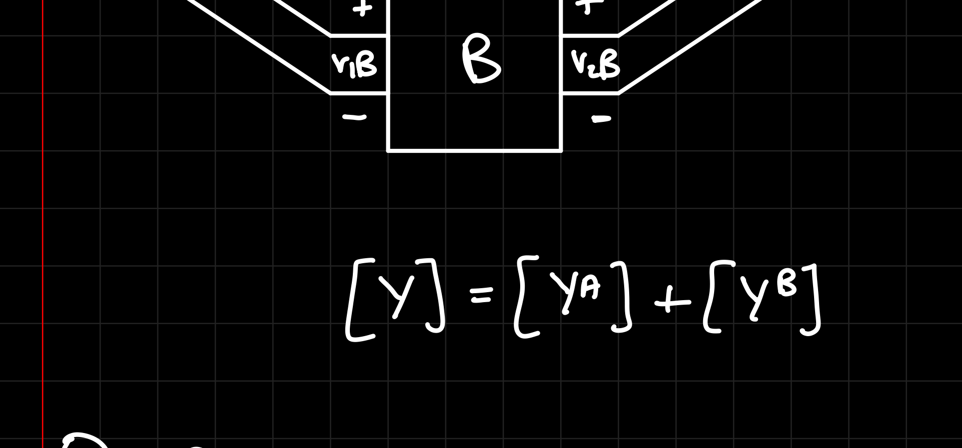


$V_1 = V_1A + V_1B = \underbrace{Z_{11}}_{Z_{11}A + Z_{11}B} I_1 + \underbrace{Z_{12}}_{Z_{12}A + Z_{12}B} I_2$
$V_2 = V_2A + V_2B$

$I_1 = I_1A = I_1B$
$I_2 = I_2A = I_2B$

$$[Z] = [Z_A] + [Z_B]$$

② Parallel

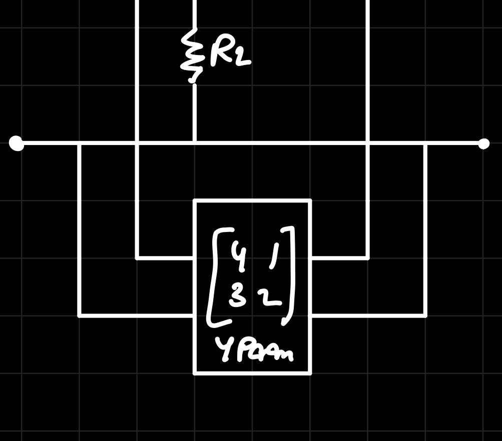


$$[Y] = [Y^A] + [Y^B]$$

③ CASCADING

$$[t] = [t_A][t_B]$$

eg}



① find Y_a

② $Y = Y_a + Y_B$

③ $Y \rightarrow Z$

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FOURIER CIRCUIT ANALYSIS

* Laplace Transform

$$F(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt \quad \text{unilateral LT}$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt \quad \text{bilateral LT}$$

$s = \sigma + j\omega$

* FOURIER TRANSFORM

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

LT but $\sigma = 0$ i.e. $s = j\omega$

FT is a subset of LT

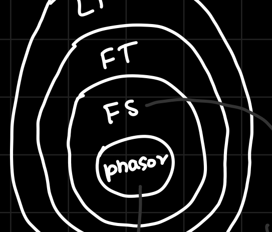
eg: $f(t) = e^{5t} u(t)$

fourier transform doesn't exist because it does not satisfy absolute integribility

i.e. $\int_{-\infty}^{\infty} |e^{5t} u(t)| \not< \infty$

but laplace transfrm exists

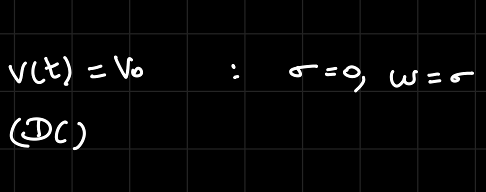
$$F(s) = \frac{1}{s-5}$$



because FS exists only for periodic sign

because sinusoids are periodic

eg)



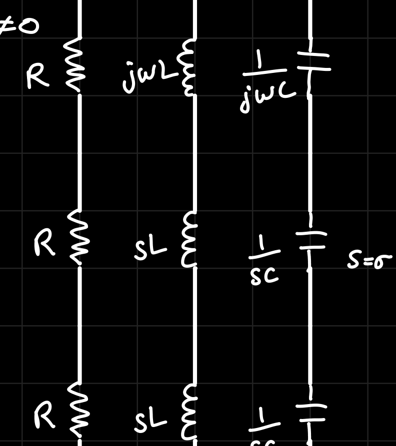
$$v(t) = V_0 e^{-\sigma t} \cos(\omega t + \theta)$$

① $v(t) = V_0$: $\sigma = 0,\ \omega = \sigma$
(DC)

② $v(t) = V_0 \cos(\omega t + \theta)$: $\sigma = 0,\ \omega \neq 0$
$V = V_0 \angle \theta$
sinusoidal steady state

③ $v(t) = V_0 e^{-\sigma t}$: $\sigma \neq 0,\ \omega = 0$
exponential

④ $v(t) = V_0 e^{-\sigma t} \cos(\omega t + \theta)$: $\sigma, \omega \neq 0$



⑤ Switches

initial conditions

LT	FT
+ initial conditions	- not convient for initial conditions
+ stability analysis	- doesn't exist for unstable systems
- not convenient	+ freq. domain analysis (power spectrum)

→ FOURIER SERIES

if $f(t)$ is a periodic signal

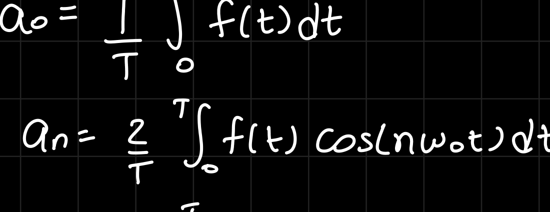
$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)\,dt$$

eg)



$T = 2s$

$$a_0 = \frac{1}{2}\int_0^1 1\,dt + \frac{1}{2}\int_1^2 -1\,dt = 0$$

$$a_n = \frac{2}{2}\int_0^1 \cos(n\omega_0 t) - \int_1^2 \cos(n\omega_0 t)$$

$$= \left[\frac{\sin(n\omega_0 t)}{n\omega_0}\right]_0^1 - \left[\frac{\sin(n\omega_0 t)}{n\omega_0}\right]_1^2$$

x ↓ $= \left(-\cos(n\pi)^{+1} + \cos(2n\pi) - \cos(n\pi)\right)\frac{1}{n\omega_0}$

$$= \frac{1}{n\pi}\left(\frac{e^{jn\omega_0} - e^{-jn\omega_0}}{2j} + \frac{e^{j2n\omega_0} + e^{-j2n\omega_0}}{2} - \frac{e^{jn\omega_0} - e^{-jn\omega_0}}{2}\right)$$

$$= \left(\frac{-1}{j2n\pi} - \frac{1}{2}\right)e^{-jn\omega_0} + \left(\frac{1}{j2n\pi} - \frac{1}{2}\right)e^{jn\omega_0}$$

x ↑ $+ \frac{1}{2}e^{j2n\omega_0} + \frac{1}{2}e^{-j2n\omega_0}$

OR

$$a_n = \frac{2}{T}\int_0^T \underbrace{\underbrace{x(t)}_{\text{odd}} \times \underbrace{\cos(n\omega_0 t)}_{\text{even}}}_{\text{odd}}$$

So, $a_n = 0 \ \forall n$

$$b_n = \begin{cases} 0 & : n\%2 == 0 \\ \frac{4}{n\pi} & : \text{else} \end{cases}$$

So, $v(t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4}{n\pi}\sin(n\omega_0 t)$

Complex representation of FS

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt$$

can be complex

Fourier Transform

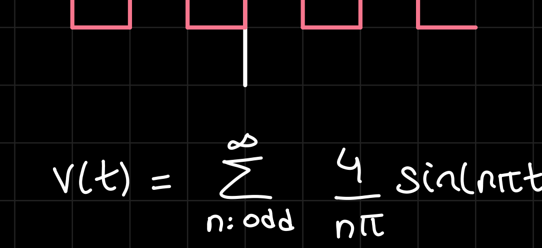
$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

note: periodic signals → Discrete fourier transform
(impulse at every ω_0)

non periodic signals → continuous fourier transform

→ FT for circuit analysis



$$v(t) = \sum_{n:\text{odd}}^{\infty} \frac{4}{n\pi}\sin(n\pi t)$$

$\sin(n\pi t) \rightleftharpoons j\delta(\omega - n\pi)$

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- LAPLACE TRANSFORM : $F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$: works for unstable systems as well and convenient for when we have initial conditions
  ↓ if $\sigma = 0$ i.e. $s = 0 + j\omega = j\omega$
- FOURIER TRANSFORM : $F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega} dt$: convenient for freq. spectrum analysis
  holds only if absolute integrability holds
- FOURIER SERIES : $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$: works for periodic signals only

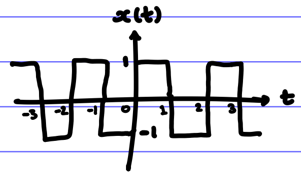
where → $a_0 = \frac{1}{T}\int_0^T f(t)dt$; $a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)dt$; $b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)dt$

OR exponential form : $f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$ where $a_k = \frac{1}{T}\int_0^T f(t) e^{-jk\omega_0 t} dt$

here, a_k can be complex numbers

note: $c_0 = a_0$
$c_n = \frac{1}{2}(a_n - jb_n) \ \forall n > 0$
$c_{-n} = c_n^* = \frac{1}{2}(a_n + jb_n)$
↳ for $c_{-1}, c_{-2}, c_{-3}, \dots$

- odd functions : $f(-t) = -f(t)$: even functions : $f(-t) = f(t)$
  eg: sine function eg: cosine function



$T = 2 \Rightarrow \omega = 2\pi f = \frac{2\pi}{T} = \pi$

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$a_k = \frac{1}{2}\int_0^2 x(t) e^{jk\omega_0 t} = \frac{1}{2}\int_0^1 e^{jk\omega_0 t} - \frac{1}{2}\int_2^1 e^{jk\omega_0 t} = \frac{1}{2}\left[\frac{e^{jk\omega_0 t}}{jk\omega_0}\right]_0^1 - \frac{1}{2}\left[\frac{e^{jk\omega_0 t}}{jk\omega_0}\right]_1^2$

$a_k = \frac{1}{2jk\omega_0}\left[e^{jk\omega_0} - 1 - e^{j2k\omega_0} + e^{jk\omega_0}\right]$ X WRONG APPROACH

try the sin/cos form since we can classify them as even/odd functions and since this is not SNS, we need not fully prove our answer

$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt$

$= \frac{1}{T}\int_{-T/2}^{0} f(t)dt + \frac{1}{T}\int_0^{T/2} f(t)dt = -\frac{1}{T}\int_0^{T/2} f(t)dt + \frac{1}{T}\int_0^{T/2} f(t)dt = 0$

$a_n = \frac{2}{T}\int_0^2 \underbrace{f(t)\cos(n\omega_0 t)}_{} dt$ # note: $f(t)$ is an odd signal

odd x even function → odd function and as we have seen earlier → $\int \text{odd} = 0$
$\Rightarrow a_n = 0$

$b_n = \frac{2}{T}\int_0^2 f(t)\sin(n\omega_0 t)dt = \frac{2}{T}\left\{\int_0^1 \sin(n\omega_0 t)dt - \int_1^2 \sin(n\omega_0 t)dt\right\}$

$= \frac{2}{T}\left\{\left[\frac{-\cos(n\omega_0 t)}{n\omega_0}\right]_0^1 + \left[\frac{\cos(n\omega_0 t)}{n\omega_0}\right]_1^2\right\} = \frac{2}{\pi n T}\left\{-\cos(n\pi) + 1 + \underbrace{\cos(2n\pi)}_{1} - \cos(n\pi)\right\}$ $\because \omega_0 = \pi$

$b_n = \frac{2}{T}\left\{\frac{2 - 2\cos(n\pi)}{n\pi}\right\} = \frac{\cancel{2}}{\cancel{2}}\left\{\frac{2 - 2\cos(n\pi)}{n\pi}\right\} = \begin{cases} \frac{4}{n\pi} & : n \% 2 \,!= 0 \ \{\text{odd}\} \\ \text{zero} & : \text{otherwise} \end{cases}$

So, $f(t) = \sum_{n:\text{odd}}^{\infty} \frac{4}{n\pi}\sin(n\pi t)$ ⇐ ANSWER