tr5frbj 12/08/24

Circuit Theory and Devices
→ tut as well

Lab: LT Spice (2nd week onwards)

book: Engineering Circuit Analysis (9th Edition)

Relative grading

Labs	20%
Quiz (every 2 weeks (n-1))	20%
Mid sem	30%
End sem	30%

Mid sem, End sem ⇐ Scientific calc

Course: 9 modules chapter 10 onwards
{continuation of BE}

* Lecture : 1

⇒ Linear Circuits

x (input) → [system] → y (output)

system is linear if

kx → [sys] → ky

where k is constant

"R" Resistor: linear element ✓
$V = iR$

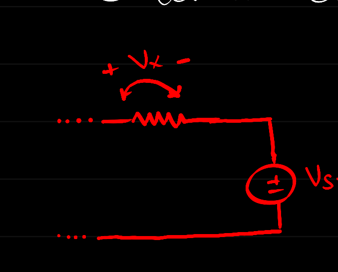
"L" Inductor: linear element ✓
$V = L\frac{di}{dt}$

"C" Capacitor: linear element ✓
$i = C\frac{dv}{dt}$

* Linear Electric circuits:

consists of ⇒

1. R, L, C → linear elements
2. Independent voltage & current sources
3. linear dependent sources



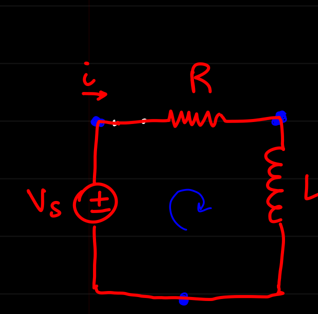
example
would not have been linear if $V_s = kV_x^2$

NOTE: diode and transistors are non-linear elements

* Response of a linear circuit

① full response = transient response + Steady State

transient response	Steady State
temporary	persists $t \to \infty$
due to sudden change	due to source
also called natural response	also called forced response
depends on linear elements R, L, C	depends on current/voltage source



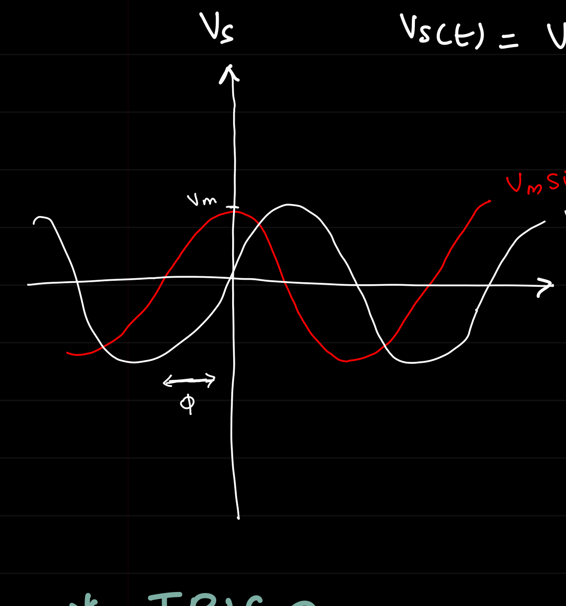
KVL : $-V_s + iR + L\frac{di}{dt} = 0$

$i_{final} = i_{transient} + i_{steady\ state}$

as $t \to \infty$, $i_{final} = i_{steady\ state}$

* Sinusoids & complex numbers

sinusoidally varying voltage source
$V_s(t) = V_m \cos(\omega t)$
$V_s(t) = V_m \sin(\omega t)$
$V_s(t) = V_m \sin(\omega t + \phi)$



② is leading ①
① is lagging ②
by angle ϕ

* TRIGO

$\sin(-\theta) = -\sin(\theta)$
$\cos(-\theta) = \cos(\theta)$
$\sin(A+B) = \sin A \cos B + \cos A \sin B$
$\cos(A+B) = \cos A \cos B - \sin A \sin B$

sin +ve	All +ve
tan +ve	cos +ve

* Complex Numbers

Real : x
complex : $\underbrace{x + iy}_{\text{rectangular form}}$

real part, imaginary part
$i = \sqrt{-1}$ a.k.a. j

⇒ Euler's identity : $\underbrace{e^{j\theta}}_{\text{exponential form}} = \underbrace{\cos\theta + j\sin\theta}_{\text{rectangular form}}$

Complex Number
- rectangular: $x + iy$
- exponential: $re^{j\theta}$ ($\equiv$ polar form : $r\angle\theta$)

$r = \sqrt{x^2 + y^2}$
$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$(r_1 e^{j\theta_1}) \times (r_2 e^{j\theta_2}) = (r_1 \cdot r_2)\, e^{j(\theta_1 + \theta_2)}$

AC	DC
• we can use transformers to vary voltage	• appliances need constant current
• transported easily over large distances	

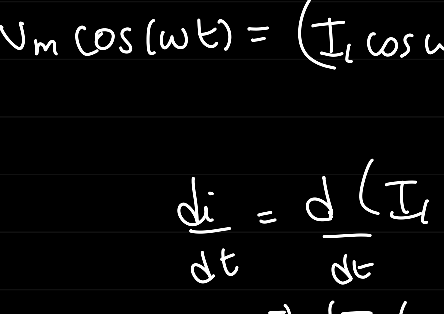
AC → DC : using rectifiers

* MODULE: 1 {CH = 10} 10:30 am

⇒ Sinusoidal steady state analysis

assuming transient has died out long back

Consider,



$V_s(t) = V_m \cos(\omega t)$
find $i(t)$

$V_s(t) = iR + L\frac{di}{dt}$

$\Rightarrow V_m \cos(\omega t) = iR + L\frac{di}{dt}$

both will take cos(ωt) form

General form Solution : $i = I_1 \cos\omega t + I_2 \sin\omega t$

$\Rightarrow V_m \cos(\omega t) = (I_1 \cos\omega t + I_2 \sin\omega t)R + L\frac{di}{dt}$

$\frac{di}{dt} = \frac{d}{dt}(I_1 \cos\omega t + I_2 \sin\omega t)$
$\Rightarrow \omega(I_1(-\sin\omega t) + I_2 \cos\omega t)$
$\Rightarrow \omega(I_2 \cos\omega t - I_1 \sin\omega t)$

$V_m \cos(\omega t) = (I_1 \cos\omega t + I_2 \sin\omega t)R + (I_2 \cos\omega t - I_1 \sin\omega t) L\omega$

$\Rightarrow \cos\omega t \cdot (I_1 R + I_2 L) + \sin\omega t \cdot (I_2 R - I_1 L)$

$\Rightarrow (I_1 R + I_2 L - V_m)\cos(\omega t) + (I_2 R - I_1 L)\sin(\omega t) = 0$

* at $t = 0$,
$(I_1 R + I_2 L - V_m) \cdot 1 + (I_2 R - I_1 L) \cdot 0 = 0$

$$\boxed{I_1 R + I_2 L = V_m}$$

* at $t = \frac{\pi}{2\omega}$, $(I_1 R + I_2 L - V_m) \cdot 0 + (I_2 R - I_1 L) \cdot 1 = 0$

$$\boxed{I_1 = I_2 \cdot \frac{R}{L}}$$

$I_1 R + I_2 L = V_m \Rightarrow I_2 \frac{R^2}{L} + I_2 L = V_m$
$\Rightarrow I_2\left(\frac{R^2 + L}{L}\right) = V_m \Rightarrow I_2 = \frac{V_m L^2}{R^2 + L}$

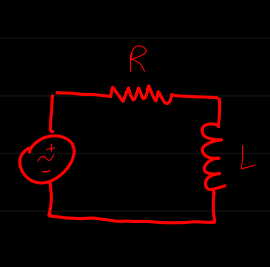
$I_1 = \frac{V_m L R}{R^2 + L}$

14/08/24 **Lecture : 2** 9:30 AM

Wednesday → Quiz : Chp 1, 2, 3

* Sinusoidal Steady State analysis

AC sources ✓ ↳ not considering any transient response



$V_s = V_m \cos(\omega t)$ ← angular freq

$freq\,(hz) = \frac{\omega}{2\pi}$

$$V_s(t) = i(t)R + L\frac{di(t)}{dt}$$

general solⁿ: $i = I_1\cos(\omega t) + I_2\sin(\omega t)$

put i from gen solⁿ in above eqⁿ

$$I_1 = \frac{RV_m}{R^2+\omega^2L^2}, \quad I_2 = \frac{\omega LV_m}{R^2+\omega^2L^2}$$

$$i(t) = \frac{RV_m}{R^2+\omega^2L^2}\cos(\omega t) + \frac{\omega LV_m}{R^2+\omega^2L^2}\sin(\omega t)$$

Note: use $\cos(A+B) = \cos A\cos B - \sin A\sin B$

⇒ So, $i(t) = A\cos(\omega t + \phi)$: general solⁿ
becomes ⇒ $i(t) = A[\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)]$

$$\frac{RV_m}{R^2+\omega^2L^2}\cos(\omega t) + \frac{\omega LV_m}{R^2+\omega^2L^2}\sin(\omega t) = A\cos(\omega t)\cos(\phi) - A\sin(\omega t)\sin(\phi)$$

⇓

$$A\cos(\phi) = \frac{RV_m}{R^2+\omega^2L^2}, \quad A\sin(\phi) = \frac{-\omega LV_m}{R^2+\omega^2L^2}$$

$$A^2\cos^2(\phi) + A^2(\sin^2\phi) = A^2(1) = A^2 = \frac{R^2V_m^2 - \omega^2L^2V_m^2}{R^2+\omega^2L^2}$$

⇓

$$\frac{V_m^2(R^2-\omega^2L^2)}{R^2+\omega^2L^2}$$

$$A = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}$$

$$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

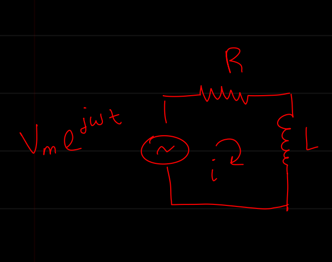
$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

⇒ Consider a complex source

$$V_m\cos(\omega t) + jV_m\sin(\omega t) = V_me^{j\omega t}$$

Real Voltage source $= V_m\cos(\omega t) = Re\{V_me^{j\omega t}\}$

Note: Euler's formula → $e^{j\theta} = \cos\theta + j\sin\theta$



$$V_me^{j\omega t} = iR + L\frac{di}{dt}$$

dont integrate ✗
general solⁿ ✓

Assume $i = I_me^{j(\omega t+\phi)}$
put eqⁿ

$$V_me^{j\omega t} = I_me^{j\omega t+j\phi}R + L(j\omega)I_me^{j\omega t+j\phi}$$

$$\Rightarrow I_me^{j\omega t+j\phi} = \frac{V_me^{j\omega t}}{R+j\omega L}$$

is this the real current? No
current cannot be complex

$$\text{Real } i(t) = Re\{I_me^{j\omega t+j\phi}\} = Re\left\{\frac{V_m}{R+j\omega L}e^{j\omega t}\right\}$$

find the real part of this complex number by simplifying
{multiply by the conjugate ≡ factorize}

$$i(t) = \frac{V_m}{R+j\omega L}\left(\frac{R-j\omega L}{R-j\omega L}\right)e^{j\omega t} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}e^{-j\tan^{-1}(\frac{\omega L}{R})}\cdot e^{j\omega t}$$

⇒ Note: rect form → polar form

$x + jy \rightleftarrows re^{j\theta}$

$x = r\cos\theta, \quad y = r\sin\theta$

$r = \sqrt{x^2+y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$\text{Real } i(t) = Re\left\{\frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cdot e^{j(\omega t - \tan^{-1}(\frac{\omega L}{R}))}\right\}$$

$$* \quad i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

Same answer as when we were not considering complex numbers

* PHASOR

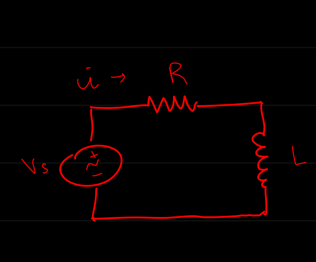
$$i(t) = I_m\cos(\omega t + \phi) = Re\{I_me^{j\omega t+j\phi}\}$$

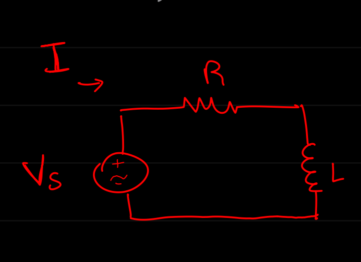
⇓ remove $e^{j\omega t}$, $Re\{\dots\}$

not changing with time
complex func ✗
complex number ✓

$$\underbrace{I_me^{j\phi}}_{\text{phasor}} = I_m\angle\phi$$

time domain	phasor representation
$i(t) = I_m\cos(\omega t + \phi)$	$I_m\angle\phi$
$v(t) = V_m\cos(\omega t + \theta)$	$V_m\angle\theta$
⇒ real function of t	⇒ complex, constant
⇒ notations: small letters	⇒ Bold, capital letters





$i(t) = I_m\cos(\omega t + \phi)$ ✓
$i(t) = Re\{I_me^{j\omega t+j\phi}\}$ ✓
$i(t) = I_m\angle\phi$ ✗ complex constant
$i(t) = I_me^{j(\omega t+\phi)}$ ✗ complex
$i(t) = I_m\angle\omega t+\phi$ ✗ complex phasor notation
phasor notation

but we can write ⇒ phasor notation of $i(t) = I_m\angle\phi = I_m\angle(\omega t + \phi)$

Note: $\angle\phi = e^{j\phi}$ or $\angle\omega t+\phi = e^{j\omega t+j\phi}$

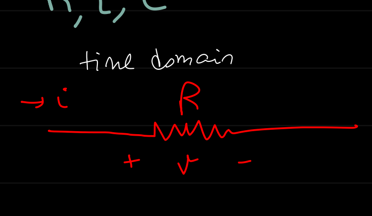
$$\underbrace{V_me^{j\theta}}_{V} = (R + j\omega L)\underbrace{I_me^{j\phi}}_{I}$$

$$\boxed{V = (R + j\omega L)I}$$

↳ phasor notation

* phasor representation of relationship between i and v for R, L, C

① RESISTOR



$v = iR$

consider complex voltage & current

$v = V_me^{j(\omega t+\theta)}$
$i = I_me^{j(\omega t+\phi)}$

$$V_me^{j\omega t+j\theta} = R\cdot I_me^{j\omega t+j\phi}$$

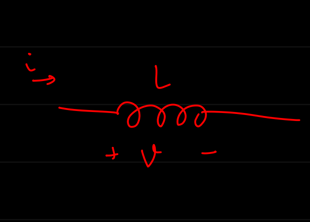
$$V_m\cancel{e^{j\omega t}}\cdot e^{j\theta} = R\cdot I_m\cancel{e^{j\omega t}}e^{j\phi}$$

$$V_me^{j\theta} = R\cdot I_me^{j\phi}$$

$$\underbrace{V_m\angle\theta}_{V} = \underbrace{I_m\angle\phi}_{I}\cdot R$$

$$\boxed{V = IR}$$

② INDUCTOR



$v = L\frac{di}{dt}$

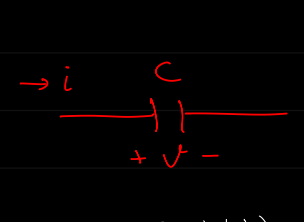
$v \to V_me^{j(\omega t+\theta)}$ $\quad i \to I_me^{j\omega t+j\phi}$

$$V_me^{j\omega t}e^{j\theta} = L\,I_me^{j\omega t}\cdot e^{j\phi}(j\omega)$$

$$\underbrace{V_me^{j\theta}}_{V} = j\omega L\cdot\underbrace{I_me^{j\phi}}_{I}$$

$$\boxed{V = j\omega L\cdot I}$$

③ CAPACITOR



$i = C\frac{dv}{dt}$

$i \to I_me^{j(\omega t+\phi)}$
$v \to V_me^{j(\omega t+\theta)}$

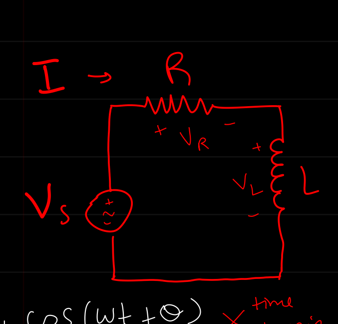
$$I_m\cancel{e^{j\omega t}}\cdot e^{j\phi} = C\cdot V_m\cancel{e^{j\omega t}}e^{j\theta}\cdot j\omega$$

$$\underbrace{I_me^{j\phi}}_{I} = C\cdot j\omega\cdot\underbrace{V_me^{j\theta}}_{V}$$

$$I = j\omega C\cdot V$$

$$\boxed{V = \frac{1}{j\omega C}\cdot I}$$

* PHASOR Representation using KVL



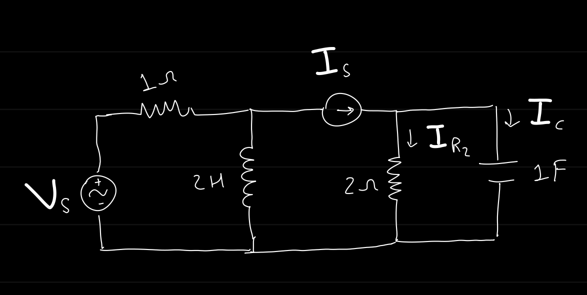
$V_s = V_m\cos(\omega t + \theta)$ ✗ time domain
$V_s = V_me^{j(\omega t+\theta)}$ ✓ phasor notation

$$V_s = V_R + V_L$$
$$= IR + (j\omega L)I$$
$$= I(R + j\omega L)$$

$$I = \frac{V_s}{R+j\omega L} = \frac{V_m\angle\theta}{R+j\omega L}$$

Time domain		Phasor (Frequency domain)
R	$v = iR$	$V = IR$
L	$v = L\frac{di}{dt}$	$V = j\omega L\,I$
C	$i = C\frac{dv}{dt}$	$V = \frac{1}{j\omega C}I$

⇒ Example



① Given: both AC sources, V_s & I_s operate at $\omega = 2$ rad/s

② Given: $I_c = 2\angle 28°$ A

Find (i) V_c
(ii) I_{R_2}
(iii) I_s

① $V_c = \frac{I_c}{j\omega C} = \frac{1}{j(2)(1)}\cdot 2\angle 28°$

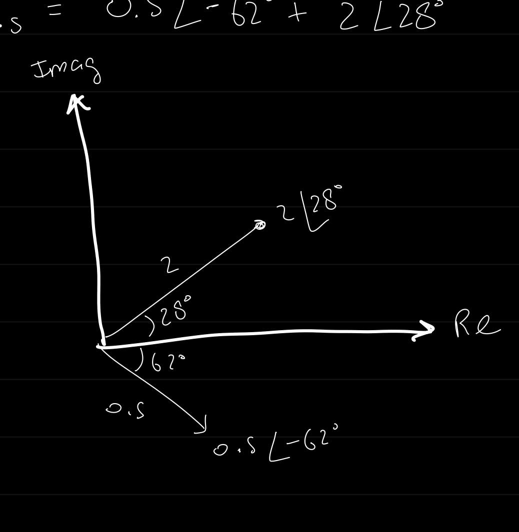
Note: $\frac{1}{j} = -j = -90°$

we add the angles directly which are being multiplied

$$V_c = (0.5\angle -90°)(2\angle 28°) = 1\angle -62°\ V$$

② $I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_{R_2}}{2} = \frac{1\angle -62°}{2} = 0.5\angle -62°$ A

③ $I_s = 0.5\angle -62° + 2\angle 28°$



$2.06\angle 14°$ A