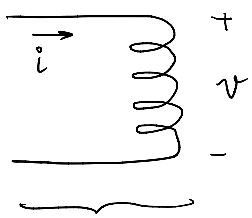
Module 5

Magnetically coupled circuits

(Chapter 13 of textbook)

" Pure linear inductors "
→ no resistance



$v = L\frac{di}{dt}$

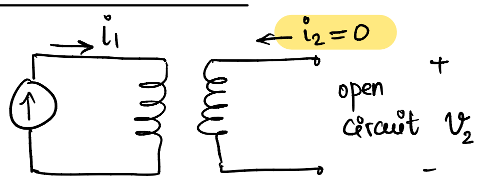
i leads to magnetic flux $\boxed{i \propto \phi}$

Magnetic flux leads to induced v $\boxed{v \propto d\phi/dt}$ $\Rightarrow$ $v \propto di/dt$

$v = L\frac{di}{dt}$

DC → const i → const. mag. flux → $v = 0$

Mutual Inductance :



$v_2 \propto \frac{di_1}{dt}$

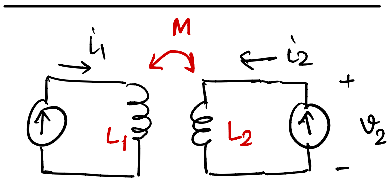
$\Rightarrow v_2 = M\frac{di_1}{dt}$

↳ mutual inductance

Henry

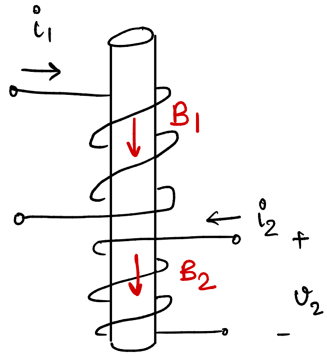
L : self inductance

Now consider : $i_2 \neq 0$



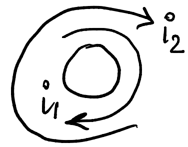
$v_2 \propto \frac{di_1}{dt}$, $v_2 \propto \frac{di_2}{dt}$

$v_2 = \underbrace{L_2\frac{di_2}{dt}}_{\text{Self inductance}} + \underbrace{M\frac{di_1}{dt}}_{\text{Mutual inductance}}$

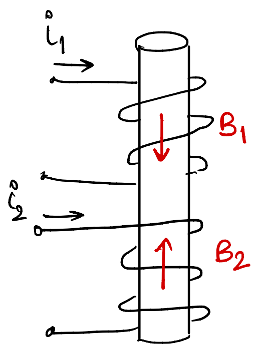


$$\boxed{v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}}$$
$$\underbrace{\phantom{L_2 \frac{di_2}{dt}}}_{B_2} \quad \underbrace{\phantom{M \frac{di_1}{dt}}}_{B_1}$$

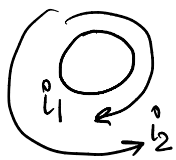
Top view



Additive magnetic flux



Top view

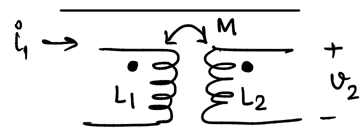


Substractive magnetic flux

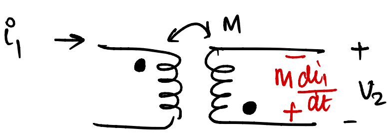
$$v_2 = \frac{L_2 di_2}{dt} - \frac{M di_1}{dt}$$

Dot convention

Assume $i_2 = 0$

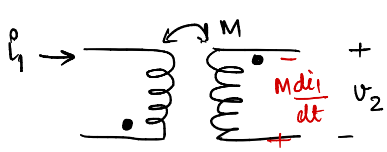


$$v_2 = M \frac{di_1}{dt}$$

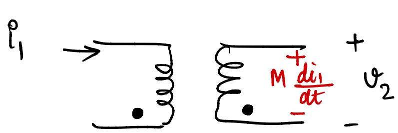


$$v_2 = - M \frac{di_1}{dt}$$

Current entering at dotted terminal
↓ leads to
+ve voltage reference at dotted terminal.



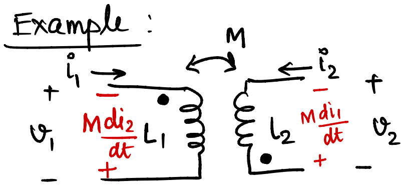
$$v_2 = - M \frac{di_1}{dt}$$



$$v_2 = M \frac{di_1}{dt}$$

Current entering at un dotted terminal
↓ leads to
+ve voltage reference at un dotted terminal.

Example :



$M = 2H$

(A) find v_2 if $i_1 = -8e^{-t}$ A, $i_2 = 0$

$$v_2 = -M\frac{di_1}{dt} + \underbrace{L_2\frac{di_2}{dt}}_{0}$$

$$v_2 = -M\frac{di_1}{dt} = -2\frac{d}{dt}(-8e^{-t}) = -2\times(-8)\times(-1)\,e^{-t}$$

$$= -16\,e^{-t} \text{ V}$$

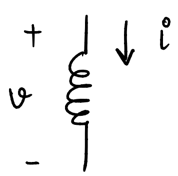
(B) Find v_1 if $i_1 = 0$ and $i_2 = 5\sin 45t$ A.

$$v_1 = \underbrace{L_1\frac{di_1}{dt}}_{0} - M\frac{di_2}{dt}$$

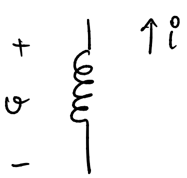
$$v_1 = -M\frac{d}{dt}(5\sin 45t) = -2\times 5\times 45\cos 45t$$

$$= -450\cos 45t \text{ V}$$

Sign Convention for self inductance :



$v = L\frac{di}{dt}$ (current entering from +ve terminal)



$v = -L\frac{di}{dt}$ (current leaving from +ve terminal)

Phasor Representation :

$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

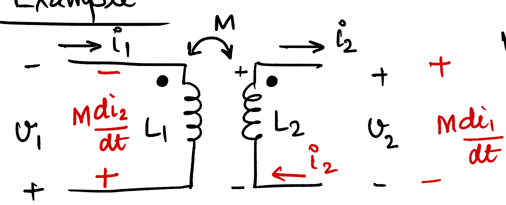
$v = L\frac{di}{dt}$ time domain

$V = j\omega L I_1$ freq. domain

For sinusoidally varying sources

$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$
$$V_2 = j\omega L_2 I_2 + j\omega M I_1$$

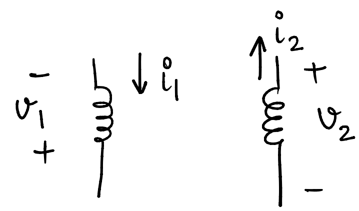
Example



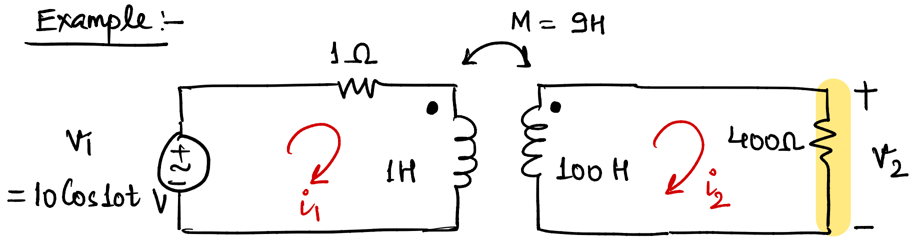
Write phasor equations for V_1 and V_2.

$$V_1 = -j\omega L_1 I_1 + j\omega M I_2$$

$$V_2 = -j\omega L_2 I_2 + j\omega M I_1$$

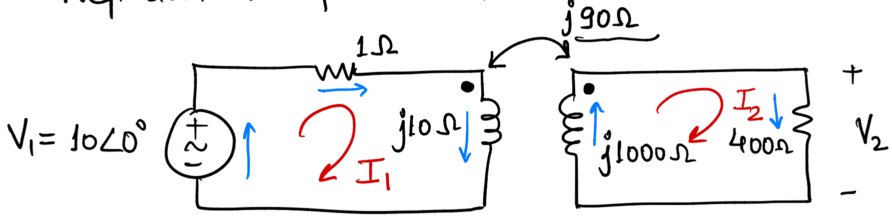


Example :-



Find v_2.

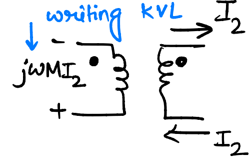
Freq. domain representation



Mesh 1: $-10\angle 0° + I_1(1) + (j10)I_1 - (j90)I_2 = 0$

$$\Rightarrow 10\angle 0° = (1 + j10)I_1 - j90 I_2$$

direction in which writing KVL

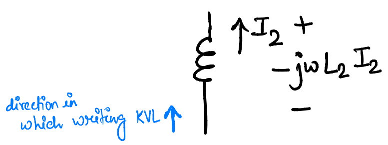


Last term

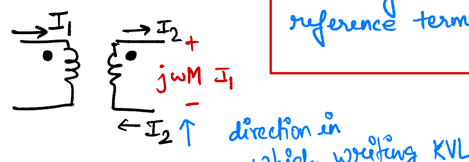
Mesh 2 : $-(-j1000)I_2 - (j90)I_1 + (400)I_2 = 0$

$$\Rightarrow -j90\,I_1 + (400 + j1000)\,I_2 = 0$$

Always remember to put a minus sign if while writing KVL terms you encounter a −ve reference terminal first.



First term



Second term

$$(1+j10)\,I_1 - j90\,I_2 = 10$$
$$(-j90)\,I_1 + (400+j1000)\,I_2 = 0$$

Cramer's rule : $I_1 = \dfrac{D_1}{D_0}$, $I_2 = \dfrac{D_2}{D_0}$

$$D_0 = \begin{vmatrix} 1+j10 & -j90 \\ -j90 & 400+j1000 \end{vmatrix}$$

$$D_1 = \begin{vmatrix} 10 & -j90 \\ 0 & 400+j1000 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} 1+j10 & 10 \\ -j90 & 0 \end{vmatrix}$$

$$V_2 = 400\,I_2$$