Quick Recap

Capacitors (C) $\qquad C = \dfrac{\varepsilon A}{d}$

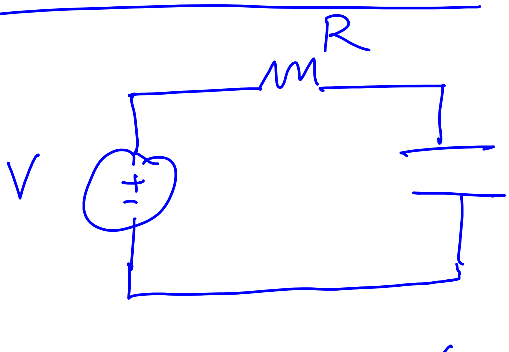
$$E = \underbrace{\int_0^t}_{} P\, dt = \int_0^t V I\, dt = \int_0^t V \cdot C \frac{dV}{dt} \cdot dt$$

Energy $\qquad$ Power

$$= \int_{V(0)}^{V(t)} C V\, dV$$

$$= \frac{1}{2} C V^2(t) - \frac{1}{2} C V^2(0)$$

Charging of a capacitor



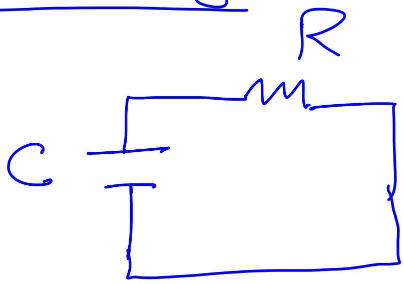
$V_c(0) = 0$

$$V_c(t) = V\left(1 - e^{-t/RC}\right) \qquad \forall\, t \geq 0$$

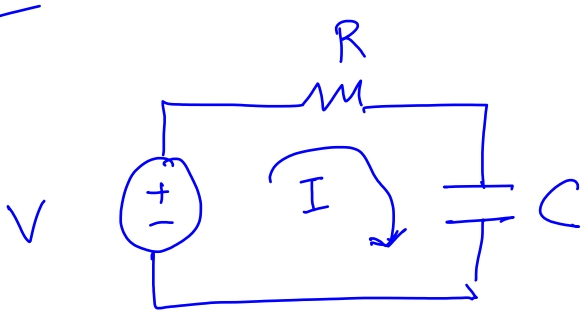
$$\boxed{V_c(\infty) = V}$$

Capacitor discharging

$V_c(0) = V$



$$V_c(t) = V e^{-t/RC} \quad \forall t \geq 0$$



$, \quad V_c(0) = 0$

$$V = \underbrace{IR} + \underbrace{\frac{1}{C}\int I\,dt}$$

$$0 = \frac{dI}{dt} \cdot R + \frac{1}{C} I$$

$$\int_{I(0)}^{I(t)} \frac{dI}{I} = -\frac{1}{RC}\int_0^t dt$$

$$\ln \frac{I(t)}{I(0)} = -\frac{1}{RC} t$$

$$\Rightarrow \quad I(t) = I(0)\, e^{-t/RC}$$

$$V = V_R(t) + V_c(t) \quad \forall t \geq 0$$

$$V = V_R(0) + V_c(0) \Rightarrow V_R(0) = V$$

$$I(t) = \frac{V_R(t)}{R} \qquad \forall\, t \geq 0$$

$$I(0) = \frac{V_R(0)}{R} = \frac{V}{R}$$

$$I(t) = I(0)\, e^{-t/RC}$$

$$= \frac{V}{R}\, e^{-t/RC}$$

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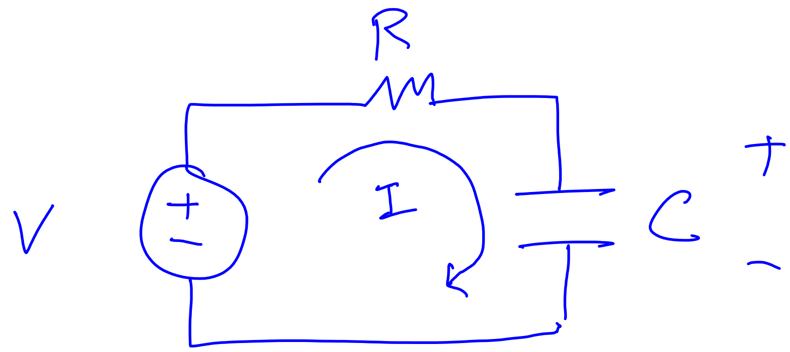


$$V_c(0) = V_0$$

$$V = IR + V_c$$

$$= C\,\frac{dV_c}{dt}\cdot R + V_c$$

$$V - V_c = RC\,\frac{dV_c}{dt}$$

$$\Rightarrow \int_{V_0}^{V_c(t)} \frac{dV_c}{V - V_c} = \int_0^t \frac{dt}{RC}$$

$$-\ln \frac{V - V_c(t)}{V - V_0} = \frac{t}{RC}$$

$$\frac{V - V_c(t)}{V - V_0} = e^{-t/RC}$$

$$V - V_c(t) = V e^{-t/RC} - V_0 e^{-t/RC}$$

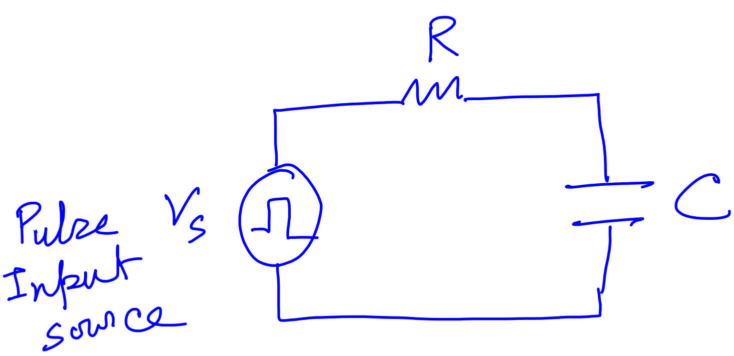
$$V_c(t) = \underbrace{V\left(1 - e^{-t/RC}\right)}_{\text{Forced Response}} + \underbrace{V_0 e^{-t/RC}}_{\text{Natural Response}}$$

Due to the initial conditions

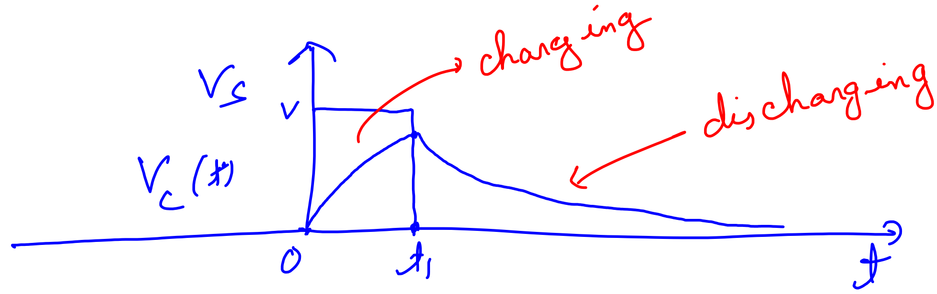
$$= \underbrace{V}_{\text{steady state}} \underbrace{- V e^{-t/RC} + V_0 e^{-t/RC}}_{\text{transient response}}$$

Due to the external source

(*) Superposition principle can also be applied to derive the above.



$$\rightarrow V_s = \begin{cases} V & 0 \le t \le t_1 \\ 0 & t > t_1 \end{cases}$$

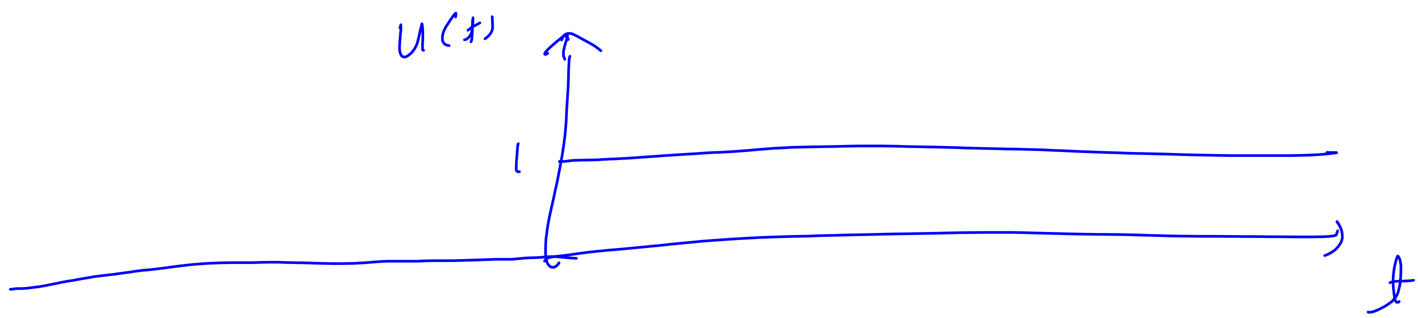


$$V_C(t) = V\left(1 - e^{-t/RC}\right) \qquad 0 \le t \le t_1$$

$$= \underbrace{V\left(1 - e^{-t_1/RC}\right)}_{V_C(t_1)} e^{-(t-t_1)/RC} \qquad t \ge t_1$$

Unit Step Function

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \ge 0 \end{cases}$$



$$V_s(t) = \underbrace{V\,u(t)} - \underbrace{V\,u(t-t_1)}$$

$$V_C(t) = V\left(1 - e^{-t/RC}\right)u(t) - V\left(1 - e^{-(t-t_1)/RC}\right)u(t-t_1).$$

for $0 \le t \le t_1$

$$V_c(t) = V\left(1 - e^{-t/RC}\right)$$

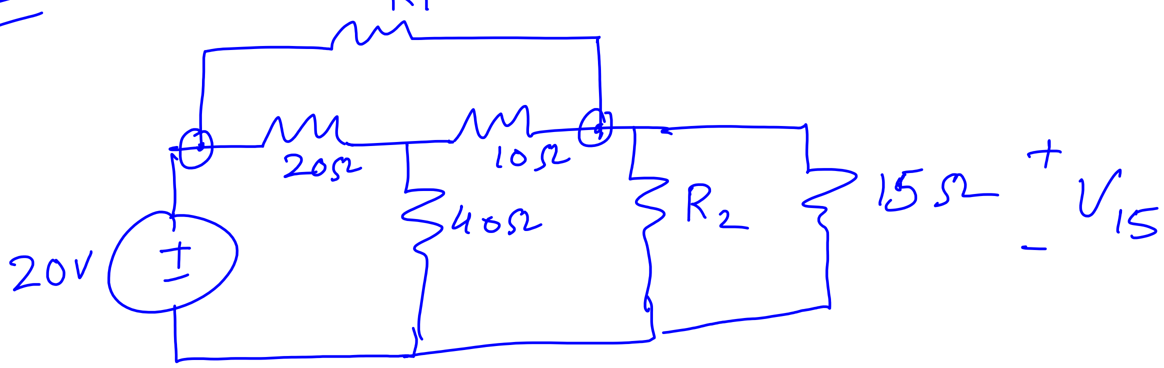
$t \ge t_1$

$$V_c(t) = V\left(1 - e^{-t/RC}\right) - V\left(1 - e^{-(t-t_1)/RC}\right)$$

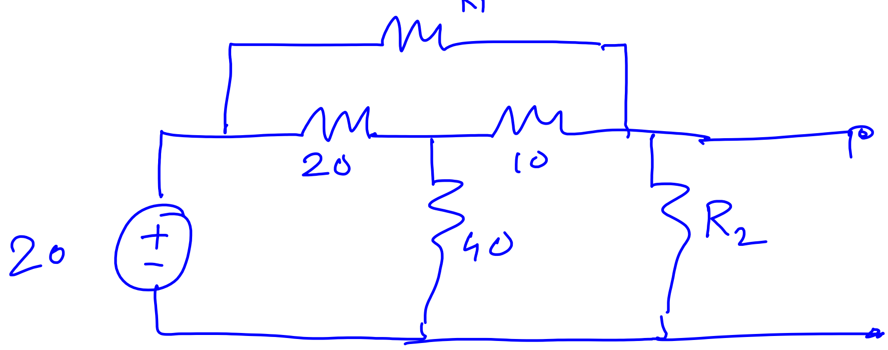
$$= V\left(e^{-(t-t_1)/RC} - e^{-t/RC}\right)$$

$$= V e^{-(t-t_1)/RC}\left(1 - e^{-t_1/RC}\right)$$

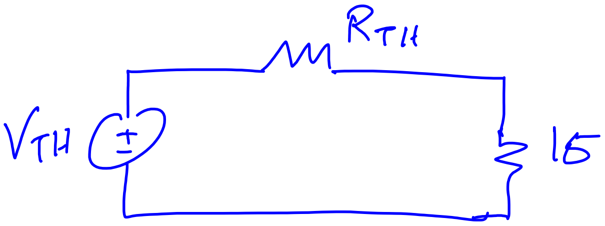
Q.1 (Midsem)



$R_1 = 0 \Rightarrow V_{15} = 20$



(R_2 can be any value, since V_{15} is independent of R_2 when $R_1 = 0$)



$$\Rightarrow P_{15} = \frac{V_{TH}^2}{(R_{TH}+15)^2} \times 15$$