PHYSICS OF SEMICONDUCTOR DEVICES
ASSIGNMENT- 1
Aditya Gautam - 2023043

Q1) $\bar{A} = \left(\dfrac{a\sqrt{3}}{2}\hat{\imath} + \dfrac{a}{2}\hat{\jmath}\right)$, $\bar{B} = \left(-\dfrac{a\sqrt{3}}{2}\hat{\imath} + \dfrac{a}{2}\hat{\jmath}\right)$, $\bar{C} = c\hat{k}$

We know that the volume of primitive unit cell with the primitive cell vector, $\bar{R} = u\bar{a} + v\bar{b} + w\bar{c}$

is $V_c = \bar{a}\cdot(\bar{b}\times\bar{c})$

So, Volume $= \bar{A}\cdot(\bar{B}\times\bar{C})$

$$(\bar{B}\times\bar{C}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -\dfrac{a\sqrt{3}}{2} & \dfrac{a}{2} & 0 \\ 0 & 0 & c \end{vmatrix}$$

$$= \hat{\imath}\left(\dfrac{ac}{2} - 0\right) - \hat{\jmath}\left(-\dfrac{ac\sqrt{3}}{2} - 0\right) + \hat{k}(0 - 0)$$

$$= \left(\dfrac{ac}{2}\right)\hat{\imath} + \left(\dfrac{ac\sqrt{3}}{2}\right)\hat{\jmath}$$

now, $\bar{A} \cdot (\bar{B} \times \bar{C}) = \left(\dfrac{a\sqrt{3}}{2} \cdot \dfrac{ac}{2} \right) + \left(\dfrac{ac\sqrt{3}}{2} \cdot \dfrac{a}{2} \right)$

$\qquad \qquad = 2 \left(\dfrac{a^2 c\sqrt{3}}{4} \right) = \dfrac{\sqrt{3}\, a^2 c}{2}$

Hence, we conclude that the volume of the primitive unit cell is indeed → $\dfrac{\sqrt{3}\, a^2 c}{2}$

Primitive Translational Vectors →

for 3d crystal structure,

$$A^* = \dfrac{2\pi \cdot (B \times C)}{A \cdot (B \times C)}$$

$$\qquad = \dfrac{2\pi \left(\frac{a}{2} \right) \hat{i} + 2\pi \left(\frac{ac\sqrt{3}}{2} \right) \hat{j}}{\sqrt{3}/2 \; a^2 c}$$

$$\qquad = \dfrac{2\pi}{a} \left(\dfrac{1}{\sqrt{3}} \hat{j} + \hat{i} \right)$$

$$B^* = \dfrac{2\pi \cdot (C \times A)}{A \cdot (B \times C)}$$

$$(C \times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & C \\ \frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$= \hat{i}(0 - \frac{ac}{2}) - \hat{j}(-\frac{ac\sqrt{3}}{2})$$

$$= \left(-\frac{ac}{2}\right)\hat{i} + \left(\frac{ac\sqrt{3}}{2}\right)\hat{j}$$

$$B^* = \frac{-\pi ac\, \hat{i} + \pi ac\sqrt{3}\, \hat{j}}{\sqrt{3}/_2 \, a^2 c}$$

$$= -\frac{2\pi}{\sqrt{3}} \cdot \frac{1}{a}\hat{i} + \frac{2\pi}{a}\hat{j}$$

$$= \frac{2\pi}{a}\left(\frac{-1}{\sqrt{3}}\hat{i} + \hat{j}\right)$$

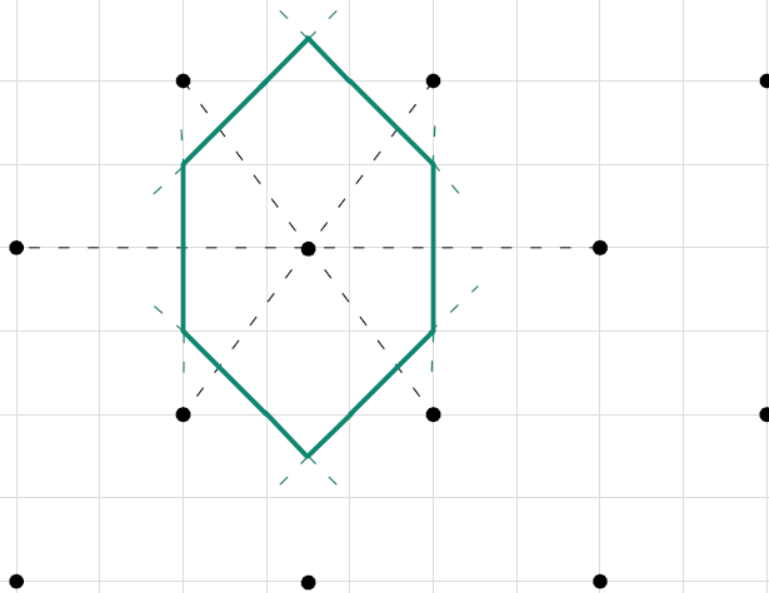$$C^* = \frac{2\pi \cdot (A \times B)}{A \cdot (B \times C)}$$

$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \\ -a\sqrt{3} & \frac{a}{2} & 0 \end{vmatrix}$$

$$= \hat{k} \left(\frac{a^2\sqrt{3}}{4} + \frac{a^2\sqrt{3}}{4} \right)$$

$$= \frac{\sqrt{3}}{2} a^2 \, \hat{k}$$

$$C^* = \frac{\pi\sqrt{3}\,a^2 \, \hat{k}}{\sqrt{3}/2\,a^2 c} = \frac{2\pi}{c} \, \hat{k}$$

Brilluion Zone of the hexagonal
Space lattice

Q2) lattice constant $= 4.3 \times 10^{-10}$ m $= a$

for (3 2 1), miller indices are
$h = 3$; $k = 2$; $l = 1$

we know that the interplanar distance is,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{4.3 \times 10^{-10}}{\sqrt{9 + 4 + 1}}$$

$$= \frac{4.3}{3.74} \mathring{A} \approx 1.15 \mathring{A}$$

we also know that constructive diffraction occurs only if $2d \sin \theta = n\lambda$

for first order reflection, $n = 1$

$$\text{So,} \quad \lambda = 2d \sin \theta = 2 \times 1.15 \times 10^{-10} \times \sin(10°)$$

$$= 2.3 \times 0.1736 \times 10^{-10}$$

$$\approx 0.4 \times 10^{-10} \text{ m}$$

g3)   KE = 0.2 MeV
       V = 20 MeV

note $1 eV = 1.602 \times 10^{-19} J$

$E = 3.2 \times 10^{-14} J$
$V = 3.2 \times 10^{-12} J$
$L = 2.97 \times 10^{-18} m$
$\hbar = 1.054 \times 10^{-34} Js$
$m = 6.68 \times 10^{-27} kg$

We know,

$$T = \frac{1}{1 + \dfrac{V^2 \sinh^2(K'L)}{4E(V-E)}}$$

where $K' = \sqrt{\dfrac{2m(V-E)}{\hbar^2}} = \sqrt{\dfrac{4.23 \times 10^{-38}}{1.11 \times 10^{-68}}}$

$= \sqrt{3.81 \times 10^{30}} = 1.95 \times 10^{15}$

$K'L = 5.8 \times 10^{-3}$

$\sinh^2(K'L) = 3.36 \times 10^{-5}$

$$T = \frac{1}{1 + \frac{3.44 \times 10^{-28}}{4.05 \times 10^{-21}}} = \frac{1}{1 + (0.85 \times 10^{-7})}$$

$$= \boxed{0.99}$$

g4) $\Delta x = 4\overset{\circ}{A} = 4 \times 10^{-10} m$

(a) Heisenberg's uncertainity principle ?

$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{0.527 \times 10^{-34}}{4 \times 10^{-10}}$$

$$\Delta p \geq 0.13175 \times 10^{-24} \simeq 1.32 \times 10^{-25} \, kg \, m \, s^{-1}$$
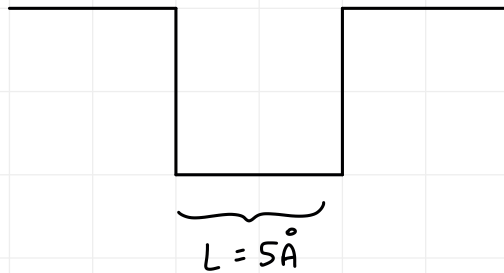
(b) $KE = \frac{p^2}{2m}$

$$\frac{d(KE)}{dp} = \frac{1}{2m} \cdot 2p = \frac{p}{m} \rightarrow \Delta KE = \frac{p}{m} \Delta p$$

$$\Delta KE = \frac{2.4 \times 10^{-23}}{9.1 \times 10^{-31}} \times 1.32 \times 10^{-25}$$

$$= 0.26 \times 10^8 \times 1.32 \times 10^{-25}$$

$$= 3.432 \times 10^{-18} \text{ J}$$

Q5)

$$L = 5\text{Å}$$

(a) for ∞ potential well,

$$E_n = \frac{k^2 \hbar^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

$$E_1 = \frac{\pi^2 \cdot (1.054 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 25 \times 10^{-20}}$$

$$= \frac{\pi^2 \cdot (1.11) \times 10^{-68} \times 10^{51}}{455} = 2.4 \times 10^{-19} \text{ J}$$

$$E_2 = 4 \times 2.4 \times 10^{-19} = 9.6 \times 10^{-19} \text{ J}$$

$$E_3 = 9 \times 2.4 \times 10^{-19} = 21.6 \times 10^{-19} \text{ J}$$

(b) $\Delta E = E_3 - E_2 = \dfrac{hc}{\lambda}$

$\lambda = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{(21.6 - 9.6)\, 10^{-19}} = \dfrac{19.878 \times 10^{-26}}{12 \cdot 10^{-19}}$

$\lambda = 1.656 \times 10^{-7} \text{ m}$