Non cartesian coordinates

→ Spherical coords → r, θ, ϕ

θ → angle with z axis

ϕ → projection's agle with x-axis

$dl_r = dr$, $dl_\theta = r\,d\theta$, $dl_\phi = r\sin\theta\,d\phi$

Gradient:

$$\nabla T = \frac{\partial T}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}}.$$

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}.$$

Curl:

$$\begin{aligned}\nabla\times\mathbf{v} &= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} \\ &+ \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}.\end{aligned}$$

Laplacian:

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}.$$

→ Cylindrical coords → ρ, ϕ, z

$dl_\rho = d\rho$, $dl_\phi = \rho\,d\phi$, $dl_z = dz$

Gradient:

$$\nabla T = \frac{\partial T}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}.$$

Divergence:

$$\nabla\cdot\mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}.$$

Curl:

$$\nabla\times\mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}.$$

Laplacian:

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial\phi^2} + \frac{\partial^2 T}{\partial z^2}.$$

Dirac Delta Function

let $\bar{v} = \frac{1}{r^2}\hat{r}$

$$\bar{\nabla}\cdot\bar{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot \frac{1}{r^2}\right) = 0$$

reason → we used the divergence formula for spherical coordinates
↳ not cartesian.

sphere of radius : R ↖ S

$$\oint_S v \cdot da = \int \frac{1}{R^2}\hat{r} \cdot R^2 \sin\theta \, d\theta d\phi \, \hat{r}$$

$$= \left(\int_0^{\pi} \sin\theta \, d\theta\right)\left(\int_0^{2\pi} d\phi\right)$$

$$= [-\cos\theta]_0^{\pi} \cdot [\phi]_0^{2\pi}$$

$$= (1+1)(2\pi) = \underline{4\pi}$$

now, $\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(r)$

quiz 2 - questions →

Q2) (a) $\int_{-1}^{+1} (r^2+2)\, \bar{\nabla}\cdot\left(\frac{\hat{r}}{r^2}\right) dv$

$$= \int_{-1}^{+1} (r^2+2)\, 4\pi\delta^3(r)\, dv$$

$$= (0+2)(4\pi) = \underline{8\pi}$$

(b) $\int_1^{\infty} (r^2+2)\, \bar{\nabla}\left(\frac{\hat{r}}{r^2}\right) dv$

$$= \int_1^{\infty} (r^2+2)\, \underbrace{4\pi\delta^3(r)}_{\downarrow}\, dv = \underline{0}$$

always = 0
∵ $r=0$ not included
in the integral