$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\oint E \cdot dl = -\frac{\partial}{\partial t} \int B \cdot ds$$

$$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$$

$$\bar{\nabla} \cdot (\bar{\nabla} \times \bar{J}) = \mu_0 \bar{\nabla} \cdot \bar{J} = 0$$

so, $\bar{\nabla} \cdot \bar{J} = 0$

conservation of charge →

$$\bar{\nabla} \cdot \bar{J} = -\frac{\partial \rho}{\partial t}$$

$$\bar{\nabla} \cdot \bar{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\bar{\nabla} \cdot \bar{J} + \bar{\nabla} \cdot \frac{\partial \bar{D}}{\partial t} = 0$$ where $\bar{D} = \epsilon \bar{E}$

$$\nabla \cdot \left(\bar{J} + \frac{\partial \bar{D}}{\partial t} \right) = 0$$

so, $\bar{\nabla} \times \bar{B} = \mu_0 \left(J + \frac{\partial \bar{D}}{\partial t} \right)$

so, we just need a time varying electric field for magnetic field. No need for even a conduction current

Maxwell's modification:

time varying $\bar{E}$ gives rise to $\bar{B}$

Faraday's Law of Induction:

time varying $\bar{B}$ gives rise to $\bar{E}$

remember: Faraday's Law

$$\bar{\nabla} \times \bar{E} = -\frac{d\bar{B}}{dt} \qquad \oint E \cdot dl = -\frac{\partial}{\partial t} \iint_S B \cdot ds$$

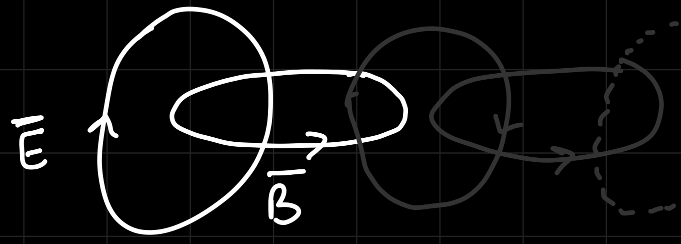
So, yes, $\bar{E}$ can be generated with just a time varying $\bar{B}$ even without charge/conductor.

How will the $\bar{E}$ field lines look like without a conductor/charges

$$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J} + \mu_0 \varepsilon \frac{\partial \bar{E}}{\partial t}$$

$$= \mu_0 (\underbrace{\bar{J}_D}_{\text{displacement current}} + \underbrace{\bar{J}_c}_{\text{conduction current}}) + \mu_0 \varepsilon \frac{\partial \bar{E}}{\partial t}$$

the $\bar{B}$ field lines will always loop and be $\perp$ to $\bar{E}$ field lines



self reproducing $\bar{E}$ and $\bar{B}$ fields from just a single initial $\bar{E}$ or $\bar{B}$ on all sides

These imaginary chains are Electromagnetic waves

Static case (no time variation)

$$\oint E \cdot ds = \frac{q_{enc}}{\varepsilon_0} \qquad \bar{\nabla} \cdot \bar{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint E \cdot dl = 0 \qquad \bar{\nabla} \times \bar{E} = 0 \Leftarrow \bar{E} = -\bar{\nabla}\phi$$

$$\oint \bar{B} \cdot d\bar{s} = 0 \qquad \bar{\nabla} \cdot \bar{B} = 0$$

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 \bar{I}_{enclosed} \qquad \bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$$

Dynamic case

$$\bar{\nabla} \cdot \bar{E} = \frac{\rho(t)}{\varepsilon_0}$$

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{\nabla} \cdot \bar{B} = 0$$

$$\bar{\nabla} \times \bar{B} = \mu_0 J + \mu_0 \left(\frac{\partial \bar{E}}{\partial t}\right) \varepsilon \quad \text{— maxwell}$$

EM Waves

Potential in Electrodynamics

we can still write $\bar{B} = \bar{\nabla} \times \bar{A}$

but not $\bar{E} = -\bar{\nabla}\phi$

from (2) $\rightarrow$ $\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\frac{\partial (\bar{\nabla} \times \bar{A})}{\partial t}$

$$\Rightarrow \nabla \times \left(E + \frac{\partial A}{\partial t}\right) = 0$$

from 2 statics $\rightarrow$ $\nabla \times E = 0$

where $E = -\nabla\phi$

so, $\bar{E} + \frac{d\bar{A}}{dt} = -\nabla\phi$

so, $\bar{E} = -\left(\bar{\nabla}\phi + \frac{d\bar{A}}{dt}\right)$

everything defined till now in dynamics is in time domain. We will be moving to the frequency domain now.

square $\rightleftharpoons$ sinc

if $x(t)$ given as square wave and system's freq response given, we find $y(t)$ by →

$x(t) \rightarrow X(\omega)$ {sinc}

multiply with $H(\omega)$

then IFT to get $y(t)$

This works for a linear system

Time → Freq (maxwell's Equations)

$$\bar{\nabla}\bar{E}(t) = \frac{\rho(t)}{\varepsilon_0}$$

$$\bar{\nabla} \times \bar{E}(t) = -\frac{\partial B(t)}{\partial t}$$

$$\bar{\nabla} \cdot \bar{B}(t) = 0$$

$$\bar{\nabla} \times \bar{B}(t) = \mu_0 \left(J(t) + \frac{\partial D(t)}{\partial t} \right)$$

Just take Fourier Transform

notation : $\bar{E}(t) \rightleftharpoons \bar{E}(\omega)$

$$\bar{\nabla} \cdot \bar{E}(\omega) = \frac{\rho(\omega)}{\varepsilon}$$

$$\bar{\nabla} \times \bar{E}(\omega) = -i\omega \bar{B}(\omega)$$

$$\bar{\nabla} \bar{B}(\omega) = 0$$

$$\bar{\nabla} \times \bar{B}(\omega) = \mu \left(\bar{J}(\omega) + i\omega \varepsilon \bar{E}(\omega) \right)$$

we know $\bar{J} = \sigma \bar{E}$

$$\boxed{\bar{\nabla} \times \bar{B} = \mu \bar{E}(\sigma + i\omega\varepsilon)}$$

In a source free region →

Time
$$\begin{cases} \bar{\nabla} \cdot \bar{E} = 0 & \bar{\nabla} \cdot \bar{B} = 0 \\ \bar{\nabla} \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t} & \bar{\nabla} \times \bar{B} = \mu \varepsilon \dfrac{\partial \bar{E}}{\partial t} \end{cases}$$

freq
$$\begin{cases} \bar{\nabla} \cdot \bar{E} = 0 & \bar{\nabla} \cdot \bar{B} = 0 \\ \bar{\nabla} \times \bar{E} = -i\omega \bar{B} & \bar{\nabla} \times \bar{B} = -i\omega \mu \varepsilon \bar{E} \end{cases}$$

wave eqn in 1d

↳ $f(x-vt)$

$$\frac{\partial^2 f}{\partial x^2} = f''(x-vt) = \frac{\partial^2 f(x-vt)}{\partial (x-vt)^2}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f(x-vt)}{\partial (x-vt)} \cdot \frac{\partial (x-vt)}{\partial t}$$

$$= f'(x-vt) \times (-v)$$

$$\frac{\partial^2 f}{\partial t^2} = v^2 f''(x-vt)$$

so, $$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

for 3d →

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

$$\Rightarrow \quad \nabla^2 \bar{f} = \frac{1}{v^2} \frac{\partial^2 \bar{f}}{\partial t^2}$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times B)$$

$$= \frac{-\partial}{\partial t}\left(\mu \varepsilon \frac{\partial \bar{E}}{\partial t}\right)$$

considering source free region

$$\bar{\nabla} \cdot \cancel{(\bar{\nabla} \cdot \bar{E})}^{0} - \bar{\nabla}^2 \bar{E} = -\mu \varepsilon \left(\frac{\partial^2 \bar{E}}{\partial t^2}\right)$$

$$\Rightarrow \quad \nabla^2 E = \mu \varepsilon \left(\frac{\partial^2 \bar{E}}{\partial t^2}\right)$$

So, the velocity $\Rightarrow \frac{1}{v^2} = \mu \varepsilon$

$$v = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$\bar{\nabla} \times \bar{B} = \mu\varepsilon \frac{\partial \bar{E}}{\partial t}$$

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{B}) = \mu\varepsilon \frac{\partial}{\partial t} (\nabla \times \bar{E})$$

$$\cancel{\bar{\nabla}(\bar{\nabla} \cdot \bar{B})}^{\,0} - \nabla^2 B = -\mu\varepsilon \frac{\partial^2 \bar{B}}{\partial t^2}$$

$$\nabla^2 B = \mu\varepsilon \frac{\partial^2 B}{\partial t^2}$$

velocity: $\frac{1}{\sqrt{\mu\varepsilon}}$

no material involved
this wave still propagates
even through vacuum
we just need time varying
magnetic field and electric field
for EM waves

$$\nabla^2 E = \mu\varepsilon\left(\frac{\partial^2 \bar{E}}{\partial t^2}\right)$$

$$\nabla^2 B = \mu\varepsilon \frac{\partial^2 B}{\partial t^2}$$

in frequency domain:

$$\bar{\nabla}^2 \bar{E} = \mu\varepsilon(-\omega^2 \bar{E})$$

$$\nabla^2 \bar{E} + (\mu\varepsilon\omega^2)\bar{E} = 0$$

let $K = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v}$ or $\frac{\omega}{c}$ velocity

$$\nabla^2 \bar{E} + K^2 \bar{E} = 0 \longrightarrow \text{Helmholtz eq}^n$$