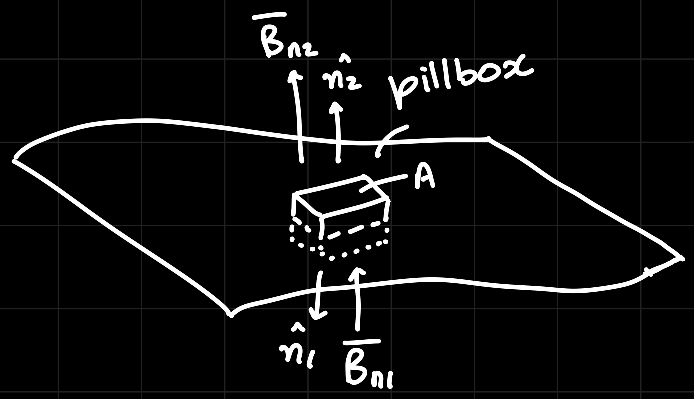
Boundary Conditions (magnetostatics)



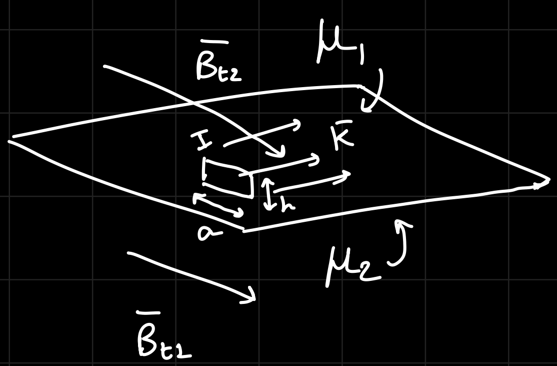
flux through sides = 0

because area of sides → 0

Gauss's Law: $\oint \bar{B}\cdot d\bar{s} = 0$

$$B_{n2}A - B_{n1}A = 0$$

$$B_{n1} = B_{n2}$$ normal component: equal



$\bar{I}_{enc} = \bar{K}a$

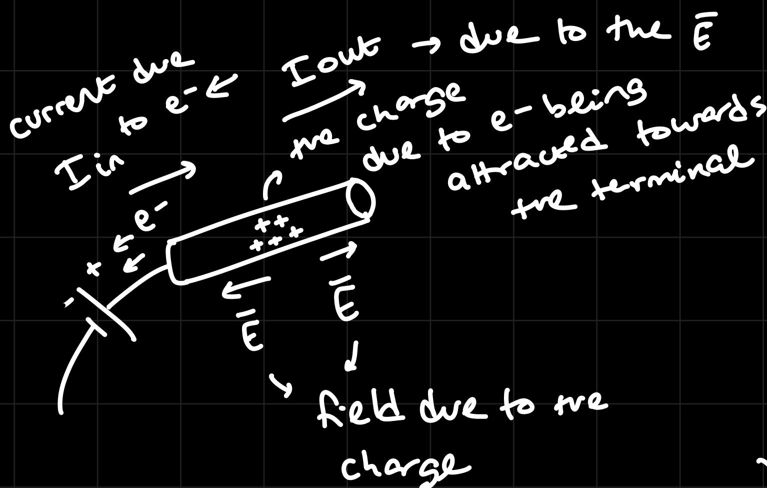
line current density

Ampere's Law ⇒ $\oint \bar{H}\cdot d\bar{l} = I_{enc} = \bar{K}a \Rightarrow H_{t1}a - H_{t2}a = K\bar{a}$

$$H_{t1} - H_{t2} = K$$ tangential component

* Electrodynamics

EMF : Voltage across a cell without the internal resistance.



This process is repeated throughout

I_{in} is stopped and
I_{out} is allowed due to $\bar{E}$

$$\bar{f} = \bar{f_s} + \bar{E} \rightarrow \text{drives the current}$$

total force ($\bar{f}$)

$\bar{f_s}$: effect of this force is confined to the close vicinity of the source

taking closed loop integral

$$\oint \bar{f}\cdot d\bar{l} = \oint \bar{f_s}\cdot d\bar{l} + \underbrace{\oint \bar{E}\cdot d\bar{l}}_{0}$$

assumption: electrostatic field

because DC source

for a circuit with no resistance, we dont need any force to push the current

So, for zero resistance,

$$\bar{f}_s + \bar{E} = 0$$

$$\Rightarrow \oint f_s \cdot dl = -\oint E \cdot dl = 0$$

but for a non closed loop

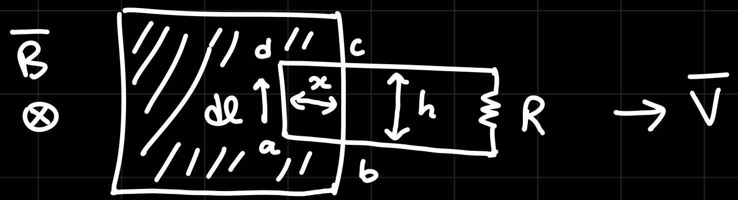
$$\underbrace{\int_A^B \bar{f}_s \cdot dl}_{\text{EMF } (\mathcal{E})} = -\int_A^B E \cdot dl = V_{BA}$$

So, EMF is the voltage when there is no internal resistance

now, we can also say

$$\oint \bar{F}_s \cdot d\bar{l} = \underset{\text{closed loop EMF}}{\mathcal{E}MF} = 0$$

→ Proving Faraday's Law using an example



$$\bar{f}_{mag} = q\bar{v}B = vB$$

$$\oint \bar{f}_{mag} \cdot d\bar{l} = vBh = EMF$$

assuming $q=1$

note: only ad contribute to the closed loop integral because ab and cd have $\bar{B} \perp dl$ and so dot product $= 0$ and rest of the ckt is considered outside the magnetic field $\bar{B}$

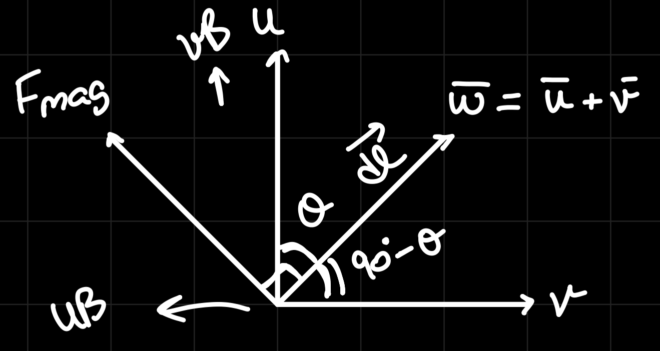
magnetic force doesn't do any work
current will flow through the resistance and there is EMF induced

but f_{mag} can't do any work

fmag is not the source of the EMF then

the current flowing through R is not due to fmag but due to the ckt being pulled to right with velocity $\bar{v}$

let the force due to the motion be equal to $\vec{f}_{pull}$



$\otimes \bar{B}$

fmag would now be $\perp$ to $\bar{w}$ (motion)

$$\bar{f}_{mag} = (\bar{w} \times \bar{B}) \quad \text{assuming } q = 1$$

$$= (\bar{u} + \bar{v}) \times \bar{B}$$

$$\text{work done} = \int \bar{f}_{pull} \cdot d\bar{l}$$

$$= \int uB \sin\theta \, dl$$

$$= uB \sin\theta \frac{h}{\cos\theta}$$

$$= u \tan\theta \, Bh = vBh = \underline{EMF}$$

$$\Phi = B \overbrace{xh}^{\text{area}} \leftarrow \int \bar{B} \cdot d\bar{s}$$

rate of decrease of flux $\rightarrow \dfrac{d\Phi}{dt} = B \dfrac{dx}{dt} h = Bvh = EMF$

(x is decreasing) $\rightarrow$ rate of change $= -EMF$

$$\oint \bar{E}\cdot d\bar{l} = \frac{-d\Phi}{dt} = \frac{-d}{dt}\left(\int B\cdot ds\right) = -\int \frac{\partial B}{\partial t}\cdot ds$$

if B is not varying with time $\rightarrow \oint \bar{E}\cdot d\bar{l} = 0$

electrostatics

$$\oint E\cdot dl = \int (\nabla\times E)\, ds = -\int \frac{\partial B}{\partial t}\, ds$$

$$\boxed{\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}}$$

So, Charge in magnetic field induces and electric field

$+(\ldots)$ for electrodynamics

$\bar{\nabla}\times\bar{B} = \mu_0 J$

So, $\bar{\nabla}\cdot\bar{J} = 0$ (div of curl)

} only valid in case of -statics

↳ electro/magneto

but $\bar{\nabla}\cdot\bar{J} = -\frac{\partial \rho}{\partial t} \rightarrow$ conservation of charge

now we need a modified ampere's Law for electrodynamics