

PHYSICS OF SEMICONDUCTOR DEVICES

ASSIGNMENT - 1

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$$Q1) \quad \bar{A} = \left(\frac{a\sqrt{3}}{2} \hat{i} + \frac{a}{2} \hat{j} \right), \quad \bar{B} = \left(-\frac{a\sqrt{3}}{2} \hat{i} + \frac{a}{2} \hat{j} \right), \quad \bar{C} = c \hat{k}$$

We know that the volume of primitive unit cell with the primitive cell vector, $\bar{R} = u\bar{a} + v\bar{b} + w\bar{c}$ is $V_c = \bar{a} \cdot (\bar{b} \times \bar{c})$

$$\text{So, volume} = \bar{A} \cdot (\bar{B} \times \bar{C})$$

$$(\bar{B} \times \bar{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \\ 0 & 0 & c \end{vmatrix}$$

$$= \hat{i} \left(\frac{ac}{2} - 0 \right) - \hat{j} \left(-\frac{ac\sqrt{3}}{2} - 0 \right) + \hat{k} (0 - 0)$$

$$= \left(\frac{ac}{2} \right) \hat{i} + \left(\frac{ac\sqrt{3}}{2} \right) \hat{j}$$

$$\begin{aligned} \text{now, } \vec{A} \cdot (\vec{B} \times \vec{C}) &= \left(\frac{a\sqrt{3}}{2} \cdot \frac{ac}{2} \right) + \left(\frac{ac\sqrt{3}}{2} \cdot \frac{a}{2} \right) \\ &= 2 \left(\frac{a^2 c \sqrt{3}}{4} \right) = \frac{\sqrt{3}}{2} a^2 c \end{aligned}$$

Hence, we conclude that the volume of the primitive unit cell is indeed $\rightarrow \frac{\sqrt{3}}{2} a^2 c$

Primitive Translational Vectors \rightarrow

for 3d crystal structure,

$$\begin{aligned} \vec{A}^* &= \frac{2\pi \cdot (\vec{B} \times \vec{C})}{\vec{A} \cdot (\vec{B} \times \vec{C})} \\ &= \frac{2\pi (a/2) \hat{i} + 2\pi (ac\sqrt{3}/2) \hat{j}}{\sqrt{3}/2 a^2 c} \end{aligned}$$

$$= \frac{2\pi}{a} \left(\frac{1}{\sqrt{3}} \hat{j} + \hat{i} \right)$$

$$\vec{B}^* = \frac{2\pi \cdot (\vec{C} \times \vec{A})}{\vec{A} \cdot (\vec{B} \times \vec{C})}$$

$$(C \times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & c \\ \frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$= \hat{i}(0 - \frac{ac}{2}) - \hat{j}(-\frac{ac\sqrt{3}}{2})$$

$$= (-\frac{ac}{2})\hat{i} + (\frac{ac\sqrt{3}}{2})\hat{j}$$

$$B^* = \frac{-\pi ac \hat{i} + \pi ac\sqrt{3} \hat{j}}{\sqrt{3}/2 a^2 c}$$

$$= \frac{-2\pi}{\sqrt{3}} \cdot \frac{1}{a} \hat{i} + \frac{2\pi}{a} \hat{j}$$

$$= \frac{2\pi}{a} \left(-\frac{1}{\sqrt{3}} \hat{i} + \hat{j} \right)$$

$$C^* = \frac{2\pi \cdot (A \times B)}{A \cdot (B \times C)}$$

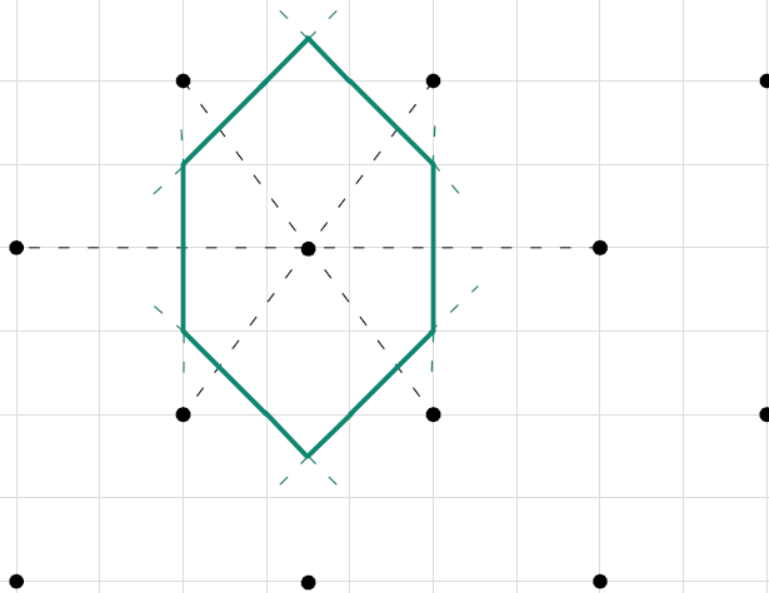
$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \\ -\frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$= \hat{k} \left(\frac{a^2 \sqrt{3}}{4} + \frac{a^2 \sqrt{3}}{4} \right)$$

$$= \frac{\sqrt{3}}{2} a^2 \hat{k}$$

$$C^* = \frac{\pi \sqrt{3} a^2 \hat{k}}{\sqrt{3} \frac{1}{2} a^2 c} = \frac{2\pi}{c} \hat{k}$$

Brilluion zone of the hexagonal space lattice



Q2) lattice constant = $4.3 \times 10^{-10} \text{ m} = a$

for (3 2 1), miller indices are
 $h=3$; $k=2$; $l=1$

we know that the interplanar distance is,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{4.3 \times 10^{-10}}{\sqrt{9 + 4 + 1}}$$

$$= \frac{4.3}{3.74} \text{ \AA} \approx \underline{1.15 \text{ \AA}}$$

we also know that constructive diffraction occurs only if $2d \sin \theta = n\lambda$

for first order reflection, $n = 1$

$$\begin{aligned} \text{So, } \lambda &= 2d \sin \theta = 2 \times 1.15 \times 10^{-10} \times \sin(10^\circ) \\ &= 2.3 \times 0.1736 \times 10^{-10} \\ &\approx 0.4 \times 10^{-10} \text{ m} \end{aligned}$$

Q3)

$$KE = 0.2 \text{ MeV}$$

$$V = 20 \text{ MeV}$$

$$\text{note } 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$E = 3.2 \times 10^{-14} \text{ J}$$

$$V = 3.2 \times 10^{-12} \text{ J}$$

$$L = 2.97 \times 10^{-18} \text{ m}$$

$$\hbar = 1.054 \times 10^{-34} \text{ Js}$$

$$m = 9.11 \times 10^{-31} \text{ kg}$$

We know,

$$T = \frac{1}{1 + \frac{V^2 \sinh^2(k'L)}{4E(V-E)}}$$

$$\text{where } k' = \sqrt{\frac{2m(V-E)}{\hbar^2}} = \sqrt{\frac{4.23 \times 10^{-38}}{1.11 \times 10^{-68}}}$$

$$= \sqrt{3.81 \times 10^{30}} = 1.95 \times 10^{15}$$

$$k'L = 5.8 \times 10^{-3}$$

$$\sinh^2(k'L) = 3.36 \times 10^{-5}$$

$$T = \frac{1}{1 + \frac{3.44 \times 10^{-28}}{4.05 \times 10^{-21}}} = \frac{1}{1 + (0.85 \times 10^{-7})}$$

$$= \boxed{0.99}$$

this is incorrect since we cannot get ~ 1 tunnelling probability. Check last page ↓

Q4) $\Delta x = 4 \text{ \AA} = 4 \times 10^{-10} \text{ m}$

(a) Heisenberg's uncertainty principle ↴

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{0.527 \times 10^{-34}}{4 \times 10^{-10}}$$

$$\Delta p \geq 0.13175 \times 10^{-24} \simeq 1.32 \times 10^{-25} \text{ kg m s}^{-1}$$

(b) $KE = \frac{p^2}{2m}$

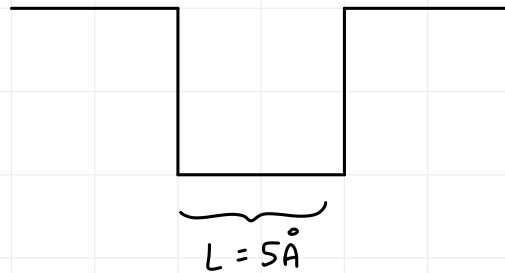
$$\frac{d(KE)}{dp} = \frac{1}{2m} \cdot 2p = \frac{p}{m} \rightarrow \Delta KE = \frac{p}{m} \Delta p$$

$$\Delta KE = \frac{2.4 \times 10^{-23}}{9.1 \times 10^{-31}} \times 1.32 \times 10^{-25}$$

$$= 0.26 \times 10^8 \times 1.32 \times 10^{-25}$$

$$= 3.432 \times 10^{-18} \text{ J}$$

Q5)



(a) for ∞ potential well,

$$E_n = \frac{k^2 \hbar^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

$$E_1 = \frac{\pi^2 \cdot (1.054 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 25 \times 10^{-20}}$$

$$= \frac{\pi^2 \cdot (1.11) \times 10^{-68} \times 10^{51}}{455} = 2.4 \times 10^{-19} \text{ J}$$

$$E_2 = 4 \times 2.4 \times 10^{-19} = 9.6 \times 10^{-19} \text{ J}$$

$$E_3 = 9 \times 2.4 \times 10^{-19} = 21.6 \times 10^{-19} \text{ J}$$

$$(b) \quad \Delta E = E_3 - E_2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{(21.6 - 9.6) 10^{-19}} = \frac{19.878 \times 10^{-26}}{12 \cdot 10^{-19}}$$

$$\lambda = \underline{1.656 \times 10^{-7} \text{ m}}$$

Q3) note: $1\text{eV} = 1.602 \times 10^{-19} \text{ J}$

$$E = 3.2 \times 10^{-14} \text{ J}$$

$$V = 3.2 \times 10^{-12} \text{ J}$$

$$L = 2.97 \times 10^{-18} \text{ m}$$

$$\hbar = 1.054 \times 10^{-34} \text{ Js}$$

$$m = 6.68 \times 10^{-27} \text{ kg}$$

Since $V \gg E \rightarrow$

$$T = 16 \left(\frac{E}{V} \right) \left(1 - \frac{E}{V} \right) e^{-2KL}$$

(according to the book)

where $K = \sqrt{\frac{2m(V-E)}{\hbar^2}}$

$$K = 1.95 \times 10^{15}$$

$$T = 16 \left(\frac{1}{100} \right) \left(1 - \frac{1}{100} \right) e^{-2(1.95 \times 10^{15})(2.97 \times 10^{-18})}$$

$$T = (0.16)(0.99) e^{-11.583 \times 10^{-3}}$$

$$T = (0.16)(0.99)(0.988) = \underline{0.1565}$$