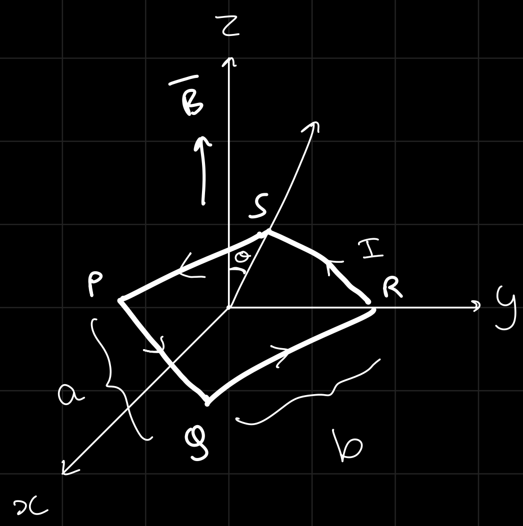
Magnetic field in a matter

note: $\bar{\nabla}\cdot\bar{B} = 0 \rightarrow \int(\nabla\cdot B)\,dv = \oint B\,ds = 0$

Gauss Law

Diamagnets
Paramagnets
Ferromagnets

assuming a rectangular closed loop



θ: angle of plane's normal with z-axis

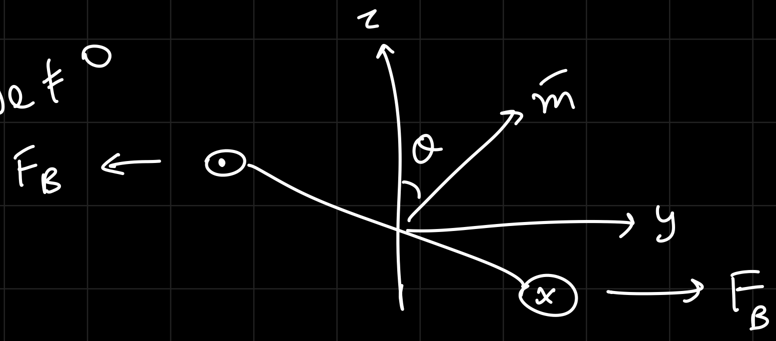
$\bar{F}_B = I(\bar{l} \times \bar{B})$

from lorentz Law

$\bar{F}_B = q(\bar{v} \times \bar{B})$

The PQ and RS sides' force will cancel out but the force due to the other two sides will make the rectangle rotate

net Force = 0
but net torque ≠ 0



at equilibrium
$\tau = 0 \Rightarrow \bar{m}$ aligns paralleles with $\bar{B}$

$$\bar{\tau} = \bar{F}.a\sin\theta\,\hat{x}$$
$$= IbBa\sin\theta\,\hat{x}$$
$$= IAB\sin\theta\,\hat{x}$$

area

$IA = \bar{m}$: magnetic moment

$$= |\bar{m}||\bar{B}|\sin\theta\,\hat{x} \Rightarrow \boxed{\tau = \bar{m}\times\bar{B}}$$

assuming the loop is kept in a uniform field

In case of dipoles →

{ constant thermal fluctuation trows dipole out of equilibrium }

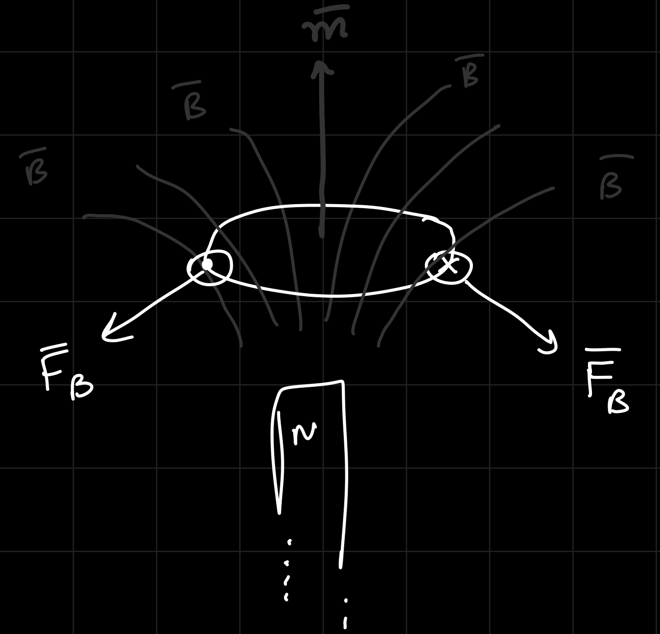
Similar reason for "out of equilibrium" state in this case as well requirements:

liquid (not gas)
low temperature

or else these small loops wont respond to the magnetic field

non-uniform magnetic field $\equiv$ $\bar{B}$ varying in space

eg:



Diamagnetic material
↳ tries to align $\bar{m}$ in the opposite direction to $\bar{B}$

Paramagnetism
↳ $\bar{m}$ and $\bar{B}$ align in the same direction

\$ cannot use gas because of not enough density for it to exhibit enough magnetization

hence we go for the next lightest type of materials → liquid

magnetic susceptibility
↳ determines para/ferro/diamagnetism

liquid nitrogen → diamagnetic
doesn't hold

liquid oxygen → paramagnetic
sticks b/w poles because of very high magnetic susceptibility

aluminium → paramagnetic
but doesn't hold because gravitational pull is greater because of high mass (solid)

$\bar{E}$ arises due to q

$\bar{B}$ arises due to I

$\bar{B} = \bar{\nabla} \times \bar{A}$

↳ magnetic vector potential

$\bar{A}$ and $\bar{V}$ vary with $\frac{1}{r}$ but for dipole → $\frac{1}{r^2}$

$\bar{A}$ ↓ magnetic vec potential

$\bar{V}$ ↓ electrostatic potential

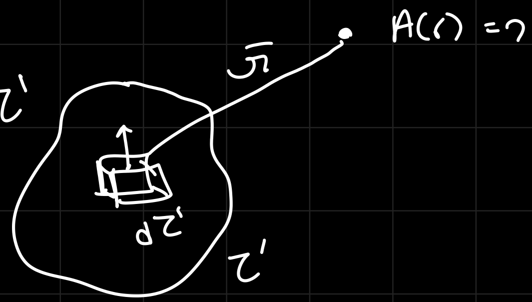
magnetic dipole moment

$$\vec{A}(r) = \frac{\mu_0}{4\pi} \frac{\bar{m} \times \hat{r}}{r^2}$$

just like $\bar{P}$: polarization,
we have $\bar{M}$: magnetization
($\bar{m}$ / volume)

$$\bar{A}(r) = \frac{\mu_0}{4\pi} \int_V \frac{\bar{m} \times \hat{r}}{r^2} d\tau'$$ due to continuous material

$$= \frac{\mu_0}{4\pi} \int_V \left[\bar{m} \times \bar{\nabla}' \left(\frac{1}{r} \right) \right] d\tau'$$



$$(\bar{\nabla}' \times f\bar{p}) = f(\bar{\nabla} \times \bar{P}) - \bar{P} \times (\bar{\nabla}' f)$$

$$\Rightarrow \quad \bar{P} \times \bar{\nabla}' f = f(\bar{\nabla}' \times \bar{P}) - \bar{\nabla}(f\bar{P})$$

$\bar{P} = \bar{m}$, $f = \frac{1}{r}$

$$\bar{A}(r) = \frac{\mu_0}{4\pi} \int \left[\frac{1}{\mathscr{r}} (\bar{\nabla}' \times \bar{m}) - \bar{\nabla} \times \left(\frac{\bar{m}}{\mathscr{r}} \right) \right] d\tau'$$

remember →

$$\bar{J}_b (\mathscr{r}') = \bar{\nabla} \times \bar{m}(r')$$

↳ bound volume current density

Gauss's divergence theorem: $\int \bar{\nabla} \bar{P} \, dV = \oint P \cdot ds$

if $\bar{P} = \bar{V} \times \bar{C}$
where $\bar{V}$ varies with space and
$\bar{C}$ doesnt varies with space
then $\therefore$ $\bar{P}$ varies with space

$$\int_V \bar{\nabla} \cdot (\bar{V} \times \bar{C}) \, dv = \oint_S (\bar{V} \times \bar{C}) \, ds$$

$$\int \bar{C} \cdot (\bar{\nabla} \times \bar{V}) - \bar{V} \cdot \underbrace{(\bar{\nabla} \times \bar{C})}_{\text{zero because C doesnt vary with space}} dv = \oint \bar{C} \cdot (d\bar{S} \times \bar{V})$$

$$\int C \cdot (\nabla \times V) \, dv = - \oint \bar{C} \cdot (\bar{V} \times d\bar{S})$$

$$\int C \cdot (\bar{\nabla} \times V) \, dV = -C \oint_S (\bar{V} \times d\bar{S})$$

$$\int (\bar{\nabla} \times \bar{V})\, dV = -\oint (\bar{V} \times d\bar{s})$$

$$\frac{\mu_0}{4\pi} \int \frac{\bar{J}(r')\, d\tau'}{\mathscr{r}}$$

$$= \frac{\mu_0}{4\pi} \int \left(\frac{\bar{\nabla} \times \bar{m}}{\mathscr{r}} \right) d\tau'$$

$$= -\frac{\mu_0}{4\pi} \oint \left(\frac{\bar{m}}{\mathscr{r}} \right) ds$$

$$\bar{A}(r) = \frac{\mu_0}{4\pi} \int \left[\frac{1}{\mathscr{r}} (\bar{\nabla}' \times \bar{m}) - \bar{\nabla} \times \left(\frac{\bar{m}}{\mathscr{r}} \right) \right] d\tau'$$

$$= \frac{\mu_0}{4\pi} \int \frac{\overbrace{J_b(\mathscr{r}')}^{\bar{\nabla}' \times \bar{m}(r')}\, d\tau'}{\mathscr{r}} + \frac{\mu_0}{4\pi} \oint \frac{\overbrace{K_b(r')}^{\bar{m} \times \hat{n}}}{\mathscr{r}}\, ds$$

volume current density

surface current density

$$\bar{A}(\mathscr{r}) = \frac{\mu_0}{4\pi} \int \frac{J_b(\mathscr{r}')\, d\tau'}{\mathscr{r}} + \frac{\mu_0}{4\pi} \oint \frac{K_b(\mathscr{r}')}{\mathscr{r}}\, ds$$