Lec (Revision) 17/03/25

Ampere's Law

$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$ $\oint B \cdot dl = \mu_0 \bar{I}_{enc}$

$\bar{m} = I\bar{A}$: magnetic moment

magnetic vector potential : $\bar{A}(r)$

from multiple expansion

due to a single magnetic dipole : $\bar{A}(r) = \frac{\mu_0}{4\pi} \frac{\bar{m} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$

$\bar{M}$: magnetic dipole moment per unit volume

for a continuous distribution :

$$\bar{A}(r) = \int \frac{\mu_0}{4\pi} \frac{\bar{m}(r') \times \hat{\mathscr{r}}}{\mathscr{r}^2} dV'$$

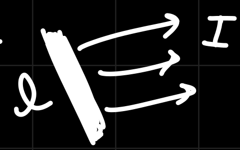
$$= \frac{\mu_0}{4\pi} \int \frac{J_b(\mathscr{r}')}{\mathscr{r}} dV' + \frac{\mu_0}{4\pi} \oint \frac{K_b(\mathscr{r}')}{\mathscr{r}} ds'$$

summation of and bound volume current density and bound surface current density where, $J_b(r) = \bar{\nabla} \times \bar{M}$, $K_b(r) = \bar{M} \times \hat{n}$

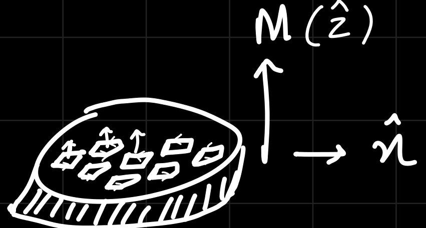
J_b: Am^{-2} K_b: Am^{-1}

$$K = \frac{I}{l_\perp}$$

$l_\perp$: length perpendicular to flow of current



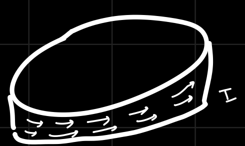
Uniform magnetized material



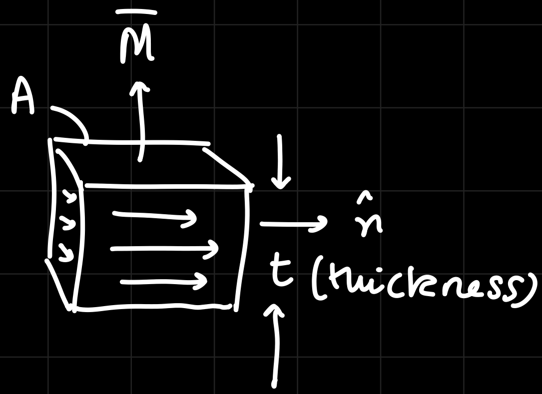
$\bar{m} = I\bar{A}$

internal currents are cancelled out

$\downarrow$



called bound surface current like a ribbon wrapped around the material carrying the current



$\bar{m} = \bar{M}\bar{A}t$

$I\bar{A} = \bar{M}\bar{A}t$

$\bar{M} = \dfrac{\bar{I}}{t} = K_b$

So, $\boxed{\bar{K_b} = \bar{M} \times \hat{n}}$

because $\bar{m}, \hat{n}, \bar{I}$ are mutually $\perp$ and dir of K_b is = dir of current

NOTE: volume current density is zero for uniform magnetized material

NON-UNIFORM MAGNETIZATION

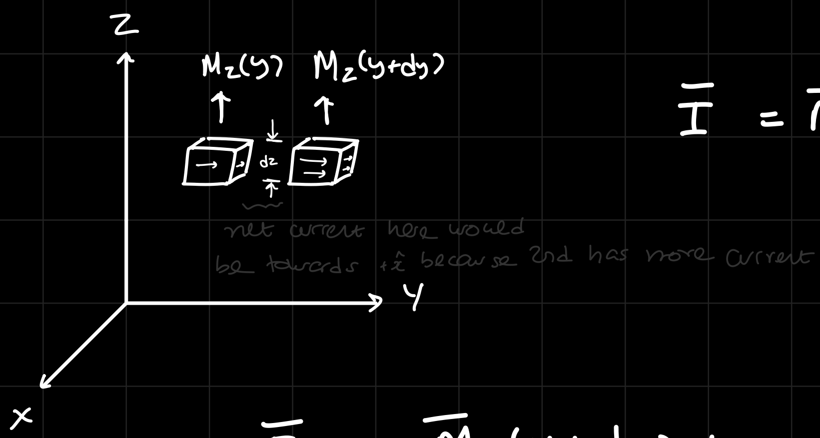
$\bar{M}$ will not be same in 2 different dipoles and hence, same goes for current

$\bar{M}$ varies with pos^n

from above $\rightarrow \bar{I} = \bar{M}t$

$\bar{J_b} = \bar{J}_{b-x}\hat{x} + \bar{J}_{b-y}\hat{y} + \bar{J}_{b-z}\hat{z}$

consider slabs



$$\bar{I} = \bar{M}t$$

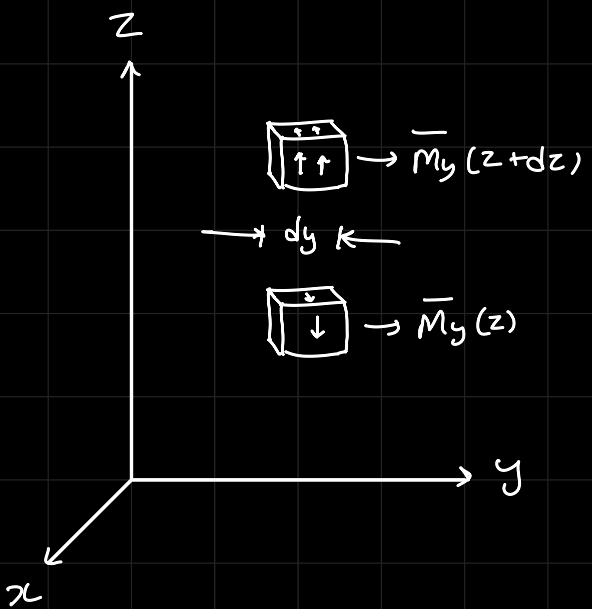
$$\bar{I}_x = \bar{M}_z(y+dy)\,dz - \bar{M}_z(y)\,dz$$
$$= [\bar{M}_z(y+dy) - \bar{M}_z(y)]\,dz$$

taylor series expansion

$$\bar{I}_x = \left[M_z(y) + \frac{\partial M_z}{\partial y}dy\right]dz - M_z(y)\,dz$$
$$= \frac{\partial M_z}{\partial y}\,dy\,dz$$

So, $\boxed{J_{b\_x} = \frac{\partial M_z}{\partial y}}$

but magnetization can have a x and y component as well



$$I_x = -\bar{M}_y(z+dz)\,dy + \bar{M}_y(z)\,dy$$

again taylor series expansion

$$= -\frac{\partial M_y}{\partial z}\,dy\,dz$$

$$\boxed{J_{b\_x} = -\frac{\partial M_y}{\partial z}}$$

Total volume current density : $\boxed{J_{b_x} = \left(\frac{\partial \bar{M}_z}{dy} - \frac{\partial \bar{M}_y}{dz}\right)}$

which is equal to $\underline{\underline{\bar{\nabla} \times \bar{M}}}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \bar{M}_x & \bar{M}_y & \bar{M}_z \end{vmatrix}$$

So, $J_{b_x} = \hat{x}\left(\frac{\partial}{\partial y} M_z - \frac{\partial}{\partial z} M_y\right)$

again Ampere's Law $\rightarrow$ $\bar{\nabla} \times \bar{B} = \mu_0 J_{total}$

$$\frac{\bar{\nabla} \times \bar{B}}{\mu_0} = \bar{J}_b + \bar{J}_{free}$$

$$\bar{J}_{free} = \nabla \times \left(\frac{\bar{B}}{\mu_0} - \bar{M} \right)$$

let $\vec{H} = \frac{\bar{B}}{\mu_0} - \bar{M}$

So, $\bar{J}_f = \bar{\nabla} \times \bar{H}$

and $\oint H \cdot dl = I_{free}$

$\bar{B} = \mu_0 \bar{H} + \bar{M}$

$\bar{M} = \chi_m \bar{H}$

$\hookrightarrow$ magnetic susceptibility

$$\bar{B} = \mu_0 (\bar{H} + \chi_m \bar{H}) = \underbrace{\mu_0 (\chi_m + 1)}_{\mu} \bar{H}$$

$\mu = \mu_0 (1 + \chi_m)$ $\rightarrow$ $\bar{B} = \mu \bar{H}$

magnetic materials

- linear ($\bar{M} = \chi_m \bar{H}$)
  - Para-magnetic ($\chi_m > 0$)
  - Dia-magnetic ($\chi_m < 0$)
- non-linear (ferromagnetic)

  $\chi_m \sim 10^5$

$$\mu = \mu_0(1+\underbrace{\chi_m}_{<0})$$

$$\mu \simeq \mu_0$$

very low magnetization (at room temp) in general for both linear elements