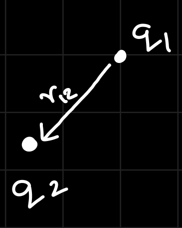
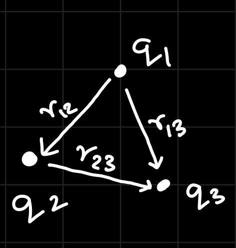
bringing charges one at a time

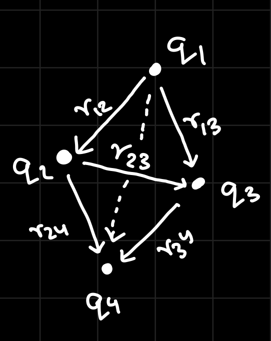
q_1 — $W_1 = 0$ work done because no elec field



$$W_2 = \frac{1}{4\pi\varepsilon_0} q_2\left(\frac{q_1}{r_{12}}\right)$$



$$W_3 = \frac{1}{4\pi\varepsilon_0} q_3\left(\frac{q_2}{r_{23}} + \frac{q_1}{r_{13}}\right)$$



$$W_4 = \frac{1}{4\pi\varepsilon_0} q_4\left(\frac{q_3}{r_3 r_4} + \frac{q_2}{r_2 r_4} + \frac{q_1}{r_1 r_4}\right)$$

Total work done $= W = W_1 + W_2 + W_3 + W_4$

$$= \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N} q_i \sum_{j<i} \frac{q_j}{\underbrace{r_{ij}}_{\text{magnitude (not vector)}}}$$

$$= \frac{1}{8\pi\varepsilon_0}\sum_{i=1}^{N} q_i \sum_{\substack{j=1\\ j\neq i}}^{N} \frac{q_i q_j}{r_{ij}}$$

eg ⇒ took double deliberately

$$W_3 = \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\left(q_3\left\{\frac{q_2}{r_{23}} + \frac{q_1}{r_{13}}\right\} + \left\{\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right\}\right)\right)$$

$$= \frac{1}{2}\left(2kq_3\left\{\frac{q_2}{r_{23}} + \frac{q_1}{r_{13}}\right\}\right)$$

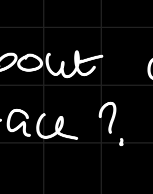
same

$$= \frac{1}{2}\sum_{i=1}^{N} q_i \left\{\sum_{\substack{j=1\\ j\neq i}}^{N} \frac{1}{4\pi\varepsilon_0}\frac{q_j}{r_{ij}}\right\}$$

Total work done by N individual charges

$$= \boxed{\frac{1}{2}\sum_{i=1}^{N} q_i \underbrace{V(r_i)}_{\text{potential at the position of } i^{th} \text{ charge due to all other charges}}} \quad \text{—— ①}$$

What about a surface?



$\to q_i = \rho\, dv$

$$dW = \frac{1}{2}\rho V dv \;\; dv$$

The energy of a continuous charge distribution

$$\boxed{W = \frac{1}{2}\int \rho V dV} \quad \text{—— ②}$$

vector Property

$$\bar{\nabla}(f\bar{A}) = f(\bar{\nabla}\cdot\bar{A}) + \bar{A}\cdot(\bar{\nabla} f)$$

gauss's law

$$\int \bar{\nabla}(f\bar{A})\, dv = \int f(\bar{\nabla}\cdot\bar{A})\, dv + \int \bar{A}(\bar{\nabla} f)\, dv$$

$$\oint f\cdot\bar{A}\, d\bar{s} - \int \bar{A}\cdot(\bar{\nabla} f)\, dv = \int f(\bar{\nabla}\cdot\bar{A})\, dv \quad ③$$

divergence of a scalar and vector field

scalar and vector multiplication yields vector then divergence of that results in a scalar

Note: $\rho = \bar{\nabla}E\cdot\varepsilon_0$

on ① →

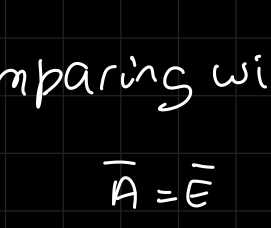
$$W = \frac{1}{2}\int \rho V dv$$

$$W = \frac{\varepsilon_0}{2}\int (\bar{\nabla}E) V dv$$

comparing with RHS of ③

$\bar{A} = \bar{E}$ and $V = f$

$$\text{So, } W = \frac{\varepsilon_0}{2}\left[\oint_{S_1} V\bar{E}\, d\bar{s} - \int_{V_1} \bar{E}\cdot(\bar{\nabla} V)\, dv\right]$$



→ big region
finite surface/volume

we know $\bar{\nabla} V = -\bar{E}$

$$\text{So, } W = \frac{\varepsilon_0}{2}\left[\oint_{S_1} V\bar{E}\, d\bar{s} + \int_{V_1} |E|^2 dv\right] \quad \text{— ④}$$

surface is very large
↳ r is very large
↳ r^2 grows very fast
↳ $\frac{1}{r^2} \to 0$ so, $\bar{E} \to 0$
↳ also $\frac{1}{r} \to$ very small so, $V \to 0$

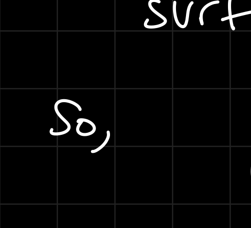
Note: ④ is the general form

but if you take the distance between the charge as very big, the surface integral tends → 0
↳ converges

So,

$$W = \frac{1}{2}\varepsilon_0 \int_{\text{all space}} |E|^2 dv \quad \to \text{converges?}$$

further you are from the surface, lesser this integral becomes



$$\boxed{W = \frac{1}{2}\varepsilon_0 \int |E|^2 dv}$$

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

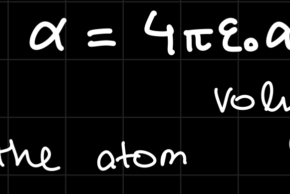
$\varepsilon_{r_{\text{water}}} = 80$

we have written Gauss's Law for vacuum.

but what about a specific medium? Next Class →

→ CHAPTER 4: Polarization

Dipole



dipole moment → $\bar{p} = q\bar{d}$

linear materials: where $\bar{p} \propto \bar{E}$

$$\bar{p} = \alpha\bar{E}$$

↳ atomic polarizability

at equilibrium → $\alpha = 4\pi\varepsilon_0 a^3 = 3\varepsilon_0 v$ ← volume of the atom

a: radius of the atom

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

polarizability matrix or polarizability tensor

So, $p_x = \alpha_{xx}E_x + \alpha_{xy}E_y + \alpha_{xz}E_z$

$p_y = \alpha_{yx}E_x + \alpha_{yy}E_y + \alpha_{yz}E_z$

$p_z = \alpha_{zx}E_x + \alpha_{zy}E_y + \alpha_{zz}E_z$

for isotropic elements,

diagonal α_s are α and rest are zero

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$\bar{p} = \hat{x}p_x + \hat{y}p_y + \hat{z}p_z$$

$$\bar{p} = \hat{x}(\alpha E_x) + \hat{y}(\alpha E_y) + \hat{z}(\alpha E_z)$$

$$\bar{p} = \alpha(\hat{x}E_x + \hat{y}E_y + \hat{z}E_z)$$

$$\bar{p} = \alpha\bar{E}$$

```
# include <stdio.h>
int main(int argc, char** argv){
    printf("4ΣM4 4V<K4");
    return 0;
}
```

constant thermal fluctuation throws dipole out of equilibrium

In Induced dipole moment, Electric field is mostly in the direction of the dipole only.