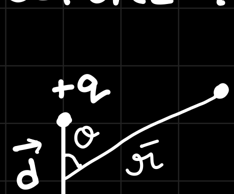
* LECTURE 9

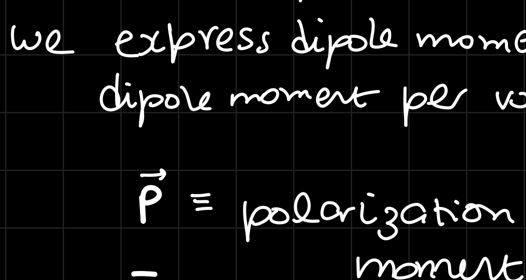


$\bar{p} = q\bar{d}$ $\quad V(r,\theta) = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \left(\dfrac{p}{4\pi\varepsilon_0}\cdot\dfrac{\hat{r}}{r^2}\right)$

net dipole moment due to induced and inherent dipole moment in the direction of electric field.

Instead of dipole moment for individual particles, we express dipole moment as polarization dipole moment per volume for a surface.

$\vec{P} \equiv$ polarization dipole moment per unit volume



$f(x-x', y-y', z-z')$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial(x-x')}\cdot\frac{\partial(x-x')}{\partial x} = \frac{\partial f}{\partial(x-x')}$$

$x \to$ const
$x' \to$ variable

$$\frac{\partial f}{\partial x'} = \frac{\partial f}{\partial(x-x')}\cdot\frac{\partial(x-x')}{\partial x'} = (-1)\frac{\partial f}{\partial(x-x')}$$

So, $\bar{\nabla}' f = -\bar{\nabla} f$

$$\bar{\nabla}\left(\frac{1}{r}\right) = -\bar{\nabla}'\left(\frac{1}{r}\right) = \frac{\hat{r}}{r^2}$$

* remember to use the vector notation ($\rightarrow$) for quiz

$\vec{p} = \vec{P}\,dv'$

$$dV = \frac{1}{4\pi\varepsilon_0}\left(\frac{\vec{P}\cdot\hat{r}}{r^2}\right)dv'$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\vec{P}\cdot\hat{r}}{r^2}\,dv'$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_{V'}\vec{P}\cdot\bar{\nabla}\left(\frac{1}{r}\right)dv'$$

remember last lecture →

$$\int \vec{A}(\bar{\nabla} f)\,dv = \underbrace{\oint f\bar{A}\,d\bar{s}}_{\text{surface integral}} - \int f(\bar{\nabla}\bar{A})\,dv$$

here $\bar{P} = \bar{A}$ and $f = \frac{1}{r}$ } magnitude

$$V = \frac{1}{4\pi\varepsilon_0}\left[\oint_{S'}\frac{\bar{P}\cdot d\bar{s}}{r} - \int\frac{1}{r}(\bar{\nabla}\bar{P}(r'))\,dv'\right]$$

$$V = \frac{1}{4\pi\varepsilon_0}\oint_{S'}\frac{\bar{P}\cdot d\bar{s}}{r} - \int_{V'}\frac{1}{r}(\bar{\nabla}\bar{P}(r'))\,dv'$$

Potential due to a surface charge density:

$$= \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma_s\,ds'}{r}$$

Potential due to a volume charge density

$$= \frac{1}{4\pi\varepsilon_0}\int\frac{\rho\,dv'}{r}$$

for a material kept in an electric field →

We are assuming a uniform polarization vector

So, potential of the surface charge is $\sigma_B = \bar{P}\cdot\hat{n} = \vec{P}\cos\theta$

$$V = \frac{1}{4\pi\varepsilon_0}\oint_{S'}\frac{\overbrace{(\bar{P}\cdot\hat{n})}^{\sigma_B}ds'}{r} + \int_{V'}\frac{1}{r}(\overbrace{-\bar{\nabla}\bar{P}(r')}^{\rho_B})\,dv'$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\oint\frac{\sigma_b\,ds'}{r} + \frac{1}{4\pi\varepsilon_0}\int\frac{\rho_B\,dv'}{r}$$

[where $\underbrace{\sigma_B = \bar{P}\cdot\hat{n}}_{\text{total surface charge?}}$ and $\underbrace{\rho_B = -\bar{\nabla}\cdot\bar{P}}_{\text{total volume charge?}}$]

B: bound in σ_B, ρ_B

$$q_{\substack{\text{volume}\\\text{(bound)}}} = \int \rho_B(r)\,dv = \int(-\bar{\nabla}\bar{P})\,dv = -\oint\bar{P}\cdot d\bar{s} = -\oint(\bar{P}\cdot\hat{n})\,d\bar{s} = -\oint\sigma_B\,ds = -q_{\substack{\text{surface}\\\text{(bounded)}}}$$

So, total surface charge is equal to negative of total volume charge.

irrespective of the type of polarization vector, whether uniform or not, the total surface and total vol charge add up to zero
Note: We are considering bounded charge
↓
none of them are mobile
They are tied to the molecules and molecules are tied to their posn.
They are bound to the material.

Remember → $\bar{\nabla}\cdot E = \frac{\rho}{\varepsilon_0}$

$$\bar{\nabla}\bar{E} = \frac{\rho_{free} + \rho_{bound}}{\varepsilon_0}$$

$$\bar{\nabla}(\varepsilon_0 E) = \rho_{free} - \bar{\nabla}\cdot\bar{P}$$

$$\rho_{free} = \bar{\nabla}(P + \varepsilon_0 E)$$

$\chi_e \Rightarrow$ Electric susceptibility

easier to polarize if chi/zai (χ_e) is higher

$$\bar{P} = \underbrace{\alpha}_{\text{atomic polarizability}}\bar{E} = \varepsilon_0\chi_e\bar{E}$$

So, $\rho_{free} = \bar{\nabla}(\varepsilon_0\chi_e\bar{E} + \varepsilon_0\bar{E})$

$$= \varepsilon_0(1+\chi_e)\bar{\nabla}\bar{E}$$

$$\bar{\nabla}E = \frac{\rho_{free}}{\varepsilon_0\underbrace{(1+\chi_e)}_{\varepsilon_r}} = \frac{\rho_{free}}{\varepsilon_0\varepsilon_r} = \frac{\rho_{free}}{\varepsilon}$$

$\varepsilon_r = 1 + \chi_e$
for free space → $\chi_e = 0$
so $\varepsilon_r = 1$

$$\boxed{\bar{\nabla}\bar{E} = \frac{\rho_{free}}{\varepsilon}}$$

$\bar{D}$: displacement density

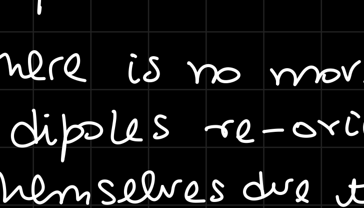
$\bar{I}_D$: displacement current

$$I_D = \varepsilon_0\frac{\partial E}{\partial t} = \frac{\partial D}{\partial t}$$

$\vec{D} = \varepsilon_0\vec{E}$

displacement current is the reason current flows between the plates of the capacitor.

→ $i_c = 0$
because $Z_c = \frac{1}{j\omega C} = 0$
∵ $\omega \to \infty$ for DC



there is no movement of charge dipoles re-orienting/rotating themselves due to $\bar{E}$ and there is a change of charge density and so there is an illusion of charge movement and that's how current moves