• Lecture: 2 — 8/01/25

→ Gradient operator
→ Level surface

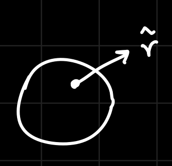
Directional derivative : $\frac{d\phi}{dr} = (\bar{\nabla}\phi)\cdot\hat{r}$

$= |\bar{\nabla}\phi| \cos\theta$

$\left|\frac{d\phi}{dr}\right|_{max} = |\bar{\nabla}\phi|$

→ for a level surface, directional derivative is along the surface normal

eg. for a sphere, the direction $\hat{r}$ at a specific point will be outwards from that point



$\bar{r} = x\hat{x} + y\hat{y} + z\hat{z}$
$d\bar{r} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

$\phi(x,y,z)$: func of x, y, z

$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz$

• Potential

1 dimension : $\bar{E} = -\frac{d\Phi}{dx}\hat{x}$ (→ electrostatic potential)

3 dimension : $\bar{E} = -\left(\frac{\partial\Phi}{\partial x}\hat{x} + \frac{\partial\Phi}{\partial y}\hat{y} + \frac{\partial\Phi}{\partial z}\hat{z}\right)$

• Cylindrical coordinates

$\rho, \ \phi, \ z$

differential length
differential area
differential solid angle

• Spherical coordinates

$0 \le r < \infty \qquad 0 \le \theta \le \pi \qquad 0 \le \phi \le 2\pi$

differential length
differential area
differential $\underbrace{\text{solid angle}}_{\frac{\text{length of arc}}{\text{area}}} = \frac{dS}{r^2} = \sin\theta\, d\theta\, d\phi$

• Line Integral

$\left.\begin{array}{l}\text{WORK} : \int_A^B F\,dl \to \text{for any force} \\ \text{Electrostatic potential} : \Phi_{AB} = -\int_A^B E\,dl\end{array}\right\}$ path dependent

• Note: some are path independent

assume : $\bar{V} = \bar{\nabla}u$

$\int_A^B V\cdot dl = \int_A^B (\bar{\nabla}u)\,dl = \int_A^B du = \underbrace{U_B - U_A}$

here, path independent ← only depends on end points

• A field which can be written as the gradient of a scalar is called **conservative field**

• note: gravitational (→ represented as gradient of gravitational potential) and electric fields (→ represented as gradient of electrostatic potential) are conservative whereas magnetic field cannot be represented as the gradient of a scalar.

• for closed loop → $\oint_A^B \bar{V}\cdot dl = \int_A^B du + \int_B^A du$

$= U_B - U_A + U_A - U_B = 0$

So, work done by a conservative field in a closed loop is zero.

i.e. closed loop line integral = 0

• Flux

• Open surface : $\int_{S_{closed}} V\cdot ds$

• closed surface : $\oint_{S_{open}} V\cdot ds$

$\oint_{S_{closed}} V\cdot ds = \lim$

• Divergence operator

$\bar{\nabla} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)$

Gradient of
- dot product of vector ≡ Divergence operator
- Cross product of vector ≡ CURL
- scalar ≡ Conservative field

if $\bar{\nabla}\times\bar{A} = 0$ it does not necessarily mean they are parallel because you cannot practically determine direction of $\frac{\partial}{\partial x} / \frac{\partial}{\partial y} / \frac{\partial}{\partial z} / \bar{\nabla}$ } Same goes for dot product as well

divergence for a very small surface $= \frac{\text{total outward flux}}{\text{volume}}$