

# Physics of Semiconductor devices

Books

semiconductor physics and devices by D.A. Neaman

physics of semiconductor devices by S.M. Sze and K.K. Ng

inverter can be used to create all logical operations / gates

increase voltage → amplifier eg: op Amp

decrease voltage → rectifier

let the threshold current for 'on' state be 10mA.

even then, 9mA will be considered 'on' because there isn't sufficient difference and hence difficult to sense by hardware

$$V = IR \xrightarrow{\text{input output constant}} \text{consequence of the Ohm's Law}$$

$$\text{Ohm's Law} \rightarrow \bar{J} = \sigma \bar{E}$$

which leads to  $V = IR$ 

Resistance is different for different metals because of mobility

Current : no. of electrons flowing per unit density area per length

$$\sigma \propto \frac{1}{R}$$

$$\sigma = \frac{\text{no. of electrons}}{\tau} \frac{q}{m_e} \mu_e$$

mobility  
charge

generally, a system with high  $\sigma$  = metalSystems with lower  $\sigma$  = semiconductors

electrons are bound with the atom due to electrostatic forces

$$\bar{E} = -\frac{dV}{dx}$$

electric field goes from higher potential to lower potential

intel i7 processor has 7 bil transistors

processor has → contact pins

global interconnect } vias

local interconnect

(metals)

transistors

The vias enable us to access all billions of transistors using limited ~100 contact pins

imp question : Which material to be used

## TRANSISTOR

Gate metal

SiO<sub>2</sub> insulator

silicon substrate semiconductor

10<sup>-9</sup> Å = 1nm

diameter of an atom: ~1-2 Å

dimension of a transistor: ~20nm

metal gate

insulator

inversion channel

②

depletion layer

8/04

18/04

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## Lecture 3

\* **Wave Function**  $\psi(x)$  = varies with time  
(Born interpretation) or  $\psi(r, t)$

for 1 particle system:  $\psi = \psi(x, y, z, t)$

for 2 particle system:  $\psi = \psi(x_1, y_1, z_1, x_2, y_2, z_2, t)$

wave function contains all info about the system

wave function  $\leftrightarrow$  classical trajectory  
(quantum mechanics)  $\leftrightarrow$  (newtonian mechanics)

Probability density

$$\hookrightarrow P(r) = |\psi|^2 = \int \psi^* \psi \, d\tau \quad \text{--- (1)}$$

### ① COPENHAGEN/BORN'S INTERPRETATION

The probability that a particle can be found at a point  $x$  at time  $t$  is given by (1)

→ Psi can be represented as a linear combination of other Psis

② To every physical property, there corresponds a linear, Hermitian operator in quantum mechanics

	SYMBOL	OPERATOR
Position	$x$	$\hat{x}$
	$y$	$\hat{y}$
Momentum	$p_x$	$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$
	$p_z$	$\hat{p} = -i\hbar \left( \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right)$
KE	$T_x$	$\hat{T}_x = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$
	$T$	$\hat{T} = -\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) = -\frac{\hbar^2}{2m} \nabla^2$
$P_E$	$V(x)$ $V(x, y, z)$	$\hat{V}(x)$ $\hat{V}(x, y, z) \times V(x, y, z)$
Total energy	$E$ $= E_k + P_E$	$\hat{H} = \frac{-\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + V(x, y, z) = -\frac{\hbar^2}{2m} \nabla^2 + V(x, y, z)$

NOTE: Hermitian matrix:

$$\text{if } A = A^T$$

$$\hat{A}\psi = a\psi \quad \text{--- (2)}$$

In any measurement of the observable associated with operator  $\hat{A}$ , the only values that will be observed are the eigenvalues 'a' which satisfy: (2)

$$\hat{A}f(x) = kf(x)$$

$f(x)$ : eigenfunction of  $A$   
with eigenvalue  $k$

the eigenvalue gives the real observable

e.g.:  $e^{ikx}$  is an eigenfunction of the operator  $\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$

if I operate  $\hat{p}_x$  on  $\psi$ , i.e.  $(-i\hbar \frac{\partial}{\partial x})\psi$

and we get: #  $\psi$

↳ some number

then # is eigenvalue

here  $\psi = e^{ikx}$

$$-i\hbar \frac{\partial}{\partial x} e^{ikx} = -i^2 \hbar k^2 e^{ikx} = \hbar k^2 e^{ikx}$$

$$k = \text{wave number} : k = \frac{2\pi}{\lambda}$$

$$\lambda = \text{wave length}$$

$$P = \frac{h}{\lambda} = \frac{h}{2\pi k} = \frac{\hbar \cdot k}{\lambda} \quad \begin{matrix} \text{reduced plank constant} \\ \text{wave number} \end{matrix}$$

$$\hbar = \frac{h}{2\pi}$$

$$P = \hbar \cdot k$$

hence, Momentum space is also called K-space and we can swap between the two using a real number  $k$

$$\Psi = e^{ikx} \quad \text{time independent wave func}$$

$$\psi = e^{ikx - \omega t} \quad \text{time dependent wave func}$$

④ For a system in a state described by a normalized wave function  $\psi$ , the avg or expectation value of the observable corresponding to  $A$  is given by

$$\langle A \rangle = \int \psi A \psi^* dz$$

### ⑤ SCHRÖDINGER EQUATION

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi \quad \text{time dependent}$$

$$E = \hat{H} \psi \quad \text{time independent}$$

$$\hat{H} = \hat{T} + \hat{V} + V_{ext} \quad \text{external potential energy}$$

$$T_E = KE = p_E$$

$$\text{total energy}$$

$$\text{symmetric}$$

$$\text{symmetric}$$

$$\text{antisymmetric}$$

## • Lecture : 4

16/01/25

$$\hat{H} = \hat{T} + \hat{V}$$

↓  
hamiltonian operator  
associated the overall  
energy of the system

$$\hat{H}\Psi = \hat{T}\Psi + \hat{V}\Psi = E\Psi$$

↓      ↓

$$\text{1d motion} = -\frac{\hbar^2}{2m} \left( \frac{\partial^2 \Psi}{\partial x^2} \right) + V(x)\Psi = E\Psi$$

↓

Note: if there is Coulomb potential

btw the particle, the

$$\text{potential energy} \rightarrow V(x) \propto \frac{1}{x}$$

NOTE 3: Potential energy

~ stored energy

~ energy due to position

gravitational potential = mgh

NOTE 4: Elastic PE:  $U = \frac{1}{2}kx^2$

k: spring constant : N/m

x: how much the spring

been stretched : m

NOTE 2: Kinetic energy is defined for individual objects and anything that moves has KE.

Any object at rest has  $KE = 0$

$$\text{Generally: } KE = \frac{1}{2}mv^2$$

NOTE 5: wave particle duality principle:

momentum of photon →

$$\text{momentum } p = \frac{h}{\lambda} \begin{matrix} \text{plank's constant} \\ \text{de broglie's wavelength} \end{matrix}$$

→ When can an object that appears as particle behave as wave?

↳ when the dimension(r) over which the change of potential energy  $V(r)$  of a particle becomes smaller as compared to its wavelength, its wave nature reveals.

## • Heisenberg's Uncertainty Principle

$$\Delta p \Delta x \geq \hbar$$

$$\# \text{ note: } \hbar = \frac{h}{2\pi}$$

both momentum and position cannot be precisely determined at the same time

$$\Delta E \Delta t \geq \hbar$$

energy of the particle

↳ the time instant the particle had

# another form of

that Energy

the uncertainty principle

Using this version, Energy cannot be zero minimum Energy  $\equiv$  ZPE  $\rightarrow$  Zero Point Energy  $\neq 0$

## • Wave Function $\Psi$

describes the quantum state of an isolated system of one or more particles.

There exists only one wave function containing info for an entire system

$$\Psi(x) / \Psi(r, t)$$

$$\text{Probability Density: } P(r) = |\Psi|^2 = \int \Psi^* \Psi d\tau$$

↓

probability that a particle can be found at a point x in time t

## • POSTULATES OF QUANTUM MECHANICS

① The state of a quantum mechanical system is completely specified by a wave function  $\Psi(\mathbf{r}, t)$  that depends on the particle's position  $\mathbf{r}$  and time  $t$

$$\Psi(x_1, y_1, z_1, \dots, x_i, y_i, z_i, t)$$

$$P(\mathbf{r}) = |\Psi|^2 = \int \Psi^* \Psi d\mathbf{r}$$

the probability that the particle can be found at point  $\mathbf{r}$  at time  $t$

$$\Rightarrow P(\mathbf{r}) \geq 0$$

$$\Rightarrow \int_{\text{all space}} \Psi^* \Psi d\mathbf{r} = 1$$

② To every physical property observable in classical mechanics, there corresponds a linear, hermitian operator in quantum mechanics

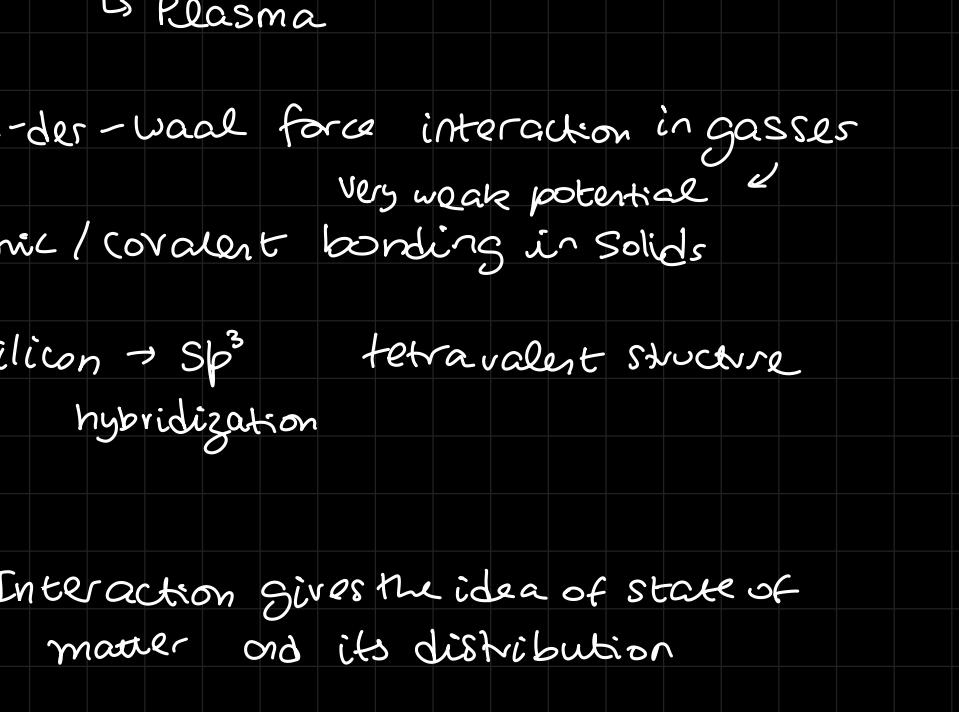
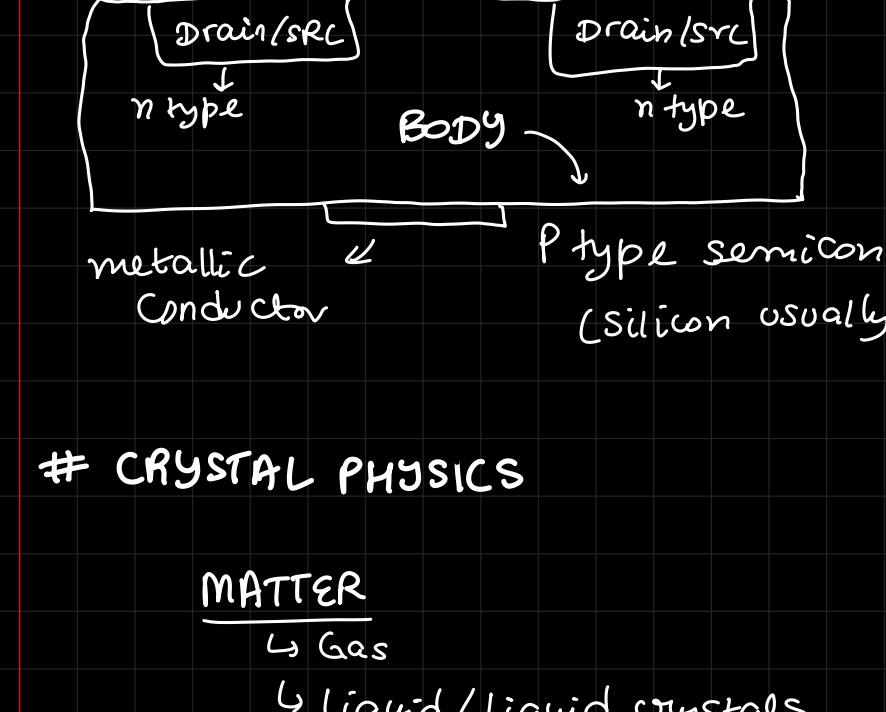
<u>OBSERVABLE</u>		<u>OPERATOR</u>	
Position	$x$	$\hat{x}$	multiply by $x$
	$\mathbf{r}$	$\hat{\mathbf{r}}$	multiply by $\mathbf{r}$
Momentum	$p_x$	$\hat{p}_x$	$-i\hbar \frac{\partial}{\partial x}$
	$\mathbf{p}$	$\hat{\mathbf{p}}$	$-i\hbar \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right)$
Kinetic Energy	$T_x$	$\hat{T}_x$	$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$
	$T$	$\hat{T}$	$-\frac{\hbar^2}{2m} \underbrace{\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)}_{\nabla^2}$
Potential Energy	$V(x)$	$\hat{V}(\hat{x})$	multiply by $V(x)$
	$V(x, y, z)$	$\hat{V}(\hat{x}, \hat{y}, \hat{z})$	multiply by $V(x, y, z)$
Total Energy	$E$	$\hat{H}$	$-\frac{\hbar^2}{2m} \nabla^2 + V(x, y, z)$
Angular Momentum	$L_x$	$\hat{L}_x$	$-i\hbar \left( y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right)$
	$L_y$	$\hat{L}_y$	$-i\hbar \left( z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right)$
	$L_z$	$\hat{L}_z$	$-i\hbar \left( x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right)$

③

## # Lecture: PSD 7

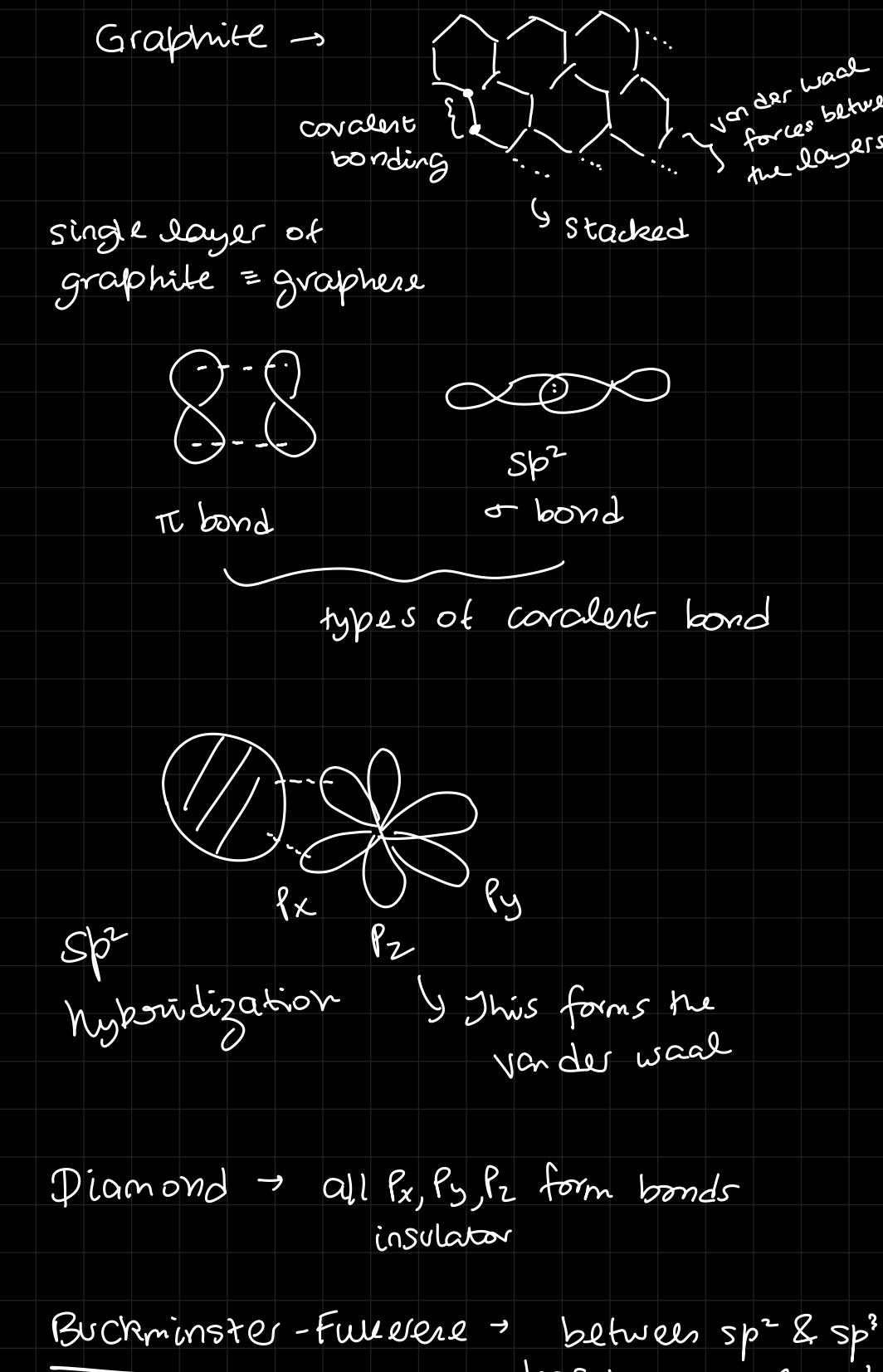
### → CRYSTAL STRUCTURE OF SOLIDS

PN junction acts as a diode when we provide a bias to the terminals using a "connection"/electrodes



### MOS Transistor

↓  
metal oxide semiconductor



## # CRYSTAL PHYSICS

### MATTER

- ↳ Gas
- ↳ Liquid / Liquid crystals
- ↳ Solid
- ↳ Plasma

Van-der-Waal force interaction in gasses  
Very weak potential  
Ionic / covalent bonding in solids

Silicon →  $sp^3$  tetrahedral structure  
hybridization

Interaction gives the idea of state of matter and its distribution

### CARBON

- ↳  $sp^2$  Graphite → metal
- ↳  $sp^3$  Diamond → insulator
- ↳ Buckminster-Fullerene → semiconductor

Some element but displays different properties because of different crystal structure

Graphite →  
single layer of graphite = graphene



$\pi$  bond

$\sigma$  bond

types of covalent bond

$sp^2$  hybridization

This forms the van der waal

Diamond → all  $P_x, P_y, P_z$  form bonds

insulator

Buckminster-Fullerene → between  $sp^2$  &  $sp^3$  has hexagonal and pentagonal structure

Super conductor

↳ conducts electricity without any resistance

= zero resistance conduction

↳ electrical resistivity

↳  $T$  [kelvin]

↳ if we dope polycrystalline structure, it behaves as metal (conductor)

" " amorphous structure, it behaves as oxide (insulator)

↳  $sp^2$  monocrystalline structure, it behaves as metal (conductor)

" " polycrystalline structure, it behaves as oxide (insulator)

↳  $sp^3$  amorphous structure, it behaves as metal (conductor)

" " monocrystalline structure, it behaves as oxide (insulator)

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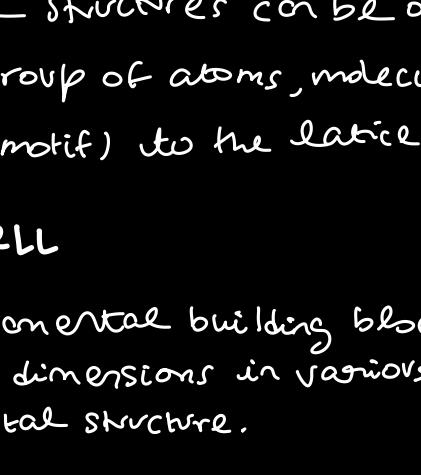
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## #Lecture: 8

### → CRYSTAL LATTICE

replacing each atom by a geometrical point because only their geometry is of use.



POLYCRYSTALLINE → finite array of points in space

Crystal structure = Crystal Lattice + Basis

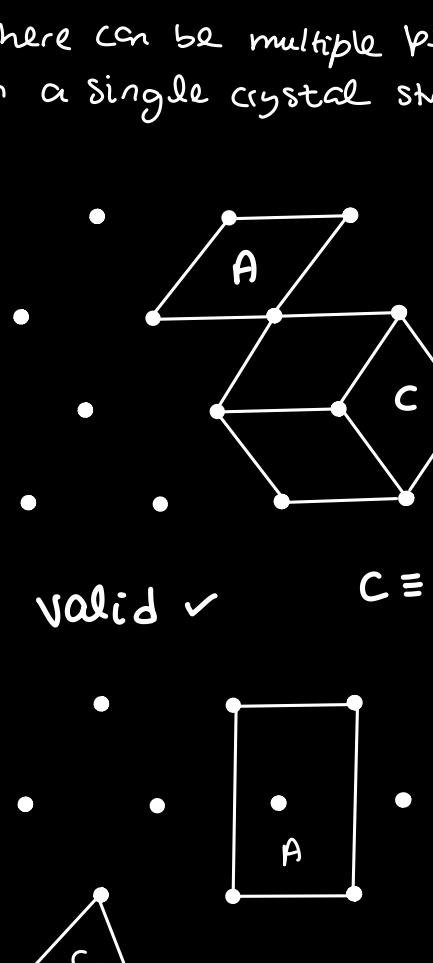
$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix} + \text{Atom} = \begin{matrix} \text{Atom} & \text{Atom} & \text{Atom} \\ \text{Atom} & \text{Atom} & \text{Atom} \end{matrix}$$

Crystal structures can be obtained by attaching atom, group of atoms, molecules which are called basis (motif) to the lattice sites of the lattice points.

### #UNIT CELL

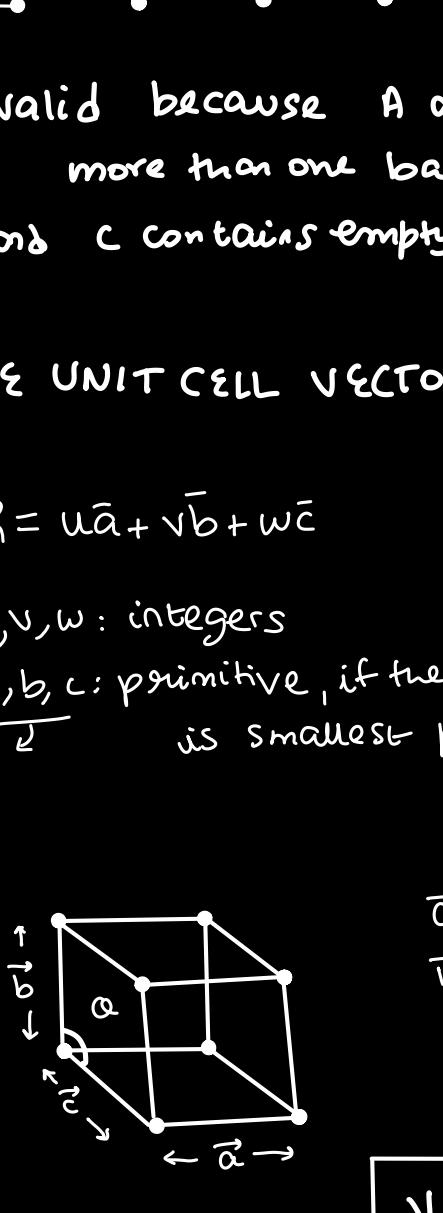
fundamental building block which repeats its own dimensions in various directions and gives crystal structure.

unit cell size should be constant throughout the crystal structure



e.g.: 2dimensional NaCl

e.g.:



→ empty space is not allowed  
→ periodicity not maintained in context of translational and rotational symmetry.

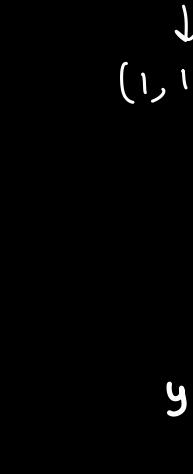
e.g.)



is this fractal a valid crystal structure?

No, because even though we can find a repeating pattern, we cannot link up the unit cells.  
i.e. there is no translational symmetry

e.g.)

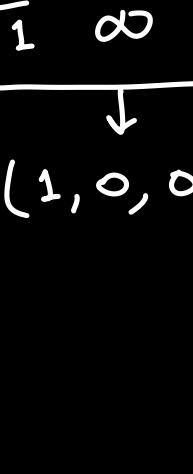


No, because even though it maintains rotational symmetry but it does not maintain translational symmetry

### QUASI CRYSTAL

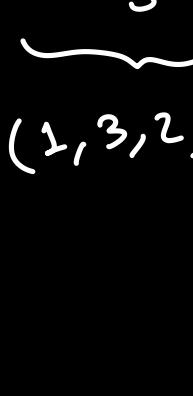
aperiodic, yet exhibits orientational symmetry

e.g.)



valid ✓

e.g.)



invalid because A and B contain more than one basis and C contains empty space

#PRIMITIVE UNIT CELL VECTOR

$$\vec{R} = u\vec{a} + v\vec{b} + w\vec{c}$$

u, v, w: integers

$\vec{a}, \vec{b}, \vec{c}$ : primitive, if the volume of cell is smallest possible

lattice constant



$\vec{a}$ : along x axis  
 $\vec{b}$ : along y axis  
 $\vec{c}$ : along z axis

$$V_c = \vec{a} \cdot (\vec{b} \times \vec{c})$$

### RULES

- $\vec{a}$  should be the shortest period of the lattice
- $\vec{b}$  to shortest period and non-parallel to  $\vec{a}$
- $\vec{c}$  to shortest period and non-parallel to  $(\vec{a}, \vec{b})$

### BRAVAIS LATTICE

There are just 14 3d crystal lattices out of which 7 are main and other are subdivision of those main.

Seven main divisions:

Cubic, Tetragonal, Orthorhombic, Hexagonal, Trigonal, Monoclinic, Triclinic

Simple Cubic

Body Centered

Face Centered



In 1D, we have just one Bravais Lattice



In 2D, we have 5 main divisions.

→ Square :  $a=b$ ,  $\theta=90^\circ$

→ Rectangle :  $a \neq b$ ,  $\theta=90^\circ$

→ Rhombus :  $a=b$ ,  $\theta=60^\circ$

→ Parallelogram :  $a \neq b$ ,  $\theta=60^\circ$

→ Body centered cube



invalid because A and B contain more than one basis and C contains empty space

and C contains empty space



#CONVENTIONAL/PRIMITIVE UNIT CELL

→ minimum volume unit cell.

→ always consists of only ONE basis.

→ There can be multiple primitive unit cells in a single crystal structure.

e.g.)



## → Schrodinger's wave equation

Total energy = Potential + Kinetic

$$\hat{H}\Psi = \hat{V}\Psi + \hat{E}\Psi \quad (\frac{P^2}{2m} - \frac{\hbar^2}{2m} \nabla^2\Psi)$$

$$E\Psi = V(x,y,z)\Psi - \frac{\hbar^2}{2m} \nabla^2\Psi$$

## → Particle moving in 1D

Side track →  $\Psi$  the solution of a differential eqn

$$\Psi = A\sin(kx) + B\cos(kx)$$

$$\frac{\partial\Psi}{\partial x} = k(A\cos(kx) - B\sin(kx))$$

$$\frac{\partial^2\Psi}{\partial x^2} = k^2(-A\sin(kx) - B\cos(kx))$$

$$\frac{\partial^2\Psi}{\partial x^2} = -k^2\Psi$$

### ① Free Particle : $V(x) = 0$

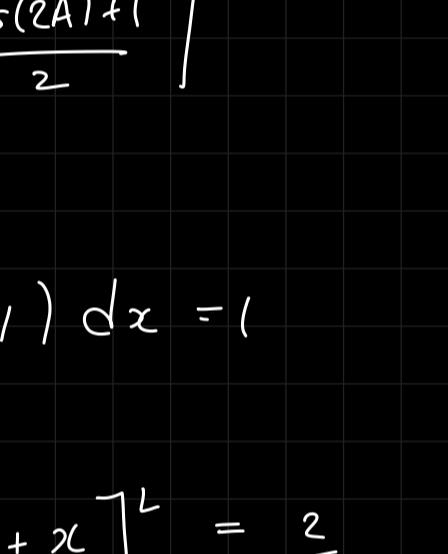
$$E\Psi = -\frac{\hbar^2}{2m} \frac{\partial^2\Psi}{\partial x^2} = -\frac{\hbar^2}{2m} (-k^2\Psi)$$

$$E = \frac{\hbar^2 k^2}{2m} \quad \text{---} \quad ①$$

Total Energy depends on wavenumber

## POTENTIAL WELL

$$-\frac{\hbar^2}{2m} \frac{\partial^2\Psi(x)}{\partial x^2} + V(x)\Psi = E\Psi$$



Schrodinger's time independent equation

$$\text{R} \rightarrow \text{I and III} : -\frac{\hbar^2}{2m} \frac{\partial^2\Psi}{\partial x^2} + \infty\Psi = E\Psi \quad \Psi \text{ must be zero in regions I and III}$$

$$\text{R} \rightarrow \text{II} : -\frac{\hbar^2}{2m} \frac{\partial^2\Psi}{\partial x^2} + 0 = E\Psi$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2\Psi}{\partial x^2} = E\Psi$$

from ①, we know  $k \propto E$

so, we can conclude that energy is discrete / quantized

And,

$$\Psi = A\sin\left(\frac{n\pi}{L}x\right)$$

using probability density formula,

$$\int_{-\infty}^{\infty} |\Psi|^2 dx = 1$$

$$\int_0^L A^2 \sin^2\left(\frac{n\pi}{L}x\right) dx = 1$$

$$\left\{ \begin{array}{l} \sin^2(x) = 1 - \cos^2(x) \\ \cos(A+B) = \cos A \cos B - \sin A \sin B \\ \cos(2A) = \cos^2 A - \sin^2 A \\ \cos(2A) = 2\cos^2 A - 1 \\ \cos^2(A) = \frac{\cos(2A) + 1}{2} \end{array} \right\}$$

$$\int_0^L \left( \cos\left(\frac{2n\pi}{L}x\right) + 1 \right) dx = 1$$

$$\left[ \left( \sin\left(\frac{2n\pi}{L}x\right) \right) \frac{2n\pi}{L} + x \right]_0^L = \frac{2}{A^2}$$

$$\sin\left(2n\pi\right) \cdot \frac{2n\pi}{L} + L - 0 - 0 = \frac{2}{A^2}$$

$$A^2 = \frac{2}{L} \rightarrow A = \sqrt{\frac{2}{L}}$$

$$\Psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right)$$

$$\text{allowed energies} \rightarrow E_n = \frac{k^2 \hbar^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

$E \neq 0$  ever that would mean  $\Psi = 0 \forall x$  (not allowed)

and  $E_{\min} = ZPE$  lowest energy of the particle

$$E_n \propto \frac{n^2}{L^2}$$

$n \equiv \text{number of times curve cuts } x\text{-axis}$

Region I: the potential  $V = 0$  hence Schrodinger's equation reduces to that of a free particle. Therefore, its wave function

$$\Psi_I = \Psi_{I,x}\Psi_{I,y} = A e^{ikx} + B e^{-ikx} \quad \text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II: finite potential barrier  $V$

$$\frac{\hbar^2}{2m} \frac{\partial^2\Psi_{II}}{\partial x^2} = (V - E)\Psi_{II}$$

Therefore, its wave function is

$$\Psi_{II} = C e^{-k'x} + D e^{k'x} \quad \text{where } k' = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

Region III: the potential  $V = 0$  and the transmitted particle is moving towards the right and has a positive momentum

$$\Psi_{III} = F e^{ikx} \quad \text{where } k = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$



$$\text{Transmission coefficient } T = \frac{|\Psi_{III}|^2}{|\Psi_{II}|^2} = \frac{|F|^2}{|C|^2}$$

$$\text{Tunneling probability: } T = \frac{|F|^2}{|A|^2} = \frac{1}{1 + \frac{V^2 \sinh^2(k'L)}{4E(V-E)}}$$

## # Schrodinger's wave equation

$$-\frac{\hbar^2}{2m} \frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) = i\hbar \frac{\partial\Psi(x,t)}{\partial t}$$

⇒ Time Independent

$$-\frac{\hbar^2}{2m} \frac{\partial^2\Psi(x)}{\partial x^2} + V(x)\Psi(x) = E\Psi(x)$$

$$\hat{H}\Psi(x,t) = i\hbar \frac{\partial\Psi(x,t)}{\partial t}$$

## \* BORN'S Interpretation (CRITERIAS)

Note (a)  $\Psi$  must be continuous (no breaks)

(b)  $\nabla\Psi = \frac{\partial\Psi}{\partial x}$  must be continuous (no kinks)

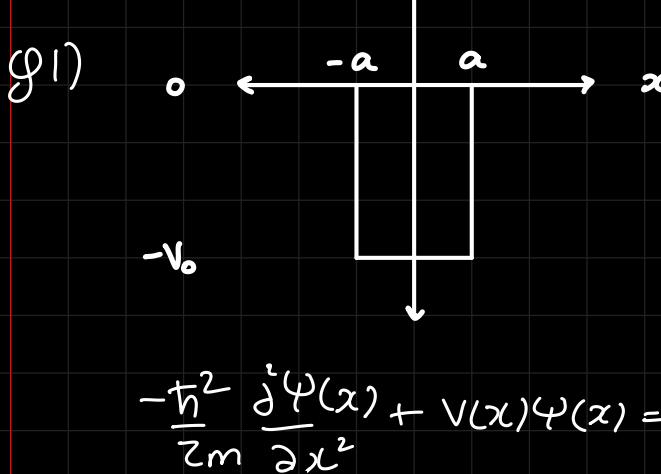
gradient of Psi we will have a divergence if gradient is discontinuous and so it will violate continuity

(c)  $\Psi$  must have a single value at any pt. in space since for  $\Psi$  same for  $\Psi$  and  $\Psi$  is discontinuous and so it will violate continuity

(d)  $\Psi$  must be finite everywhere

(e)  $\Psi$  cannot be zero everywhere

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$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + V(x) \psi(x) = E \psi(x)$$

at  $x = |a|$  :  $V(x) = 0$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} = E \psi(x)$$

$$\frac{\partial^2 \psi}{\partial x^2} - \left( \frac{2m}{\hbar^2} \cdot E \right) \psi = 0$$

$$\text{let } \psi(x) = A \sin(kx) + B \cos(kx)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -k^2 \psi$$

$$\text{so, } E = \frac{k^2 \hbar^2}{2m}$$

in the region II :

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} - V_0 \psi(x) = E \psi(x)$$

$$\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + (V_0 + E) \psi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} (V_0 + E) \psi = 0$$

$$\text{let } \psi(x) = C e^{-kx} + D e^{kx}$$

for  $x < -a \rightarrow \psi \neq \infty$

$$\text{so, } C = 0$$

for  $x > a \rightarrow \psi \neq 0$

$$\text{so, } D = 0$$

$$\text{so, } \psi = \begin{cases} D e^{-kx} & : x < -a \\ A \sin(k'x) + B \cos(k'x) & : -a < x < a \\ C e^{-kx} & : x > a \end{cases}$$

# at  $x = -a \rightarrow \psi(x)$  and  $\frac{\partial \psi(x)}{\partial x}$  must be continuous

$$-A \sin(k'a) + B \cos(k'a) = D e^{-ka} \quad \text{--- (2)}$$

$$+ k' A \cos(k'a) - k' B \sin(k'a) = k \cdot D e^{-ka} \quad \text{--- (3)}$$

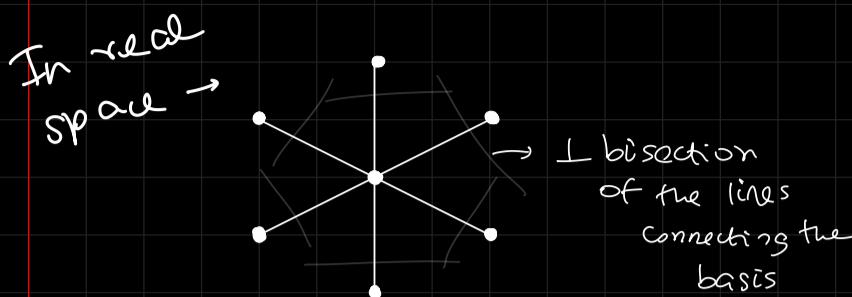
$$(2) + (3)$$

$$2B \cos(k'a) = (C + D) e^{-ka}$$

$$(2) - (3)$$

$$2A \sin(k'a) = (C - D) e^{-ka}$$

# Wigner-Szilz cell → another primitive cell in the real space



draw the line between the nearest neighbour basis taking the least length and then take the perpendicular bisection of those lines and you will get the Wigner Szilz cell

## Reciprocal Lattice

(= Fourier space / K space / momentum space)

Electronic number density =  $n(r)$  in the crystal is periodic

$$n(r) = n(r+R), \quad R: \text{primitive unit vector} \\ \equiv \text{period}$$

$R$  equal to a direct lattice translational vector :

$$R = u\bar{a} + v\bar{b} + w\bar{c}$$

$u, v, w \rightarrow$  numbers (const)

$n(r)$  can be expressed as a Fourier series

$$n(r) = \sum_G n_G e^{ikr}$$

$$\text{where } n_G = \frac{1}{V_c} \int_0^a dV n(r) e^{-ikr}$$

where  $\bar{k} =$  Reciprocal Lattice Vectors

reciprocal lattice represents the Fourier transform of another lattice like Bravais Lattice in real space

$$\boxed{\bar{k} \cdot \bar{R} = 2\pi \delta_{ij}} \quad \leftarrow e^{ikr} = 1$$

for unique  $\bar{R}$ , we have unique  $\bar{k}$

We have 14 lattices for 3d  
 " 5 lattices for 2d  
 1 lattice for 1d

How about in reciprocal lattice?

What is the purpose of moving into the reciprocal lattice?

It decreases the complexity of theoretical & practical analysis

Electronic number density ( $n(r)$ ) must be periodic  $\rightarrow n(r) = n(r+R)$

$$\sum_{\mathbf{G}} n_{\mathbf{G}} e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_{\mathbf{G}} n_{\mathbf{G}} e^{i(\mathbf{k}+\mathbf{R})\cdot\mathbf{r}} \quad \begin{matrix} \text{Fourier} \\ \text{transform} \end{matrix}$$

$$\sum_{\mathbf{G}} n_{\mathbf{G}} e^{i\mathbf{k}\cdot\mathbf{r}} = \sum_{\mathbf{G}} n_{\mathbf{G}} e^{i\mathbf{k}\cdot\mathbf{r}} e^{i\mathbf{R}\cdot\mathbf{r}}$$

So,  $e^{i\mathbf{R}\cdot\mathbf{r}}$  must be equal to 1

$$R \cdot R = 2\pi \quad e^{i\mathbf{R}\cdot\mathbf{R}} = 1 \quad \begin{matrix} \# R \rightarrow \text{periodicity} \\ \# \mathbf{R} \rightarrow \text{Reciprocal lattice vector} \end{matrix}$$

$$\bar{R} \cdot \bar{R} = 2\pi \delta_{ij} \quad i, j = x, y, z \quad \delta_{ij} = 1 : i=j / 0 : \text{else}$$

When we move from real space to  $k$ /momentum space, our differential equations turn into quadratic eqn which is easier to solve

But,

$$\bar{R} = u\bar{a} + v\bar{b} + w\bar{c}$$

$$\text{and, } \bar{k} = h\bar{a}^* + k\bar{b}^* + l\bar{c}^*$$

where  $h, k, l$  are Miller indices

Reciprocal of the intersections

eg:

$$\begin{array}{ccccccc} \leftarrow & \rightarrow & \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$$

[1d]

$$\text{then } \bar{a}\bar{a}^* = 2\pi$$

since they are in the same direction,

$$a\bar{a}^* = 2\pi$$

$$\boxed{\bar{a}^* = \frac{2\pi}{a}}$$

So, reciprocal lattice  $\rightarrow$

$$\begin{array}{ccccccc} \vdots & \rightarrow & \bullet & \bullet & \bullet & \bullet & \bullet \end{array}$$

for [3d]

$$a\bar{a}^* = 2\pi$$

$$b\bar{b}^* = 2\pi$$

$$c\bar{c}^* = 2\pi$$

but  $b\bar{a}^* = 0$

because of  $\delta_{ij}$

$$a^* = \frac{2\pi}{a} \frac{(b \times c)}{a \cdot (b \times c)}$$

$$b^* = \frac{2\pi}{b} \frac{(c \times a)}{b \cdot (c \times a)}$$

$$c^* = \frac{2\pi}{c} \frac{(a \times b)}{c \cdot (a \times b)}$$

Direct Lattice Vector:

$$\bar{R} = u\bar{a} + v\bar{b} + w\bar{c}$$

Reciprocal Lattice Vector:

$$\bar{k} = m\bar{a}^* + n\bar{b}^* + o\bar{c}^*$$

Reciprocal lattice is the Fourier transform of the real lattice

Note: Set of reciprocal lattice vectors  $\bar{k}$  is also known as lattice

since  $b\bar{a}^* = 0$

and  $c\bar{a}^* = 0$

so,  $a^*$  must be linear multiple of  $(b \times c)$

i.e.  $\perp$  to both  $b$  and  $c$

so,  $a^* = 2\pi \frac{a}{(b \times c)}$

$$a^* = \frac{2\pi}{a} (i+j)$$

$$b^* = \frac{2\pi}{b} (j+k)$$

$$c^* = \frac{2\pi}{c} (k+i)$$

Note: The Real space vector for FCC is:

$$\bar{a} = \frac{a}{2} (i+j)$$

$$\bar{b} = \frac{a}{2} (j+k)$$

$$\bar{c} = \frac{a}{2} (k+i)$$

We notice that the Reciprocal lattice of FCC is FCC and vice versa.

Face centered cube  $\xrightarrow{\text{Fourier transform}} \text{Body centered cube}$

If we move into the Reciprocal space, the Wigner Seitz cell gives the first Brillouin zone.

Note: Wigner Seitz cell has square or hexagon structure

if we can find the energies in the Brillouin zone, we can find the energy of holes in the, say, silicon structure

In the Brillouin zone (FCC) we have a square at  $[001], [010], [100]$  whereas we have a hexagon at  $[111]$

# Remember last question of the quiz

↳ Work function

↳ if it is diff for a particular material, then the min energy must be too

#  $E = \frac{h^2 k^2}{8m} \rightarrow$  quantum mechanics

$\frac{-h^2}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + V(x) \psi(x) = E \psi(x)$

time independent Schrödinger's wave equation

Let  $\psi(x) = A \sin(kx) + B \cos(kx)$

then  $\frac{\partial^2 \psi(x)}{\partial x^2} = -k^2 \psi(x)$

$\therefore -\frac{h^2}{2m} k^2 \psi(x) + V(x) \psi(x) = E \psi(x)$

for the 2D potential well problem,  $V(x) = 0$

$$E = \frac{h^2 k^2}{2m}$$

$$k^2 = \frac{2m}{h^2} E$$

$$k = \sqrt{\frac{2m}{h^2} E}$$

$$k = \sqrt{\frac{2m}{h^2} E} \hat{x}$$

$$k = \sqrt{\frac{$$

## \* BAND THEORY OF SOLIDS {3rd Chp}

Bragg zone = Wigner-Seitz cell in the reciprocal space

### → FREE ELECTRON THEORY

↳ no potential energy

$$E\psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \quad \text{from Schrodinger's wave eqn}$$

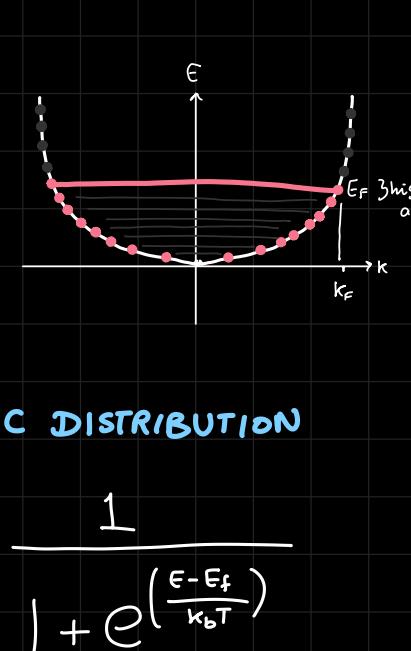
free e- are plane waves  $\therefore \psi = A e^{\pm ikx}$

$$\rightarrow \text{momentum: } i\hbar \frac{\partial \psi}{\partial x} = \pm \hbar k \psi$$

$$\rightarrow \text{Energy: } -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = \frac{\hbar^2 k^2}{2m} \psi$$

$$\rightarrow \text{Grp velocity: } \frac{\partial \omega}{\partial k} = \frac{1}{\hbar} \frac{\partial E}{\partial k} = \frac{\hbar k}{m}$$

Very closely packed energy bands if container of e- is very large, we can conclude that the energy band graph looks continuous.



wave packets: superposition of different waves of separate electrons

group velocity: velocity of a whole group e- / wave packets.

phase velocity: velocity of individual particles like e-

properties like conductivity, temperature etc. can be figured out using this free e-theory but this theory is incomplete on its own without the crystal nature.

### → FERMI-SURFACE

We have seen in the "particle in a box" problem, that energy levels are discretized. But as the width approaches  $\rightarrow \infty$ , the  $\Delta E$  (energy band gap) tends to  $\rightarrow 0$ .

$$\frac{\hbar^2 k^2}{2m} = E_F \rightarrow \text{Fermi-level}$$



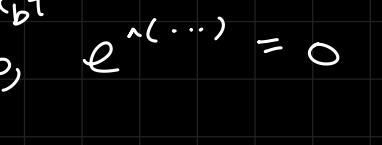
**FERMI-DIRAC DISTRIBUTION**

The probability that a particle has energy  $E$  is

$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{k_b T}\right)}}$$

$k_b$ : Boltzmann constant

$T$ : temperature



At  $T=0$ ,  $E_F$  is the highest energy where we can find a valence electron with energy  $= E$

When we increase the temperature, the valence electrons move to higher energy levels

↳ valence e-  $\rightarrow$  conduction e-

LM : The e- occupy this space

LM : Holes occupy this space which was earlier occupied by electrons

In case of METAL, the gap between the new energy band (conduction) and  $E_F$  (Fermi level) is very small.

↳ # of free e- for metals is very high

In 12th, we were taught that there was an overlap between the conduction band and valence band but the idea was that they were indistinguishable

In case of SEMICONDUCTOR, the gap is high and hence conduction e- and valence e- can be distinguished easily.

Some case for insulators as well

FERMI LINE

1d

FERMI SURFACE

2d

FERMI SPHERE

3d



$$\rightarrow k_F = \sqrt{\frac{2mE_F}{\hbar}}$$

k-space

$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{k_b T}\right)}}$$

$$E - E_F = -\infty$$

$$\text{so, } e^{(E-E_F)/k_b T} = 0$$

$$\text{and } f(E) = 1$$

$$\text{but at } E > E_F,$$

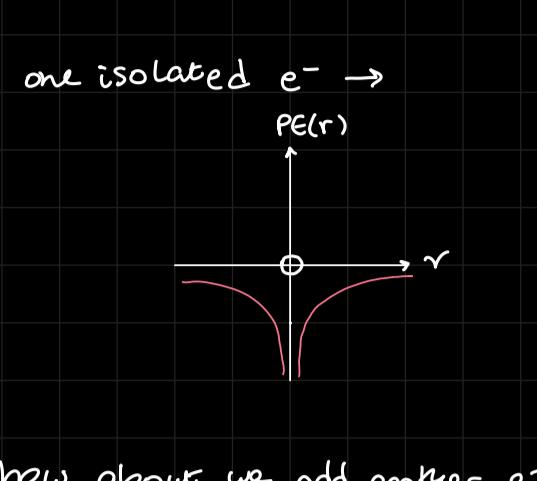
$$\text{we have } e^{(E-E_F)/k_b T} = e^{\infty}$$

$$\text{and so, } f(E) = 0$$

fermi energy = chemical potential ( $E_F$ )

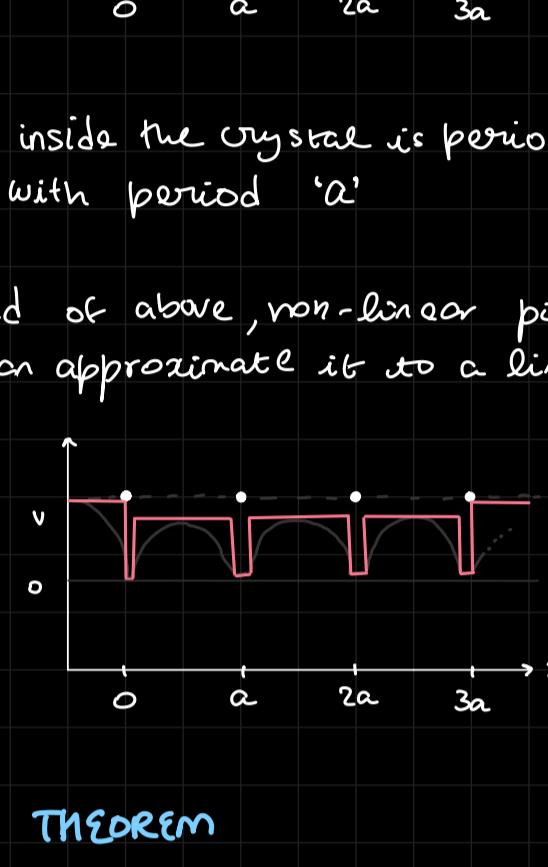
$$\text{the probability that a particle has energy } E \} f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{k_B T}\right)}}$$

### PE of $e^-$ in crystal

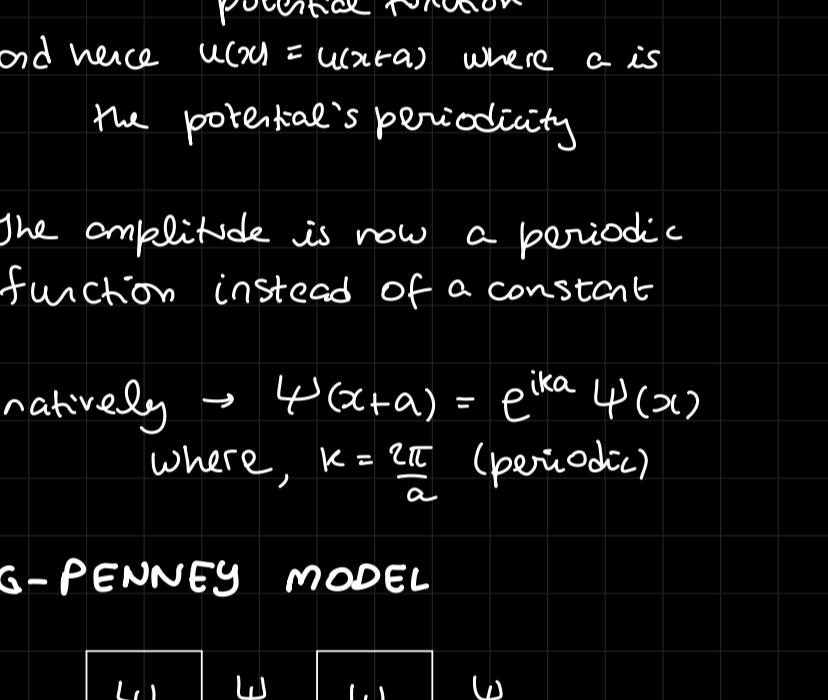


$$\text{for } 1d \rightarrow a\alpha^* = 2\pi \Rightarrow \alpha^* = \frac{2\pi}{a}$$

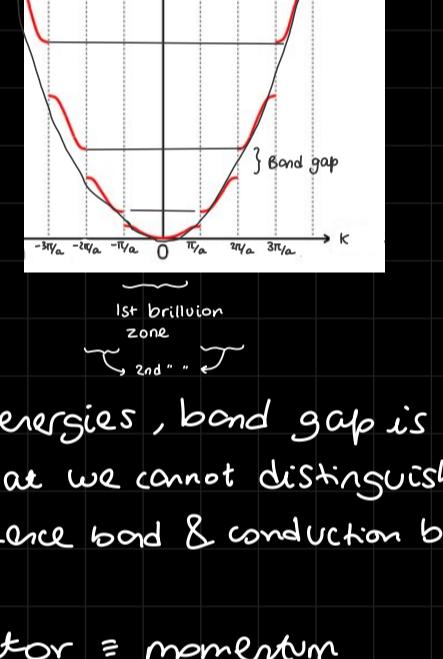
in k space  $\rightarrow$  we have Brillouin zone



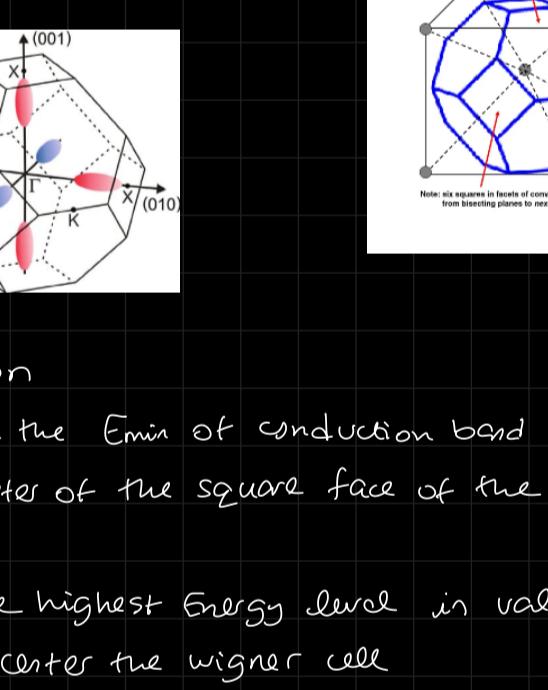
for one isolated  $e^- \rightarrow$



how about we add another  $e^-$ ?

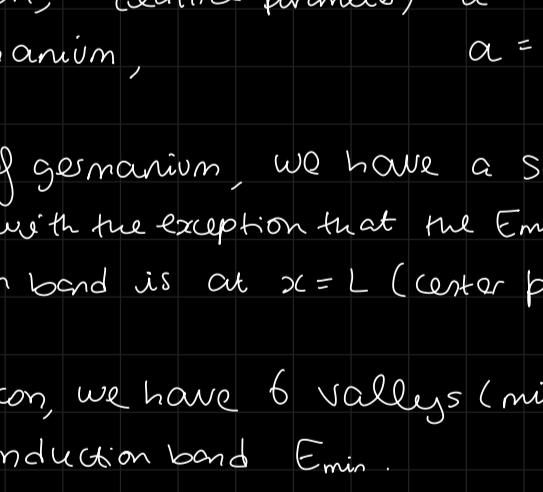


and we can have a long chain of particles as well



PE inside the crystal is periodic with period 'a'

instead of above, non-linear potential, we can approximate it to a linear potential.



**BLOCH THEOREM**

$$\text{for a general crystal: } \psi(x) = e^{ikx} u(x)$$

where  $u(x) = \text{periodic just like potential function}$

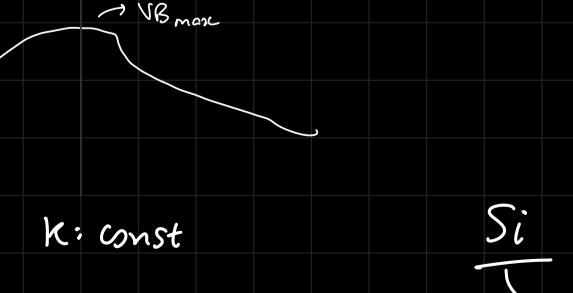
and hence  $u(x) = u(x+a)$  where  $a$  is the potential's periodicity

note: The amplitude is now a periodic function instead of a constant

$$\text{alternatively } \psi(x+a) = e^{ika} \psi(x)$$

$$\text{where, } k = \frac{2\pi}{a} \text{ (periodicity)}$$

**KRONIG-PENNEY MODEL**



Solution: D.A. Neeman pg 63-70

E-K relation  $\rightarrow$

$$ka = \pm n\pi$$



initial assumption,

continuous graph

as soon as we bring periodicity, we observe the band gap

at some energies, band gap is so less that we cannot distinguish between valence band & conduction band.

wave vector = momentum

In a Wigner-Seitz cell, center point of the hexagonal face  $\equiv L$

center point of the square face  $\equiv X$

center point of the cell  $\equiv r$  (gamma)

All these are wave vectors



for Silicon

we observe the  $E_{\min}$  of conduction band at  $k=X$  i.e. the center of the square face of the Wigner Seitz cell.

we achieve highest energy level in valence band at  $k=r$ , center the Wigner cell

$$\text{and hence, band gap} = E(X) - E(r)$$

for Si,

$$\rightarrow CB_{\min}$$

$$\rightarrow VB_{\max}$$

for Ge,

$$\rightarrow CB_{\min}$$

$$\rightarrow VB_{\max}$$

also called energy band valleys

energy band structure gives relation

between E and k (momentum space)

Energy band diagram gives relation between E and x (real dimension)

for silicon, (lattice parameter)  $a = 5.45 \text{ \AA}$

for germanium,  $a = 5.68 \text{ \AA}$

in case of germanium, we have a similar E-k relation with the exception that the  $E_{\min}$  for the conduction band is at  $x=L$  (center pt of hexagonal face)

for silicon, we have 6 valleys (minima condition).

for conduction band  $E_{\min}$ .

$CB_{\min}$  at every center point of a square face of the Wigner cell

and 8 valleys for germanium

Since we have larger # of valleys,  $e^-$  will flow "more" in case of Ge and hence more current

Si and Ge are indirect band gap semiconductors

$\hookrightarrow CB_{\min}$  and  $VB_{\max}$  are not at the same momentum point

for Si, particle moves in momentum axis and then changes energy to move from  $CB_{\min} \rightarrow VB_{\max}$

LEDs use direct band gap semiconductor

$\hookrightarrow$  not losing energy when transitioning from  $VB \rightarrow CB$ . e.g. GaAs

DBGS very useful in optical electronic devices.



GaAs

$k: \text{const}$

$$\rightarrow CB_{\min}$$

$$\rightarrow VB_{\max}$$

Si

$$\rightarrow CB_{\min}$$

$$\rightarrow VB_{\max}$$

# PHYSICS OF SEMICONDUCTOR DEVICES

## ASSIGNMENT - 1

Aditya Gautam - 2023043

$$q1) \bar{A} = \left( \frac{a\sqrt{3}}{2} \hat{i} + \frac{a}{2} \hat{j} \right), \bar{B} = \left( -\frac{a\sqrt{3}}{2} \hat{i} + \frac{a}{2} \hat{j} \right), \bar{C} = c \hat{k}$$

We know that the volume of primitive unit cell with the primitive cell vector,  $\bar{R} = u\bar{a} + v\bar{b} + w\bar{c}$  is  $V_c = \bar{a} \cdot (\bar{b} \times \bar{c})$

$$\text{So, volume} = \bar{A} \cdot (\bar{B} \times \bar{C})$$

$$(\bar{B} \times \bar{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \\ 0 & 0 & c \end{vmatrix}$$

$$= \hat{i} \left( \frac{ac}{2} - 0 \right) - \hat{j} \left( -\frac{ac\sqrt{3}}{2} - 0 \right) + \hat{k} (0 - 0)$$

$$= \left( \frac{ac}{2} \right) \hat{i} + \left( \frac{ac\sqrt{3}}{2} \right) \hat{j}$$

$$\begin{aligned} \text{now, } \bar{A} \cdot (\bar{B} \times \bar{C}) &= \left( \frac{a\sqrt{3}}{2} \cdot \frac{ac}{2} \right) + \left( ac\sqrt{3} \cdot \frac{a}{2} \right) \\ &= 2 \left( \frac{a^2 c \sqrt{3}}{4} \right) = \frac{\sqrt{3} a^2 c}{2} \end{aligned}$$

Hence, we conclude that the volume of the primitive unit cell is indeed  $\rightarrow \frac{\sqrt{3}}{2} a^2 c$

Primitive Translational Vectors  $\rightarrow$

for 3d crystal structure,

$$A^* = \frac{2\pi \cdot (B \times C)}{A \cdot (B \times C)}$$

$$= \frac{2\pi \left( \frac{ac}{2} \hat{i} + \frac{ac\sqrt{3}}{2} \hat{j} \right)}{\sqrt{3}/2 a^2 c}$$

$$= \frac{2\pi}{a} \left( \frac{1}{\sqrt{3}} \hat{j} + \hat{i} \right)$$

$$B^* = \frac{2\pi \cdot (C \times A)}{A \cdot (B \times C)}$$

$$(C \times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & c \\ \frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$= \hat{i}(0 - \frac{ac}{2}) - \hat{j}(-\frac{ac\sqrt{3}}{2})$$

$$= \left(-\frac{ac}{2}\right)\hat{i} + \left(\frac{ac\sqrt{3}}{2}\right)\hat{j}$$

$$B^* = \frac{-\pi ac \hat{i} + \pi ac\sqrt{3} \hat{j}}{\sqrt{3}/2 a^2 c}$$

$$= \frac{-2\pi}{\sqrt{3}} \cdot \frac{1}{a} \hat{i} + \frac{2\pi}{a} \hat{j}$$

$$= \frac{2\pi}{a} \left( \frac{-1}{\sqrt{3}} \hat{i} + \hat{j} \right)$$

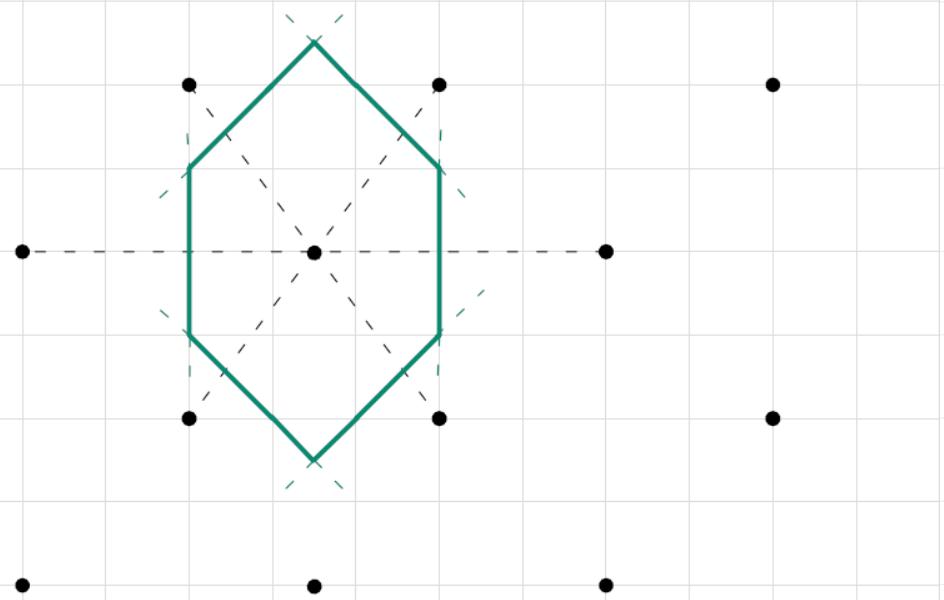
$$C^* = \frac{2\pi \cdot (A \times B)}{A \cdot (B \times C)}$$

$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \\ -\frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$\begin{aligned}
 &= \hat{k} \left( \frac{\alpha^2 \sqrt{3}}{4} + \frac{\alpha^2 \sqrt{3}}{4} \right) \\
 &= \frac{\sqrt{3}}{2} \alpha^2 \hat{k}
 \end{aligned}$$

$$C^* = \frac{\pi \sqrt{3} \alpha^2 \hat{k}}{\sqrt{3}/2 \alpha^2 c} = \frac{2\pi}{c} \hat{k}$$

# Brillouin zone of the hexagonal Space lattice



(Q2) Lattice constant =  $4.3 \times 10^{-10}$  m = a

for (321), miller indices are  
 $h=3$ ;  $k=2$ ;  $l=1$

We know that the interplanar distance is,

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{4.3 \times 10^{-10}}{\sqrt{9+4+1}}$$
$$= \frac{4.3}{3.74} \text{ \AA} \approx 1.15 \text{ \AA}$$

We also know that constructive diffraction occurs only if  $2ds\sin\theta = n\lambda$

for first order reflection,  $n=1$

$$\begin{aligned} \text{So, } \lambda &= 2ds\sin\theta = 2 \times 1.15 \times 10^{-10} \times \sin(10^\circ) \\ &= 2.3 \times 0.1736 \times 10^{-10} \\ &\approx 0.4 \times 10^{-10} \text{ m} \end{aligned}$$

g3)

$$KE = 0.2 \text{ MeV}$$

$$V = 20 \text{ MeV}$$

note  $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$

$$E = 3.2 \times 10^{-14} \text{ J}$$

$$V = 3.2 \times 10^{-12} \text{ J}$$

$$L = 2.97 \times 10^{-18} \text{ m}$$

$$\hbar = 1.054 \times 10^{-34} \text{ Js}$$

$$m = 6.68 \times 10^{-27} \text{ kg}$$

We know,

$$T = \frac{1}{1 + \frac{v^2 \sinh^2(k'L)}{4E(v-E)}}$$

$$\text{where } k' = \sqrt{\frac{2m(v-E)}{\hbar^2}} = \sqrt{\frac{4.23 \times 10^{-38}}{1.11 \times 10^{-68}}}$$

$$= \sqrt{3.81 \times 10^{30}} = 1.95 \times 10^{15}$$

$$k'L = 5.8 \times 10^{-3}$$

$$\sinh^2(k'L) = 3.36 \times 10^{-5}$$

$$T = \frac{1}{1 + \frac{3.44 \times 10^{-28}}{4.05 \times 10^{-21}}} = \frac{1}{1 + (0.85 \times 10^{-7})}$$

$= \boxed{0.99}$  This is incorrect since we cannot get  $\sim 1$  tunnelling probability. Check last page ↓

(q4)  $\Delta x = 4\text{\AA} = 4 \times 10^{-10}\text{m}$

(a) Heisenberg's uncertainty principle ↴

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{0.527 \times 10^{-34}}{4 \times 10^{-10}}$$

$$\Delta p \geq 0.13175 \times 10^{-24} \simeq 1.32 \times 10^{-25} \text{ kg m s}^{-1}$$

(b)  $KE = \frac{p^2}{2m}$

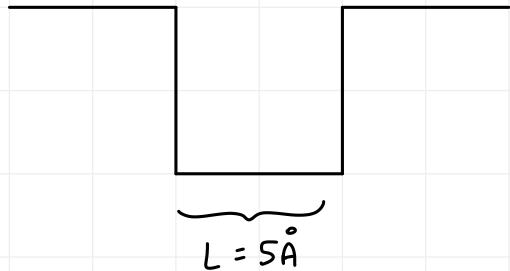
$$\frac{d(KE)}{dp} = \frac{1}{2m} \cdot 2p = \frac{p}{m} \rightarrow \Delta KE = \frac{p}{m} \Delta p$$

$$\Delta KE = \frac{2.4 \times 10^{-23}}{9.1 \times 10^{-31}} \times 1.32 \times 10^{-25}$$

$$= 0.26 \times 10^8 \times 1.32 \times 10^{-28}$$

$$= 3.432 \times 10^{-18} \text{ J}$$

(Q5)



(a) for  $\infty$  potential well,

$$E_n = \frac{k^2 \hbar^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

$$E_1 = \frac{\pi^2 \cdot (1.054 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 25 \times 10^{-20}}$$

$$= \frac{\pi^2 \cdot (1.11) \times 10^{-68} \times 10^{51}}{455} = 2.4 \times 10^{-19} \text{ J}$$

$$E_2 = 4 \times 2.4 \times 10^{-19} = 9.6 \times 10^{-19} \text{ J}$$

$$E_3 = 9 \times 2.4 \times 10^{-19} = 21.6 \times 10^{-19} \text{ J}$$

$$(b) \quad \Delta E = E_3 - E_2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{(21.6 - 9.6) 10^{-19}} = \frac{19.878 \times 10^{-26}}{12 \cdot 10^{-19}}$$

$$\lambda = \underline{1.656 \times 10^{-7} \text{ m}}$$

(Q3) note:  $1\text{eV} = 1.602 \times 10^{-19} \text{J}$

$$E = 3.2 \times 10^{-14} \text{ J}$$

$$V = 3.2 \times 10^{-12} \text{ J}$$

$$L = 2.97 \times 10^{-18} \text{ m}$$

$$\hbar = 1.054 \times 10^{-34} \text{ Js}$$

$$m = 6.68 \times 10^{-27} \text{ kg}$$

Since  $V \gg E \rightarrow$

$$T = 16 \left( \frac{E}{V} \right) \left( 1 - \frac{E}{V} \right) e^{-2KL}$$

(according to  
the book)

where  $K = \sqrt{\frac{2m}{\hbar^2} (V - E)}$

$$K = 1.95 \times 10^{15}$$

$$T = 16 \left( \frac{1}{100} \right) \left( 1 - \frac{1}{100} \right) e^{-2(1.95 \times 10^{15})(2.97 \times 10^{-18})}$$

$$T = (0.16)(0.99) e^{-11.583 \times 10^{-3}}$$

$$T = (0.16)(0.99)(0.988) = \underline{0.1565}$$

$n(r)$  : electronic density

$$n(r) = n(r+R)$$

$$n(r) = \sum n_q e^{ikr}$$

$$n(r+R) = \sum n_q e^{ik(r+R)} = \sum n_q e^{ikr} e^{ikR}$$

and since  $n(r) = n(r+R)$ ,

$$e^{ikR} = 1$$

$$\cos(kR) + j\sin(kR) = 1$$

$$\begin{matrix} \bar{k} \cdot \bar{R} = 2\pi \\ \text{k-space vector} \end{matrix} \quad \text{OR} \quad \bar{k}_i \bar{R}_j = 2\pi \delta_{ij} \quad \begin{matrix} \bar{k} \cdot \bar{R} = 2\pi \\ \text{Real space vector} \end{matrix}$$

$$1d: \quad R = \bar{a} \\ k = \bar{a}^*$$

$$aa^* = 2\pi \rightarrow a^* = \frac{2\pi}{a}$$

$$2d: \quad R = \bar{a} + \bar{b} \\ k = \bar{a}^* + \bar{b}^*$$

$$\begin{aligned} \bar{R} &= u\bar{a} + v\bar{b} + w\bar{c} \\ \bar{k} &= h\bar{a}^* + k\bar{b}^* + l\bar{c}^* \end{aligned}$$

3d

$$aa^* = 2\pi \quad bb^* = 2\pi \quad cc^* = 2\pi$$

but  $ba^* = 0$  and so on ...

$$ca^* = 0$$

$$\text{So, } a^* = x(b \times c) \\ \text{i.e. } a^* \perp b \text{ and } c$$

$$aa^* = x a \cdot (b \times c) = 2\pi$$

$$x = \frac{2\pi}{a \cdot (b \times c)}$$

$$a^* = \frac{2\pi(b \times c)}{a \cdot (b \times c)}$$

2d

$$aa^* = 2\pi$$

$$ba^* = 0$$

$$bb^* = 2\pi$$

$$ab^* = 0$$

$$a^* \perp b$$

$$a \parallel a^*$$

$$a^* = \frac{2\pi}{a} \hat{a}$$

$$b^* = \frac{2\pi}{b} \hat{b}$$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$= \frac{a}{\sqrt{b^2 + 36 + 9}} = \frac{a}{\sqrt{61}}$$

## # Effective mass :

$$\frac{1}{m_{ij}^*} = \frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k_i \partial k_j}$$

Curvature of the band determines the effective mass

If  $m^* = \infty$ , we are referring to  $e^-$   
for -ve, we are referring to holes

for Ge, 8 valley degeneracy for  $e^-$   
1 " for holes

$\rightarrow E_0$  ( $C_B \text{ min}$  is at  $x = L$ ) so,  
valley degeneracy = # of hexagon faces  
in the Wigner cell

for Si, 6  $\rightarrow e^-$   
1  $\rightarrow$  hole

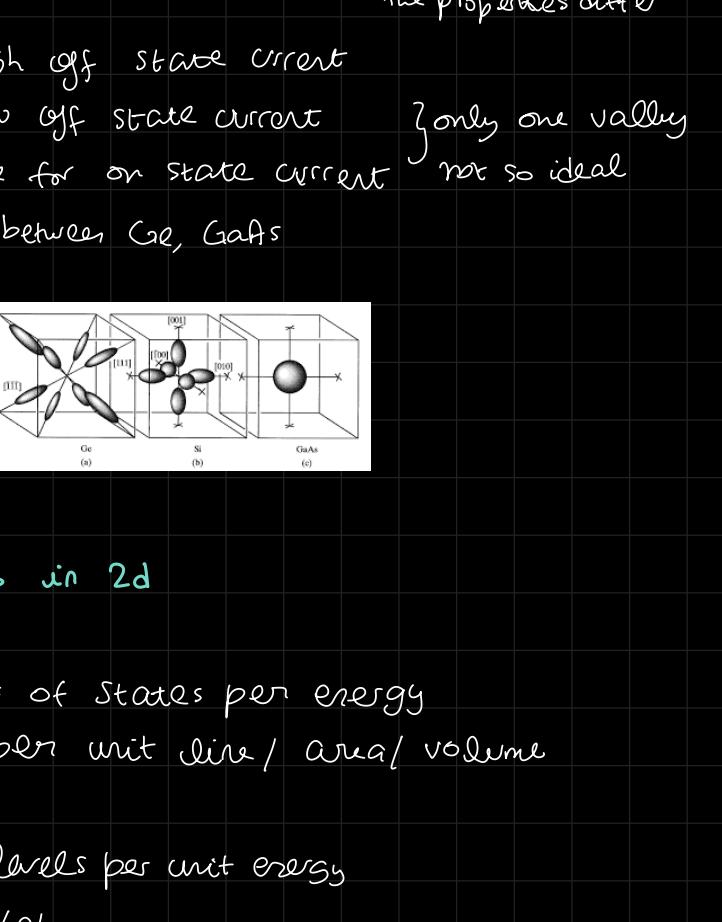
$\rightarrow$  # of square faces in the Wigner Seitz cell  
because  $C_B \text{ min} \Rightarrow x = r$

for GaAs, 1  $\rightarrow e^-$  (Direct band gap)  
1  $\rightarrow$  hole semiconductor

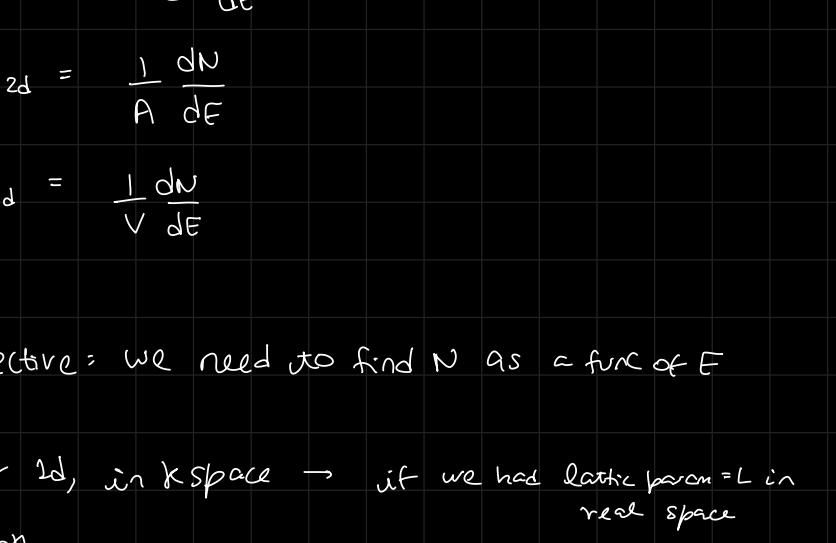
$\rightarrow$  # = 1 because  $C_B \text{ min} \Rightarrow x = r$  and we are  
at the center of the Wigner cell

$\rightarrow$  for all 3 cases,  $V_B \text{ max}$  is at  $x = r$  i.e. the  
center of the Wigner cell  $\rightarrow$  so, # valley  
degeneracy for holes = 1

how to get more current?



narrow curvature  $\rightarrow$  low effective mass  
wider curvature  $\rightarrow$  higher  $m^*$



In case of Ge, we have an equienergy surface

$\hookrightarrow$  isotropic effective mass

$\hookrightarrow$  direction doesn't matter

$$m_e^* = 0.98 m_0 \quad m_t^* = 0.19 m_0$$

longitudinal

transverse

very high  $m^*$

low  $m^*$

low mobility

higher mobility

band gap: Si  $\rightarrow$  1.12 eV      Ge  $\rightarrow$  0.7 eV      GaAs  $\rightarrow$  1.45 eV      } Crystal structure is almost similar but properties differ

Ge  $\rightarrow$  very high off state current

GaAs  $\rightarrow$  very low off state current and some for on state current } only one valley not so ideal

Si  $\rightarrow$  balance between Ge, GaAs



$\# \text{ of energy levels per unit energy per unit } l/a/v$

$$\overline{n} = \int \text{DOS}(E) f(E) dE \quad \xrightarrow{\text{fermi-dirac distribution}}$$

number of  $e^-/\text{holes}$  limit:  $0 \rightarrow \infty$  for  $e^-$

$-\infty \rightarrow 0$  for holes

$$\text{DOS}_{1d} = \frac{1}{L} \frac{dN}{dE}$$

$$\text{DOS}_{2d} = \frac{1}{A} \frac{dN}{dE}$$

$$\text{DOS}_{3d} = \frac{1}{V} \frac{dN}{dE}$$

\* Objective: we need to find N as a func of E

for 1d, in k space  $\rightarrow$  if we had lattice parameter = L in real space

lowest length in k-space (1st Brillouin zone)

$$-\frac{\pi}{L} \quad \frac{\pi}{L}$$

we just need the # of  $e^-$  in  $[-\frac{\pi}{L}, \frac{\pi}{L}]$

for 2d  $\rightarrow$  circle and 3d  $\rightarrow$  sphere

as we move on the Brillouin zones (1st  $\rightarrow$  2nd  $\rightarrow$  ...), we are increasing the # of states as well

for 2d  $\rightarrow$  area in one quadrant  $\rightarrow \pi L^2 \times \frac{1}{4}$

for 3d  $\rightarrow$  volume in 1 quad  $\rightarrow \frac{4}{3} \pi L^3 \times \frac{1}{8}$

$2e^-$  in every state/cube

$$\text{for 3d} \rightarrow N = 2 \times \left( \frac{1}{8} \times \frac{4\pi}{3} L^3 \right)$$

lowest volume of each state

$$\text{DOS}_{3d}(E) = \frac{\sqrt{2m}}{\pi^2 \hbar^3} \sqrt{E}$$

$$\text{DOS}_{3d} \propto m^{3/2} \text{ and also } \propto \sqrt{E}$$

$$f(E) \xrightarrow{\text{Fermi-Dirac distribution}}$$

$$g_{2d}(E) = \frac{m^*}{\pi^2 \hbar^2} \quad \} \text{ independent of } E$$

just moving from 3d  $\rightarrow$  2d, we obtain differently behaving DOS.

STRU

$f(E)$  we are multiplying  $f(E)$  and  $\text{DOS}(E)$

and we set #  $e^-/\text{hole}$  density

$$\text{DOS}(E) \cdot f(E) dE$$

$$n(E) = p(E) = n_i$$

intrinsic carrier concentration

$$\int \text{DOS}(E) \cdot f(E) dE$$

$$n_i^2 = n_i^2$$

# # Lecture after midsem

$$n = \int_{E_{\text{cmir}}}^{\infty} D(E) f(E) dE$$

$e^-$  concentration

$$p = \int_{-\infty}^{E_{\text{vmax}}} D(E) (1 - f(E)) dE$$

hole concentration

for an intrinsic semiconductor with effective mass of  $e^-$  = effective mass of holes

$$\hookrightarrow n = p \text{ because } f(E) = \frac{E_c + E_v}{2} = 1 - f(E)$$

$$f(E) = \frac{1}{1 + e^{(E - E_f)/k_B T}}$$

$$n_i = N_c f(E_c) \simeq N_c e^{- (E_c - E_f)/kT}$$

$\hookrightarrow$  equilibrium  $e^-$  carrier concentration

$N_c \rightarrow$  effective density of states for  $e^-$

$$N_c = 2 \left( \frac{2\pi m_n^* kT}{\hbar^2} \right)^{3/2} \rightarrow \text{for 3d}$$

$$(cm^{-3}/m^{-3})$$

$$p_i = N_v f(E_v) \simeq e^{- (E_f - E_v)/kT}$$

$\hookrightarrow$  equilibrium hole carrier concentration

$$N_v = 2 \left( \frac{2\pi m_p^* kT}{\hbar^2} \right)^{3/2} \rightarrow \text{for 3d}$$

$$(cm^{-3}/m^{-3})$$

$$m_n^* / m_p^*$$

$$m_{\text{DOS}}^* = \left( g_v^2 m_x m_y m_z \right)^{1/3}$$

#  $g_v \rightarrow$  valley degeneracy

$$g_{\text{od}}(E) = g_v 2SCE - E_c)$$

$$g_{\text{id}}(E) = g_v \frac{1}{\pi \hbar} \sqrt{\frac{m^*}{2(E - E_c)}}$$

$$g_{\text{2d}}(E) = g_v \frac{m^*}{\pi \hbar^2}$$

$$g_{\text{3d}}(E) = g_v \frac{(2m^*)^{2/3}}{2\pi^2 \hbar^3} \sqrt{E - E_c}$$

Si, Ge : Electric and Anisotropic in nature

for Si  $\rightarrow m_x = m_l$

$$m_y = m_z = m_t$$

$$g_v = 6$$

$$\text{So, } m_{\text{DOS}}^* = (6m_l m_t^2)^{1/3}$$

$$m_{\text{transport}}^* = \frac{3}{\frac{1}{m_x} + \frac{1}{m_y} + \frac{1}{m_z}}$$

↓  
transport  
effective

mass

if  $m_x = m_y = m_z \rightarrow m_{\text{transport}}^* = m_x$

MUSTANG

(  
isotropic case like GaAs

	<u>Si</u>	<u>Ge</u>	<u>GaAs</u>
$N_c$	$2.8 \times 10^{19}$	$1.04 \times 10^{19}$	$4.7 \times 10^{17}$
$N_v$	$1.04 \times 10^{19}$	$6 \times 10^{18}$	$7 \times 10^{18}$

off state current = leakage current

Silicon's off state  $I$  is less than that of Ge

for intrinsic semiconductor  $\rightarrow n_0 = p_0 = n_i = p_i \rightarrow$

$$n_i^2 = n_0 p_0 = N_c N_v e^{-(E_c - E_v)kT}$$

$$= N_c N_v e^{-E_g/kT} \quad \text{Law of mass Action}$$

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

→ exponentially decreasing with factor of band gap /  $2kT$   
 if  $E_g \gg \rightarrow n_i \ll$  very rapidly

equilibrium  $\rightarrow$  thermodynamically static/stable System

extrinsic = doped / impurity

now, intrinsic  $\rightarrow$  extrinsic : how?

trivalent: p type doping | pentavalent: n type doping

$f_p > n$       ↴       $n > f_p$       ↴

# Charge Neutrality condition (for any system)

$$\underbrace{p - n}_{\substack{\text{# of} \\ \text{carriers}}} + \underbrace{N_D^+ - N_A^-}_{\substack{\text{# of} \\ \text{ions}}} = 0 \quad \rightarrow p + N_D = n + N_A$$

Total charge = 0  
Electrically Neutral

$$\text{Intrinsic: } n_i p_i = n_i^2$$

$$\text{Extrinsic: } n p = n_i^2$$

Amphoteric Dopant: element which can act either as a donor or an acceptor

e.g.: Silicon for GaAs

acts as donor on Ga site

acts as acceptor on As site

for n type system  $\rightarrow n \gg p$

if  $N_D > N_A \rightarrow N_D + p = N_A + n \quad \leftarrow$   
 $N_D = n$

$$n = N_C e^{-\frac{(E_C - E_F)/kT}{}} = N_D$$

$$\frac{-(E_C - E_F)}{kT} = \ln\left(\frac{N_D}{N_C}\right)$$

$$\ln\left(\frac{N_C}{N_D}\right) = \frac{E_C - E_F}{kT}$$

$$E_C - E_F = kT \ln\left(\frac{N_C}{N_D}\right)$$

for higher donor conc, smaller the energy difference ( $E_C - E_F$ ), fermi level moves closer to the bottom of conduction band

for a p type system, if  $N_A > N_D$ ,

$N_A = p$  and similar like before,

$$E_V - E_F = kT \ln\left(\frac{N_A}{N_V}\right)$$

effective mass  $\rightarrow$  average  
transport mass  $\rightarrow$  harmonic mean

$$\mu = \frac{e}{m^*} \tau \rightarrow \text{relaxation}$$

mobility  $\leftarrow$

$m^*$   $\hookrightarrow$  effective mass

### # Carrier generation and Recombination Process

↳ Photogeneration light

↳ Phonon generation

↳ Impact Ionization collision

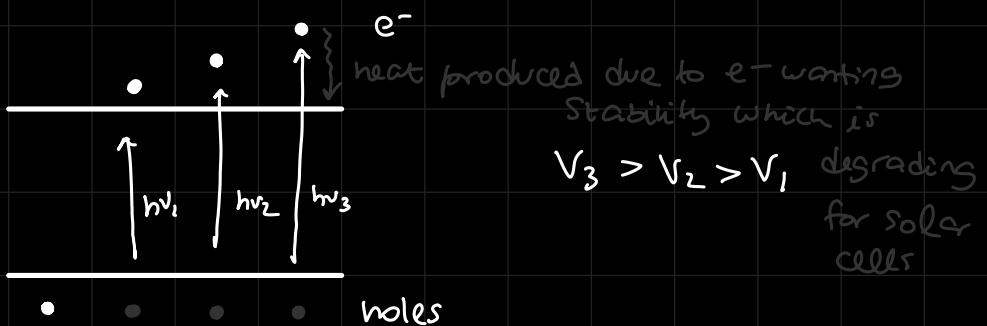
$\rightarrow$  Carrier generation: Process through which holes and  $e^-$  are generated

$\rightarrow$  Recombination: Process in which  $e^-$  and holes are annihilated

Temp  $\uparrow$   $\xrightarrow{\text{how?}}$  Energy  $\uparrow$   $\rightarrow e^-$  move to diff  $E$  states

① Photogeneration : light energy ( $h\nu$ ) is absorbed by  $e^-$  and if  $h\nu > E_g$ ,  $e^-$  jump from VB to CB and an  $e^-$ -hole pair is formed

if  $h\nu > E_g$



exciton :  $e^-$  and hole pair



if  $E_g < h\nu \rightarrow$  This coulomb attraction is overcome and this pair breaks

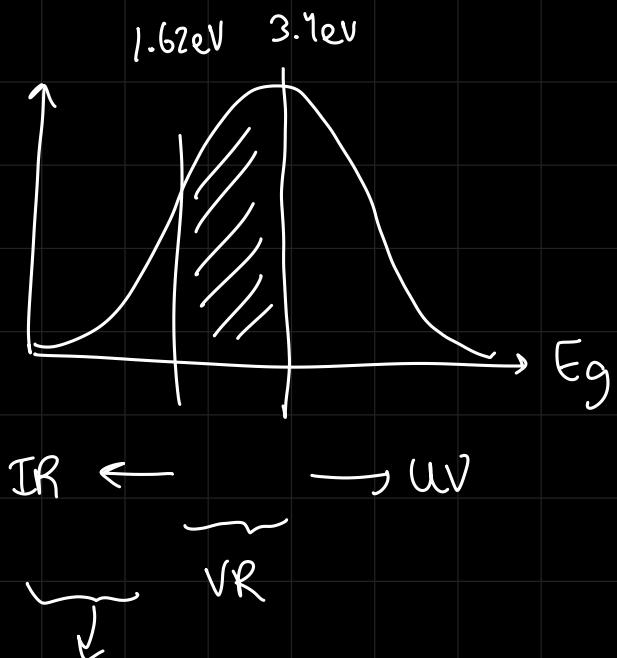
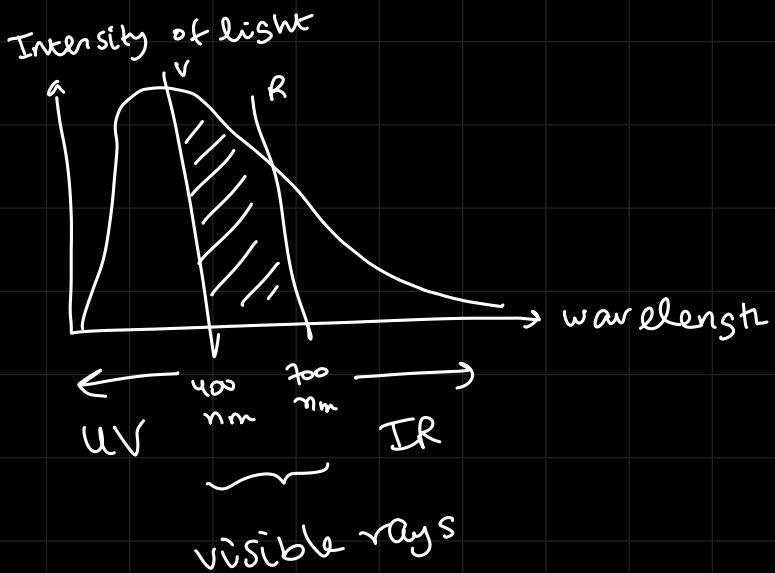
IP m<sub>2</sub> IHCl DC com / DSA BE P&S CO ISA / ELD M4 S&S CTD API FNW IE PSD ESP APA  
9 7 8 9 7 7 5 6 8 7 10 8 7 8 9 8 10 8 10 8 10 7

?

# # SOLAR CELL

Solar spectrum

$$E = hv = \frac{hc}{\lambda}$$



the order flips because

$$E \propto \frac{1}{\lambda}$$

Energy loss  $\rightarrow$  absorption of light energy ↓

$e^-$  and hole generation ↗  
less

for Si,  $E_g = 1.12$  eV

not good enough

## # PHONON GENERATION

occurs when a semiconductor is under thermal excitation.

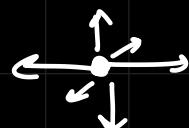
Temp  $\uparrow \rightarrow$  lattice vibrations increase leading to more photons

due to more lattice vibrations, covalent bonds in semiconductors break and hence e--hole pairs are generated

phonons  $\rightarrow$  quantized quasi particles like electrons

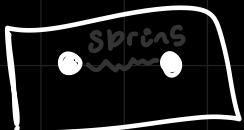
phonon branch/modes

In a 3 dimensional space, a particle can vibrate in atleast 3 directions ( $x/-x, y/-y, z/-z$ )

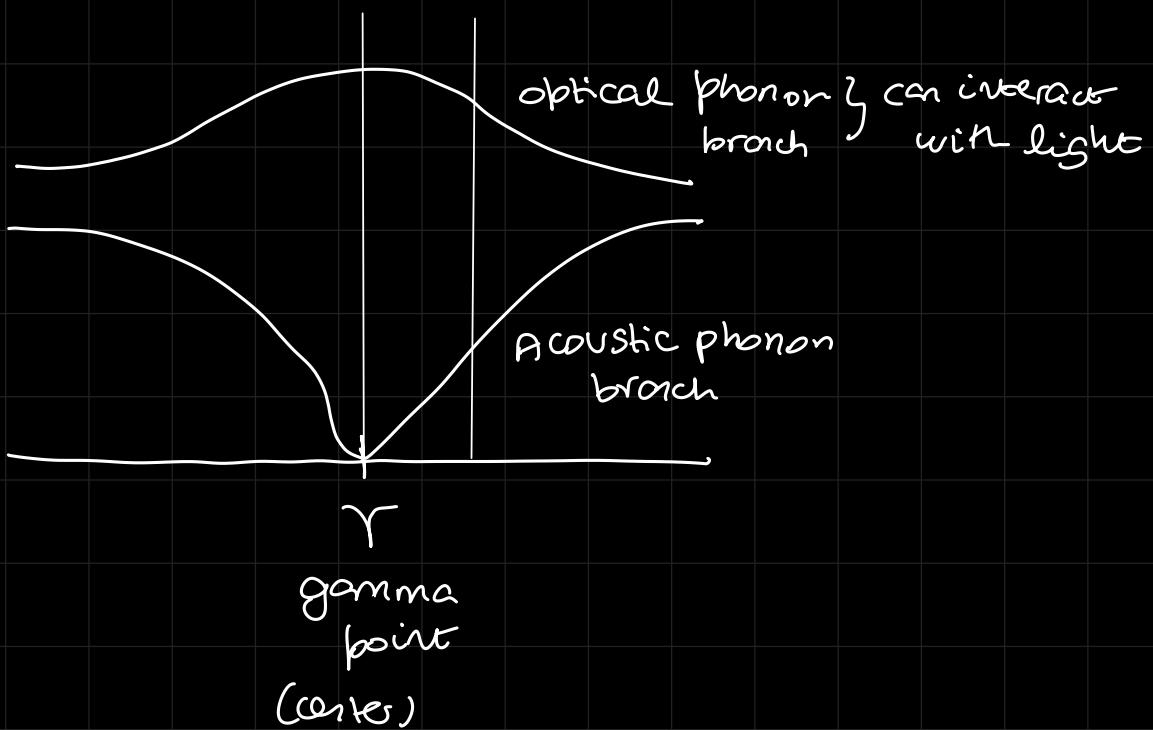
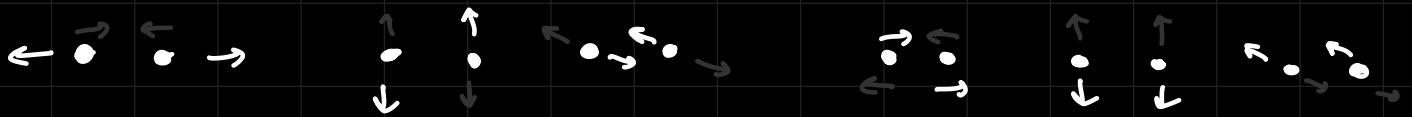


for a two particle system,

6 directions  $\rightarrow$  3 as a whole body



3 in a harmonic motion wrt each other



### ③ Impact Ionization :

When a semiconductor is under an electric field,  $e^-$  gain energy from the applied  $\bar{E}$  and hit other atoms.

A bond breaks generating more carriers

# Recombination

Free  $e^-$  in  
conduction band + Hole  $\xrightarrow{\text{recombination}}$  bound  $e^-$  in  
valence band

## ↳ Radiative Recombination

occurs for direct band gap semiconductors  
like GaAs

$e^-$  from CBM  $\rightarrow$  VBM w/o changing  
momentum and one photon of energy  
( $h\nu$ ) is emitted.

Energy is always released as radiation  
in case of Radiative Recombination

$e^-$  which are at higher energy  
states come down to CB by releasing  
energy as heat. Then from CB,  
photon is released when coming down  
to valence band.

Also called direct recombination.

for a blue LED, we need a material with a band gap of 3.4 eV

$$\text{GaN} \rightarrow E_g = 3.3 / 3.5 \text{ eV}$$

↳ p type doping is very difficult because Mg doping can only give P type GaN

Mg is transient element

↳ has d state

↳ but GaN has p state

↳ defect states are observed

↳ defect states are almost flat

↳ curvature  $\rightarrow 0$

↳ effective mass  $\rightarrow \infty$

↳ current  $\rightarrow 0$

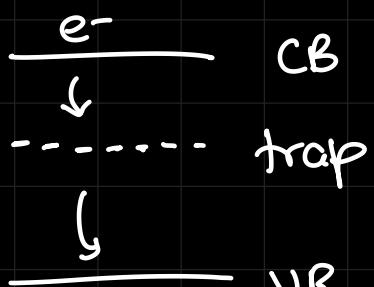
\* Instead of jumping from CB<sub>min</sub>  $\rightarrow$  VB<sub>max</sub>,

↓  
e<sup>-</sup> might jump to Defect states,  
stays there for some time  
and then it jumps down VB<sub>max</sub>

Shockley

Read

Hall Recombination



Slow process, generates

heat. not ideal for LED since light emitted ↓

### (C) Auger Recombination

occurs for heavily doped materials

3 carriers are involved.

an  $e^-$  and a hole recombine and the energy generated is given to the other  $e^-$  and it jumps to higher energy state, then it releases heat energy to come back to CB

MOSFET : surface dominating transport device

Hence the need to smoothen / minimize the irregularities on the surface

$$E \downarrow \rightarrow \mu \downarrow \rightarrow I \downarrow$$

for a DBGS  $\rightarrow$  Direct Recombination is efficient since  $k$  is conserved but not for an Indirect Band Gap semiconductor.

# # Contact and Junctions

17/03/25

$$\left. \begin{array}{l} \text{Si : } 1.12 \text{ eV} \\ \text{Ge : } 0.7 \text{ eV} \end{array} \right\} \text{band gap}$$

MOSFET

if the junction isn't prepared properly, we can observe a voltage barrier - i.e. voltage drop (not ideal)

2x

semiconductor

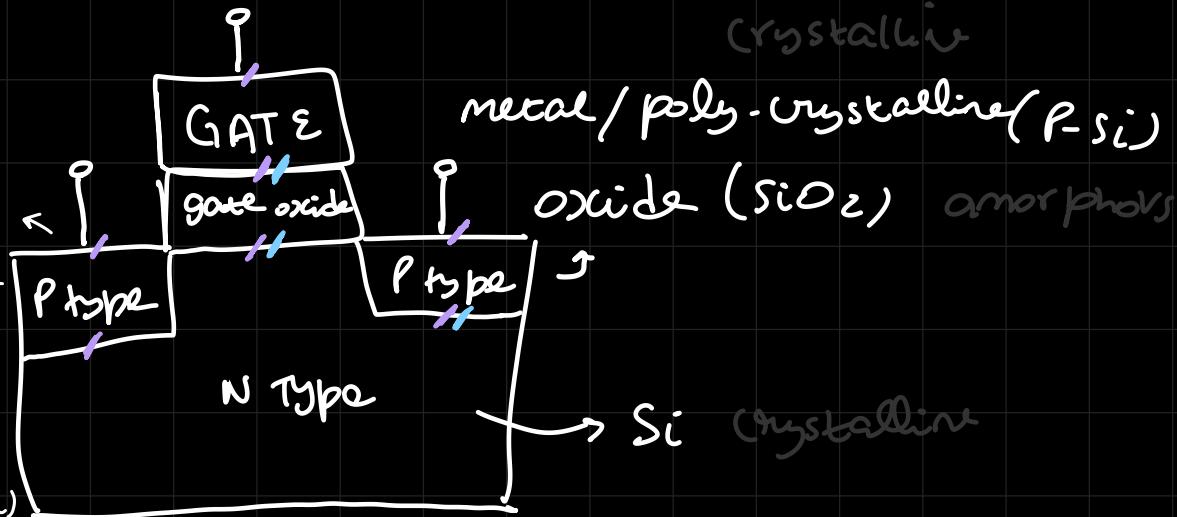
- semiconductor  $\rightarrow$  semiconductor  $\rightarrow$  metal - insulator

$$\text{# of Junctions} = 4 + 3$$

$$\text{# of Contacts} = 3$$

Junction resistance  $\propto$  voltage drop

for an ideal MOSFET, if we reverse the polarity of the current, it will change the direction but magnitude remains the same. But if there is junction resistance, we may or may not get same results



## # JUNCTIONS

- HOMOJUNCTIONS: Junction b/w 2 differently doped regions of the same semiconductor
- HETEROJUNCTION: between 2 different types of materials
- METAL-SEMICONDUCTOR JUNCTIONS

Metal - Semiconductor →



Ohmic ~ non rectifying contact



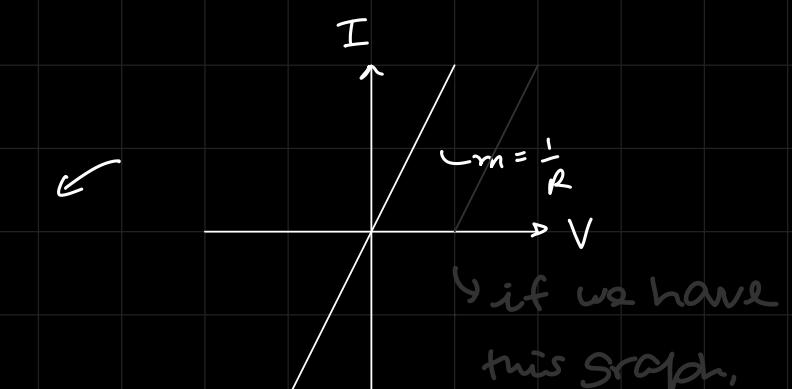
M-S junction

Schottky → Rectifying contact

Ideal Ohmic

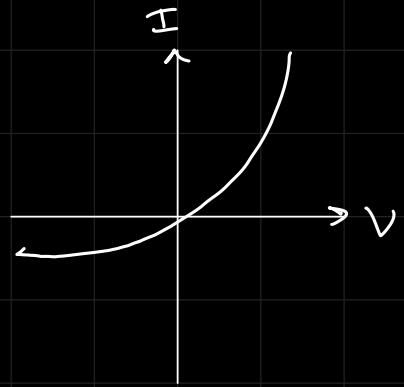
Contact

(Au - P-type Si)

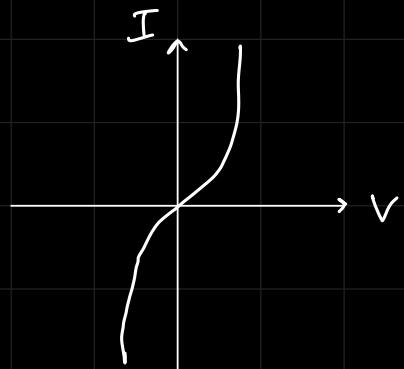


If we have this graph, this is also Ohmic but with some voltage drop  $V_c$ .

Schottky contact  
(Al - n type Si)



non-linear "ohmic" contact  
(Al -  $n^+$  type Si)  
heavily doped

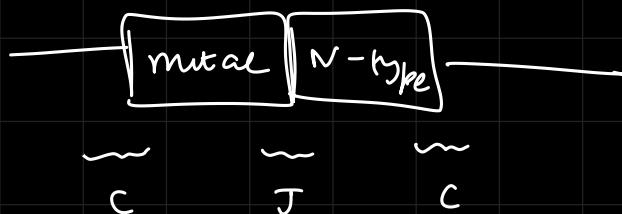


↳ linear for  
very small  $V$

Energy dissipation relation  
≡ E-K relation

visualized with an Energy Band Diagram

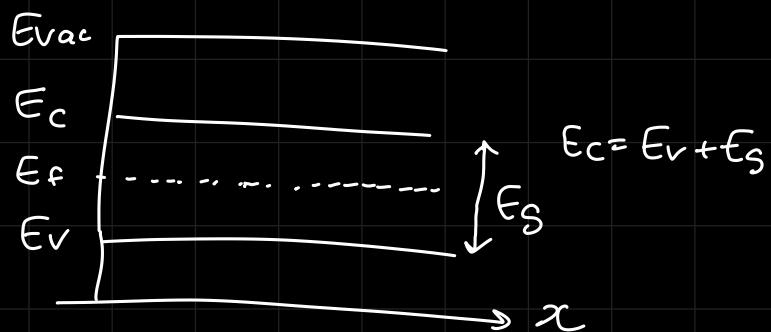
closely related to E- $\chi$   
(real space distance)



reference point for  $E$ - $\chi$  graph

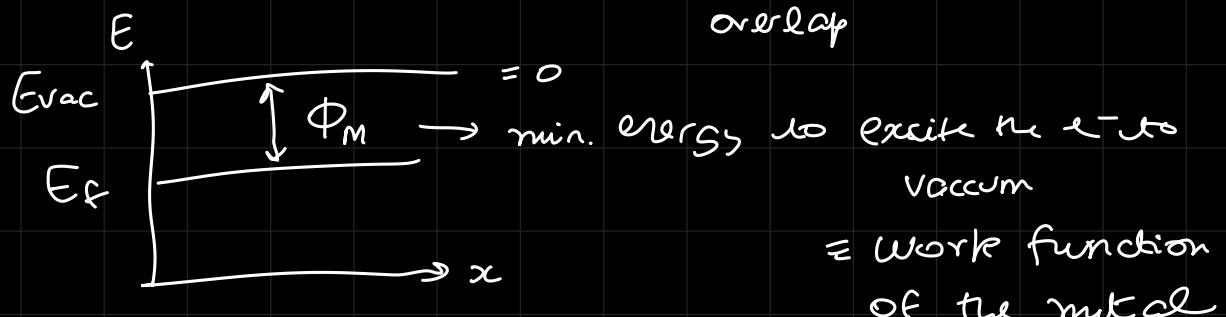
$$\hookrightarrow E_{\text{vac}} \Rightarrow E = 0$$

vaccum

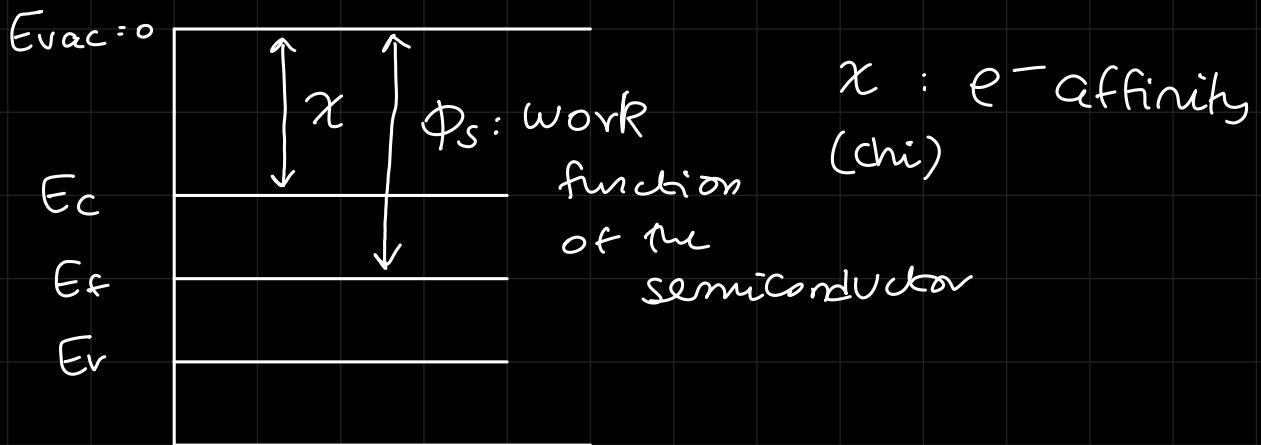


lets take a metal

$$(E_c \approx E_v \approx E_f)$$



Now, going back to the semiconductor

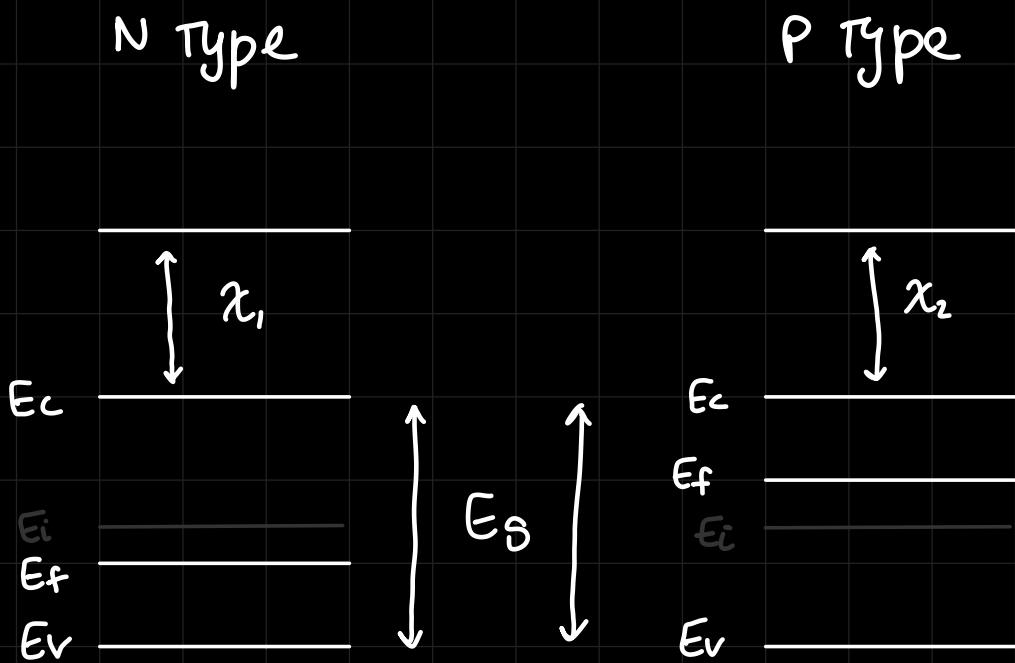


for semiconductors, we need 3 params  
 $\phi_s$ ,  $\chi$ ,  $E_g$

$\phi_s$  : work func can change with doping

$\chi$  and  $E_s$  : uniform properties

$$\phi_{s \text{ p-type}} > \phi_{s \text{ n-type}}$$

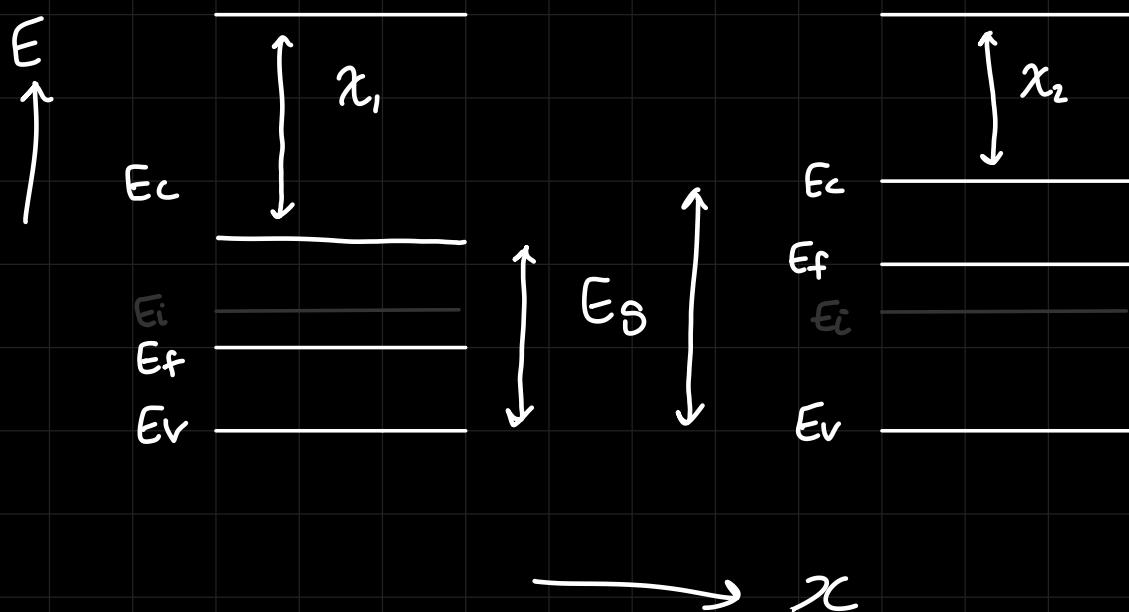


Some band gap

uniform  $\chi$  :  $\chi_1 = \chi_2$

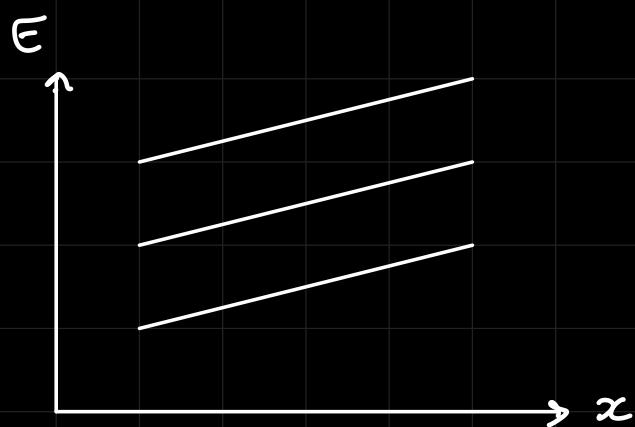
N Type

P Type



different  $E_g$

→ What if  $E_C, E_V, E_F$  are varying with  $x$

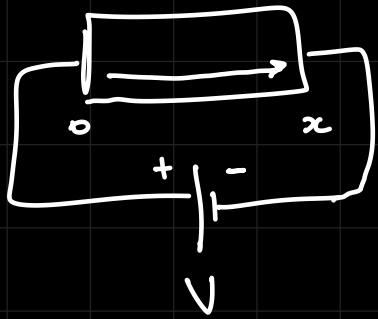


$$E = -\frac{dU}{dx}$$

if  $U = \text{const} \rightarrow E = 0$

elif  $U(x) \rightarrow E = \text{constant}$

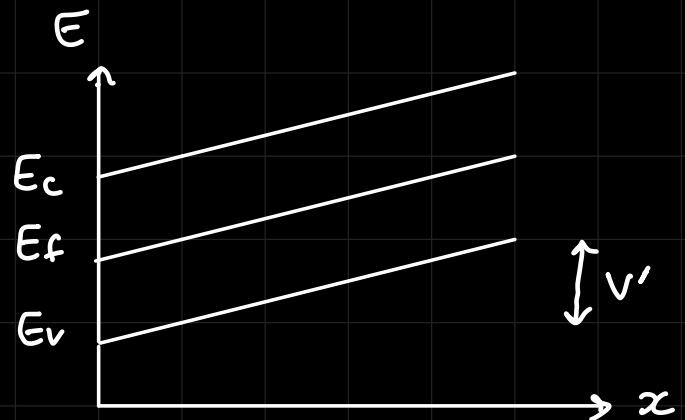
$$U = Kx$$



$$V = 0$$



$$V = V'$$



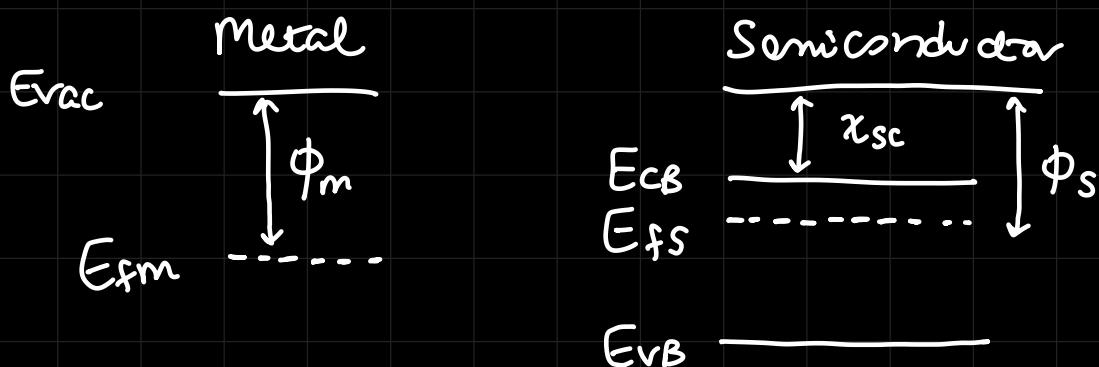
Energy Band Bending

Slope determines direction of  $\bar{E}$

## # Ideal MS contact

- no oxide layer between the metal & the semiconductor (no gap)
- no inter mixing and no inter diffusion between the metal and the semiconductor
- no impurities at the MS interface

Individually, M and S are at equilibrium but as a single system (junction), it is at non equilibrium state because there are 2  $E_F$  levels.



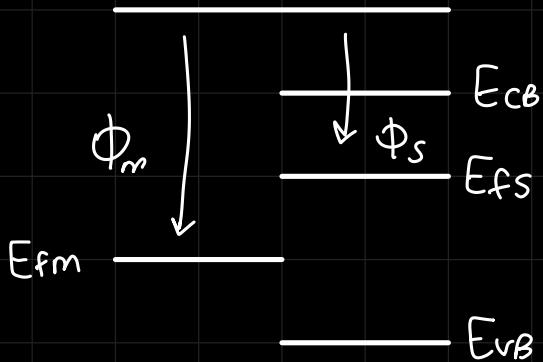
for equilibrium  $\rightarrow E_{fM} = E_{fS}$   
 ↴ tries to attain lowest energy level if not  
 at equilibrium

either  $E_{fM} \uparrow$  or  $E_{fS} \downarrow$  or simultaneously

metals have  $e^-$  and hence taking out  $e^-$   
 will not make much diff in  $E_{fM}$

but for semiconductors,  $E_{fS}$  can easily move

So, we assume  $E_{fm}$  as static and  $E_{fs}$  moves ↓

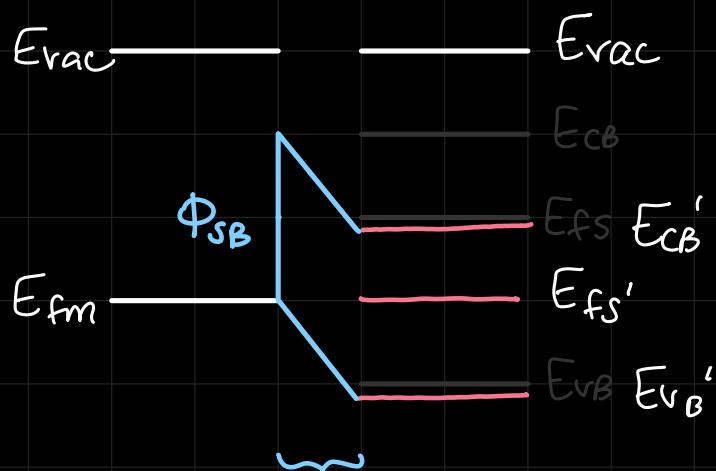


assuming n type semiconductor so,

$$\phi_m > \phi_s$$

$E_g, \chi_e$

if we want  $E_{fs}$  to go down,  $E_{CB}, E_{VB}$ , and even intrinsic fermi level should also go down that much so that the properties of the semiconductor remains the same.



We need to

remove e- from SC

Junction

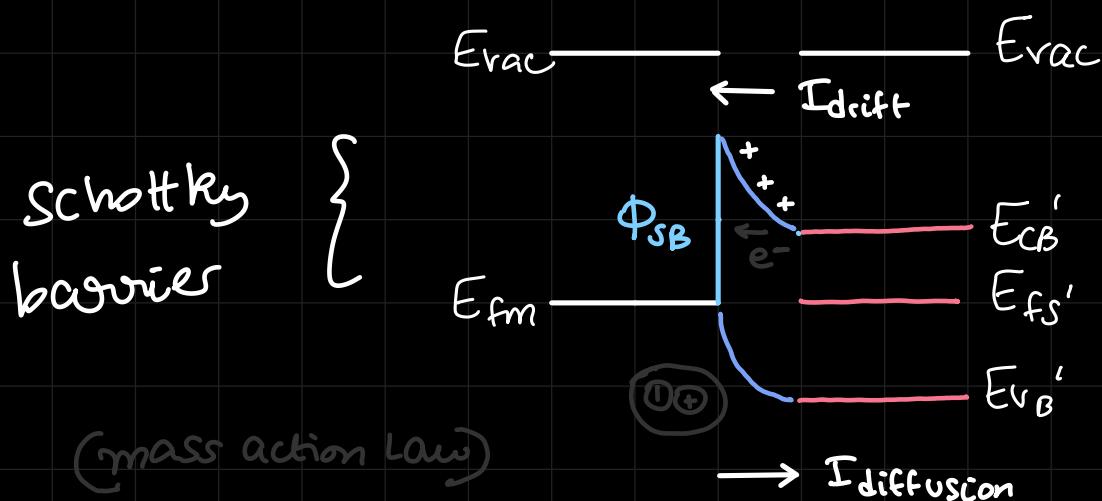
to make Ef go ↓

and those e- move from SC to e- until equilibrium is reached

- $e^-$  from semiconductor side diffuse to the metal side
- ↳ static ions appear
  - ↳ bond bending takes place

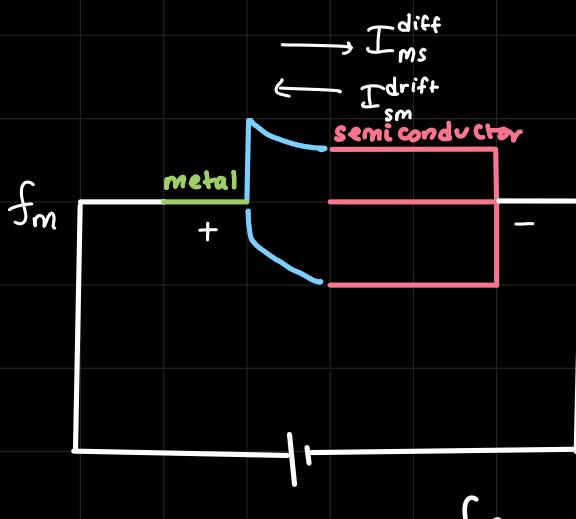
diffusion current is generated from  $SC \rightarrow M$   
(opposite to flow of  $e^-$ )

drift current generated from  $M \rightarrow S$



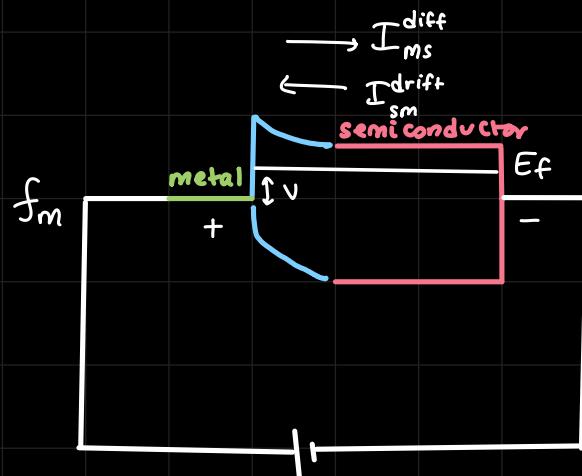
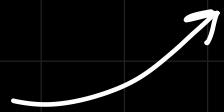
$M \rightarrow S$

To maintain equilibrium,  
holes are being generated  
at the  $E_{VB}$  side

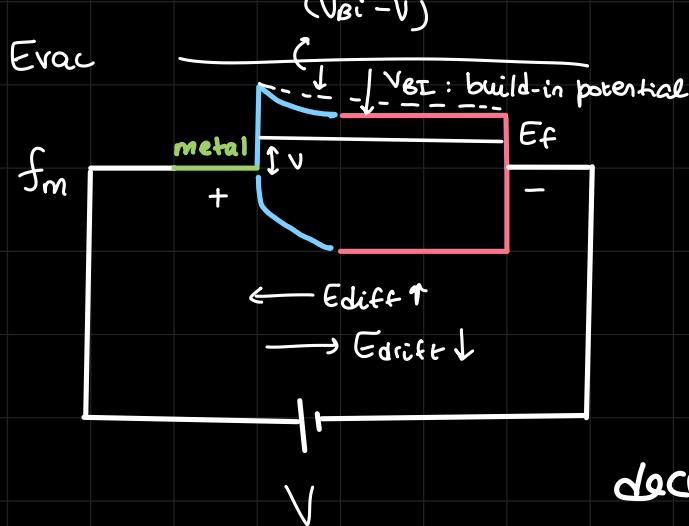


fermi level shifts due to this  
chemical potential

$$f(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

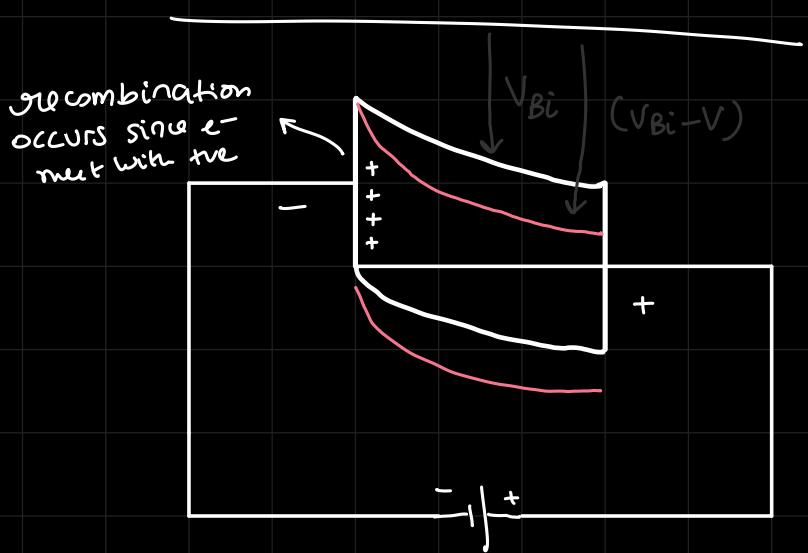


So other properties must change as well like the band bending



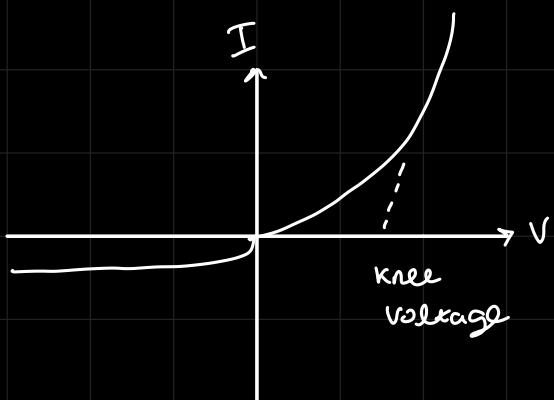
$$\bar{E} = \frac{dE_c(k)}{dk}$$

## # Reverse Bias Potential



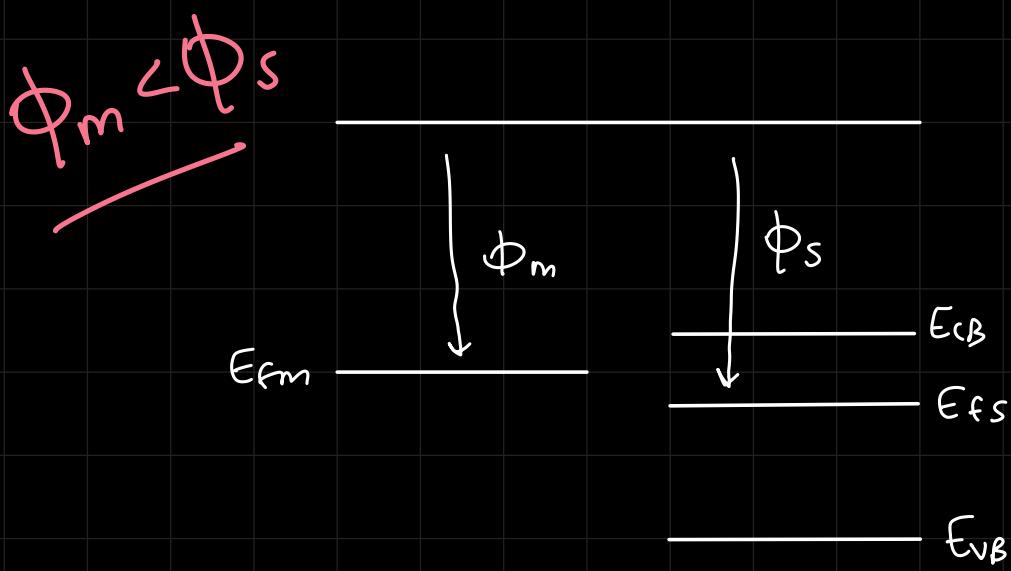
Reverse bias: intuitively fm↑ but it is eq to  
see  $fsc \downarrow$  rather than  $fme \uparrow$  and hence  
band bending ↑

IDiffusion: negligible



Schottky  
barrier

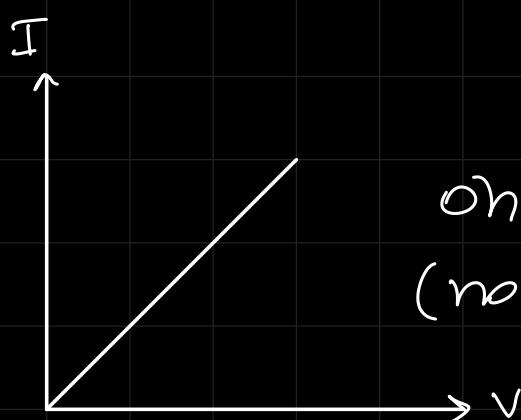
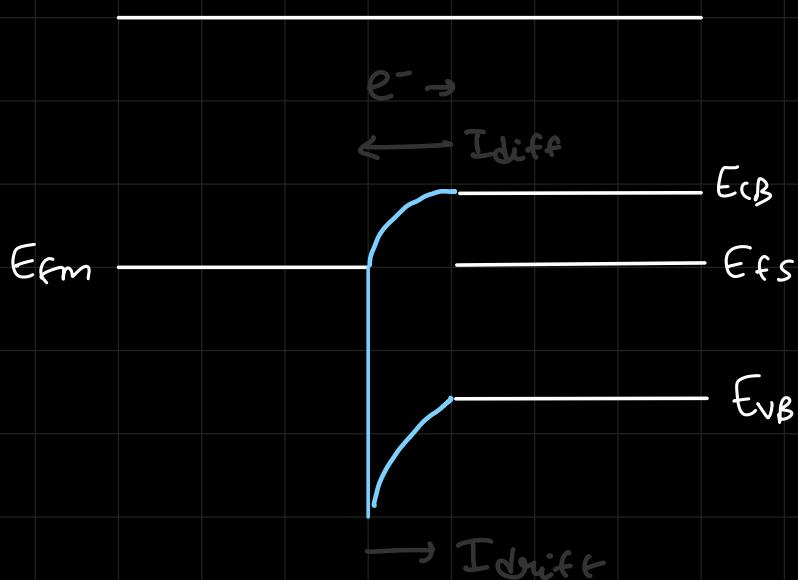
A PN Junction  
has a similar  
IV characteristic  
graph but a higher  
knee voltage



now  $E_{fs}$  should go up ↑

need  $e^-$

so,  $e^-$  travels from  $M \rightarrow S$



Ohmic  
(non rectifying)

$\Phi_m < \Phi_s \rightarrow$  Ohmic

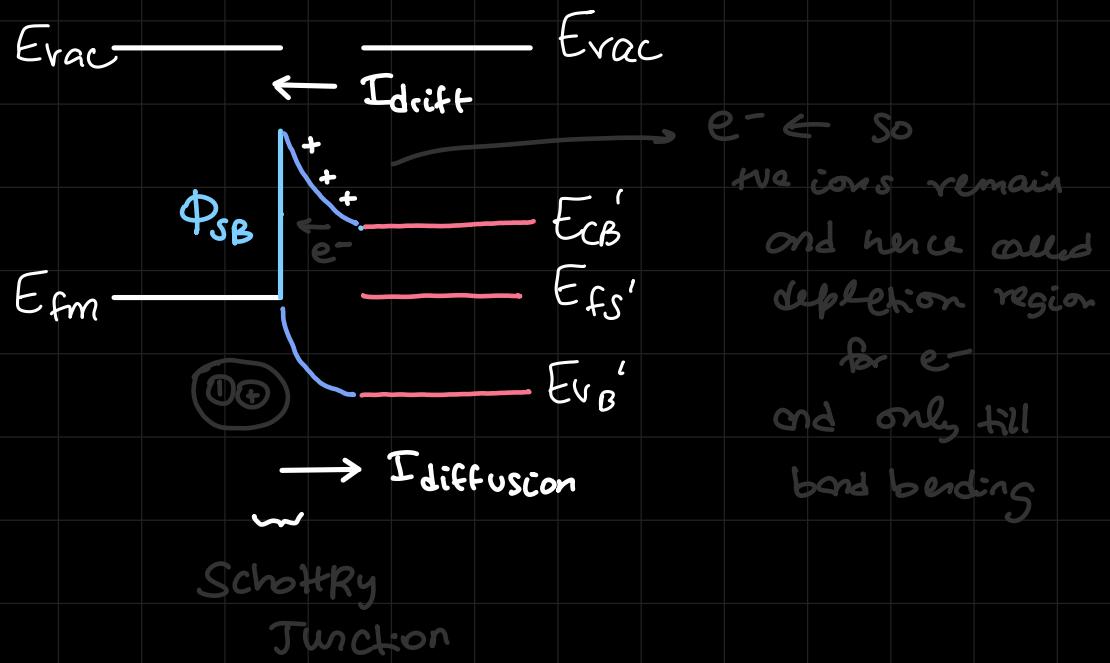
$\Phi_m = \Phi_s \rightarrow$  Ohmic ( $\sim 0$  drift current)

$\Phi_m > \Phi_s \rightarrow$  Schottky

We have taken lightly doped n type  
semiconductor

$\phi_m > \phi_s \rightarrow$  Schottky effect

SB  $\rightarrow$  Schottky barrier



Since  $I_{diffusion} > I_{drift}$

due to  $\leftarrow$   
motion of  $e^-$

$\hookrightarrow$  hence called a majority

carrier device because we  
have a N-type semiconductor

for a Schottky barrier  $\rightarrow$  majority carrier  
based device

Whereas most other are minority " " eg: MOSFET

$$\phi_{SB} = \phi_m - \phi_s = V_{bi}$$

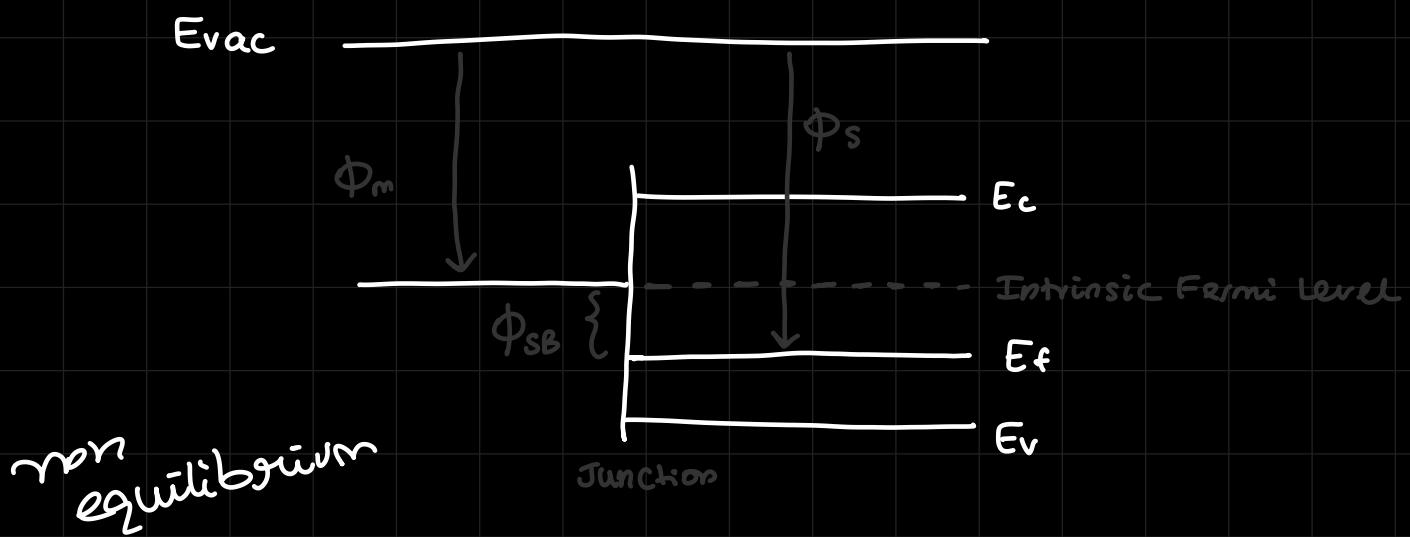
forward bias

- ↳  $V_{bi}$  decreases with  $V$
- ↳ depletion width decreases
  - ↳ barrier decreases
  - ↳  $e\tau$  for  $e^-$  to pass through
  - ↳  $I_{diff} > I_{drift}$
- ↳ low band bending

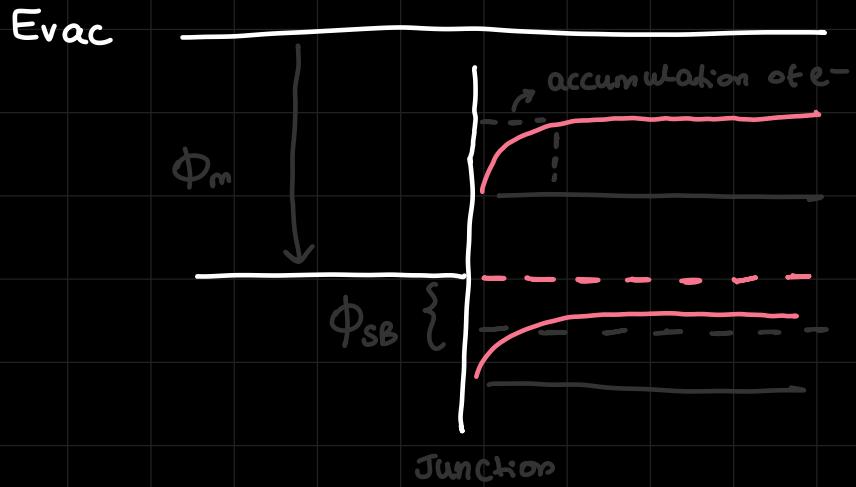
# for OHMIC CONTACT

- ↳  $\phi_m \leq \phi_s$

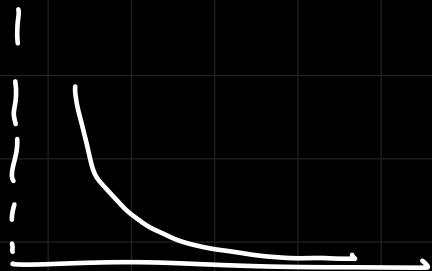
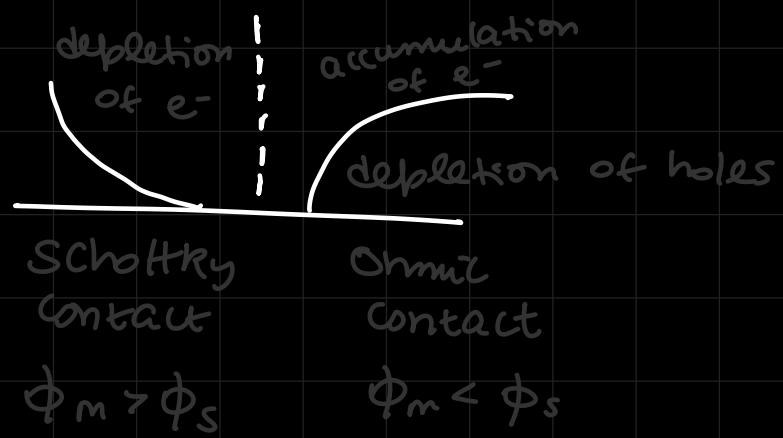
for P type sc  $\rightarrow$



equilibrium

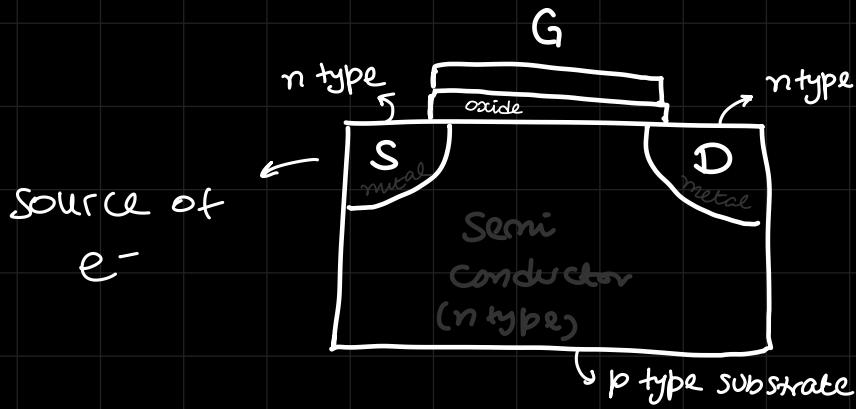


upwards bending



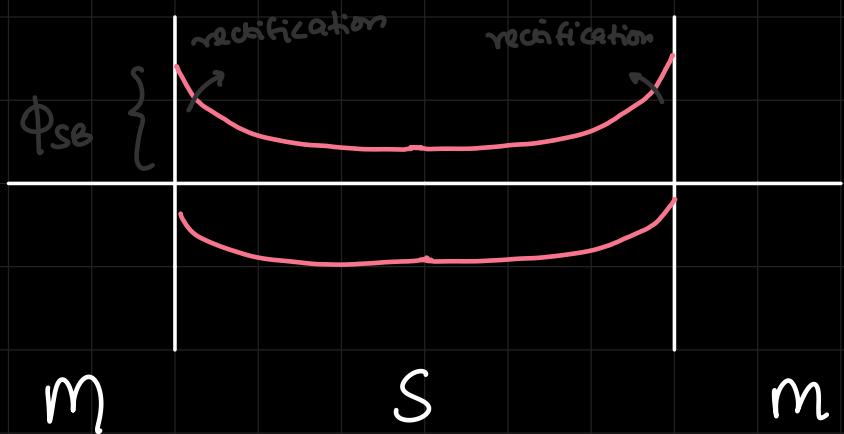
for ptype  $\rightarrow$

$$\Phi_m > \phi_s$$



lets replace the SC  
in SRL & drain by a  
metal in a MOSFET  
Which contact would we  
need?

↳ Schottky  
(~PN Junction)



### NPN Junction

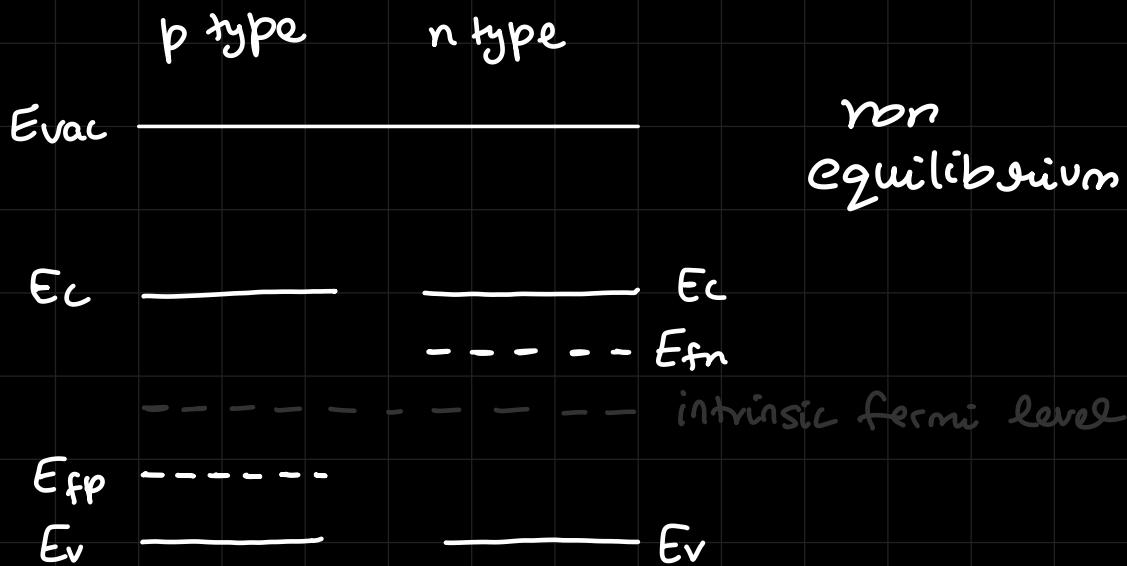
similar to BJT besides  
the C, B, E region width

Quasi Fermi Level :  $E_f$  splits when bias applied

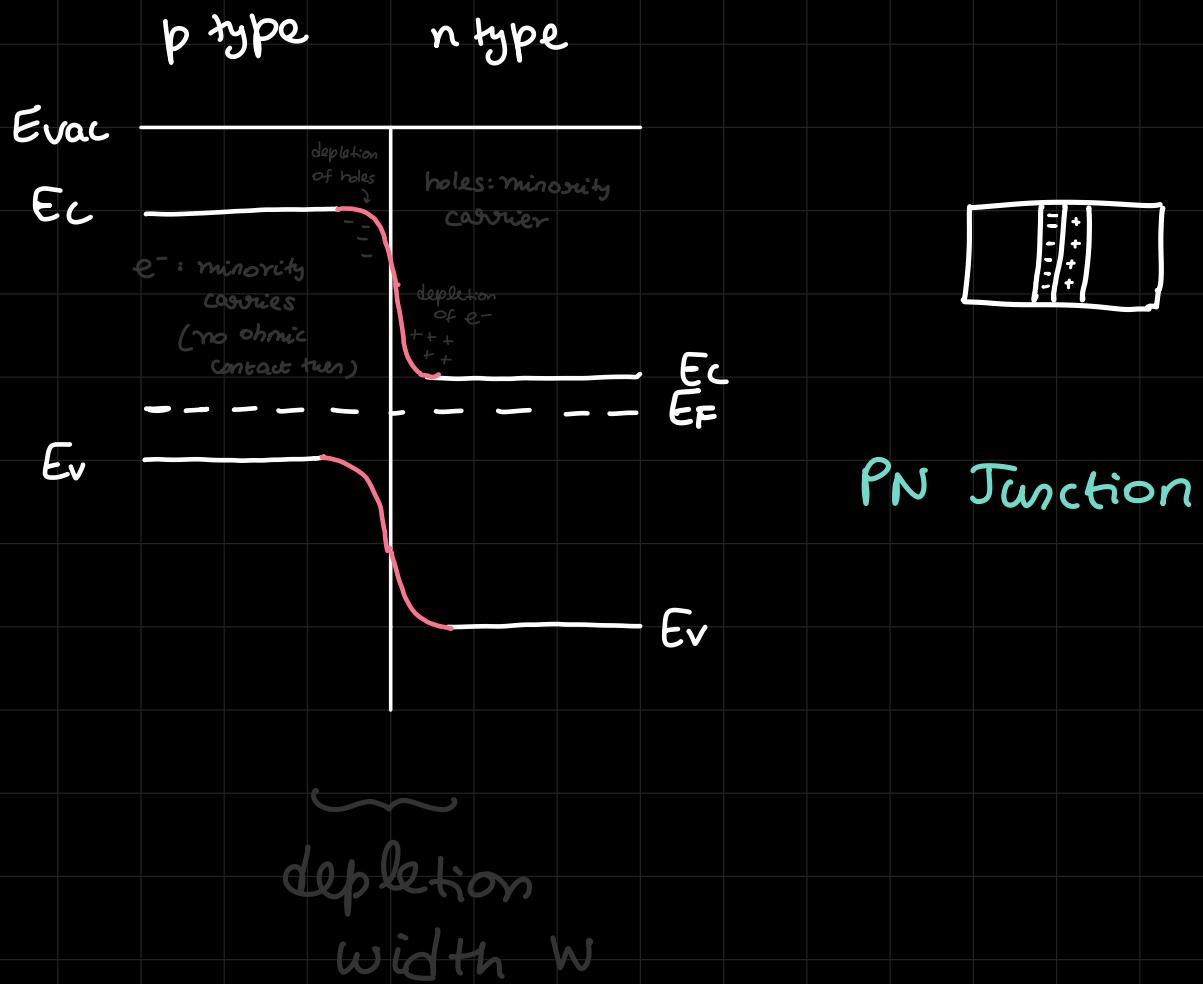
# HOMO JUNCTION : between same SC but could have  
different doping

some material  $\rightarrow$  some e- affinity  
and some  
band gap

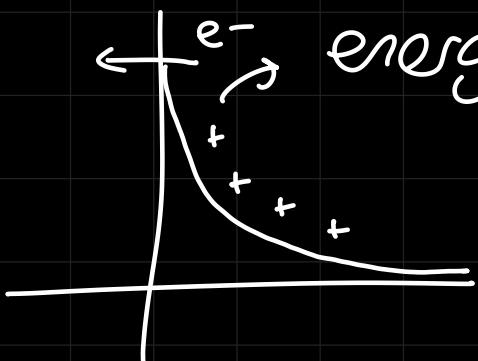
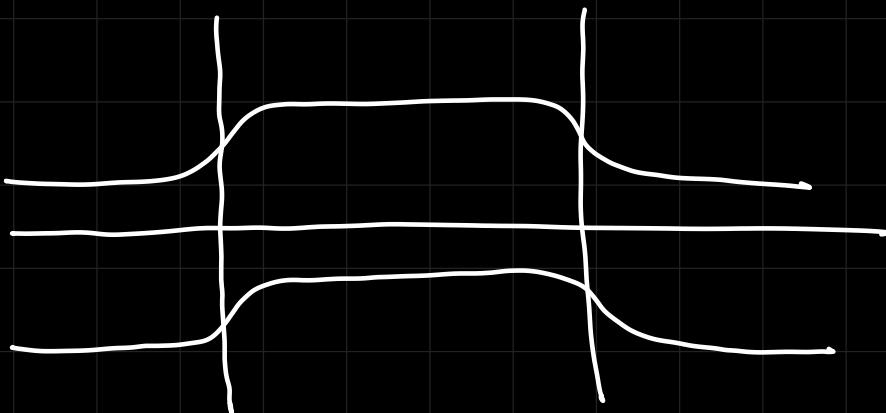
$$\chi_e \\ (E_{vac} - E_C)$$



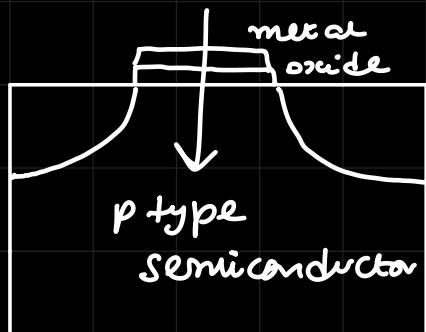
equilibrium



for npn MOSFET



energy goes down  
e<sup>-</sup> would want to go to the other side  
hence depletion of e<sup>-</sup>



Longitudinally,

M → I → S  
(insulator)

In MSM, replace S → I and 2nd M → S  
and we achieve the MIS junction

metal



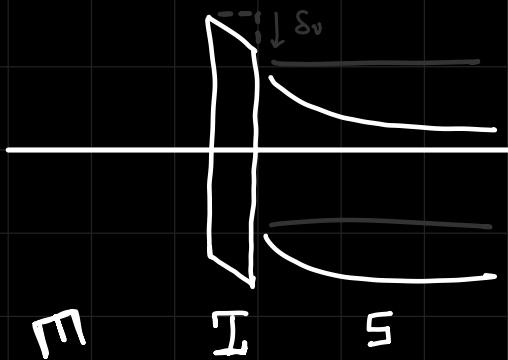
Semi-conductor

Insulator  $\rightarrow$  barrier  $\rightarrow$  potential

drop

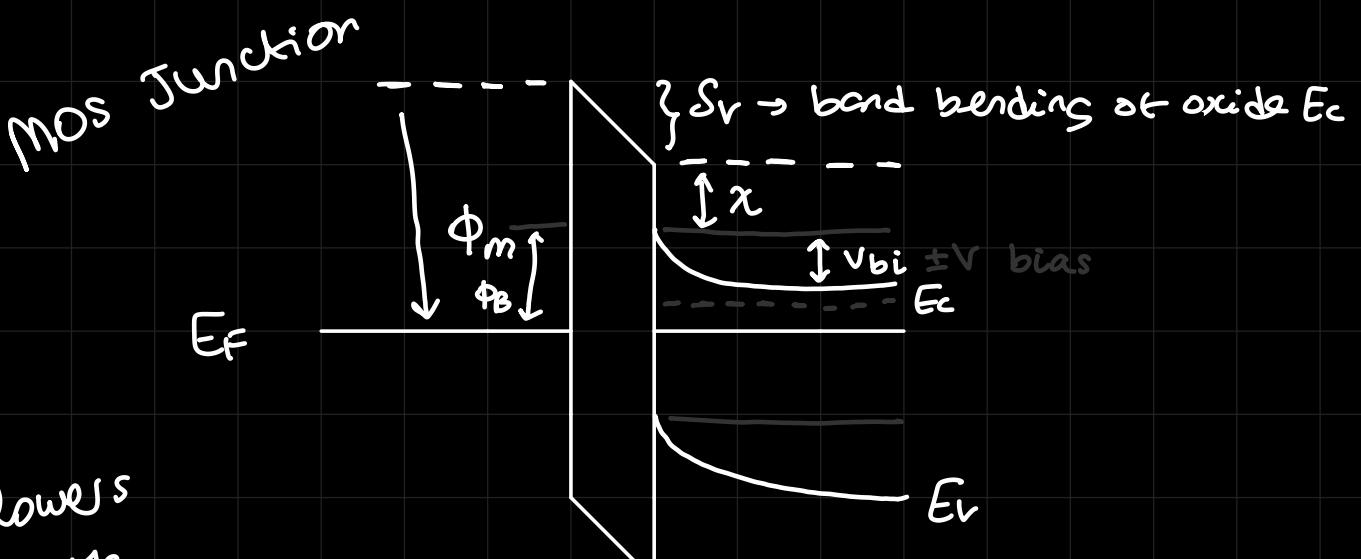
earlier:  $\phi_{SB} = \phi_m - \phi_s$

now:  $\phi_{SB} = \phi_m - \phi_s - \delta_v$



# Defects: Any impurity

Fermi Level pinning



lowers  
schottky  
barrier  
height  
(not ideal)

metal oxide n type  
(insulator) semi conductor

## # Fermi Level Pinning

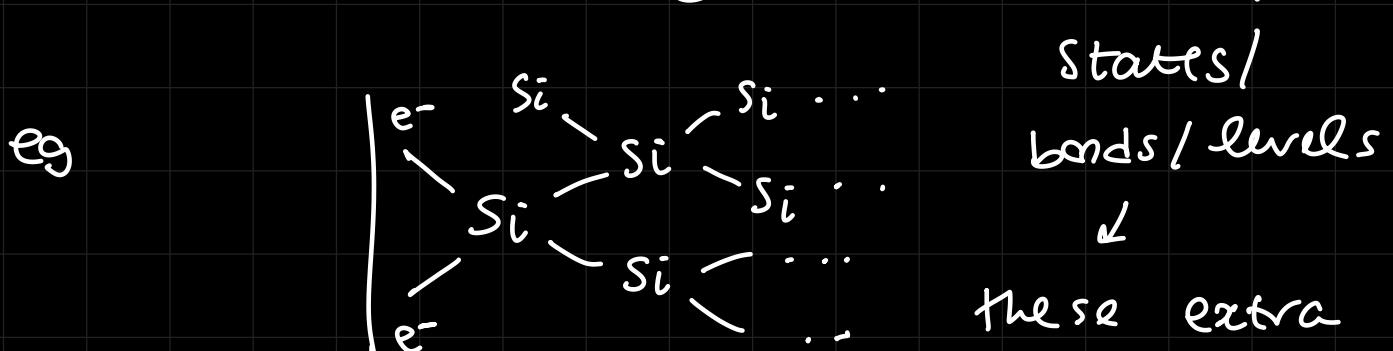
metal induced gap states ( $\approx$  impurities)

↳ another origin of interface states that

results in "metallic" screening

↳ FERMI LEVEL PINNING

Interface unsaturated bonds ↳  
possibility of dangling bonds at the  
surface → excess e- → results in extra



Surface

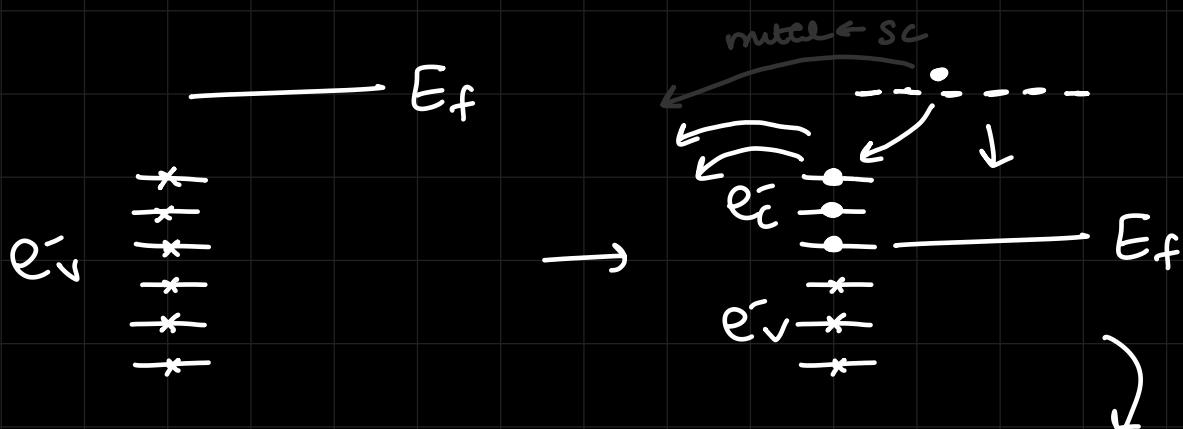
these extra  
levels remain  
within the band gap

it is observed that the impurity states take up  $\frac{2}{3}$ rd of the band gap

We have the Fermi level at the mid band of band gap  $\rightarrow$  at  $\frac{E_g}{2}$

and we are shifting this  $E_f$

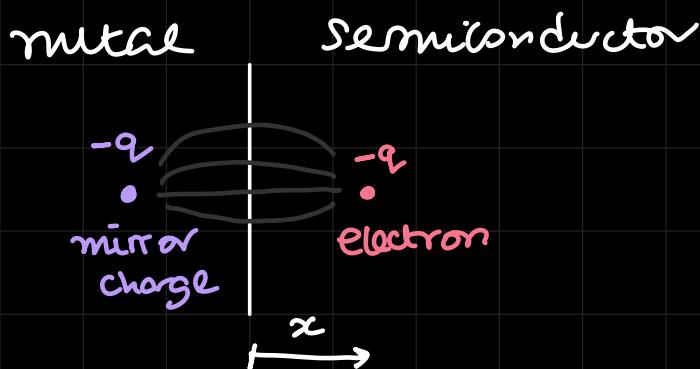
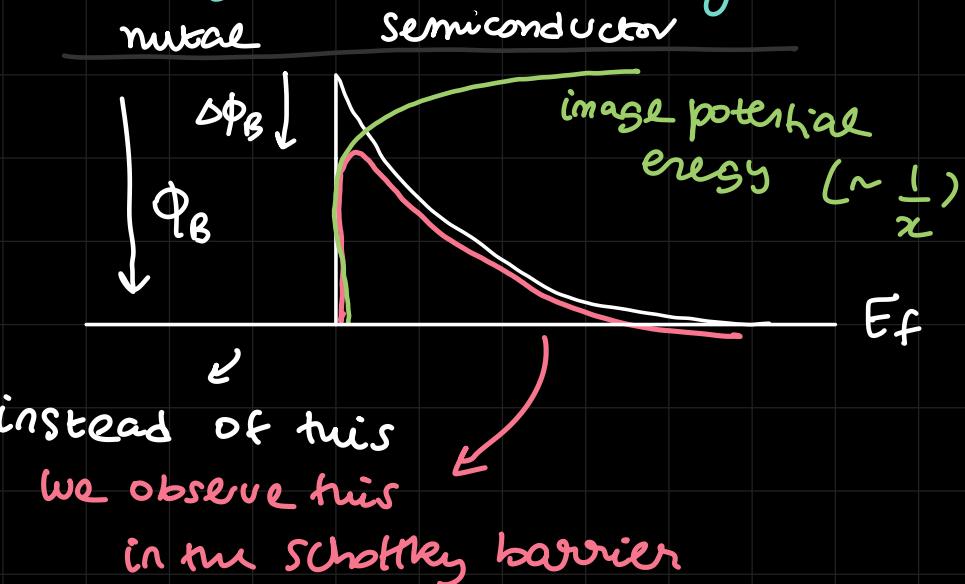
At equilibrium in the M-S junction, the system must maintain charge neutrality and hence some of the conduction  $e^-$  (free  $e^-$ ) go to the metal side after the Fermi level converts the valence  $e^-$  to conduction  $e^-$  when coming down.



those  $e^-$  that leave  $E_f$  and fill the upper positions, due to them the  $E_f$  which it is filled and hence  $E_f$  goes up ↑

Due to this, the Fermi level is restricted

# # modification to the Barrier Potential due to Image Force Lowering

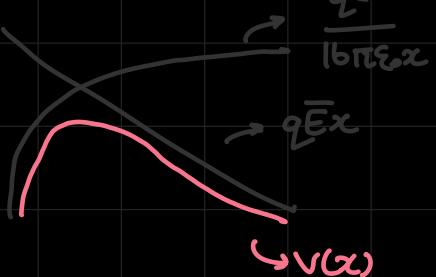


$$F = \frac{1}{4\pi\epsilon_0} \times \frac{q^2}{(2x)^2} = \frac{q^2}{16\pi\epsilon_0 x^2}$$

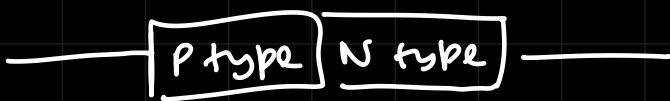
$$V = - \int \vec{F} \cdot d\vec{x}$$

$$V(x) = \left( \frac{q^2}{16\pi\epsilon_0 \cdot x} - qEx \right)$$

Under an Electric Field

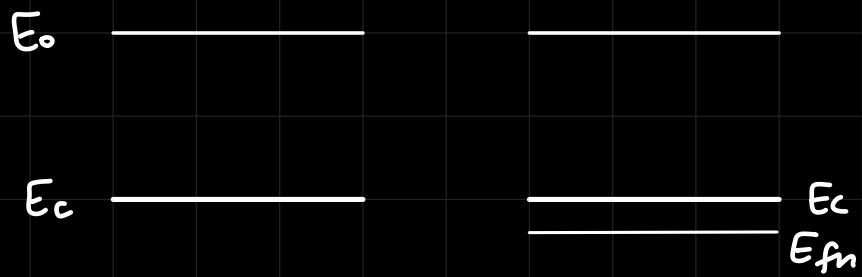


# Homojunction

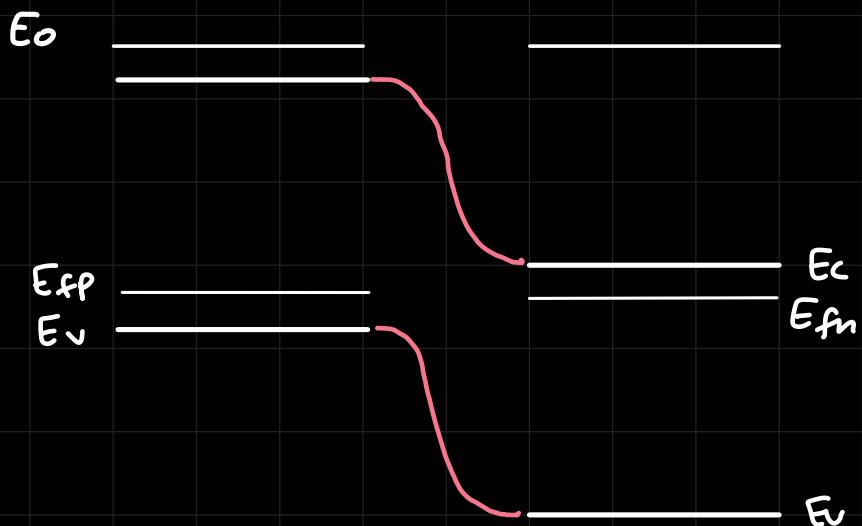


Heavily Doped Case

NON EQUILIBRIUM

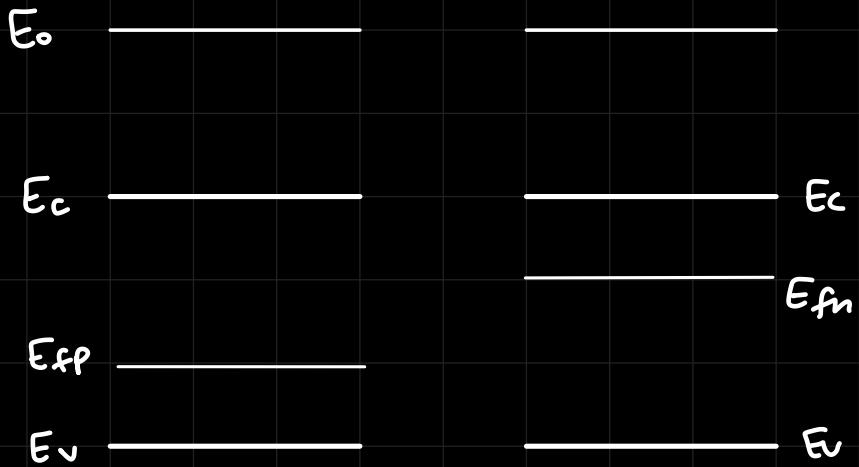


EQUILIBRIUM

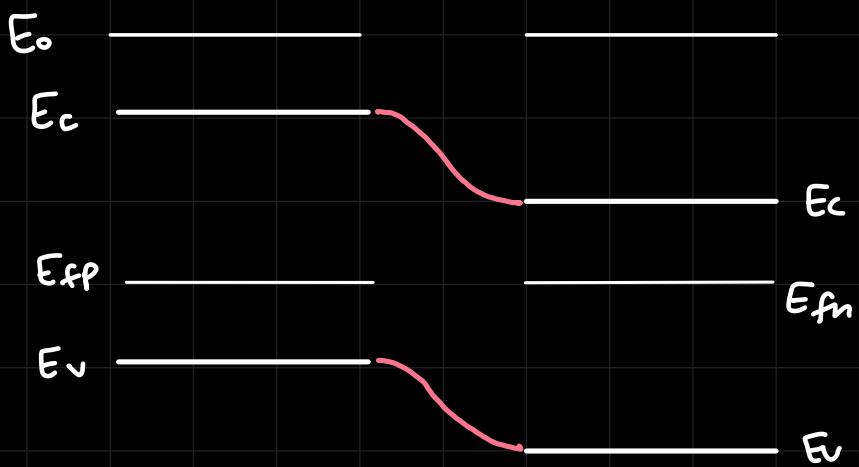


moderately doped case

NON EQUILIBRIUM



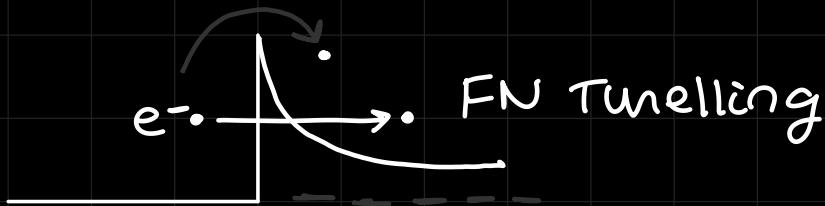
EQUILIBRIUM



The heavily doped one will have a higher  $\bar{E}$  and drift current because of more band bending

In the case of MS Junction, heavily doped  $\rightarrow$  band bending  $\uparrow \rightarrow \bar{E} \uparrow \rightarrow$  drift current  $\uparrow$  (easier to drift to the SC side)  
 $\hookrightarrow$  high amount of force on  $e^-$  on metal side  
 $\downarrow$

Instead of all  $e^-$  passing the potential barrier from above, some "tunnel" through barrier



Homojunctions usually don't have dangling bonds and hence <sup>(less)</sup> no impurity  
 $\hookrightarrow$  lesser probability of observing fermi level pinning

\$ GaAs  $\rightarrow$  3,5 system  
 $\hookrightarrow$  too much impurity levels  
 $\hookrightarrow$  not ideal

$e^-$  movement

↳ drift

↳ diffusion

↳ tunnelling

Zener diode: reverse bias

heavily doped

very high band bending

very high  $E$

tunnelling ✓

in breakdown region

non equilibrium

$E_C$  —————

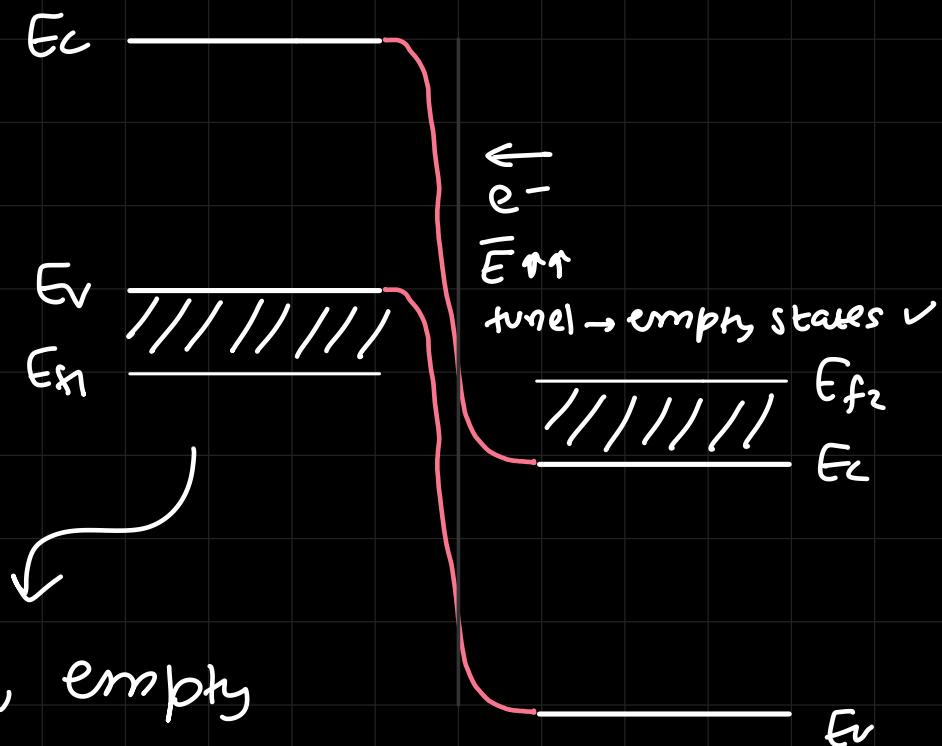
—————  $E_{f2}$

$E_C$

$E_V$  —————

—————  $E_V$

$E_{X1}$  —————



above  $E_F$ , empty  
states

(no holes)

below  $E_F$ , filled  $e^-$  states

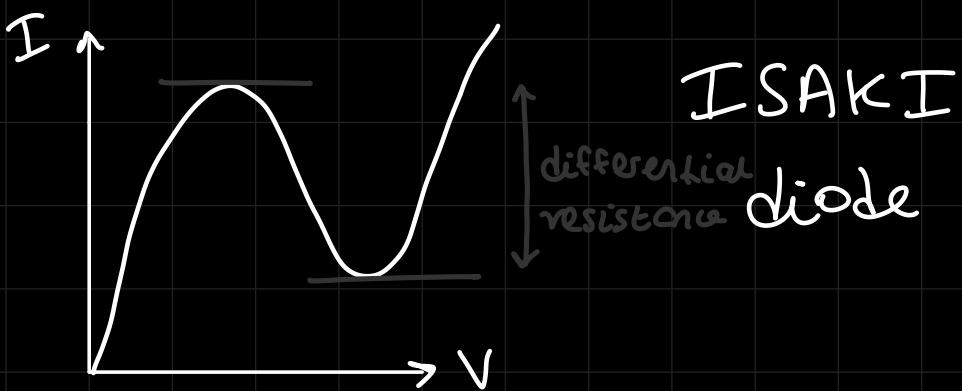
barrier height is very high ( $V_{bi}$ )

↳  $\sim$  no drift current

Apply a small amount of forward bias,  
 $E_F$  at the n side goes up along with it  
 $E_C$  and  $E_V$  to maintain band gap

or with n type at reference, p type  
goes down

due to the high  $\bar{E}$  the free  $e^-$  go toward  
the p types' empty states



non linear characteristics

$$\text{absolute Resistance} = \frac{V}{I}$$

$$\text{differential Resistance} = \frac{dV}{dI}$$

for the above characteristic, we have a region where current is reducing with increase in potential

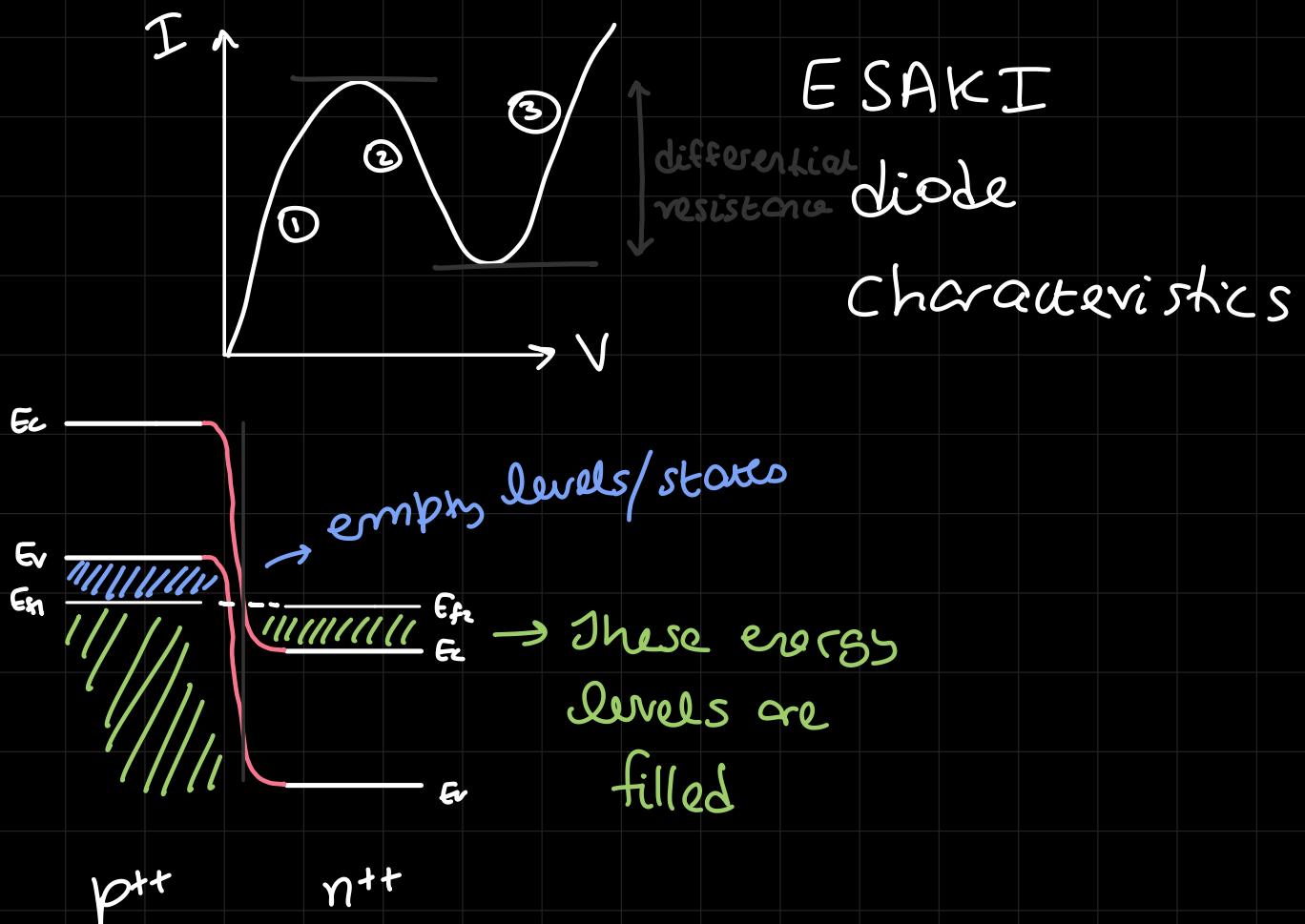
$$\text{so, we get } R = \frac{dV}{dI} (+)$$

$\equiv$  Negative Differential Resistance  
 $\equiv$  NDR diode

useful for amplifiers

Esaki diode  $\rightarrow$  degenerate semiconductor  
 (heavily doped  $\sim 10^{21}/\text{cm}^3$ )  
 $n^{++}/p^{++}$

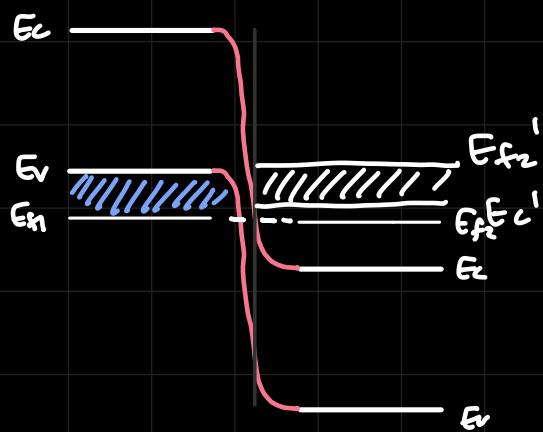
$E_F$  lies outside the band gap



When we potential applied,  $n^{++}$  side's fermi level goes up and some  $e^-$  from  $n$  side "tunnel" through the barrier towards  $p$  side and we observe maximum current

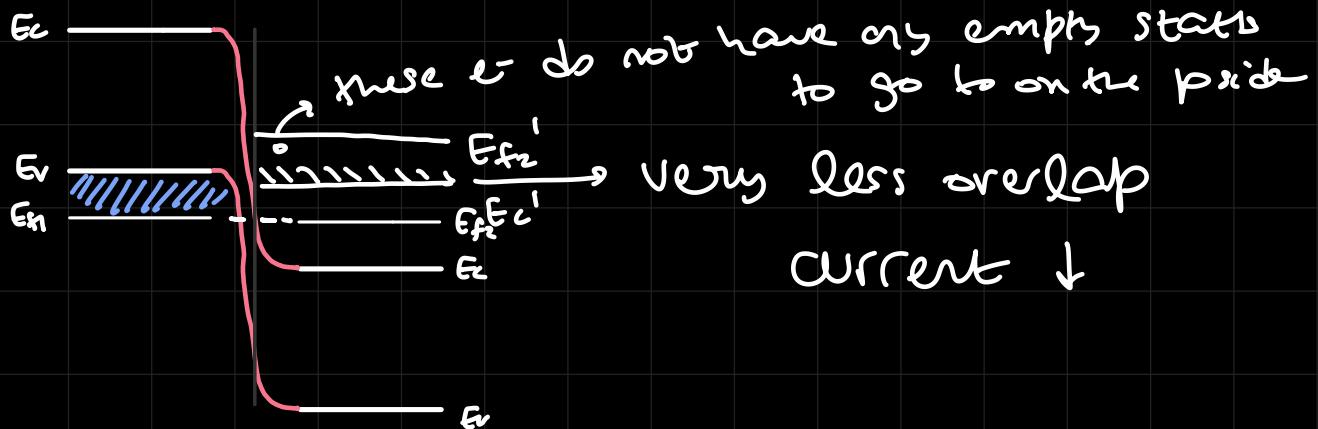
①

initial potential bias

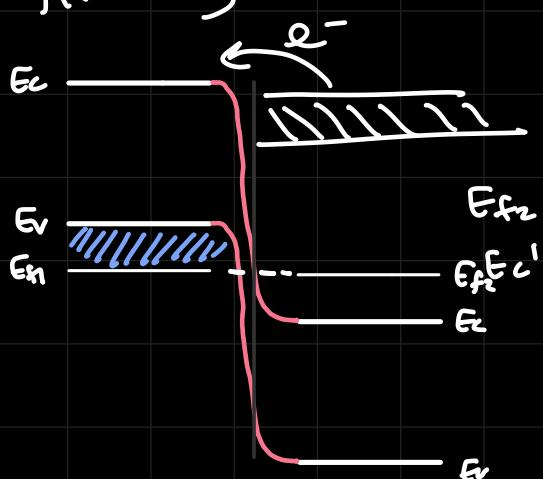


all  $e^-$  can tunnel through

when increasing  $V$  further ②



finally



③

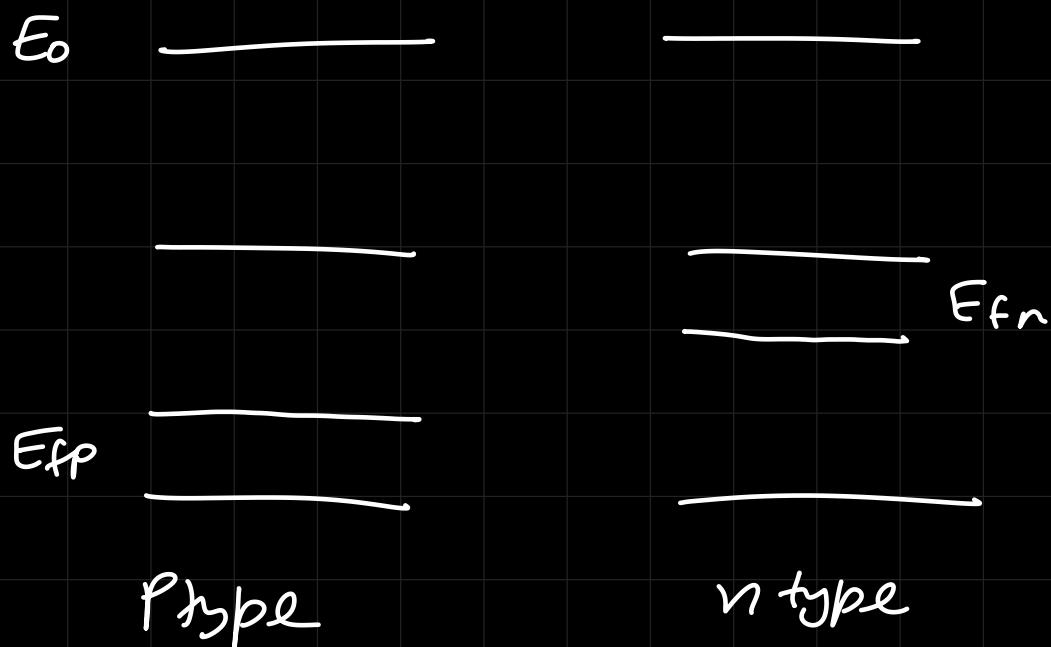
jumps over the barrier  
at very high  $V$

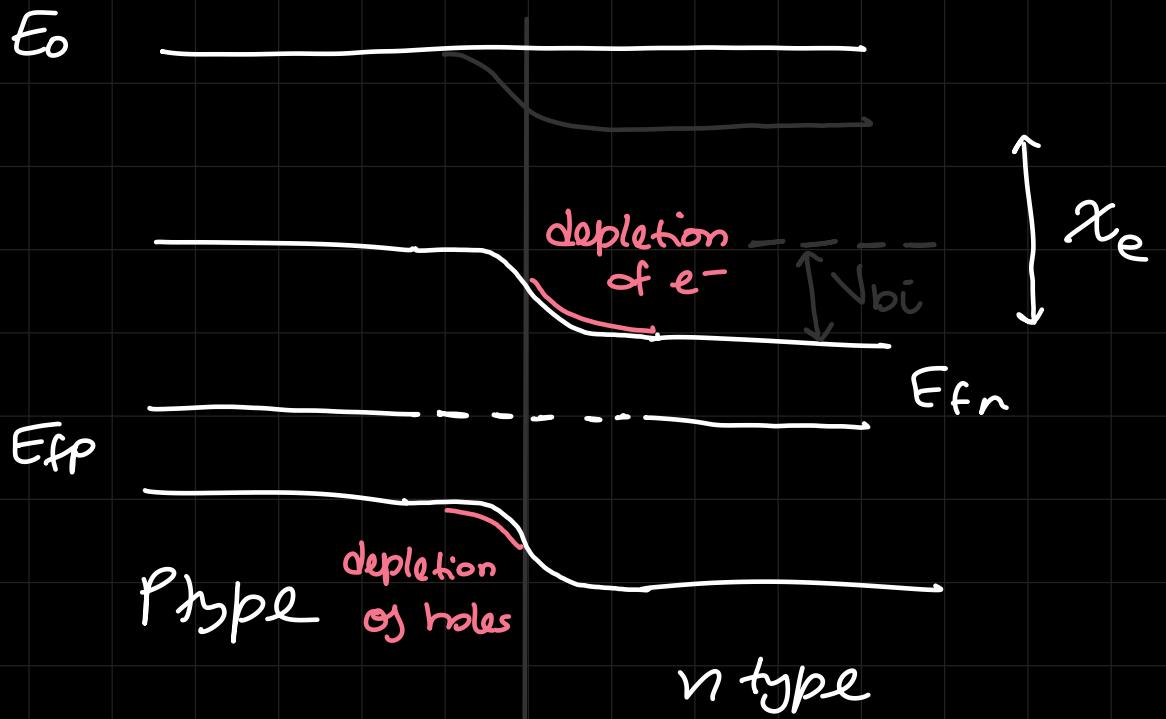
ESAKI diode works in forward bias  
Zener diode (PN) works in reverse bias

diffusion current : current due to difference  
in concentration

# PN Junction : at room temp, diff and  
drift current cancel each  
other out and hence net  
current = 0

note: There exist NN and PP Junctions as well.





exponentially varying  
band bending

(excess) acceptor ions  $\nwarrow$  donor ions  $\nearrow$

	-	+	
P	-	+	$n$
	-	+	

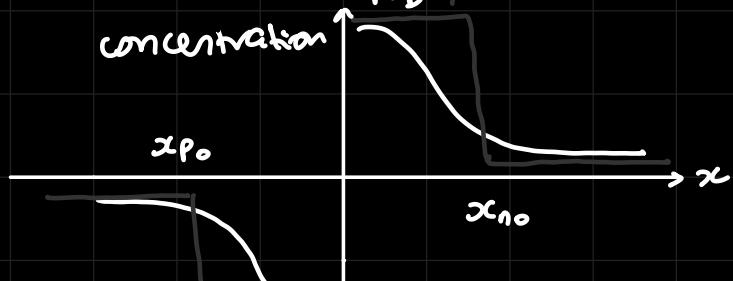
charge:

$$p - N_A$$

$\downarrow$

free ions

ion concentration  $N_D \rightarrow$  ideally



$$\mathcal{J}_n = q N_D$$

$$\mathcal{J}_p = -q N_A$$

but  $\mathcal{J}(x)$

$$x: 0 \rightarrow x_{n_0}$$

$\mathcal{J}(x)$

$$x: -x_{p_0} \rightarrow 0$$

$$f_0(x) = \begin{cases} -qN_a & : -x_{p_0} < x \leq 0 \\ qN_D & : 0 < x < x_{n_0} \\ 0 & : -x_{p_0} > x \text{ or } x > x_{n_0} \end{cases}$$

# Gauss law

$$\oint_S E \cdot d\alpha = \frac{1}{\epsilon} \oint_V f \cdot dV = \frac{Q_{enc}}{\epsilon_0}$$

$$\frac{dE}{dx} = \frac{f}{\epsilon_0}$$

$$E(x) - E(x_0) = \frac{1}{\epsilon} \int_{x_0}^x f(x) dx$$

$$\frac{\partial^2 \phi(x)}{\partial x^2} = -\nabla E = -\frac{f(x)}{\epsilon_0}$$

$$\phi(x) - \phi(x_0) = \int_{x_0}^x -E(x) dx$$

$$\text{Laplace's eqn} \rightarrow \frac{\partial^2 \phi(x)}{\partial x^2} = 0$$

$\left\{ \begin{array}{l} \text{no net} \\ \therefore \text{excess} \\ \text{charge} \end{array} \right\}$

for  $f = -qN_A : -x_{p_0} < x \leq 0$

$$\bar{E} = ?$$

$$\frac{dE}{dx} = -\frac{qN_A}{\epsilon}$$

$$\int dE = -\frac{qN_A}{\epsilon} \int dx$$

$$\bar{E}_{(x)} = -\frac{qN_A x}{\epsilon} + C$$

find using

we know  $f = 0$

boundary condition

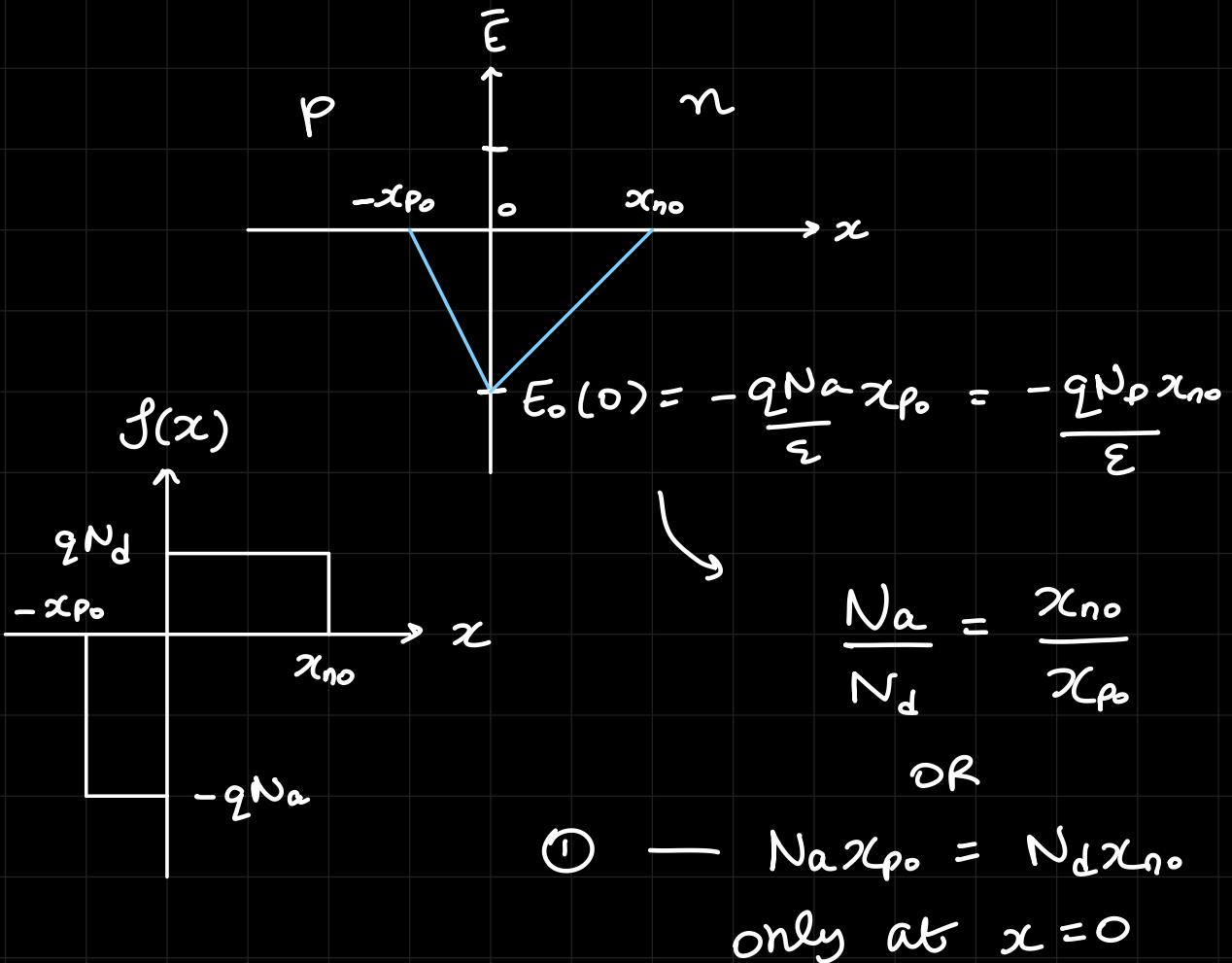
for  $x < -x_{p_0}$

so at  $x = -x_{p_0}$  (boundary)

$$\hookrightarrow \bar{E} = 0$$

$$\bar{E}(x) = \begin{cases} -\frac{qN_A}{\epsilon}(x + x_{p_0}) & : -x_{p_0} < x < 0 \\ \frac{qN_A}{\epsilon}(x - x_{p_0}) & : 0 < x < x_{p_0} \end{cases}$$

max  $\bar{E}$  is  $E_0$  at  $x = 0$



note:

hole density > e- density

→ p side less doped wrt n side

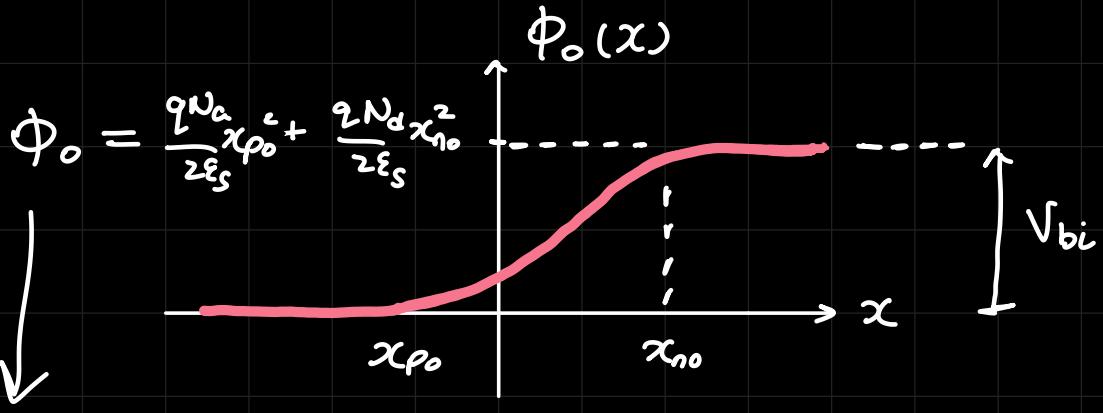
hence  $\bar{E}$  graph is not a perfect triangle

Poisson's Eqn  $\rightarrow \phi(x) = -\int E(x) dx + C$

$\bar{E}$  and potential are continuous at  $x=0$

so,

$$\phi(x) = \begin{cases} \frac{qN_a}{\epsilon_s} \left[ x_{p0}x + \frac{x^2}{2} + \frac{x_{p0}^2}{2} \right] & : -x_{p0} < x < 0 \\ \frac{qN_d}{\epsilon_s} \left( x_{n0}x - \frac{x^2}{2} \right) + \frac{qN_a x_{p0}^2}{2\epsilon_s} & : 0 < x < x_{n0} \end{cases}$$



$\Phi_0$  is the  
built in  
voltage

$$\Phi_0 = V_{bi}$$

$$V_{bi} = \frac{qN_a}{2\epsilon_s} x_{p0}^2 + \frac{qN_d}{2\epsilon_s} x_{n0}^2$$

$$\text{depletion width} = x_{n0} - (-x_{p0})$$

$$= x_{n0} + x_{p0}$$

$$\text{from } \textcircled{1} \rightarrow N_a x_{p0} = N_d x_{n0}$$

$$x_{p0} = \frac{N_d}{N_a} x_{n0}$$

$$W = x_{n0} \left( 1 + \frac{N_d}{N_a} \right)$$

if  $N_a, N_d \gg$ ,  
 $W \downarrow$

Low concentration  
 $\equiv$  wider width



so, for high dopings  
we have small depletion width

if  $N_A, N_D \uparrow \rightarrow W \downarrow \rightarrow$  band bending  $\downarrow$

$$W = \frac{2\epsilon_s V_{bi}}{q} \left[ \frac{N_A + N_D}{N_A \cdot N_D} \right]^{1/2}$$

note :  $W \propto V_{bi}$

Reverse bias  $\rightarrow V_{bi}' = V_{bi} - qV \rightarrow W \uparrow \rightarrow E \downarrow I \downarrow$

Forward bias  $\rightarrow V_{bi}' = V_{bi} + qV \rightarrow W \downarrow \rightarrow E \uparrow I \uparrow$

diffusion current = - drift current  
for pn junction

$$V_{bi} = \frac{kT}{q} \ln \left[ \frac{N_A N_D}{n_i^2} \right]$$

drift current =  $q P \mu p E$

diffusion current =  $q D_p \frac{dp}{dx}$

} for p side

$$q \mu_F E = q D_F \frac{dp}{dx}$$

$$\mu_F \left[ -\frac{dv}{dx} \right] = D_F \frac{dp}{dx}$$

$$-\mu_F \int_{x_1}^{x_2} dv = D_F \int_{p_n}^{p_p} \frac{1}{p} dp$$

$$V(x_{p_0} - x_{n_0}) = \frac{D_F}{\mu_F} \ln \left( \frac{p_p}{p_n} \right)$$

→ diffusion coeff  
↓ mobility

$$\frac{D_F}{\mu_F} = \frac{kT}{q} : \text{Einstein relation}$$

$$\text{and } p_p \approx N_A$$

$$\text{and } p_n n = n_i^2 \quad (\text{mass action Law})$$

$$p_n N_D = n_i^2$$

$$V(x_{p_0} - x_{n_0}) = \frac{kT}{q} \ln \left[ \frac{N_A}{n_i^2 / N_D} \right]$$

$$(\text{Unit: Volts}) \quad V_{bi} = \frac{kT}{q} \ln \left[ \frac{N_A N_D}{n_i^2} \right]$$

now we have 2 formulae for  $V_{bi}$

# # Lecture after Quiz-3

## → HETERO Junctions

Hetero structure devices: devices with a position dependent alloy composition

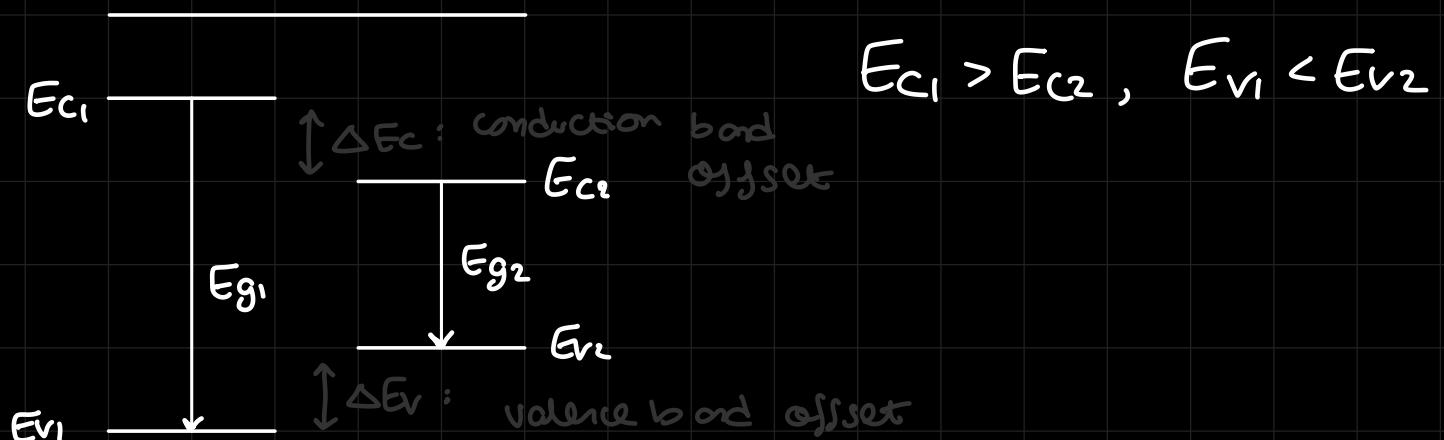
## → Type of heterojunctions

Straddling Gap

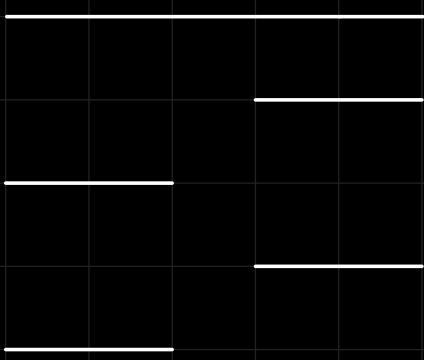
Staggered gap

Broken gap

### ① Staggered gap



## (2) Staggered Gap



$$E_{C1} < E_{C2}$$

$$E_{V1} < E_{V2}$$

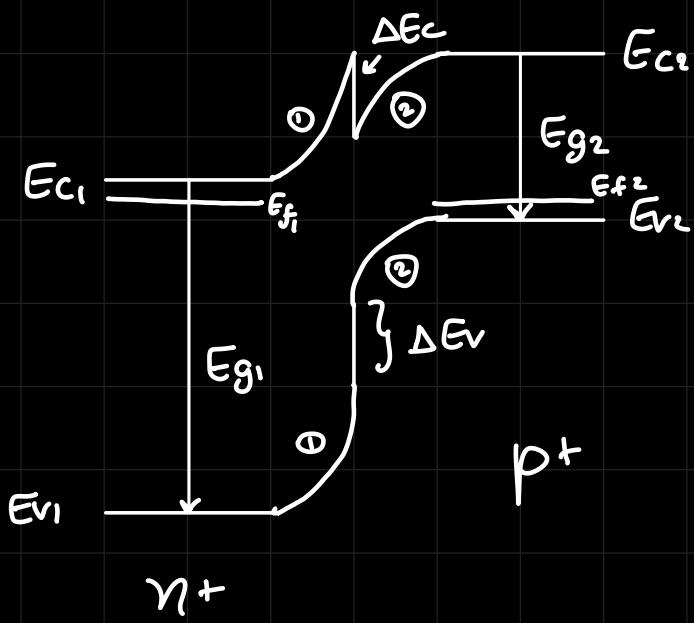
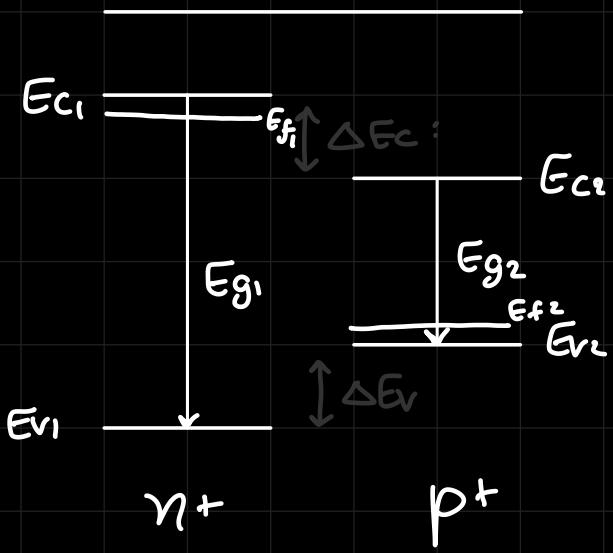
## (3) Broken Gap



$$E_{C2} > E_{V2} > E_{C1} > E_{V1}$$

eg:  $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$

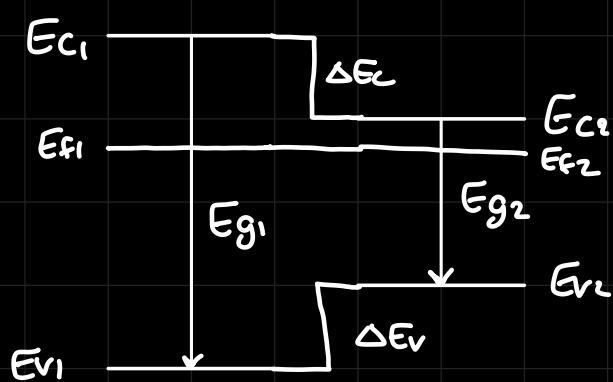
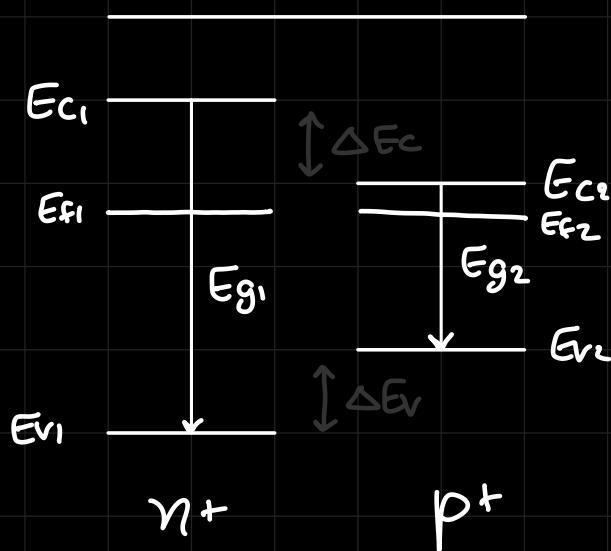
before contact



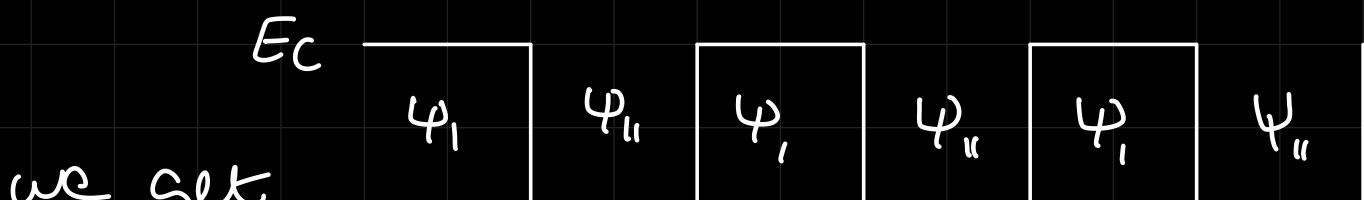
① : e- depleted  
② : e- accumulated

how to determine  $\bar{E}$  for the sudden jump due to the offsets?

How about we choose doping so that no band bending required ( $E_F$  const)



what if we add more such SC pairs?



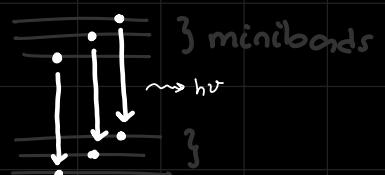
we get  
quantum  
well

Kroneig Penny Model  
quantized periodicity

This conduction bond will form mini bonds. We can also think about valence bond similarly.

minibond (super lattice design)

quantum cascade laser

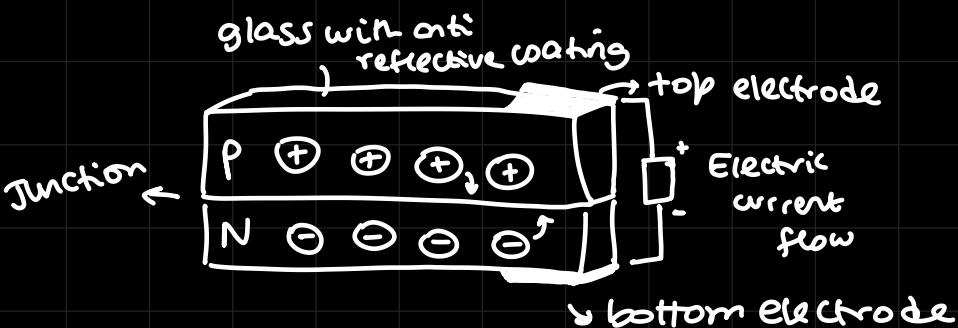


If the distance of traversal is kept constant, we can achieve a radiation of some freq with very high intensity (LASER)

# # SOLAR CELL

$h\nu$  absorbed :  $e^- \xrightarrow{\text{free}} \text{electron hole pair generated}$

Utilizes a PN junction



This charge is held in a solar cell

works in unbiased mode

The charges in the depletion region must be immediately separated ( $h\nu$ ) and stored

$$I = I_{ph} - I_d$$

photosogenerated current      like leakage current  
diode/dark current due to thermal excitation inside the device  
need to minimize  $I_d$  and maximize  $I_{ph}$

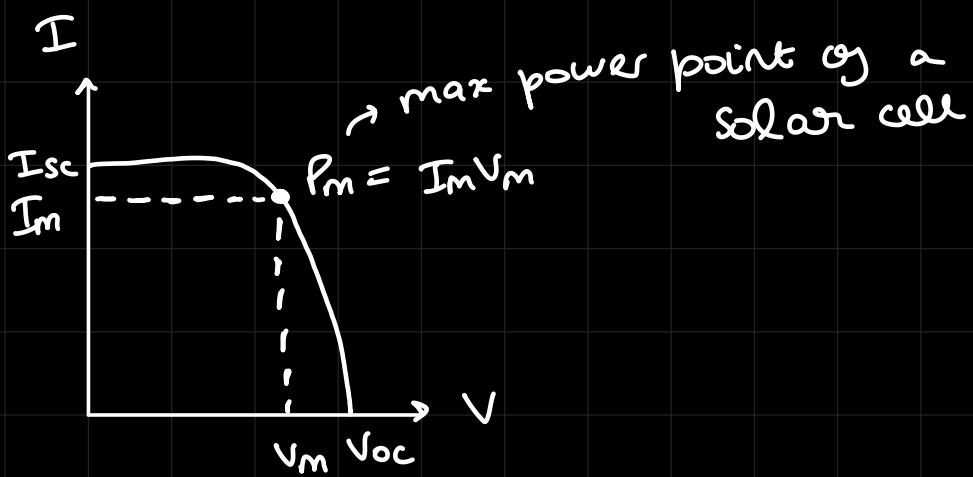
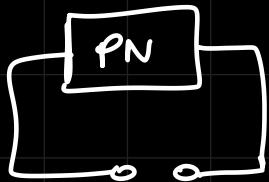
$$I = I_{ph} - I_o \left( e^{\frac{qV}{k_B T}} - 1 \right)$$

\* Short circuit current: max  $I$  generated with no bias/resistor

P-N

$I_{sc}$

\* Open circuit voltage : max V generated (V<sub>oc</sub>) in open circuit condition



# SOLAR Efficiency (Shockley - Queisser Limit)

The max efficiency for a single pn junction solar cell is 33.7%.

at  $E_g = 1.4 \text{ eV}$

but  $\text{Si} \rightarrow E_g = 1.1 \text{ eV}$   
resulting in  $\eta_{\text{max}} = 32\%$  only

visible range :  $1.6 \text{ eV} \rightarrow 3.1 \text{ eV}$

but  $n_{33.7} \rightarrow E_g = 1.4 \text{ eV} < 1.6 \text{ eV}$

We get better  $\eta$  efficiency by stacking layers of diff materials (with varying band gaps) to capture wider range of frequency spectrum.

### = Tandem Solar Cell

Theoretically, we got 50% - 52%  $\eta$

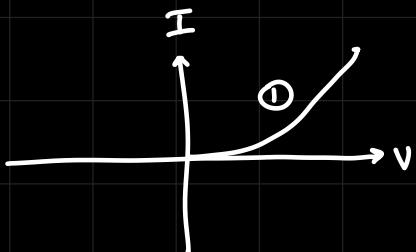
Still we use Si because practical use case wise, it offers mix of  $\eta$  and complexity.

## # 1st charge transport mechanism

$$\bar{v}_h = \mu_p \bar{E} , \quad \bar{v}_e = \mu_n \bar{E}$$

\* { primary transport : drift  
secondary transport : diffusion  
→ tunnelling is also a transport mechanism  
in case of highly doped junctions  
→ Thermionic

PN :



①  $I = I_s e^{\frac{eV}{k_b T_n}}$  ↴ non ideality factor

## # DRIFT TRANSPORT MECHANISM

Spintronics: spin motion related to electrons

movement of charges because of  
an electric field  $\equiv$  DRIFT

drift velocity:  $\bar{v}_h = \mu_p \bar{E}$

$\bar{v}_e = -\mu_n \bar{E}$  ↴ mobility

## # DRUDE Model

When  $e^-$  move, we can visualize interaction as that of a gas i.e. very less

$\tau$  { relaxation time approximation : time b/w 2 successive  $e^-$  collisions  
OR mean free time  
OR collision time

→ (25-30 nm for Si/Ge)

mean free path = some thing but the distance between 2 successive collisions

$\tau$  is independent of  $e^-$  position and velocity

ballistic missile : travels without deviation

ballistic motion :  $e^-$  travelling through the mean free path without deviation

but if  $e^-$  goes beyond mean free path,  
there are collisions and hence resistance.

probability that an  $e^-$  faces

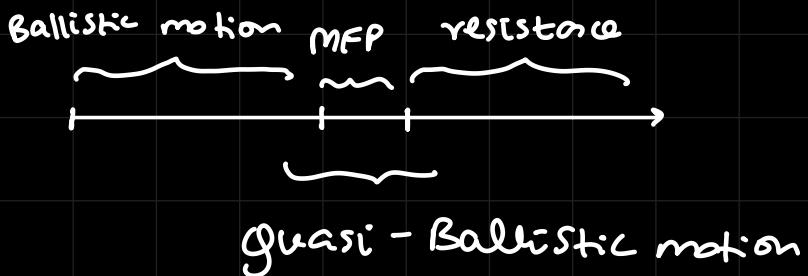
collision per time →  $\frac{1}{\tau}$

within small time,

$$\frac{dt}{\tau}$$

# Quasi-Ballistic Motion

↪ near the mean free path,  
midway between collisions and  
no-collisions



$$m_e \frac{d\bar{v}}{dt} = -e\bar{E}$$

integration ↴

$$\bar{v}(t) = -\frac{e\bar{E}t}{m_e}$$

drift velocity ↗      mean free time ↗

$$\boxed{\bar{v} = -\frac{e\bar{E}\tau}{m_e}} \quad \textcircled{1}$$



(e-) charges passing in  
unit time:  $n|\bar{v}|A$

current:  $-en|\bar{v}|A$

current density:  $J = n\bar{v}(-e)$

from ① →  $\bar{J} = \frac{n e^2 \tau \bar{E}}{m_e^*}$

$\underbrace{\phantom{...}}$   
 $\sigma$

$\bar{J} = \sigma \bar{E}$  conductivity

$$\mathcal{J} = \sigma E = \frac{E}{\tau}$$

$$\sigma = \frac{n e^2 \tau}{m_e} = n \mu e \xrightarrow{\substack{\text{# of } e^- \\ \downarrow \\ \text{mobility}}} \text{charge}$$

$$\text{resistivity, } \rho = \frac{m_e}{n e^2 \tau}$$

mobility = avg of relaxation time  $\tau$

$$\mu = \frac{e \tau}{m_e^*}$$

$$|\vec{V}| = \mu |\vec{E}|$$

$$\rightarrow \sigma = n \mu e \xrightarrow{\substack{\text{unit: } cm^2/vs \\ \downarrow \\ \frac{e \tau}{m^*}}} \left\{ \begin{array}{l} \text{we can find } n \\ \text{from } m^* \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{carrier density} \propto \frac{1}{\text{mobility}} \end{array} \right\} \text{we need to balance}$$

$m^* \downarrow \rightarrow \mu \uparrow$  but  $m^* \propto \text{curvature}$

and so DOS goes down

Si balances  $\mu$  and carrier density, and hence  $I \uparrow$   
 $\hookrightarrow$  hence preferred

$m_l = 0.98m_0$  } GaAs      and Si  $\{ m_* = 0.15m_0$   
 $m_t = 0.19m_0$       not much difference  
 BUT

$$\mu_{\text{GaAs}} = 5000 \text{ cm}^2/\text{Vs} \quad \downarrow$$

$$\mu_{\text{Si}} = 1200 \text{ cm}^2/\text{Vs}$$

So,  $\tau$  plays a major role

$$J_n = \mu_n \bar{E} \cdot n \cdot q$$

$$J_{\text{total}} = q(\mu_n n + \mu_p p) \bar{E}$$

note:  $\mu_{n_{\text{Si}}} = 1200 \text{ cm}^2/\text{Vs}$  ( $e^-$ )

$$\mu_{p_{\text{Si}}} = 600 \text{ cm}^2/\text{Vs}$$
 (hole)

because of difference in effective mass  
 $(m_e > m_h)$

NMOS: MOSFET with n channel ( $e^-$ )

PMOS: P channel (holes)

We take size of PMOS as double of NMOS

because of  $\mu_p$  and  $\mu_n$  ( $\mu_p \approx \frac{1}{2}\mu_n$ )

to balance current from PMOS & NMOS

$(m_* \rightarrow \mu \rightarrow \tau \rightarrow J \rightarrow I)$

# # SCATTERING MECHANISM IN A SC DEVICE

→ Matthiessen's Rule: Resistivity is the sum of all

individual resistivity  
in the system

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{phonon}}} + \frac{1}{\tau_{\text{impurity}}}$$

$$\frac{1}{\mu} = \frac{1}{\mu_{\text{phonon}}} + \frac{1}{\mu_{\text{impurity}}}$$

↓  
conductivity  $\sigma$

$$= \frac{1}{f}$$

## SCATTERING MECHANISMS

↳ Defect

↳ Crystal

↳ Impurity

↳ Neutral

↳ Ionized

↳ Alloy

↳ carrier - carrier

↳ Lattice

↳ Intravalley

↳ Acoustic

↳ Deformation Potential

↳ Piezo electric

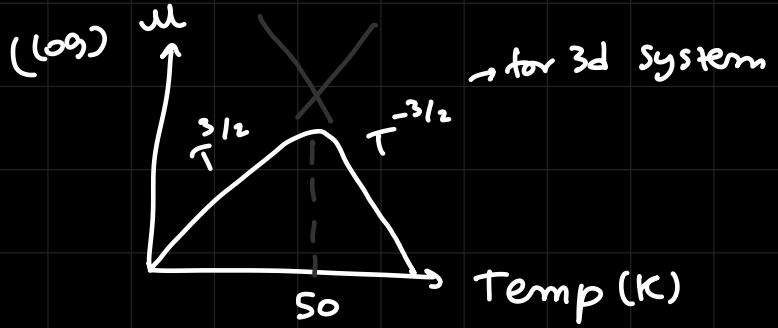
↳ Optical

↳ Non Polar

↳ Polar

↳ Intervalley

↳ Acoustic / Optical



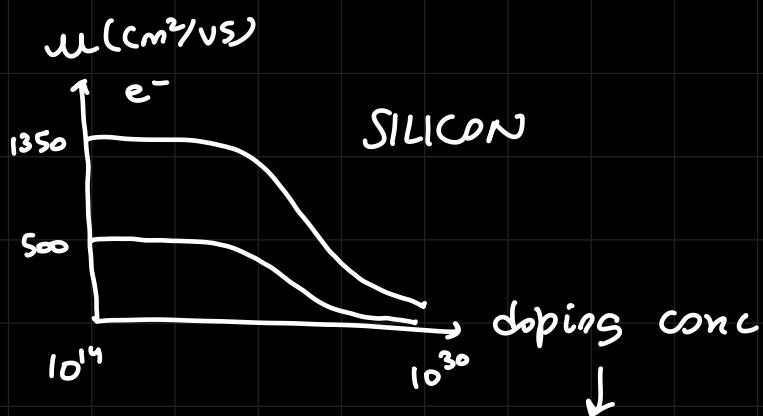
related to doping

impurity scattering dominates at low temps

lattice scattering dominates at high temps

remember phonon generation  
temp ↑, vibrations ↑,  
scattering ↑, μ ↓

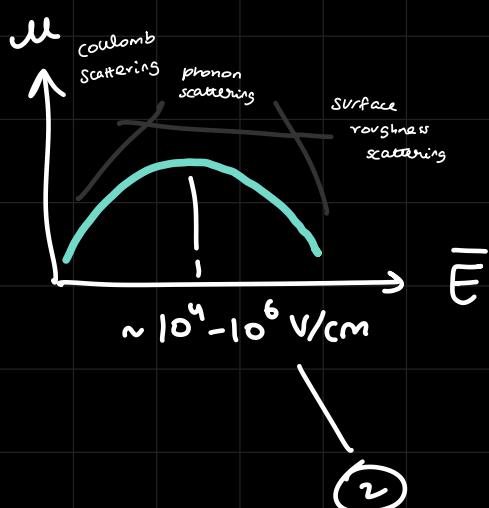
below 50K, lattice vibrations are very small



carrier density ↑  
but μ ↓  
tradeoff ↗

because τ ↓↓

(more carriers in limited space)

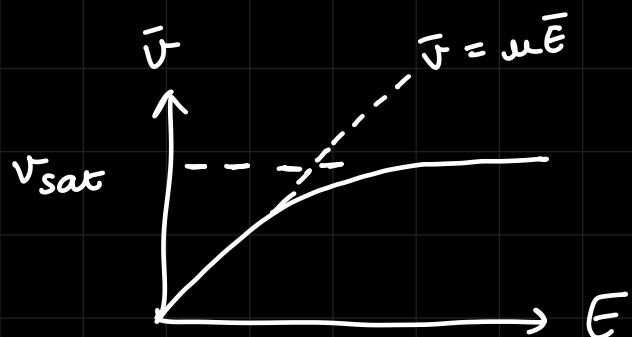


$$V_t = \mu \bar{E}$$

but practically,  $V$  does not increase linearly with  $\bar{E}$

since  $\mu$  does not linearly increase as  $\bar{E}$  ↑ in ②

$$V = \frac{\mu_0}{1 + \frac{\mu_0 E}{V_{sat}}} E$$

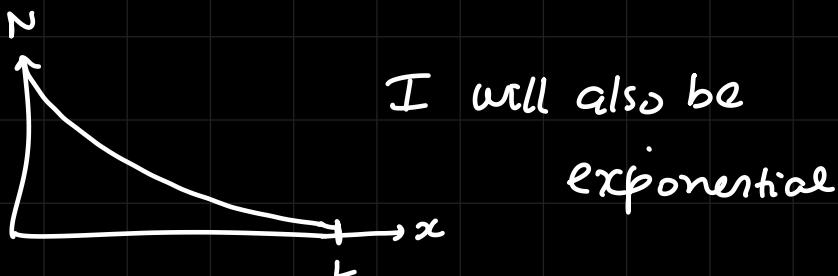


Velocity  
Saturation

## # Second Transport Mechanism : Diffusion

$$J_n = q D_n \frac{dn}{dx}, \quad J_p = -q D_p \frac{dp}{dx}$$

if current density is non linear, current will also be non-linear



forward bias: diffusion (major)

reverse bias: drift (major)

$$\frac{D}{\mu} = \frac{kT}{q}$$

Einstein's  
Relation

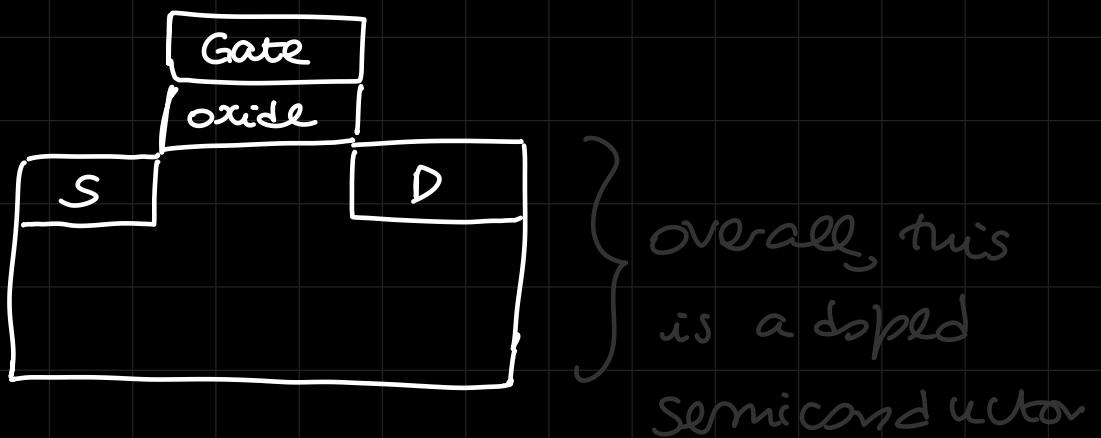
# # MOS capacitor & MOSFET

since 180nm node ↓ metal gate  $\xrightarrow[\text{by}]{\text{replaced}}$  Poly Crystalline (P-Si)

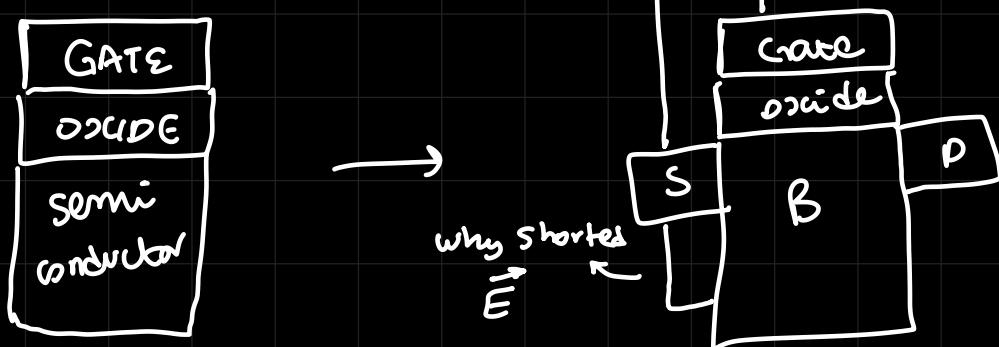
since 90nm node ↓ again back to metal because of performance issues

now also,  $\text{SiO}_2$  (oxide)  $\rightarrow$  high 'K' dielectric  
 $\text{HfO}_2$   
↙ (gate dielectric)

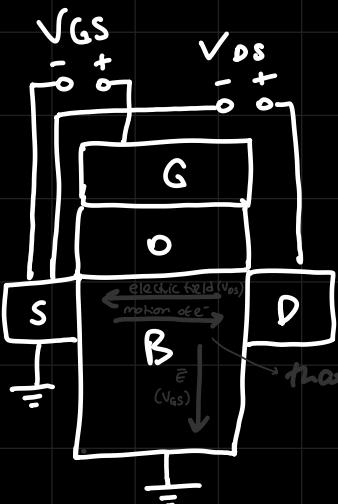
With less and less space with decrease in size (Moore's Law)  
 $\text{SiO}_2$  couldn't provide optimum performance.



Now what if S and D are not doped wrt body / substrate



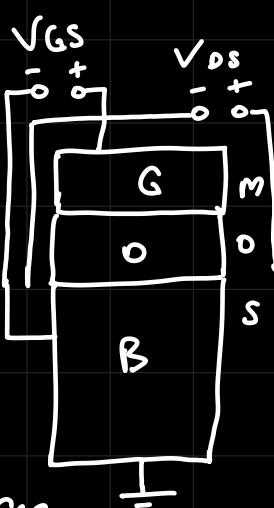
typically :



that's why terminals named source and drain

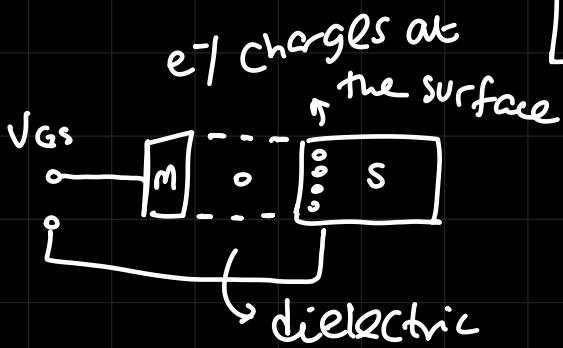
go back to

when S, B, D had  
some doping



capacitor

charges separated  
by a dielectric  
coaxial



Variable  
capacitor

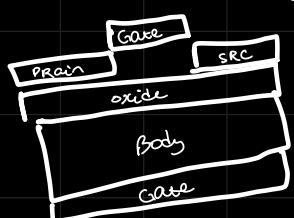
for NMOS  $\rightarrow$  S: p type  $\rightarrow V_{GS} > 0$

PMOS  $\rightarrow$  S: n type  $\rightarrow V_{GS} < 0$  (reversed)



Classical  $\rightarrow$  Planar  
double  
gate

$\rightarrow$  ultra  
thin  
body SOI



T-gate  
Fin FET

$\rightarrow$  Gate-All-Around

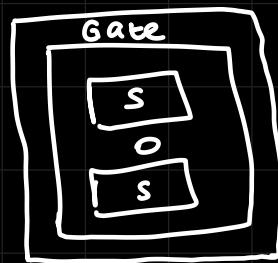
NWFET



complementary MOS  $\rightarrow$  connection between  
(CMOS) PMOS & NMOS

complementary FET  $\rightarrow$  a single device acting  
as both PMOS, NMOS

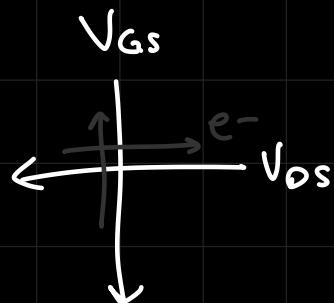
Sheet FET



over the evolution

of MOSFETs, # of gate is observed to have been  
increasing

$V_{ds}$  is pulling  
 $e^-$  from SRC  
to DRAIN  
and that flow  
is controlled  
by  $V_{gs}$



avg Laptop Voltage rating  $\rightarrow$  12V  
for phones  $\rightarrow$  5V

but there are multiple devices  
we need to control power consumption

At very small dimension and very high  $\bar{\epsilon}$ , the dielectric creates a channel allowing the flow of  $e^-$  hence acting as a conductor (not intended)

## Dielectric Breakdown

and therefore we moved from



$\downarrow$

$4\epsilon_0$

$\downarrow$

$\sim 25 - 27 \epsilon_0$

(high  $K\sqrt{}$ )

$\rightarrow \text{const}$

$$\text{we know, } C = \frac{\epsilon A}{d} \rightarrow C_{\text{hf}} \gg C_{\text{si}}$$

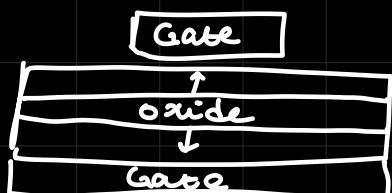
$$\text{and } Q = CV \xrightarrow{\text{const}} \text{high } Q \checkmark$$

holds more surface charge

Going back:

charge density  $\checkmark$

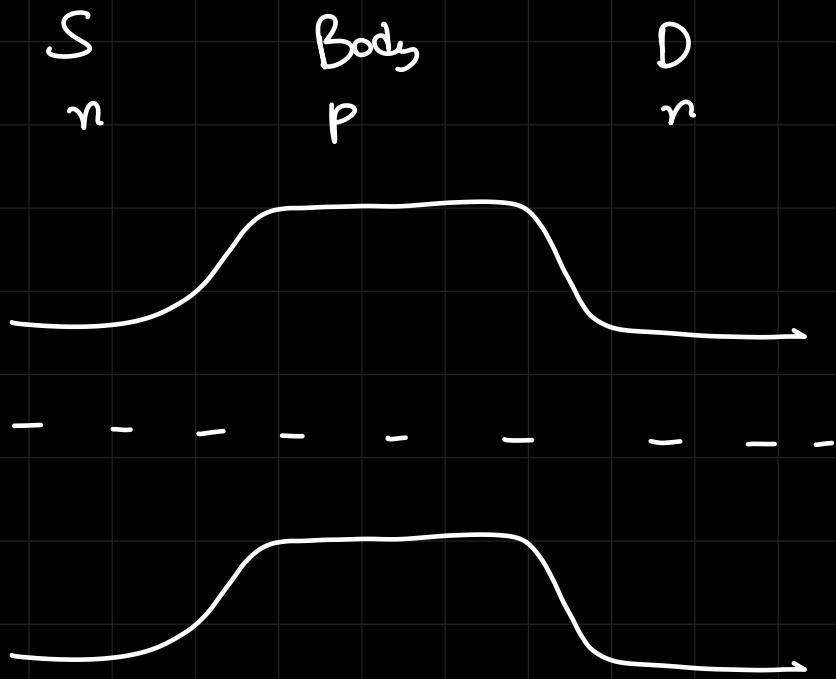
double gate:



$e^-$  are being pulled from both sides. Helps in keeping the charges at the surface  
good capacitor  $\checkmark$  (wrt classical)

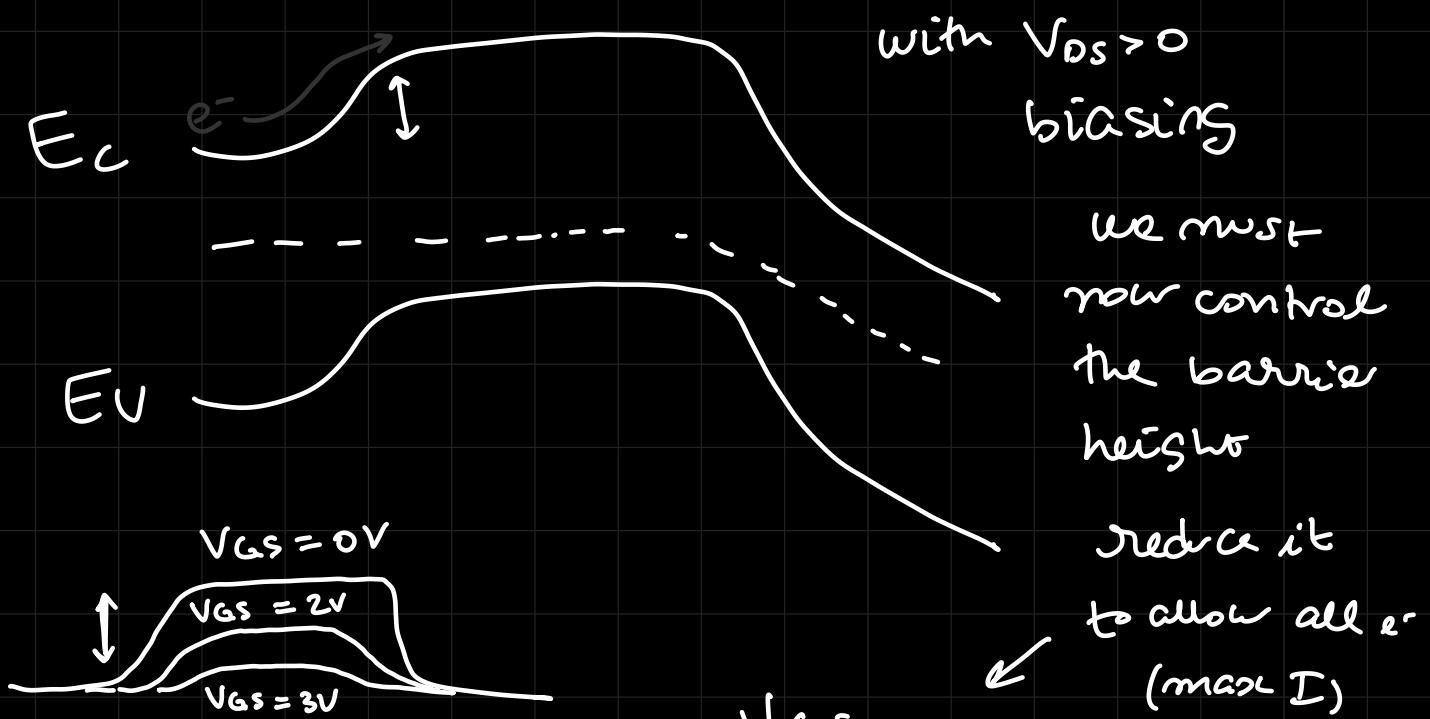
Therefore, # gates  $\uparrow \rightarrow$  carrier density  $\uparrow$

NOTE: This is a surface mode device



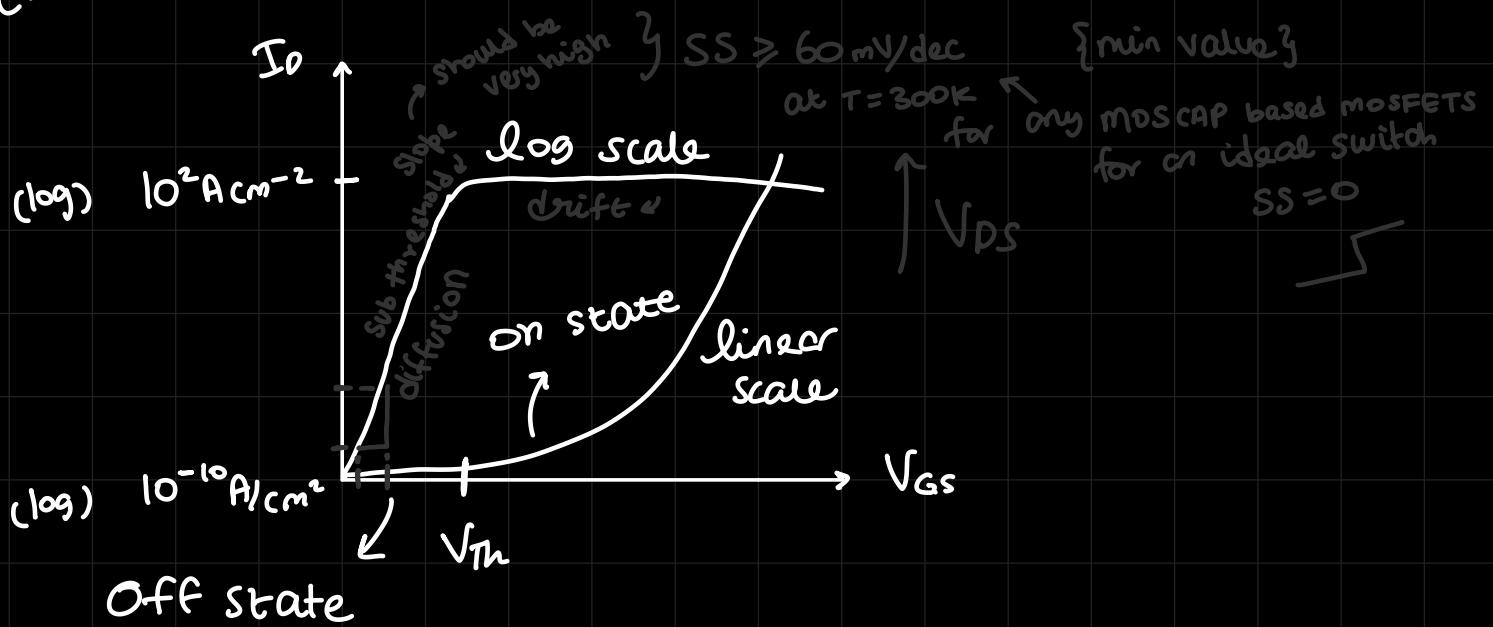
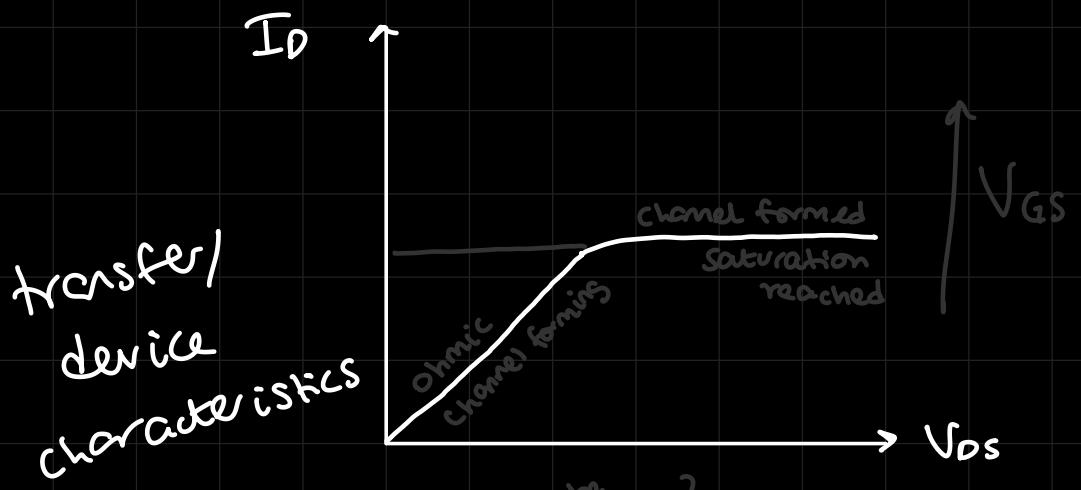
When the  $V_{DS}$  is applied:

over the barrier



$V_{GS}$  is the threshold voltage  
to trigger for better flow of  $e^-$

$V_{GS}$  does this



minimum ON-OFF state ratio  $\sim 10^4 - 10^5$

Compare characteristics with PN Junction

want better off state: improve diffusion  
on state: drift transport mechanism

MOS CAP action: loosing carriers

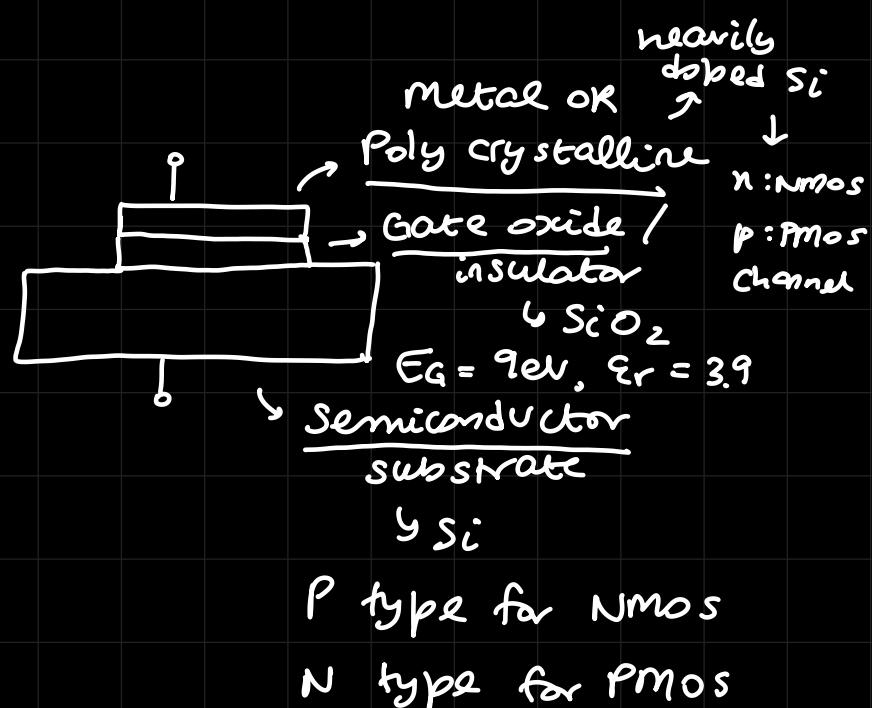
# # Lecture

MOSCAP  $\rightarrow$  MOSFET w/o S and D

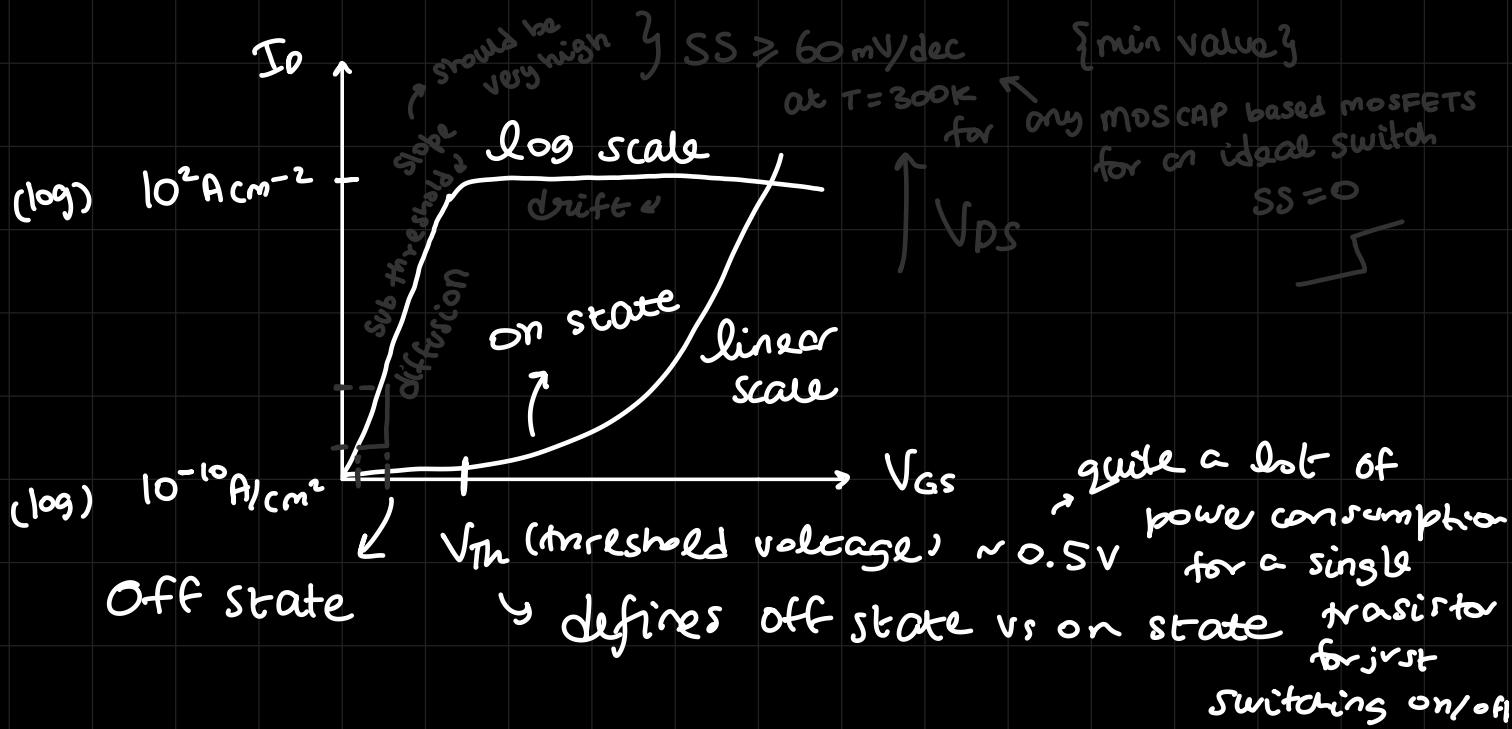
output characteristic  $\rightarrow$   $I_D$  vs  $V_{DS}$

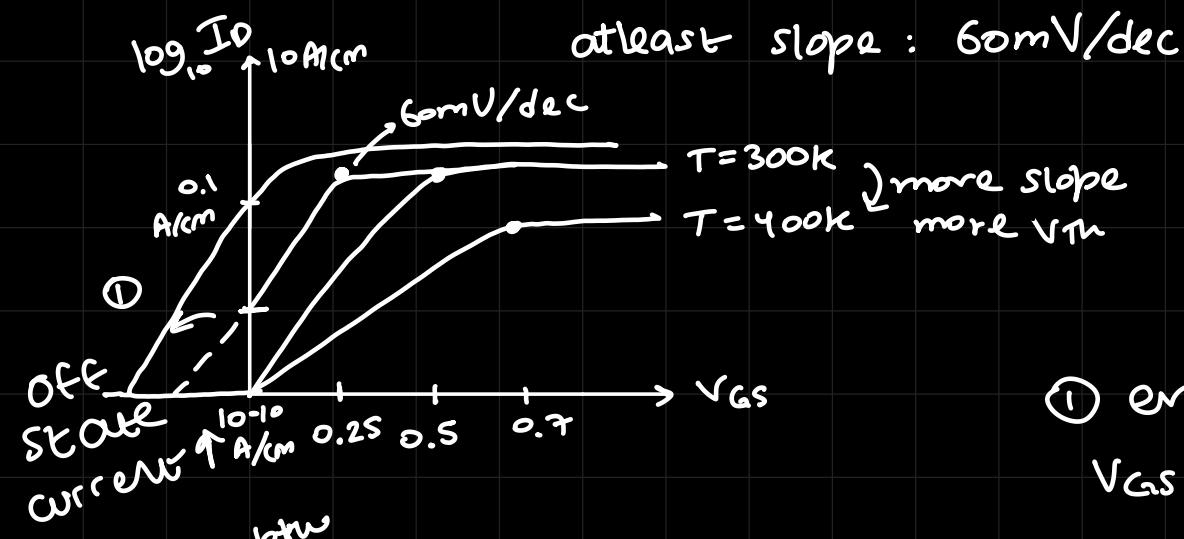
transfer characteristic  $\rightarrow$   $I_D$  vs  $V_{GS}$

## #MOSCAP



Remember  $\rightarrow$





if we go even left,  
the off state current ↑  
and hence difficult to differentiate  
between HIGH & LOW  
↪ Not Good

① even at  $V_{GS} = 0 \rightarrow$  there is some current

$I_d$   
i.e. charge movement

### ENHANCEMENT MODE MOSFET :

no channel formed at no  $V_G$

↪ off state without any gate voltage

### DEPLETION MODE MOSFET :

-ve potential required for OFF state

there is some current even for no  $V_G$

channel already formed without  $V_G$

depletion of charge carriers

depletion of holes in the n channel [nmos]

because of tve  $E$ , the holes move down

from the channel hence depletion

M O S

we consider -

$$\phi_m - \phi_s$$

Why not  
considering  
stuff like  
fermi level?

even though  $E_g = 8-9 \text{ eV}$  (big)

$$\text{because } n_i = \sqrt{n_c n_v} e^{-E_g/k_B T}$$

with so much  $E_g$ ,  $n_i \sim \text{negligible}$   
hence  $E_F, \phi, \chi$  is not relevant

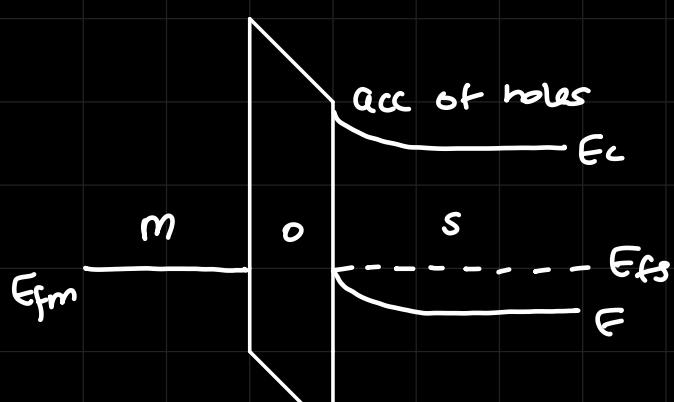
Also, we observed potential drop across  
the oxide and hence  $\sim$  resistance

depletion  
of  $e^-$   
CB

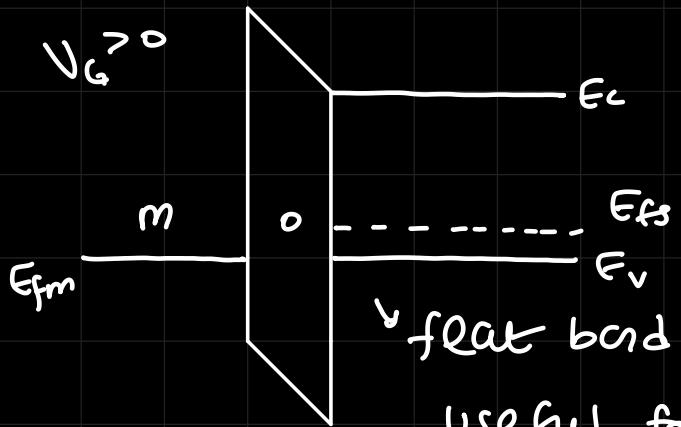
accumulation  
of holes  
VB

MOS

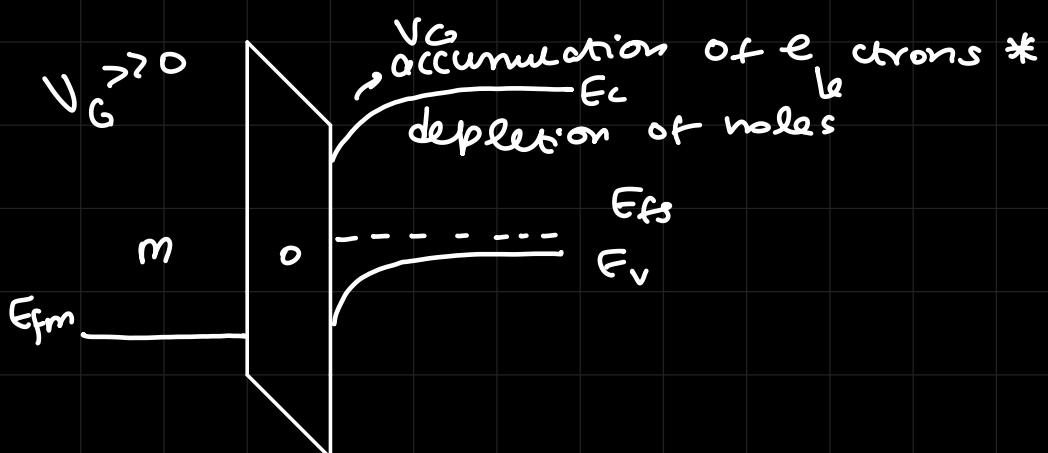
equilibrium



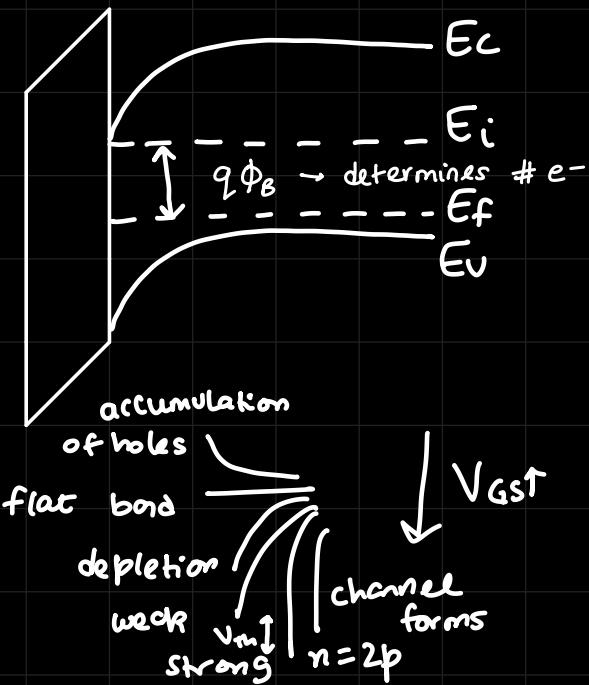
(+)ve bias



useful for determining  
threshold voltage



acc of holes → → → depletion of holes



$V$  at which inversion goes from weak to strong  $\equiv V_m$  (threshold voltage)

$$\phi_B = V_m \ln\left(\frac{N_a}{N_i}\right)$$

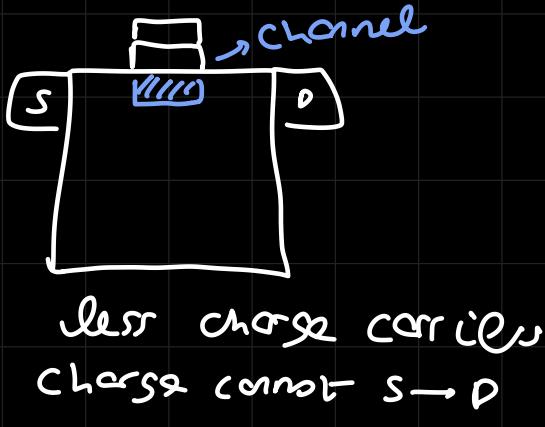
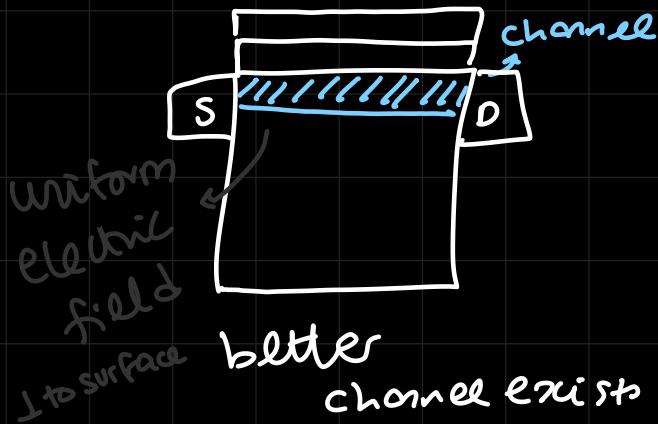
flat band :  $\phi_s = 0$

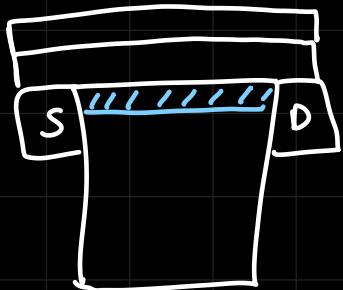
hole accumulation :  $\phi_s < 0$

$e^-$  accumulation :  $\phi_s > 0$

inversion :  $\phi_s > \phi_B$

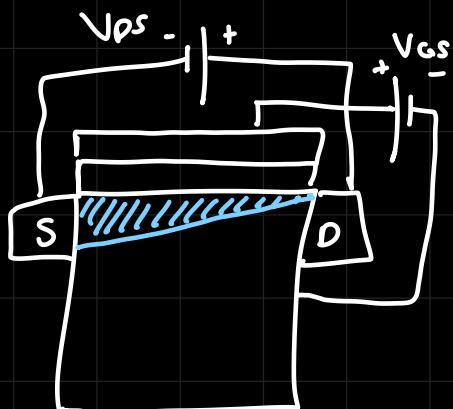
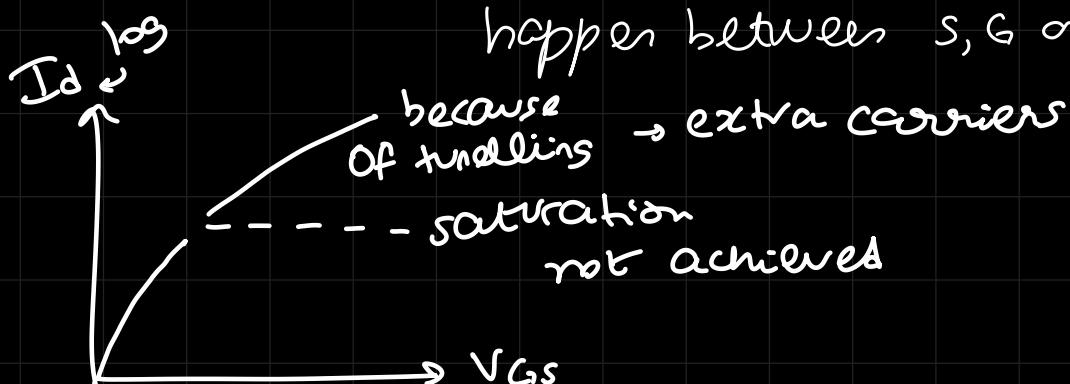
$q\phi_s \rightarrow$  bond bending at the surface





This is also not optimum because of parasitic capacitors

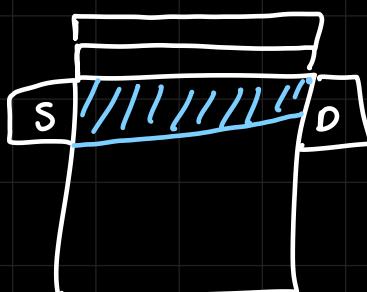
and also some tunnelling can happen between S, G and G, D



Pinch off condition  
When the corner touches the end point

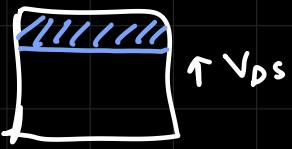
happens at the threshold voltage  
 $V_{GS} = V_T$

below threshold voltage  
no need for longitudinal  $E$  to push charges,  
the carrier concentration drives the carriers  
hence the linear property  
 $\#_{eff} \uparrow \uparrow$  diffusion  $\rightarrow$

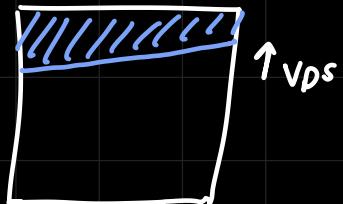


At  $V_{DS} = 0$ , uniform channel

for  $V_{DS} = 0.1 - 0.5V$  (eg)



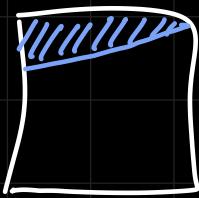
linear



#e- $\propto$

$$\bar{E} \sim 0$$

for  $V_{DS} > V_{GS} - V_T$



#e-  $\rightarrow$  const

very high  $\bar{E}$  due to  
so,  $I_D$  saturates

Delay

- ↳ Intrinsic : within devices
- ↳ Extrinsic : within circuit

## #TRANSCONDUCTANCE

$$g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{GS}} \quad \begin{matrix} \text{dependent on} \\ \text{temp} \end{matrix}$$

$\equiv$  idea of speed of "switching"

Channel conductance

$$g_d = \left. \frac{\partial I_d}{\partial V_{DS}} \right|_{V_{GS}}$$

Used for finding drive current  
for small bias input