Syllabus : lec-1 to Electrostatic boundary conditions

Reference : Griffith
↳ ch 1, 2, 4

* FORMULAS :

CH1

→ Condensed form of maxwell's equations

$$\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\varepsilon_0}$$

$$\bar{\nabla}\cdot\bar{B} = 0$$

$$\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{\nabla}\times\bar{B} = \mu_0\left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right)$$

→ Electrostatic Potential :

1d → $\bar{E} = -\frac{d\phi_x}{dx}\hat{x}$

3d → $E = -\left(\frac{\partial \phi}{\partial x}\hat{x} + \frac{\partial \phi}{\partial y}\hat{y} + \frac{\partial \phi}{\partial z}\hat{z}\right)$

$= -\bar{\nabla}\phi$

→ Cylindrical coords

$\bar{dl} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$

$\bar{ds} = \rho\, d\phi\, dz\, a_\rho \;/\; dz\, d\rho\, \hat{a}_\phi \;/\; \rho\, d\rho\, d\phi\, a_{\hat{z}}$

$dv = \rho\, d\rho\, d\phi\, dz$

→ Spherical coords

$\bar{dl} = dr\, \hat{a}_r + r\, d\theta\, \hat{a}_\theta + r\sin\theta\, d\phi\, \hat{a}_\phi$

$\bar{ds} = r^2\sin\theta\, d\theta\, d\phi\, \hat{a}_r \;/$
$r\sin\theta\, d\phi\, dr\, \hat{a}_\theta \;/$
$r\, dr\, d\theta\, \hat{a}_\phi$

differential solid angle : $d\Omega = \frac{ds_r}{r^2} = \sin\theta\, d\theta\, d\phi$

$dV = r^2 \sin\, dr\, d\phi\, d\theta$

→ LINE INTEGRAL

$W = \int_A^B \bar{F}\cdot\bar{dl}$ $\qquad \phi_{AB} = -\int_A^B \bar{E}\cdot\bar{dl}$

↳ path dependent

→ CONSERVATIVE FIELD
↳ A field that can be expressed as a gradient of scalar.
↳ path independent
↳ Closed loop integral = 0
↳ eg: gravitational and electric field

→ Laplacian operator

$$\nabla^2 \equiv \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

for a field ϕ if $\nabla^2\phi = 0$,
ϕ cannot have a local minima/maxima

* Tutorial 5

Q1) $\bar{A} = \rho\cos\phi\,\hat{a}_\rho + \sin\phi\,\hat{a}_\phi$

$\oint A \cdot dl$

for cylindrical $\rightarrow dl = d\rho\,\hat{a}_\rho + \rho\, d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$

$$\oint A\cdot dl = \int_{60^\circ}^{30^\circ} 2\sin\phi\, d\phi\,\hat{a}_\phi + \int_2^5 \rho\cos(30^\circ)\, d\rho\,\hat{a}_\rho + \int_{30^\circ}^{60^\circ} 5\sin\phi\, d\phi\,\hat{a}_\phi + \int_5^2 \rho\cos(60^\circ)\, d\rho\,\hat{a}_\rho$$

$$= 2\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2}\cdot\frac{1}{2}\cdot(21) - 5\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\cdot\frac{1}{2}(-21)$$

$$= 1-\sqrt{3} + \frac{21\sqrt{3}}{4} - \frac{5}{2} + \frac{5\sqrt{3}}{2} - \frac{21}{4}$$

$$= \frac{4-4\sqrt{3}+21\sqrt{3}-10+10\sqrt{3}-21}{4}$$

$$= \frac{27\sqrt{3}-27}{4} = \frac{27}{4}(\sqrt{3}-1) \checkmark$$

Using STOKE'S THEOREM :

$$\oint \bar{A}\cdot \bar{dl} = \int_s (\bar{\nabla}\times\bar{A})\, ds$$

$\bar{ds} = \rho\, d\rho\, d\phi$

$$\oint A\cdot dl = \iint (\bar{\nabla}\times A)\, \rho\, d\rho\, d\phi$$

$$\bar{\nabla}\times\bar{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\,\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$\frac{\partial}{\partial\rho}(\rho\sin\phi) - \frac{\partial}{\partial\phi}(\rho\cos\phi)$

$\sin\phi + \rho\sin\phi$

$$= \frac{1}{\rho}\left(\hat{a}_\rho\left(0 - \frac{\partial}{\partial z}(\rho\sin\phi)\right)\right) - \frac{1}{\rho}\left(\rho\,\hat{a}_\phi\left(0-\frac{\partial}{\partial z}(\rho\cos\phi)\right)\right) + \frac{1}{\rho}\left(\hat{a}_z\left(\frac{\partial}{\partial\rho}(\rho\sin\phi) - \frac{\partial}{\partial\phi}(\rho\cos\phi)\right)\right)$$

$$= \frac{1}{\rho}\hat{a}_z\left(\sin\phi + \rho\sin\phi\right)$$

$$= \left(\frac{1}{\rho}\sin\phi + \sin\phi\right)\hat{a}_z$$

$$\oint \bar{A}\cdot\bar{dl} = \int_{30}^{60}\int_2^5 \sin\phi\,(\rho+1)\, d\rho\, d\phi$$

$$= \int_{30}^{60}\left[\sin\phi\left(\frac{\rho^2}{2}+\rho\right)\right]_2^5 d\phi$$

$$= \int_{30}^{60}\sin\phi\left(\frac{27}{2}\right) d\phi$$

$$= -\frac{27}{2}\left[\cos\phi\right]_{30}^{60}$$

$$= -\frac{27}{2}\left[\frac{1}{2}-\frac{\sqrt{3}}{2}\right]$$

$$= \frac{27}{4}(\sqrt{3}-1) \checkmark$$

Chapter: 2

Electric Field :

① distinct points → $\bar{E} = k \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}$

② continuous charge → $\bar{E} = k \int \frac{1}{r^2} \hat{r}\, dq$

along a line : $dq = \lambda\, dl$
on a surface : $dq = \sigma\, ds$
in a volume : $dq = \rho\, dv$

Divergence and Curl of Electrostatic fields

→ flux of $\bar{E}$ through surface S,

$$\Phi_E = \int_S E \cdot \bar{ds}$$

Gauss's Law : $\oint_S E \cdot ds = \frac{q_{enclosed}}{\varepsilon_0}$

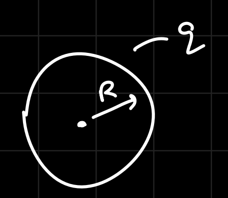
Divergence Theorem : $\oint_S E \cdot \bar{ds} = \int_V (\bar{\nabla} \cdot \bar{E})\, dv$

note : $q = \int_V \rho\, dv$

so, $\int \frac{\rho\, dv}{\varepsilon_0} = \int \bar{\nabla} \bar{E}\, dv$

$$\bar{\nabla} E = \frac{\rho}{\varepsilon_0}$$

Ex 2.2 →



field = ?

$$\oint_S E \cdot ds = \frac{1}{\varepsilon_0} q_{enclosed} = \frac{q}{\varepsilon_0}$$

$$|E| \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$|E| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

$$E = k \frac{q}{r^2} \hat{r}$$

CURL of $\bar{E}$

$$\bar{E} \cdot \bar{dl} = 0$$

→ Stoke's theorem : $\oint \bar{E} \cdot \bar{dl} = \int_S (\nabla \times E)\, ds$

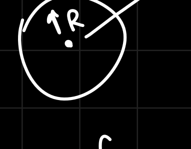
so, $\bar{\nabla} \times \bar{E} = 0$

Electric Potential

$$\bar{E} = -\bar{\nabla} \bar{V}$$

$\bar{V}$ ↳ electric potential

ex 2.6 →



outside ?

$\bar{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$ $\quad \oint E \cdot ds = \frac{q_{enclosed}}{\varepsilon_0}$

Poisson's Equation

we know → $E = -\nabla V$, $\nabla E = \frac{\rho}{\varepsilon_0}$, $\nabla \times E = 0$

so → $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

LAPLACE'S EQN → $\nabla^2 V = 0$ in regions with no charge $\therefore \rho = 0$

$V = \frac{kq}{r}$ → single charge

$V = k \sum_{i=1}^{n} \frac{q_i}{r_i}$ → multiple charge

$V = k \int \frac{1}{r} dq$ → continuous distribution

$V = k \int \frac{1}{r} \rho\, dv$ → volume charge

WORK DONE : $W = \int_a^b F \cdot dl$

$W = 0$ for single charge

$W_2 = \frac{1}{4\pi\varepsilon_0} q_2 \left(\frac{q_1}{r_{12}} \right)$ for 2 charges

$W = \frac{1}{2} \sum_{i=1}^{n} q_i V(r_i)$ for multiple charges

continuous → $W = \frac{1}{2} \int \rho V\, dv$