Charge density → $\rho = \varepsilon_0 \nabla E$

ε_0 = permittivity = 8.854×10^{-12}

$\rho = 8.854$

$\bar{\nabla}\bar{E} = \dfrac{\rho}{\varepsilon_0}$ — gauss's Law

$\bar{\nabla}(-\bar{\nabla}\phi) = \dfrac{\rho}{\varepsilon_0}$

$\bar{\nabla}^2\phi = -\dfrac{\rho}{\varepsilon_0}$ — Possoin's Eqn

if $\rho = 0$ → $\nabla^2\phi = 0$ — laplacian eqn

Q1) (a) 20

(b) $\ln(3) = 1.098$

Q2) $E = Kr^3\hat{r}$ spherical coords

(a) $\rho = 8.854 \times 10^{-12} \times 3r^2 \times K$

note: $\bar{\nabla}E = \dfrac{\rho}{\varepsilon_0}$

$\rho = \bar{\nabla}\cdot\bar{D}$ note: $\bar{D} = \varepsilon\bar{E}$

↳ electric field density ↓ $\varepsilon_0\varepsilon_r$

here $\varepsilon_r = 1$ because free space

(b) $dq = \rho\, dv$

$q = \int \rho\, dv = 3\varepsilon_0 \int_0^R r^2\, dr$

$= \varepsilon_0 R^3$

Q3) $\rho_v = \begin{cases} \dfrac{\rho_0 r}{R} & : 0 \le r \le R \\ 0 & : r > R \end{cases}$

spherical symmetry

	h_1	h_2	h_3
xyz	1	1	1
cylindrical	1	ρ	1
spherical	1	r	$r\sin\theta$

$\Big\}\ \dfrac{1}{h_1h_2h_3} = \dfrac{1}{r^2\sin\theta}$

$\nabla\cdot D = \dfrac{1}{r^2}\left\{\dfrac{\partial(r^2\ \ r^3)}{\partial r}\right\}$

$= \dfrac{1}{\cancel{r^2}}\, 5\cancel{r^4} = 5r^2$

$\rho = 5Kr^2\varepsilon_0$

(b) $\varphi = \int \rho_v\, dv$

$= \int 5K\varepsilon_0 r^2\, dv$

$dv = 4\pi r^2 dr$

$= \int 5kr^2\varepsilon_0\, 4\pi r^2 dr$

$= 20k\pi\varepsilon_0 \int_0^R r^4\, dr$

$= 4K\pi\varepsilon_0 R^5$

Q3) $\rho_v = \begin{cases} \dfrac{\rho_0 r}{R} & : 0 \le r \le R \\ 0 & : r > R \end{cases}$

$E = ?$ for both regions

Q4) $\bar{J} = \int (r^2+2)\cdot\bar{\nabla}\left(\dfrac{1}{r^2}\right)\hat{r}\, dr$

$= \int (r^2+2)(-2r^{-3})\, dr$

$= 2\int (r^{-1} + 2r^{-3})\, dr$

$= \dfrac{-1}{R^4}$

$\bar{\nabla}F = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}(r^2F_r) + \dfrac{1}{r\sin\theta}\dfrac{\partial(\sin\theta F_\theta)}{\partial\theta} +$

$\dfrac{1}{r\sin\theta}\dfrac{\partial F_\phi}{\partial\phi}$

Q3) (a) $Q_{enclosed} = \varepsilon_0 E \times 4\pi r^2$

for $r \le R$ $= \int_0^{2\pi}\int_0^{\pi}\int^{r} \rho_v r^2 \sin\theta\, dr\, d\theta\, d\phi$

$= \dfrac{1}{R}\int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_0 r^3 \sin\theta\, dr\, d\theta\, dr$

$= \pi\dfrac{\rho_0 r^4}{R}$

$E = Q_{enclosed} \times \dfrac{1}{4\pi r^2\varepsilon_0}$

$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{|r|^2}\hat{r} = \dfrac{1}{4\cancel{\pi}\varepsilon_0}\dfrac{\rho_0\cancel{\pi}r^2}{R}$

$= \dfrac{\rho_0 r^2}{4\varepsilon_0 R}\hat{a}_r$

(b) $E = k\dfrac{q}{r^2}$

$q = \int \rho_v \dfrac{4\pi}{3} r^3\, dr$

Q4) $\bar{J} = \int_r (r^2+2)\,\bar{\nabla}\left(\dfrac{\hat{r}}{r^2}\right) dr$