PHYSICS OF SEMICONDUCTOR DEVICES

ASSIGNMENT - 1

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Q1) $\bar{A} = \left(\frac{a\sqrt{3}}{2}\hat{i} + \frac{a}{2}\hat{j}\right)$, $\bar{B} = \left(-\frac{a\sqrt{3}}{2}\hat{i} + \frac{a}{2}\hat{j}\right)$, $\bar{C} = c\hat{k}$

We know that the volume of primitive unit cell with the primitive cell vector, $\bar{R} = u\bar{a} + v\bar{b} + w\bar{c}$ is $V_c = \bar{a}\cdot(\bar{b}\times\bar{c})$

So, volume $= \bar{A}\cdot(\bar{B}\times\bar{C})$

$$(\bar{B}\times\bar{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \\ 0 & 0 & c \end{vmatrix}$$

$$= \hat{i}\left(\frac{ac}{2} - 0\right) - \hat{j}\left(-\frac{ac\sqrt{3}}{2} - 0\right) + \hat{k}(0 - 0)$$

$$= \left(\frac{ac}{2}\right)\hat{i} + \left(\frac{ac\sqrt{3}}{2}\right)\hat{j}$$

now, $\bar{A}\cdot(\bar{B}\times\bar{C}) = \left(\frac{a\sqrt{3}}{2}\cdot\frac{ac}{2}\right) + \left(\frac{ac\sqrt{3}}{2}\cdot\frac{a}{2}\right)$

$$= 2\left(\frac{a^2 c\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{2}a^2c$$

Hence, we conclude that the volume of the primitive unit cell is indeed $\rightarrow \frac{\sqrt{3}}{2}a^2c$

Primitive Translational Vectors $\rightarrow$

for 3d crystal structure,

$$A^* = \frac{2\pi\cdot(B\times C)}{A\cdot(B\times C)}$$

$$= \frac{2\pi(ac/2)\hat{i} + 2\pi(ac\sqrt{3}/2)\hat{j}}{\sqrt{3}/2\, a^2c}$$

$$= \frac{2\pi}{a}\left(\frac{1}{\sqrt{3}}\hat{j} + \hat{i}\right)$$

$$B^* = \frac{2\pi\cdot(C\times A)}{A\cdot(B\times C)}$$

$$(C\times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & c \\ \frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$= \hat{i}\left(0-\frac{ac}{2}\right) - \hat{j}\left(-\frac{ac\sqrt{3}}{2}\right)$$

$$= \left(-\frac{ac}{2}\right)\hat{i} + \left(\frac{ac\sqrt{3}}{2}\right)\hat{j}$$

$$B^* = \frac{-\pi ac\,\hat{i} + \pi ac\sqrt{3}\,\hat{j}}{\sqrt{3}/_2\, a^2 c}$$

$$= -\frac{2\pi}{\sqrt{3}}\cdot\frac{1}{a}\hat{i} + \frac{2\pi}{a}\hat{j}$$

$$= \frac{2\pi}{a}\left(\frac{-1}{\sqrt{3}}\hat{i} + \hat{j}\right)$$

$$C^* = \frac{2\pi\cdot(A\times B)}{A\cdot(B\times C)}$$

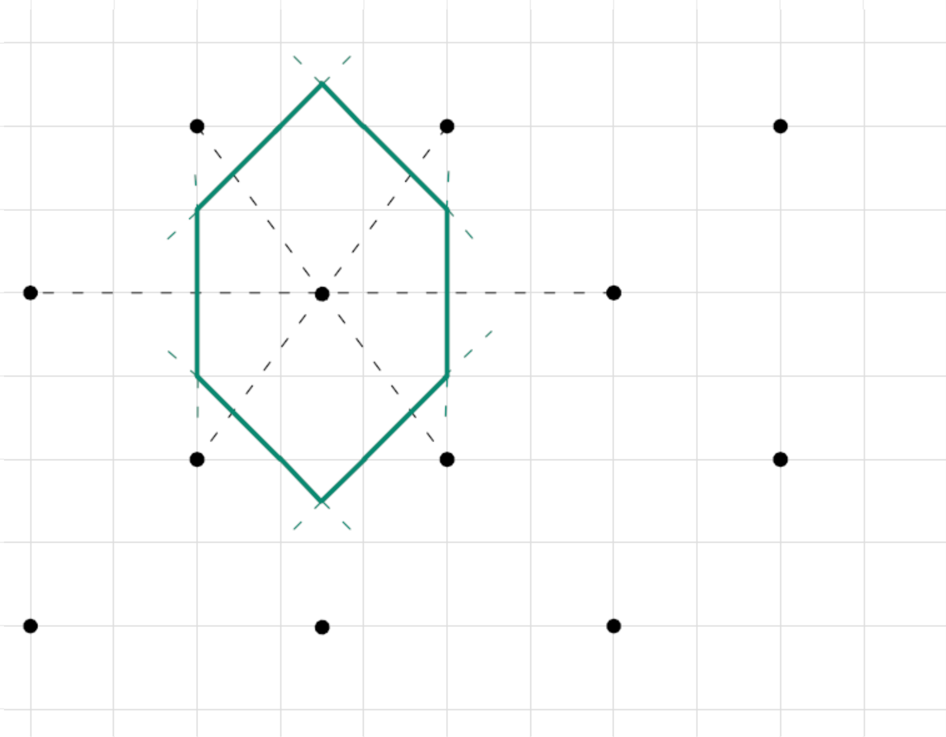
$$A\times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \\ -\frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$= \hat{k}\left(\frac{a^2\sqrt{3}}{4} + \frac{a^2\sqrt{3}}{4}\right)$$

$$= \frac{\sqrt{3}}{2} a^2 \hat{k}$$

$$C^* = \frac{\pi\sqrt{3}a^2 \hat{k}}{\sqrt{3}/2\, a^2 c} = \frac{2\pi}{c} \hat{k}$$

Brillouin zone of the hexagonal space lattice



Q2) lattice constant $= 4.3 \times 10^{-10}$ m $= a$

for (3 2 1), miller indices are
$h = 3$; $k = 2$; $l = 1$

we know that the interplanar distance is,

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}} = \frac{4.3 \times 10^{-10}}{\sqrt{9+4+1}}$$

$$= \frac{4.3}{3.74} \text{Å} \simeq \underline{1.15 \text{ Å}}$$

we also know that constructive diffraction occurs only if $2d\sin\theta = n\lambda$

for first order reflection, $n = 1$

So, $\lambda = 2d\sin\theta = 2 \times 1.15 \times 10^{-10} \times \sin(10^\circ)$

$= 2.3 \times 0.1736 \times 10^{-10}$

$\simeq 0.4 \times 10^{-10}$ m

Q3) $KE = 0.2\,MeV$
$V = 20\,MeV$

note $1eV = 1.602 \times 10^{-19}\,J$

$E = 3.2 \times 10^{-14}\,J$
$V = 3.2 \times 10^{-12}\,J$
$L = 2.97 \times 10^{-18}\,m$
$\hbar = 1.054 \times 10^{-34}\,Js$
$m = 6.68 \times 10^{-27}\,kg$

We know,

$$T = \frac{1}{1 + \frac{V^2 \sinh^2(k'L)}{4E(V-E)}}$$

where $k' = \sqrt{\frac{2m(V-E)}{\hbar^2}} = \sqrt{\frac{4.23 \times 10^{-38}}{1.11 \times 10^{-68}}}$

$$= \sqrt{3.81 \times 10^{30}} = 1.95 \times 10^{15}$$

$k'L = 5.8 \times 10^{-3}$

$\sinh^2(k'L) = 3.36 \times 10^{-5}$

$$T = \frac{1}{1 + \frac{3.44 \times 10^{-28}}{4.05 \times 10^{-21}}} = \frac{1}{1 + (0.85 \times 10^{-7})}$$

$$= \boxed{0.99}$$

Q4) $\Delta x = 4\mathring{A} = 4 \times 10^{-10}\ m$

(a) Heisenberg's uncertainity principle ?

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{0.527 \times 10^{-34}}{4 \times 10^{-10}}$$

$$\Delta p \geq 0.13175 \times 10^{-24} \simeq 1.32 \times 10^{-25}\ kg\ m\ s^{-1}$$

(b) $KE = \frac{p^2}{2m}$

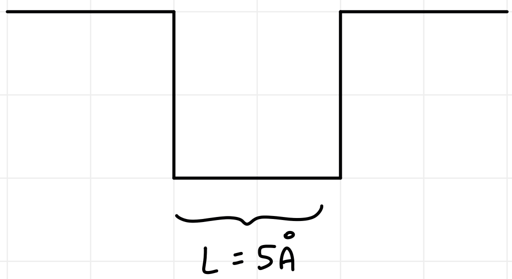
$$\frac{d(KE)}{dp} = \frac{1}{2m} \cdot 2p = \frac{p}{m} \rightarrow \Delta KE = \frac{p}{m} \Delta p$$

$$\Delta KE = \frac{2.4 \times 10^{-23}}{9.1 \times 10^{-31}} \times 1.32 \times 10^{-25}$$

$$= 0.26 \times 10^{8} \times 1.32 \times 10^{-25}$$

$$= 3.432 \times 10^{-18} \text{ J}$$

Q5)



(a) for ∞ potential well,

$$E_n = \frac{k^2\hbar^2}{2m} = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$E_1 = \frac{\pi^2 \cdot (1.054 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 25 \times 10^{-20}}$$

$$= \frac{\pi^2 \cdot (1.11) \times 10^{-68} \times 10^{51}}{455} = 2.4 \times 10^{-19} \text{ J}$$

$$E_2 = 4 \times 2.4 \times 10^{-19} = 9.6 \times 10^{-19} \text{ J}$$

$$E_3 = 9 \times 2.4 \times 10^{-19} = 21.6 \times 10^{-19} \text{ J}$$

(b) $\Delta E = E_3 - E_2 = \dfrac{hc}{\lambda}$

$$\lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{(21.6 - 9.6)\, 10^{-19}} = \frac{19.878 \times 10^{-26}}{12 \cdot 10^{-19}}$$

$$\lambda = \underline{1.656 \times 10^{-7}\ m}$$

Q3) note : 1eV = 1.602×10^{-19} J

$$E = 3.2 \times 10^{-14} \text{ J}$$
$$V = 3.2 \times 10^{-12} \text{ J}$$
$$L = 2.97 \times 10^{-18} \text{ m}$$
$$\hbar = 1.054 \times 10^{-34} \text{ Js}$$
$$m = 6.68 \times 10^{-27} \text{ kg}$$

Since $V >> E$ →

$$T = 16\left(\frac{E}{V}\right)\left(1-\frac{E}{V}\right)e^{-2KL}$$ (according to the book)

where $$k = \sqrt{\frac{2m}{\hbar^2}(V-E)}$$

$$K = 1.95 \times 10^{15}$$

$$T = 16\left(\frac{1}{100}\right)\left(1-\frac{1}{100}\right)e^{-2(1.95\times10^{15})(2.97\times10^{-18})}$$

$$T = (0.16)(0.99)e^{-11.583\times10^{-3}}$$

$$T = (0.16)(0.99)(0.988) = \underline{0.1565}$$