⇒ Tutorial 6 :

Q1)

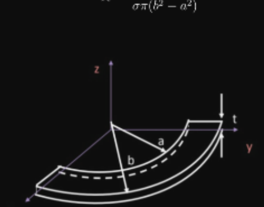
Ques 1. A metal bar of conductivity σ is bent to form a flat 90° sector of inner radius a, outer radius b, and thickness t as shown in Figure 1. Show that

(a) The resistance of the bar between the vertical curved surfaces at ρ =a and ρ =b is

$$R = \frac{2\ln\frac{b}{a}}{\sigma\pi t}$$

(b) The resistance between the two horizontal surface at z=0 and z=t is

$$R' = \frac{4t}{\sigma\pi(b^2-a^2)}$$



↳ cylindrical cords

(a) $R = \frac{V}{I}$ → laplacian

$$\nabla^2 V = 0$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$$

$$\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$$

Integrate

$$\rho \frac{\partial V}{\partial \rho} = 0 + A$$

$$\frac{\partial V}{\partial \rho} = \frac{A}{\rho}$$

$$\partial V = A \cdot \frac{1}{\rho}\partial \rho$$

Integrate

$$V = A \ln \rho + B$$

how to find A & B?

boundary condition

let $V = 0$ at $\rho = a$

$V = V_0$ at $\rho = b$

$$0 = A\ln(a) + B$$

$$B = -A\ln(a)$$

at $\rho = b$,

$$V_0 = A\ln(b) + B = A\ln\left(\frac{b}{a}\right)$$

$$A = \frac{V_0}{\ln(b/a)}$$

$$V = A \ln\rho + B$$

$$A = V_0 \ln(a/b), \quad B = -\frac{V_0}{\ln(b/a)}\ln(a)$$

$$V = V_0 \ln(a/b)\left(\ln\rho - \ln a\right)$$

$I = ?$

$$\bar{J} = \sigma \bar{E}$$

$$\bar{E} = -\bar{\nabla} V = -\frac{dV}{d\rho}\hat{a}_\rho$$

$$= \frac{-A}{\rho}\hat{a}_\rho$$

$$= \frac{-V_0}{\rho}\ln(a/b)\,\hat{a}_\rho$$

$$\bar{J} = -\frac{\sigma V_0}{\rho}\ln(a/b)\,\hat{a}_\rho$$

$$I = \int \bar{J}\cdot d\bar{s}$$

$$d\bar{s} = -\rho\, d\phi\, dz\, \hat{a}_\rho$$

$$I = \int_0^t \int_{0°}^{90°} \frac{V_0\sigma}{\rho}\ln(a/b)\,\rho\, d\phi\, dz$$

$$= \int_{0°}^{90°} (t V_0 \sigma)\ln(a/b)\, d\phi$$

$$= \frac{\pi}{2}\cdot t\cdot V_0\cdot \sigma\cdot \ln(a/b)$$

$$R = \frac{V}{I} = \frac{\cancel{V_0}\,\cancel{\ln(a/b)}\left(\ln\frac{\rho}{a}\right)}{\frac{\pi}{2}\cdot t\cdot \cancel{V_0}\cdot\sigma\cdot\cancel{\ln(a/b)}}$$

$$= \frac{2\ln\left(\frac{\rho}{a}\right)}{\pi t\sigma}$$

(b) Similar but just boundary conditions different

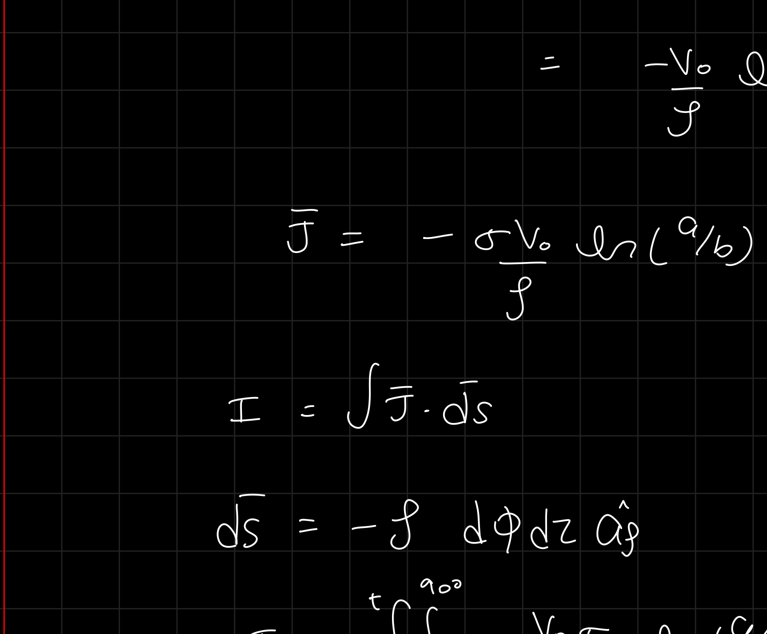
at $z = 0$, $V = 0$

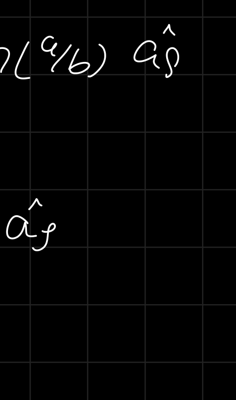
$z = t$, $V = V_0$

Q2)

Ques 2. A rectangular loop carrying current I_2 is placed parallel to an infinitely long filamentary wire carrying current I_1 as shown in Figure 2. Show that the force experienced by the loop is given by

$$\vec{F} = -\frac{\mu_o I_1 I_2 b}{2\pi}\left[\frac{1}{\rho_o} - \frac{1}{\rho_o + a}\right]\hat{a}_\rho \text{ N}$$





use biot savart

Q4)

$$\nabla^2 V = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$r^2 \frac{\partial V}{\partial r} = 0 + A$$

$$\frac{\partial V}{\partial r} = \frac{A}{r^2}$$

$$V = \frac{-A}{r} + B$$

find A, B

$$E = -\nabla V$$

$$Q = \int \varepsilon_0 E\cdot ds$$

$$C = \frac{Q}{V}$$