PHYSICS OF SEMICONDUCTOR DEVICES
ASSIGNMENT - 1
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Q1) $\bar{A} = \left(\dfrac{a\sqrt{3}}{2}\hat{\imath} + \dfrac{a}{2}\hat{\jmath}\right)$, $\bar{B} = \left(-\dfrac{a\sqrt{3}}{2}\hat{\imath} + \dfrac{a}{2}\hat{\jmath}\right)$, $\bar{C} = c\hat{k}$

We know that the volume of primitive unit cell with the primitive cell vector, $\bar{R} = u\bar{a} + v\bar{b} + w\bar{c}$

is $V_c = \bar{a}\cdot(\bar{b}\times\bar{c})$

So, Volume $= \bar{A}\cdot(\bar{B}\times\bar{C})$

$$(\bar{B}\times\bar{C}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -\dfrac{a\sqrt{3}}{2} & \dfrac{a}{2} & 0 \\ 0 & 0 & c \end{vmatrix}$$

$$= \hat{\imath}\left(\dfrac{ac}{2} - 0\right) - \hat{\jmath}\left(-\dfrac{ac\sqrt{3}}{2} - 0\right) + \hat{k}(0 - 0)$$

$$= \left(\dfrac{ac}{2}\right)\hat{\imath} + \left(\dfrac{ac\sqrt{3}}{2}\right)\hat{\jmath}$$

now, $\bar{A} \cdot (\bar{B} \times \bar{C}) = \left(\dfrac{a\sqrt{3}}{2} \cdot \dfrac{ac}{2} \right) + \left(\dfrac{ac\sqrt{3}}{2} \cdot \dfrac{a}{2} \right)$

$\qquad\qquad = 2 \left(\dfrac{a^2 c \sqrt{3}}{4} \right) = \dfrac{\sqrt{3} \, a^2 c}{2}$

Hence, we conclude that the volume
of the primitive unit cell is indeed $\rightarrow \dfrac{\sqrt{3}}{2} a^2 c$

Primitive Translational Vectors $\rightarrow$

for 3d crystal structure,

$\qquad A^* = \dfrac{2\pi \cdot (B \times C)}{A \cdot (B \times C)}$

$\qquad\qquad = \dfrac{2\pi \left(\frac{ac}{2} \right) \hat{i} + 2\pi \left(\frac{ac\sqrt{3}}{2} \right) \hat{j}}{\sqrt{3}/2 \, a^2 c}$

$\qquad\qquad = \dfrac{2\pi}{a} \left(\dfrac{1}{\sqrt{3}} \hat{j} + \hat{i} \right)$

$\qquad B^* = \dfrac{2\pi \cdot (C \times A)}{A \cdot (B \times C)}$

$$(C \times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & C \\ \frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$= \hat{i}\left(0 - \frac{ac}{2}\right) - \hat{j}\left(-\frac{ac\sqrt{3}}{2}\right)$$

$$= \left(-\frac{ac}{2}\right)\hat{i} + \left(\frac{ac\sqrt{3}}{2}\right)\hat{j}$$

$$B^* = \frac{-\pi ac\,\hat{i} + \pi ac\sqrt{3}\,\hat{j}}{\sqrt{3}/_{2}\,a^2 c}$$

$$= -\frac{2\pi}{\sqrt{3}} \cdot \frac{1}{a}\hat{i} + \frac{2\pi}{a}\hat{j}$$

$$= \frac{2\pi}{a}\left(\frac{-1}{\sqrt{3}}\hat{i} + \hat{j}\right)$$

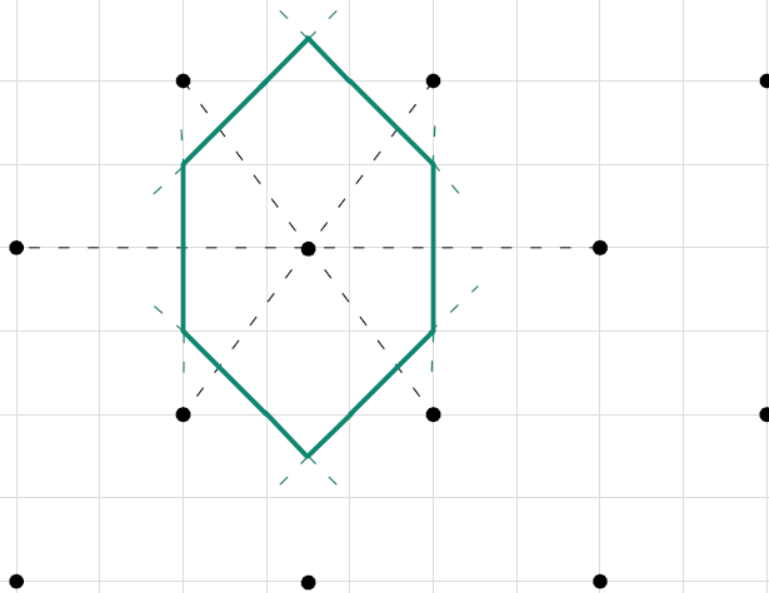$$C^* = \frac{2\pi \cdot (A \times B)}{A \cdot (B \times C)}$$

$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \\ -\frac{a\sqrt{3}}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$= \hat{k}\left(\frac{a^2\sqrt{3}}{4} + \frac{a^2\sqrt{3}}{4}\right)$$

$$= \frac{\sqrt{3}}{2}a^2\,\hat{k}$$

$$C^* = \frac{\pi\sqrt{3}a^2\,\hat{k}}{\sqrt{3}/2\,a^2 c} = \frac{2\pi}{c}\,\hat{k}$$

Brilluion Zone of the hexagonal Space lattice

Q2) lattice constant $= 4.3 \times 10^{-10}$ m $= a$

for $(3\ 2\ 1)$, miller indices are
$h = 3; \quad k = 2; \quad l = 1$

we know that the interplanar distance is,

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{4.3 \times 10^{-10}}{\sqrt{9 + 4 + 1}}$$

$$= \frac{4.3}{3.74} \overset{\circ}{A} \approx \underline{1.15\ \overset{\circ}{A}}$$

we also know that constructive diffraction occurs only if $2d\sin\theta = n\lambda$

for first order reflection, $n = 1$

So, $\lambda = 2d\sin\theta = 2 \times 1.15 \times 10^{-10} \times \sin(10°)$

$$= 2.3 \times 0.1736 \times 10^{-10}$$

$$\approx 0.4 \times 10^{-10}\ m$$

Q3) KE = 0.2 MeV

V = 20 MeV

note $1eV = 1.602 \times 10^{-19} J$

$E = 3.2 \times 10^{-14} J$

$V = 3.2 \times 10^{-12} J$

$L = 2.97 \times 10^{-18} m$

$\hbar = 1.054 \times 10^{-34} Js$

$m = 6.68 \times 10^{-27} kg$

We know,

$$T = \frac{1}{1 + \dfrac{V^2 \sinh^2(K'L)}{4E(V-E)}}$$

where $K' = \sqrt{\dfrac{2m(V-E)}{\hbar^2}} = \sqrt{\dfrac{4.23 \times 10^{-38}}{1.11 \times 10^{-68}}}$

$$= \sqrt{3.81 \times 10^{30}} = 1.95 \times 10^{15}$$

$K'L = 5.8 \times 10^{-3}$

$\sinh^2(K'L) = 3.36 \times 10^{-5}$

$$T = \cfrac{1}{1 + \cfrac{3.44 \times 10^{-28}}{4.05 \times 10^{-21}}} = \cfrac{1}{1 + (0.85 \times 10^{-7})}$$

$$= \boxed{0.99}$$

This is incorrect since we cannot get ~1 tunelling probability. Check last page ↓

g4) $\Delta x = 4\mathring{A} = 4 \times 10^{-10} m$

(a) Heisenberg's uncertainity principle ₹

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{0.527 \times 10^{-34}}{4 \times 10^{-10}}$$

$$\Delta p \geq 0.13175 \times 10^{-24} \simeq 1.32 \times 10^{-25} \text{ kg m s}^{-1}$$
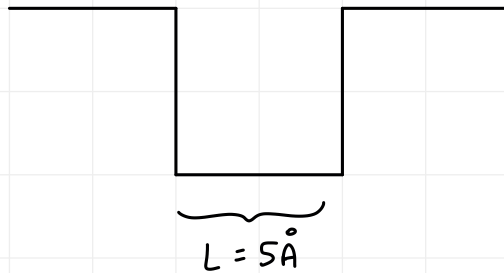
(b) $KE = \dfrac{p^2}{2m}$

$$\frac{d(KE)}{dp} = \frac{1}{2m} \cdot 2p = \frac{p}{m} \quad \rightarrow \quad \Delta KE = \frac{p}{m} \Delta p$$

$$\Delta KE = \frac{2.4 \times 10^{-23}}{9.1 \times 10^{-31}} \times 1.32 \times 10^{-25}$$

$$= \ 0.26 \times 10^8 \ \times \ 1.32 \times 10^{-25}$$

$$= \ 3.432 \times 10^{-18} \ J$$

Q5)

$L = 5\text{Å}$

(a) for ∞ potential well,

$$E_n = \frac{k^2 \hbar^2}{2m} \ = \ \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

$$E_1 \ = \ \frac{\pi^2 \cdot (1.054 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times 25 \times 10^{-20}}$$

$$= \ \frac{\pi^2 \cdot (1.11) \times 10^{-68} \times 10^{51}}{455} \ = \ 2.4 \times 10^{-19} \ J$$

$$E_2 \ = \ 4 \times 2.4 \times 10^{-19} \ = \ 9.6 \times 10^{-19} \ J$$

$$E_3 \ = \ 9 \times 2.4 \times 10^{-19} \ = \ 21.6 \times 10^{-19} \ J$$

(b) $\Delta E = E_3 - E_2 = \dfrac{hc}{\lambda}$

$\lambda = \dfrac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{(21.6 - 9.6)\,10^{-19}} = \dfrac{19.878 \times 10^{-26}}{12 \cdot 10^{-19}}$

$\lambda = \underline{1.656 \times 10^{-7}\ m}$

Q3)

note : $1eV = 1.602 \times 10^{-19} J$

$$E = 3.2 \times 10^{-14} J$$
$$V = 3.2 \times 10^{-12} J$$
$$L = 2.97 \times 10^{-18} m$$
$$\hbar = 1.054 \times 10^{-34} Js$$
$$m = 6.68 \times 10^{-27} kg$$

Since $V >> E \rightarrow$

$$T = 16\left(\frac{E}{V}\right)\left(1-\frac{E}{V}\right)e^{-2KL} \qquad \text{(according to the book)}$$

where $K = \sqrt{\dfrac{2m}{\hbar^2}(V-E)}$

$$K = 1.95 \times 10^{15}$$

$$T = 16\left(\frac{1}{100}\right)\left(1-\frac{1}{100}\right)e^{-2(1.95 \times 10^{15})(2.97 \times 10^{-18})}$$

$$T = (0.16)(0.99)e^{-11.583 \times 10^{-3}}$$

$$T = (0.16)(0.99)(0.988) = \underline{0.1565}$$