

$n(r)$: electronic density

$$n(r) = n(r+R)$$

$$n(r) = \sum n_g e^{ikr}$$

$$n(r+R) = \sum n_g e^{ik(r+R)} = \sum n_g e^{ikr} e^{ikR}$$

and since $n(r) = n(r+R)$,

$$e^{ik\bar{R}} = 1$$

$$\cos(kR) + j\sin(kR) = 1$$

$$\begin{array}{ccc} \text{k-space} & \bar{k} \cdot \bar{R} = 2\pi & \text{OR} \quad \bar{k}_i \bar{R}_j = 2\pi \delta_{ij} \\ \text{vector} & \begin{array}{c} \leftarrow \quad \rightarrow \\ \text{Real space} \\ \text{vector} \end{array} & \end{array}$$

1d: $R = \bar{a}$
 $k = \bar{a}^*$

$$a a^* = 2\pi \rightarrow a^* = \frac{2\pi}{a}$$

2d: $R = \bar{a} + \bar{b}$
 $k = \bar{a}^* + \bar{b}^*$

$$\begin{aligned} \bar{R} &= u\bar{a} + v\bar{b} + w\bar{c} \\ \bar{k} &= h\bar{a}^* + k\bar{b}^* + l\bar{c}^* \end{aligned}$$

3d

$$\begin{array}{lll} a a^* = 2\pi & b b^* = 2\pi & c c^* = 2\pi \\ \text{but } b a^* = 0 & \text{and so on ...} & \\ c a^* = 0 & & \end{array}$$

So, $a^* = \chi(b \times c)$
i.e. $a^* \perp b$ and c

$$\begin{aligned} a a^* &= \chi a \cdot (b \times c) = 2\pi \\ \chi &= \frac{2\pi}{a \cdot (b \times c)} \end{aligned}$$

$$a^* = \frac{2\pi(b \times c)}{a \cdot (b \times c)}$$

2d

$$\begin{array}{ll} a a^* = 2\pi & b b^* = 2\pi \\ b a^* = 0 & a b^* = 0 \end{array}$$

$$\begin{aligned} a^* &\perp b \\ a &\parallel a^* \end{aligned}$$

$$a^* = \frac{2\pi}{a} \hat{a}$$

$$b^* = \frac{2\pi}{b} \hat{b}$$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$= \frac{a}{\sqrt{6 + 36 + 9}} = \frac{a}{\sqrt{61}}$$