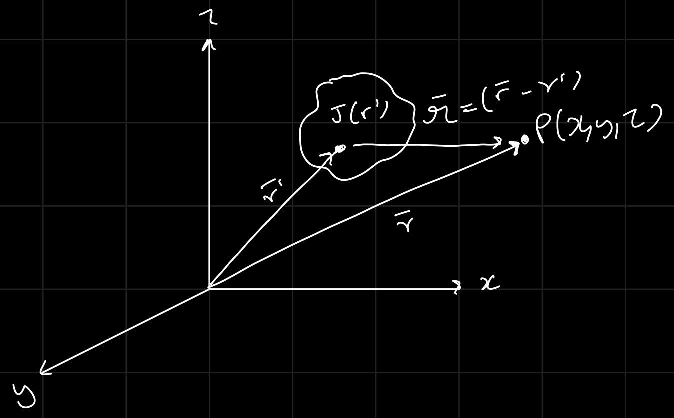
Lecture after midsem



$$J(\bar{r}') \rightarrow A/m^2 = \frac{dI}{da_\perp}$$

→ dependent on x', y', z'

$$\bar{B}(r) = \frac{\mu_0}{4\pi} \int \frac{\bar{J}(r') \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\bar{\nabla} f = -\bar{\nabla}' f$$

note: assuming steady current

$$\bar{\nabla} \cdot \bar{B} = \frac{\mu_0}{4\pi} \int \nabla \left(\frac{\bar{J} \times \hat{\mathscr{r}}}{\mathscr{r}^2} \right) = \frac{\mu_0}{4\pi} \int \underbrace{\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} (\bar{\nabla} \times \bar{J}(r')}_{\text{zero because div wrt r but } J(r')} - \bar{J} \left(\bar{\nabla} \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \right)$$

$$\bar{\nabla} \cdot (\bar{P} \times \bar{Q}) = \bar{Q} (\bar{\nabla} \times \bar{P}) - \bar{P} (\bar{\nabla} \times \bar{Q})$$

and since $\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = \nabla \left(\frac{1}{r} \right) \rightarrow$ curl $= 0$

So, $\boxed{\nabla \cdot B = 0}$

Curl of magnetic field

$$\bar{B} = \bar{\nabla} \times \bar{A}$$

↳ magnetic vector potential

(need not be unique)

$\bar{A}_{new} = \bar{A} + \bar{\nabla}\theta$ ← gauge freedom

$\bar{\nabla} \times \bar{A}_{new} = \bar{\nabla} \times A + 0 = \bar{B}$ always

if $\bar{B}$ given, find the magnetic vector potential

or given, $\bar{J}$, find the magnetic field

since $B \propto \frac{1}{r^2} \rightarrow A \propto \frac{1}{\mathscr{r}}$ because $\bar{B} = \bar{\nabla} \times \bar{A}$

$$\bar{\nabla} \times \bar{A} = \frac{\mu_0}{4\pi} \int \left(\bar{J}(r') \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \right) dV'$$

we know $\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = -\nabla\left(\frac{1}{\mathscr{r}}\right)$

$$\bar{\nabla} \times \bar{A} = \frac{\mu_0}{4\pi} \int \underbrace{\left(\bar{J}(r') \times \bar{\nabla}\left(\frac{1}{\mathscr{r}}\right) \right)}_{(\bar{P} \times \bar{\nabla} f)} dv'$$

$$\bar{\nabla} \times (f\bar{P}) = f(\bar{\nabla}\times\bar{P}) - \bar{P}\times(\bar{\nabla} f)$$

$$\rightarrow \quad P\times(\bar{\nabla} f) = f(\bar{\nabla}\times\bar{P}) - \bar{\nabla}\times(f\bar{P})$$

here $\bar{\nabla}$ wrt $\mathscr{r}$ but $J(r')$

so, $f(\bar{\nabla}\times\bar{P}) = 0$

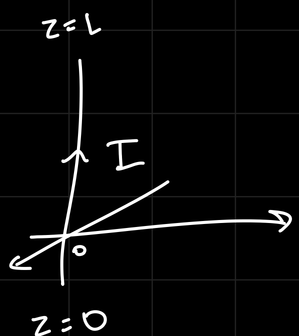
$$\bar{\nabla}\times\bar{A} = \frac{\mu_0}{4\pi}\int \left(\bar{J}(r') \times \bar{\nabla}\left(\frac{1}{\mathscr{r}}\right)\right) dv'$$

$$\bar{\nabla}\times\bar{A} = \frac{\mu_0}{4\pi}\int \bar{\nabla}\times\left(\frac{\bar{J}(r')}{\mathscr{r}}\right) dv'$$

$$\boxed{\bar{A} = \frac{\mu_0}{4\pi}\int \frac{\bar{J}(r')\,dv'}{\mathscr{r}}}$$

current carrying filamentary conductor

$\hat{k} \uparrow$



$\bar{J}(r') = ?$

$$\bar{J} = \frac{dI}{da_\perp}$$

$$J = \bar{I}\,\delta(x)\,\delta(y)$$

$$\bar{A} = \frac{\mu_0}{4\pi} \iiint_{V'} \frac{\bar{J}(r')}{\mathscr{r}} \, dx' \, dy' \, dz'$$

$$\bar{J}(r') \times dx' dy' = I$$

for $x, y \to$ infinitesimally small

$$\bar{A} = \frac{\mu_0}{4\pi} \underbrace{\int_0^\infty \int_0^\infty \delta(x)\delta(y)\,dx\,dy}_{1} \cdot I\hat{z} \int_0^L \frac{dz}{\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{1/2}}$$

$\hookrightarrow \mathscr{r}$

$=$ function of x, y, z

curl$(E) = 0$ but

div$(E) \Rightarrow$ Gauss's Law

div$(B) = 0$ but

curl of $B \Rightarrow$ Ampere's Law

$$\bar{B} = \frac{\mu_0}{4\pi} \int \bar{J}(r') \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \, dV'$$

$$\bar{\nabla} \times \bar{B} = ?$$

problem $\to$ cannot apply chain rule to $\nabla \times (\bar{P} \times \bar{\mathscr{r}})$

$$\bar{\nabla}\times(\bar{P}\times\bar{\mathscr{r}}) = (\bar{\mathscr{r}}\cdot\bar{\nabla})\bar{P} - (\bar{P}\cdot\bar{\nabla})\mathscr{r} + \bar{P}(\bar{\nabla}\cdot\bar{\mathscr{r}}) - \bar{\mathscr{r}}(\bar{\nabla}\cdot\bar{P})$$

$$\left(\mathscr{r}_x \frac{\partial P}{\partial y} + \mathscr{r}_y \frac{\partial P}{\partial y} + \mathscr{r}_z \frac{\partial P}{\partial z}\right)$$

$\bar{P} = \bar{J}$ but since $\bar{J}$ depends on r', not $\mathscr{r}$

so, 1st and last term $= 0$

non zero term:

$$\bar{\nabla}\times(\bar{P}\times\bar{\mathscr{r}}) = \bar{P}(\bar{\nabla}\cdot\bar{\mathscr{r}}) - (\bar{P}\cdot\bar{\nabla})\bar{\mathscr{r}}$$

$$\bar{\nabla}\times\bar{B} = \frac{\mu_0}{4\pi}\left[\int \bar{J}\left(\bar{\nabla}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\right) dV' + \int (\bar{J}\cdot\bar{\nabla}')\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\, dV'\right]$$

not minus

because $\bar{\nabla} f = -\bar{\nabla}' f$

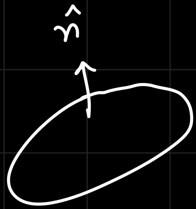
$$= \frac{\mu_0}{4\pi}\left[\int \bar{J}(\bar{r}')\cdot 4\pi\delta^3(\bar{r}-\bar{r}')\, dv + \int (\bar{J}\cdot\bar{\nabla})\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\, dv'\right]$$

$$= \frac{\mu_0}{4\pi}\times 4\pi\times J(\bar{\mathscr{r}}) + \frac{\mu_0}{4\pi}\int (\bar{J}\cdot\bar{\nabla})\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\, dV'$$

$$= \mu_0 J(\bar{\mathscr{r}}) + \frac{\mu_0}{4\pi}\int (\bar{J}\cdot\bar{\nabla})\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\, dV'$$

assuming that 2nd term is zero

$$\bar{\nabla} \times \bar{B} = \mu_0 J(\bar{r}) \rightarrow$$ differential form of Ampere's Law



$$\int (\bar{\nabla} \times \bar{B}) \cdot ds = \mu_0 \int \bar{J} \cdot d\bar{s}$$

$$\boxed{\oint \bar{B} \cdot d\bar{l} = \mu_0 I_{enclosed}}$$

Integral form of Ampere's Law

Proving the previous assumption

steady current $\rightarrow$ $\bar{\nabla} \bar{J}(r') = 0$

so, $\bar{\nabla}' \bar{J}(r') = 0$ also

We need to prove $\rightarrow$ $\displaystyle\int (\bar{J}(r') \cdot \bar{\nabla}) \frac{\hat{\mathfrak{r}}}{\mathfrak{r}^2} \, dv$

$$\frac{\hat{\mathfrak{r}}}{\mathfrak{r}^2} = \frac{\mathfrak{r}\hat{\mathfrak{r}}}{\mathfrak{r}^3} = \frac{\hat{x}(x-x') + \hat{y}(y-y') + \hat{z}(z-z')}{\mathfrak{r}^3}$$

we need contribution of entire system but we will be checking for individual components

$$= \int_{V'} \left(\bar{J}(r')\cdot\bar{\nabla}\right)\frac{(x-x')}{r^3}\,dv'$$

$$\bar{A}\cdot(\bar{\nabla}f) = \bar{\nabla}'\cdot(f\bar{A}) - \underbrace{\bar{f}(\bar{\nabla}'\cdot\bar{A})}_{0 \text{ because } \bar{\nabla}'\cdot J = 0}$$

$$= \int_{V'} \bar{\nabla}'\cdot\left(\frac{J(r')(x-x')}{r^3}\right)dv'$$

$$= \oint_{S'} \frac{\bar{J}(r')(x-x')}{r^3}\,ds$$

since $J(r) = 0$ outside v'

this $\int$ holds for $v' + \Delta$ as well

$$\oint_{S'+\Delta} \underbrace{\bar{J}(r')\Big|_{\text{surface}}}_{\text{zero}} \frac{(x-x')}{r^3}\,ds'$$

$\rightarrow$ Assumption correct