$$\underset{(r,t)}{\bar{\nabla}^2 \bar{E}} = \frac{1}{c^2}\frac{\partial^2 \bar{E}(r,t)}{\partial t^2}$$

let $K = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v}$ or $\frac{\omega}{c}$ velocity

$$\underset{(r,\omega)}{\nabla^2 \bar{E}} + \underset{(r,\omega)}{K^2 \bar{E}} = 0 \longrightarrow \text{Helmholtz eq}^n$$

freq domain → time ?

IFT

$$\bar{E}(r,t) = \int_{-\infty}^{\infty} \bar{E}(r,\omega)\, e^{i\omega t}\, d\omega$$

(for multiple freq)

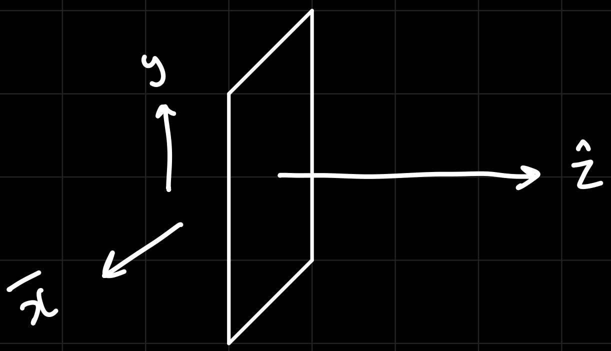
$$\delta(x) \overset{IFT}{\rightleftharpoons} 1$$

for single frequency

$$E_0(r)\,\delta(\omega-\omega_0) \rightleftharpoons E_0(r)\, e^{i\omega t}$$

↳ only a func of space

1D → Uniform Plane wave



$\bar{E}$ and $\bar{B}/\bar{H}$ only vary with z
they are uniform in the xy plane

considering a monochromatic wave
i.e. only one frequency

$$\text{So,} \quad \bar{E}(z,t) \rightleftharpoons \bar{E}(z)e^{i\omega t}$$

helmoltz equation →

$$\frac{\partial^2 \bar{E}(z)}{\partial z^2} + k^2 \bar{E}(z) = 0$$

$$\text{assume } \bar{E}(z) \simeq e^{mz}$$

$$m^2 e^{mz} + k^2 e^{mz} = 0$$
$$m^2 + k^2 = 0$$
$$m = \pm ik$$

$$E(z) = e^{\pm ikz}$$

$$\bar{E}(z,\omega) = \underset{(-\hat{z})}{\bar{A}e^{ikz}} + \underset{(\hat{z})}{\bar{B}e^{-ikz}}$$

$$\bar{E}(z,t) = \bar{A}e^{i(\omega t + kz)} + \bar{B}e^{i(\omega t - kz)}$$

lets take the case of forward travelling wave

$$\bar{E}(z,t) = \underset{\text{(Amplitude)}}{\bar{E}_0}\, e^{i(\omega t - kz)} \rightarrow \text{(phase)}$$

direction of EM wave $(\bar{E})$ = polarization of the wave

$\underline{t=t_1}$ Locus of the points at the same phase = ?
we will look at it now.

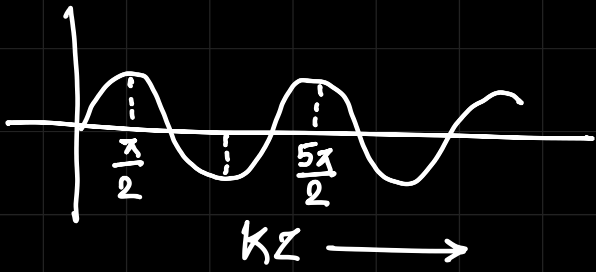
$\omega t_1 - kz = \underline{M}$

↳ same phase M at $t = t_1$.
Locus = ?

$$z = \frac{\omega t_1 - M}{k} \rightarrow \text{totally a constant}$$

Locus ⇒ Z = constant
Plane parallel to xy - plane

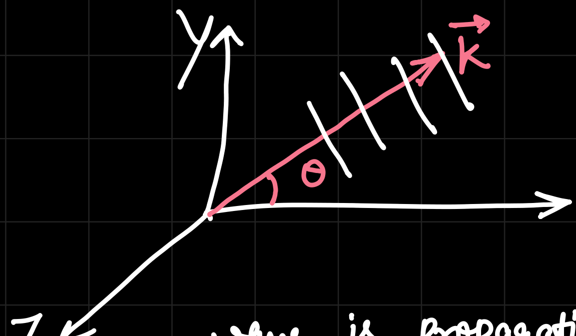
$$\bar{E}(z,t) = \bar{E}_0\, e^{i(\omega t - kz)}\, e^{i 2n\pi}$$



$2n\pi$ gap in Kz axis.

* wave = planar but not uniform. Possible?

YES



Still a plane wave but not uniform.

wave is propagating but tilted (at an angle θ) w.r.t. x-axis.

There can be other examples but this is simplest one.

Basic Helmh. eqn we studied till now,
we assumed $\sigma = 0$ (source-free)
↳ No conduction current.

$$\boxed{\begin{aligned} \nabla \cdot \bar{E} &= 0 \\ \bar{\nabla} \times \vec{E} &= -i\omega \bar{B} \\ \bar{\nabla} \times \bar{B} &= \mu\sigma \vec{E} + i\omega\mu\epsilon E \\ \bar{\nabla} \cdot \bar{B} &= 0 \end{aligned}}$$

↳ freq. domain

$\sigma = 0$

If not this, it will be very tough to analyze.

$$\begin{aligned} \bar{\nabla} \times (\bar{\nabla} \times \bar{B}) &= i\omega\mu\epsilon(\bar{\nabla} \times \bar{E}) \\ &= i\omega\mu\epsilon(-i\omega \bar{B}) \\ &= -i^2 \omega^2 \mu\epsilon \vec{B} \end{aligned}$$

$$\bar{\nabla}\times\bar{B} = i\omega\mu\epsilon_c\vec{E} = i\omega\mu\epsilon\vec{E}$$ (written in simpler)

$$i\omega\mu\epsilon_c = \mu\sigma + i\omega\mu\epsilon$$ (equating both $\bar{\nabla}\times\bar{B}$ eqn.)

$$\epsilon_c = \epsilon + \frac{\sigma}{i\omega}$$

$$\boxed{\epsilon_c = \epsilon - i\frac{\sigma}{\omega}}$$

Permittivity of a medium becomes complex no.

$$\epsilon_c = \epsilon\left(1 - i\frac{\sigma}{\omega\epsilon}\right)$$

* $K = \omega\sqrt{\mu\epsilon_c} = K' - i\underbrace{K''}_{>0}$

Initially, K was simpler. ($K = \omega\sqrt{\mu\epsilon}$)
Now, it is complex. ($\epsilon \rightarrow \epsilon_c$)

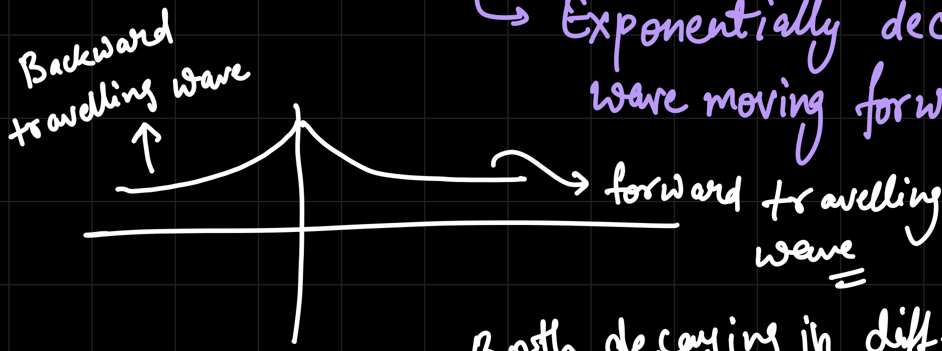
Just ϵ changed to a complex no. (ϵ_c).

* $$\vec{E}_0 e^{i(\omega t - Kz)} = \vec{E}_0 e^{i\omega t} \cdot e^{-i(K' - iK'')z}$$

$$= E_0 e^{i\omega t} e^{-iK'z} e^{-K''z}$$

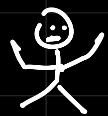
Nature of this wave = ?

↳ Exponentially decaying wave moving forward



Both decaying in diff. dirxns.

If we had taken +ve from $k' + ik''$, we would have an exp. increasing wave and at an ∞ amplitude, Energy will become ∞ (which is not possible obv.)



* When to assume ϵ_c = real and ϵ_c = complex

$\sigma \neq 0$, current $\neq 0$

Lost energy will be released in form of heat because of conduction current.

what if $\sigma \ll \omega\epsilon$

↳ Perfect dielectric
$\epsilon_c = \epsilon$ (real ϵ)

$\sigma \sim \omega\epsilon$

↳ Lot of heat will be generated.
medium with decreasing constant

$\sigma \gg \omega\epsilon$

EM wave will die inside a P.E.C.

It will decay very fast.

* Static $\vec{E}$
Time-varying $\vec{E}$ } inside P.E.C. $= 0$

* Static $\vec{B}$ can exist inside P.E.C.

Q: Can Time-varying $\vec{B}$ exist inside P.E.C ?

NO, as T-V $\vec{B}$ will generate $\vec{E}$ but $\vec{E} = 0$ inside P.E.C.

Ag, Cu → good conductors at microwave frequencies
→ not good conductors at optical frequencies