$n(r)$: electronic density

$$n(r) = n(r+R)$$

$$n(r) = \sum n_G e^{ikr}$$
$$n(r+R) = \sum n_G e^{ik(r+R)} = \sum n_G e^{ikr} e^{ikR}$$

and since $n(r) = n(r+R)$,

$$e^{i\bar{k}\bar{R}} = 1$$
$$\cos(KR) + j\sin(KR) = 1$$

$$\underbrace{\bar{K}}_{\text{k-space vector}} \cdot \underbrace{\bar{R}}_{\text{Real space vector}} = 2\pi \qquad \text{OR} \qquad \bar{k}_i \bar{R}_j = 2\pi \delta_{ij}$$

1d:
$$R = \bar{a}$$
$$k = \bar{a}^*$$

$$aa^* = 2\pi \rightarrow a^* = \frac{2\pi}{a}$$

2d:
$$R = \bar{a} + \bar{b}$$
$$k = \bar{a}^* + \bar{b}^*$$

$$\bar{R} = u\bar{a} + v\bar{b} + w\bar{c}$$
$$\bar{k} = h\bar{a}^* + k\bar{b}^* + l\bar{c}^*$$

$\boxed{3d}$

$$aa^* = 2\pi \qquad bb^* = 2\pi \qquad cc^* = 2\pi$$
but $ba^* = 0$ and so on...
$$ca^* = 0$$

So, $a^* = x(b \times c)$

i.e. $a^* \perp b$ and c

$$aa^* = x\,a\cdot(b\times c) = 2\pi$$
$$x = \frac{2\pi}{a\cdot(b\times c)}$$

$$a^* = \frac{2\pi(b\times c)}{a\cdot(b\times c)}$$

$\boxed{2d}$

$$aa^* = 2\pi \qquad bb^* = 2\pi$$
$$ba^* = 0 \qquad ab^* = 0$$

$$a^* \perp b$$
$$a \parallel a^*$$

$$a^* = \frac{2\pi}{a}\hat{a} \qquad b^* = \frac{2\pi}{b}\hat{b}$$

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

$$= \frac{a}{\sqrt{16+36+9}} = \frac{a}{\sqrt{61}}$$