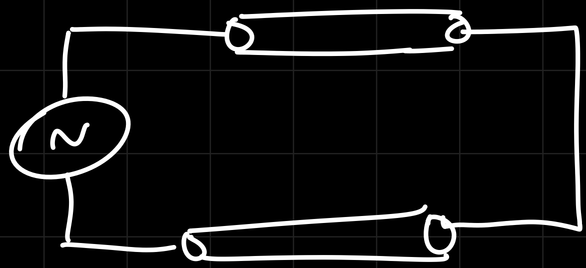
Lecture

transmission line ended with short circuit



for low freq ckts, KVL and KCL applies and $Z_{in} = 0$

for high freq → $Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tan(\gamma L)}{Z_0 + Z_L \tan(\gamma L)} \right]$

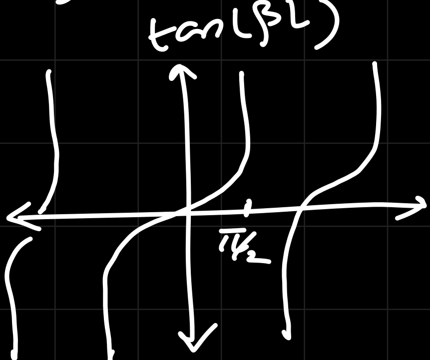
but $Z_L = 0$

$$Z_{in} = Z_0 \tan(\gamma L)$$

for no transmission loss:

$$Z_{in} = j Z_0 \tan(\beta L)$$

inductive or capacitive? depends on L



reflection coefficient?

we know $\Gamma_L(z) = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

and $Z_L = 0$

so $\Gamma_V(z) = -1$

and $\Gamma_I(z) = -\Gamma_V(z) = 1$

if $\beta L = \dfrac{\pi}{2}$

$\beta = \dfrac{2\pi}{\lambda} \rightarrow \dfrac{2\cancel{\pi}}{\lambda} L = \dfrac{\cancel{\pi}}{2}$

$\boxed{L = \dfrac{\lambda}{4}}$ and $Z_{in} = \infty$ because $\tan(\pi/2) = \infty$

for $L = \dfrac{\lambda}{2} \rightarrow \beta L = \pi \rightarrow Z_{in} = 0$

what if transmission line ended by open circuit ?

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tan(\gamma L)}{Z_0 + Z_L \tan(\gamma L)} \right]$$

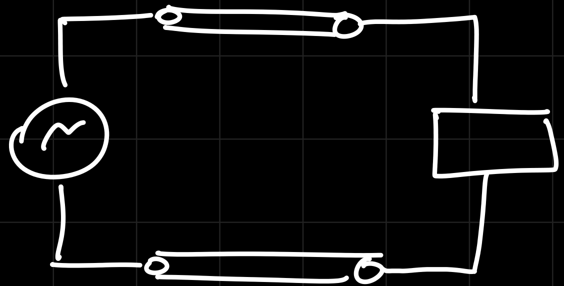
but $Z_L = \infty$

$$Z_{in} = Z_0 \left[\frac{1 + Z_0/Z_L \tan(\gamma L)}{Z_0/Z_L + \tan(\gamma L)} \right]$$

$$= Z_0 \left[\frac{1}{j\tan(\beta L)} \right] = \underline{\underline{-jZ_0 \cot(\beta L)}}$$

$$Z_{in} = -jZ_0 \cot(\beta L)$$

So if we really want ∞ input impedance we dont just do oc we short output and put $L = \frac{\lambda}{2}$



$Z_L = j\beta$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{j\beta - Z_0}{j\beta + Z_0}$$

$|\Gamma_L|$ = reflection coefficient = 1

perfect / pure reflection
no absorption of energy
goes back to SRC

Now prove,
A Perfect Electric conductor is a perfect reflector

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_1}}$ | $\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}}$

1 | 2

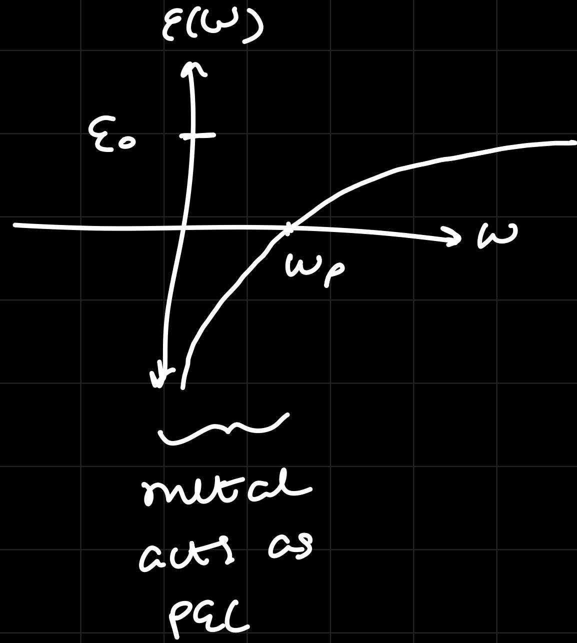
PEC

plasma frequency: $\omega_p \sim$ UV Range

metal $\rightarrow \varepsilon(\omega) = \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right)$

for $\omega < \omega_p$ and lossless transmission,

$$\varepsilon(\omega) = -ve$$



So, $\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}} = \sqrt{\frac{\mu_0}{-a}}$

$$= jm$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{jm - \eta_1}{jm + \eta_1}$$

$$|\Gamma| = \frac{m^2 + n^2}{m^2 + n^2} = \underline{\underline{1}}$$

hence for $\omega < \omega_p$ and lossless transmission, a metal acts as a perfect reflector