

# Midsem Syllabus

↳ ch2, ch4, ch5

## # Non cartesian coordinates

→ Spherical coords →  $r, \theta, \phi$   
↗ angle with z axis  
↘ projection's angle with x-axis

$$d\ell_r = dr, \quad d\ell_\theta = r d\theta, \quad d\ell_\phi = r \sin\theta d\phi$$

Gradient:

$$\nabla T = \frac{\partial T}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial T}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi} \hat{\phi}.$$

Divergence:

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}.$$

Curl:

$$\begin{aligned} \nabla \times \mathbf{v} = & \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{r} + \frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta} \\ & + \frac{1}{r} \left[ \frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi}. \end{aligned}$$

Laplacian:

$$\nabla^2 T = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2}.$$

→ Cylindrical coords →  $\rho, \phi, z$

$$d\ell_\rho = d\rho, \quad d\ell_\phi = \rho d\phi, \quad d\ell_z = dz$$

Gradient:

$$\nabla T = \frac{\partial T}{\partial \rho} \hat{s} + \frac{1}{\rho} \frac{\partial T}{\partial \phi} \hat{\phi} + \frac{\partial T}{\partial z} \hat{z}.$$

Divergence:

$$\nabla \cdot \mathbf{v} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho v_\rho) + \frac{1}{\rho} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}.$$

Curl:

$$\nabla \times \mathbf{v} = \left( \frac{1}{\rho} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right) \hat{s} + \left( \frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial \rho} \right) \hat{\phi} + \frac{1}{\rho} \left[ \frac{\partial}{\partial \rho} (\rho v_\phi) - \frac{\partial v_\rho}{\partial \phi} \right] \hat{z}.$$

Laplacian:

$$\nabla^2 T = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial T}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}.$$

## # Dirac Delta Function

$$\text{let } \vec{v} = \frac{1}{r^2} \hat{r}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \cdot \frac{1}{r^2} \right) = 0$$

reason  $\rightarrow$  we used the divergence formula for spherical coordinates  
 $\hookrightarrow$  not cartesian.

sphere of radius  $R \leftarrow s$

$$\oint \vec{v} \cdot d\vec{a} = \int_{R^2} \frac{1}{r^2} \hat{r} \cdot R^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$$
$$= \left( \int_0^\pi \sin\theta \, d\theta \right) \left( \int_0^{2\pi} d\phi \right)$$
$$= [-\cos\theta]_0^\pi \cdot [\phi]_0^{2\pi}$$
$$= (1+1)(2\pi) = \underline{4\pi}$$

now,

$$\vec{\nabla} \cdot \left( \frac{\hat{r}}{r^2} \right) = 4\pi \delta^3(r)$$

quiz 2 - questions  $\rightarrow$

Q2) (a)  $\int_{-1}^{+1} (r^2 + 2) \vec{\nabla} \cdot \left( \frac{\hat{r}}{r^2} \right) dv$

$$= \int_{-1}^{+1} (r^2 + 2) 4\pi \delta^3(r) dv$$
$$= (0+2)(4\pi) = \underline{8\pi}$$

(b)  $\int_1^\infty (r^2 + 2) \vec{\nabla} \cdot \left( \frac{\hat{r}}{r^2} \right) dv$

$$= \int_1^\infty (r^2 + 2) \underbrace{4\pi \delta^3(r)}_{\text{always } = 0} dv = \underline{0}$$

$\therefore r=0$  not included in the integral