Lecture: TRANSMISSION LINES

$$\frac{\partial I}{\partial z} = -VY \qquad \frac{\partial V}{\partial z} = -IZ$$

Y: admittance

Z: impedance (same as X)

$$\frac{\partial^2 I}{\partial z^2} - \gamma^2 I = 0 \qquad \frac{\partial^2 V}{\partial z^2} - \gamma^2 V = 0$$

$$\gamma = \sqrt{XY}$$

$$X = R + j\omega L$$

$$Y = G + j\omega C$$

$\frac{1}{R_s}$

$$I(z) = \underbrace{I_0^+ e^{-\gamma z}}_{\text{forward travelling}} + \underbrace{I_0^- e^{\gamma z}}_{\text{backward travelling}}$$

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$= V_0^+ e^{-i\beta z} e^{-\alpha z} + V_0^- e^{i\beta z} e^{\alpha z}$$

$$v = \frac{1}{\sqrt{LC}} \quad : \text{Velocity}$$

Tx line with no reflected wave

$$V(z) = V_0^+ e^{-\gamma z}$$
$$I(z) = I_0^+ e^{-\gamma z}$$

$$\frac{V(z)}{I(z)} = Z_0 = \frac{V_0^+}{I_0^+}$$

Characteristic Impedance
$\hookrightarrow Z_0$

$$\frac{\partial V}{\partial z} = \frac{\partial (V_0 e^{-\gamma z})}{\partial z} = -XI = -X I_0 e^{-\gamma z}$$

$$-\gamma V_0^+ e^{-\gamma z} = -X I_0^+ e^{-\gamma z}$$
$$\frac{V_0^+}{I_0^+} = \frac{X}{\gamma} = \frac{X}{\sqrt{XY}} = \sqrt{\frac{X}{Y}}$$

So, $$Z_0 = \sqrt{\frac{X}{Y}} = \frac{\sqrt{R+j\omega L}}{\sqrt{G+j\omega C}}$$

In lossless transmission:

$$Z_0 = \sqrt{\frac{X}{Y}} = \sqrt{\frac{0+j\omega L}{0+j\omega C}} = \sqrt{\frac{L}{C}}$$

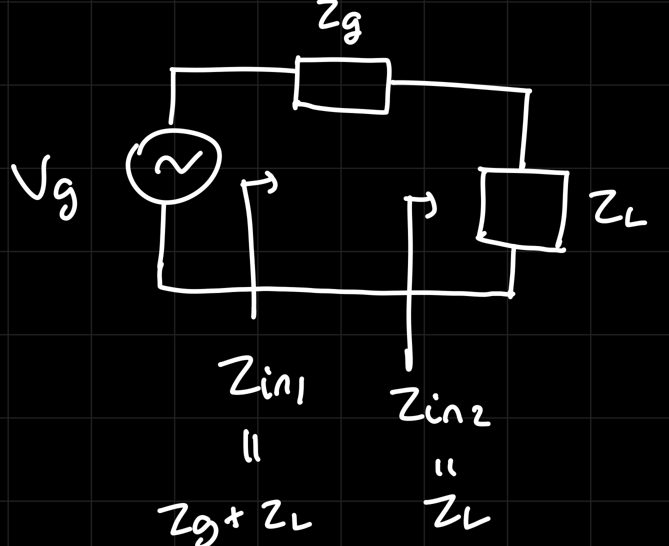
$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

for backward travelling wave:

$Z_0 \rightarrow -Z_0$

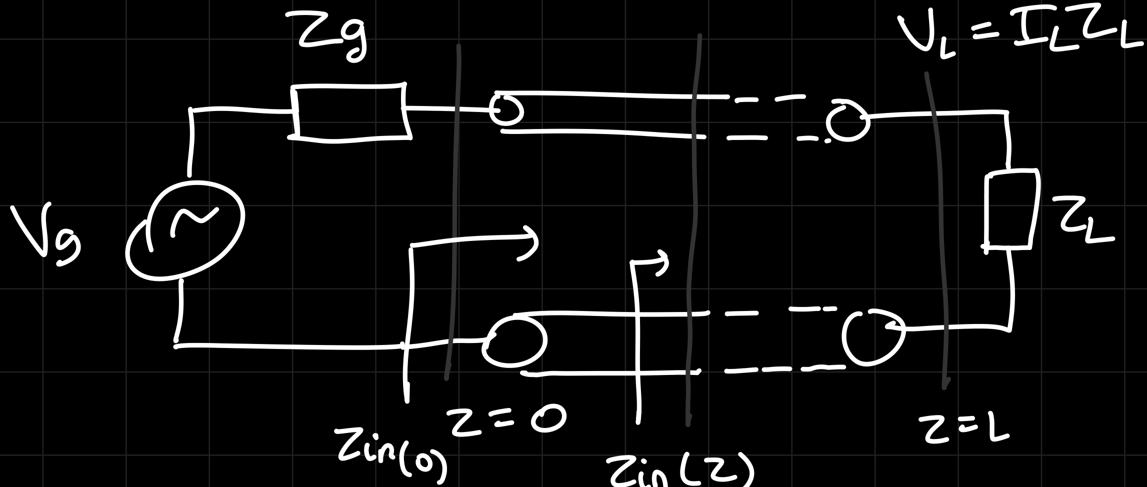
① * $\begin{cases} V_0^- = -Z_0 I_0^- \\ V_0^+ = Z_0 I_0^+ \end{cases}$

$$\frac{V_0^-}{I_0^-} = -Z_0$$



for a small circuit like this ←
it ez to determine input impedances

for transmission lines we have to consider the π ckts of R, L, C, Rs as well



Q) given L; what is Zin?

$$Z_{in}(z) = \frac{V(z)}{I(z)}$$

$$Z_{in} = Z_{in}(0) = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{I_0^+ + I_0^-}$$

$$\{\text{from } 1\} = \frac{V_0^+ + V_0^-}{\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}} = Z_0\left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right)$$

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$$I_L = I_0^+ e^{-\gamma L} + I_0^- e^{\gamma L}$$

$$-/+ \begin{cases} V_L = V_0^+ e^{-\gamma L} + V_0^- e^{\gamma L} \\ Z_0 I_L = V_0^+ e^{-\gamma L} - V_0^- e^{\gamma L} \end{cases}$$

$\downarrow$

$$V_L + Z_0 I_L = 2V_0^+ e^{-\gamma L}$$
$$V_L - Z_0 I_L = 2V_0^- e^{\gamma L}$$

$$V_0^+ = \frac{1}{2}(Z_L + Z_0) I_L e^{\gamma L}$$

$$V_0^- = \frac{1}{2}(Z_L - Z_0) I_L e^{-\gamma L}$$

back to ②

$$Z_{in} = Z_0\left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right)$$

$$= Z_0\left(\frac{(Z_L+Z_0)e^{\gamma L} + (Z_L-Z_0)e^{-\gamma L}}{(Z_L+Z_0)e^{\gamma L} - (Z_L-Z_0)e^{-\gamma L}}\right)$$

$$= Z_0\left(\frac{Z_L(e^{\gamma L}+e^{-\gamma L}) + Z_0(e^{\gamma L}-e^{-\gamma L})}{Z_L(e^{\gamma L}-e^{-\gamma L}) + Z_0(e^{\gamma L}+e^{-\gamma L})}\right)$$

$$= Z_0\left(\frac{Z_L\cdot\cosh(\gamma L) + Z_0\cdot\sinh(\gamma L)}{Z_L\cdot\sinh(\gamma L) + Z_0\cdot\cosh(\gamma L)}\right)$$

$Z_{in} = Z_0\left[\frac{Z_L + Z_0\tanh(\gamma L)}{Z_L\tanh(\gamma L) + Z_0}\right]$

for a lossless transmission: $\gamma = j\beta$

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tanh(\beta L)}{Z_0 + jZ_L\tanh(\beta L)}\right]$$

remember : reflection coefficient

$$\Gamma(z) = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^+}{V_0^-} e^{2\gamma z}$$

$$\Gamma_L(z) = \frac{V_0^- e^{\gamma L}}{V_0^+ e^{-\gamma L}}$$

$$\left.\begin{aligned} \tfrac{1}{2}(V_L + Z_0 I_L) &= V_0^+ e^{-\gamma L} \\ \tfrac{1}{2}(V_L - Z_0 I_L) &= V_0^- e^{\gamma L} \end{aligned}\right\} \text{ from before}$$

$$\Gamma_L(z) = \frac{V_L - Z_0 I_L}{V_L + Z_0 I_L} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$\Gamma_L(z) = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

reflection coeff right at the load

we wont have any reflected wave

① when: $\Gamma_L(z) = 0$ i.e. $Z_L = Z_0$

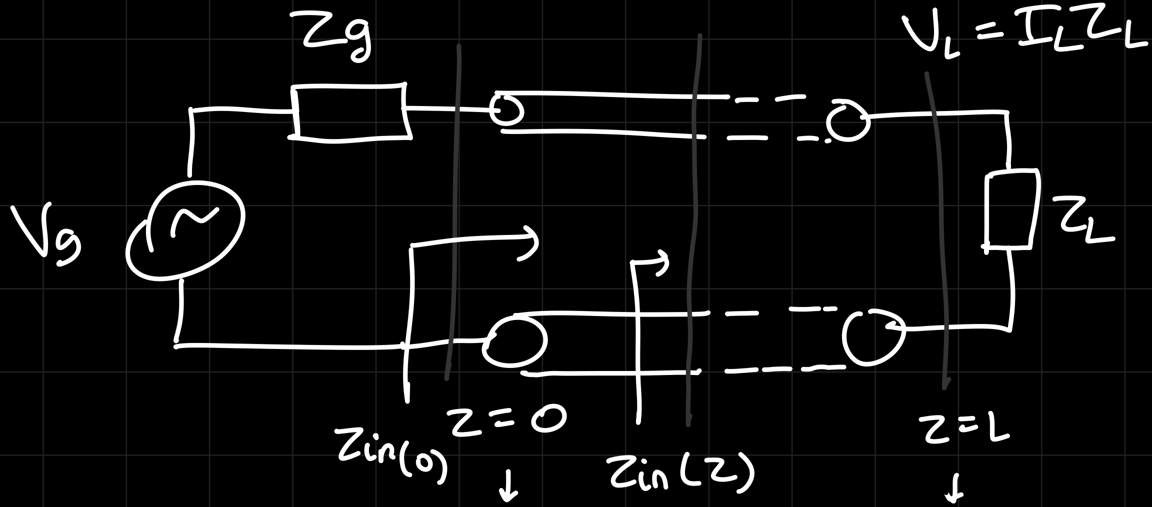
(matched impedance/Load)

RFIC

② or when the 1st medium is infinitely large

i.e. the interface is at ∞

$(L \to \infty)$



let : $\quad l' = l \quad\quad l' = l - z$

$$\Gamma(z) = \frac{V_0^- e^{\gamma L}}{V_0^+ e^{-\gamma L}} \times \frac{e^{-\gamma L}}{e^{-\gamma L}} \times e^{2\gamma z}$$

$$= \Gamma_L e^{-2\gamma(l-z)} = \Gamma_L e^{-2\gamma l'}$$

$\Gamma(z) = \Gamma_0 e^{-2\gamma l'}$

$$\rightarrow Z_{in}(z) = \frac{V(z)}{I(z)} = \frac{V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}}{I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}}$$

$$\times \frac{e^{\gamma z}}{e^{\gamma z}}$$

$$= \frac{V_0^+ + V_0^- e^{2\gamma z}}{\frac{V_0^+}{Z_0} - \frac{V_0^- e^{2\gamma z}}{Z_0}}$$

$$\div \frac{V_0^+}{V_0^+}$$

$$= \left(\frac{1 + V_0^-/V_0^+ \, e^{2\gamma z}}{1 - V_0^-/V_0^+ \, e^{2\gamma z}} \right) Z_0$$

$Z_{in} = Z_0 \left(\frac{1 + \Gamma(z)}{1 - \Gamma(z)} \right)$

reflection coeff (current)

$$\Gamma_{current_L} = \frac{I_0^- e^{\gamma L}}{I_0^+ e^{-\gamma L}} = -\frac{V_0^-}{V_0^+} e^{2\gamma L} = -\Gamma_{voltage_L}$$

$\Gamma_{I_L} = -\Gamma_{V_L}$

Considering lossless transmission

$$V(z) = V_0^+ e^{-i\beta z} + V_0^- e^{i\beta z}$$

$$= V_0^+ \cos\beta z - jV_0^+ \sin\beta z + V_0^- \cos\beta z + jV_0^- \sin\beta z$$

$$= \cos\beta z (V_0^+ + V_0^-) - j\sin\beta z (V_0^+ - V_0^-)$$

amplitude $= |V(z)| = \sqrt{\cos^2\beta z (V_0^+ + V_0^-)^2 + \sin^2\beta z (V_0^+ - V_0^-)^2}$

$$= V_0^{+2} + V_0^{-2} + 2V_0^+ V_0^- \cos(2\beta z)$$

maxima : $\cos(2\beta z) = \cos(2n\pi) = 1$

$\downarrow$ $\quad z = \frac{n\pi}{\beta}$

$\boxed{|V(z)|_{max} = V_0^+ + V_0^-}$

note: $\beta = \omega\sqrt{LC}$

minima : $\cos(2\beta z) = \cos((2n+1)\pi) = -1$

$z = \frac{(2n+1)\pi}{\beta} \longrightarrow \boxed{|V(z)|_{min} = V_0^+ - V_0^-}$

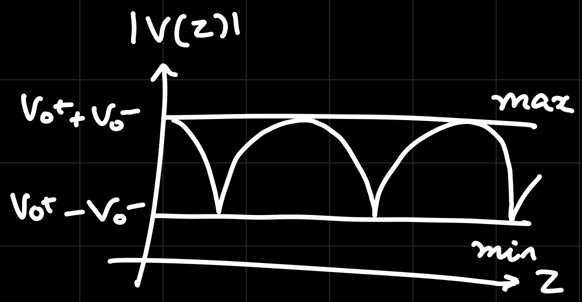
$\beta \to \frac{2\pi}{\lambda}$

max : at $z = \frac{n\pi}{\beta} = \frac{n\lambda}{2}$

min : at $z = (n+\frac{1}{2})\lambda$

$\beta \equiv$ wave numbe

and hence $\beta = \frac{2\pi}{\lambda}$



Standing wave pattern

if $V_0^+ = V_0^- \to$ pure standing wave

if no reflected back i.e $V_0^- = 0$,

we get a straight line standing

wave pattern

VSWR (voltage standing wave ratio)

$$\hookrightarrow s = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

if $\Gamma_L = 1/-1$ i.e. everything is getting reflected back

$\downarrow$

$s = \infty$ (standing wave)

if $|\Gamma_L| = 0 \to s = 1$

$$\boxed{1 \leq s \leq \infty}$$