lecture

$$\frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = \eta : \Omega$$

$$w_e = \frac{1}{2}\varepsilon|E|^2$$

$$w_m = \frac{1}{2}\mu|E|^2$$

$$w_e + w_m = \frac{1}{2}\left(\varepsilon|E|^2 + \mu|H|^2\right)$$

$$W_e = \frac{1}{2}\varepsilon\int|E|^2 dz$$

$$W_m = \frac{1}{2}\mu\int|H|^2 dz$$

$$\bar{\nabla}\cdot(\bar{E}\times\bar{H}) = \bar{H}\cdot(\bar{\nabla}\times\bar{E}) - \bar{E}(\bar{\nabla}\times\bar{H})$$

$$= \bar{H}\left(-\mu\frac{\partial\bar{H}}{\partial t}\right) - \bar{E}(\cdots)$$

$$= -\mu\,\bar{H}\cdot\frac{\partial\bar{H}}{\partial t} - \bar{E}\left(\bar{J} + \varepsilon\frac{\partial\bar{E}}{\partial t}\right)$$

$$= -\frac{\mu}{2}\frac{\partial|H|^2}{\partial t} - \bar{E}\cdot\bar{J} - \frac{\varepsilon}{2}\frac{\partial|E|^2}{\partial t}$$

$$= -\frac{\partial}{\partial t}\left\{\frac{1}{2}\left(\mu|\bar{H}|^2 + \varepsilon|\bar{E}|^2\right) - \bar{E}\cdot\bar{J}\right\}$$

$$-\frac{\partial}{\partial t}(W_e + W_m) = \underset{\text{(heat)}}{\bar{E}\cdot\bar{J}} + \underset{\text{(outgoing)}}{\bar{\nabla}(\bar{E}\times\bar{H})}$$

decay rate of stored energy in an electromagnetic field

we said in the last class

$\bar{E}$ = Voltage

$\bar{B}$ = current

So, $\bar{E}\cdot\bar{J} \simeq$ Power

$$dw\ \bar{F}\cdot d\bar{l} = q(\bar{E} + \underbrace{(\bar{v}\times\bar{B}))\cdot\bar{v}}_{0}\,dt$$

$$dw\ \bar{F}\cdot d\bar{l} = q\bar{E}\bar{v}\,dt$$

$$\frac{dw}{dt} = \rho\, d\tau\ \bar{E}\cdot\bar{v}$$

$$\frac{dw}{dt} = \bar{E}\cdot\bar{J}\,d\tau$$

The Energy is either used by doing some work $(\bar{E}\cdot\bar{J})$ or comes out of the surface $(\bar{\nabla}\cdot(\bar{E}\times\bar{H}))$

$$-\frac{\partial}{\partial t}\int (w_e + w_m)\, d\tau = \int \bar{E}\cdot\bar{J}\, d\tau + \int \bar{\nabla}\cdot(\bar{E}\times\bar{H})\, d\tau$$

$$= \int \bar{E}\cdot\bar{J}\, d\tau + \oiint (\bar{E}\times\bar{H})\, d\bar{s}$$

So, $P_{outgoing} = \oiint \underbrace{(\bar{E}\times\bar{H})}_{\vec{S}}\, d\bar{s}$

(Poynting vector)

$$\bar{S} = \bar{E}\times\bar{H}$$

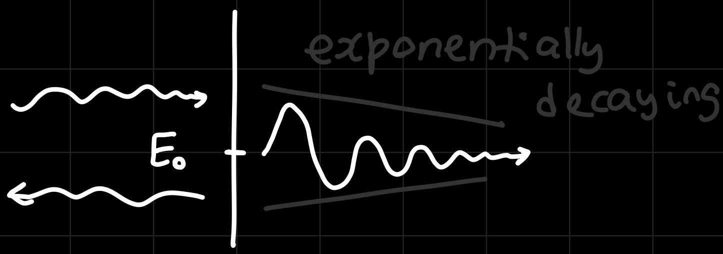
$|\bar{S}| = |E||H| = |E|\cdot\frac{|E|}{\eta} = \frac{|E|^2}{\eta}$: $\simeq$ power per unit area (Watt/m²)

last topic for EM waves →

Next : transmission lines

SOMETHINGIDK123

$$\varepsilon_c = \varepsilon\left(1 - \frac{i\sigma}{\omega\varepsilon}\right)$$



$$k = \omega\sqrt{\mu\varepsilon_c}$$

$$k = k' - ik''$$

$$e^{-ikz} = e^{-i(k'-ik'')z} = e^{-ik'z}e^{-k''z}$$

Skin depth of a medium:

$$\delta = \frac{1}{k''}$$

↳ 0 for a PEC $(\infty \times)$

distance for the signal to decay

for $z = \delta$,

$$\bar{E} = E_0 e^{-ik'z} e^{-k''/k''} = \frac{E_0}{e} e^{-ik'z}$$

$$k^2 = \omega^2\mu\varepsilon\left(1 - \frac{i\sigma}{\omega\varepsilon}\right)$$

$$(k'-ik'')^2 = k'^2 - k''^2 - 2ik'k'' = \omega^2\mu\varepsilon - i\omega\sigma\mu$$

$$2k'k'' = \omega\sigma\mu$$

$$k'^2 - k''^2 = \omega^2\mu\varepsilon \quad \text{— ①}$$

for simplicity, we find

$$(k'^2 + k''^2)^2 = (k'^2 - k''^2)^2 + 4(k'k'')^2$$

$$= \omega^4 \mu^2 \varepsilon^2 + \omega^2 \mu^2 \sigma^2$$

$$(k'^2 + k''^2) = \omega^2 \mu \varepsilon \left[1 + \frac{\sigma^2}{\omega^2 \varepsilon^2} \right]^{1/2} \quad \text{— } 2$$

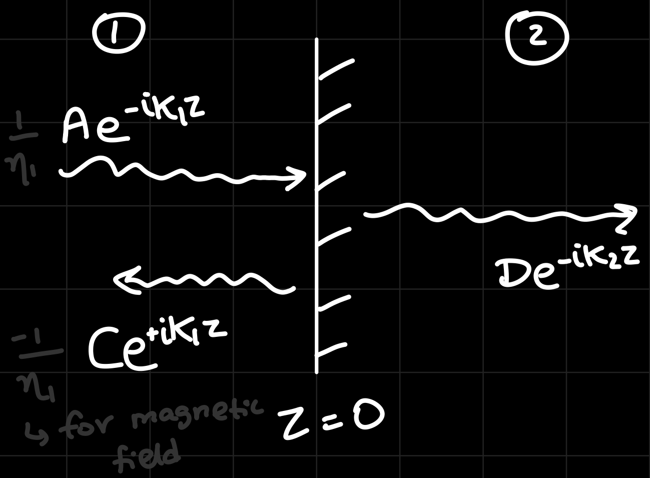
using 1 and 2 →

$$k' = \omega \sqrt{\frac{\mu \varepsilon}{2}} \left[1 + \left(1 + \frac{\sigma^2}{\omega^2 \varepsilon^2} \right)^{1/2} \right]^{1/2}$$

$$k'' = \omega \sqrt{\frac{\mu \varepsilon}{2}} \left[\left(1 + \frac{\sigma^2}{\omega^2 \varepsilon^2} \right)^{1/2} - 1 \right]^{1/2}$$

Note: Lossless medium → no decay

We figure out reflection and trasmission of the em waves



(μ_1, ε_1) (μ_2, ε_2)

$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$ $k_2 = \omega\sqrt{\mu_2\varepsilon_2}$

assuming SRC ≡ medium ①

transmitted ≡ medium ②

$$\Gamma(z) = \frac{Ce^{-ik_1z}}{Ae^{-ik_2z}}$$

Reflection coefficient:

(gamma)

$$\Gamma(z) = \frac{C}{A} e^{i2k_1z}$$

at the interface → $z=0$

$$\Gamma_0 = \frac{C}{A}$$

$$\boxed{\Gamma(z) = \Gamma_0 e^{i2k_1z}}$$

lossless medium ≡ no conduction current (pure dielectric)

for a lossless medium → Γ_0 decreases as we move away from the interface and $|\Gamma(z)| = |\Gamma_0|$

periodicity:

$$e^{i2\pi} = 1 \rightarrow 2k_1 z = 2\pi$$

$$2 \cdot \frac{2\pi}{\lambda} \cdot z = 2\pi \rightarrow \boxed{z = \frac{\lambda}{2}}$$

Transmission coefficient : $T = \frac{D}{A}$

defined at the interface only, not all points because there exists only the transmitted wave in medium 2.

at $Z=0$: $\frac{E_{t1}}{H_{t1}} = \frac{E_{t2}}{H_{t2}}$

$$\frac{A+C}{A/\eta_1 + C/(-\eta_1)} = \eta_2$$

$$\frac{1+(\Gamma_0)}{\frac{1}{\eta_1}(1-\Gamma_0)} = \eta_2$$

$$\frac{\eta_2}{\eta_1} = \frac{1+\Gamma_0}{1-\Gamma_0}$$

$$A + C = D$$

$$1 + C/A = D/A$$

$$\boxed{1 + \Gamma_0 = T}$$

$$T = 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \rightarrow \boxed{T = \frac{2\eta_2}{\eta_1 + \eta_2}}$$