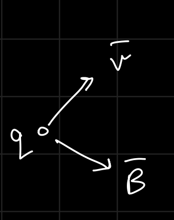
Lorentz's Force

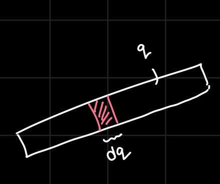
$$F = q\cdot(\bar{V}\times\bar{B}) + q\bar{E}$$



$q\cdot(\bar{V}\times\bar{B})$: magnetic component

$q\bar{E}$: electric field component (ignoring for now)

magnetic force on continuous charge



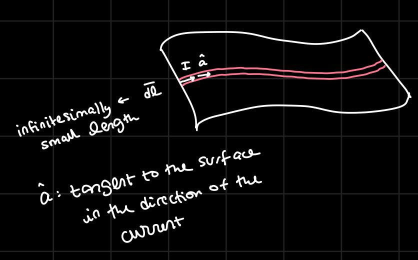
$$d\bar{F}_{mag} = dq(\bar{V}\times\bar{B})$$
$$\downarrow$$
$$\bar{F}_{mag} = \int d\bar{F}_{mag}$$

$$\bar{F}_{mag} = \int dq\,(\bar{V}\times\bar{B})$$
$$= \int \frac{dq}{dt}(d\bar{l}\times\bar{B})$$
$$= \boxed{\int I\cdot(d\bar{l}\times B)}$$

line charge $\rightarrow$ C/m

surface " $\rightarrow$ C/m^2

volume " $\rightarrow$ C/m^3



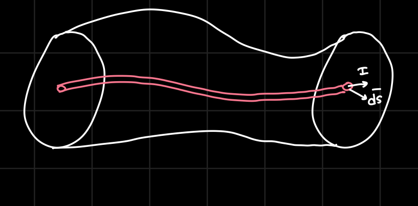
$\hat{k}$ = surface current density

$$\boxed{\hat{k} = \frac{dI}{dl_\perp}\hat{a}}$$

unit: Ampere per metre
Am^{-1}

NOTE: There is no concept of line current density

VOLUME CURRENT DENSITY : $\bar{J}$



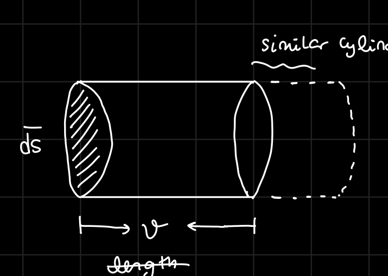
note: $\hat{n}$ ≡ direction of current
$\hat{n}$ and $\bar{ds}$ are $\perp$

$$\boxed{\bar{J} = \frac{dI}{ds_\perp}\hat{n}}$$

unit: A/m^2

note: $\bar{J} = \sigma\bar{E}$ is the same $\bar{J}$

> Now we can relate current density with charge density



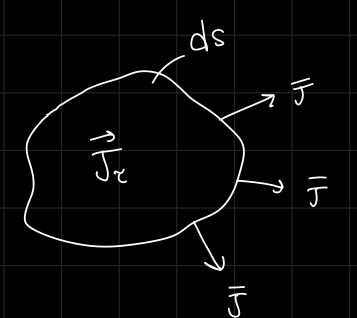
if ρ ≡ vol. charge density
now find I from ρ

$$I = \rho\,\underbrace{\bar{ds}\cdot v}_{\text{volume}}$$

$$\frac{I}{ds} = \rho v \quad\rightarrow\quad \boxed{\bar{J} = \rho\bar{v}}$$

$$\bar{F}_{mag} = \int dq\,(V\times B)$$
$$= \int \rho\, d\tau\,(V\times B) = \int(\rho\bar{v}\times\bar{B})\,d\tau$$

$$\boxed{\bar{F}_{mag} = (\bar{J}\times\bar{B})\,d\tau}$$



$$\boxed{\oint \bar{J}\cdot\bar{ds} = -\frac{dQ}{dt}}$$

↳ law of conservation of charge

using divergence theorem

$$\oint \bar{J}\cdot\bar{ds} = \int(\nabla\cdot J)\,d\tau = -\frac{\partial}{\partial t}\int\rho\, d\tau$$
$$= -\int\frac{\partial\rho}{\partial t}d\tau$$

$$\boxed{\bar{\nabla}\bar{J} + \frac{\partial\rho}{\partial t} = 0} \rightarrow \text{eq}^n \text{ of continuity}$$

also,

$$\bar{J} = \rho\bar{v}$$
$$\bar{\nabla}(\rho\bar{v}) + \frac{\partial\rho}{\partial t} = 0$$

for small Electric field, we can assume v varies with $\bar{E}$ linearly $\rightarrow$ $\bar{v} = \mu\bar{E}$

$$\bar{J} = \rho\bar{v} = (\rho\mu)\bar{E} = \sigma\bar{E}$$

↳ conductivity
also, reciprocal of resistivity

$$\boxed{\vec{J} = \sigma\vec{E}}$$

OHM'S LAW

from eq^n of continuity,

$$\sigma(\nabla\bar{E}) = -\frac{\partial\rho}{\partial t} \quad\rightarrow\quad \sigma\frac{\rho}{\varepsilon_0} = -\frac{\partial\rho}{\partial t}$$

$$\int\frac{\partial\rho}{\rho} = \frac{-\sigma}{\varepsilon_0}\int\partial t$$

$$\ln\rho + c = \frac{-\sigma}{\varepsilon_0}t$$

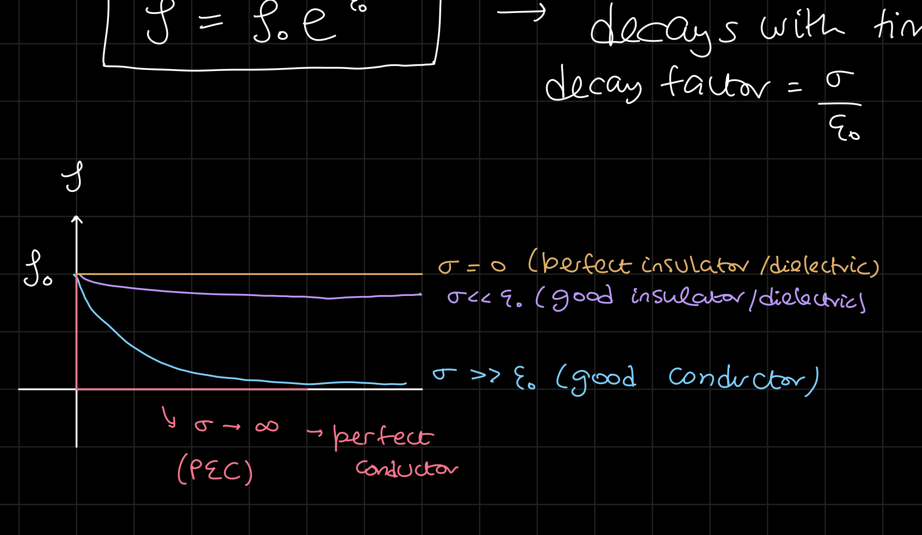
at $t = 0$, $\rho = \rho_0$

$$\ln\rho_0 = -c \quad\rightarrow\quad c = -\ln\rho_0$$

so,
$$\ln\rho - \ln\rho_0 = \frac{-\sigma}{\varepsilon_0}t$$
$$\ln\left(\frac{\rho}{\rho_0}\right) = -\frac{\sigma}{\varepsilon_0}t$$

$$\boxed{\rho = \rho_0 e^{-\frac{\sigma}{\varepsilon_0}t}} \rightarrow \text{decays with time}$$

decay factor $= \frac{\sigma}{\varepsilon_0}$



$$\rho = \rho_0 e^{(-t/\tau_{relaxation})}$$

where $\tau_{relaxation} = \frac{\varepsilon_0}{\sigma}$

time variation involved

at $t = \tau$, $\rho = \rho_0 e^{-1} = \frac{\rho_0}{e}$

Significance of τ: time taken to reduce the ρ to $\frac{1}{e}$ times of ρ_0

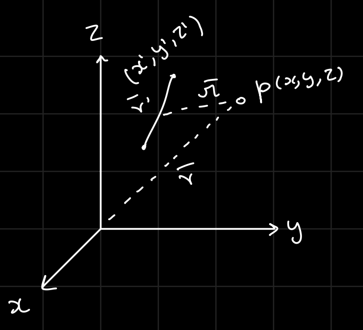
Magnetostatics

$$\bar{\nabla}\bar{J} = -\frac{\partial \rho}{\partial t}$$

let ρ = const with time
≡ steady current
so, $\frac{\partial \rho}{\partial t} = 0 \quad \rightarrow \quad \boxed{\bar{\nabla}\bar{J} = 0}$

* A point charge cannot give steady current
because $\rho \equiv$ not constant
because ρ is a function of space

→ BIOT SAVART'S LAW



$$\bar{B}(r) = \frac{\mu_0}{4\pi}\int \frac{I \cdot d\bar{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$= \frac{\mu_0}{4\pi}\int \frac{\bar{J}(r') \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, dv'$$

(x',y',z')

volume integral

$$(\bar{\nabla}\cdot\bar{B}) = \frac{\mu_0}{4\pi}\int \nabla\cdot\left(\bar{J}(r') \times \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\right)$$

divergence wrt (x,y,z)

$$= \frac{\mu_0}{4\pi}\int \left[\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}(\bar{\nabla}\times\bar{J}(r')) - \bar{J}(r')\left(\bar{\nabla}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\right]$$

note: $\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = \nabla\cdot\left(\frac{1}{\mathscr{r}}\right)$

So, its curl is 0

also, J is dependent on $\mathscr{r}'$ / x', y', z'
but the curl is wrt $\mathscr{r}$ / x,y,z
so, that is zero as well

so, $\boxed{\bar{\nabla}\cdot\bar{B} = 0}$

$$\int(\nabla\cdot B)\,dv = \boxed{\oint \bar{B}\cdot d\bar{s} = 0}$$

Gauss's Law for magnetic field

Remember: This is only for steady current / pure magnetostatics

but it is true for magnetic field varying with time as well.

↑ MIDSEM SYLLABUS ↑