$H = X(X^T X)^{-1} X^T$

$\Rightarrow$ Symmetric $\quad H^T = H$

$\Rightarrow$ Idempotent $\quad HH = H$

Symmetric, $H' = \left( X (X^T X)^{-1} X^T \right)^T$

$= (X^T)^T \left[ (X^T X)^{-1} \right]^T X^T$

$= X \left( (X' X)^T \right)^{-1} X^T$

$= X (X' X)^{-1} X^T$

$= H$

$(ABC)' = C'B'A'$

H is idempotent

$$HH = H^2 = \left(X\,(X^TX)^{-1}X^T\right)_{p\times n}\left(X\,(X^TX)^{-1}X^T\right)_{n\times p}$$

$$= X\,\underbrace{(X^TX)^{-1}(X^TX)}_{I}\,(X^TX)^{-1}X^T$$

$$= X\,I\,(X^TX)^{-1}X^T$$

$$= X\,(X^TX)^{-1}X^T$$

$$= \underline{\underline{H}}.$$

if we have N sample points $\{(x_1, y_1), (x_2, y_2), \ldots\}$

$$\boxed{Y_i = \beta_0 + \beta_1 x_i}$$

Error of Sample i :

$$e_i = y_i - \hat{Y}_i$$

and we have to minimize

$$SSE = \sum_{i=1}^{n} (y_i - \hat{Y}_i)^2$$

$$\arg\min_{\beta_0 \beta_1} \sum_{i=1}^{n} SSE = \arg\min_{\beta_0 \beta_1} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 =$$

$$= \arg\min_{\beta_0 \beta_1} \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 x_i)^2$$

Diff wrt $\beta_0$

$$2 \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 x_i)(-1) = 0$$

$$\beta_0 = \frac{\sum_{i=1}^{n} (Y_i - \beta_1 x_i)}{n}$$

$$\beta_0 = \sum_{i=1}^{n} \frac{Y_i}{n} - \beta_1 \sum_{i=1}^{n} \frac{x_i}{n} \Rightarrow \beta_0 = \bar{Y} - \beta_1 \bar{X}$$

means.

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$$\arg\min_{\beta_0, \beta_1} \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)^2$$

Diff wrt $\beta_1$

$$\Rightarrow 2 \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i)(-x_i) = 0$$

$$\Rightarrow \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_i) x_i = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i y_i - \beta_0 \sum_{i=1}^{n} x_i - \beta_1 \sum_{i=1}^{n} x_i^2 = 0$$

put value of $\beta_0$ here

$$\Rightarrow \sum_{i=1}^{n} x_i y_i - \left( \sum_{i=1}^{n} \frac{y_i}{n} - \beta_1 \sum \frac{x_i}{n} \right) \sum x_i - \beta_1 \sum x_i^2 = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i y_i - \sum \frac{y_i}{n} \sum x_i + \beta_1 \sum \frac{x_i}{n} \sum x_i - \beta_1 \sum x_i^2 = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i y_i - \frac{1}{n} \sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i + \beta_1 \frac{1}{n} \sum x_i \sum x_i - \beta_1 \sum x_i^2 = 0$$

$$\beta_1 = \frac{\sum x_i y_i - (1/n) \sum y_i x_i}{\sum x_i^2 - \frac{1}{n} \left(\sum x_i\right)^2}$$

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$$\beta_1 = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2}$$

multip Var Normal Eqn

$$\Rightarrow \beta = (X^T X)^{-1} X^T Y$$

$$\boxed{y = X\beta}$$

predicted value.