EE2703 Applied Programming Lab - Assignment 6

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1 Introduction

In this assignment, we model a tubelight as a one dimensional space of gas in which electrons are continually injected at the cathode and accelerated towards the anode by a constant electric field. The electrons can ionize material atoms if they achieve a velocity greater than some threshold, leading to an emission of a photon. This ionization is modeled as a random process. The tubelight is simulated for a certain number of timesteps from an initial state of having no electrons. The results obtained are plotted and studied.

2 The simulation

A function to simulate the tubelight given certain parameters is written below:

```
def simulateTubelight(n,M,nk,u0,p,Msig,accurateCollisions=False):
    """
    Simulate a tubelight and return the electron positions and velocities,
    and positions of photon emissions.

    n: integer length of tubelight
    M: average number of electrons generated per timestep
    nk: total number of timesteps to simulate
    u0: threshold voltage for ionization
    p: probability of ionization given an electron is faster than the threshold
    Msig: stddev of number of electrons generated per timestep
    accurateCollisions: whether to consider accurate updates after collisions

    """

    xx = zeros(n*M)
    u = zeros(n*M)
    dx = zeros(n*M)

    I = []
    X = []
    V = []
```

```python
for k in range(nk):

    # add new electrons
    m=int(randn()*Msig+M)
    jj = where(xx==0)
    xx[jj[0][:m]]=1

    # find electron indices
    ii = where(xx>0)

    # add to history lists
    X.extend(xx[ii].tolist())
    V.extend(u[ii].tolist())

    # update positions and speed
    dx[ii] = u[ii]+0.5
    xx[ii]+=dx[ii]
    u[ii]+=1

    # anode check
    kk = where(xx>=n)
    xx[kk]=0
    u[kk]=0

    # ionization check
    kk = where(u>=u0)[0]
    ll=where(rand(len(kk))<=p);
    kl=kk[ll];

    # ionize
    dt = rand(len(kl))
    xx[kl]=xx[kl]-dx[kl]+((u[kl]-1)*dt+0.5*dt*dt)
    u[kl]=0

    if accurateCollisions:
        u[kl]+=1-dt
        xx[kl]+=0.5*(1-dt)**2

    # add emissions
    I.extend(xx[kl].tolist())

return X,V,I
```

3 Plots

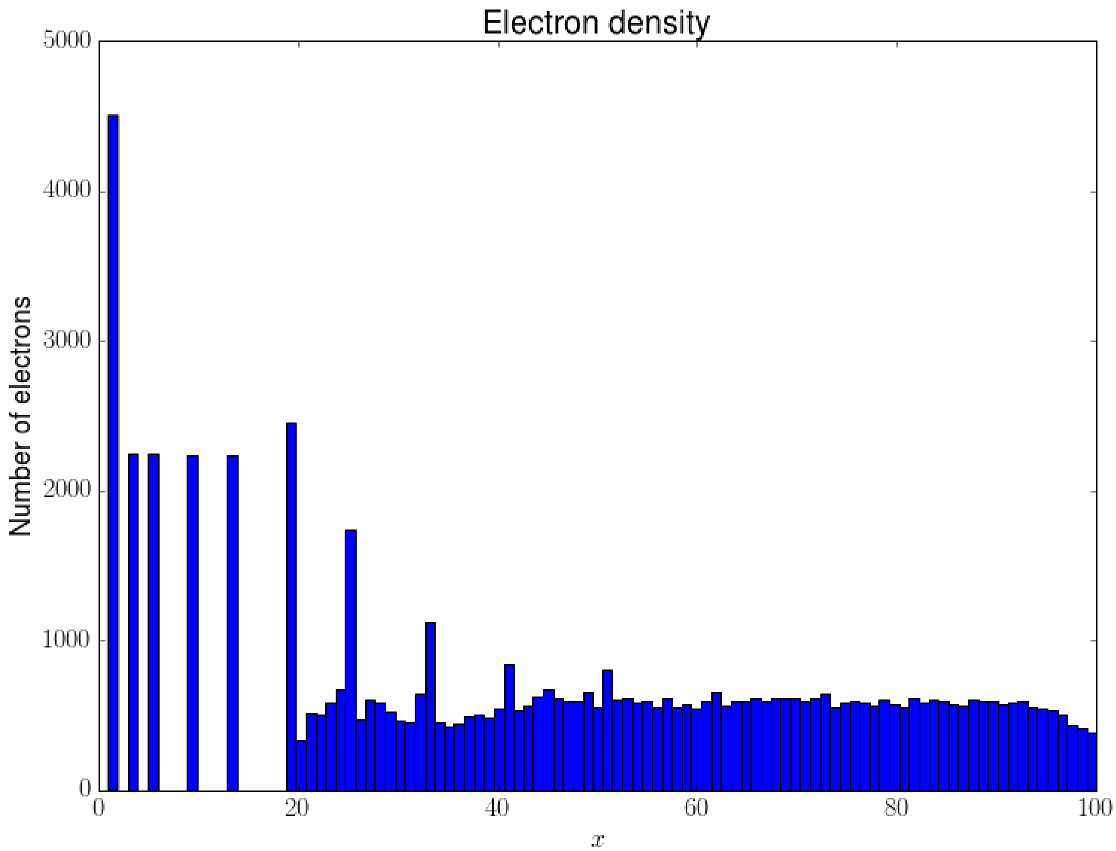
A function to plot the required graphs is written below:

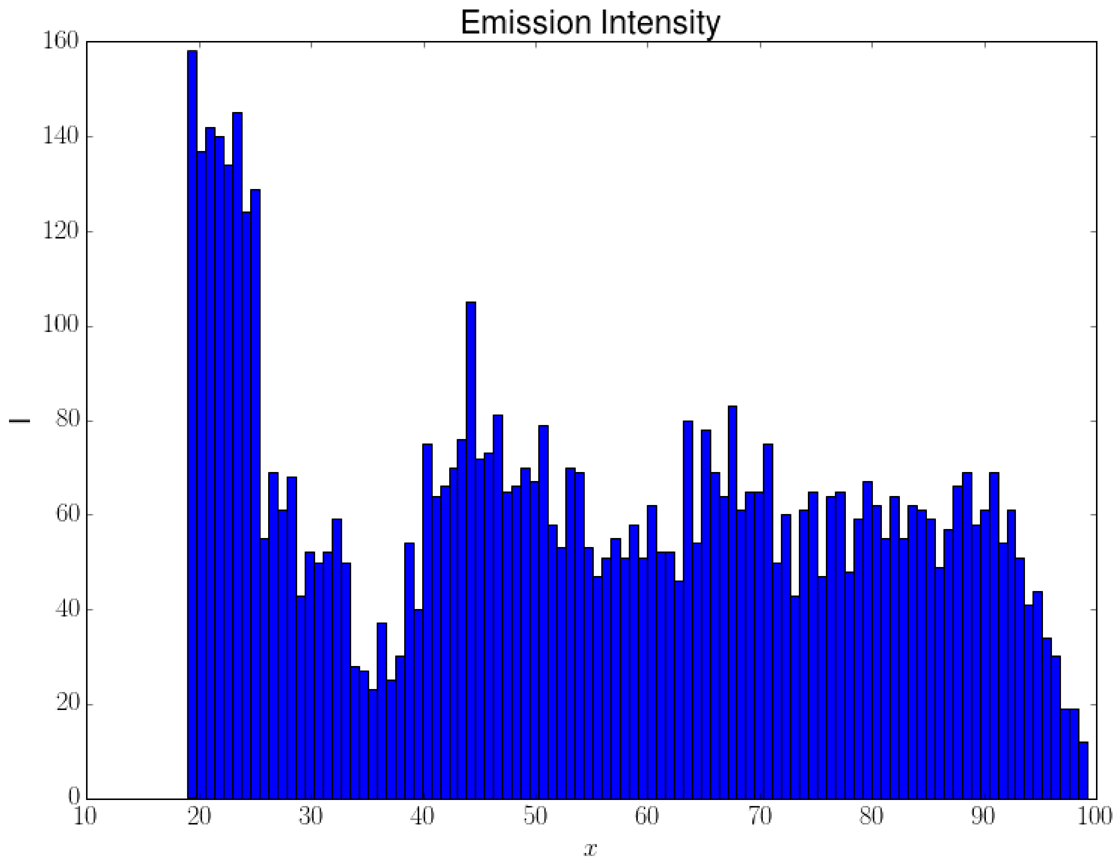
```
def plotGraphs(X,V,I):
    """
    Plot histograms for X and I, and a phase space using X and V.
    Returns the emission intensities and locations of histogram bins.
    """

    # electron density
    figure()
    hist(X,bins=n,cumulative=False)
    title("Electron density")
    xlabel("$x$")
    ylabel("Number of electrons")
    show()

    # emission instensity
    figure()
    ints,bins,trash = hist(I,bins=n)
    title("Emission Intensity")
    xlabel("$x$")
    ylabel("I")
    show()

    # electron phase space
    figure()
    scatter(X,V,marker='x')
    title("Electron Phase Space")
    xlabel("$x$")
    ylabel("$v$")
    show()

    return ints,bins
```

4 Running the simulation

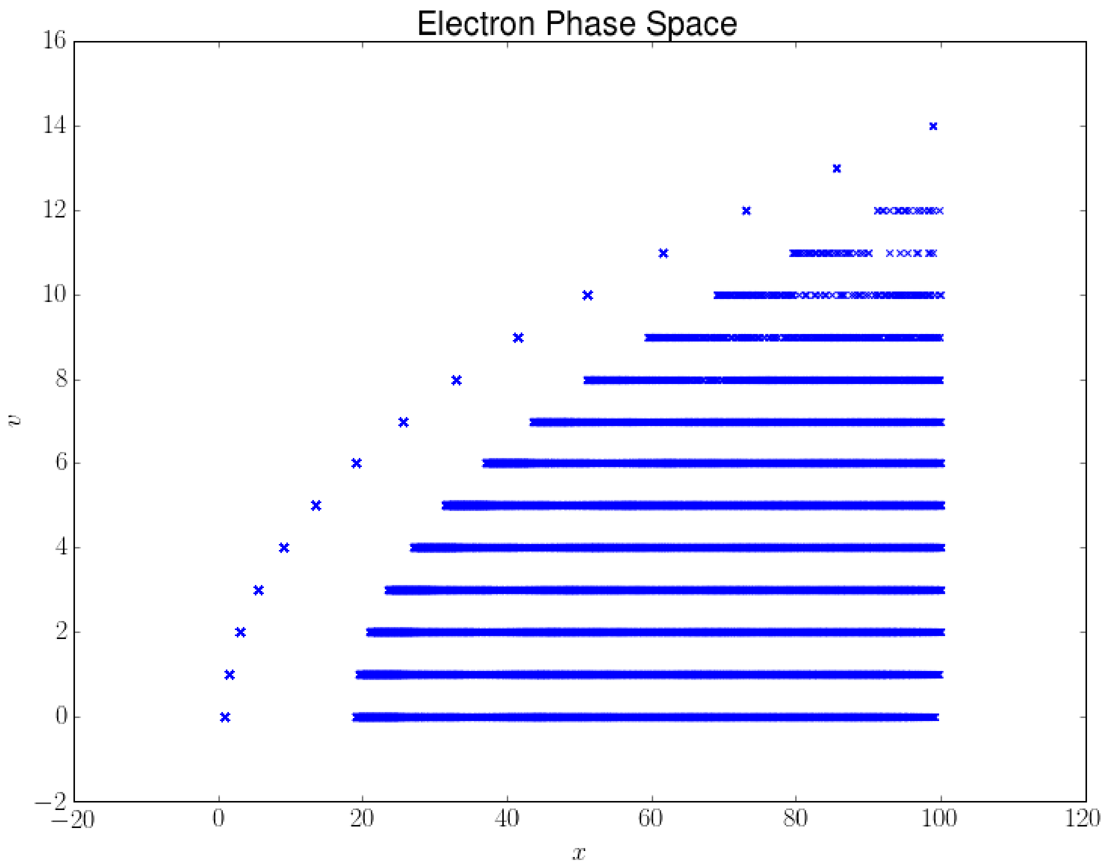
The tubelight is simulated with the default parameters of $n = 100$, $M = 5$, $nk = 500$ and $Msig = 1$. A threshold speed of $u0 = 7$, and an ionization probability of $p = 0.5$ are chosen.

```
X,V,I = simulateTubelight(n,M,nk,u0,p,Msig,False)

ints, bins = plotGraphs(X,V,I)
```


Electron density
Number of electrons
x
0
1000
2000
3000
4000
5000
0
20
40
60
80
100



Emission Intensity
I
x
0
20
40
60
80
100
120
140
160
10
20
30
40
50
60
70
80
90
100



We can make the following observations from the above plots:

- The electron density is peaked at the initial parts of the tubelight as the electrons are gaining speed here and are not above the threshold. This means that the peaks are the positions of the electrons at the first few timesteps they experience.
- The peaks slowly smoothen out as x increases beyond 19. This is because the electrons achieve a threshold speed of 7 only after traversing a distance of 19 units. This means that they start ionizing the gas atoms and lose their speed due to an inelastic collision.
- The emission intensity also shows peaks which get diffused as x increases. This is due the same reason as above. Most of the electrons reach the threshold at roughly the same positions, leading to peaks in the number of photons emitted there.
- This phenomenon can also be seen in the phase space plot. Firstly, the velocities are restricted to discrete values, as the acceleration is set to 1, and we are not yet performing accurate velocity updates after collisions.
- One trajectory is separated from the rest of plot. This corresponds to those electrons which travel until the anode without suffering any inelastic collisions with gas atoms. This can be seen by noticing that the trajectory is parabolic. This means that $v = k\sqrt{x}$, which is precisely the case for a particle moving with constant acceleration.

- The rest of the plot corresponds to the trajectories of those electrons which have suffered at least one collision with an atom. Since the collisions can occur over a continuous range of positions, the trajectories encompass all possible positions after $x = 19$.

Now, if we consider the fact that an electron will collide after a dt amount of time, and then accelerate after its collision for the remaining time period, we need to perform a more accurate update step. This is done by taking time as the uniformly distributed random variable. Say dt is a uniformly distributed random variable between 0 and 1. Then, the electron would have traveled an actual distance of dx' given by

$$dx_i' = u_i + \frac{1}{2}dt^2$$

as opposed to $dx_i = u_i + 0.5$

We update the positions of collisions using this displacement instead. We also consider that the electrons accelerate after the collision for the remaining $1 - dt$ period of time. We get the following equations for position and velocity updates:

$$dx_i'' = \frac{1}{2}(1 - dt)^2$$

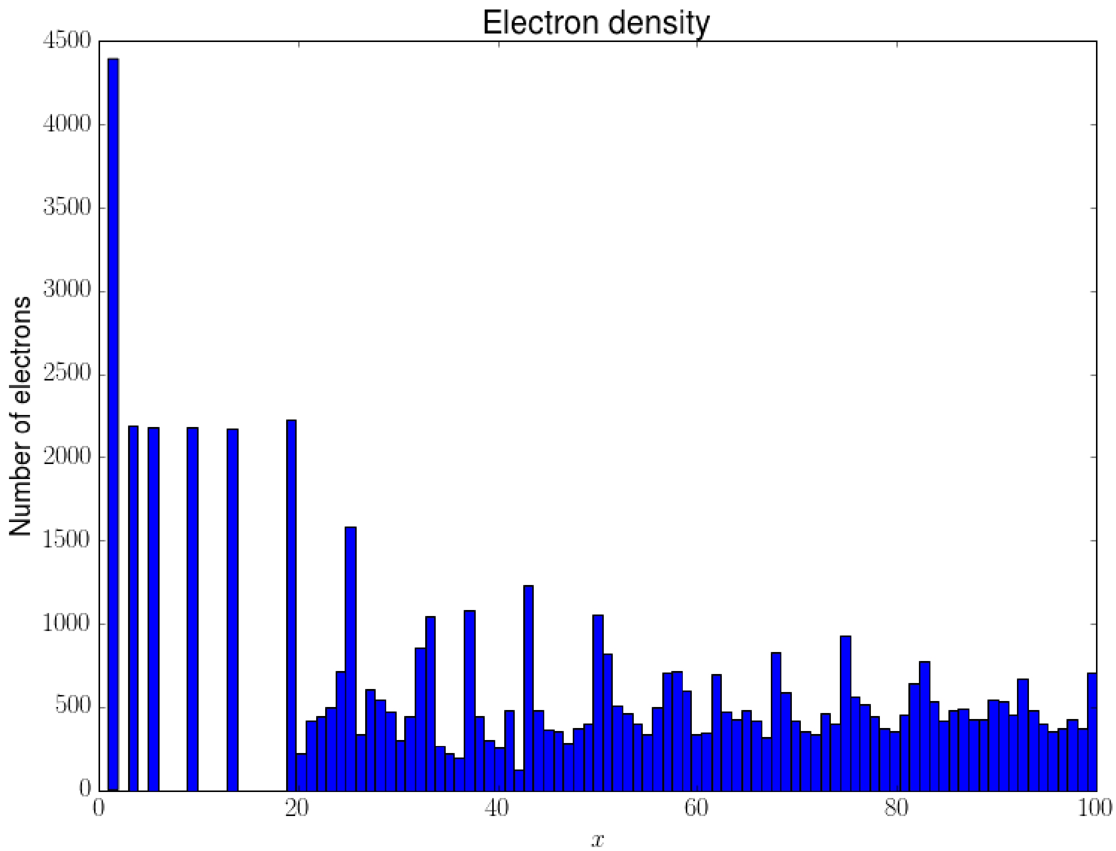
$$u_{i+1} = 1 - dt$$

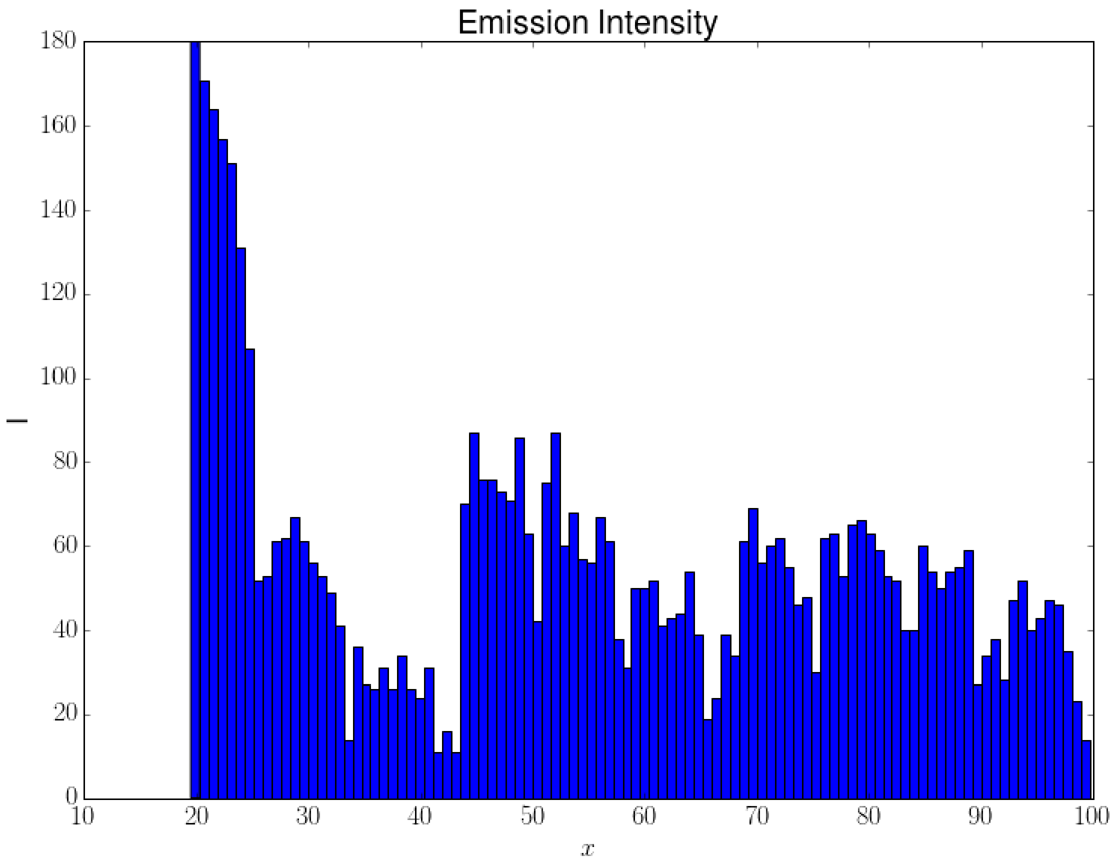
With the following update rule:

$$xx_{i+1} = xx_i + dx_i' + dx_i''$$

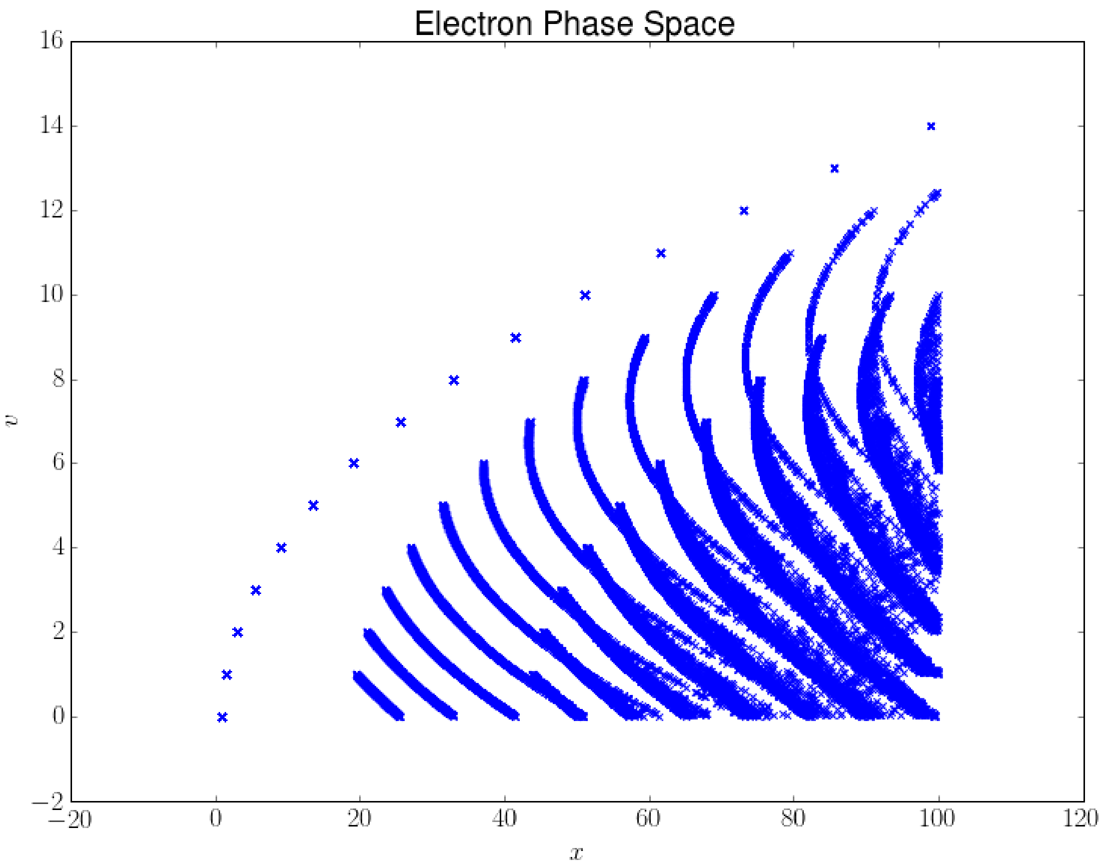
Using these updates, we get the following plots:

```
X,V,I = simulateTubelight(n,M,nk,u0,p,Msig,True)

ints, bins = plotGraphs(X,V,I)
```


Electron density
Number of electrons
4500
4000
3500
3000
2500
2000
1500
1000
500
0
0
20
40
60
80
100
x



Emission Intensity
I
x



We now observe that the phase space can have electrons with a continuous range of velocities as well. The peaks in the emission intensity plot also stand out more with this correction.

We can also notice that there are distinct families of curves in the phase space. Each family corresponds to a peak in emission intensity. A new family of curves is created at the position where the previous one reaches the threshold velocity. We can see from the emission instensity graphs that these positions line up with the positions of the diffused peaks fairly well. These families of curves can also be distinguished by noticing that they correspond to those electrons which have suffered 0,1,2,.. collisions before reaching the anode.

The emission count for each value of x is tabulated below:

```
xpos=0.5*(bins[0:-1]+bins[1:])
from tabulate import *
print("Intensity Data")
print(tabulate(stack((xpos,ints)).T,["xpos","count"]))
```

```
Intensity Data
   xpos    count
-------  -------
19.9015      180
20.7033      171
21.5052      164
```

22.307	157
23.1088	151
23.9107	131
24.7125	107
25.5144	52
26.3162	53
27.118	61
27.9199	62
28.7217	67
29.5236	61
30.3254	56
31.1272	53
31.9291	49
32.7309	41
33.5327	14
34.3346	36
35.1364	27
35.9383	26
36.7401	31
37.5419	26
38.3438	34
39.1456	26
39.9474	24
40.7493	31
41.5511	11
42.353	16
43.1548	11
43.9566	70
44.7585	87
45.5603	76
46.3622	76
47.164	73
47.9658	71
48.7677	86
49.5695	63
50.3713	42
51.1732	75
51.975	87
52.7769	60
53.5787	68
54.3805	57
55.1824	56
55.9842	67
56.7861	61
57.5879	38
58.3897	31
59.1916	50
59.9934	50

60.7952	52
61.5971	41
62.3989	43
63.2008	44
64.0026	54
64.8044	39
65.6063	19
66.4081	24
67.2099	39
68.0118	34
68.8136	61
69.6155	69
70.4173	56
71.2191	60
72.021	62
72.8228	55
73.6247	46
74.4265	48
75.2283	30
76.0302	62
76.832	63
77.6338	53
78.4357	65
79.2375	66
80.0394	63
80.8412	59
81.643	53
82.4449	52
83.2467	40
84.0486	40
84.8504	60
85.6522	54
86.4541	50
87.2559	54
88.0577	55
88.8596	59
89.6614	27
90.4633	34
91.2651	38
92.0669	28
92.8688	47
93.6706	52
94.4724	40
95.2743	43
96.0761	47
96.878	46
97.6798	35
98.4816	23

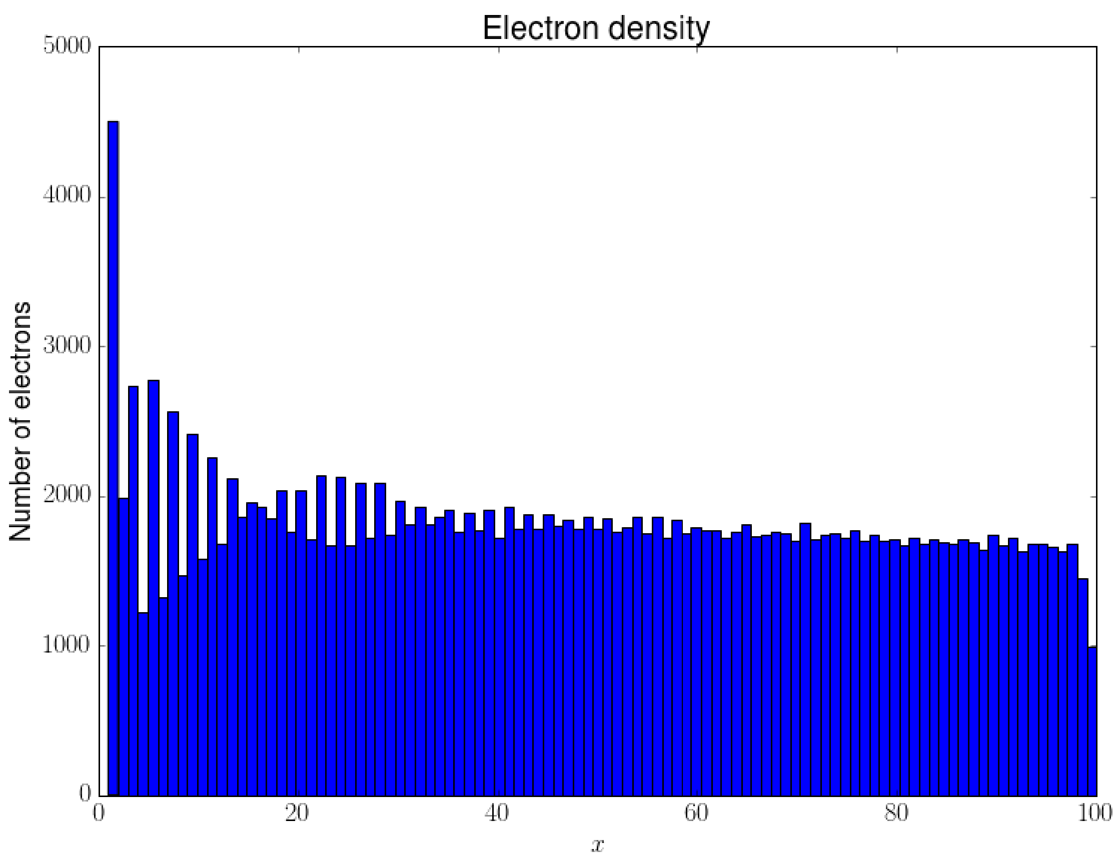
```
99.2835        14
```

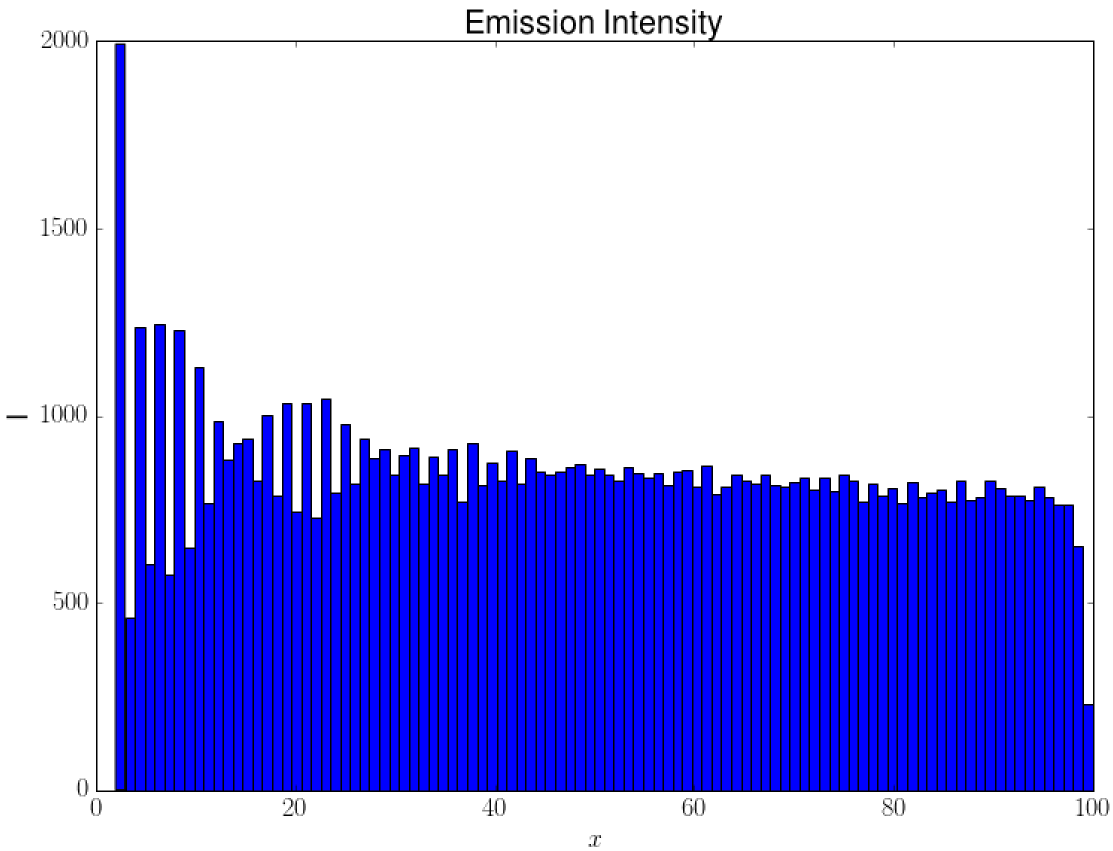
5 Different set of parameters

The simulation is repeated using a different set of parameters. Namely, the threshold velocity is greatly reduced to $u0 = 2$, and the ionization probability is increased to 0.9.

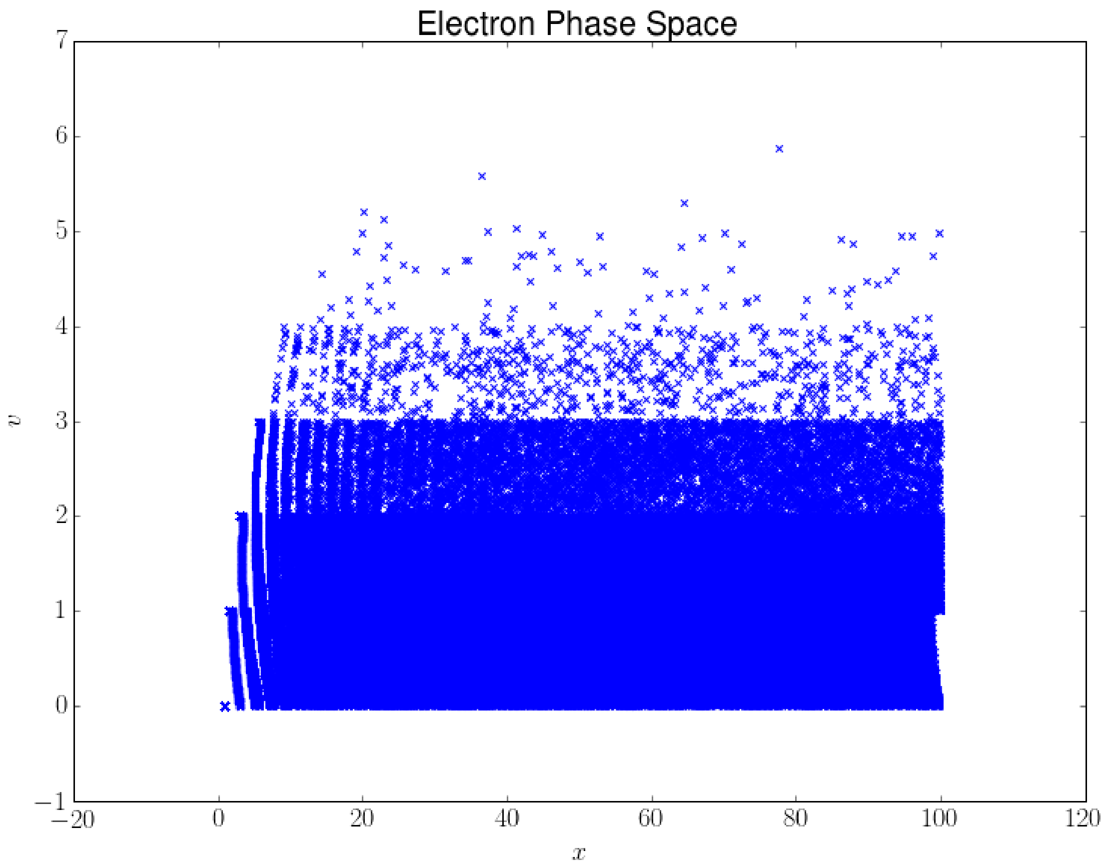
```
X,V,I = simulateTubelight(n,M,nk,u0,p,Msig,True)
```

```
ints, bins = plotGraphs(X,V,I)
```





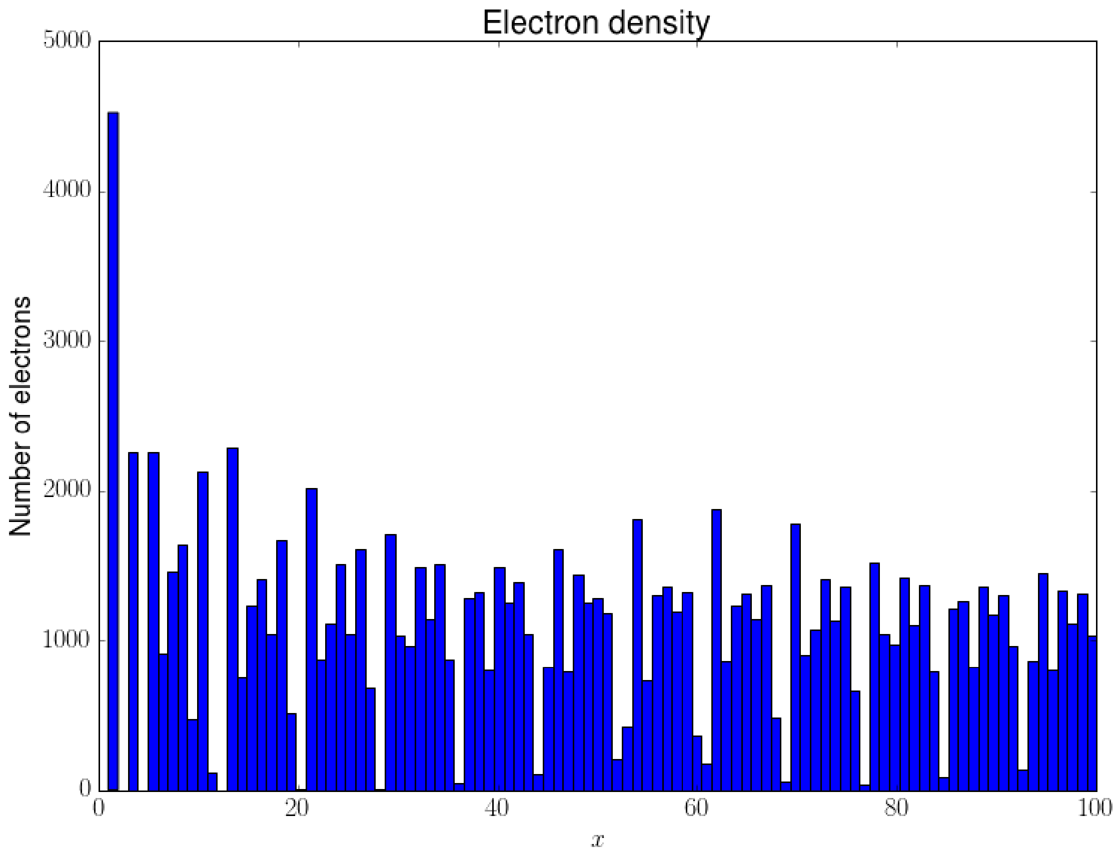
Emission Intensity
2000
1500
1000
500
0
I
0
20
40
60
80
100
x

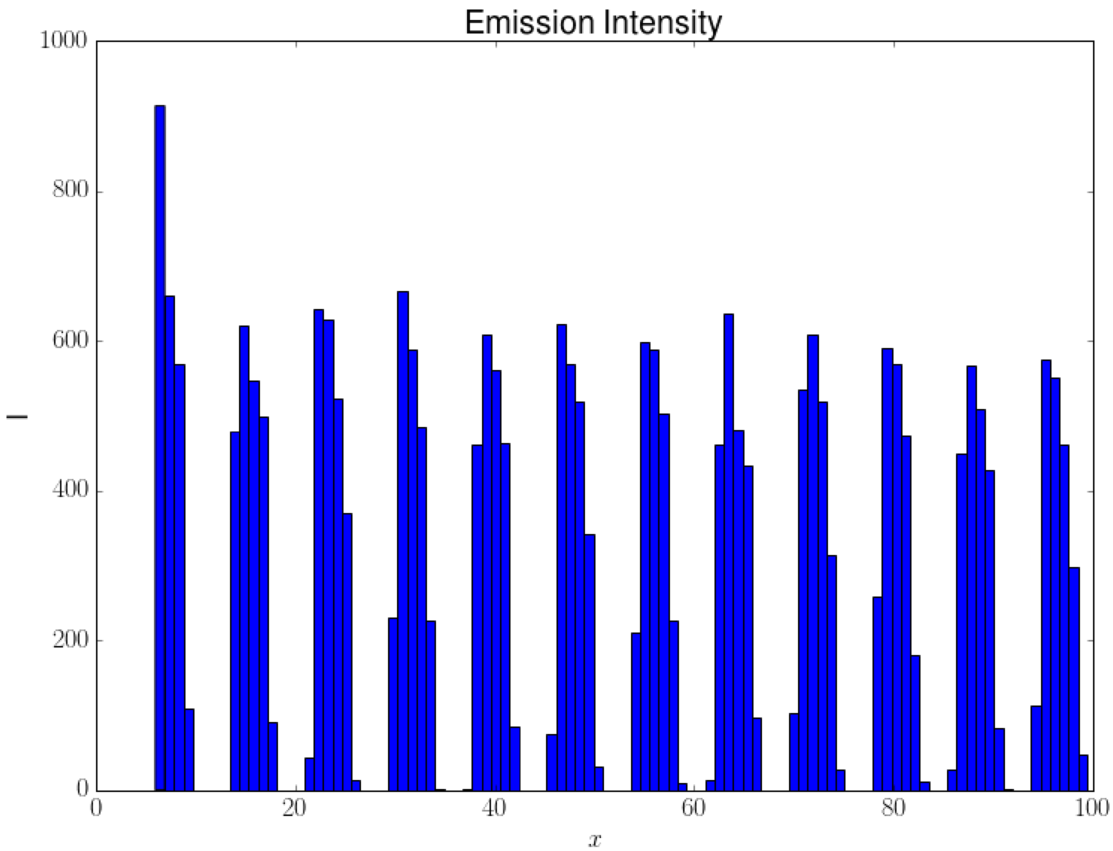


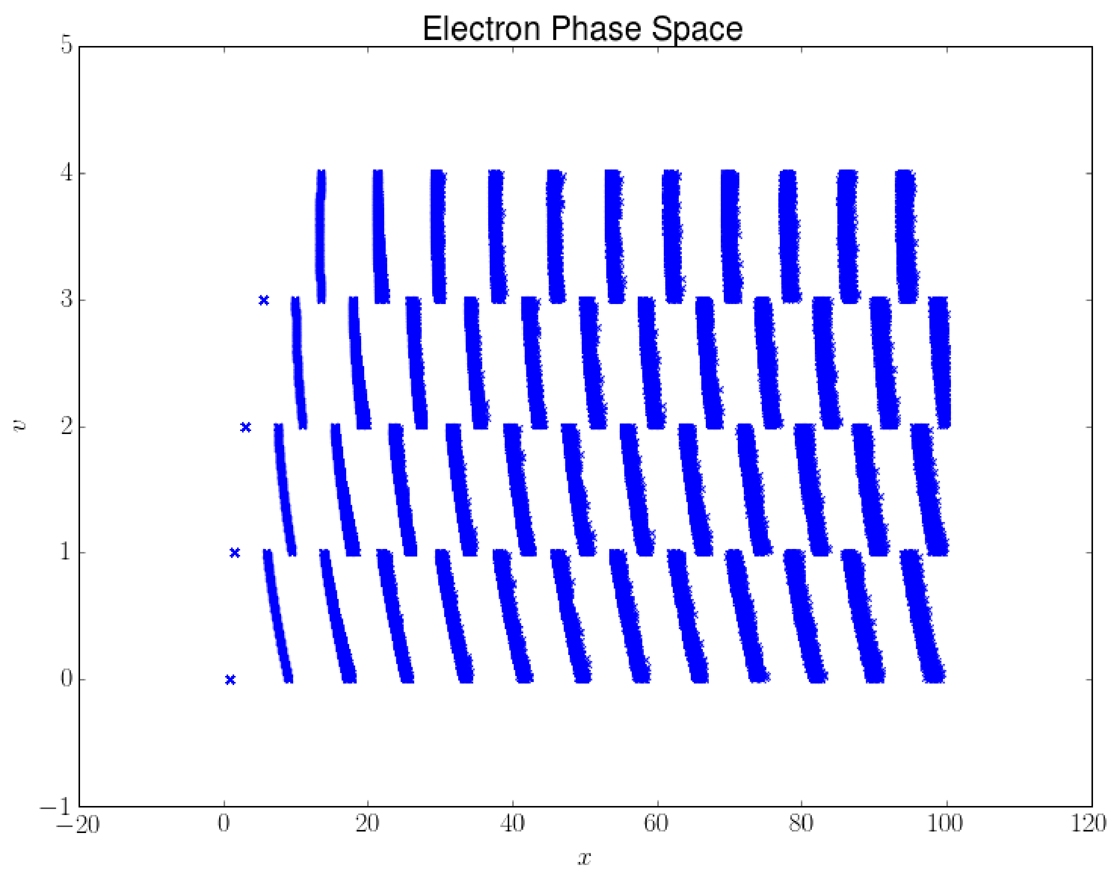
Now we simulate the tubelight with an unrealistic set of parameters, namely that the probability of photon emission is 1. This results in the following plots:

```
X,V,I = simulateTubelight(n,M,nk,u0,p,Msig,True)
```

```
ints, bins = plotGraphs(X,V,I)
```


Electron density
Number of electrons
5000
4000
3000
2000
1000
0
0
20
40
60
80
100
x


Emission Intensity
1000
800
600
400
200
0
I
0
20
40
60
80
100
x



6 Conclusion

We can make the following conclusions from the above sets of plots:

- Since the threshold speed is much lower in the second set of parameters, photon emission starts occuring from a much lower value of x. This means that the electron density is more evenly spread out. It also means that the emission intensity is very smooth, and the emission peaks are very diffused.
- Since the probability of ionization is very high, total emission intensity is also relatively higher compared to the first case.
- We can conclude from the above observations that a gas which has a lower threshold velocity and a higher ionization probability is better suited for use in a tubelight, as it provides more uniform and a higher amount of photon emission intensity.
- Coming to the case where the ionization probability is 1, we observe that the emission intensity consists of distinct peaks. The reason that these peaks are diffused is that we perform the actual collision at some time instant within the interval between two time steps. This also explains the slightly diffused phase plot as well.

- The families of phase space plots corresponding to the peaks is also more evident in this unrealistic case.