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find $\underline{y(x)}$ → (admissible)

└ s.t. → $y(x)$ gives stationary value to a fn^{xal} (integral)

Procedure
(i) assume y exists
(ii) introduce a variation in $y(x)$

$$\bar{y}(x) = y(x) + \alpha\,\eta(x)$$

(iii) find stationary condition using

$$\boxed{I'(0) = 0}$$

(iv) differentiate, integrate by parts to get rid of $\eta'(x)$, factor out $\eta(x)$ & Euler's eqⁿ.

$$\boxed{\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial y} = 0}$$

$$y'' f_{y'y'} + y' f_{y'y} + f_{y'x} - f_y = 0$$

A] x, y missing,

$$y'' \cancel{f_{y'y'}} = 0 \Rightarrow y = mx + c$$

(+ve)

→ y missing,

$$\frac{\partial f}{\partial y'} = \text{const.}$$

→ x missing, use Beltrami's identity,

$$y' \frac{\partial f}{\partial y'} - f = \text{const.}$$

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Beltrami's

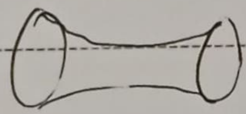
$$\frac{d}{dx}\left(\frac{\partial f}{\partial y'}y' - f\right) = y'\underbrace{\left[\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial y}\right]}_{\therefore\ 0 \text{ from Euler's eq}^n} - \underbrace{\frac{\partial f}{\partial x}}_{\therefore\ 0 \text{ since } x \text{ missing}}$$

Examples:

I

$$\int dz = \int \sqrt{1 + (y')^2}\, dx$$

II minimal surface of revolu^n

 → Catenary (Appendix B, blind Belgian phy J. Plateau)

III Brachistochrone

$$f = \frac{ds}{(\sqrt{2gy})} \rightarrow \text{velocity at each } y.$$

→ Cycloid

Generaliza^n's → if system also has a $z(x)$

→ Euler's eq^n for both y & z separately, simultaneously.

Double integrals

$$\frac{\partial f}{\partial z} - \left[\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial z_x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial z_y}\right)\right] = 0$$

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Lagrange Multipliers :

Minimize I subject to a side condition $J = \text{const.}$

I) Isoperimetric:

Now, minimizing I with a condiⁿ J

≡ minimizing $\int F\,dx$ where,

$$F = \underset{\downarrow \atop I}{f} + \lambda \underset{\downarrow \atop J}{g}.$$

∴ Applying Euler's eqⁿ on F,

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) - \frac{\partial F}{\partial y} = 0$$

* eg:- isoperimetric problems,
(vary area, keep perimeter const.)

II) finite side condiⁿs: (Geodesic surface)

If the condiⁿ is $G(x,y,z)=0$,

$\lambda \rightarrow \lambda(t)$

$$\boxed{\begin{aligned} \frac{d}{dt}\left(\frac{\partial f}{\partial \dot{z}}\right) &= \lambda(t)\, G_z \\ \frac{d}{dt}\left(\frac{\partial f}{\partial \dot{x}}\right) &= \lambda(t)\, G_x \\ \frac{d}{dt}\left(\frac{\partial f}{\partial \dot{y}}\right) &= \lambda(t)\, G_y \end{aligned}}$$

$$z = g(x,y)$$

$$\dot{z} = \frac{\partial g}{\partial x}\dot{x} + \frac{\partial g}{\partial y}\dot{y}$$

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Hamilton's Priciple:

A particle moves from P_1 to P_2 such that its path makes the Action stationary.

$$\text{Action} = \int_{t_1}^{t_2} \underbrace{(KE - PE)}_{\text{called the Lagrangian}} dt$$

we get $\vec{F} = m\vec{a}$ from this,

Hamilton's Principle $\equiv$ Newton's 2^{nd} Law.

similarly,

Hamilton's Principle can be used to derive the laws of Electromagnetism, ~~Wave Equa~~ Quantum Theory, and Relativity.

∴ Max Planck, "the principle of least action may claim to come nearest to this ideal final aim of theoretical (Physics) research, which is to condense all natural phenomena into one simple principle."