RBE 501: Homework Assignment 1

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1 Frame Transformations

1.1 Problem 1

Let R be the rotation matrix for transforming the frame F_0 to F_1. Then,

F_1 = [R] F_0

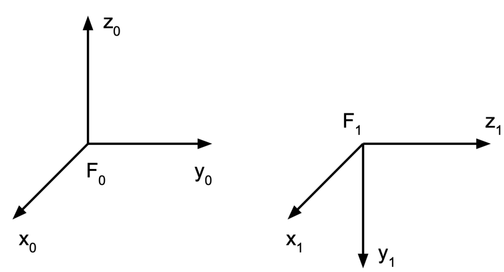

Figure 1: Two cartesian reference frames.

Where,

$$R = \begin{bmatrix} x_1.x_0 & x_1.y_0 & x_1.z_0 \\ y_1.x_0 & y_1.y_0 & y_1.z_0 \\ z_1.x_0 & z_1.y_0 & z_1.z_0 \end{bmatrix}$$

Since the rotation is about x_0 axis, $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

For ZYX Euler angles, $R_Z(\phi)R_Y(\theta)R_X(\psi)$

$$= \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\phi cos\theta & \cos\phi sin\theta sin\psi - sin\phi cos\psi & \cos\phi sin\theta cos\psi + sin\phi sin\psi \\ \sin\phi cos\theta & \sin\phi sin\theta sin\psi + cos\phi cos\psi & \sin\phi sin\theta cos\psi - cos\phi sin\psi \\ -\sin\theta & \cos\theta sin\psi & \cos\theta cos\psi \end{bmatrix}$$

On comparing the above matrix with the rotation matrix, the corresponding Euler angles [ϕ, θ, ψ] come out to be [0, 0, -$\pi/2$].

Since rotation matrices are orthonormal,

$$R^{-1} = R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Assuming the frame F_0 translates to $[0, 10, 0]^T$, the homogeneous transformation matrix can be given as

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.2 Problem 2

Given,

$$\text{K} = \begin{bmatrix} 0 & -0.0875 & 0.5670 \\ 0.0875 & 0 & -0.8190 \\ -0.5670 & 0.8190 & 0 \end{bmatrix}, \text{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \theta = 30°$$

Then if we substitute the values in $R = I + sin(\theta)K + [1 - cos(\theta)]K^2$

$$\text{We get, R} = \begin{bmatrix} 1.0000 & -0.0352 & 0.2281 \\ 0.0352 & 1.0000 & -0.3295 \\ -0.2281 & 0.3295 & 1.0000 \end{bmatrix}$$

Two important properties of a Rotation Matrix are:

1. The determinant of R equals 1.

2. Its product with its transpose is I.

On calculating the determinant,

$det(R) = 1 * [1 * 1 - 0.3295 * (-.3295)] - (-0.0352) * [0.0352 * 1 - -.2281 * (-.3295)] + 0.2281 * 0.0352 * 0.3295 - (-.2281 * 1) = 1$

Also, clearly $R^{-1} = R^T$ and $R.R^T = I$.

2 Robot Kinematics and D-H frames

2.1 Problem 3

Frames have been assigned to the SCARA Robot according to the Denavit-Hartenberg (D-H) convention in Fig 2.

Furthermore, Table 1 describes the joint and link D-H parameters for the robot.

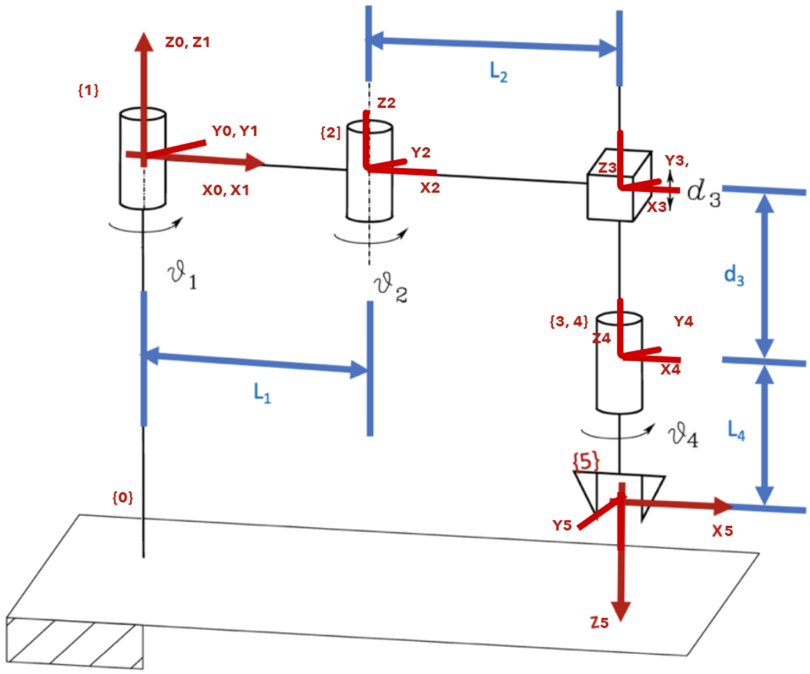

Figure 2: SCARA Robot reference frames.

Link	ϑ	**d**	**a**	α
1	ϑ_1	0	L_1	0
2	ϑ_2	0	L_2	π
3	0	d_3	0	0
4	$-\vartheta_4$	L_4	0	0

Table 1: DH parameters for SCARA Robot.

2.2 Problem 4

Frames have been assigned to the RPP (Revolute-Prismatic-Prismatic) robotic manipulator according to the Denavit-Hartenberg (D-H) convention in Fig 3.

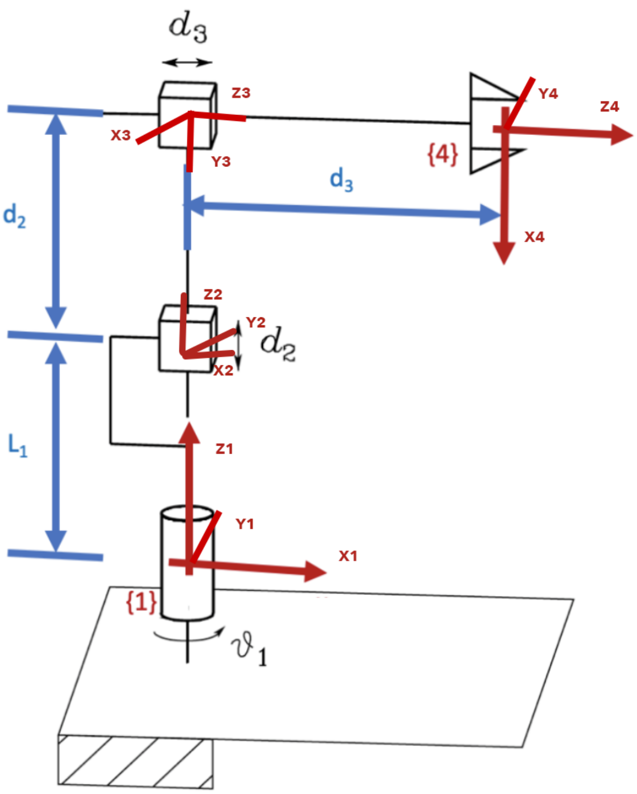

Figure 3: 3-DoF RPP (Revolute-Prismatic-Prismatic) robotic manipulator.

Furthermore, Table 2 describes the joint and link D-H parameters for the robot.

Link	ϑ	**d**	**a**	α
1	ϑ_1	0	L_1	0
2	$-\pi/2$	d_2	0	$-\pi/2$
3	$\pi/2$	d_3	0	0

Table 2: DH parameters for SCARA Robot.

3 MATLAB Programming

The remaining solutions are submitted on Matlab Grader.