RBE 549: Homework Assignment 9

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Problem 1

a Let E = 12 and M = 5 Since $E > M(if E \geq 12, M \leq 5)$
$E^M < M^E \quad (12^5 < 5^{12})$
$\therefore$ Less computation. It is easier to match model with edges.

b When using a null element, the value of M may increase by 1, which gives us M = 6. However, the relationship still holds, i.e. $E^M < M^E \quad (12^6 < 6^{12})$. $\quad \therefore$ Less computation.

Problem 2

a Assuming center at origin, an ellipse can be represent by the following equation:
$(\frac{x}{a})^2 + (\frac{y}{b})^2 = 1$
or
$(\frac{x}{a})^2 + (\frac{y}{a\sqrt{1-e^2}})^2 = 1$
$x = rcos(\theta)$ and $y = rsin(\theta)$ can be used, where θ is the angle subtended by the elliptical arc.
$r = \frac{a(1-e^2)}{1+ecos(\theta)}$

b Ellipses can be decomposed into circular arcs and line segments and can be detected using Hough Transforms and edge-following methods or simply use an Elliptical Hough Transform.

c Features can be matched by correlating image with the model.

d This representation is invariant to scale if divided by the circumference and segment lengths. It is also invariant to translation if features are represented with respect to the center of the dog face. However, it may not be invariant to rotation. It can be made invariant by normalizing with orientations after finding the axis of the said orientation.

e This representation can only handle occlusion if a null vector is created and perform matching with the model. The present model is incapable of handling occlusion.

Problem 3

Let, $\overrightarrow{\mu_a} = [a_x, \quad a_y]$ & $\overrightarrow{\mu_b} = [b_x, \quad b_y]$

The decision boundary is equidistant from $\overrightarrow{\mu_a}$, $\overrightarrow{\mu_b}$.

Let (x, y) be any point on the decision boundary.

$\Rightarrow (x - a_x)^2 + (y - a_y)^2 = (x - b_x)^2 + (y - b_y)^2$ ($\because distancefrom\mu_a =$ distance from μ_b)

$\Rightarrow x^2 + 2a_x x + a_x^2 + y^2 - 2a_y y + a_y^2 = x^2 + 2b_x x + b_x^2 + y^2 - 2b_y y + b_y^2$

$\Rightarrow y = -\frac{a_x - b_x}{a_y - b_y} x + \frac{a_x^2 + a_y^2 - b_x^2 - b_y^2}{2a - y - 2b_y}$

$m_1 =$ Slope of decision boundary $= -\frac{a_x - b_x}{a_y - b_y}$

$m_2 =$ Slope of line joining μ_a & $\mu_b = \frac{b_y - a_y}{b_x - a_x}$

$\Rightarrow -\frac{a_x - b_x}{a_y - b_y} \frac{b_y - a_y}{b_x - a_x} = -1$ ($\because$m$_1 m_2 =$ -1)

$\therefore$ it's perpendicular to the line segment connecting μ_a, μ_b

Equation of decision boundary,

$f(x, y) \Rightarrow y(a_y - b_y) + x(a_x - b_x) = \frac{a_x^2 + a_y^2 - b_x^2 - b_y^2}{2}$

Mid-point of the line segment joining $\mu_a, \mu_b = \left(\frac{a_x + b_x}{2}, \frac{a_y + b_y}{2}\right)$

$f\left(\frac{a_x + b_x}{2}, \frac{a_y + b_y}{2}\right) = \frac{a_x + b_x}{2}(a_x - b_x) + \frac{a_y + b_y}{2}(a_y - b_y) = \frac{a_x^2 + a_y^2 - b_x^2 - b_y^2}{2}$

$\therefore$ it passes through the mid-point joining μ_a, μ_b

Problem 4

We have,

$\overrightarrow{V}^I = \frac{1}{\overrightarrow{F}\overrightarrow{X}^C}(\overrightarrow{F}.\overrightarrow{X}^C)\overrightarrow{V}^C - \overrightarrow{X}^C(\overrightarrow{F}.\overrightarrow{V}^C)$

$\Rightarrow -\frac{\overrightarrow{F}.\overrightarrow{V}^C}{\overrightarrow{F}.\overrightarrow{X}^C}(\overrightarrow{X}^I - \frac{(\overrightarrow{F}.\overrightarrow{X}^I\overrightarrow{V}^C)}{\overrightarrow{F}.\overrightarrow{V}^C})$

We know that,

$\overrightarrow{X}^I = \frac{|F|^2.\overrightarrow{X}^C}{\overrightarrow{F}.\overrightarrow{X}^C}$

$\Rightarrow \overrightarrow{F}.\overrightarrow{X}^I = \frac{|F|^2.\overrightarrow{X}^C}{\overrightarrow{F}.\overrightarrow{X}^C}\overrightarrow{F} = |F|^2\frac{\overrightarrow{F}.\overrightarrow{X}^C}{\overrightarrow{F}.\overrightarrow{X}^C} = |F|^2$

Hence,

$\overrightarrow{V}^I = -\frac{\overrightarrow{F}.\overrightarrow{V}^C}{\overrightarrow{F}.\overrightarrow{X}^C}(\overrightarrow{X}^I - \frac{|F|^2\overrightarrow{V}^C)}{\overrightarrow{F}.\overrightarrow{V}^C})$

Now, since

$\overrightarrow{X}_{FOE} = \frac{|F|^2\overrightarrow{V}^C}{\overrightarrow{F}.\overrightarrow{V}^C}$

$\therefore \overrightarrow{V}^I = -\frac{\overrightarrow{F}.\overrightarrow{V}^C}{\overrightarrow{F}.\overrightarrow{X}^C}(\overrightarrow{X}^I - \overrightarrow{X}_{FOE})$

So,
$k = -\frac{\overrightarrow{F}.\overrightarrow{V}^C}{\overrightarrow{F}.\overrightarrow{X}^C} = \frac{Projection of \overrightarrow{V}^C in \overrightarrow{F}}{Distance of \overrightarrow{X}^C in \overrightarrow{F}} = \frac{\overrightarrow{v_z}^C}{-z^C}$