RBE 549: Homework Assignment 12

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January 2, 2022

Problem 1

We have the brightness constraint $E_1 = \int\int (I(x_1 y) - R(p_1 q))^2 dxdy$

And the smoothness constraint as $E_2 = \int\int (p_x^2 + p_y^2 + q_x^2 + q_y^2) dxdy$

The energy function $E = \int\int (E_1 + \lambda E_2) dxdy = \int\int ((I(x_1 y) - R(p_1 q))^2 - \lambda(p_x^2 + p_y^2 + q_x^2 + q_y^2)) dxdy$

Using the Eurler-Lagrange equation:

$F_p + \frac{\delta F_{p_x}}{\delta x} + \frac{\delta F_{p_y}}{\delta y} = 0$

$-2(I - R)\frac{dR}{dp} - 2\lambda p_{xx} - 2\lambda p_{yy} = 0 \Rightarrow \nabla^2 p = \frac{1}{\lambda}(R - I)\frac{dR}{dp} \qquad \cdots \qquad (1)$

And similarly,

$F_q + \frac{\delta F_{q_x}}{\delta x} + \frac{\delta F_{q_y}}{\delta y} = 0$

$-2(I - R)\frac{dR}{dq} - 2\lambda q_{xx} - 2\lambda q_{yy} = 0 \Rightarrow \nabla^2 q = \frac{1}{\lambda}(R - I)\frac{dR}{dq} \qquad \cdots \qquad (2)$

Where, $\nabla^2 p = p_{xx} + p_{yy}$ and $\nabla^2 q = q_{xx} + q_{yy}$

From (1),

$\nabla^2 p = p_{x+1,y} - p_{x,y} + p_{x-1,y} - p_{x,y} + p_{x,y+1} - p_{x,y} + p_{x,y-1} - p_{x,y} = p_{x+1,y} + p_{x-1,y} + p_{x,y+1} + p_{x,y-1} - 4p_{x,y}$

$p_{x+1,y} + p_{x-1,y} + p_{x,y+1} + p_{x,y-1} - 4p_{x,y} = \frac{1}{\lambda}(R(p_{x,y}, q_{x,y}) - I(x,y))\frac{dR}{dp} \qquad \cdots \qquad (3)$

Similarly,

$q_{x+1,y} + q_{x-1,y} + q_{x,y+1} + q_{x,y-1} - 4q_{x,y} = \frac{1}{\lambda}(R(p_{x,y}, q_{x,y}) - I(x,y))\frac{dR}{dq} \qquad \cdots \qquad (3)$

From (3),

$p_{x,y} = p_{avg} + \frac{1}{4\lambda}(I - R)\frac{dR}{dp}$

From (4),

$q_{x,y} = q_{avg} + \frac{1}{4\lambda}(I - R)\frac{dR}{dq}$

Problem 2

$tanh(\Sigma) = \frac{sinh(\Sigma)}{cosh(\Sigma)}$

$\Rightarrow= \frac{e^{(\Sigma)} - e^{-(\Sigma)}}{e^{(\Sigma)} + e^{-(\Sigma)}}$

$\Rightarrow= \frac{e^{(\Sigma)} + e^{-(\Sigma)} - 2e^{-(\Sigma)}}{e^{(\Sigma)} + e^{-(\Sigma)}}$

$\Rightarrow= 1 - \frac{2}{e^{(2\Sigma)} + 1}$

Now from the logistic function's perspective we have,

$tanh(\Sigma) = 1 - \frac{2}{e^{(2\Sigma)}+1} = 1 - 2\sigma(-2\Sigma)$

$\Rightarrow = 1 - 2(1 - \sigma(2\Sigma)$

$\Rightarrow = 1 - 2 - 2\sigma(2\Sigma)$

$\Rightarrow = 2\sigma(2\Sigma) - 1$

Hence, we can conclude that the tanh function is just a rescaled version of the logistic sigmoid function.

Problem 3

We know that,

$|A \bigotimes B| = |A \bigcup B| \bigcap \neg |A \bigcap B|$

$\Rightarrow |A \bigcup B| - |A \bigcap B|$

$\Rightarrow |A \bigcap B| = |A \bigcup B| - |A \bigotimes B|$

$\therefore IoU(A, B) = \frac{|A \bigcap B|}{|A \bigcup B|}$

$\Rightarrow \frac{|A \bigcup B| - |A \bigotimes B|}{|A \bigcup B|}$

$\Rightarrow IoU(A, B) = 1 - \frac{|A \bigotimes B|}{|A \bigcup B|}$

Problem 4

a C_3 is a convolution layer with 16 feature maps of size 10 x 10, and each activation unit in C_3 is connected to several 5 x 5 receptive fields at identical locations in S_2.

On referring the table we can say that 3 feature maps in S_2 are combined with 6 feature maps in C_3 and 4 feature maps in S_2 are combined with 9 feature maps in C_3. Similarly, 6 feature maps in S_2 are combined with 1 feature maps in C_3.

(3 x 5 x 5 + 1) x 6 x 10 x 10 + (4 x 5 x 5 + 1) x 9 x 10 x 10 + (6 x 5 x 5 + 1) x 1 x 10 x 10 = 151,600

Thus, there are 151,600 connections in total.

b Similarly,

(3 x 5 x 5 + 1) x 6 + (4 x 5 x 5 + 1) x 9 + (6 x 5 x 5 + 1) = 1,516

Therefore, there are 1,516 trainable parameters.