



### INSTRUCTIONS

1. This homework assignment includes three sections:

Section 1: Frame transformations

Section 2: Forward kinematics

Section 3: MATLAB programming

2. Submit your solutions for sections 1 and 2 as a single PDF file through Canvas. The PDF should include your calculations (step by step) and any supporting drawings or sketches. Use of computer software for calculations (e.g. MATLAB) is allowed.

3. Submit your solutions for section 3 online through Matlab grader:

<https://grader.mathworks.com/courses/51899-rbe-501-fall-2021>

4. Due date: Tuesday 14-Sep-21 at 5:00pm (start of class)

### Section 1: Frame transformations (30 points total) – 5 points for each correct answer

1. Given the reference frames  $F_0$  and  $F_1$  as shown in Fig. 1:

- Without doing any calculations, write the rotation matrix  $\mathbf{R}$  required to align  $F_0$  with  $F_1$ .
- Given the rotation matrix  $\mathbf{R}$  previously calculated in step a, calculate a corresponding set of ZYX Euler angles  $\boldsymbol{\phi} = [\varphi, \vartheta, \psi]$ .
- Calculate the inverse rotation  $\mathbf{R}^{-1}$ .
- Calculate the homogeneous transformation matrix  $\mathbf{T}$  between  $F_0$  and  $F_1$ . Assume that the translational component of the transformation is given by  $\mathbf{p} = [0 \ 10 \ 0]^T$ .

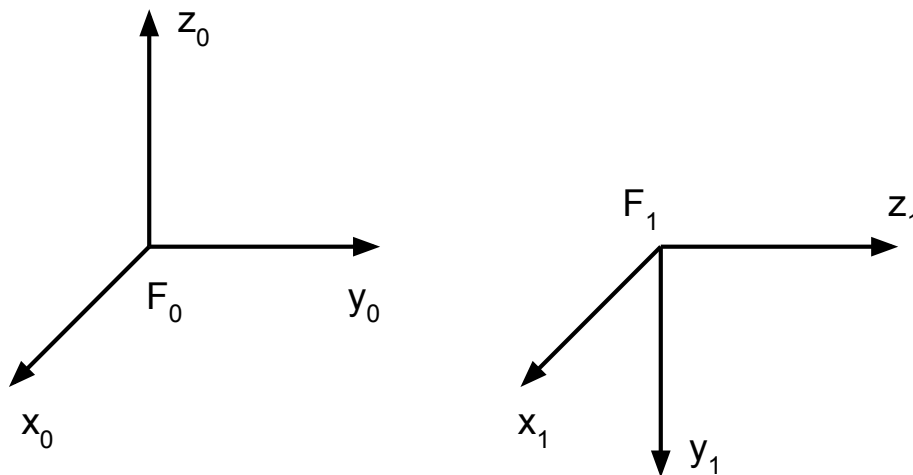


Figure 1: Two cartesian reference frames.

2. Given the *skew-symmetric* matrix  $K = \begin{bmatrix} 0 & -0.0875 & 0.5670 \\ 0.0875 & 0 & -0.8190 \\ -0.5670 & 0.8190 & 0 \end{bmatrix}$  and angle value  $\theta = 20^\circ$ :
- Calculate the 3x3 matrix given by  $R = I + \sin(\theta) K + [1 - \cos(\theta)] K^2$ .
  - Prove that  $R$  is a valid rotation matrix.

**Section 2: Robot kinematics and D-H frames (20 points total) – 5 points for each correct answer**

3. Given the SCARA manipulator shown below:
- Assign reference frames to each of the joint axes following the Denavit-Hartenberg convention<sup>1</sup>. The frame at the first joint {1} and the end effector frame {5} were pre-assigned for your convenience (red ink).
  - Create the table of D-H parameters. Use link lengths as noted in the picture (blue ink).

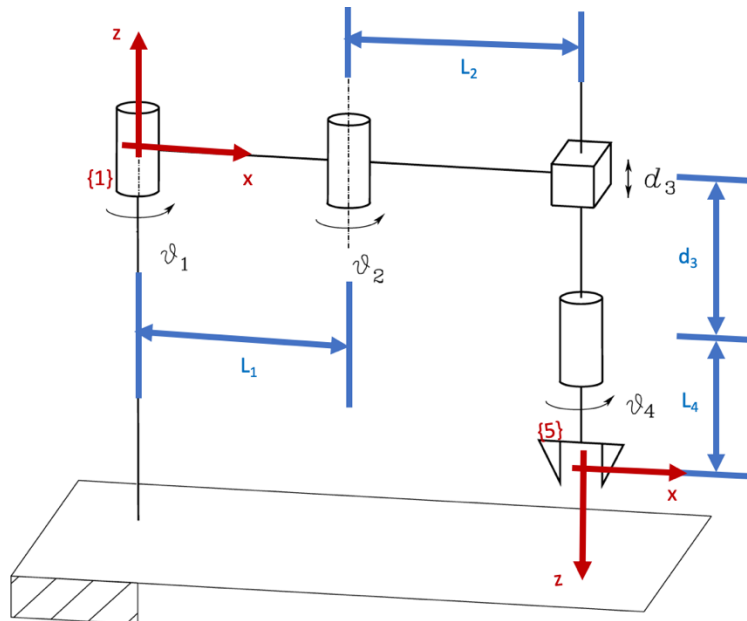


Figure 2: 4 DoF SCARA Manipulator

$\theta$	$d$	$a$	$\alpha$

<sup>1</sup> Multiple variants of the Denavit-Hartenberg convention exist. For this homework, use the version described in Siciliano et al. (2010), *Robotics: modelling, planning and control* (§2.8.2). Link: <https://www.springer.com/us/book/9781846286414>.

4. Given the RPP (Revolute-Prismatic-Prismatic) robotic manipulator illustrated in Fig. 3:
- Assign reference frames to each of the joint axes following the Denavit-Hartenberg convention. Frames at the first joint of the robot and at the end effector were pre-assigned for your convenience (red ink).
  - Create the table of D-H parameters. Use link lengths as noted in the picture (blue ink).

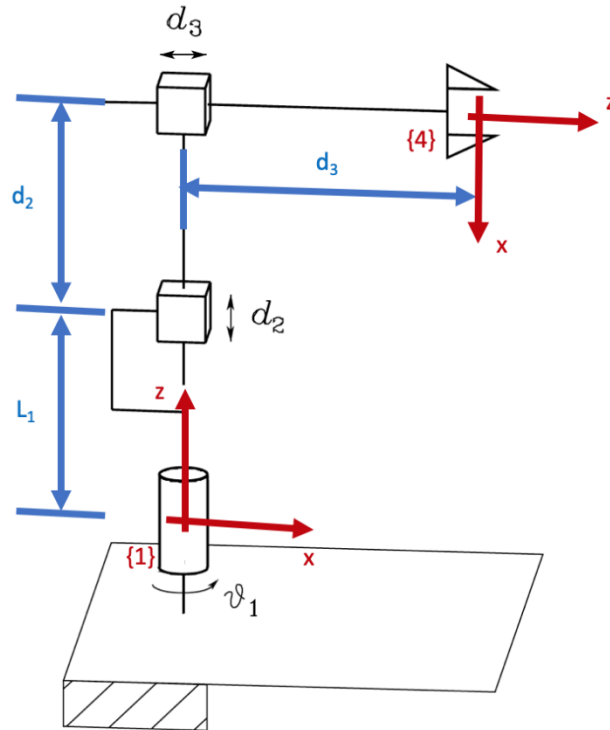


Figure 3: 3-DoF RPP (Revolute-Prismatic-Prismatic) robotic manipulator.

$\theta$	$d$	$a$	$\alpha$

### **Section 3: MATLAB programming (50 points total)**

This section must be completed online at:

<https://grader.mathworks.com/courses/51899-rbe-501-fall-2021>

- 5. Rotation matrices in Matlab (10 points)**
- 6. DH Parameters in Matlab (10 points)**
- 7. Forward kinematics of the SCARA robot (20 points)**
- 8. Forward kinematics of the RPP robot (10 points)**