RBE 549: Homework Assignment 10

Aditya Mehrotra
amehrotra@wpi.edu

November 18, 2021

Problem 1

For a line $ax + by + c = 0$, the shortest distance from a point (x', y') is given as $= \left|\frac{ax'+by'+c}{\sqrt{a^2+b^2}}\right|$

For the origin at (0, 0), this distance would be $= \left|\frac{c}{\sqrt{a^2+b^2}}\right|$
In the given equation of Optical Flow Constraint, we have, $I_x u + I_y v + I_t = 0$
So, the shortest magnitude of velocity, $|V| = \left|\frac{I_t}{\sqrt{I_x^2+I_y^2}}\right|$
The unit vector perpendicular to the constraint line is
$\hat{v} = \frac{(I_x, I_y)}{\sqrt{I_x^2+I_x^2}}$
So, we can find the smallest optical flow as:
$\overrightarrow{V_{min}} = \frac{|I_t|}{I_x^2+I_y^2}(I_x, I_y)$
$\Rightarrow \overrightarrow{V_{min}} = \frac{|I_t|}{I_x^2+I_y^2}\overrightarrow{\nabla} I$

Problem 2

a Given $I(x, y, y) = I_0 + k\left(tan^{-1}(\frac{x}{y}) - st\right)$
Using chain rule,
$I_x = \frac{\delta I}{\delta x} = k\left(\frac{y}{x^2+y^2}\right)$
$I_y = \frac{\delta I}{\delta y} = k\left(\frac{-x}{x^2+y^2}\right)$
$I_t = \frac{\delta I}{\delta t} = -ks$

b $I_x u + I_y v + I_t = 0$
$\Rightarrow k\frac{y}{x^2+y^2}u - k\frac{x}{x^2+y^2}v - ks = 0$
$\Rightarrow \frac{y}{x^2+y^2}u - \frac{x}{x^2+y^2}v - s = 0$
$\therefore uy = vx + s(x^2 + y^2) = 0$

c If $u = sy$ and $v = -sx$, we get,
$\therefore (sy^2)y = (-sx^2)x + s(x^2 + y^2) = 0$
So, the values are a solution to the OFCE.

Problem 3

a $\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix} = \begin{bmatrix} \lambda I_x^2 + 4 & \lambda I_x I_y \\ \lambda I_x I_y & \lambda I_y^2 + 4 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n \in neightbors(x,y)} u^{old}(n) & -\lambda I_x I_t \\ \sum_{n \in neightbors(x,y)} v^{old}(n) & -\lambda I_y I_t \end{bmatrix}$

Let A $= \begin{bmatrix} \lambda I_x^2 + 4 & \lambda I_x I_y \\ \lambda I_x I_y & \lambda I_y^2 + 4 \end{bmatrix}$

So, $A^{-1} = \begin{bmatrix} \lambda I_x^2 + 4 & \lambda I_x I_y \\ \lambda I_x I_y & \lambda I_y^2 + 4 \end{bmatrix}^{-1}$

$\Rightarrow \frac{1}{det(A)} \begin{bmatrix} \lambda I_y^2 + 4 & -\lambda I_x I_y \\ -\lambda I_x I_y & \lambda I_x^2 + 4 \end{bmatrix}$

$\Rightarrow \frac{1}{4\lambda I_x^2 + 4\lambda I_y^2 + 16} \begin{bmatrix} \lambda I_y^2 + 4 & -\lambda I_x I_y \\ -\lambda I_x I_y & \lambda I_x^2 + 4 \end{bmatrix}$

b $\sum u^{old}/4 = \overrightarrow{u^{old}}$ and $\sum v^{old}/4 = \overrightarrow{v^{old}}$
So from part **a**,
$u^{new} = \frac{1}{4\lambda I_x^2 + 4\lambda I_y^2 + 16} \left[(\lambda I_y^2 + 4)(\sum u^{old} - \lambda I_x I_t) + (-\lambda I_x I_y)(\sum v^{old} - \lambda I_y I_t) \right]$
$\Rightarrow u^{new} = \frac{1}{4\lambda I_x^2 + 4\lambda I_y^2 + 16} \left(\lambda I_y^2 \sum u^{old} + 4 \sum u^{old} - \lambda^2 I_x I_y^2 I_t - 4\lambda I_x I_t - \lambda I_x I_y \sum v^{old} + \lambda^2 I_x I_y^2 I_t) \right)$
$\Rightarrow u^{new} = \frac{1}{I_x^2 + I_y^2 + \frac{4}{\lambda}} \frac{1}{4\lambda} \left(\lambda I_y^2 \sum u^{old} + 4 \sum u^{old} - 4\lambda I_x I_t - \lambda I_x I_y \sum v^{old}) \right)$
$\Rightarrow u^{new} = \frac{1}{I_x^2 + I_y^2 + \frac{4}{\lambda}} \left(I_y^2 \sum u^{old}/4 + \frac{\sum u^{old}}{\lambda} - I_x I_t - I_x I_y \frac{\sum v^{old}}{4}) \right)$
$\Rightarrow u^{new} = \frac{1}{I_x^2 + I_y^2 + \frac{4}{\lambda}} \left((I_x^2 + I_y^2 + \frac{4}{\lambda}) \sum u^{old}/4 - I_x (I_x \sum u^{old} + I_y \sum v^{old} + I_t) \right)$
$\Rightarrow u^{new} = \overrightarrow{u^{old}} - \frac{I_x}{I_x^2 + I_y^2 + \frac{4}{\lambda}} \left(I_x \sum u^{old} + I_y \sum v^{old} + I_t \right)$

Similarly, $v^{new} = \overrightarrow{v^{old}} - \frac{I_y}{I_x^2 + I_y^2 + \frac{4}{\lambda}} \left(I_x \sum u^{old} + I_y \sum v^{old} + I_t \right)$

c $u^{new} = u^{old} - \frac{I_x}{I_x^2 + I_y^2 + \frac{4}{\lambda}} \left(I_x u^{old} + I_y v^{old} + I_t \right)$
$\Rightarrow u^{new} = u^{old} - \frac{\lambda I_x}{\lambda I_x^2 + \lambda I_y^2 + 4} \left(I_x u^{old} + I_y v^{old} + I_t \right)$
When $\lambda = 0$,
$\Rightarrow u^{new} = u^{old} - \frac{0 I_x}{0 I_x^2 + 0 I_y^2 + 4} \left(I_x u^{old} + I_y v^{old} + I_t \right)$
$\Rightarrow u^{new} = u^{old}$
Similarly, $v^{new} = v^{old}$
So, $\begin{bmatrix} \overrightarrow{u^{new}} \\ \overrightarrow{v^{new}} \end{bmatrix} = \begin{bmatrix} \overrightarrow{u^{old}} \\ \overrightarrow{v^{old}} \end{bmatrix}$

Problem 4

a Its coordinates in the Hough-type velocity space are u & v.

b A point I(x, y, t) in the Image space map corresponds to $I_x u + I_y v + I_t = 0$ or simply $\frac{dI}{dt} = 0$

c P1 has brightness $I(x, y, t) = 250 + 10x + 20y - 70t$
Similarly, P2 has brightness $I(x, y, t) = 150 + 30x - 20y + 30t$
For P1, $I_x = \frac{\delta I}{\delta x} = 10, I_y = \frac{\delta I}{\delta y} = 20, I_t = \frac{dI}{dt} = -70$
Which gives us the OFCE, $10u + 20v - 70 = 0$
And for P2, $I_x = \frac{\delta I}{\delta x} = 30, I_y = \frac{\delta I}{\delta y} = -20, I_t = \frac{dI}{dt} = 30$
Which gives us the OFCE, $30u - 20v + 30 = 0$
On solving the two above equation, we get
$10u + 20v - 70 = 0$
$30u - 20v + 30 = 0$
$40u - 40 = 0$
$u = 1$
$10 + 20v - 70 = 0$
$v = 3$
So, $u = 1$ and $v = 3$.