1.

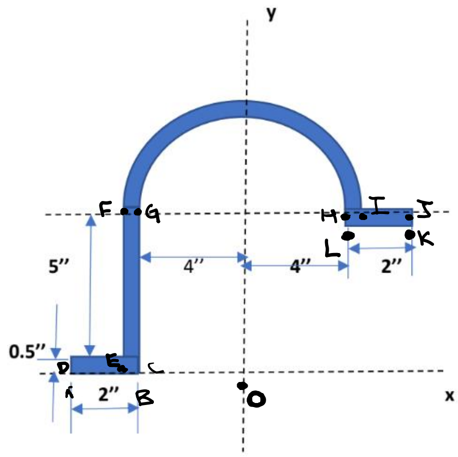

Figure 1

Let O be the origin (0,0).

We split the area into 4 regions:

ABCD (Region 1), ECFG (Region 2), FGHI (Region 3) and HJKL (Region 4)

We know,

X_{CM} (Total area) = $(\Sigma_{I=1}^{4} X_{CM}$(Region i) x Area(Region i))/ $(\Sigma_{I=1}^{4}$ Area(Region i))

Y_{CM} (Total area) = $(\Sigma_{I=1}^{4} Y_{CM}$(Region i) x Area(Region i))/ $(\Sigma_{I=1}^{4}$ Area(Region i))

Region 1:

X_{CM} = -4-(2/2) = -5"

Y_{CM} = 0.5/2 = 0.25"

Area = 2x0.5 = 1 sq.inch

Region 2:

X_{CM} = -4-(0.5/2) = -4.25"

Y_{CM} = 0.5 + 2.5 = 3"

Area = 5x0.5 = 2.5 sq.inch

Region 3:

X_{CM} = 0

Y_{CM} -

We take semi-circular elements of infinitesimal area $dA = \pi r dr$ (r ranging from 4" to 4.5")

$\therefore Y_{CM} = (\int_4^{4.5} y dA)/(\text{Area}) = (\int_4^{4.5} (2r/\pi)(\pi r dr))/13.352$

$= 18.0833/6.675 = 2.709"$

From origin, $Y_{CM} = 5.5 + 2.709 = 8.209"$

Area = $(\pi(4+0.5)^2 - \pi(4)^2)$

= 6.675 sq.inch

Region 4:

X_{CM} = 4 + (2/2) = 5"

Y_{CM} = 5.5 - (0.5/2) = 5.25"

Area = 2x0.5 = 1 sq.inch

X_{CM} of the entire area = ((-5)(1) + (-4.25)(2.5) + 0 + (5)(1))/(1+2.5+6.675+1)

= -10.625/11.175

= -0.9508"

Y_{CM} of the entire area = ((0.25)(1)+(3)(2.5)+(8.209)(6.675)+(5.25)(1))/11.175

= 6.066"

2.

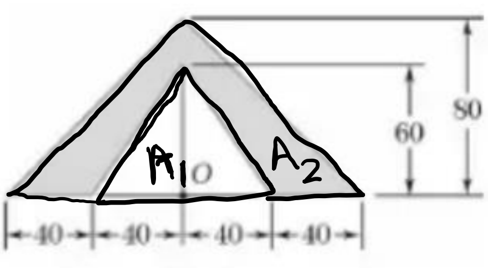

Figure 2

(a)

$A_3 = A_1 + A_2$ (Entire triangle)

We know I_{A1}(about 0) + I_{A2}(about 0) = I_{A3}(about 0)

I_A (A being any isosceles triangle of base = b and height = h):

I_x (taking centroid as origin) = $bh^3/36$

I_y (taking centroid as origin) = $hb^3/48$

∴I (about origin) = $I_x + I_y = bh/12(h^2/3 + b^2/4)$ [Perpendicular Axis Theorem]

We know the center of mass of an isosceles triangle lies at h/3

∴ I (about base center) = I (about origin) + $bh/2(h/3)^2$ [Parallel Axis Theorem]

$= bh(b^2 + 4h^2)/48$

I_{A1} (b = 80, h = 60) = $80x60x((80)^2 + 4(60)^2)/48$

= 2080000

I_{A3} (b = 160, h = 80) = $160x80x((160)^2 + 4(80)^2)/48$

$= 13653333.3333$

$\therefore I_{A2} = 13653333.3333 - 2080000 = 11573333.3333 \text{ mm}^4 = 1157.333 \text{ cm}^4$

(b) We know that $A_1X_{cm(1)} + A_2X_{cm(2)} = A_3X_{cm(3)}$.

Similarly, $A_1Y_{cm(1)} + A_2Y_{cm(2)} = A_3Y_{cm(3)}$

$X_{cm(1)} = X_{CM(3)} = 0$ (Symmetry)

$\therefore X_{CM(2)} = 0$

$Y_{CM(1)} = h_1/3 = 60/3$ (from 0)

$= 20$

$Y_{CM(3)} = h_3/3 = 80/3$ (from 0)

$= 26.66$

$A_1 = (80)(60)/2 = 2400$

$A_3 = (160)(80)/2 = 6400$

$A_2 = A_3 - A_1 = 4000$

$\therefore Y_{CM(2)} = (A_3Y_{cm(3)} - A_1Y_{cm(1)})/A_2$

$= 30.666$ mm (from 0)

3.

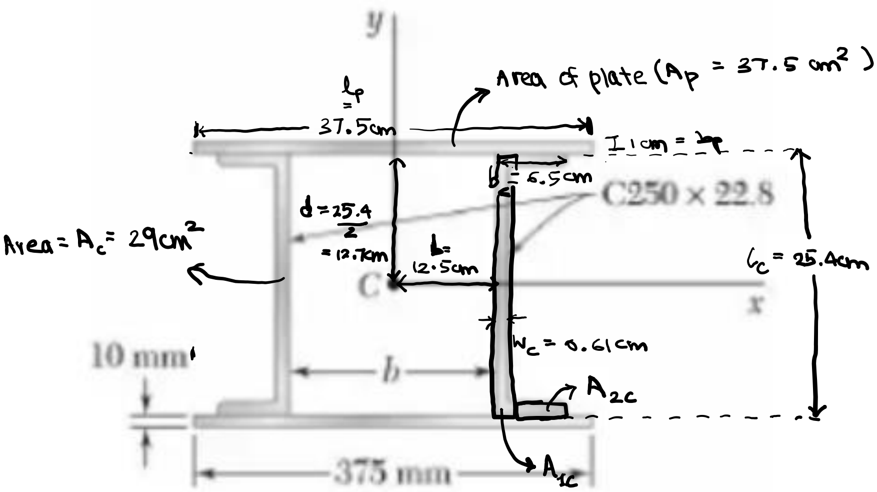

Figure 3

First, we will calculate I_X:

I_x of one plate = $A_p b_p^2/12 + A_p(d+b_p/2)^2$

$= (37.5)(1)^2/12 + (37.5)(13.2)^2$

$= \mathbf{6537.125\ cm^4}$

$\therefore I_X$ of both plates = 2x6537.125 = $\mathbf{13074.25\ cm^4}$

I_X of one C-Section = $I_X(\text{of } A_{1C}) + 2I_X(\text{of } A_{2C})$

$\therefore I_X$ of both C-Sections = $2I_X(\text{of } A_{1C}) + 4I_X(\text{of } A_{2C})$ [Due to symmetry]

$= 2(A_{1C}l_c^2/12) + 4(A_{2C}w_c^2/12 + A_{2C}((l_c-w_c)/2)^2)$

$= 2(15.494\text{x}25.4^2/12) + 4(3.5929(0.61^2/12 + 12.395^2))$

$= 1666.018 + 2208.441$

$= \mathbf{3874.4591\ cm^4}$

I_X of entire section = 13074.25 + 3874.4591

$= \mathbf{16948.7091\ cm^4}$

Total Area of section = $2\text{x}(A_P + A_C) = 2(37.5 + 22.6798) =$

Radius of gyration about X axis = $(I_X/\text{Total Area})^{0.5}$

$= (16948.7092/120.3596)^{0.5}$

$= \mathbf{11.8666\ cm}$

Now we will calculate I_Y:

I_Y of one plate = $A_P l_p^2/12$

$= 37.5\text{x}(37.5)^2/12$

$= 4394.53125\ cm^4$

$\therefore I_Y$ of both plates = 2x4394.53125

$= \mathbf{8789.0625\ cm^4}$

I_Y of one C-Section = I_Y(of A_{1C}) + $2I_Y$(of A_{2C})

$\therefore I_Y$ of both C-Sections = $2I_Y$(of A_{1C}) + $4I_Y$(of A_{2C}) [Due to symmetry]

= A

I_Y of entire section = 8789.0625 + 11648.0488SS

= **20437.1113 cm^4**

The radius of Gyration about Y-axis = $(I_Y/\text{Total Area})^{0.5}$

= $(20437.1113/133)^{0.5}$

= **12.396 cm**

4.

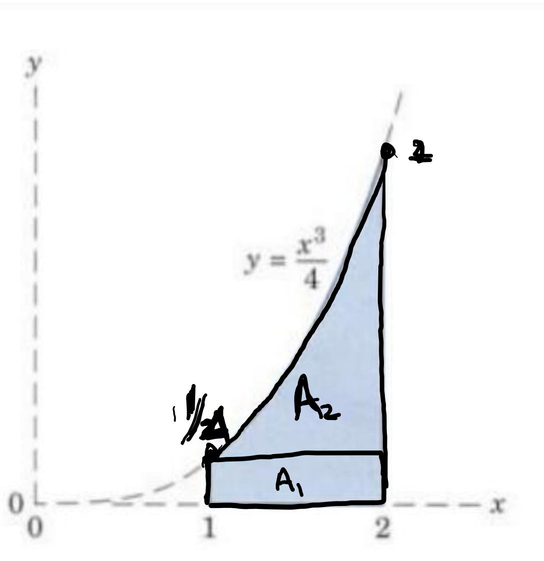

Figure 4

Area of figure = $\int_1^2 dA$ = $\int_1^2 y\,dx$ = $(\int_1^2 x^3 dx)/4$

= $(2^4 - 1^4)/16$

= 15/16 mm^2

= **0.9375 mm^2**

I_y = $\int_1^2 x^2 dA$ (dA = ydx)

$= \int_1^2 x^2 y dx$

$= \int_1^2 x^2 (x^3/4) dx$

$= (\int_1^2 x^5 dx)/4$

$= 1/24(2^6 - 1^6)$

$= 63/24 = 21/8$

$= \mathbf{2.625\ mm^4}$

$\therefore$Rectangular radius of gyration (r_Y) about this axis $= (I_Y/A)^{0.5}$

$= ((21/8)/(15/16))^{0.5}$

$= 2.8^{0.5}$

$= \mathbf{1.6733\ mm}$

$I_x = I_{A1(\text{about X})} + I_{A2(\text{about X})}$

$I_{A1(\text{about X})} = \int_0^{0.25} y^2 dA$ ($dA = 1dy$)

$= \int_0^{0.25} y^2 dy$

$= 1/(64x3)$

$I_{A2(\text{about X})} = \int_{0.25}^2 y^2 dA$ ($dA = (2-x)dy = (2-4^{1/3}y^{1/3})dy$)

$= 2\int_{0.25}^2 y^2 dy - 2^{2/3}\int_{0.25}^2 y^{7/3} dy$

$= 2/3(8 - 1/64) - 3.2^{2/3}(2^{10/3} - 2^{-20/3})/10$

$= 16/3 - 2/(64x3) - 3(2^4 - 2^{-6})/10$

$I_x = 1/(64x3) + 16/3 - 2/(64x3) - 3(16 - 1/64)/10$

$= (16 - 1/64)/3 - 3(16 - 1/64)/10$

$= (1/3 - 3/10)\ x\ (16 - 1/64)$

$= (1/30)\ x\ (16 - 1/64)$

$= \mathbf{0.5328125\ mm^4}$

$\therefore$Rectangular radius of gyration (r_X) about this axis $= (I_X/A)^{0.5}$

$= (0.5328125/0.9375)^{0.5}$

$= (0.568333)^{0.5}$

$= 0.75387$ mm