1. $P_\nu \propto \Omega(E - E_\nu)$

By Taylor-Series Expansion,

$$\ln[\Omega(E-E_\nu)] = \ln[\Omega(E)] - E_\nu \frac{\partial \ln \Omega}{\partial E} + \ldots\ldots$$ ($E_\nu \ll E$ so latter terms are neglected)

$$= \ln[\Omega(E)] - E_\nu \times \left(\frac{1}{k_B T}\right)$$

$$\beta = \frac{1}{\Omega}\frac{\partial \Omega}{\partial E} \Rightarrow \int_E \beta \, \partial E = \int_\Omega \frac{\partial \Omega}{\Omega} \Rightarrow \text{Integrating,}$$

$$\therefore \boxed{\ln \Omega \propto \beta E} \Rightarrow \frac{\partial \ln \Omega}{\partial E} = \beta = \frac{1}{k_B T}$$

$\therefore$

$$\therefore \; \therefore \Omega(E - E_\nu) \approx e^{\ln[\Omega(E)] - E_\nu \beta}$$

$$= \Omega(E) e^{-\beta E_\nu}$$

Since $P_\nu \propto \Omega(E - E_\nu)$,

$$P_\nu \propto \Omega(E) e^{-\beta E_\nu} \Rightarrow P_\nu \propto e^{-\beta E_\nu}$$

2. No. of microstates =

(No. of combinations of N rooks placed in a straight line with $2V$ spaces) $\times$ (permutations of all these rooks in a st line of $2V$ spaces)

$$= {}^{2V}C_N \times {}^{2V}P_N$$

$$= \boxed{\frac{(2V!)^2}{((2V-N)!)^2 \, N!}}$$

2. $Q(N,\beta) = \sum_i e^{-\beta E_n}$

For each E_n, $\exists$ only ONE microstate (due to the given constraint)

$\therefore E_n = n\Delta$

$$Q(N,\beta) = \sum_{n=0}^{N} e^{-\beta n \Delta}$$

This is a geometric series

$$\boxed{Q(N,\beta) = \frac{1-e^{-\beta\Delta(N+1)}}{1-e^{-\beta\Delta}}}$$

$$\langle n\rangle = \sum_i P_n \times n$$

$$= \sum_{n=0}^{N} \frac{e^{-\Delta\beta n}}{Q} \times n$$

$$= \frac{1}{Q} \sum_i n e^{-\Delta\beta n}$$

↳ Arithmetico - Geometric Series

$$Q\langle n\rangle = e^{-\Delta\beta} + 2e^{-2\Delta\beta} + 3e^{-3\Delta\beta} + \dots + Ne^{-N\Delta\beta}$$

$$e^{-\Delta\beta}Q\langle n\rangle = e^{-2\Delta\beta} + 2e^{-3\Delta\beta} + \dots (N-1)e^{-N\Delta\beta} + Ne^{-(N+1)\Delta}$$

(-)

$$(1-e^{-\Delta\beta})Q\langle n\rangle = \frac{1-e^{-\beta\Delta(N+1)}}{1-e^{-\beta\Delta}} + Ne^{-(N+1)\beta\Delta}$$

$$\therefore \langle n\rangle = \frac{1}{Q}\left[\frac{(1-e^{-\beta\Delta(N+1)})}{(1-e^{-\beta\Delta})^2} + \frac{Ne^{-(N+1)\beta\Delta}}{1-e^{-\Delta\beta}}\right]$$

~~At very large~~

$$\langle n \rangle = \frac{1}{1-e^{-\beta\Delta}} + \frac{Ne^{-(N+1)\beta\Delta}}{1-e^{-(N+1)\beta\Delta}}$$

At very large N,

$$Ne^{-(N+1)\beta\Delta} \rightarrow 0$$

$$e^{-(N+1)\beta\Delta} \rightarrow 0$$

$$\therefore \boxed{\langle n \rangle = \frac{e^{\beta\Delta}}{1-e^{\beta\Delta}}}$$

$$\boxed{\beta\Delta = 3.894 \times 10^{-2}}$$

$\therefore \langle n \rangle = 26.183$ links are open on avg

at $T = 298K$ & $\Delta = 0.001 eV$

4. For these BIONS,

$$Z = \sum_{n_1, n_2 \dots n_j} \prod_j e^{-\beta(\epsilon_j - \mu)(n_j)}$$

$$= \prod_j \sum_{n_j=0}^{\infty} e^{-\beta(\epsilon_j - \mu)(2 \times n_j)}$$

[since it takes only even number of particles]

(This is a geometric series)

$$= \boxed{\prod_j \frac{1}{1 - e^{-2\beta(\epsilon_j - \mu)}}}$$

$$\langle n_j \rangle = \frac{\sum (2n_j) \times \left(\prod_j e^{-\beta(\epsilon_j - \mu)(2n_j)}\right)}{Z}$$

$$= \frac{1}{Z} \frac{\partial Z}{\partial(-\beta\epsilon_j)}$$

$$= \frac{\partial(\ln Z)}{\partial(-\beta\epsilon_j)}$$

$$\frac{\ln\left(\frac{1}{1 - e^{-2\beta(\epsilon_j - \mu)}}\right)}{\partial(-\beta\epsilon_j)}$$

$$= \boxed{\frac{2}{e^{2\beta(\epsilon_j - \mu)} - 1}}$$

5.

$$\langle n_1 \rangle = \frac{1}{1 + e^{\beta(\varepsilon_1 - \mu)}}$$

$$\boxed{\beta = \frac{1}{K_B T} = \frac{1}{8.617 \times 10^{-5} \times 300} = 38.683 eV^{-1}}$$

$$= \frac{1}{1 + e^{(0.5 - 0.7) \times 38.683}} = \frac{1}{1.0004365533}$$

$$= \boxed{0.9995636372 \text{ particle}}$$

$$\cancel{= \frac{1}{1 + 0.99848}}$$

$$\cancel{= 0.5013 \text{ particles}}$$

$$\langle n_2 \rangle = \frac{1}{1 + e^{(0.3)\beta}} = \frac{1}{1.09634 \times 10^{5}} = \boxed{9.1212 \times 10^{-6} \text{ particles}}$$

6.

$$G = \begin{cases} a\left(1-\frac{T}{T_c}\right)^2 & T < T_c \\ a\left(1-\frac{T}{T_c}\right)^2 + b\left(1-\frac{T}{T_c}\right)^3 & T \geq T_c \end{cases}$$

- G is continuous at $T = T_c$. [$G = 0$ when $T = T_c$]

$$\frac{\partial G}{\partial T} = \begin{cases} -\frac{2a}{T_c}\left(1-\frac{T}{T_c}\right) & T < T_c \\ -\frac{2a}{T_c}\left(1-\frac{T}{T_c}\right) - \frac{3b}{T_c}\left(1-\frac{T}{T_c}\right)^2 & T \geq T_c \end{cases}$$

- $\frac{\partial G}{\partial T}$ is continuous at $T = T_c$

$$\frac{\partial^2 G}{\partial T^2} = \begin{cases} \frac{2a}{T_c^2} & T < T_c \\ \frac{2a}{T_c^2} + \frac{6b}{T_c^2}\left(1-\frac{T}{T_c}\right) & T \geq T_c \end{cases}$$

- $\frac{\partial^2 G}{\partial T^2}$ is continuous at $T = T_c$

$$\frac{\partial^3 G}{\partial T^3} = \begin{cases} 0 & T < T_c \\ -\frac{6b}{T_c^3} & T \geq T_c \end{cases}$$

$\frac{\partial^3 G}{\partial T^3}$ is DISCONTINUOUS at $T = T_c$.

$\therefore$ $\boxed{\text{The order of phase transition} = 3}$

7. (i) Energy gap $= \dfrac{hc}{\lambda} = \dfrac{1242}{632.8} = \boxed{1.9627 \text{ eV}}$

(ii) We know ~~In/Spon~~

$$A = \frac{1}{\text{lifetime}} = 10^{10} \, s^{-1}$$

Also,

$$\frac{A}{B} = \frac{2h\nu^3 n_0^3}{c^3} = \frac{2hn_0^3}{\lambda^3}$$

1.3252×10^{-4} / $6.328^3 \times 10^{9}$

0.05229×10^{-13}

5.229×10^{-15}

$$= \frac{2 \times 6.626 \times 10^{-34} \times 1}{10^{-30} \times (6328)^3}$$

$$= \cancel{13.282 \times 10^{-4}} \quad \cancel{1.325 \times 10^{-3} \, Js/m^3} \quad = \boxed{5.229 \times 10^{-15} \, kg/ms}$$

$$\therefore B = \frac{A}{1.3252 \times 10^{-3}} = \cancel{7.546 \times 10^{12}} \quad \cancel{= 7.546 \times 10^{12} \, m^3/Js^2}$$

$$B = \frac{10^{10}}{5.229 \times 10^{-15}} = 1.912 \times 10^{24} \, m/kg$$

$\dfrac{kgm^2}{s^2} \cdot \dfrac{s}{m^3}$

(iii) No. of resonant frequencies $= \dfrac{2d}{\lambda}$

$\dfrac{E \times s}{m^3} = \dfrac{kgm^2/s}{m^3}$

$$= \frac{2 \times 1}{6328 \times 10^{-10}}$$

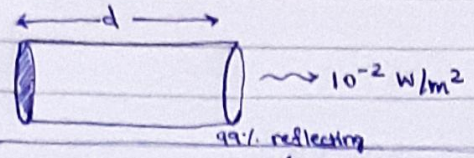
$\dfrac{Js}{m^3}$

$$= \frac{10^{10}}{3164}$$

$\dfrac{m^3}{\frac{kgm^2}{s^2} \cdot s^2}$

$\cong$ ~~31660~~ $\boxed{3160566 \text{ frequencies}}$

8.



$\cancel{1\%\ \text{of Power}} = I \times A \times \cancel{99\%}$ [Consumed Power]

$= 10^{-2} \times 10^{-4}\ \cancel{99\%}$

$= 10^{-6}\,W = \text{Energy/Time}$

$\therefore\ \cancel{P = 10^{-2} \times 10^{-4} = 10^{-6}\,W}\ \cancel{\boxed{0.99 \times 10^{-6}\,W}}$

$P = \dfrac{nE_p}{t} = \dfrac{nE_p}{(d/c)} = \dfrac{nE_pc}{d}$ [E_p = energy of photon]

$\therefore\ n = \dfrac{Pd}{E_pc} = \dfrac{10^{-4} \times 10^{-1} \times \cancel{0.99}}{10^{-1} \times 1.6 \times 10^{-19} \times 3 \times 10^{8}}$

$= \dfrac{10^{7} \times 0.99}{4.8} = \cancel{\boxed{2.065 \times 10^{6}\ \text{photons}}}$

→ Power consumed by the laser

$= 99 \times$ Power Output

$= 99 \times 10^{-6}$

$= \boxed{9.9 \times 10^{-5}\,W}$

→ Energy of beam = Intensity × Area × Time

$= 10^{-2} \times 10^{-4} \times \dfrac{2d}{c}$

$= 10^{-6} \times \dfrac{10^{-1}}{3 \times 10^{8}} = 6.66 \times 10^{-16}\,J$

$= \cancel{\dfrac{3.33 \times 10^{-16}}{1.6 \times 10^{-19}}\ eV}$

$=$

$\therefore$ Energy of beam inside cavity $(E_B) = \dfrac{6.66 \times 10^{-16}}{1\%}$

$= 6.66 \times 10^{-14}\,J = \dfrac{6.66 \times 10^{-14}}{1.6 \times 10^{-19}}\,eV = \boxed{4.16 \times 10^{5}\,eV}$

$\therefore$ No. of photons $= \dfrac{E_B}{E_p\ (\text{Energy of 1 photon})} = \dfrac{4.16 \times 10^{5}}{0.1} = \boxed{4.16 \times 10^{6}\ \text{photons}}$