Homework 06

ME1020 - Engineering Mechanics

1.

(a)

$x(cm) = 2t - 6t^3$ → Eqn. 1

Differentiating wrt time, we get

$v_x(cm/s) = dx/dt = 2 - 18t^2$ → Eqn. 2

$a_x\ (cm/s^2) = d^2x/dt^2 = dv_x/dt$

$= -36t$ → Eqn. 3

∴At t = 5s,

Velocity of head = v_x (at t = 5)

$= 2 - 18*5^2$ (From Eqn 2)

$= -448$ cm/s

Acceleration of head = a_x (at t = 5)

$= -36*5$ (From Eqn 3)

$= -180\ cm/s^2$

(b)

Initial velocity of car = 20km/h = 50/9 = 5.555 m/s

Acceleration of car after 2 seconds = a = $3m/s^2$

First, let us see how long the car takes to reach the signal which is 100m away (say 't' seconds).

If this time is **less than 5 seconds** (time taken to turn from yellow to red), that means **he reaches the signal before it turns red**. If the calculated time is **more than 5 seconds**, he **doesn't reach the signal on time**.

Total distance = $(v_{initial})(2s) + (v_{initial})(t-2) + {}^1/_2\ a(t-2)^2$

$100m = (5.555\ m/s)(2s) + (5.555)(t-2) + {}^1/_2\ (3\ m/s^2)(t-2)^2$

$1.5(t-2)^2 + 5.555(t-2) - 88.89$

Solving, we get

$(t-2) = 6.065s$

t = 8.065s > 5s

∴The car doesn't reach the signal on time.

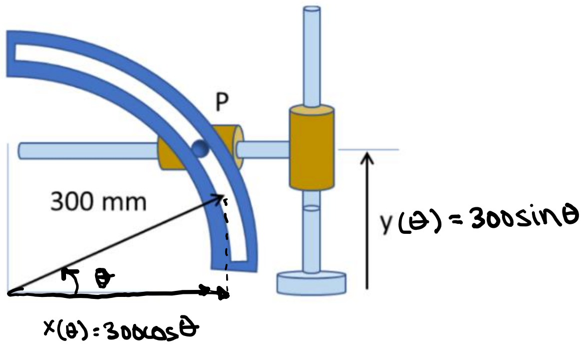
The velocity of the car when it reaches the signal = $v_{initial} + a(t-2)$

$= 5.555 + 3(6.065)$

$= 23.75$ m/s

2.

(a)



$y = 300\sin\theta$ → Eqn 1

Differentiating wrt t,

dy/dt (mm/s) $= (300\cos\theta)d\theta/dt$ → Eqn 2

Differentiating again wrt t,

d^2y/dt^2 (mm/s^2) $= 300[(\cos\theta)(d^2\theta/dt^2) - (\sin\theta)(d\theta/dt)^2]$ → Eqn 3

When $y = 200$mm, $\sin\theta = y/300 = 2/3 = 0.666$ (From Eqn 1)

$\therefore \cos\theta = \sqrt{5}/3 = 0.745$

$\tan\theta = 0.894$

At $y = 200$mm,

(i) $dy/dt = 200$mm/s (Given)

$\therefore 300\cos\theta d\theta/dt = 200$

$d\theta/dt = 2/(3\cos\theta)$ [Eqn 1]

$= 0.894$ rad/s

$\therefore$ Radial acceleration (towards centre) at point P (a_R) $= (d\theta/dt)^2R$

$= 240$ mm/s^2

(ii) $d^2y/dt^2 = 0$

$(\cos\theta)(d^2\theta/dt^2) = (\sin\theta)(d\theta/dt)^2$

$d^2\theta/dt^2 = 0.894(0.894)^2$

$= \mathbf{0.7155\ rad/s^2}$ **(Angular acceleration)**

$\therefore$Tangential acceleration $(a_T) = (d^2\theta/dt^2)R$

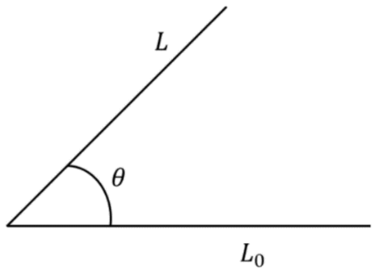
$= 0.7155 \times 300$

$= \mathbf{214.65\ mm/s^2}$

$\therefore$Net acceleration $= \sqrt{(a_r^2 + a_T^2)}$

$= \mathbf{321.98\ mm/s^2}$

(b)



$\alpha = d\omega/dt = 2-1.5t$

$d\omega = (2 - 1.5t)dt$

Integrating with suitable limits,

$\int_5^{\omega(t)} d\omega = \int_0^t (2 - 1.5t)dt$

$\omega(t) - 5 = 2t - 0.75t^2$

$\boldsymbol{\omega(t) = 5 + 2t - 0.75t^2 \rightarrow}$ **Eqn. 1**

$d\theta/dt = \omega(t) = 5 + 2t - 0.75t^2$

$d\theta = (5 + 2t - 0.75t^2)dt$

Integrating with suitable limits,

$\int_0^{\theta(t)} d\theta = \int_0^t (5 + 2t - 0.75t^2)$

$\boldsymbol{\theta(t) = 5t + t^2 - 0.25t^3 \rightarrow}$ **Eqn 2**

$\therefore\omega(3s) = 5 + 2 \times 3 - 0.75 \times 3^2$

$= \mathbf{4.25\ rad/s}$

$$\theta(3s) = 5 \times 3 + 3^2 - 0.25 \times 3^3$$

$$= 17.25 \text{ rad}$$