

- ① (a) To do : Find and tabulate the 'd' spacings of different orientations of the given x-ray diffraction pattern.

Radiation used = $\text{Cu}(K_\alpha)$ with $\lambda = 1.5406 \text{ \AA}$

Bragg's Law : $n\lambda = 2d \sin \theta$

(i) $2\theta = 22.88^\circ \Rightarrow \theta = 11.44^\circ$

$\sin(11.44^\circ) = 0.198$

So, $1 \times 1.5406 \times 10^{-10} \text{ m} = 2 \times d \times \sin(11.44^\circ)$

$\Rightarrow d = \frac{1.5406}{2 \times 0.198} \text{ \AA} \Rightarrow d = 3.89 \text{ \AA}$

(ii) $2\theta = 32.54^\circ \Rightarrow \theta = 16.27^\circ$

$\sin(16.27^\circ) = 0.28$

So, $1 \times 1.5406 \text{ \AA} = 2d \sin(16.27^\circ)$

$\Rightarrow d = \frac{1.5406}{2 \times 0.28} \text{ \AA} \Rightarrow d = 2.75 \text{ \AA}$

(iii) $2\theta = 40.10^\circ \Rightarrow \theta = 20.05^\circ$

$\sin(20.05^\circ) = 0.342$

So, $1 \times 1.5406 \text{ \AA} = 2d \sin(20.05^\circ)$

$\Rightarrow d = \frac{1.5406}{2 \times 0.342} \text{ \AA} \Rightarrow d = 2.25 \text{ \AA}$

(iv) $2\theta = 46.60^\circ \Rightarrow \theta = 23.30^\circ$

$\sin(23.30^\circ) = 0.395$

So, $1 \times 1.5406 \text{ \AA} = 2d \sin(23.30^\circ)$

$\Rightarrow d = \frac{1.5406}{2 \times 0.395} \text{ \AA} \Rightarrow d = 1.95 \text{ \AA}$

$$(v) \quad 2\theta = 52.48^\circ \Rightarrow \boxed{\theta = 26.24^\circ}$$

$$\sin(26.24^\circ) = 0.442$$

$$\text{So, } 1 \times 1.5406 \text{ \AA} = 2d \sin(26.24^\circ)$$

$$\Rightarrow d = \frac{1.5406}{2 \times 0.442} \text{ \AA} \Rightarrow \boxed{d = 1.74 \text{ \AA}}$$

$$(vi) \quad 2\theta = 57.92^\circ \Rightarrow \boxed{\theta = 28.96^\circ}$$

$$\sin(28.96^\circ) = 0.484$$

$$\text{So, } 1 \times 1.5406 \text{ \AA} = 2d \sin(28.96^\circ)$$

$$\Rightarrow d = \frac{1.5406}{2 \times 0.484} \text{ \AA} \Rightarrow \boxed{d = 1.59 \text{ \AA}}$$

$$(vii) \quad 2\theta = 67.94^\circ \Rightarrow \boxed{\theta = 33.97^\circ}$$

$$\sin(33.97^\circ) = 0.558$$

$$\text{So, } 1 \times 1.5406 \text{ \AA} = 2d \sin(33.97^\circ)$$

$$\Rightarrow d = \frac{1.5406}{2 \times 0.558} \text{ \AA} \Rightarrow \boxed{d = 1.38 \text{ \AA}}$$

$$(viii) \quad 2\theta = 72.68^\circ \Rightarrow \boxed{\theta = 36.34^\circ}$$

$$\sin(36.34^\circ) = 0.592$$

$$\text{So, } 1 \times 1.5406 \text{ \AA} = 2d \sin(36.34^\circ)$$

$$\Rightarrow d = \frac{1.5406}{2 \times 0.592} \text{ \AA} \Rightarrow \boxed{d = 1.30 \text{ \AA}}$$

$$(ix) \quad 2\theta = 77.30^\circ \Rightarrow \boxed{\theta = 38.65^\circ}$$

$$\sin(38.65^\circ) = 0.624$$

$$\text{So, } 1 \times 1.5406 \text{ \AA} = 2d \sin(38.65^\circ)$$

$$\Rightarrow d = \frac{1.5406}{2 \times 0.624} \text{ \AA} \Rightarrow \boxed{d = 1.23 \text{ \AA}}$$

Table

S. No.	2θ	θ	d (in Å)
1	22.88°	11.44°	3.89 Å
2	32.54°	16.27°	2.75 Å
3	40.10°	20.05°	2.25 Å
4	46.60°	23.30°	1.95 Å
5	52.48°	26.24°	1.74 Å
6	57.92°	28.96°	1.59 Å
7	67.94°	33.97°	1.38 Å
8	72.68°	36.34°	1.30 Å
9	77.30°	38.65°	1.23 Å

(b) Radiation used = Fe(α) with $\lambda = 1.93604 \text{ Å}$

$$(i) \quad 1 \times 1.93604 \times 10^{-10} \text{ m} = 2 \times 3.89 \times 10^{-10} \times \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1.93604}{2 \times 3.89} = 0.248$$

$$\Rightarrow \theta = \sin^{-1}(0.248) \Rightarrow \boxed{\theta = 14.35^\circ}$$

$$(ii) \quad 1 \times 1.93604 \text{ Å} = 2 \times 2.75 \text{ Å} \times \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1.93604}{5.5} = 0.352$$

$$\Rightarrow \theta = \sin^{-1}(0.352) \Rightarrow \boxed{\theta = 20.60^\circ}$$

$$(iii) \quad 1 \times 1.92604 \text{ Å} = 2 \times 2.25 \text{ Å} \times \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1.93604}{4.5} = 0.430$$

$$\Rightarrow \theta = \sin^{-1}(0.43) \Rightarrow \boxed{\theta = 25.46^\circ}$$

$$(iv) \quad 1 \times 1.93604 \text{ A} = 2 \times 1.95 \text{ A} \times \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1.93604}{2.9} = 0.496$$

$$\Rightarrow \theta = \sin^{-1}(0.496) \Rightarrow \boxed{\theta = 29.73^\circ}$$

$$(v) \quad 1 \times 1.93604 \text{ A} = 2 \times 1.74 \text{ A} \times \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1.93604}{3.48} = 0.556$$

$$\Rightarrow \theta = \sin^{-1}(0.556) \Rightarrow \boxed{\theta = 33.77^\circ}$$

$$(vi) \quad 1 \times 1.93604 \text{ A} = 2 \times 1.59 \text{ A} \times \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1.93604}{3.18} = 0.6088$$

$$\Rightarrow \theta = \sin^{-1}(0.6088) \Rightarrow \boxed{\theta = 37.5^\circ}$$

$$(vii) \quad 1 \times 1.93604 \text{ A} = 2 \times 1.38 \text{ A} \times \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1.93604}{2.76} = 0.701$$

$$\Rightarrow \theta = \sin^{-1}(0.701) \Rightarrow \boxed{\theta = 44.5^\circ}$$

$$(viii) \quad 1 \times 1.93604 \text{ A} = 2 \times 1.30 \text{ A} \times \sin \theta$$

$$\Rightarrow \sin \theta = \frac{1.93604}{2.6} = 0.744$$

$$\Rightarrow \theta = \sin^{-1}(0.744) \Rightarrow \boxed{\theta = 48.04^\circ}$$

$$(ix) \quad 1 \times 1.93604 \text{ A} = 2 \times 1.23 \text{ A} \times \sin \theta$$

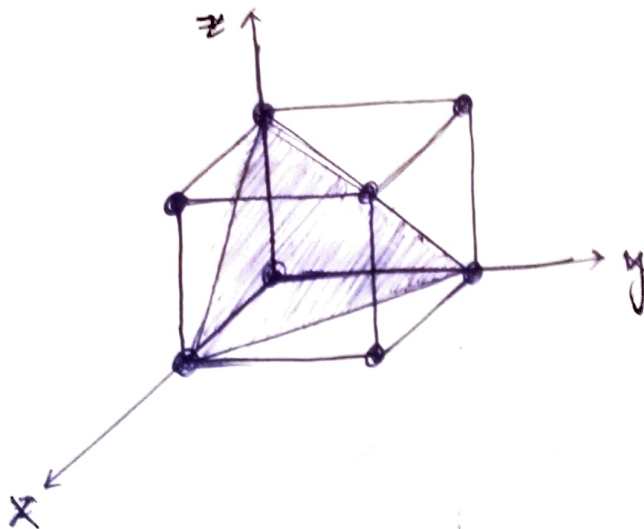
$$\Rightarrow \sin \theta = \frac{1.93604}{2.46} = 0.787$$

$$\Rightarrow \theta = \sin^{-1}(0.787) \Rightarrow \boxed{\theta = 51.9^\circ}$$

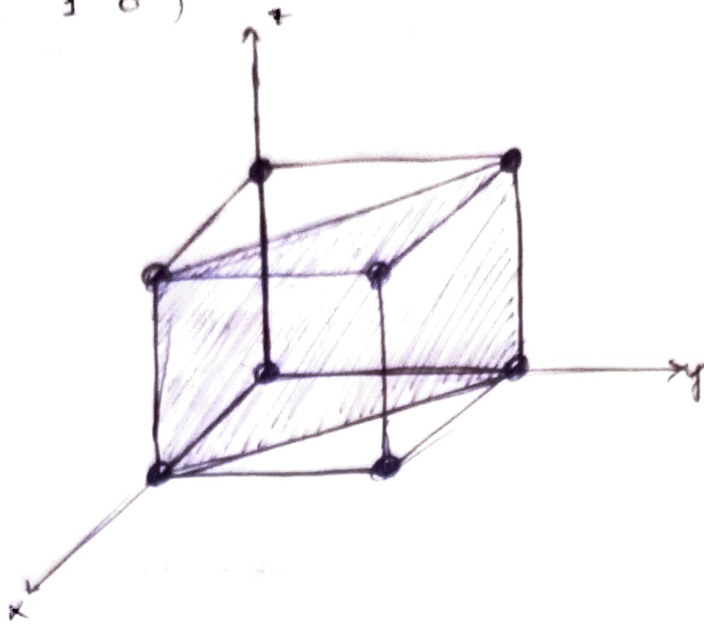
Table

S.No.	d (in Å)	θ	2θ
1	3.89 Å	14.25°	28.5°
2	2.45 Å	20.60°	41.2°
3	2.25 Å	25.46°	50.92°
4	1.95 Å	29.73°	59.46°
5	1.74 Å	33.49°	66.98°
6	1.59 Å	37.5°	75°
7	1.38 Å	44.5°	89°
8	1.30 Å	48.07°	96.14°
9	1.23 Å	51.9°	103.8°

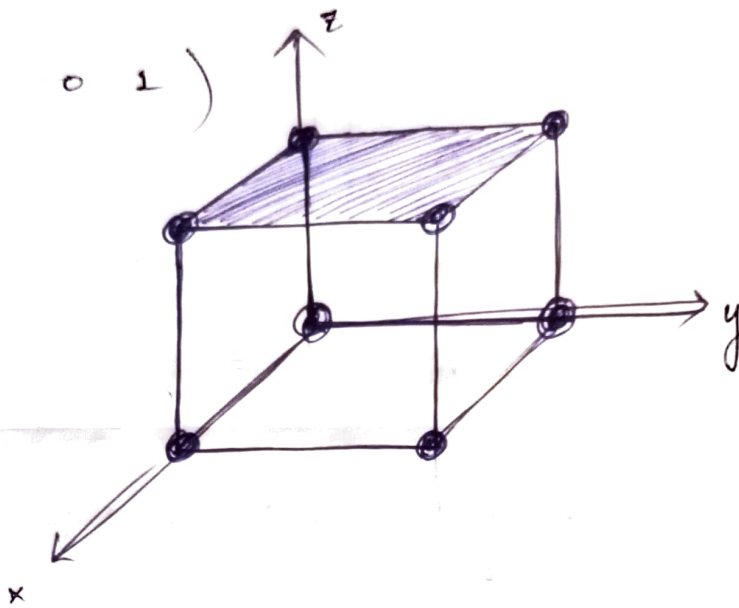
② (a) $(2\ 2\ 2) \equiv 2(1\ 1\ 1)$



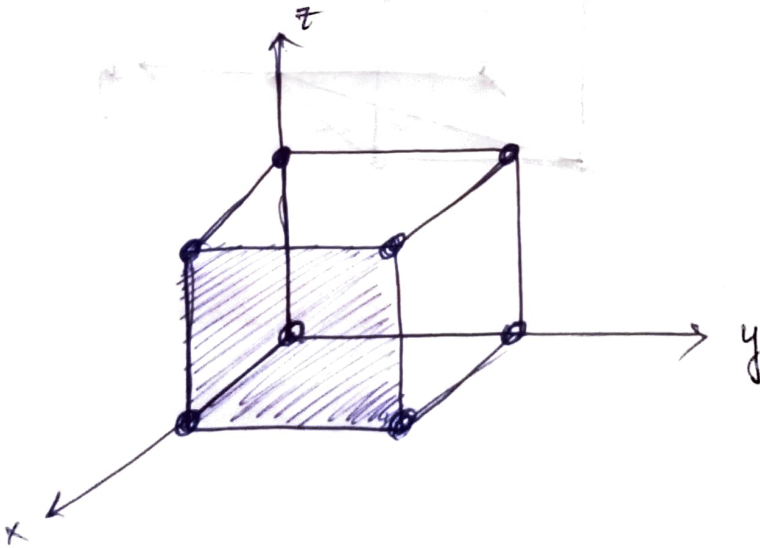
$$(b) \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$$



$$(c) \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$



$$(d) \begin{pmatrix} 2 & 0 & 0 \end{pmatrix} \equiv 2 \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$



(4) (a) When a coin is tossed once, the outcomes are either a head (H) or a tail (T).

Two eigen states = H, T

When 3 coins are tossed simultaneously
the total no. of outcomes = 2^3
= 8

The eight eigen states are as follows

(H H H), (H H T), (H T H),

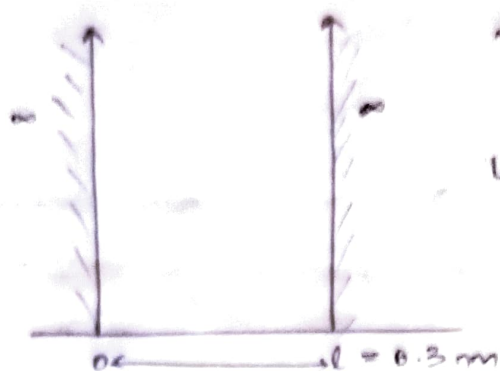
(T H H), (T T H), (H T T),

(T H T), (T T T)

(b) If 3 indistinguishable coins are tossed, there will be 2 degenerate states.

Degenerate states : $(H H H)$, $(T T T)$

- ⑤ Given: An e^- is confined in a 1-D potential well of width 0.3 nm .



$$V(x) = \begin{cases} 0, & \text{if } 0 < x < l \\ \infty, & \text{otherwise} \end{cases}$$

Using Schrödinger's eq.

$$\boxed{\hat{H}\psi(x) = E\psi(x)}$$

$$\Rightarrow -\frac{\hbar^2}{2m} \cdot \frac{d^2\psi(x)}{dx^2} + \cancel{V(x)\psi(x)} = E\psi(x)$$

$$\Rightarrow -\frac{\hbar^2}{2m} \cdot \frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

$$\Rightarrow \boxed{\frac{d^2\psi(x)}{dx^2} = -\frac{2mE}{\hbar^2} \cdot \psi(x)} \quad \text{--- ①}$$

Now, solving the above differential eqⁿ, we have

$$\psi(x) = A \cos(kx) + B \sin(kx)$$

Using the boundary conditions,

$$\psi(x \leq 0) = 0 \rightarrow \psi(0) = 0$$

$$\psi(x \geq l) = 0 \rightarrow \psi(l) = 0$$

$$\text{So, } \psi(0) = A \cos(k \cdot 0) + B \sin(k \cdot 0)$$

$$\Rightarrow \psi(0) = 0 = A \cos(0) \Rightarrow \boxed{A = 0}$$

and,

$$\psi(l) = B \sin(kl) = 0$$

$$\Rightarrow \sin(kl) = 0 \Rightarrow kl = n\pi$$

$$\Rightarrow \boxed{k = \frac{n\pi}{l}}, n \in \mathbb{Z}$$

$$\text{So, } \boxed{\Psi_n(x) = B \sin\left(\frac{n\pi x}{l}\right)}$$

(2)

Now, on comparing

$$\frac{n\pi}{l} = k = \frac{p}{\hbar} = \frac{\sqrt{2mE}}{\hbar}$$

$$\Rightarrow \frac{n^2\pi^2}{l^2} = \frac{2mE}{\hbar^2}$$

$$\Rightarrow E = \frac{n^2\pi^2\hbar^2}{2m \cdot l^2}$$

$$\Rightarrow \boxed{E_n = \frac{n^2\hbar^2}{8ml^2}}$$

(2)

Electron at ground state $\Rightarrow n = 1$

$$\text{So, } E_1 = \frac{(1)^2 \cdot \hbar^2}{8 \cdot m_e \cdot (0.3 \times 10^{-9})^2}$$

$$\Rightarrow E_1 = \frac{(1)^2 \cdot (6.63 \times 10^{-34})^2}{8 \cdot (9.31 \times 10^{-31}) \cdot (0.3 \times 10^{-9})^2}$$

$$\Rightarrow E_1 = \frac{43.9569 \times 10^{-68} \text{ J}}{6.7032 \times 10^{-49}}$$

$$\Rightarrow \boxed{E_1 = 6.557 \times 10^{-19} \text{ J} = 4.09 \text{ eV}}$$

\therefore The kinetic energy of the e^- when it is in the ground state = 4.09 eV.

Now, the energy at ground state = 6.557×10^{-19} J

The energy at 2nd excited state
 $\hookrightarrow n = 3$

$$E_3 = \frac{(3)^2 \cdot h^2}{8 m_e (l)^2}$$

$$\Rightarrow E_3 = \frac{9 \times 43.7567 \times 10^{-09}}{8 \times 9.31 \times 10^{-31} \times 0.09 \times 10^{-18}}$$

$$\Rightarrow \boxed{E_3 = 59.01 \times 10^{-19} \text{ J}}$$

Now, $\Delta E = h\nu$

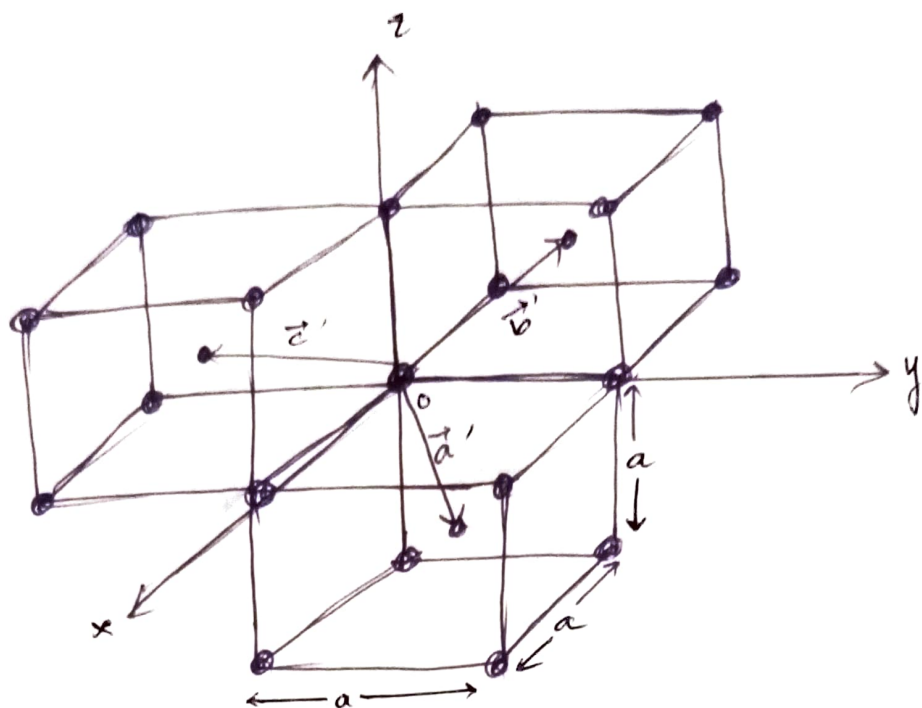
$$\Rightarrow E_3 - E_1 = h\nu$$

$$\Rightarrow (59.01 - 6.557) \times 10^{-19} \text{ J} \\ = 6.63 \times 10^{-34} \text{ J s} \cdot \nu$$

$$\Rightarrow \frac{52.453 \times 10^{-19}}{6.63 \times 10^{-34}} \text{ s}^{-1} = \nu$$

$$\Rightarrow \boxed{\nu = 7.911 \times 10^{15} \text{ Hz}}$$

\therefore The spectral frequency resulting from a transition between the 2nd excited possible state to the ground level is $7.911 \times 10^{15} \text{ Hz}$



To prove : The reciprocal lattice of BCC is FCC.

Let \vec{a}' , \vec{b}' , \vec{c}' be the primitive translational vectors of BCC crystal lattice.

Let \hat{i} , \hat{j} , \hat{k} be the orthogonal unit vectors along x, y and z axis respectively.

So,
$$\vec{a}' = \frac{a}{2}\hat{i} + \frac{a}{2}\hat{j} - \frac{a}{2}\hat{k} = \frac{a}{2}(\hat{i} + \hat{j} - \hat{k})$$

$$\Rightarrow \boxed{\vec{a}' = \frac{a}{2}(\hat{i} + \hat{j} - \hat{k})} \quad \text{--- (1)}$$

Similarly,
$$\boxed{\vec{b}' = \frac{a}{2}(-\hat{i} + \hat{j} + \hat{k})} \quad \text{--- (2)}$$

and,
$$\boxed{\vec{c}' = \frac{a}{2}(\hat{i} - \hat{j} + \hat{k})} \quad \text{--- (3)}$$

Let \vec{a}^* , \vec{b}^* and \vec{c}^* be the primitive translational vectors of reciprocal lattice.

$$\text{So, } \vec{a}^* = \frac{2\pi \cdot (\vec{b}' \times \vec{c}')}{\vec{a}' \cdot (\vec{b}' \times \vec{c}')} \quad \text{--- (4)}$$

$$\vec{b}^* = \frac{2\pi \cdot (\vec{c}' \times \vec{a}')}{\vec{a}' \cdot (\vec{b}' \times \vec{c}')} \quad \text{--- (5)}$$

$$\vec{c}^* = \frac{2\pi \cdot (\vec{a}' \times \vec{b}')}{\vec{a}' \cdot (\vec{b}' \times \vec{c}')} \quad \text{--- (6)}$$

$$\vec{b}' \times \vec{c}' = \left[\frac{a}{2} (-\hat{i} + \hat{j} + \hat{k}) \right] \times \left[\frac{a}{2} (\hat{i} - \hat{j} + \hat{k}) \right]$$

$$= \frac{a^2}{4} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \frac{a^2}{4} \left[\hat{i} (1+1) - \hat{j} (-1-1) + \hat{k} (1-1) \right]$$

$$= \frac{a^2}{4} [2\hat{i} + 2\hat{j}] = \frac{a^2}{2} (\hat{i} + \hat{j})$$

$$\Rightarrow \boxed{\vec{b}' \times \vec{c}' = \frac{a^2}{2} (\hat{i} + \hat{j})} \quad \text{--- (7)}$$

$$\text{So, } \vec{a}' \cdot (\vec{b}' \times \vec{c}') =$$

$$= \left[\frac{a}{2} (\hat{i} + \hat{j} - \hat{k}) \right] \cdot \left[\frac{a^2}{2} (\hat{i} + \hat{j}) \right]$$

$$= \frac{a^3}{4} (1+1) = \frac{a^3}{2}$$

$$\Rightarrow \boxed{\vec{a}' \cdot (\vec{b}' \times \vec{c}') = \frac{a^3}{2}} \quad \text{--- (8)}$$

Now, using (7) and (8) in (4), we have

$$\vec{a}^* = \frac{2\pi \cdot \frac{a^2}{2} (\hat{i} + \hat{j})}{\frac{a^3}{2}}$$

$$\Rightarrow \boxed{\vec{a}^* = \frac{2\pi}{a} (\hat{i} + \hat{j})} \quad \text{--- (9)}$$

Similarly, $\boxed{\vec{b}^* = \frac{2\pi}{a} (\hat{j} + \hat{k})} \quad \text{--- (10)}$

and $\boxed{\vec{c}^* = \frac{2\pi}{a} (\hat{k} + \hat{i})} \quad \text{--- (11)}$

But these are the primitive translational vectors of FCC lattice. So, reciprocal lattice of BCC is FCC (Hence, proved)

⑧ (a) The phase velocity is given by,

$$\boxed{V = V_g = \frac{d\omega}{dk}} \quad \text{--- (1)}$$

Now, $E = h\nu \Rightarrow E = \frac{h}{2\pi} \cdot 2\pi\nu$

$$\Rightarrow \boxed{E = \hbar\omega} \quad \text{--- (2)}$$

Diff. (2) w.r.t k , we have

$$\frac{dE}{dk} = \hbar \cdot \frac{d\omega}{dk} \Rightarrow \boxed{\frac{d\omega}{dk} = \frac{1}{\hbar} \cdot \frac{dE}{dk}} \quad \text{--- (3)}$$

Comparing (1) and (3), we have

$$\boxed{V = \frac{1}{\hbar} \cdot \frac{dE}{dk}} \quad \text{--- (4)}$$

We know that, $E = \frac{p^2}{2m}$

Using de-Broglie hypothesis

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda}$$

$$\Rightarrow \boxed{p = \hbar \cdot k}$$

$$\text{So, } \boxed{E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}} \quad \text{--- (5)}$$

Diff. (5) w.r.t. k , we have

$$\boxed{\frac{dE}{dk} = \frac{\hbar^2}{2m} \cdot 2k = \frac{\hbar^2 \cdot k}{m}} \quad \text{--- (6)}$$

From (1), (4) and (6), we have

$$v = \frac{1}{\hbar} \cdot \frac{dE}{dk} = \frac{1}{\hbar} \cdot \frac{\hbar^2 k}{m}$$

$$\Rightarrow \boxed{v = \frac{\hbar k}{m}} \iff v \propto k$$

On applying \vec{E} , the e^- gets accelerated

The Force acting on it is given by,

$$F = m \cdot a \Rightarrow \boxed{a = \frac{F}{m}} \quad \text{--- (7)}$$

$$\text{Now, } a = \frac{dv}{dt}$$

$$\Rightarrow a = \frac{d}{dt} \left(\frac{1}{\hbar} \cdot \frac{dE}{dk} \right)$$

$$\Rightarrow a = \frac{d}{dt} \left(\frac{1}{\hbar} \cdot \frac{dE}{dk} \right) \frac{dk}{dk}$$

$$\Rightarrow \boxed{a = \frac{1}{\hbar} \cdot \frac{dk}{dt} \cdot \frac{d^2 E}{dk^2}} \quad \text{--- (8)}$$

Now, $p = \hbar k$

$$\Rightarrow \frac{dp}{dt} = \hbar \cdot \frac{dk}{dt}$$

$$\Rightarrow \boxed{\frac{dk}{dt} = \frac{1}{\hbar} \cdot \frac{dp}{dt} = \frac{1}{\hbar} \cdot F} \quad \text{--- (9)}$$

So, $a = \frac{1}{\hbar} \cdot \frac{1}{\hbar} \cdot F \cdot \frac{d^2 E}{dk^2}$

$$\Rightarrow \boxed{a = \frac{F}{\hbar^2} \cdot \frac{d^2 E}{dk^2}} \quad \text{--- (10)}$$

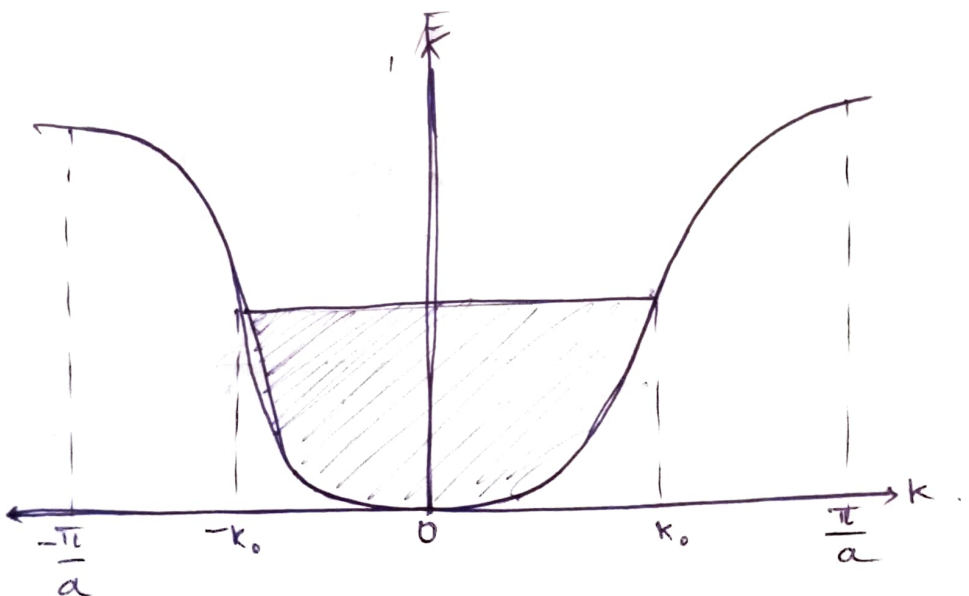
from (7) and (10), we have

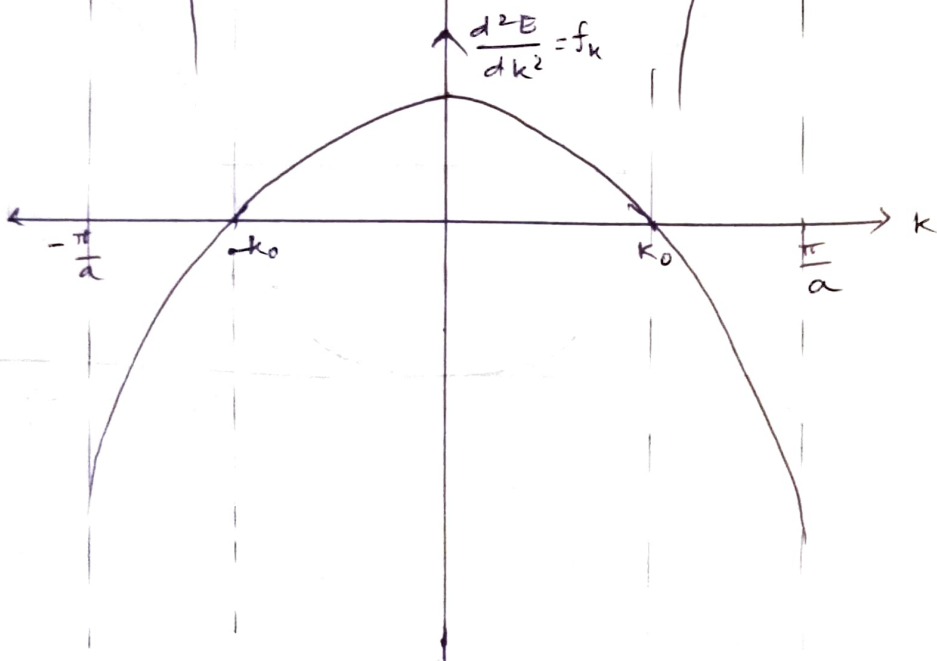
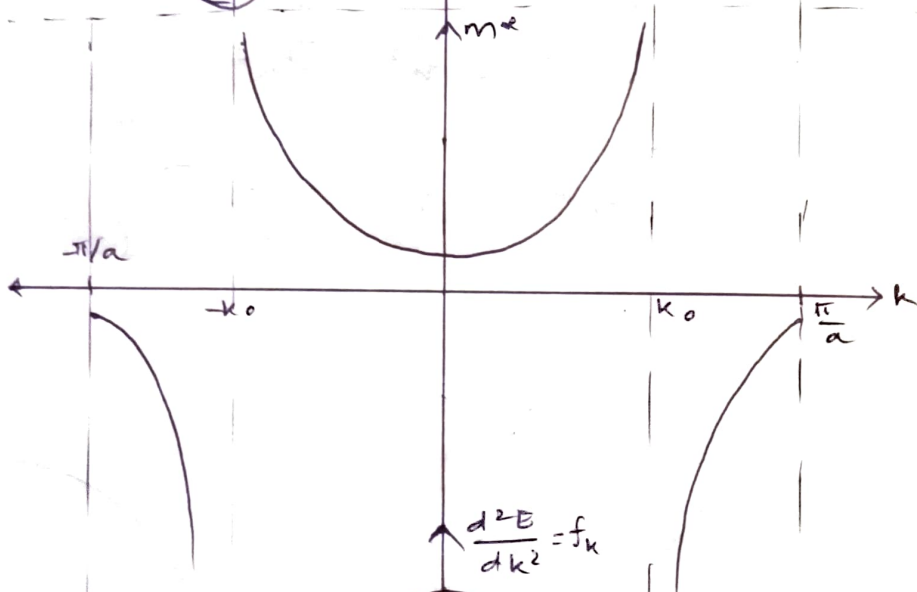
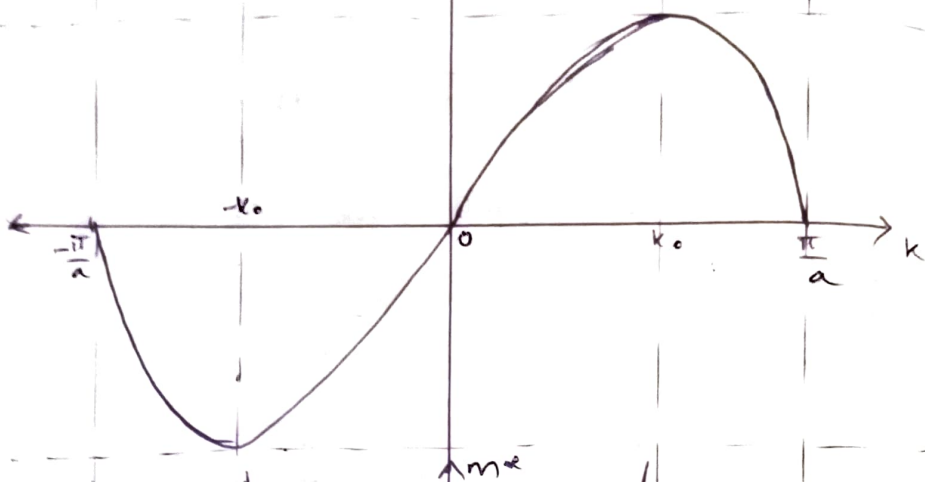
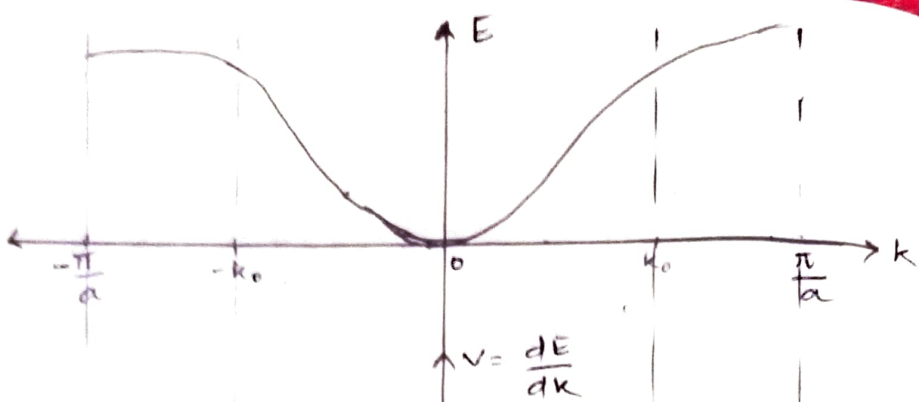
$$\frac{F}{m^*} = \frac{F}{\hbar^2} \cdot \frac{d^2 E}{dk^2}$$

$$\Rightarrow \frac{1}{m^*} = \frac{1}{\hbar^2} \cdot \frac{d^2 E}{dk^2}$$

$$\Rightarrow \boxed{m^* = \hbar^2 \cdot \left(\frac{d^2 E}{dk^2} \right)^{-1}} \quad \text{--- (11)}$$

(b)





⑨ The effective mass is represented as a function of k ($m^* = \hbar^2 \left(\frac{d^2E}{dk^2} \right)^{-1}$). In the lower portion of $E-k$ graph, $\left(\frac{d^2E}{dk^2} \right)$ is positive. Hence m^* is positive. With increase in value of k , m^* attains a maximum value at the point of inflection (say k_0). Now, for higher values of k , $\left(\frac{d^2E}{dk^2} \right)$ becomes negative. As $k \rightarrow \frac{\pi}{a}$, the m^* approaches to a smaller negative value. At the point of inflection $\left[\left(\frac{d^2E}{dk^2} \right) = 0 \right]$, m^* becomes infinite.

As the electron approaches the zone boundary, the m^* becomes negative (can be seen from the graph). The external force is still positive, means that the acceleration on electron now becomes negative, i.e., the electron decelerates. The negative m^* tells us that the electron responds to the Electric field opposite to how a free electron would. Physically, the fact that the electron ~~responds~~ accelerates opposite to the direction of the force is because the electron must reflect off the zone boundary. As it approaches the boundary, the electron must decelerate.

(10) 1 g of Li = $\frac{1}{7}$ mol atom Li

$$= \frac{1}{7} \times 6.022 \times 10^{23} \text{ atom Li}$$

The number density of conduction electron in Li is

$$n = \frac{N_{\text{Na}}}{M_{\text{Li}}} = \frac{0.534 \text{ g cm}^{-3} \times 6.022 \times 10^{23} \text{ mol}^{-1}}{7 \text{ g mol}^{-1}}$$

$$\Rightarrow n = 6.459 \times 10^{23} \text{ cm}^{-3}$$

$$\Rightarrow \boxed{n = 45.9 \text{ nm}^{-3}}$$

$$E_F = \left[\frac{(hc)^2}{8mc^2} \right] \left[\frac{3}{\pi} \right]^{2/3} \cdot n^{2/3}$$

on calculating $\left(\frac{3}{\pi} \right)^{2/3} \cdot \frac{1}{8} \approx 0.121$

$$\text{So, } E_F = \frac{0.121 (hc)^2}{m_e c^2} \cdot n^{2/3}$$

Now, $hc = 1240 \text{ eV nm}$

$$\text{So, } E_F = \frac{(0.121) \cdot (1240)^2 \text{ eV}^2 \text{ nm}^2}{9.31 \times 10^{-31} \text{ kg} (3 \times 10^8)^2 \text{ m}^2 \text{ s}^{-2}} \times (45.9 \text{ nm}^{-3})^{2/3}$$

$$\Rightarrow E_F = \frac{(0.121) (1240)^2 \text{ eV}^2}{511 \times 10^3 \text{ eV}} \times (45.9)^{2/3} \text{ e}$$

$$\Rightarrow E_F = 0.364 \times (45.9)^{2/3} \text{ eV}$$

$$\Rightarrow E_F = 0.364 \times 12.5 \text{ eV}$$

$$\Rightarrow \boxed{E_F = 4.55 \text{ eV}}$$

$$1 \text{ g of Na} = \frac{1}{23} \text{ mol atom of Na}$$

$$= \frac{1}{23} \times 6.022 \times 10^{23} \text{ atom Na}$$

The number density of conduction electron in Na is

$$n = \frac{\rho_{\text{Na}}}{M_{\text{Na}}} = \frac{0.968 \text{ g cm}^{-3} \times 6.022 \times 10^{23}}{23 \text{ g mol}^{-1}}$$

$$\Rightarrow n = 0.253 \times 10^{23} \text{ cm}^{-3}$$

$$\Rightarrow \boxed{n = 25.3 \text{ nm}^{-3}}$$

$$\text{Now, } E_F = \left[\frac{(hc)^2}{8m_0 c^2} \right] \left[\frac{3}{\pi} \right]^{2/3} (n)^{1/3}$$

$$\Rightarrow E_F = \frac{(0.121) (hc)^2}{511 \times 10^3 \text{ eV}} \times (25.3 \text{ nm}^{-3})^{1/3}$$

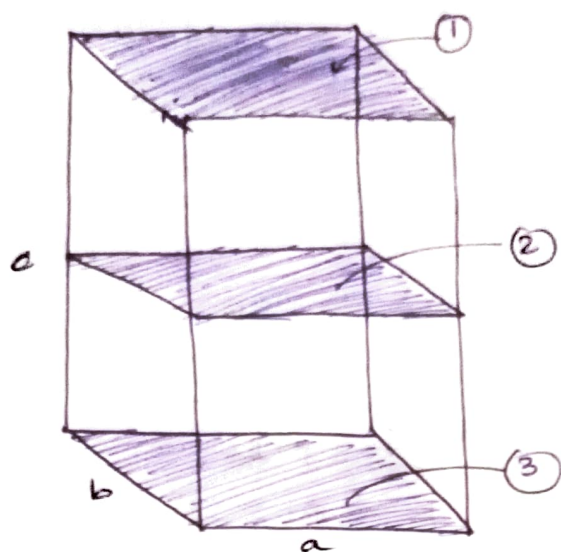
$$\Rightarrow E_F = \frac{(0.121) (1240 \text{ eV}^{1/2} \text{ nm})^2}{511 \times 10^3 \text{ eV}} \times (25.3)^{1/3}$$

$$\Rightarrow E_F = (0.364) \times (8.34) \text{ eV}$$

$$\Rightarrow \boxed{E_F = 3.035 \text{ eV}}$$

\therefore The Fermi energy of Li is 4.55 eV and that of Na is 3.035 eV.

6

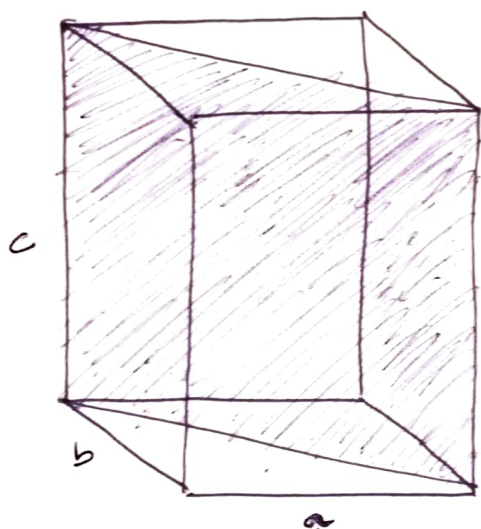


Miller indices:

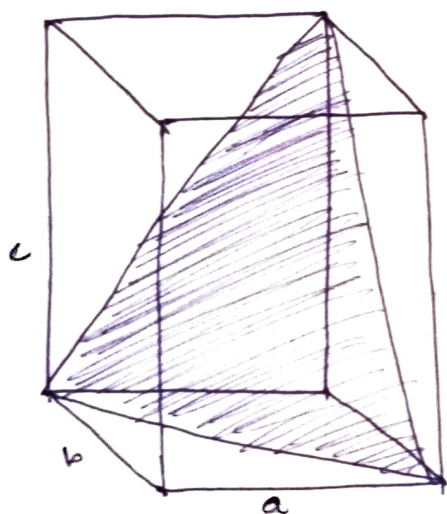
plane 1 = $(0\ 0\ 1)$

plane 2 = $(0\ 0\ 1)$

plane 3 = $(0\ 0\ \bar{1})$



Miller indices
of the plane
 $(1\ 1\ 0)$



Miller indices of
the plane
 $(1\ 1\ 1)$

Q (a) Fermi energy of copper = 7 eV (given)
To find: Probability of finding electrons
at energy = 8 eV at room T

Probability of finding electrons is given
by,

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

$$= \frac{1}{e^{\frac{(8-7)}{0.0259}} + 1} \quad \left(\begin{array}{l} \text{at room temp} \\ kT = 0.0259 \text{ eV} \end{array} \right)$$

$$= \frac{1}{e^{1/0.0259} + 1}$$

$$\text{Now, } e^{1/0.0259} = e^{38.61}$$

$$= 5.046 \times 10^{16} \gg 1$$

$$\text{So, } f(E) = \frac{1}{5.046 \times 10^{16}}$$

$$= \frac{1}{5.046} \times 10^{-16}$$

$$= 0.198 \times 10^{-16}$$

\therefore Probability of finding electron at 8 eV
at room T is 0.198×10^{-16} if
fermi energy is 7 eV .

(b) Energy = 8 eV

Probability = 0.3 = $f(E)$

So, A/Q $0.3 = \frac{1}{e^{(8-7)/kT} + 1}$

$$\Rightarrow \frac{10}{3} = e^{1/kT} + 1$$

$$\Rightarrow e^{1/kT} = \frac{7}{3}$$

$$\Rightarrow \frac{1}{kT} = \ln\left(\frac{7}{3}\right) = 0.85$$

$$\Rightarrow T = \frac{1}{k \times 0.85}$$

$$\Rightarrow \boxed{T = 13675.338 \text{ K}}$$

\therefore If the probability of finding an e^- at 8 eV is 0.3, then temperature should be 13675.338 K.