MA1140, Elementary Linear Algebra

ASSIGNMENT - I

1. $$\begin{bmatrix} 3 & 4 & -1 & 2 \\ 1 & -2 & 3 & 1 \\ 0 & 10 & -10 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ x \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

2. $$\begin{bmatrix} 1 & 1 & 1 \\ -4 & -3 & -2 \\ 3 & 2 & 1 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 + 4R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad R_1 \to R_1 - R_2$$

(Note : This is echleon form of matrix)

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$: Row reduced form of matrix.

Note : $x \Rightarrow x$ is a pivot point.

3. $$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 2 & 5 & -1 & 0 & 1 & 0 \\ -1 & 4 & 0 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{array}$$

Pivot I:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & -2 & -1 & 1 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 + 2R_2 \end{array}$$

Pivot II

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 5 & -2 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -3 & 2 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + 3R_3 \\ R_2 \to R_2 - R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & 4 & 3 \\ 0 & 1 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -3 & 2 & 1 \end{array}\right]$$

$\therefore$ Inverse of matrix $= \begin{bmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & 1 \end{bmatrix}$

4. Gaussian Elimination of matrix:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & x & 0 & 0 & 1 \end{array}\right] \qquad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -6 & -2 & -3 & 1 & 0 \\ 0 & -1 & (x-1) & -1 & 0 & 1 \end{array}\right] \qquad R_2 \to -R_2/6$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/3 & 1/2 & -1/6 & 0 \\ 0 & -1 & (x-1) & -1 & 0 & 1 \end{array}\right] \qquad \begin{array}{l} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 + R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1/3 & 0 \\ 0 & 1 & 1/3 & 1/2 & -1/6 & 0 \\ 0 & 0 & (x-2/3) & -1/2 & -1/6 & 1 \end{array}\right] \qquad R_3 \to R_3/(x-2/3)$$

$\therefore$ For Matrix to have an inverse,

$$x - 2/3 \neq 0 \Rightarrow \boxed{x \neq 2/3}$$

5. $A = \begin{bmatrix} 7 & -2 & 1 \\ 14 & -7 & -3 \\ -7 & 11 & 18 \end{bmatrix}$

Pivot I: $E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$E.A = \begin{bmatrix} 7 & -2 & 1 \\ 0 & -3 & -5 \\ 0 & 9 & 19 \end{bmatrix}$

Pivot II: $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

$(FE)A = \underbrace{\begin{bmatrix} 7 & -2 & 1 \\ 0 & -3 & -5 \\ 0 & 0 & 4 \end{bmatrix}}_{U}$

L^{-1}

$\Rightarrow LU = A$

$\therefore \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 7 & -2 & 1 \\ 0 & -3 & -5 \\ 0 & 0 & 4 \end{bmatrix}}_{U} = \underbrace{\begin{bmatrix} 7 & -2 & 1 \\ 14 & -7 & -3 \\ -7 & 11 & 18 \end{bmatrix}}_{A}$

$7a = 14 \Rightarrow a = 2$

$7b = -7 \Rightarrow b = -1$

$-2b - 3c = 11 \Rightarrow c = 3$

$$\therefore \quad A = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 3 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 7 & -2 & 1 \\ 0 & -3 & -5 \\ 0 & 0 & 4 \end{bmatrix}}_{U}$$

6. $$\begin{bmatrix} 2 & -3 & 1 & 7 \\ 2 & 8 & -4 & 5 \\ 1 & 3 & -3 & 0 \\ -5 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ x \end{bmatrix} = \begin{bmatrix} 14 \\ -1 \\ 4 \\ -19 \end{bmatrix}$$

Augmented Matrix (with row change)

$$\left[\begin{array}{cccc|c} 2 & -3 & 1 & 7 & 14 \\ 2 & 8 & -4 & 5 & -1 \\ 1 & 3 & -3 & 0 & 4 \\ -5 & 2 & 3 & 4 & -19 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - \frac{1}{2} R_1$

$R_4 \to R_4 + \frac{5}{2} R_1$

$$\left[\begin{array}{cccc|c} 2 & -3 & 1 & 7 & 14 \\ 0 & 11 & -5 & -2 & -15 \\ 0 & 4.5 & -3.5 & -3.5 & \\ & & & & \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -3 & 0 & 4 \\ 2 & 8 & -4 & 5 & -1 \\ 2 & -3 & 1 & 7 & 14 \\ -5 & 2 & 3 & 4 & -19 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 2R_1$

$R_4 \to R_4 + 5R_1$

$$\left[\begin{array}{cccc|c} 1 & 3 & -3 & 0 & 4 \\ 0 & 2 & 2 & 5 & -9 \\ 0 & -9 & 7 & 7 & 6 \\ 0 & 17 & -12 & 4 & 1 \end{array}\right] \qquad R_2 \to R_2/2$$

Pivot I:

$$\left[\begin{array}{cccc|c} 1 & 3 & -3 & 0 & 4 \\ 0 & 1 & 1 & 5/2 & -9/2 \\ 0 & -9 & 7 & 17 & 6 \\ 0 & 17 & -12 & 4 & 1 \end{array}\right] \qquad \begin{array}{l} R_1 \to R_1 - 3R_2 \\ R_3 \to R_3 + 9R_2 \\ R_4 \to R_4 - 17R_2 \end{array}$$

$9 + 27/2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -6 & -15/2 & 35/2 \\ 0 & 1 & 1 & 5/2 & -9/2 \\ 0 & 0 & 16 & 59/2 & -69/2 \\ 0 & 0 & -29 & -77/2 & 155/2 \end{array}\right] \qquad R_3 \to R_3/16$$

Pivot II:

$$\left[\begin{array}{cccc|c} 1 & 0 & -6 & -15/2 & 35/2 \\ 0 & 1 & 1 & 5/2 & -9/2 \\ 0 & 0 & 1 & 59/32 & -69/32 \\ 0 & 0 & -29 & -77/2 & 155/2 \end{array}\right] \qquad \begin{array}{l} R_1 \to R_1 + 6R_3 \\ R_2 \to R_2 - R_3 \\ R_4 \to R_4 + 29R_3 \end{array}$$

$-15/2 + 59/32 \times 6$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 57/16 & 73/16 \\ 0 & 1 & 0 & 21/32 & -75/32 \\ 0 & 0 & 1 & 59/32 & -69/32 \\ 0 & 0 & 0 & 479/32 & 479/32 \end{array}\right] \qquad R_4 \to \frac{32}{479} R_4$$

Pivot III:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 57/16 & 73/16 \\ 0 & 1 & 0 & 21/32 & -75/32 \\ 0 & 0 & 1 & 59/32 & -69/32 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \qquad \begin{array}{l} R_1 \to R_1 - \frac{57}{16} R_4 \\ R_2 \to R_2 - \frac{21}{32} R_4 \\ R_3 \to R_3 - \frac{59}{32} R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$$\underbrace{\qquad\qquad\qquad}_{\text{Row reduced Echleon form}}$$

Solution : $\begin{bmatrix} u \\ v \\ w \\ x \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ -4 \\ 1 \end{bmatrix}$ $\left(\begin{array}{l} u = 1 \\ v = -3 \\ w = -4 \\ x = 1 \end{array} \right)$