1. $P_\nu \propto \Omega(E - E_\nu)$

By Taylor-Series Expansion,

$$\ln[\Omega(E-E_\nu)] = \ln[\Omega(E)] - E_\nu \frac{\partial \ln\Omega}{\partial E} + \cdots$$

($E_\nu << E$ so latter terms are neglected)

$$= \ln[\Omega(E)] - E_\nu \times \left(\frac{1}{k_B T}\right)$$

$$\beta = \frac{1}{\Omega}\frac{\partial\Omega}{\partial E} \Rightarrow \int_E \beta\,\partial E = \int_\Omega \frac{\partial\Omega}{\Omega} \Rightarrow \text{Integrating,}$$

$$\therefore \boxed{\ln\Omega \propto \beta E} \Rightarrow \frac{\partial\ln\Omega}{\partial E} = \beta = \frac{1}{k_B T}$$

$$\therefore \Omega(E-E_\nu) \approx e^{\ln[\Omega(E)] - E_\nu\beta}$$

$$= \Omega(E)\, e^{-\beta E_\nu}$$

Since $P_\nu \propto \Omega(E - E_\nu)$,

$$\boxed{P_\nu \propto \Omega(E)\, e^{-\beta E_\nu} \Rightarrow P_\nu \propto e^{-\beta E_\nu}}$$

2. No. of microstates =

(No. of combinations of N rooks placed in a straight line with 2V spaces) × (permutations of all these rooks in a st line of 2V spaces)

$$= {}^{2V}C_N \times {}^{2V}P_N$$

$$= \boxed{\frac{(2V!)^2}{((2V-N)!)^2 \, N!}}$$

3. $Q(N,\beta) = \sum_n e^{-\beta E_n}$

For each E_n, $\exists$ only ONE microstate (due to the given constraint)

$\therefore E_n = n\Delta$

$$Q(N,\beta) = \sum_{n=0}^{N} e^{-\beta n \Delta}$$

This is a geometric series

$$\boxed{Q(N,\beta) = \frac{1 - e^{-\beta\Delta(N+1)}}{1 - e^{-\beta\Delta}}}$$

$$\langle n \rangle = \sum P_n \times n$$

$$= \sum_{n=0}^{N} \frac{e^{-\Delta\beta n}}{Q} \times n$$

$$= \frac{1}{Q} \sum n e^{-\Delta\beta n}$$

↳ Arithmetico-Geometric Series

$$Q\langle n \rangle = e^{-\Delta\beta} + 2e^{-2\Delta\beta} + 3e^{-3\Delta\beta} + \dots + Ne^{-N\Delta\beta}$$
$$e^{-\Delta\beta} Q\langle n \rangle = e^{-2\Delta\beta} + 2e^{-3\Delta\beta} + \dots (N-1)e^{-N\Delta\beta} + Ne^{-(N+1)\Delta}$$

(-)

$$(1-e^{-\Delta\beta}) Q\langle n \rangle = \frac{1 - e^{-\beta\Delta(N+1)}}{1 - e^{-\beta\Delta}} + Ne^{-(N+1)\beta\Delta}$$

$$\langle n \rangle = \frac{1}{Q}\left[\frac{(1 - e^{-\beta\Delta(N+1)})}{(1 - e^{-\beta\Delta})^2} + \frac{Ne^{-(N+1)\beta\Delta}}{1 - e^{-\Delta\beta}}\right]$$

$$\langle n \rangle = \frac{1}{1-e^{-\beta\Delta}} + \frac{N e^{-(N+1)\beta\Delta}}{1-e^{-(N+1)\beta\Delta}}$$

At very large N,

$$N e^{-(N+1)\beta\Delta} \rightarrow 0$$

$$e^{-(N+1)\beta\Delta} \rightarrow 0$$

$$\therefore \boxed{\langle n \rangle = \frac{e^{\beta\Delta}}{1-e^{\beta\Delta}}}$$

$$\boxed{\beta\Delta = 3.894 \times 10^{-2}}$$

$\therefore$ $\langle n \rangle$ = 26.183 links are open (on avgs)

at $T = 298K$ & $\Delta = 0.001eV$

4. For these bosons,

$$\mathcal{Z} = \sum_{n_1, n_2, \ldots n_j} \prod_j e^{-\beta(\varepsilon_j - \mu)(2n_j)}$$

$$= \prod_j \sum_{n_j = 0}^{\infty} e^{-\beta(\varepsilon_j - \mu)(2n_j)}$$

[Since it takes only even number of particles]

(This is a geometric series)

$$= \boxed{\prod_j \frac{1}{1 - e^{-2\beta(\varepsilon_j - \mu)}}}$$

$$\langle n_j \rangle = \frac{\sum_j (2n_j) \times \left(\prod_j e^{-\beta(\varepsilon_j - \mu)(2n_j)}\right)}{\mathcal{Z}}$$

$$= \frac{1}{\mathcal{Z}} \frac{\partial \mathcal{Z}}{\partial(-\beta \varepsilon_j)}$$

$$= \frac{\partial(\ln \mathcal{Z})}{\partial(-\beta \varepsilon_j)}$$

$$\frac{\ln\left(\frac{1}{1 - e^{-2\beta(\varepsilon - \mu)}}\right)}{\partial(-\beta\varepsilon_j)}$$

$$= \boxed{\frac{2}{e^{2\beta(\varepsilon_j - \mu)} - 1}}$$

5.

$$\langle n_1 \rangle = \frac{1}{1 + e^{\beta(\epsilon_1 - \mu)}}$$

$$\boxed{\beta = \frac{1}{K_B T} = \frac{1}{8.617 \times 10^{-5} \times 300} = 38.683 eV^{-1}}$$

$$= \frac{1}{1 + e^{(0.5 - 0.7) \times 38.683}} = \frac{1}{1.0004365533}$$

$$= \boxed{0.9995636372 \text{ particles}}$$

$$\cancel{= \frac{1}{1 + 0.99848}}$$

$$\cancel{= 0.5013 \text{ particles}}$$

$$\langle n_2 \rangle = \frac{1}{1 + e^{(0.3)\beta}} = \frac{1}{1.09634 \times 10^{5}} = \boxed{9.1212 \times 10^{-6} \text{ particles}}$$

6.

$$G = \begin{cases} a\left(1-\frac{T}{T_c}\right)^2 & T < T_c \\ a\left(1-\frac{T}{T_c}\right)^2 + b\left(1-\frac{T}{T_c}\right)^3 & T \geq T_c \end{cases}$$

- G is continuous at $T = T_c$. [$G = 0$ when $T = T_c$]

$$\frac{\partial G}{\partial T} = \begin{cases} -\frac{2a}{T_c}\left(1-\frac{T}{T_c}\right) & T < T_c \\ -\frac{2a}{T_c}\left(1-\frac{T}{T_c}\right) - \frac{3b}{T_c}\left(1-\frac{T}{T_c}\right)^2 & T \geq T_c \end{cases}$$

- $\frac{\partial G}{\partial T}$ is continuous at $T = T_c$

$$\frac{\partial^2 G}{\partial T^2} = \begin{cases} \frac{2a}{T_c^2} & T < T_c \\ \frac{2a}{T_c^2} + \frac{6b}{T_c^2}\left(1-\frac{T}{T_c}\right) & T \geq T_c \end{cases}$$

- $\frac{\partial^2 G}{\partial T^2}$ is continuous at $T = T_c$

$$\frac{\partial^3 G}{\partial T^3} = \begin{cases} 0 & T < T_c \\ -\frac{6b}{T_c^3} & T \geq T_c \end{cases}$$

$\frac{\partial^3 G}{\partial T^3}$ is DISCONTINUOUS at $T = T_c$.

$\therefore$ $\boxed{\text{The order of phase transition} = 3}$

7. (i) Energy gap $= \dfrac{hc}{\lambda} = \dfrac{1242}{632.8} = \boxed{1.9627 \text{ eV}}$

(ii) We know ~~in spon~~

$$A = \frac{1}{\text{lifetime}} = 10^{10}\ s^{-1}$$

Also,

$$\frac{A}{B} = \frac{2h\nu^3 n_0^3}{c^3} = \frac{2hn_0^3}{\lambda^3}$$

$$= \frac{2\times 6.626\times10^{-34}\times 1}{10^{-30}\times(6328)^3}$$

~~$= 1.3252\times10^{-4}$~~

~~$\boxed{1.3252\times10^{-3}}$ Js/m³~~

$$= \boxed{5.229\times10^{-15}\ \text{kg/ms}}$$

~~$\therefore B = \dfrac{A}{1.3252\times10^{-3}} = \boxed{7.546\times10^{12}}$~~

~~$= \boxed{7.546\times10^{12}\ \text{m}^3/\text{Js}^2}$~~

$$\therefore B = \frac{10^{10}}{5.229\times10^{-15}} = 1.912\times10^{24}\ \text{m/kg}$$

$\dfrac{1.3252\times10^{-24}}{6.328^3\times10^{9}}$

0.05229×10^{-13}

5.229×10^{-15}

$\dfrac{kgm^2}{s^2}\cdot\dfrac{s}{m^3}$

(iii) No. of resonant frequencies $= \dfrac{2d}{\lambda}$

$$= \frac{2\times1}{6328\times10^{-10}}$$

$$= \frac{10^{10}}{3164}$$

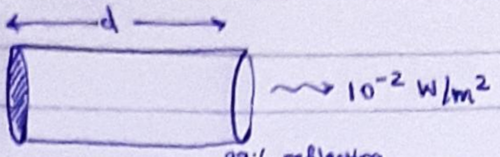
$\cong$ ~~3.160~~ $\boxed{3160566 \text{ frequencies}}$

$\dfrac{E\times s}{m^3} = \dfrac{kg\,m^2/s}{m^3}$

$\dfrac{Js}{m^3}$

$\dfrac{m^3}{\frac{kgm^2}{s^2}\cdot s}$

8.



Power = $I \times A$ [Consumed Power]

$= 10^{-2} \times 10^{-4}$

$= 10^{-6}$ W = Energy / Time

$P = \frac{nE_p}{t} = \frac{nE_p}{(d/c)} = \frac{nE_pc}{d}$ [E_p = energy of photon]

$\therefore n = \frac{Pd}{E_pc} = \frac{10^{-4} \times 10^{-1}}{10^{-1} \times 1.6 \times 10^{-19} \times 3 \times 10^{8}}$

$= \frac{10^{7} \times 0.99}{4.8} = \boxed{2.065 \times 10^{6} \text{ photons}}$

→ Power consumed by the laser

$= 99 \times$ Power Output

$= 99 \times 10^{-6}$

$= \boxed{9.9 \times 10^{-5} \text{ W}}$

→ Energy of beam = Intensity × Area × Time

$= 10^{-2} \times 10^{-4} \times \frac{d}{c}$

$= 10^{-6} \times \frac{10^{-1}}{3 \times 10^{8}} = 3.33 \times 10^{-16}$ J

∴ Energy of beam inside cavity $(E_B) = \frac{3.33 \times 10^{-16}}{1\%}$

$= 3.33 \times 10^{-14}$ J $= \frac{3.33 \times 10^{-14}}{1.6 \times 10^{-19}}$ eV $= \boxed{2.08 \times 10^{5} \text{ eV}}$

∴ No. of photons $= \frac{E_B}{E_p \text{(Energy of 1 photon)}} = \frac{2.08 \times 10^{5}}{0.1} = \boxed{2.08 \times 10^{6} \text{ photons}}$