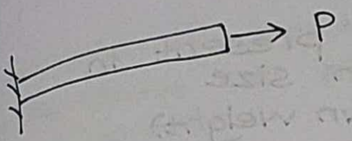
cture 1

August 1, 2022

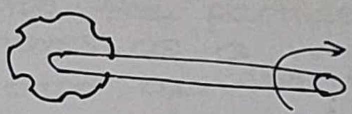


Bar

Beam

When the beam is under torsion → shaft

Ex. Axle of the car



Buckling

Column

CA 10%

A 10%

Tests 4×20%

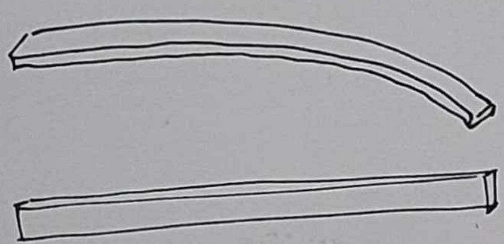
Lecture 2 August 3^{rd}, 2022

Review

- Structures: man-made and those present in nature, are limited in size (cannot support their own weight)

- History: Galileo
  Leonardo da Vinci → studied beams, expt. on strengths of beams, bars, strings

- Intuition:



Bends more in one orientation than the other

Behaviour changes acc. to orientation of the cross-sectional area (resistance to bending)

dimension in the plane of bending = thickness = d

(1) Beam bending $\propto d^{\alpha} w^{\beta}$ $\alpha > \beta$ (thickness affects much more than the width)

(2) Bar L_1 & L_2

In which case do we need to apply more force to break the bar?

→ same F in both cases, independent of length (same stress - strain curve)

More elongation?

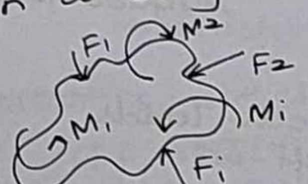
→ In L_2.

Lecture - 3 August 4^{th}, 2022

Review

- Dependence of bending on width and depth
- Independence of strength on force applied, but elongation is different
- R. Hooke's law
- Mariotte, Bernoulli's, Euler
- Difference between displacement and deformation:
  displacement: no relative motion b/w the particles of the body, the body moves as a whole.
  deformation: there is relative motion b/w the particles of the body. Internal changes in body to maintain the eq^m.

- When a body is not moving, it is under static eq^m.

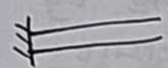


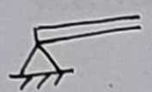
Equilibrium equations:
$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\cancel{\Sigma F_z = 0}$
$\Sigma M_z = 0$.

In nature, bodies are moving while deforming. Not in eq^m.

$y \uparrow \rightarrow x$

- Boundary conditions:

Fixed support  R_x, R_y, M_z

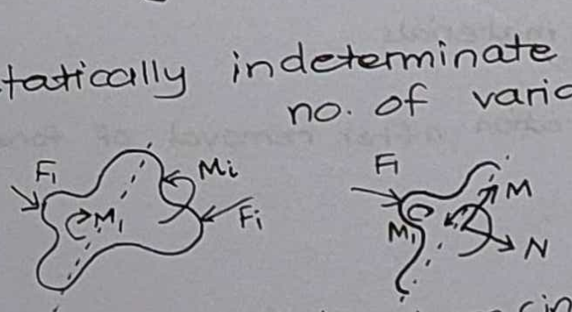
Pinned support  R_x, R_y, Free to rotate

Roller support  free to move in x-dir., R_y, Free to rotate

- Pressure - intensity distribution of force on a body.
  ↳ stress is similar thing, happening inside the body.

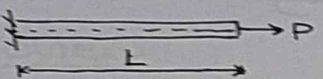
Statically determinate: we can get all the reaction forces acting on the body with using eq^m eq^ns.

Statically indeterminate: -- cannot -- no. of variables > no. of eq^m eq^ns we used earlier.



How to calculate stress (internal):
Think about external F's and M's. Cut body, take an internal reaction force & a reaction moment.
(The cut DOESN'T CHANGE the stress at a point)

Bar (axial forces acting)   Assume the wt. is negligible.

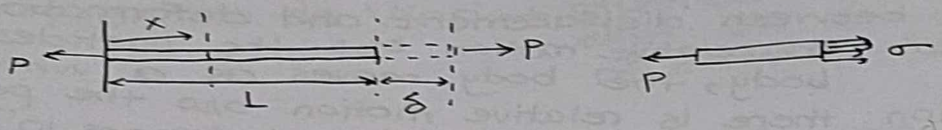


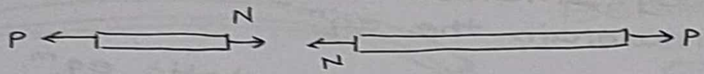
[P goes thru the COM]
⇒ no moments

Cross-section ⇒ area ⊥ axis

$R_y = 0$
$R_x = -P$
$M_z = 0$

Area is uniform ⇒ prismatic bar





Here $N = P$. (but not necessary in other cases)

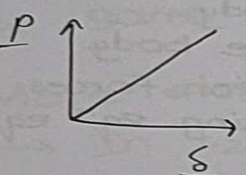
August 10^{th}, 2022

Lecture-4 (Dr. Gangadharan)

- Why or how the structures carry load?
  ↳ mech. loads
  ↳ self-wt

- Hooke: English scientist - 1676
  If a material structure is subjected to mechanical force/self-wt. and this is done by the push back force offered by the structure.
  internal
  Structure/material experiences some deflection/deformation.

$P = K\delta$, K = stiffness of the structure
$(\equiv F = kx)$



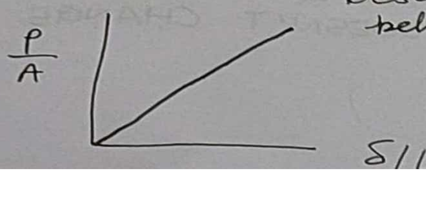
\# elasticity
↳ for some materials only
(linear elastic, ex.- alloys, metals, concrete)

Non-linear elastic - Rubber, other soft materials
Plastic - Clay
Doesn't come back to original configuration after removal of force.
Permanent deformation.

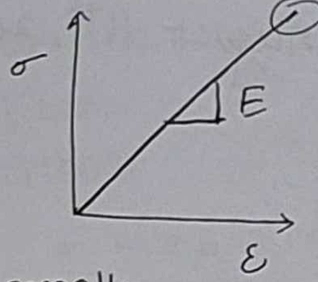
Plastic behaviour leads to failure.
Deflection depends on - material
- size and geometry of the object

Hooke designed universal joint, invented lamps

- Cauchy - 1822
\# look at internal stru. of material.



Describes mech behaviour of a material
Stress vs. stra



1 → ultimate tensile strength (N/m^2)

$\sigma = E\varepsilon$ — Hooke's law

σ: N/m^2 or Pa

E → Young's mod. of elasticity (GPa)

Normally, we use N/mm^2 or MPa.

E(Steel) = 200GPa, E(Al) = 70GPa.

In automobiles, steel is used for making frame.

For other parts, Al is used as it has less density.

The discussion above works only for 1D structures. ($L >> A$)

- Diff. b/w strength and stiffness:
  - strength ↓ max. stress the material can take (after that pt., there is fracture)
  - stiffness → related to flexibility, spring-like nature of the material

soft robotics, electronics components need flexible material.

steel - stiff, strong
jelly - flexible, weak
rubber - flexible, strong
biscuit - stiff, weak

We talk about Young's mod. & tensile strength of the material

$\tau = G\gamma$

τ: shear stress

G → Shear modulus → imp. while designing shafts (torsions)

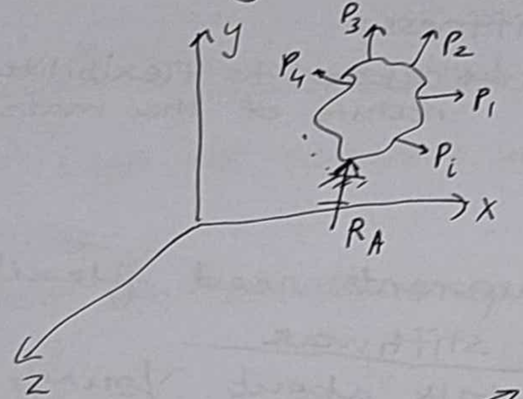
Lecture 5 August 11^{th}, 2022

Stress $\frac{P}{A} = \sigma$ $\sigma = E\varepsilon$

Strain $\frac{\delta l}{L} = \varepsilon$

Assumptions: Cross-sectional area = const.
Force is acting in one dir. only
The body is 1D

Defining coord^n system to define a body.

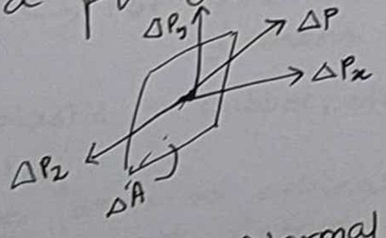


Loads acting on a body → gravity → acts on all pts
→ reaction forces

Types of loads → concentrated
→ pressure loads → acts on a surface
→ inertial loads [F = ma] → acts on all pts of the body
→ temperature loads (thermal) → gen. σ

Assumption - body under equilibrium
#method of sections to det. σ [introducing a cut ⇒ get internal forces]

Stress at a pt. on the cross-section:



[Resolve the force]

$$\sigma_{xx} = \lim_{\Delta A \to 0} \frac{\Delta P_x}{\Delta A}$$ ← Normal stress [bcz force ⊥ cross-sectional area]

(x → area normal; x → force dir.)

$$\sigma_{xy} = \lim_{\Delta A \to 0} \frac{\Delta P_y}{\Delta A}$$

$$\sigma_{xz} = \lim_{\Delta A \to 0} \frac{\Delta P_z}{\Delta A}$$

} Shear stresses

To define stress at a pt., we need 9 components
Vector - 1^{st} order tensor - 3 components in Cartesian c.s.
Stress - 2^{nd} order tensor - 3^2 comp.

$\underline{v} = v_1 e_x + v_2 e_y + v_3 e_z$ (e_1, e_2, e_3) - basis vectors

$$\boxed{T_i = \sigma_{ij} n_j}$$

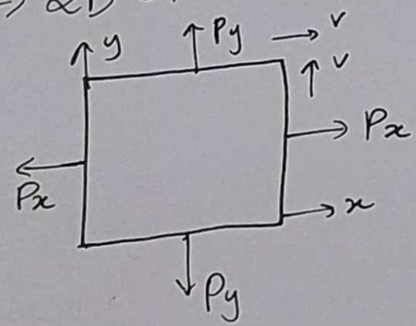
stress vector

normal vector

Ex.- $\sigma_{ij}\underbrace{\begin{bmatrix}1\\0\\0\end{bmatrix}}_{\text{normal in X-dir}}$ gives $\boxed{T^x = \sigma_{xx} e_x + \sigma_{xy} e_y + \sigma_{xz} e_z}$

stress tensor X normal to a plane = stress acting on that plane

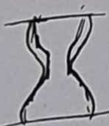
- Consider a thin planar sheet and assume that loads are acting coplanar to the sheet [ignore Poisson's problem]
$\Rightarrow$ 2D stress state



2D: $\begin{bmatrix}\sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy}\end{bmatrix}$

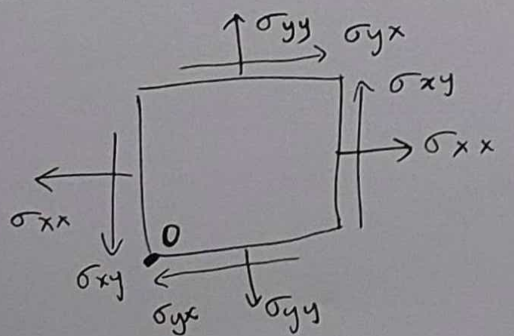
1D: $[\sigma_{xx}]$

Yield strength
Ultimate tensile strength → necking



Fibre - molecules arranged along single di → more strength
[spider silk is 5 times stronger than steel]

Stress tensor matrix - each column represents a plane
each row represents a line



Moment eq^m abt pt. 0:

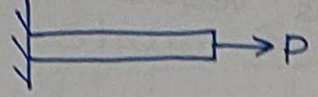
$$M_0 = \underbrace{\sigma_{yx}}_{(\sigma}\ \underbrace{dx.dy}_{A = F)} - \sigma_{xy}\,dx\,dy = 0$$

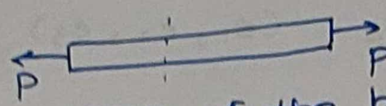
$\Rightarrow \sigma_{yx} = \sigma_{xy}$

Shear stresses are complimentary!

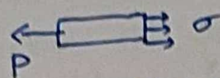
Lecture 6 August 17th, 2022

Uniaxial stress and strain





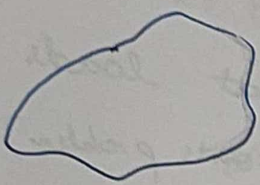
FBD of the bar



even though it is deforming, this component has to be in equilibrium.

$$\int_A \sigma\, dA = P \quad \text{[Given that } \sigma \text{ is uniform]}$$

$$\Rightarrow \sigma = P/A.$$



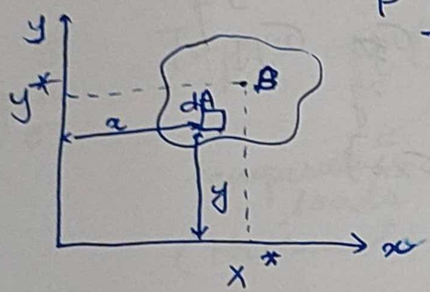
arbitrary shape

For the stress to be uniform in a body with arbitrary shape:

(1) • The forces should be acting at the centroid.

pt. B = centroid $B(x^*, y^*)$

P is coming out of the plane at pt. B.



Then, $M_x = P\cdot y^*$ and $M_y = -Px^*$

$= \int y\sigma\, dA$ $\qquad = -\int x\sigma\, dA$

We have

$$Py^* = \int y\sigma\, dA$$

$$\therefore Py^* = \sigma\int y\, dA \quad \text{[Assuming that } \sigma \text{ is uniform]}$$

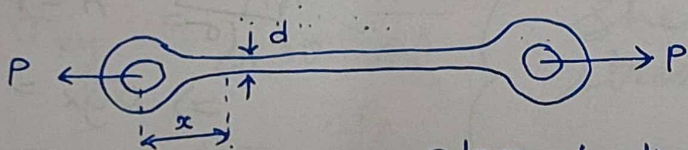
$$\therefore Py^* = \frac{P}{A}\int y\, dA$$

$$\Rightarrow y^* = \frac{\int y\, dA}{A} \quad \text{[Eq}^n \text{ for centroid]}$$

Sim., $$x^* = \frac{\int x\, dA}{A}$$

- Point load - like singularity, stress conc. is very high at that pt.

(2) • C.S. is sufficiently distant from stress concentrations.



$x > d$ ← largest dimension of the body

Stress becomes uniform as we go away from the point of application of load.

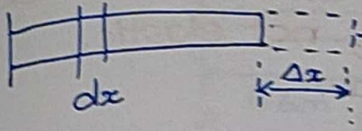
- Normal stress:

$$\sigma = \frac{P}{A}$$

Units: Pa or N/m^2
For engineering purposes, [MPa or N/mm^2] is usually used.

If P is stretching the bar → tensile stress (>0)
If P is contracting the bar → compressive stress (<0)
sign convention

- Nature of elongation at diff points along the bar:



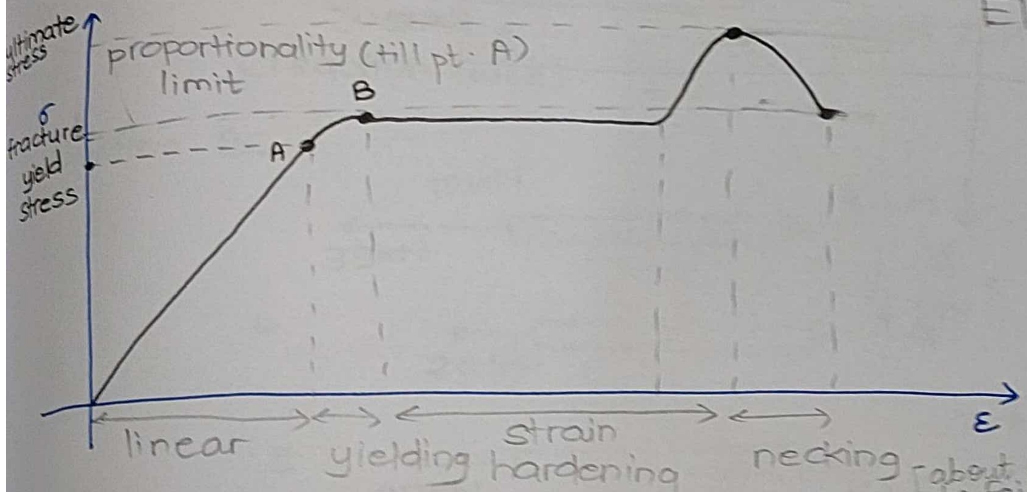
For the elongation to be same at every point:
~~① A should be uniform~~
~~②~~
③ Homogeneous material
[as deformation is property of material]

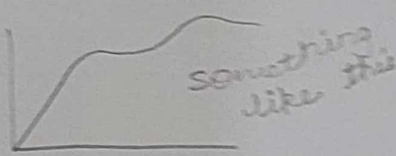
$\varepsilon = \delta / L$

- Tensile test - done on a Universal Test Machine (UTM)
↳ extentiometer

☐ Stress - strain curve:

Linear till pt. A
Elastic till pt. B



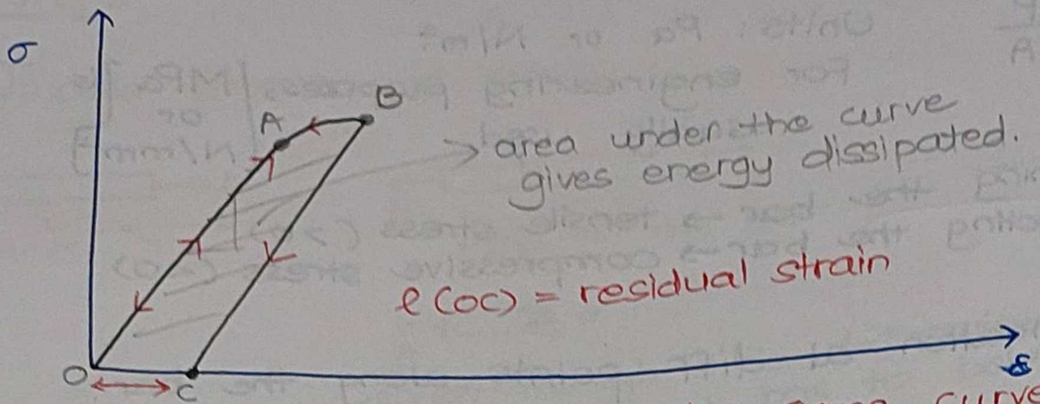


There's a dip in the curve bcz as the bar deforms, its area changes as well.

Things may not be linear, but elastic.

Elasticity : The property by which the material regains its original shape/form when the applied load is removed.

Area with which we started = Nominal area
true stress?



→ area under the curve gives energy dissipated.

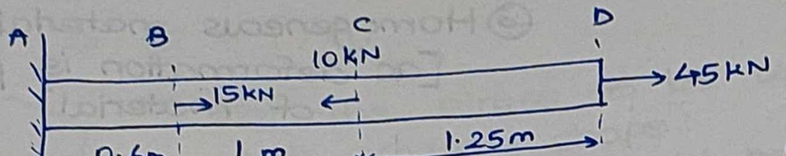
$\ell(OC)$ = residual strain

DAB → elastic

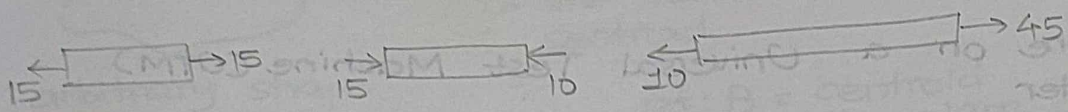
Goes along and returns along the same curve
If it returns along some curve like BC, not elastic.

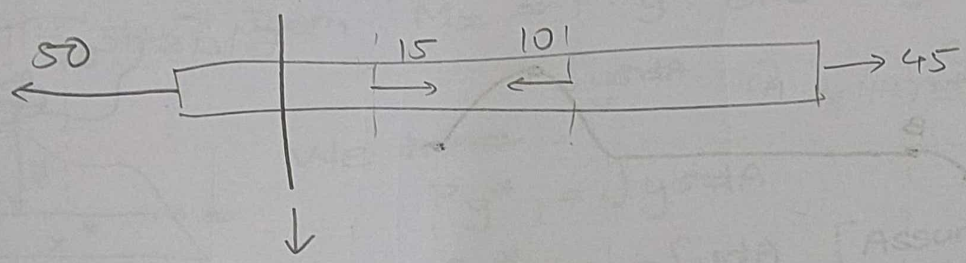
In the linear region, $\boxed{\sigma = E\epsilon}$ Hooke's law.

Q:



E = 200 GPa
C.S. = 500 mm^2





$$\Delta = \frac{P_i L_i}{AE}$$

0.3 mm

$$\Delta_1 = \frac{(50kN)(0.6)}{(500mm^2)(200GPa)} = 0.3 mm$$

$$\Delta_2 = \frac{35kN \times 1}{AE} = 0.35 mm$$

Write 0.3

$$\Delta_3 = \frac{45 \times 1.25}{AE} = 0.56 mm$$

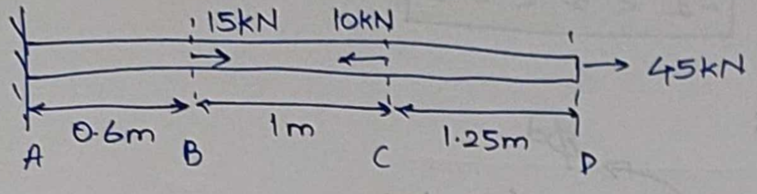
Lecture - 7

August 17th

Summary
1. Normal σ and ε
2. Assumptions for $\sigma = P/A$ and $\varepsilon = \delta/L$
   ↳ stress const.
3. Stress-strain diagram
4. Linear elasticity / Hooke's law - Modulus of Elasticity

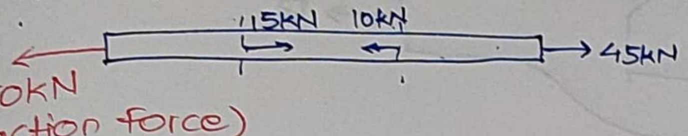
Class assignment:

Q:



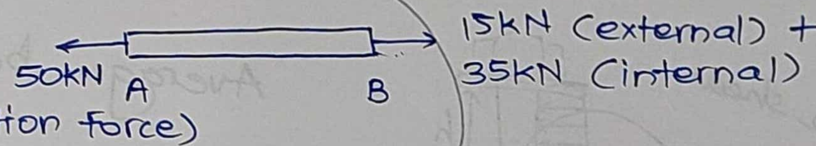
$E = 200GPa$
$C.S. = 500\ mm^2$

① Calculate the reaction force.
eqm eqns.

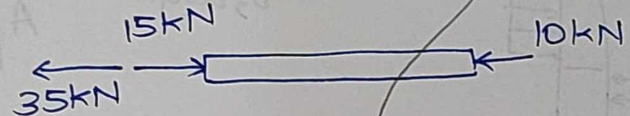


Now this reaction force should get balanced in each section.

Section AB



Section BC



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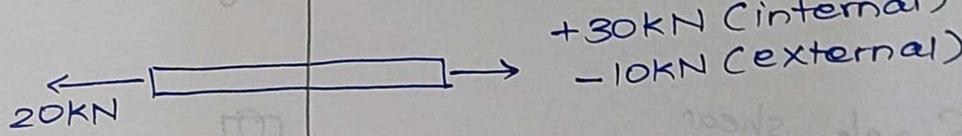
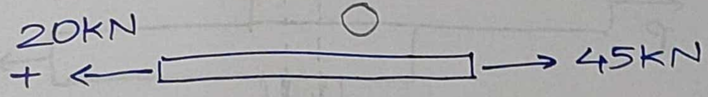
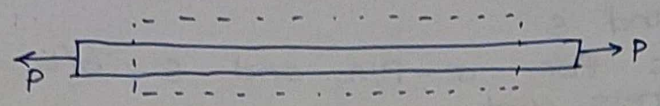


Section CD



All these elongations are in series.
How to find them? - Hooke's law

☐ Poisson's ratio :



$$\boxed{\nu = -\frac{\varepsilon'}{\varepsilon} = -\frac{\Delta d/d}{\Delta l/l}}$$

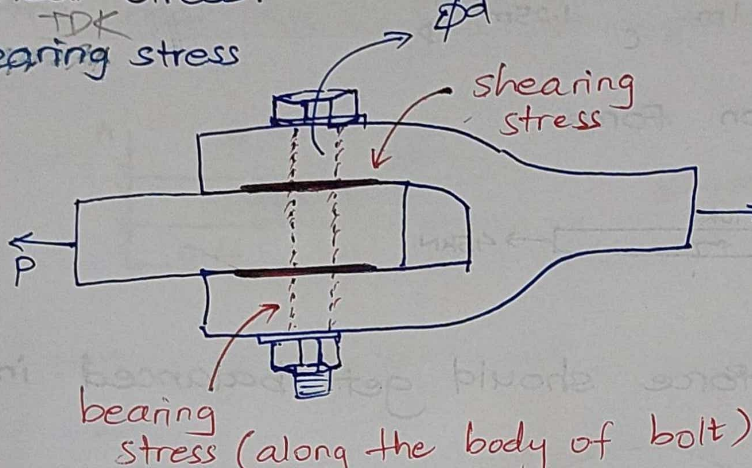
There are certain materials w/ -ve ν.

$$\boxed{-1 < \nu < 0.5}$$

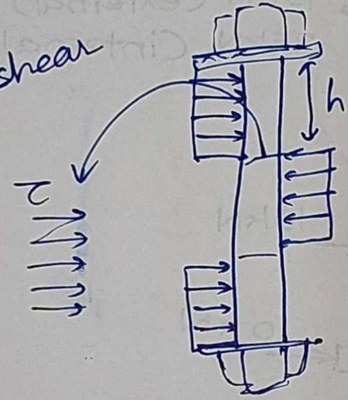
☐ Shear stress:

TDK

~~bearing~~ bearing stress



Double shear

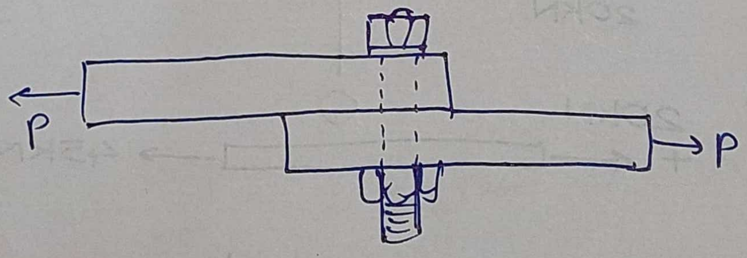


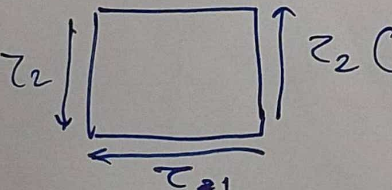
Average bearing stress

$$\sigma_{b,avg.} = \frac{P/2}{A_{projected}\ (hd)}$$

$$\tau_{avg.} = \tau_{average} = \frac{V}{A}$$

Single shear





(to balance the entire element)

$\tau_1 = \tau_2$

(to balance the $\rightarrow \tau_1$)

Lecture 8

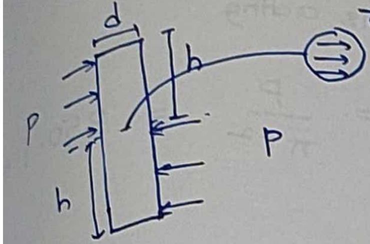
Summary:

August 24th, 22

Axial loading problem

Poisson's ratio – is const. till the elastic limit

τ_{avg} & σ_b

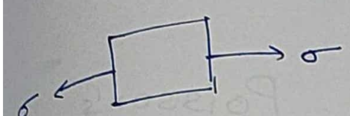


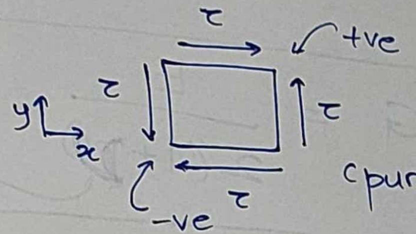
(shear)

$$\tau_{avg} = \frac{4P}{\pi d^2}$$

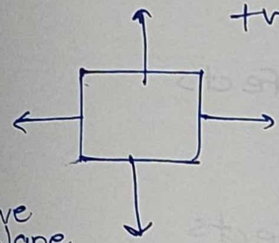
$$\sigma_{b,avg} = \frac{P}{hd}$$

hd → projected area



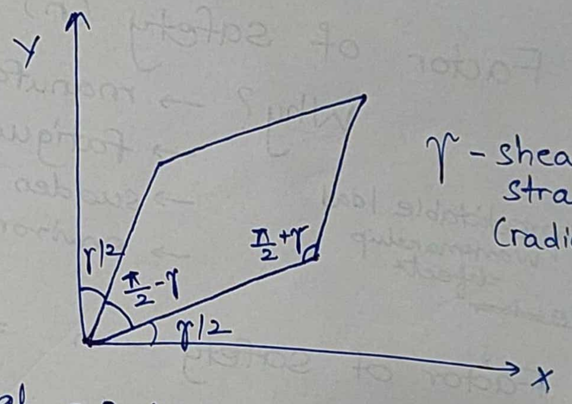


(pure shear)



+ve plane direction

-ve plane direction



γ - shear strain (radians)

$$\boxed{\tau = G\gamma}$$

→ modulus of rigidity

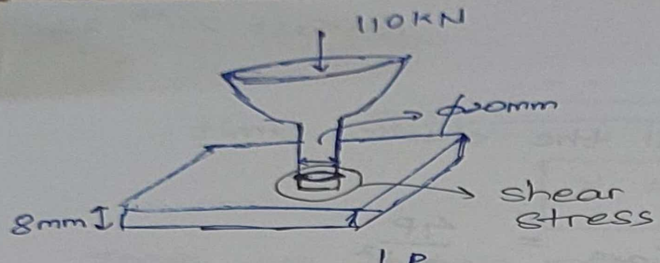
shear modulus of elasticity

normal plane direction { 2 +ve, 2 –ve } & angle ↓ b/w them

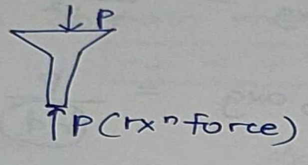
shear strain – +ve

b/w two positive face (negative) → +ve shear strain

basically if angle is reducing b/w them



$$\underbrace{\tau}_{\text{shear stress}} = \frac{P}{\underbrace{\underbrace{\pi d}_{\text{circumference}} \times \underbrace{t}_{\text{thickness}}}_{\text{Area where shear is acting}}} = 218.8\ \text{MPa}$$



$$\underbrace{\sigma}_{\text{compressive stress in punch}} = \frac{P}{\pi r^2} = 350.1\ \text{MPa}$$

$$\boxed{G = \frac{E}{2(1+\underbrace{\nu}_{\text{Poisson's ratio}})}}$$

- Factor of safety (n):

Why? → manufacturing defects
→ fatigue
→ sudden loading
→ environmental effects

unpredictable load
workmanship defects
~~[illegible]~~

$$\text{Factor of safety} = n = \frac{\text{Actual strength}}{\text{Required strength}}$$

$$\text{Margin of safety} = f = n - 1$$

Analysis	Design
What are the loads?	Find physical dimensions

Lecture 9 August 24^{th}, 2022

- Summary
  1. Pure shear (τ), ($\pm$) convention
  2. Shear strain (γ), -"- for γ, G - modulus of rigidity
  3. Punch example
  4. Factor of safety (n), f
  5. Analysis and design

If the material is brittle, we've to use ultimate σ while deciding factor of safety.

Allowable stress; ~~stra~~ [valid for all types of stress]

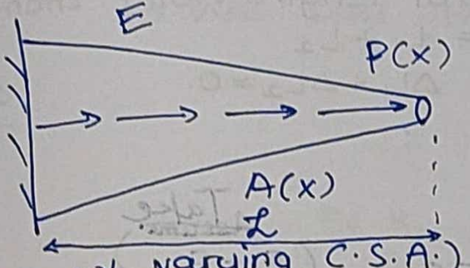
$$\sigma_{allowable} = \frac{\sigma_Y}{n_1} \quad \text{(yield stress)}$$

$$\tau_{allowable} = \frac{\tau_y}{n_2}$$

$$\frac{\sigma_u}{n_3}, \quad \frac{\tau_u}{n_4}$$

$P_{allowable} = \sigma_{allowable} \times A$
(allowable load)

Bar extension problem:

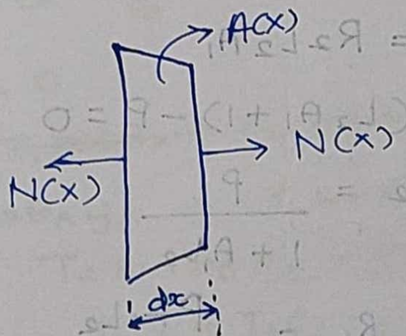


(Bar w/ varying C.S.A.)

Total elongation of the rod? (δ)

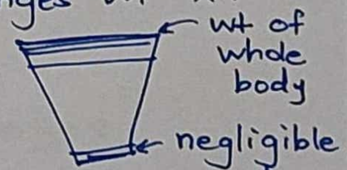
There are infinite small sections we need to consider.

Section w/ length = dx.
This section deforms by $d\delta$.



N(x) = reaction force
rx^n F changes w/ x.

Ex.-

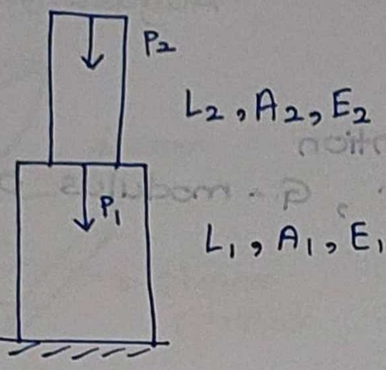


$$d\delta = \frac{N(x) \cdot dx}{E \cdot A(x)}$$

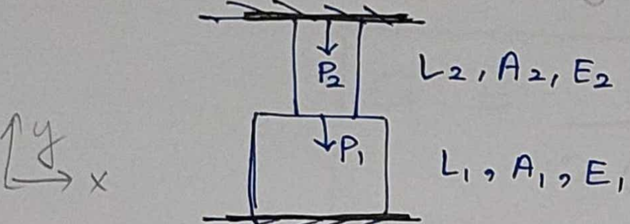
P(x) acting on the length results in N(x).

$$\delta = \int_0^L \frac{N(x)\,dx}{E A(x)}$$

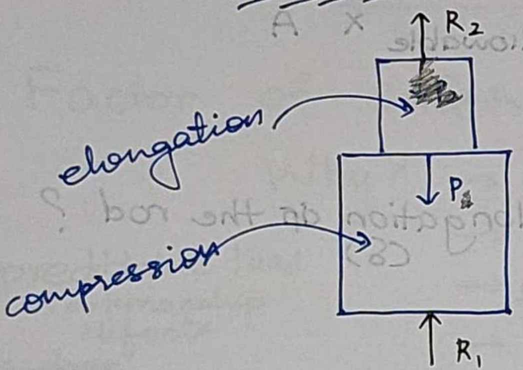
Q.



We need not worry about reaction force at the interface. It'll eventually be balanced by rxn at the joint / fixed part.



Here, there'll be two reaction forces.



Eqm eqn:

$$R_1 + R_2 - P = 0$$

At this pt., prob. is statically indeterminate.

As the body is fixed from both sides, total length won't change.

$$L = L_1 + L_2$$

and $\Delta L_1 + \Delta L_2 = 0$.

Compatibility equation

$$\left\{ -\frac{R_1 L_1}{A_1 E_1} + \frac{R_2 L_2}{A_2 E_2} = 0 \right. \quad (E_1 = E_2,\ A_1 = A_2)$$

Take ~~Assume~~

$$R_1 = R_2 L_2 A_1$$

$$R_2 (L_2 A_1 + 1) - P = 0$$

$$R_2 = \frac{P}{1 + A_1 L_2}$$

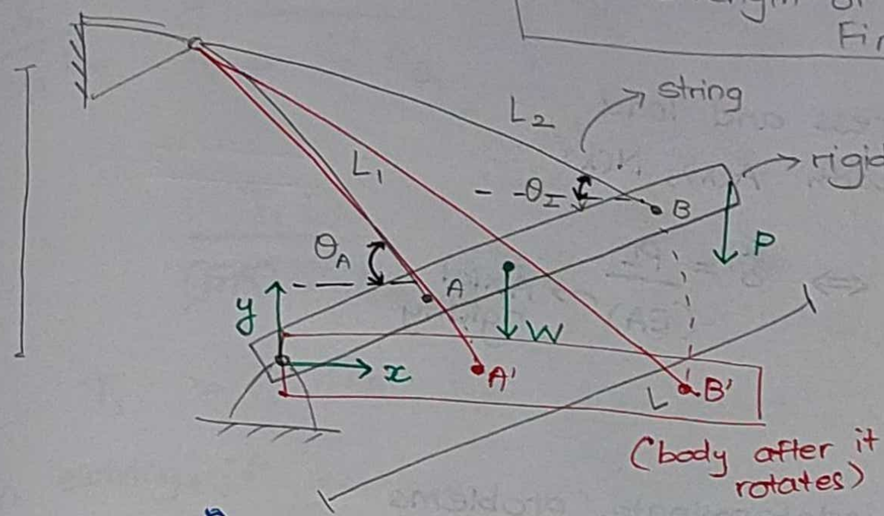
and $R_1 = \dfrac{P A_1 L_2}{1 + A_1 L_2}$

$$\boxed{R_1 = \frac{P L_2}{L} \quad \& \quad R_2 = \frac{P L_1}{L}}$$

$\therefore$

Max. strength of strings is given. Find P.

Q/



A (x_A, y_A)

B (x_B, y_B)

W (x_W, y_W)

P (x_P, y_P)

4 unknowns - 2 rxn forces, 2 tension forces

3 eqm eqns

statically indeterminate.

$\Rightarrow$ Use compatibility eqn.

The given body is rigid $\Rightarrow$ won't deform

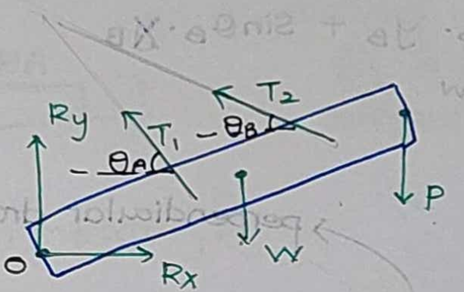
$\Rightarrow$ dist. b/w A and B (or any two points) won't change.

Or, the body is rotating along the joint.

Points A and B will lie on concentric circles

↳ We can find relation b/w θ_A and θ_B.

FBD



Eqm eqns: $\Sigma M_o = 0$

$\cancel{T_1 \cos\theta_A + T_2 \cos\theta_B}$

$$T_1 \cos\theta_A y_A + T_1 \sin\theta_A x_A + \underbrace{T_2 \cos\theta_B y_B}_{C_2} +$$

$$T_2 \sin\theta_B x_B - \underbrace{W x_W}_{C_3} - P \underbrace{x_P}_{C_4} = 0$$

$$T_1 C_1 + T_2 C_2 = C_3 + P C_4$$

$\delta_1 = L_1' - L_1$

$\delta_2 = L_2' - l_2$

$$\frac{\Delta_A}{l_A} = \frac{\Delta_B}{l_B} \quad \text{similar triangles}$$

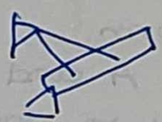
$$\Delta_A = \frac{l_A}{l_B} \Delta_B$$

Lecture 10 August 25^{th}, 2022

- Summary
  1. Allowable stress and load
  2. Elongation with $A(x)$, $N(x)$

$$x = \frac{F}{k} \iff \delta = \frac{PL}{(EA)} \to \text{Axial rigidity}$$

$$k = \frac{EA}{L}$$

  3. Statically indeterminate problems
     ↳ Additional geometric eq^n needed - which assures the compatibility eq^n
  4. Compatibility eq^n
  5. 
  6. 

Solution:

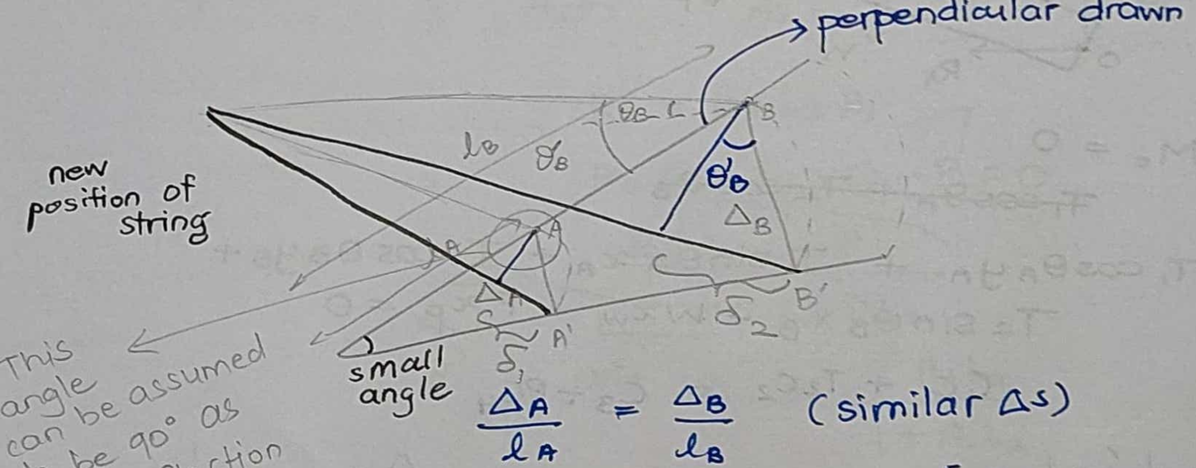
Eq^m eq^n - $T_1C_1 + T_2C_2 = C_3 + PC_4$

$$C_1 = \cos\theta_A \cdot y_A + \sin\theta_A \cdot x_A$$

$$C_2 = \cos\theta_B \cdot y_B + \sin\theta_B \cdot \cancel{x}_B$$

$$C_3 = Wx_w$$

$$C_4 = x_p$$



$$\frac{\Delta_A}{l_A} = \frac{\Delta_B}{l_B} \quad \text{(similar } \Delta\text{s)}$$

$$\delta_2 = \Delta_B \sin\theta'_B$$

[Using small angle assumption]

$$\delta_1 = \Delta_A \sin\theta'_A$$

$$\frac{\delta_1}{\sin\theta_A' \, l_A} = \frac{\delta_2}{\sin\theta_B' \cdot l_B} \qquad \delta_1 = \frac{T_1 L_1}{EA}$$

$$\Downarrow$$

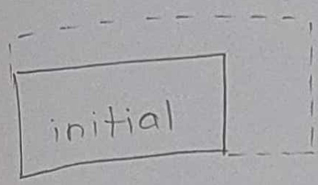
$$\frac{T_1 L_1}{\cancel{(EA)} \sin\theta_A' \cdot l_A} = \frac{T_2 L_2}{\cancel{(EA)} \cdot \sin\theta_B' \cdot l_B}$$

$$T_1 = T_2 \cdot \frac{L_2}{L_1} \cdot \frac{l_A}{l_B} \cdot \frac{\sin\theta_A'}{\sin\theta_B'}$$

Upon solving, if you get $T_1 > T_2$, write $T_2 = C_5 T_1$

and $T_1 C_1 + C_2 C_5 T_1 = C_3 + \underbrace{P}_{\text{the only unknown.}} C_4$

☐ THERMAL EFFECTS

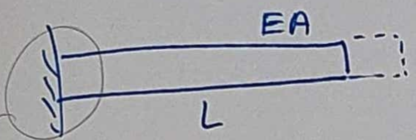


$$\boxed{\text{Thermal strain} = \varepsilon_T = \alpha \Delta T}$$

ε_T → unitless

α → coeff. of thermal expansion, units: K^{-1} or $°C^{-1}$!

Thermal stress $= E\alpha\Delta T$

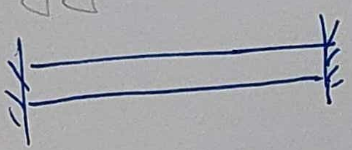


(expands on heating)

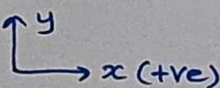
There's some expansion on this side too, but it's negligible.

$\varepsilon_T = \alpha\Delta T$

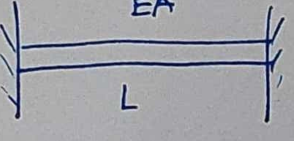
stress $= 0$ (Freely expanding, no opposing force)



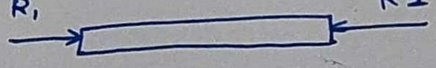
stress $\neq 0$ [there's internal rxn force, resistance to deformation]



Q.



ΔT



Eqm eqn : $R_1 - R_2 = 0$

statically indeterminate (at this pt.) {we can't find R_1 & R_2}

$R_1 = R_2$.

Heating → bar tries to expand

support → opposes expansion

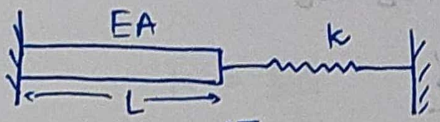
Compatibility eqn : $\Delta L = 0$ $\quad \underbrace{\delta_1}_{\text{due to expansion by heat}^n} + \underbrace{\delta_2}_{\text{suppression by rxn forces}} = 0$

$$\delta_2 = -\frac{R_1 L}{EA} \quad (\text{-ve as it is for compression})$$

$$\delta_1 = L\alpha\Delta T \quad (\text{+ve, thermal expansion})$$

$$\therefore \quad R_1 = \alpha \cdot \Delta T \cdot EA$$

Q.



Assume that the spring isn't affected by thermal effects.

① Figure out how'd the system move if there was no support.

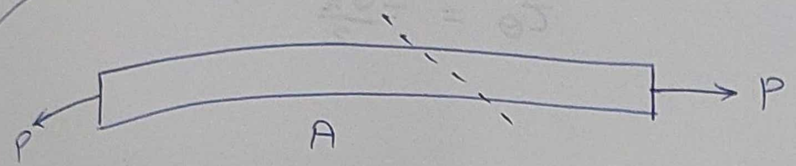
→ Spring will move along w/ the bar, no rxn force.

Lecture 11 August 29th, 2022

Summary

SIP = Statically Indeterminate Problem

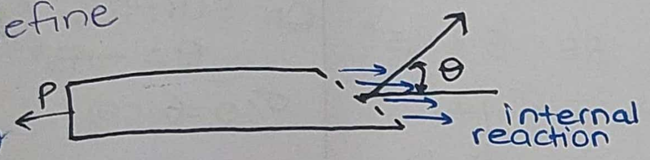
1.] SIP with one bar and two strings
2] Thermal effects, ε_T
3] SIP w/ ε_T (expanding bar)
4] SIP w/ an expanding bar and a spring



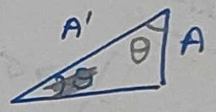
Shear + normal σ acting

Depending upon the cross-section, the stress values are changing. (Normal and shear stress exchange values, the net stress in the body stays the same).

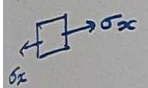
To find/define stress, magnitude + 2 dir. are needed. #tensor

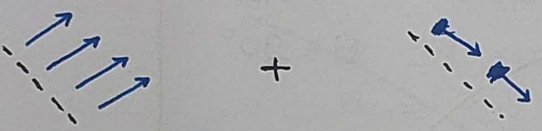


$\sigma_x = P/A$



$A' = A/\cos\theta$



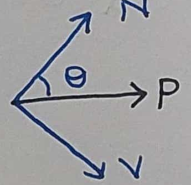


normal (N) tangential (V)

(acting in -ve dir.)

While deciding on the sign convention, assume $\theta \to 0$

$$\sigma_\theta = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta$$

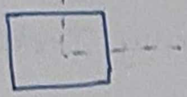


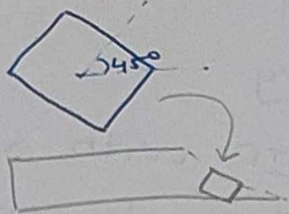
$N = P\cos\theta$
$V = P\sin\theta$

$$\tau_\theta = \frac{-P\sin\theta}{A/\cos\theta} = \frac{-P}{A}\sin\theta\cos\theta$$

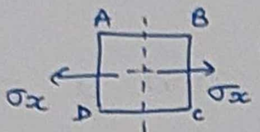
$$\boxed{\begin{aligned}\sigma_\theta &= \frac{P}{A}\left(\frac{1+\cos 2\theta}{2}\right) = \frac{\sigma_x}{2}(1+\cos 2\theta)\\ \tau_\theta &= \frac{-P}{2A}\sin 2\theta = -\frac{\sigma_x}{2}(\sin 2\theta)\end{aligned}}$$

Q.

a] 

b] 

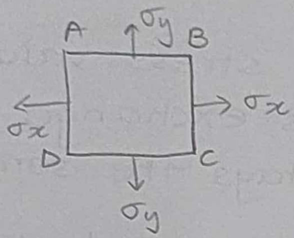
Find σ_θ and τ_θ on every surface.

→ a] 

$\theta = 0$

On BC $\begin{cases} \sigma_\theta = \frac{\sigma_x}{2} \times 2 = \sigma_x \\ \tau_\theta = 0 \end{cases}$

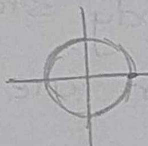
[$\sigma_\theta = \sigma_{\theta,max}$ here]



here $\sigma_y = 0$

$\sigma_x = \sigma_\theta$

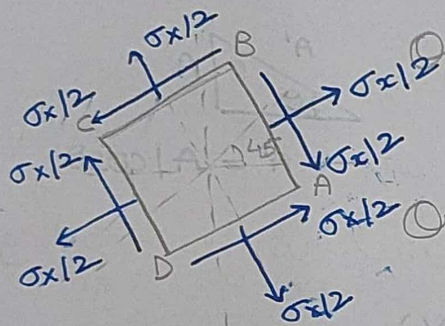
and $\tau_\theta = 0$



b] $\theta = 45°$

$\sigma_\theta = \frac{\sigma_x}{2}(1) = \frac{\sigma_x}{2}$

$\tau_\theta = -\frac{\sigma_x}{2}$



On surface AB: $\theta = 45°$

$\sigma_{(\theta=45°)} = \sigma_x/2$

$\tau_{(\theta=45°)} = -\sigma_x/2$

On surface BC: $\theta = 90°$

$\sigma_{(\theta=90°)} = 0$

$\tau_{(\theta=90°)} = 0$

On surface CD: $\theta = 135°$

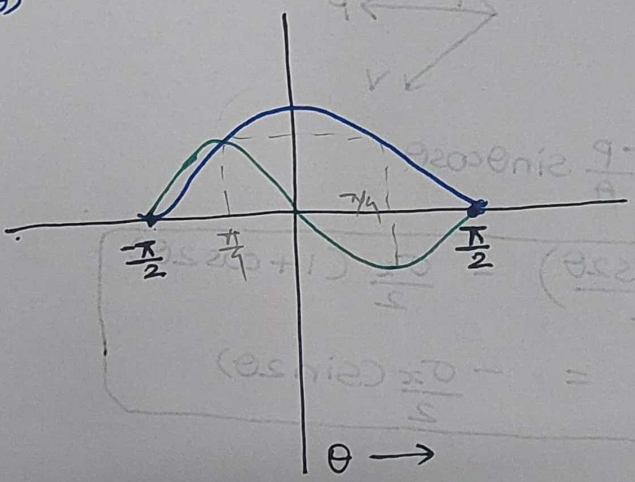
On surface AD:

$\theta = -45°$

$\sigma_{(\theta=-45°)}$

$\sigma_\theta = \frac{\sigma_x}{2}(1+\cos 2\theta)$

$\tau_\theta = -\frac{\sigma_x}{2}\sin 2\theta$



$\sigma_\theta = \frac{\sigma_x}{2}(1+\cos 2\theta)$

$\sigma_\theta' = \cancel{2}\frac{\sigma_x}{\cancel{2}}(-\sin 2\theta)$

$\tau_\theta = -\frac{\sigma_x}{2}\sin 2\theta$

$\tau_\theta' = -\sigma_x \cos 2\theta$

$\tau_\theta'' = +2\sigma_x \sin 2\theta$