STRESS & STRAIN

Strain is "measurable" ~ easily.
Stress can be something "relative".

** STRAIN IS MORE FUNDAMENTAL

↝ Stress can be calculated by strain
↝ Material response is the relation between "stress & strain"
↳ The output depends on it

⇒ HOOKE'S LAW

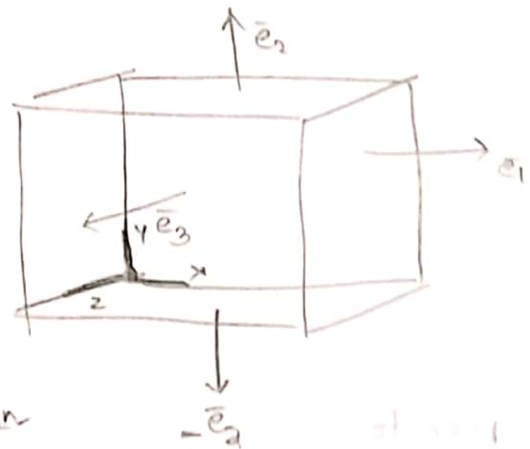
"perceived quantity"
Stress ∝ Strain — measurable

$\sigma \propto e$ (linear proportion)

$\boxed{\sigma = Ee}$ ⟶ Young's Modulus Elasticity

** BOTH STRESS & STRAIN ARE "TENSORS" (SECOND ORDER)

"There's nothing called stress at point" on a "infinitesimal cube but not point



$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

All the 9 components determine stress

σ

$\overrightarrow{I}$ ↳ direction orientation

Plane orientation

Ily Strain

$$\begin{bmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{bmatrix}$$

Generalised Hooke's Law :—

$$\sigma_{ij} \propto \epsilon_{kl}$$

L I E — Linear, Isotropic, Elastic

All the strains are dep contribute to diff σ_{ik}

$$\boxed{\sigma_{ij} = C_{ijkl}\,\epsilon_{kl}}$$

σ_{ij} → 9 Components

ϵ_{kl} → 9 Components

C_{ijkl} → STIFFNESS TENSOR (4th order Tensor)

$1/C$ → Complaince Tensor

$$\boxed{\epsilon_{ij} = S\,\sigma_{kl}}$$

81 unique attributes required to differentiate materials.

with Isotropic — 2 are only req. — Youngs Modulus; Any other like Bulk's Modulus, Poisson's Ratio, Plasticity, Viscoplasticity

Isotropic → Along every dirn the properties are same

2 are only req. → All the others are dependent on them

→ Major Symmetry :

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

Not ferro magnetic material then top & bottom are Symmetric

$$\sigma_{ij} = \sigma_{ji}$$

⇒ dilatational — dentoric

Size — Shape

* finite deformation (large)

infinite " (small) ⟹ Assumption for "linearity"

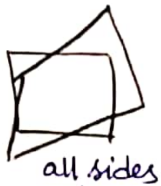
$$\epsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] = \epsilon_{ji}$$

Scanned with CamScanner

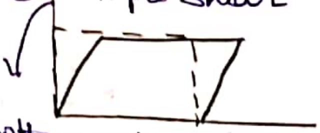
$\epsilon_{11} = \frac{\partial u_1}{\partial x_1}$ $\quad$ $\epsilon_{12} = \frac{1}{2}\left[\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right]$

$\downarrow$ Stretch $\quad$ $\downarrow$ distortion.

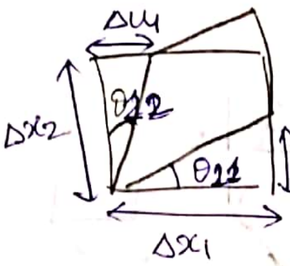
① pure Shear →



② Simple Shear



~~$\epsilon_{ij} = \frac{1}{2}\left[\frac{du_i}{dx}\right.$~~



$e_{21} = \frac{\Delta u_1}{\Delta x_2}$

$e_{12} = \epsilon_{12} = \frac{1}{2}(e_{12} + e_{21})$

⇒ due to "Linearity" — Superposition allowed (Retailer)

Non Linear — Superposition not allowed (whole sale)

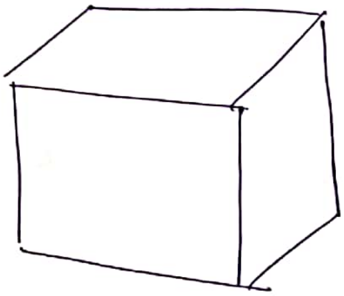
⇒ Stress is ~~Strain~~ asymmetric unless no body movements

Strain is "SYMMETRIC"

✳ 3^0 Scalar
3^1 vector
3^2 2nd order Tensor
3^4 4th order Tensor

$\overset{2}{\underline{a}} \otimes \overset{2}{\underline{b}} = \overset{2}{\underline{c}}$ $\quad$ $a \cdot b =$ 0

dot product dec by 1



e_{ij} → It is in 'j' direction in the plane

plane whose normal is in 'i' direction

$\sigma = \sigma_{ij}\, e_i \otimes e_j$ → tensor product

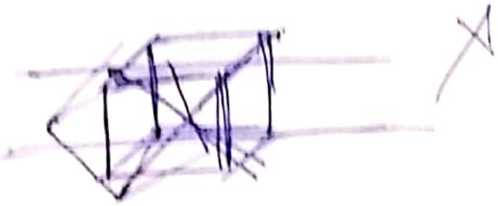
$= \sigma_{11}\, e_1 \otimes e_1 + \sigma_{12}\, e_1 \otimes e_2 + \sigma \cdots$

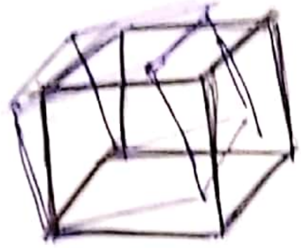
$+ \sigma_{21}\, e_2 \otimes e_1$

$\vdots$

~~f~~

$\underset{\sim}{C} = C_{ijkl}\, e_i \otimes e_j \otimes e_k \otimes e_l$



Pr: ⇒ 

$$V = v_1 e_1 + v_2 e_2 + v_3 e_3$$
$$= v_1' e_1' + v_2' e_2' + v_3' e_3'$$

⟹ for the vector ⇒ invariants: the magnitude

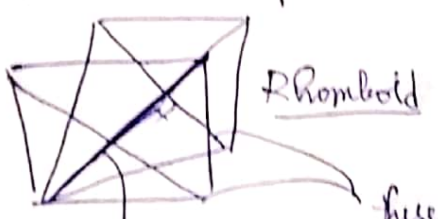
2nd order for tensor (2n order) ⇒ invariants $(\lambda_1, \lambda_2, \lambda_3)$ eigenvalues.

trace $I_1 = \lambda_1 + \lambda_2 + \lambda_3$

trace of Adjmatrix $I_2 = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1$

determinent $I_3 = \lambda_1\lambda_2\lambda_3$

✱ 6 independent stress components



⟹ $Ax = b$

in this transformation few transformations follow this relation : "few directions retain their direction"

$$\boxed{Ax = \lambda x}$$

$\Rightarrow (A - \lambda I)x = 0 \Rightarrow |A - \lambda I| = 0$

↳ characteristic equations

$\lambda^3 - I_1 \lambda^2 + I_2 \lambda - I_3 = 0$.

characteristic polynomial

<u>Caley Hamilton Theorem</u>

$$A^3 - I_1 A^2 + I_2 A - I_3 I = 0$$

Finding inverse :

$$A^2 - I_1 A + I_2 = I_3 A^{-1}$$

physical meaning of eigen values : there are certain orientation i,e in this case 3, where it "streches" but not distorted

$\Rightarrow$ 6 independent stress components : $\lambda_1, \lambda_2, \lambda_3$ & x_1, x_2, x_3

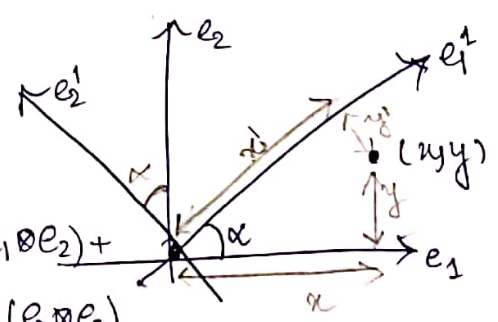
6 are ~~needed~~ present but only 3 are needed.

↳ only for the orientations which "strech" [Eigenvalues]

$*$ $e_1' = e_1 \cos\alpha + e_2 \sin\alpha$

$e_2' = e_2 \cos\alpha - e_1 \sin\alpha$

$$\underset{\sim}{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$



$$\underset{\sim}{\sigma} = \sigma_{11}(e_1 \otimes e_1) + \sigma_{12}(e_1 \otimes e_2) + \sigma_{21}(e_2 \otimes e_1) + \sigma_{22}(e_2 \otimes e_2)$$

$$= \sigma_{11}'(e_1' \otimes e_1') + \sigma_{12}'(e_1' \otimes e_2') + \sigma_{21}'(e_2' \otimes e_1') + \sigma_{22}'(e_2' \otimes e_2')$$

$$\underset{\sim}{\sigma} = \sigma_{11}'\left[(e_1 C_\alpha + e_2 S_\alpha) \otimes (e_1 C_\alpha + e_2 S_\alpha)\right] +$$

$$\sigma_{12}'\left[(e_1 C_\alpha + e_2 S_\alpha) \otimes (e_2 C_\alpha - e_1 S_\alpha)\right] +$$

$$\sigma_{21}'\left[(e_2 C_\alpha - e_1 S_\alpha) \otimes (e_1 C_\alpha + e_2 S_\alpha)\right] +$$

$$\sigma_{22}'\left[(e_2 C_\alpha - e_1 S_\alpha) \otimes (e_2 C_\alpha - e_1 S_\alpha)\right]$$

$$\Rightarrow e_1 \otimes e_1 \left[C_\alpha^2 \sigma_{11}' - \sigma_{12}' C_\alpha S_\alpha - \sigma_{21}' C_\alpha S_\alpha + \sigma_{22}' S_\alpha^2 \right]$$

* $\sigma_{12}' = \sigma_{21}'$ [Symmetric Matrix] ; $e_1 \otimes e_2 = e_2 \otimes e_1$

$$\Rightarrow \sigma_{11} = \left[C_\alpha^2 \sigma_{11}' + \sigma_{22}' S_\alpha^2 - 2\sigma_{12}' S_\alpha C_\alpha \right]$$

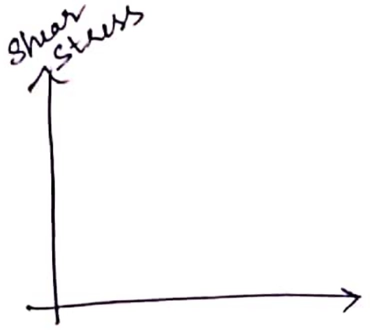
$$\sigma_{22} = \left[S_\alpha^2 \sigma_{11}' + 2\sigma_{12} C_\alpha S_\alpha + C_\alpha^2 \sigma_{22}' \right]$$

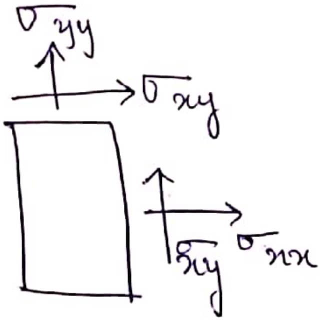
$$\sigma_{21} = \sigma_{12} = \left[\sigma_{11}' C_\alpha S_\alpha + \sigma_{12}' (C_\alpha^2 - S_\alpha^2) - \sigma_{22}' S_\alpha C_\alpha \right]$$

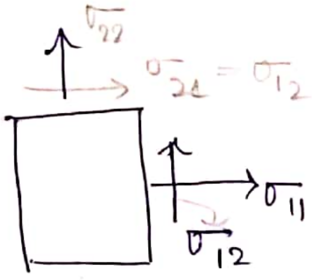
$$\Rightarrow \qquad R = \begin{bmatrix} C_\alpha & S_\alpha \\ -S_\alpha & C_\alpha \end{bmatrix}$$

$$\boxed{\sigma = R^T \sigma R}$$

* α_p : when $\sigma_{21} = \sigma_{12} = 0$

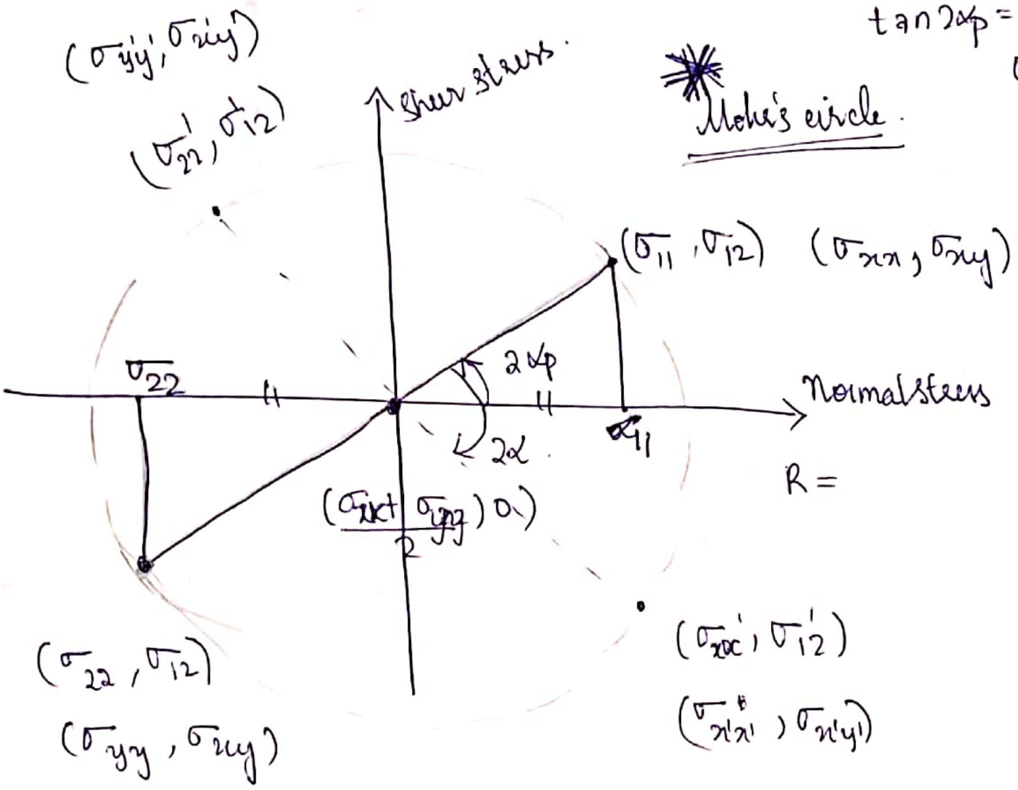






$$\tan 2\alpha_p = \frac{\sigma_{xy}}{\frac{\sigma_{xx}-\sigma_{yy}}{2}}$$

Mohr's circle.



R =

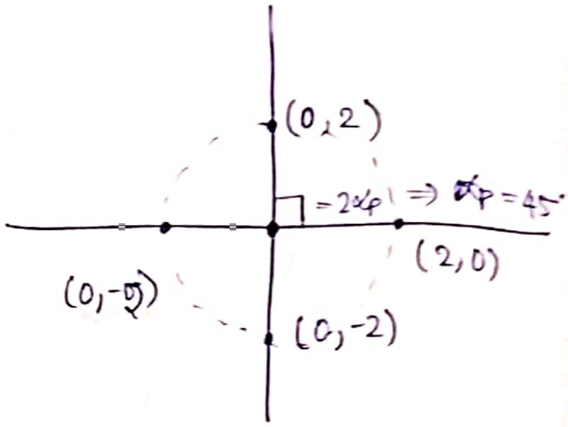
$$I_{x'x'} = \left(\frac{I_{xx}+I_{yy}}{2}\right) + \left(\frac{I_{xx}-I_{yy}}{2}\right) C_{2\alpha} - I_{xy} S_{2\alpha}$$

$$I_{y'y'} = \left(\frac{I_{xx}+I_{yy}}{2}\right) + \left(\frac{I_{xx}-I_{yy}}{2}\right) C_{2\alpha} + I_{xy} S_{2\alpha}$$

$$I_{x'y'} = \left(\frac{I_{xx}-I_{yy}}{2}\right) S_{2\alpha} + I_{xy} C_{2\alpha}.$$

$\Rightarrow$ $\Rightarrow$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}$$



Example

$\exists$

$$\underset{\sim}{\sigma} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$ Pure stress

$\sigma_1 > \sigma_2$

$\det(A - \lambda I) = 0$

$\sigma_1 = 2$

$\sigma_2 = -2$

$$\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\lambda & 2 \\ 2 & -\lambda \end{bmatrix}$$

$\Downarrow$

$\lambda = \pm 2 \Rightarrow$ this gives the principle values

$(\sigma_1, \sigma_2) = (2, -2)$

Dynamics

$\Rightarrow$ CLOSED SYSTEM: Conservation of mass!

Conservation of linear momentum!

Conservation of Angular momentum!

Conservation of Energy!

} trivial

Dissipiation Energy ≥ 0

Law of motion: $F = \frac{\partial(mv)}{dt} = m\frac{dv}{dt} = ma$