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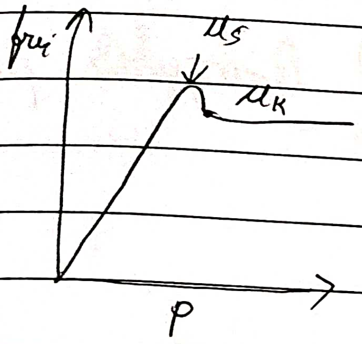
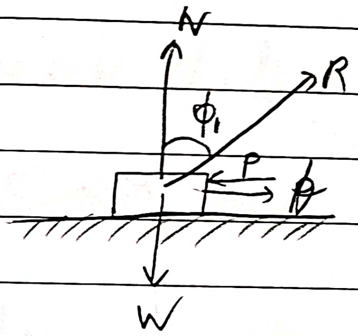
Lec 16

μ_K is always less than μ_S as interlocking is less during motion

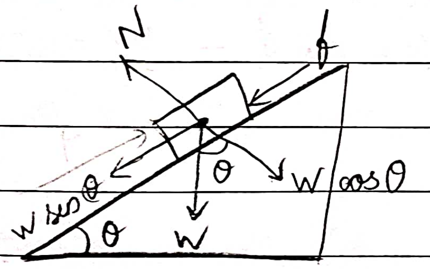


- Angle of friction (ϕ)



$R = \sqrt{N^2 + f^2}$

- Angle of Repose (θ)



$\Sigma f_x = 0 \Rightarrow W \sin\theta - f = 0$

$\Sigma f_y = 0 \Rightarrow W \cos\theta - N = 0$

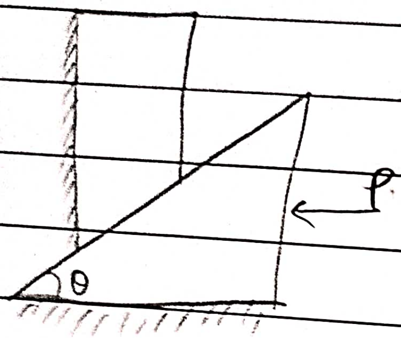
$f = W \sin\theta$ $\qquad N = W \cos\theta$

$\frac{f}{N} = \tan\theta$ $\qquad \boxed{f = \mu N}$

$\boxed{\mu = \tan\theta}$ $\qquad \boxed{\theta = \tan^{-1}\mu}$

☆ Application of friction

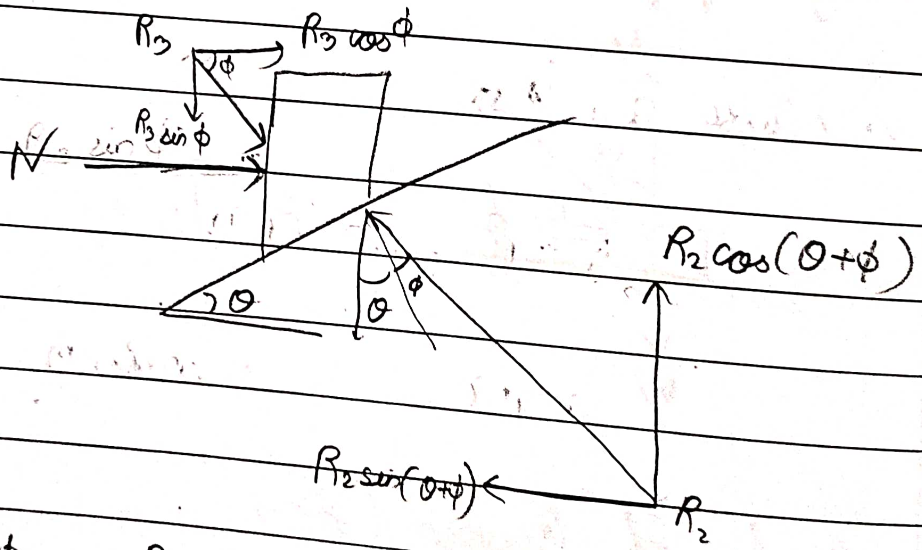
WEDGE – wooden or metal triangular piece



$W = 1500N$

$\theta = 10^\circ$

$\mu = 0.3$ (same)



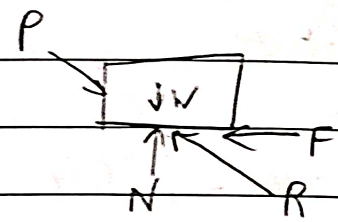
$$\Sigma F_x = R_3 \cos\phi - R_2 \sin(\phi + \theta) = 0$$

$$\Sigma F_y = R_2 \cos(\theta + \phi) - R_3 \sin\phi - W = 0$$

Lec 17

Friction

- Dry — Static μ_s ; Kinetic μ_k ; $\mu_s > \mu_k$



$\tan\phi_s = \frac{F}{N}$

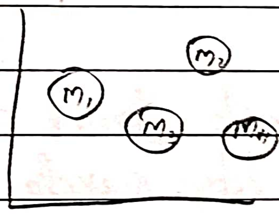
- **Moments of Area**

Classical - Rigid, point mass

Solid - Deform, mass uniformly distributed
(Continuum)

- Centroid

$M = m_1 + m_2 + m_3 + m_4$



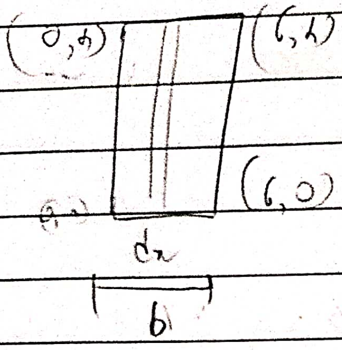
First moment $= m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4$

$M\bar{x} = M_1x_1 + M_2x_2 + M_3x_3 + M_4x_4$

$\bar{x} = \frac{\sum_{i=1}^{n} M_ix_i}{\sum M_i}$ $\qquad \bar{y} = \frac{\sum M_iy_i}{\sum M_i}$

$\bar{x} = \frac{\int dm\,x}{\int dm}$ $\qquad \bar{y} = \frac{\int dm\,y}{\int dm}$

Moments Of Inertia



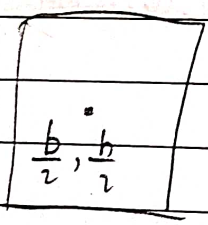
ρ = mass of the building or length + area = const

$dM = \rho h\, dx$

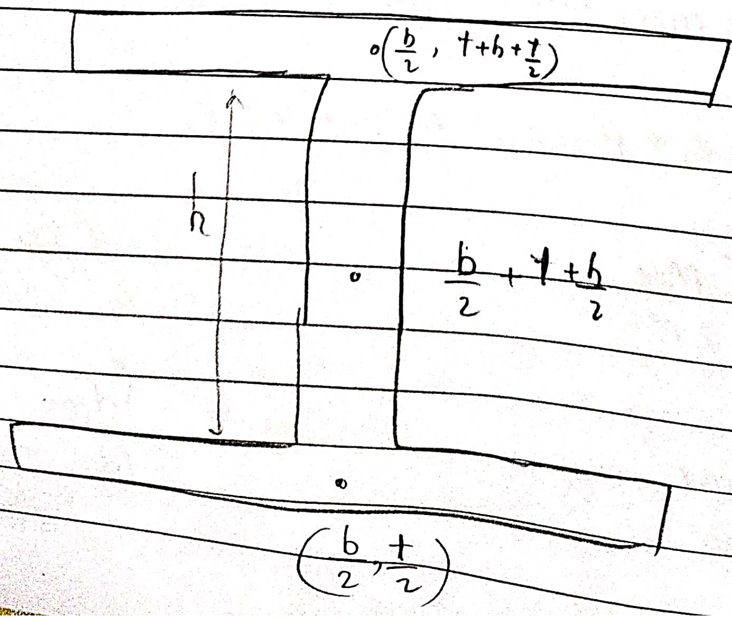
Total mass $M = \int_0^b dM = \int_{\rho}^{2\rho} \rho h\, dx$

$\bar{x} = \dfrac{\int dm\, x}{\rho h b} = \dfrac{\int \rho h x\, dx}{\rho h b}$

$= \dfrac{\left[\dfrac{x^2}{2}\right]_0^b \cancel{\rho h}}{\cancel{\rho h}\, b} = \dfrac{b}{2}$



Composite struct are those which are made from simple str like circle, rect, sq.



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$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A}$$

$$\bar{x} = \frac{b}{2}$$

$$y = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A}$$

$$= \frac{bt\left(\frac{3t}{2} + h\right) + th\left(t + \frac{h}{2}\right) + tb\left(\frac{t}{2}\right)}{t(2b+h)}$$

$$= \frac{(2t+h)(b+h/2)}{2b+h} = t + \frac{h}{2}$$

= 150×50 7500

= 300×50 15000

– 300×100 30000

Lec 18 First moment of Area

Centroid $\bar{x} = \dfrac{\sum x \Delta W_i}{\sum_i \Delta W_i}$ $\bar{x} = \dfrac{\int x\, dW}{\int dW}$

discrete continous

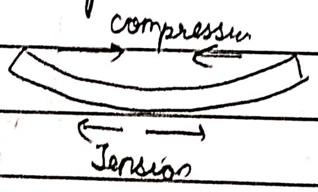
$$W = W_1 + W_2 + W_3 + \cdots$$

$$W\bar{x} = W_1 x_1 + W_2 x_2 + W_3 x_3 + \cdots$$

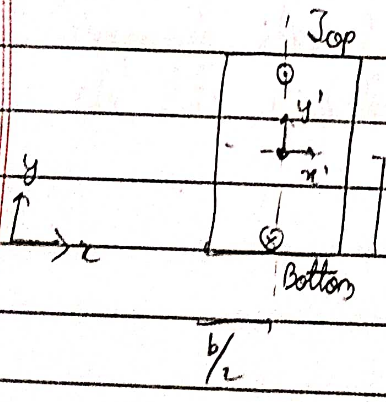
It should give same net effect (net force & net momentum

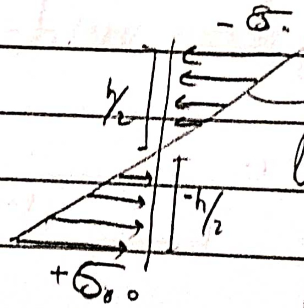
$W = \rho A t g$ $A\bar{x} = A_1 x_2 + A_2 x_2 + A_3 x_3$

• Second moment of Area
Moment of Inertia



pure bending, ∴ net force has to be 0.





$\dfrac{-2\sigma_0 y'}{h}$

linearly

$\boxed{y = y' + \frac{h}{2}}$ if axis is changed

$$\sigma = \frac{-2\sigma_0 (y - h/2)}{h}$$

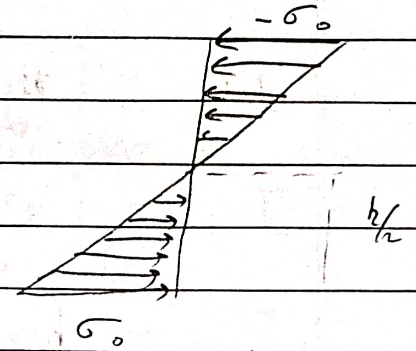
$$F = \int_0^h \sigma\, dA = \int_0^h \frac{-2\sigma_0 (y - h/2)}{h}\, b\, dy = 0$$

(area was also 0)

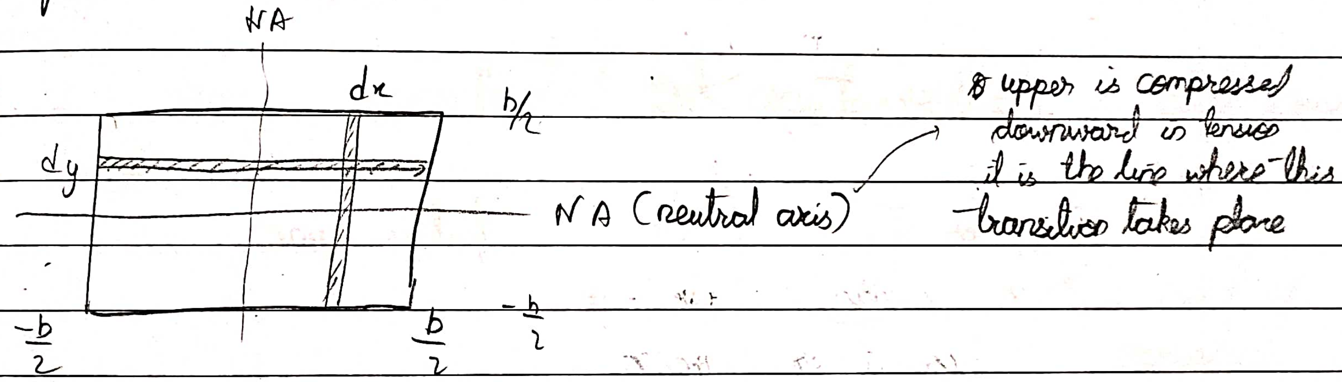
Second Moment deals with moment

$$M = \int_0^h \sigma . y . dA$$

$$= -\frac{\sigma b h^2}{6}$$



I has more moment of inertia as compare to [] i.e. it offers more resistance as it is thicker at top (where forces are max).



$$I_{xx} = \int y^2 dA = b \int_{-h/2}^{h/2} y^2 dy = b\left[\frac{y^3}{3}\right]_{-h/2}^{h/2} = \frac{bh^3}{12}$$

$$I_{yy} = \int x^2 dA = \int_{-b/2}^{b/2} x^2 h \, dx = \frac{hb^3}{12}$$

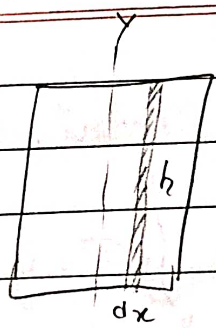
MOI is tensor so is stress cause we need to know about plane.

I_{xy} & I_{yx} will be equal only if object is symm about atleast x or y axis

measure of asymm

Lec 19

Neutral axis



$$I_{xx} = \int y^2 \, dA$$

$$I_{yy} = \int x^2 \, dA$$

$$I_{xy} = \int xy \, dA$$

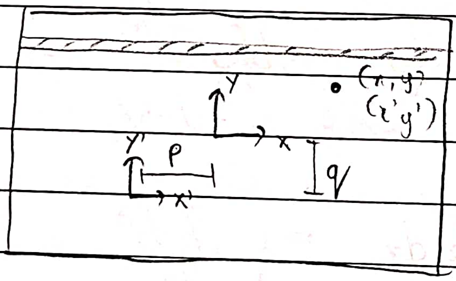
→ $yh \int x \, dx = 0$ → if it is symm about Y axis

→ $xb \int y \, dy = 0$ → if it is symm about X axis

product of Inertias

I_{xx} or $I_{yy} > 0$

I_{xy} or I_{yx} $\begin{cases} 0 & \text{if it is symm} \\ +ve \quad -ve & \end{cases}$



$$x' = x + p$$
$$y' = y + q$$

$$I_{x'x'} = \int (y')^2 \, dA$$

$$= \int (y+q)^2 \, dA$$

$$= \int (y^2 + q^2 + 2qy) \, dA$$

$2qy$ → 0 (passes through centroid)

$$\boxed{I_{x'x'} = I_{xx} + Aq^2}$$

$I_{y'y'} = I_{yy} + Ap^2$

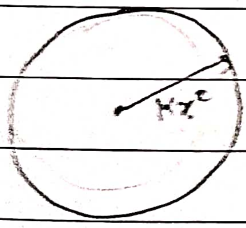
Polar Moment of Inertia ($\perp^{ar}$ axis theorem)

If x is $\perp^{ar}$ to y & both are $\perp^{ar}$ to z then

$$J = I_{xx} + I_{yy}$$

Radius of Gyration

~~We~~ Area is concentrated at anulus then its dist from centre (radius)



$I = A\,K^2$ (K is rad of gyration)

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Lec 20

Parallel axis theorem

$$I_{x'x'} = I_{xx} + Ad^2$$

I_{xx} ↳ about centroidal axis

'd' is $\perp^{ar}$ distance bet xx & x'x'

Perpendicular axis theorem

$(a \perp b) \perp c$

$$I_c = I_a + I_b$$

☆ Stress & Strain

Strain is more fundamental as it can be easily measured

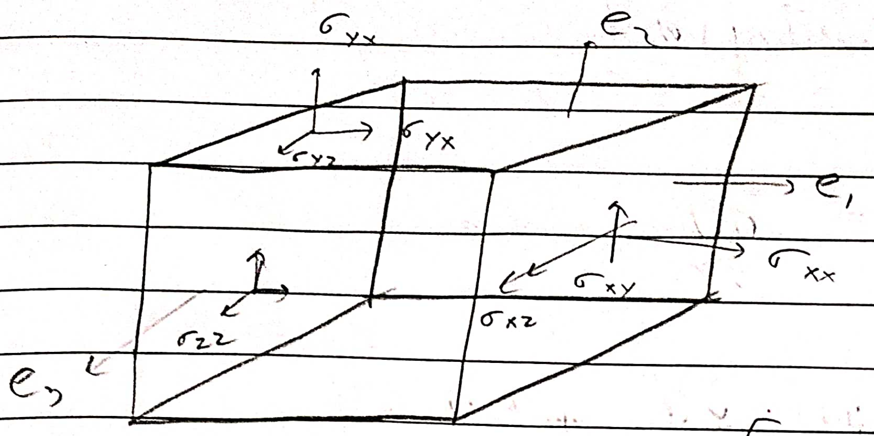
The defects in the object govern the $\underbrace{\text{strain \& stress}}_{\text{material response}}$

material response → defect, state (eg - Temp.,)

Hooke's law

$\sigma \propto \epsilon \Rightarrow \sigma = E\epsilon$

σ & ϵ both are second order tensor



$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

First we need to define the plane & then its direct"

σ_{xz} : x → plane, z → direct"

size (dilatational)

shape (devitoric)

$$\begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

This is similar for strain.

strain ⇒ volume changes along with shear distortions

* Generalized Hooke's law

$$\underset{\underset{9}{3\times3}}{\sigma_{ij}} = \underbrace{C_{ijkl}}_{\text{stiffness}} \underset{\underset{9}{3\times3}}{\epsilon_{kl}}$$

C_{ijkl} → $3\times3\times3\times3 = 81$

we can relate σ_{11} with ϵ_{21} also not only ϵ_{11}

If we assume material to be isotropic then only 2 const. are req.

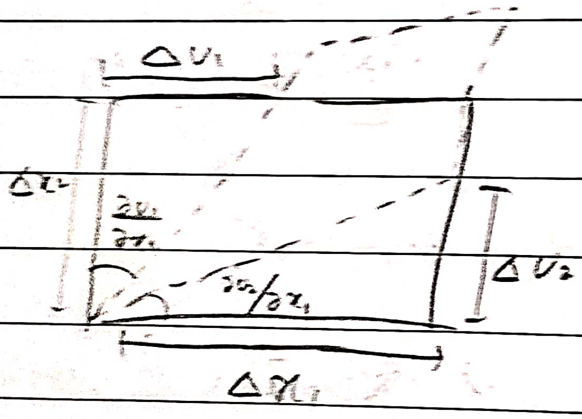
E or μ or bulk modulus or poisonn's ratio
pick any two.

$$\epsilon_{ij} = \underbrace{S_{ijkl}}_{\text{compliance}} \sigma_{kl}$$

$$\epsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]$$

$$\epsilon_{11} = \frac{\partial u_1}{\partial x_1}$$

$$\epsilon_{12} = \frac{1}{2}\left[\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right]$$



pure shear

$$e_{21} = \frac{\Delta u_1}{\Delta x_2}$$

$$e_{12} = \frac{\partial u_1}{\partial x_2}$$

$$\boxed{\epsilon_{12} = \frac{1}{2}\left(e_{12} + e_{21}\right)}$$

Lec 21

Strain $\epsilon_{ij} = \frac{1}{2}\left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right]$

Stress σ_{ij}

Constitute Behaviour (Material response) $\sigma_{ij} = C_{ijkl}\,\epsilon_{kl}$

C_{ijkl} → 4^{th} order Tensor

$\sigma_{ij} = \sigma_{ji}$ only if it is non ferromagnetic material

Small strain, by def $\epsilon_{ij} = \epsilon_{ji}$

Strain may be symm but not stress (not by def^n)

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots \\ \vdots & \vdots & \\ \vdots & \vdots & \sigma_{33} \end{bmatrix}$$ → short hand for 2^{nd} order

$P = xe_1 + ye_2 + ze_3$

(x, y, z) — short hand for co ordinate system

$\sigma = \sigma_{ij}\, e_i \otimes e_j$

$\otimes$ → tensor product

invariants

$3^0 = 1$ scalar (mag)

$3^1 = 3$ vector

$3^2 = 9$ tensor (2^{nd} order)

$3^4 = 81$ 4^{th} order

$tr(\sigma) = \lambda_1 + \lambda_2 + \lambda_3$

$= \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_1\lambda_3$

$\det(\sigma) = \lambda_1\lambda_2\lambda_3$

3 invariants

I_1, I_2, I_3 or

3 eigen values

$\lambda_1, \lambda_2, \lambda_3$

c 22

C_{ijkl} $\begin{cases} 36 \text{ independent constants for generally anisotropic material} \\ 2 \quad " \quad " \quad \text{isotropic} \end{cases}$

$A^{-1} = \dfrac{\text{adj } A}{|A|}$ —— trans of cofactor

$I_2 = \lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1$

$\sigma = \sigma_{ij} \hat{e}_i \otimes e_j$ 6 independent stress compon

$A x = \lambda I x$

$A x - \lambda I x = 0$

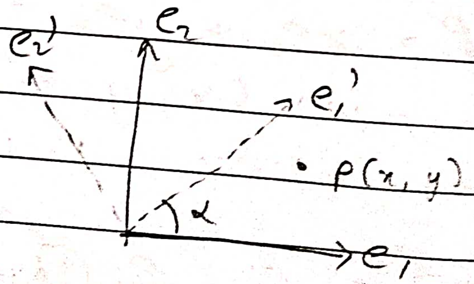
$(A - \lambda I) x = 0$

$\det (A - \lambda I) = 0$

$\lambda^3 - I_1 \lambda^2 + I_2 \lambda - I_3 = 0$

$A^3 - I_1 A^2 + I_2 A - I_3 = 0$ → Caley Hamilton theorem

$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$



$e_1' = e_1 \cos\alpha + e_2 \sin\alpha$

$e_2' = -e_1 \sin\alpha + e_2 \cos\alpha$

$\sigma = \sigma_{11} e_1 \otimes e_1 + \sigma_{12} e_1 \otimes e_2 + \sigma_{12} e_2 \otimes e_1 + \sigma_{22} e_2 \otimes e_2$

$= \sigma_{11}' e_1' \otimes e_1' + \sigma_{12}' e_1' \otimes e_2' + \sigma_{12}' e_2 \otimes e_1 + \sigma_{22} e_2' \otimes e_2'$

$$= \sigma_{11}' [e_1 C_\alpha + e_2 S_\alpha] \otimes [e_1 C_\alpha + e_2 S_\alpha]$$

$$+ \sigma_{12}' [e_1 C_\alpha + e_2 S_\alpha] \otimes [-e_1 S_\alpha + e_2 C_\alpha]$$

$$+ \sigma_{12}' [-e_1 S_\alpha + e_2 C_\alpha] \otimes [e_1 C_\alpha + e_2 S_\alpha]$$

$$+ \sigma_{22}' [-e_1 S_\alpha + e_2 C_\alpha] \otimes [-e_1 S_\alpha + e_2 C_\alpha]$$

$$\Rightarrow e_1 \otimes e_1 \Big[\underbrace{C_\alpha^2 \sigma_{11}' - 2\sigma_{12}' C_\alpha S_\alpha + S_\alpha^2 \sigma_{22}'}_{\sigma_{11}} \Big]$$

$$e_2 \otimes e_2 \Big[\underbrace{S_\alpha^2 \sigma_{11}' + 2\sigma_{12} C_\alpha S_\alpha + C_\alpha^2 \sigma_{22}'}_{\sigma_{22}} \Big]$$

$$[e_1 \otimes e_2 + e_2 \otimes e_1] \big[C_\alpha S_\alpha \sigma_{11}' + (C_\alpha^2 - S_\alpha^2) \sigma_{12}' - S_\alpha C_\alpha \sigma_{22}' \big]$$

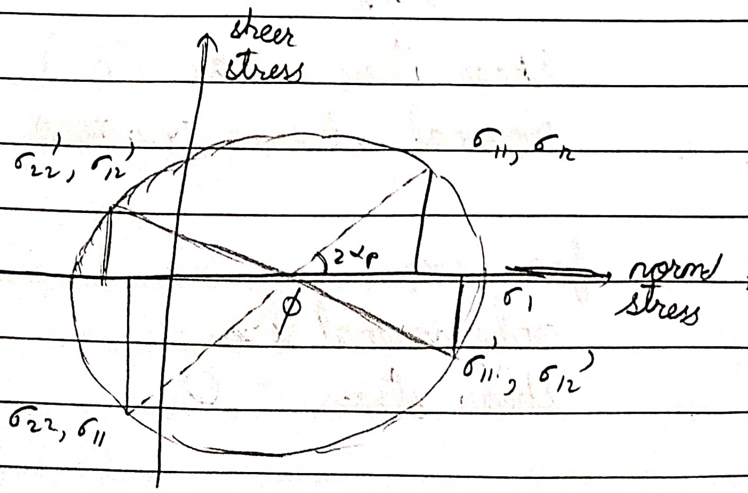
$C_\alpha = \cos\alpha$

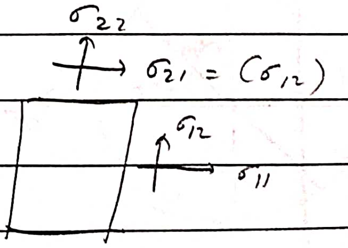
$S_\alpha = \sin\alpha$

$$R = \begin{bmatrix} C_\alpha & S_\alpha \\ -S_\alpha & C_\alpha \end{bmatrix}$$

$$\boxed{\sigma = R^T \sigma R}$$

In matrix form



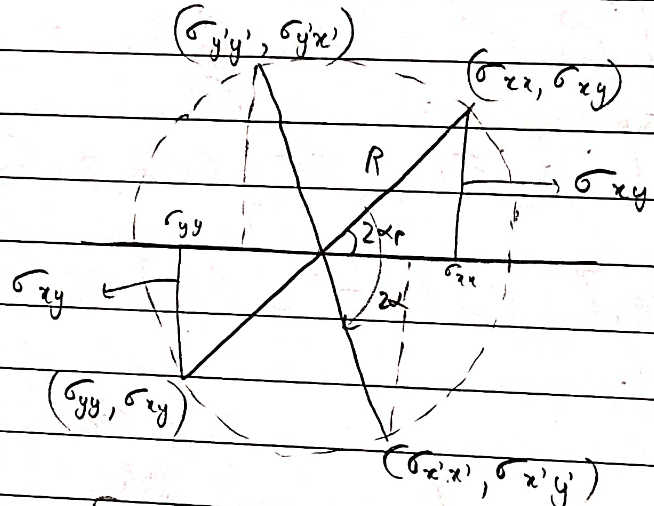


Lec 23

$$I_{x'x'} = \left(\frac{I_{xx}+I_{yy}}{2}\right) + \left(\frac{I_{xx}-I_{yy}}{2}\right) C_{2\alpha} - I_{xy} S_{2\alpha}$$

$$I_{y'y'} = \left(\frac{I_{xx}+I_{yy}}{2}\right) - \left(\frac{I_{xx}-I_{yy}}{2}\right) C_{2\alpha} + I_{xy} S_{2\alpha}$$

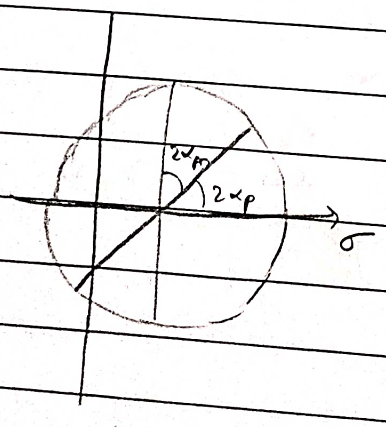
$$I_{x'y'} = \left(\frac{I_{xx}-I_{yy}}{2}\right) S_{2\alpha} + I_{xy} C_{2\alpha}$$



~~$\tan 2\alpha_p = 2\alpha_{xy}$~~

$$\tan 2\alpha_p = \frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}}$$

$$R^2 = \left[\frac{\sigma_{xx}-\sigma_{yy}}{2}\right]^2 + (\sigma_{xy})^2$$



$2\alpha_m$

Shear stress = max (R) $\sigma_{xy} = R$

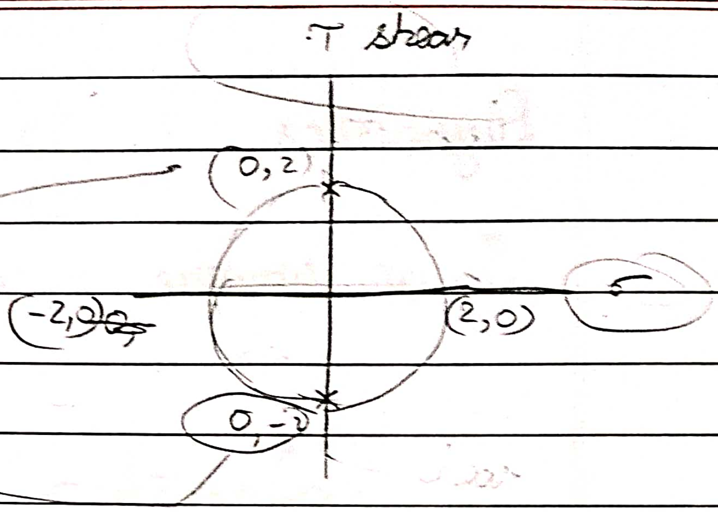
Normal stresses non zero.

for $2\alpha_p$

only normal stress

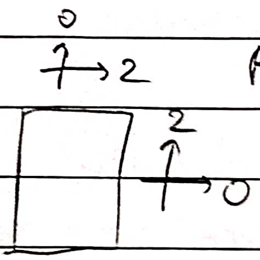
shear stress $= 0$

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{xy} \end{bmatrix}$$



eg $\sigma = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

pure shear



$\sigma_1 > \sigma_2$

$\sigma_1 = 2$

$\sigma_2 = -2$

$\det(A - \lambda I) = 0$ for generic $A = \underline{\underline{\sigma}}$

$$\det\left(\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) = \det \begin{bmatrix} -\lambda & 2 \\ 2 & -\lambda \end{bmatrix}$$

$(-\lambda)(-\lambda) - 2 \times 2 \Rightarrow \lambda^2 = 4$

$\boxed{\lambda = \pm 2}$

$\boxed{\sigma_{1/2} = \pm 2}$

Lec 24

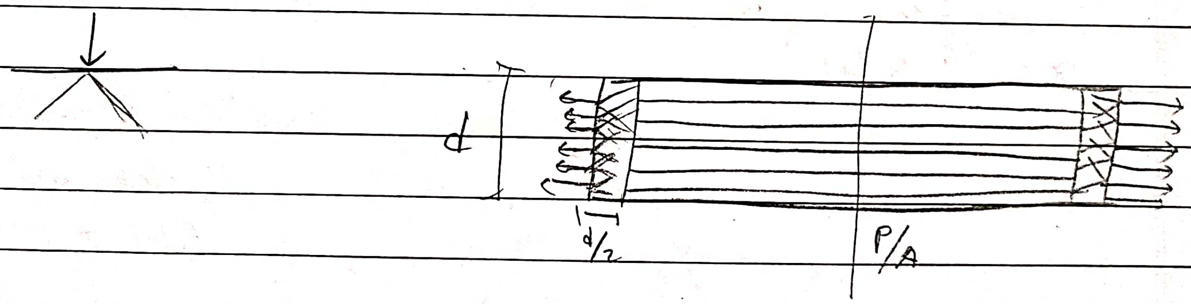
- $\lambda^3 - I_1\lambda^2 + I_2\lambda - I_3 = 0$
  $A^3 - I_1A^2 + I_2A - I_3 = 0$

λ are principle value

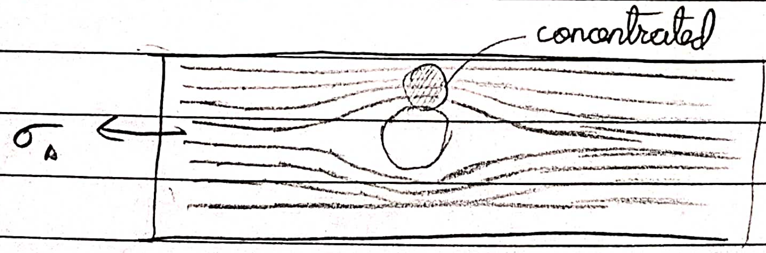
- Mohr's circle for stress transformation

- Dynamics — First law - inertia
  — Second law - $\frac{d(mv)}{dt}$
  — Third law - Action $\leftrightarrow$ React'

Stress concentration factor



$\sigma = \lim_{\Delta A \to 0} \frac{\Delta P}{\Delta A}$ edge effect



Stress factor $= \frac{\sigma_{max}}{\sigma_A}$

This is for extra safety to ensure there is no damage in struct

Study the failure