ENGINEERING MECHANICS

→ Through the course, the solids are not deformable.
→ They are so small that the deformation is negligible.

⇒ Classical & Quantum Mechanics → may have lot of differences but the general mechanics are always same

1. Conservation of mass ~ its trivial for a closed system
2. Conservation of linear momentum
3. Conservation of Angular momentum
4. Conservation of Energy.

Universally true & Applicable

1st law

Inertia (it is a characteristic of mass)

2 states → State of rest
→ State of uniform motion

2nd Law

$$F \propto \frac{dP}{dt}$$

3rd law

Every action has equal and opposite reaction

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4-5-21

Lec 3

Space defined wrt 3 coordinates (Euclidean space)

* Space $\equiv (Real)^3$

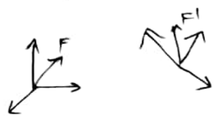
3^0 1 Scalar → magnitude (loner doesn't have any friends)

3^1 3 Vector → direction + magnitude

3^2 9 Tensor : stress

* Any changes in co-or axis (co-ordinate transformations) doesn't effect scalar because it has only 1 component

* Any co-or transformation occurs, the vector changes.



$F = F_1 \underline{e_1} + F_2 \underline{e_2} + F_3 \underline{e_3}$

$F' = F_1' \underline{e_1'} + F_2' \underline{e_2'} + F_3' \underline{e_3'}$

$F \cdot F = F' \cdot F'$ (Dot Product)

* Inner product / square of the magnitude → invariant

Components

$3^0 = 1$

$3^1 = 3$

$3^2 = 9$

No of components 3^n

Invariants

↳ doesn't change wrt co-ordinate transformations but the projection may change

$3^{0-1} \longrightarrow \times$

$3^{1-1} \longrightarrow 1$ no. of invariants of a vector

$3^{2-1} \longrightarrow 3$ " of a Tensor

No. of invariants 3^{n-1}

Vectors → needs Basis

$e_1 \; e_2 \; e_3$ → need not be mutually perpendicular

should be able to form a parallelopiped

Basis : Linearly independent vectors which can represent any point in space including zero vector

↳ if $v_1 \underline{e_1} + v_2 \underline{e_2} + v_3 \underline{e_3} = 0$

$v_1 = v_2 = v_3 = 0$

Examples of vectors

↳ Position vector

↳ Velocity

↳ acceleration

↳ Force

Scalar (a & b → scalar)

$a+b \;/\; a \div b \;/\; a \times b$ = Scalar

Vector

$\underline{a} + \underline{b}$ = Vector 3^1

dot product $\underline{a} \cdot \underline{b}$ = Scalar 3^0

cross product $\underline{a} \wedge \underline{b}$ = Vector 3^1

$\underline{a} \otimes \underline{b}$ = Tensor 3^2

Tensor product

Energy = $\underline{F} \cdot \underline{d}$ ⇒ Scalar

8-4-21 : Lec 4

Mechanics → Kinematic ~ talk about MOTION OF THE BODY

Mechanics → Kinetics → Statics / Dynamics

Kinetics: talk about ACTION OF FORCES on a body

Equilibrium → Static Equilibrium [No inertial forces present in system]
Sum of forces = 0
$\Sigma F = 0$
↳ Dynamic " [Sum of all the forces must balance inertial forces (ma)]
$\Sigma F = ma$

⇒ Force is a vector having 3 components

$$\underline{F} = F_1\underline{e}_1 + F_2\underline{e}_2 + F_3\underline{e}_3 = F_1'\underline{a} + F_2'\underline{b} + F_3'\underline{c}$$

e_1, e_2, e_3 → are orthonormal

where prod $[a\ b\ c] \neq 0$

$$\begin{bmatrix} F\cdot a \\ F\cdot b \\ F\cdot c \end{bmatrix} = \begin{bmatrix} a\cdot a & b\cdot a & c\cdot a \\ a\cdot b & b\cdot b & c\cdot b \\ a\cdot c & b\cdot c & c\cdot c \end{bmatrix} \begin{bmatrix} F_1' \\ F_2' \\ F_3' \end{bmatrix}$$

|| to $A\cdot X = B$

↳ Lot of processing must be done

⇒ $e_1\, e_2\, e_3$ → orthonormal

$$e_1\cdot e_2 = e_3\cdot e_2 = e_3\cdot e_1 = 0$$
$$e_1 e_1 = e_2\cdot e_2 = e_3\cdot e_3 = 1$$

$$\begin{bmatrix} F e_1 \\ F e_2 \\ F e_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \Rightarrow \begin{aligned} F\cdot e_1 &= F_1 \\ F\cdot e_2 &= F_2 \\ F\cdot e_3 &= F_3 \end{aligned}$$

↳ $F\cdot e_j = F_j$

$$\Rightarrow F = F_1 e_1 + F_2 e_2 + F_3 e_3 = \sum_{i=1}^{3} F_i e_j = \Sigma F_i e_i = F_i e_i$$

$$\Rightarrow \Sigma F_i e_i = \Sigma (F\cdot e_i) e_i = \underline{F}$$

→ Orthonormality is not a necessity but a convenience

De Allemberts Principle

if net $F \neq ma$ → inertial force → there is an acceleration

net $F = ma$ → no net acceleration

→ Timoshenko Model

12-04-22 : Lec 5

RIGIDBODY vs DEFORMED BODY

- Rigid body — doesn't have relative motion between the particles of the body
  ↓
  Definition : Rigid body is one for which all parts maintain same positions relative to one another wrt force
- Deformable body — it is opposite of the rigid body

Rigid Body

deformable body

Characteric of Rigid body :—

* We must define equivalent of Force & displacement at a microscopic level

Macroscopic : Force displacement
Microscopic : Stress (σ) Strain (ϵ)

$\sigma = f(\epsilon)$ → function constitution law

$$\boxed{\sigma = E\epsilon}$$

→ Youngs Modulus

Young's Modulus : (E) to deal at a microscopic level

- It defines material property
- It tells too the constitutive response of a material
- It is a characteristic of a material — unchangable.

⇒ If the stiffness (Young's Modulus) is very high (infinite) then that body is called as Rigid body.

⇒ In certain assumptions, if a body >>> than the others (Stiffness) then that body can be considered as a rigid body

compared to other
→ It can be relative

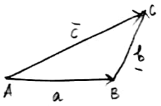
Ex: A ~~soft~~ diamond can be considered as a rigid body compared to a soft polymer

FORCE

→ Vector, needs 3 components to define
→ It has magnitude & direction
→ Point of Application of force is also important

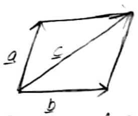
⇒ vector algebra :

i) $\underline{c} = \underline{a} + \underline{b}$ → [Vector addition]



Triangle law

$\underline{a} = \vec{AB}$
$\underline{b} = \vec{BC}$
$\underline{c} = \vec{AC}$



Parallelogram law

EQUILIBRIUM (Static Equilibrium)

I] $\underline{a} + \underline{b} \equiv 0$ [It has to zero] $\underline{a}, \underline{b}$ are two forces

* point C coincides with A ⇒ $AB = -BC$, $\underline{a} = -\underline{b}$ } equal in magnitude, opposite in direction.

* point of application on same line (collinear)

line AB & CD must coincide.

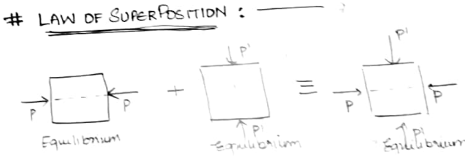
↪ $\Sigma F = 0$, $\Sigma M = 0$ } To be in equilibrium Both Forces & Moment must be equal to 'zero'

Summation of

Can be called as Generalised Forces

conjugate of energy

LAW OF SUPERPOSITION :



Definition : * "The action of a given system of forces on a rigid body will no way change, if we add or substract from them another system of forces in equilibrium"

* Law can used to split a complex problem into simpler problems

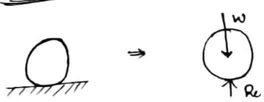
LAW OF TRANMISSIBILITY

"Point of application of a force may be transmitted along its line of action without changing the effect of the force, on rigid body"

→ but the same thing not applicable for law of Transmissibility

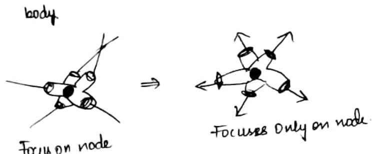
FREE BODY DIAGRAM

I]

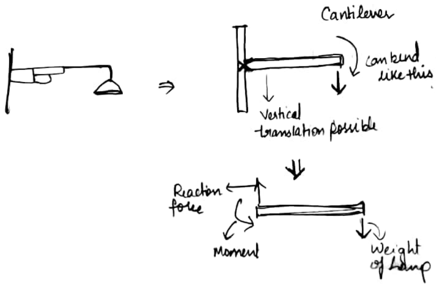


* To show net influences on the body, focusing only on the body.

II]

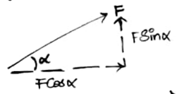


III]



RESOLUTION OF FORCE

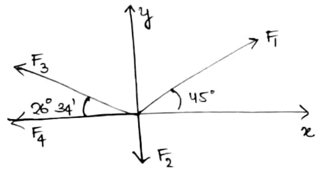
By method of projection



$F_x = F\cos\alpha$ — Horizontal component of F

$F_y = F\sin\alpha$ — Vertical Component of F

Magnitude $|F| = \sqrt{F_x^2 + F_y^2}$; $\tan\alpha = \frac{F_y}{F_x}$



$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$

10169.3345

6606.4384 +
3562.8961

$\vec{F} = F_1\cos 45\hat{i} + F_1\sin 45\hat{j} - F_3\cos(26°34')\hat{i} + F_3\sin(26°34')\hat{j} - F_2\hat{j} - F_4\hat{i}$

$\vec{F} = (F_1\cos 45 - F_3\cos(26°34') - F_4)\hat{i} + F_1\sin 45' + F_3\sin$

$150\cos 45 - 120\cos(26°34') - 80$

$150(0.707) - 120(0.8944) - 80$

$106.05 - 107.33 - 80$

$= -81.28\hat{i}$

$F_y = F_1\sin 45' + F_3\sin 26°34' - F_2$

$= 150\sin 45 + 120\sin 26°34' - 100$

$=$ ~~150 (0.4292)~~ +

$150(0.707) + 120(0.497) - 100$

$106.05 + 53.64 - 100 = 59.69$

~~$\tan(180-\alpha) = \frac{59.69}{81.28}$~~

$\tan\alpha = \frac{F_y}{F_x}$

$= \frac{59.69}{-81.28}$

0.7343

36.289

#

of a force is a me

TRUSSES

Fix – Fix Support :— $R_x \longrightarrow$ $\uparrow R_y$; Roller Support :— $\uparrow R_y$.

→ Reaction forces in both x & y - dir

→ Restrict moment in both dir

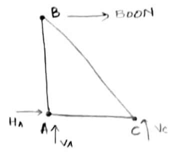
→ Moment, exist because, there is no rotation

→ Opposing in only y-direction.

→ Can move freely in x-direction.

→ R_y & $[M_z = 0]$

⟹ Reaction force is there to restrict moment



I] $\Sigma F_x = 0$

$500 + H_A = 0$

$H_A = -500$

II] $\Sigma F_y = 0$

$V_A + 0 + V_c = 0$

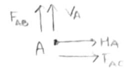
$V_A + V_c = 0$

III] $\Sigma M_z = 0$

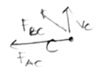
$-500 \times 2 + V_z \times 2 = 0$

$V_c = 500$

At point A



At point C



⟹

$5\hat{i} \times d$

$(2\cos 60° \hat{i} + 2\sin 60° \hat{j}) \times 5\hat{i}$

$\Sigma F_x = 0 \Rightarrow R_{Ax} + 5KN = 0$

$R_{Ax} = -5KN$

$\Sigma F_y = 0$

$R_{Ay} + R_{cy} - 5KN = 0$

$R_{Ay} + R_{cy} = 5KN$

$R_{Ay} = 5 - 5\sin 60°$

$\Sigma M_z = 0$ at A

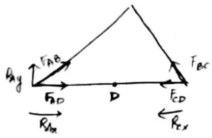
$-5 \times \cancel{2} \times \sin 60° + R_{cy} \times \cancel{2}m = 0$

$R_{cy} = 5\sin 60°$

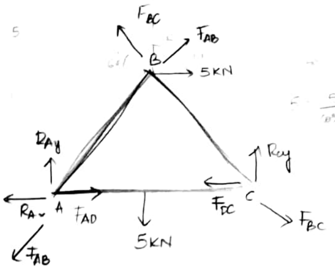
$= 4.33$

Scanned with CamScanner

At point A



$\left(\frac{5-\sin 60^\circ}{\sin 60^\circ}\right)\cos 60^\circ = 5 + 5\cos 60^\circ$



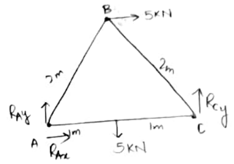
$F_{BC}\cos 60^\circ = 5KN + F_{AB}\cos 60^\circ$

$5\cos 60^\circ = 5 + \frac{5 - 5\sin 60^\circ}{\sin 60^\circ} \times \cos 60^\circ$

$$\frac{5\sin 60^\circ + 5\cos 60^\circ - 5\sin 60^\circ \cos 60^\circ}{\sin 60^\circ}$$

$\frac{5\sqrt{3}}{2} + \frac{5}{2}$

$\frac{5\sqrt{3}}{2} + \frac{5}{2} - \frac{5\sqrt{3}}{4}$



$\Sigma F_x = 0$

$R_{Ax}\hat{i} + 5KN\hat{i} = 0$

$R_{Ax}\hat{i} = -5KN\hat{i}$

$\boxed{R_{Ax} = -5KN\hat{i}}$ (i)

$\Sigma F_y = 0$

$R_{Ay} + R_{Cy} = 5KN$

ΣM_z at A

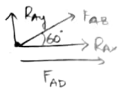
$5 \times 2 \times \sin 60 = 2 \times R_{Cy}$

$\boxed{R_{Cy} = 5\sin 60^\circ}$ (ii)

$R_{Ay} + 5\sin 60^\circ = 5$

$\boxed{R_{Ay} = 5 - 5\sin 60^\circ}$ (iii)

At point A



$\rightarrow R_{Ay} + F_{AB}\sin 60^\circ = 0$

$F_{AB} = \frac{-R_{Ay}}{\sin 60^\circ}$

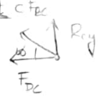
$= -\left(\frac{5 - 5\sin 60^\circ}{\sin 60^\circ}\right)$ — (iv)

$\rightarrow R_{Ax} + F_{AD} + F_{AB}\cos 60^\circ = 0$

$-5 + F_{AD} - (5 - 5\sin 60^\circ)\cot 60^\circ = 0$

$F_{AD} = (5 - 5\sin 60^\circ)\cot 60^\circ + 5$

At point C



$\rightarrow R_{Cy} + F_{BC}\sin 60^\circ = 0$

$F_{DC} + F_{BC}\cos 60^\circ = 0$

$F_{BC} = \frac{-R_{Cy}}{\sin 60} = \frac{-5\sin 60}{\sin 60^\circ} = -5$

$\rightarrow F_{DC} + F_{BC}\cos 60^\circ$

$F_{DC} = -F_{BC}\cos 60^\circ$

$= 5\cos 60^\circ$

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MOMENT

Force tends the object to translate
* Moment tends the object to rotate

Def^n: The moment of a force is a measure of its tendency to cause a body to rotate about specific axix

$$M = \text{Force} \times \perp^r \text{distance}$$
(Nm) (N) (m)

For 3-dimensional sense

$$M = r \times F = \underline{r} \wedge \underline{F} \quad (\text{Cross Product})$$

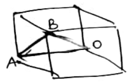
$\Rightarrow$ $\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\underline{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \Rightarrow \begin{aligned} M_x &= yF_z - zF_y \\ M_y &= zF_x - xF_x \\ M_z &= xF_y - yF_x \end{aligned}$$

* The M was. wrt Origin

$\Rightarrow$



* $M_A = \underline{r}_{B/A} \wedge \underline{F}_B$

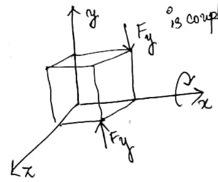
$r_{B/A} = OB - OA$

* $M_B = 0$

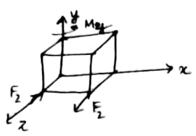
$\Rightarrow$ If a point is passing through the line of Action the moment upon that point is zero.

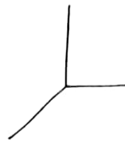
COUPLE

↳ The forces are in equilibrium but $M \neq 0$



is couple.

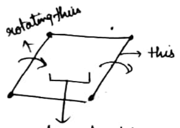




→ ※ Out of plane moment is called Torques.

WHAT IS STRUCTURE ??

⇒ Structure is a combination of links which supports or transfers the force & safely withstand the applied force.



⇒ TRUSSES : A framework composed of members – joined at this end to form a rigid structure

ASSUMPTION

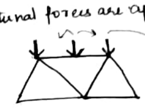
I] All members of the truss are axially loaded ⇒ only Tension & Compression

※ no bending or shear stress

II] Weight of the member is small compare to the force it supports [neglect the weight of the rods]

III] No effect of bending on member

IV] External forces are applies are at pin or joints



can be transferred in such a way that the torque is balanced

STABLE Vs UNSTABLE

Structure is statically determinate :—

Static Equilibrium Eqns : $\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M = 0$ — (1)

If no. eqn = no. variables → Statically determinent

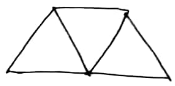
Otherwise not

⟹ $m + \underset{\text{reaction forces}}{3} = 2j$ → no. of joints in truss

m → no. of member of truss

Statically determinent

Ex:



$7 + 3 = 2(5)$

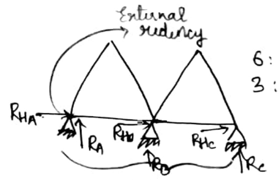
⟹ $m + 3 > 2j$ → Statically Indeterminent

$m + 3 < 2j$ → UNSTABLE

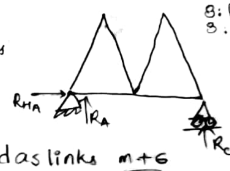
⟹ SD → solved → using easy equation — (1).

⟹ ※ First is to see if it is statically determinent

External redundancy

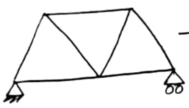


6 : forces
3 : equations



8 : forces
3 : equations

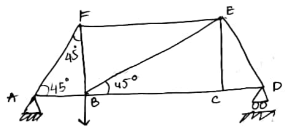
joints are also considered as links $m + 6$



→ Simple Supported Trusses

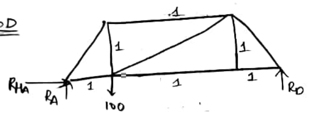
$m+3 = 2(j)$
↓
No of Reaction forces

Q]



$m+3=2j$
$9+3 = 2(6)$
$\Rightarrow 12 = 12$ ✓

FBD



Calculating Reaction forces

$\Sigma F_x = 0$

$R_{HA} = 0$

$\Sigma F_y = 0$

$R_A + R_D = 100$

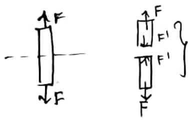
$R_A = \frac{2(100)}{3}$

$\Sigma M_A = 0$

$1(100) = R_D(3)$

$R_D = \frac{100}{3}$

→ External vs Internal

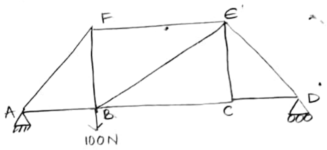


} For Equilibrium

F' → Internal Forces
F → External Forces

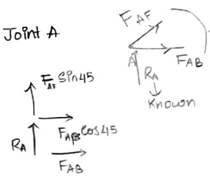
$\sigma_{ys} = 80N$ ⟶ to see if it sustains we must calculate the internal forces
↓
yield strength

↳ METHOD OF JOINTS



What joint shall we take : (i) "Over that joint we should know atleast 1 force"
(ii) "Should not have more than two unknown forces"

→ Joint A



⟹ Satisfies conditions

$\Sigma F_x = 0$
$\Sigma F_y = 0$

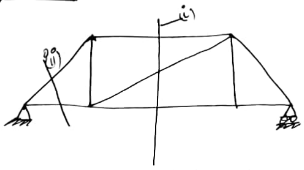
$F_{AF}\sin 45 = -R_A$

$\boxed{F_{AF} = -R_A \times \sqrt{2}} = -94.2\,N$

$\boxed{F_{AB} = \frac{200}{3}}$

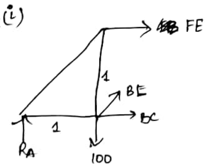
→ Joint F

Method of Section



→ The section, that we are cutting should not have more than 3 unknown forces.

→ Section (i)



$M_B = R_A + \cancel{B}FE$

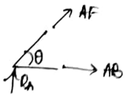
$\cancel{B}FE = -R_A = -\frac{200}{3}$ → $\Sigma F_x = 0$ & $\Sigma F_y = 0$

BE Sin θ

↓

FE + BC + BE Cos θ = 0

⇒ Section (ii)



Lec 9

6-05-22

Examples of Equilibrium

cantilever beam

* 6° of freedom : 3 translational
3 rotational

for a beam

↝ Axial loading : BEAM VS BAR

* Along the axis of the rod

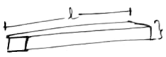
⇒ BAR

↝ Transverse loading :

* ⊥ to the axis of the rod

⇒ BEAM
Only transverse load

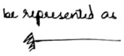
⇒



} a (cross section area) → * If a << l

Can be of any shape

Can be represented as 1D



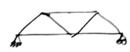
TRUSS : It is 2-D representation ↝ 2D structure

↓

made up of 1D bars

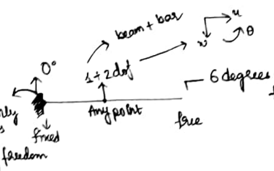
↝ It is only bars

↝ Joints are pins



TYPES OF BEAMS :

→ Cantilever beam : One end is fixed

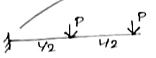


→ Simply supported beam :

→ Fixed - Fixed beam :

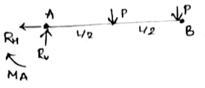
→ The crossection can be of any shape.

EXAMPLE 1:



It is fixed no movement
→ in HORIZONTAL & VERTICAL DIRECTION
→ Doesn't Allow ROTATION

If BEAM IN EQUILIBRIUM (Should mention first)



$\Sigma F_y = 0$

$R_V = 2P$

$\Sigma F_x = 0$

$R_H = 0$

⟶ No translation R_H & R_V $\quad \Sigma M_A = M_A - P.L/2 - P.l = 0$

⟶ No Rotation M_A

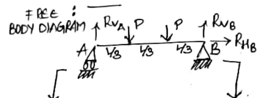
$\Rightarrow M_A = PL/2 + PL$

$= 3PL/2$

$R_H = 0$; $R_V = 2P$; $M = 3PL/2$

EXAMPLE 2:

FREE BODY DIAGRAM :



Horizontal allowed R_H ×	Horizontal not allowed R_H ✓
Vertical not allowed R_V ✓	Vertical ~~a~~ not allowed R_V ✓
Rotation allowed M ×	Rotation allowed M ×

⇒ It can sag → considering the tangent we can see it rotates

$\Sigma F_x = 0$ $\quad \Sigma F_y = 0$ $\quad \Sigma M_{\text{at any point}} = 0$ (lets take A)

↳ $R_{HB} = 0$

$-P - P + R_{VB} + R_{VA} = 0$

$R_{VB} + R_{VA} = 2P$

$P + R_{VA} = 2P$

$R_{VA} = P$

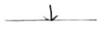
$R_{VA}(0) - P(L/3) - P(2L/3) + R_{VB}(L) + R_{HB}(0) = 0$

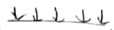
$R_{VB} \times L = PL \Rightarrow R_{VB} = P$

→ $\boxed{R_{VA} = P \;;\; R_{VB} = P \;;\; R_{HB} = 0}$

Lec10: 10-05-22

TYPES OF LOADS :—

→ Point : Act at a point/node 

→ UDL : ~~I~~ Load is distributed of the beam uniformly 

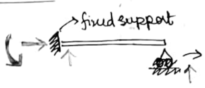
→ UVL : Load is uniformly varyling wrt length 

SHEER FORCE Vs BENDING MOMENT DIAGRAM :—

↳ Help us analyse the beam.

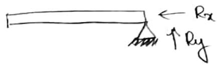
↳ Helps us understanding properties of materials that can be used

I] SIMPLE SUPPORT



II] ROLLER SUPPORT

III] HINGED SUPPORT



✱ ⟶

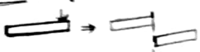
 R_x R_y M : Cant translate : both x & y dirn
Cant rotate

R_y : cant translate in y dirn
Can trdlate in x dirn
Can rotate

 R_x R_y : Can't translate both x & y dirn
Can rotate

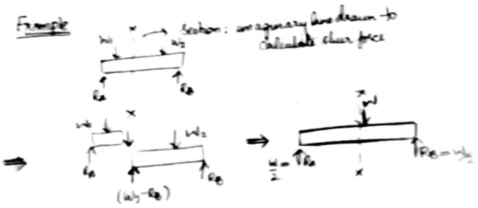
Bending moment : Moment responsible for bending of the beam

Shear force : Algebraic sum of vertical forces of a section. (Either to the left or right)



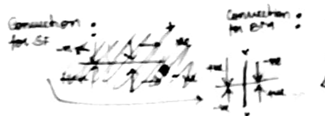
Example

Section : an imaginary line drawn to calculate shear force



Convention for SF :

Convention for BM :



CCW → +ve

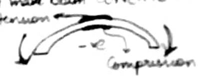
CW → −ve

SAGGING : The moment that make beam CONCAVE UPWARD

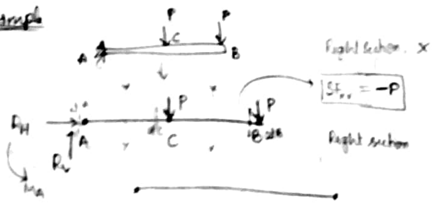
↳ +ve BM

HOGGING : The moment that make beam CONCAVE DOWNWARD

↳ −ve BM



Example



Right section. XX

$\boxed{SF_{xx} = -P}$

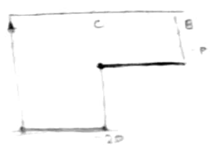
Right section

Shear Force :

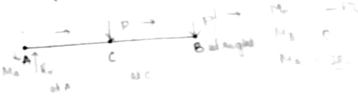
$SF_B = -P$

$SF_C = -2P$

$SF_A = -2P + R_V = 0$



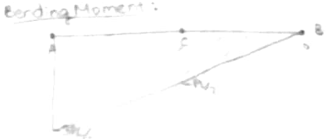
→ We must take left limit at a point that means we must be able to consider the force acting on that point as well



→ We must take right limit

~~force~~ acting not included

Bending Moment :

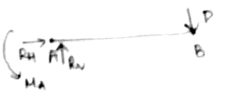


→ For BM & SF the right limit & the left limit are equal

↳ When considering left we must include the force/Moment acting at the point also

↳ When considering right limit we must not include the force/Moment

Example



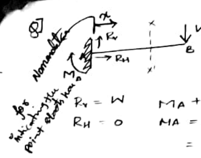
$SF_B = -P$

$SF_A = 0$

$BM_B = 0$

$BM_A = -PL$

Q]

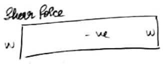


$R_V = W$ $\quad M_A + W = 0$ $\quad SF_B = -W$

$R_H = 0$ $\quad M_A = -W$ $\quad SF_A = -W$

$= Wl$

Shear Force

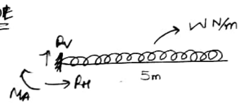


$BM_{xx} = -W \times x$

$x = 0 \Rightarrow BM_{xxB} = -W \times 0 = 0$

$x = l \Rightarrow BM_{xxA} = -W \times l = -Wl$

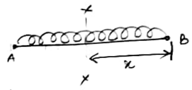
Q] UDL



$\Rightarrow$ Load $= W \times l$

$= 2 \times 5 = 10N$

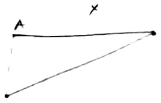
$R_V = 10N$



$SF_{xx} = -W \cdot x$

at $x = 0$ $\quad SF_{xxB} = -W \times 0 = 0$

$SF_{xxA} = -W \times l = W \times l$



$BM_{xx} = \int F\, dx$

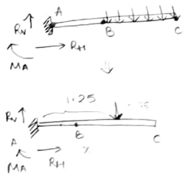
$BM_{xx} = -\frac{Wx^2}{2}$

at $x = 0$

Tutorial

$W = 1$ kN/m

(a)



$TW = 1$ kN/m $\times 1.5$

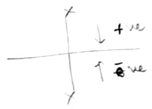
$= 1.5$ kN

$R_V = 1.5$ kN $\quad R_H = 0$

$M_A = 1.5$ kN $\times 1.25$

$= 1.1875$

b)



CCW +ve

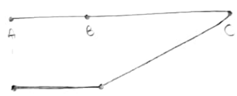
CW -ve

$R_H = 0$

$R_V = +1.5$ kN

$M_A = +1.1875$ kNm (CCW)

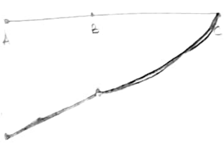
c)



$SF_{xx} = -Wx = -1$ kN x
till 1.5 (B)

$SF_{xx} = -1.5N$
from 1.5 (B to A)

d)



$BM_{xx} = \frac{Wx^2}{2} = -1x^2/2$
till 1.5

$BM_{xx} = W(1.5) \times (x - 0.75)$
from B to A

e)

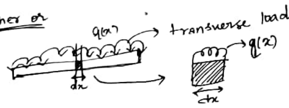
AB : -1.5 $\qquad$ $AB = -1.5(x - 0.75)$

BC : -1 kN x $\qquad$ $BC = -1$ kN $x^2/2$

Scanned with CamScanner

$u \rightarrow$ movement in x-dir
$v \rightarrow$ movement in y-dir } degrees of freedom
$\theta \rightarrow$ Rotation

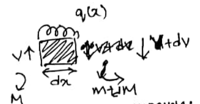
⇒ Either or

⇒ 

Taylor series ⇒ $f(x+h) = f(x) + \frac{f'(x)h}{1!} + \underbrace{\frac{f''(x)h^2}{2!} + O(h^3)}_{\text{neglected}}$

$\Downarrow$ $h \to 0$

$f(x+h) \approx f(x) + \frac{f'(x)h}{1!} = f(x) + \Delta f$

⇒ 

FOR EQUILIBRIUM:

→ $\Sigma f_x = 0$ (trivially satisfied)

→ $\Sigma f_y = 0$ $\quad V - (V + dV) - q\,dx = 0$

$-dV - q\,dx = 0$

$dV = -q\,dx \Rightarrow \boxed{\frac{dV}{dx} = -q(x)}$

→ $\Sigma M = 0$

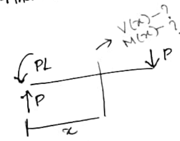
$-V\,dx + \cancel{M} - (\cancel{M} + dM) + \underbrace{(q\,dx)\frac{dx}{2}}_{\text{negligible } (dx)^2} = 0$

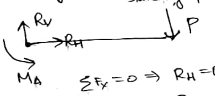
$V\,dx = -dM \Rightarrow \boxed{\frac{dM}{dx} = -V}$

Calculation of SFD & BMD

(i) Find out all the values of force acting on the system

* Can take moment about any point because Statically equilibrium

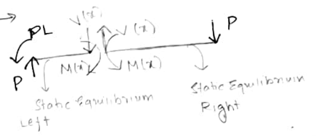
⇒ 



$\Sigma F_x = 0 \Rightarrow R_H = 0$

$\Sigma F_y = 0 \Rightarrow R_V = P$

$\Sigma M_A = 0 \Rightarrow M_A = PL$



LEFT

$\Sigma f_y = 0 \Rightarrow P - V(x) = 0$

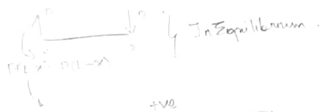
$\boxed{V(x) = P}$ *

$\Sigma M_A = 0$

$PL - Vx - M(x) = 0$

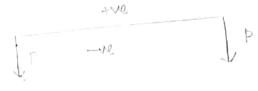
$M(x) = PL - Px$

Verification with Right

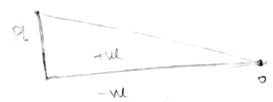


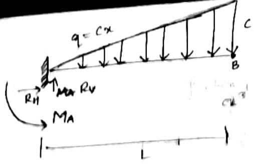
Sign Convention: ↑ -ve ↓

Diagram



BM





$Load = \frac{1}{2} \times L \times CL = \frac{CL^2}{2}$

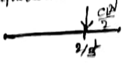
$\Sigma f_x = 0 \Rightarrow R_H = 0$

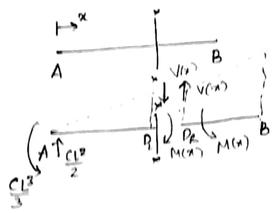
$\Sigma f_y = 0 \Rightarrow R_V = \frac{CL^2}{2}$

$M_A = \frac{CL^2}{2} \times \frac{2L}{3}$

$M_A = \frac{CL^3}{3}$

↳ Equivalent loading





LEFT

$\frac{CL^2}{2} - V(x) - \frac{Cx^2}{2} = 0$

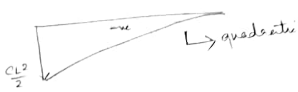
$V(x) = \frac{C}{2}(L^2 - x^2)$

$\frac{CL^3}{3} - \frac{C}{2}(L^2 - x^2) \times x - \frac{Cx^3}{3} - M(x)$

$M(x) = \frac{C}{3}(L^3 - x^3) - \frac{Cx}{2}(L^2 - x^2)$

SFD

$V(x) = \frac{C}{2}(L^2 - x^2)$



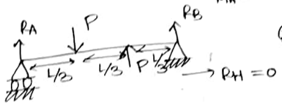
BMD

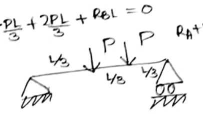
$M(x) = \frac{CL^3}{3} - \frac{CL^2x}{2} + \frac{Cx^3}{6}$

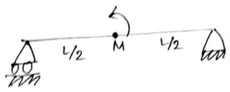
$R_A + R_B = 0$

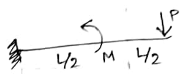
$M_A = 0 \Rightarrow -\frac{PL}{3} + \frac{2PL}{3} + R_B L = 0$

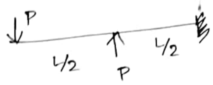
$R_A + R_B = 2P$

① 

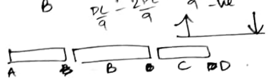
② 

③ 

④ 

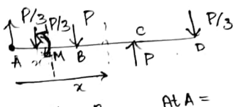
⑤ 

B $\frac{PL}{9}$ $\frac{2PL}{9}$ $-\frac{PL}{9}$ -ve



① $\frac{PL}{3} + R_B L = 0$

$R_B = -P/3$



A $-P/3$ B

B $2P/3$ C

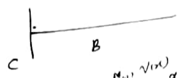
C $-P/3$ D

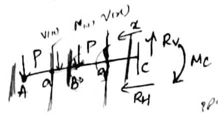
At A =

$0 - P/3\, x + M = 0$

$M = P/3\, x$.

$-3Pa + Pa$





↓ +ve ↻ +ve

$M_c = Pa + 2Pa$

$M_B =$

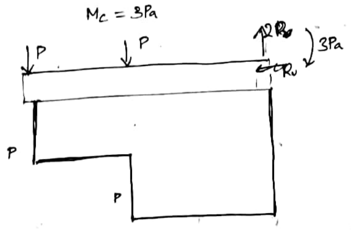
$-R_V + 2P = 0$

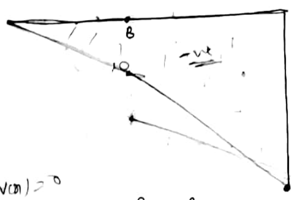
$R_V = 2P$

$R_H = 0$

$-M_c + Pa + P(2a) = 0$

$M_c = 3Pa$





$-2P + v(x) = 0$

$v(x) = 2P$

$v(x) = P$

$-R_V + P + v(x) = 0$

$-2P + P + v(x) =$

$v(x) = P$

$x = a$

$-M + Pa + v(x)(x-a) + M(x) = 0$

$(x-a)$

$-3Pa + Pa + P(x-a) + M(x) = 0$

$M(x) = +3Pa - Pa - P(x-a)$

$2Pa$

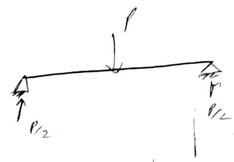
$x=0$

$x = 0$

$x = a$

$(+3Pa) - Pa$

$(2Pa)$



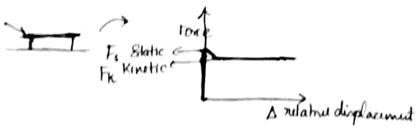
FRICTION

\# Frictionless : ① No restriction on relative motion
② Full restriction

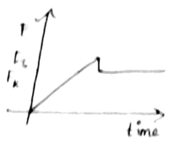
"why is friction so important ?"
↳ helps us to write, to walk . . .

⇒ FRICTION → ~~Static~~ Dry Friction → Static / ~~Dynamic~~ Kinetic/Dynamic
→ Fluid Friction

⇒



⇒



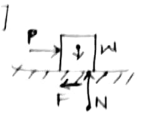
⇒ Static Friction

$$\boxed{F_s = \mu_s N}$$

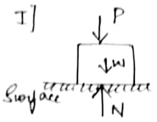
⇒ Kinetic Friction

$$\boxed{F_K = \mu_K N}$$

⇒ II]



I]

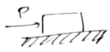


\# Force is along Δ (displacement) horizontal
Frictional force F : Opposing the force P

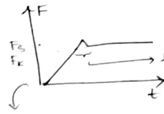
\# No relative motion because F ⊥^{r} with horizontal

→ the static friction follows few laws (Columbic laws of static friction)

⇒



Frictional force



→ happens at very short duration of time

The frictional force F is equal to the applied force uptill some extent And then F_s starts acting

STATIC/COLUMBIC FRICTION

→ $F_s \propto N$ (Effective weight)
↳ F_s also depends on the surfaces in contact

$$\boxed{F_s = \mu_s N}$$

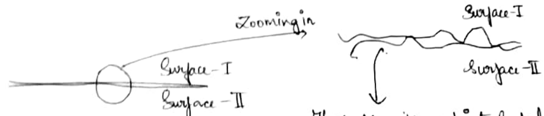
→ COEFFICIENT OF STATIC FRICTION
↳ * Depends upon the surface's characteristics

KINETIC FRICTION

$F_k \propto N \Rightarrow F_k = \mu_k N$

* Friction starts with contact stresses

⇒



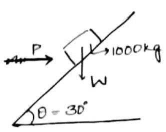
↳ These Asperities get interlocked and it resists
↳ This can be analysed in two steps
Macroslip
Microslip

Macroscopically different manifestation

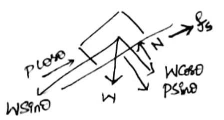
Microscopically 〃

Contact physics : K L Johnson theory

Problem I



Static Equilibrium



$\Sigma F_{\text{v wrt plane}} = 0$

$-P\sin\theta - W\cos\theta + N = 0$

$\Sigma F_{H \text{ wrt plane}}$

$P\cos\theta + f_s - W\sin\theta = 0$

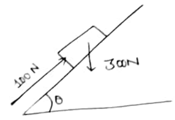
$f_s = \mu_s N \Rightarrow N = P\sin\theta + W\cos\theta$

$f_s = \mu_s(P\sin\theta + W\cos\theta)$

$P\cos\theta + \mu_s(P\sin\theta + W\cos\theta) - W\sin\theta = 0$

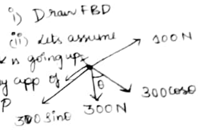
$\mu_s = 0.25$

$\mu_k = 0.20$

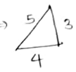


i) Draw FBD

(ii) Lets assume Block is going up by app of force P



θ ⇒



$\Sigma F_x = P - W\sin\theta - f = 0$

$\Sigma F_y = \cancel{P\sin} N - W\cos\theta = 0$

$N = \cancel{300}^{60} \times \frac{4}{\cancel{5}} = 240\text{N}$

$100 - \cancel{300}^{60} \times \frac{3}{\cancel{5}} - f = 0$

$100 - 180 = f_{equi} \Rightarrow -80$

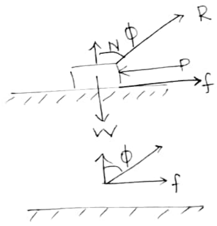
direction opposite

$f_{max} = \mu_s N = 60\text{N}$] Kinetic friction

$f_{equilibrium} = 80\text{N}$ $f_{ki} = 48\text{N}$

↳ Block will slide downwards

ANGLE OF FRICTION (ϕ) :



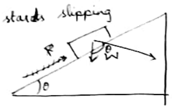
$R = \sqrt{N^2 + f^2}$

'ϕ' the angle of friction

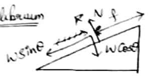
$\tan\phi = f/N$

Angle of Repose (θ/α) :

The minimum angle ~~for tetu~~ of the inclined ~~plate~~ plane, for when it starts slipping



Equilibrium



$\Sigma f_x = 0$

$\cancel{R} - W\sin\theta + f = 0$

$\Sigma f_y = 0$

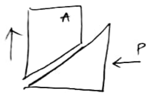
$N - W\cos\theta = 0$

$f = W\sin\theta$; $N = W\cos\theta$; $f = \mu N$

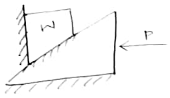
$\boxed{\mu = \tan\theta} \Rightarrow \theta = \tan^{-1}(\mu)$

APPLICATION OF WEDGE

↳ triangular block, which can move

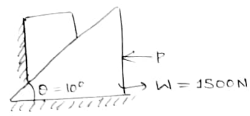


If the wedge moves ← the block A moves ↑

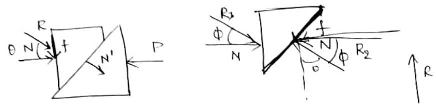


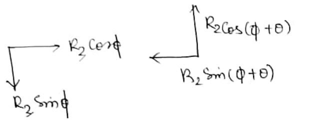
(*) Consider it a equilibrium first

Problem



$\mu = 0.3$ (on all surface)





$R_2 = 1977.45$
$R_3 = 927.54$

$$\left.\begin{array}{l} R_3\cos\phi - R_2\sin(\phi+\theta) = 0 \\ R_2\cos(\phi+\theta) - R_3\sin\phi - W = 0 \end{array}\right\} \text{2 eqn 2 unknowns}$$

$\theta = 10$
$\phi = 16°12' \approx 16$

$$R_3\cos\phi = R_2\sin(\phi+\theta)$$

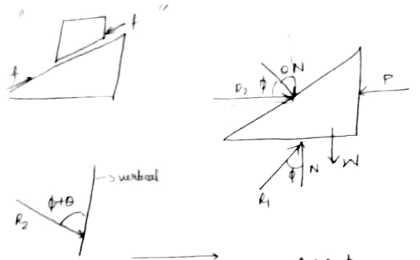
$$R_3\cos\phi\cot(\phi+\theta) - R_3\sin\phi = W$$

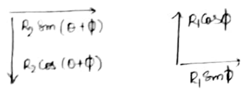
$$R_3[\cos\phi\cot(\phi+\theta) - \sin\phi] = W$$

$$R_3 = \frac{W}{\cos\phi\cot(\phi+\theta) - \sin\phi} \Rightarrow R_2 = \frac{W\cos\phi}{\cos\phi\cos(\phi+\theta) - \sin\phi\sin(\phi+\theta)}$$

~~0.9621~~ $R_3 = \dfrac{1500}{0.961 \times 2.083 - 0.275}$

$R_2 = \dfrac{W\cos\phi}{\cos(2\phi+\theta)} = \dfrac{1500 \times 0.961}{0.7431}$





$$R_1 \sin\phi + R_2 \sin(\theta+\phi) - P = 0$$

$$R_2 \cos(\theta+\phi) - R_1 \cos\phi + W = 0$$

$$R_1 = \frac{P - R_2 \sin(\theta+\phi)}{\sin\phi}$$

$$R_2 \cos(\theta+\phi) + W = \left[P - R_2 \sin(\theta+\phi)\right] \cot\phi$$

$$1977.45 \, \cancel{\cos}(0.898) + \cancel{W}\, 1500 = \frac{[P - 1977.45 \times 0.669]}{0.2867}$$

~~3678~~

939.157

$P = 2262.07$

MOMENTS OF AREA

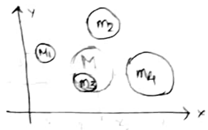
Classical Mechanics	Solid Mechanics
* Rigid	* Deform
* Point mass	* Larger bodies
* Shape doesn't matter because of lumped form	* Collection of all such point masses

⇒ What is "centroid"?

The point equivalent of a certain mass.

Centroid vs Centre of gravity

⇒ How to pick a "Centroid"?



Total Mass: Sum of all the point masses

$$M = m_1 + m_2 + m_3 + m_4$$

→ First Area moment :

$$M\bar{x} = m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4$$

$$\bar{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{M} = \frac{\sum_{i=1}^{n} m_ix_i}{\sum_{i=1}^{n} M_i}$$

||ly

$$\bar{y} = \frac{\sum_{i=1}^{n} m_iy_i}{\sum_{i=1}^{n} m_i}$$

⇒ $\Sigma \rightarrow \int$

$$\bar{x} = \frac{\int_\Omega dm\,x}{\int_\Omega dm} \qquad \bar{y} = \frac{\int_\Omega dm\,y}{\int_\Omega dm}$$

⤳ $\bar{x}$ & $\bar{y}$ map the body's translation → inertia for translation is mass

(for ROTATION?) → inertia for rotation also depends on "axis"

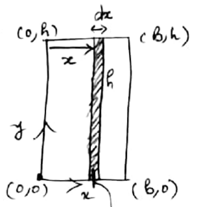
Example

Rectangular Lamina

↳ Mass defined as

Mass density (ρ)

ρ = mass / Area



$dm = \rho h dx$ ← centre of the rectangular strap

$$M = \int_0^b \rho h dx = \rho h b \Rightarrow \boxed{M = \rho h b}$$

First Moment

$$\bar{x} = \frac{\int dm\,x}{\int dm} = \frac{\int \rho h dx \times x}{\rho h b} = \int_0^b \frac{\rho h x^2}{2\rho h b}$$

$$= \frac{1}{2b}(b^2) = \frac{b}{2}$$

$$\bar{y} = \frac{\int dm\,y}{\int dm} = \text{(lly taking strip along } x\text{) (wrt } y\text{)} \; h/2,$$

$(\bar{x}, \bar{y}) = (b/2, h/2)$

Co-ordinate axes don't change the position (actual) of Centroid

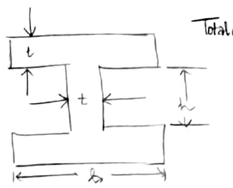
"location doesnt change, representation changes"

"composite section" — example: I

★ For simple sections memorize the values

↳ combination of simple sections

↳ Simple geometrical figures



Total area: $2bt + ht$

$(2b+h)t$

• $(b/2, t/2)$ $(b/2, t+h+t/2)$

• $(t/2, h/2)$ $(b/2, \frac{t+h}{2})$

• $(b/2, t/2)$

Global Coordinate System

⤳ If mass density is constant

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A} = b/2 \; ; \; A = A_1 + A_2 + A_3$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A}$$

$$= \frac{bt(3t/2 + h) + \cancel{t}h\left(\frac{t+h}{2}\right) + bt(t/2)}{(2b+h)t}$$

$$\neq \; 2bt^2 + \frac{3bht}{2} + \frac{bh^2}{2}$$

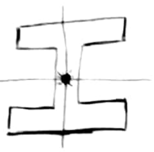
$$= \frac{\frac{3bt}{2} + bh + \frac{ht}{2} + \frac{h^2}{2} + \frac{bt}{2}}{2b+h}$$

$$= \frac{2bt + bh + \frac{ht}{2} + \frac{h^2}{2}}{2b+h}$$

$$= t + \frac{h}{2}$$

Symmetry (material & mass density uniform)

- $\bar{x}$ — lies on y symmetric axis
- $\bar{y}$ — lies on x symmetric axis



"Should always mention symmetry" if present

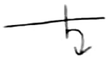
SECOND MOMENT OF AREA

⇒ Mass Vs Moment of Inertia

↓ Inertia against translation

↓ Inertia against Rotation

Beam:



Crossectional area

#

Bending

⇒ the beam deforms

While deforming, its translating

⇒ tangents at every point is diff so every point is rotating differently

#

Pure bending

⇒ Sagging

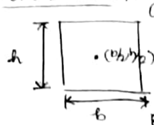
BENDING VS PURE BENDING

↓

doesn't involve Sheer Force

only Bending Moment

PURE BENDING:



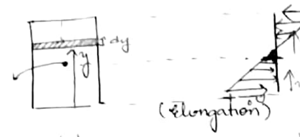
h, b

• $(b/2, h/2)$ Centroid

Compression

Expr Elongation

⇓



Centroid

dy, y

$dA = b\,dy$

$-\sigma_0$ (compression)

(Elongation)

y'

$y \Rightarrow y = y' + h/2$

$$\boxed{\sigma = \frac{-2\sigma_0}{h}(y - h/2)}$$

No net force cause its pure bending

Total force : $\int_0^h \sigma \, dA$

$$TF = \int_0^h \frac{-2\sigma_0(y - h/2)}{h} \times (b\,dy) = c\int_0^h A\,dy$$

$$= -\frac{2\sigma_0 \times b}{h} \int_0^h (y - h/2)\,dy$$

$$= \frac{-2\sigma_0 b}{h}\left[\frac{y^2}{2} - \frac{h}{2}y\right]_0^h$$

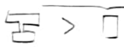
$$= \frac{-2\sigma_0 b}{h}\left[\frac{h^2}{2} - \frac{h^2}{2} - 0\right] = 0$$

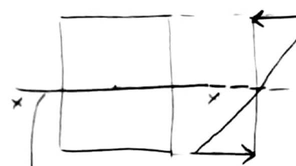
First Moment related to Force balance

Second Moment related to Moment balance.

Total moment ABOUT THE X AXIS : $\int_0^h \sigma(y)\,dA \Rightarrow \int_0^h \frac{-2\sigma_0(y - h/2)}{h}\, y\,(b\,dy) \neq 0$

More mass on the extreme ⟺ more inertia ⟺ more resistence ⟺ better

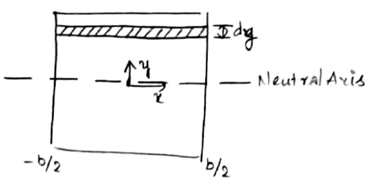




Neutral Axis : Stress is zero : line passing through centroid
or Centroidal Axis (Axial Stress)

It doesn't change the length but becomes an arc

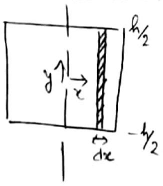
⇒



$I_{xx} = \int y^2 dA = \int y^2 b dy = by^3/3 \Big]_{-h/2}^{h/2}$

$\boxed{I_{xx} = \frac{bh^3}{12}}$ $= 2bh^3/3\times 8 = \frac{bh^3}{12}$

⇒



$I_{yy} = \int x^2 dA = \int x^2 h dx$

$= hx^3/3 \Big]_{-b/2}^{b/2} = \frac{hb^3}{12}$

$\boxed{I_{yy} = \frac{hb^3}{12}}$

⇒ Moment of Inertia is "a tensor"

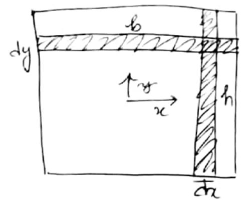
Measure of Symmetry

$I = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{bmatrix} \Rightarrow \underbrace{I_{xy} = I_{yx}}$

is a tensor

They tell about the degree of Symmetry

⇒



⇒ x symmetry

$I_{xy} = x\int y b dy = 0$

⇒ y symmetry

$I_{xy} = y\int x b dx = 0$

→ $I_{xy} = 0$ if x-symmetry / y-symmetry / both

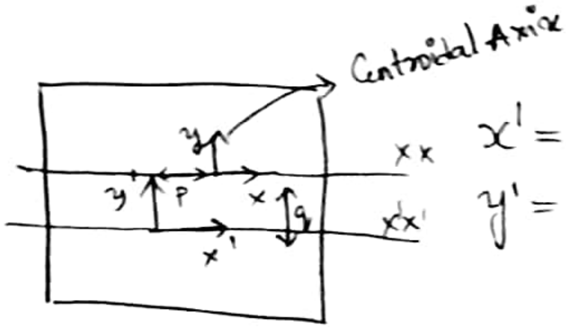
$\neq 0$ if x-symmetry and y-symmetry are absent
(maybe -ve/+ve)

I_{xx} or $I_{yy} > 0$ [$= 0$ if $A = 0$]

$I_{xx} = \int y^2 dA$

$I_{yy} = \int x^2 dA$

$I_{xy} = I_{yx} = \int xy\, dA$



$x' = x + p$

$y' = y + q$

Torque:
Moment out of plane

$$I_{x'x'} = \int (y')^2 dA$$

$$= \int (y+q)^2 dA = \int (y^2 + q^2 + \cancel{2yq}) dA$$

0 (Neutral Axis)

$$I_{x'x'} = I_{xx} + Aq^2 + 0$$

PARALLEL AXIS THEOREM

$$I_{x'x'} = I_{xx} + Aq^2$$

$$I_{y'y'} = I_{yy} + Ap^2$$

PERPENDICULAR AXIS THEOREM

(for Polar moment of Inertia)

$(x \perp y) \perp z$ then

$$J = I_{zz} = I_{xx} + I_{yy}$$

RADIUS OF GYRATION

$$I_{xx} = A K_r^2$$

equivalent annulus body