Quantum Mechanics Assignment

ADITYA SRIDHAR ES21BTECH11004

1. (a) $\lambda_{dB} = \frac{2\pi}{k} = \frac{2\pi}{50} \times 10^{-9} m = 4\pi \times 10^{-11} m$

$= \boxed{1.256 \text{ Å}}$

$$p = \frac{h}{\lambda_{dB}} = \frac{6.635 \times 10^{-34}}{1.256 \times 10^{-10}} = \boxed{5.283 \times 10^{-24} \text{ kgm/s}}$$

$$KE = \sqrt{p^2c^2 + m_0c^4} - m_0c^2$$

$$= c\sqrt{(5.283\times10^{-24})^2 + (9.11\times10^{-31}\times9\times10^{16})\times 9.11\times10^{-31}} - (9.11\times10^{-31}\times9\times10^{16})$$

$$= (3\times10^8)\sqrt{(27.91\times10^{-48}) + (74692.89\times10^{-48})}$$

$$= 3\times10^8 \times 273.351\times10^{-24} - 81.99\times10^{-15}$$

$$= 82.0053 \times 10^{-15} - 81.99\times10^{-15}$$

$$= \boxed{1.53\times10^{-17} J}$$

$$p = \gamma m_0 v$$

$$= \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

$$p^2 - \frac{p^2v^2}{c^2} = m_0^2 v^2$$

$$v^2 = \frac{p^2c^2}{p^2 + m_0^2c^2} \Rightarrow v = \frac{pc}{\sqrt{p^2 + m_0^2c^2}}$$

$$= \frac{(5.283\times\cancel{10^{-24}})(3\times10^8)}{\sqrt{273.351\times\cancel{10^{-24}}}}$$

$$= \frac{15.849}{2.73351} \times 10^6$$

$$= \boxed{5.798\times10^6 m/s}$$

(b) $\lambda_{dB} = \frac{2\pi}{50}\times 10^{-12} = \boxed{1.256\times 10^{-13}\ m}$

$p = \frac{h}{\lambda_{dB}} = \frac{6.635\times 10^{-34}}{1.256\times 10^{-13}} = \boxed{5.283\times 10^{-21}\ kgm/s}$

$KE = c\sqrt{p^2 + m_0^2c^2} - m_0c^2$

$= (3\times 10^{8})\sqrt{(27.91\times 10^{-42}) + (0.07469289\times 10^{-42})}$

$- 81.99\times 10^{-15}$

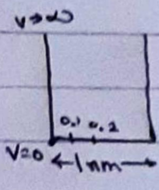
$= 15.8278\times 10^{-13} - 81.99\times 10^{-15}$

$= 1500.79\times 10^{-15}$

$= \boxed{1.5008\times 10^{-12}\ J}$

$v = \frac{pc}{\sqrt{p^2+m_0^2c^2}} = \frac{(5.283\times 10^{-21})(2.9979\times 10^{8})}{5.29\times 10^{-21}}$

$= \boxed{2.9939\times 10^{8}\ m/s}$

2. 

We know $\psi(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$ in a potential well.

$n = 1$ [ground state]

$a = 1\ nm$

$\therefore P(0.1nm \le x \le 0.2nm) = \int_{a/10}^{2a/10} \frac{2}{a}\sin^2\left(\frac{\pi x}{a}\right)dx$

$= \frac{1}{2}\cdot\frac{2}{a}\times\frac{a}{\pi}\int_{\pi/10}^{\pi/5}\sin^2 t\,dt$

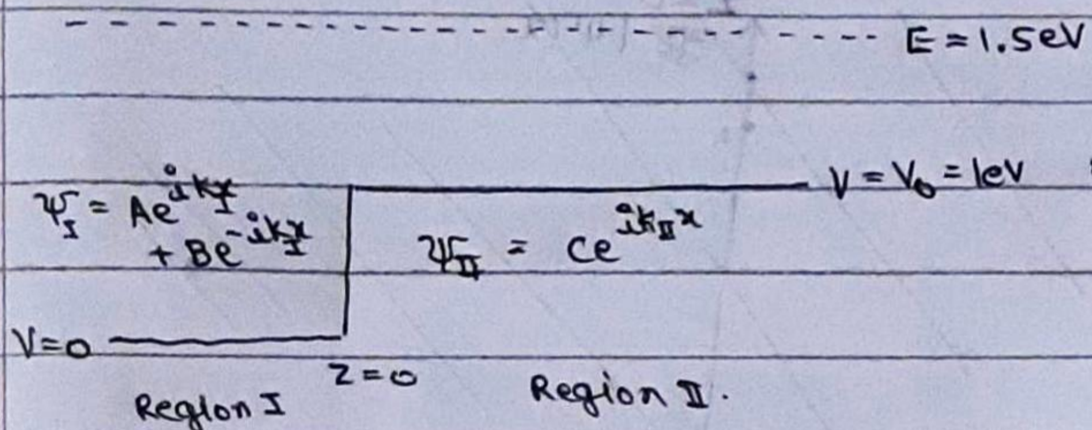
$= \frac{1}{\pi}\left[\left(\frac{\pi}{5} - \frac{1}{2}\sin\left(\frac{2\pi}{5}\right)\right) - \left(\frac{\pi}{10} - \frac{1}{2}\sin\left(\frac{\pi}{5}\right)\right)\right]$

$= \frac{1}{\pi}\left[\frac{\pi}{10} - 0.1816\right]$

$= 0.1 - 0.0578$

$= \boxed{0.0422 = 4.22\%}$

3.



$$K_I = \sqrt{\frac{2m_e E}{\hbar^2}}$$

$$= \cancel{\sqrt{2\times 9.11\times 10^{-31}\times}}$$

$$k_I = \sqrt{\frac{2m_e E}{\hbar^2}} = \frac{\sqrt{2\times 9.11\times 10^{-31}\times 1.5\times 1.6\times 10^{-19}}}{1.054\times 10^{-34}}$$

$$= \frac{6.612\times 10^{-25}}{1.054\times 10^{-34}} = \boxed{6.273\times 10^{9}\ m^{-1}}$$

$$k_{II} = \sqrt{\frac{2m_e(E-V)}{\hbar^2}} = \sqrt{\frac{1}{3}}\times k_I = \boxed{3.6219\times 10^{9}\ m^{-1}}$$

BOUNDARY CONDITIONS

(A) $\psi_I(z=0) = \psi_{II}(z=0)$

$$\boxed{A + B = C} \rightarrow 1$$

$+0.018$

(B) $\psi_I'(z=0) = \psi_{II}'(z=0)$

$$Ak_I - Bk_I = Ck_{II} = \frac{Ck_I}{\sqrt{3}}$$

$$\therefore \boxed{A - B = \frac{C}{\sqrt{3}}} \rightarrow (2)$$

From (1) & (2),

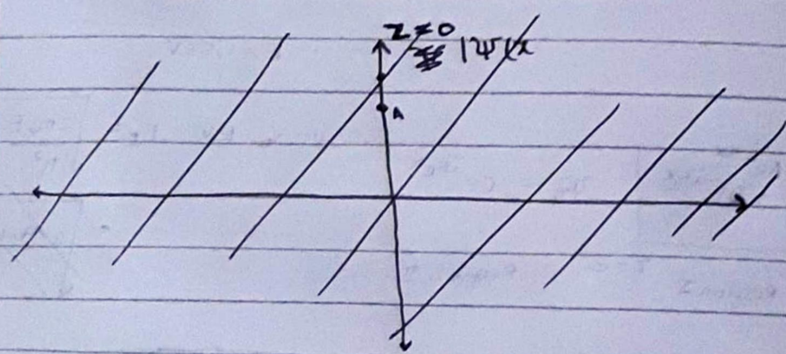
$$2A = C\left(1+\frac{1}{\sqrt{3}}\right) \Rightarrow C = \frac{2\sqrt{3}A}{1+\sqrt{3}} = \boxed{1.268A}$$

$$B = C - A = \left(\frac{2\sqrt{3}}{1+\sqrt{3}} - 1\right)A = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)A$$

$$= \boxed{0.268A}$$

$$\therefore \boxed{\begin{aligned} \psi_I &= A\left[e^{ik_I x} + 0.268e^{-ik_I x}\right] \\ \psi_{II} &= 1.268Ae^{ik_{II} x}\end{aligned}}$$

min $= 0.732$
max $= 1.268$



$$|\psi_I|^2 = A^2\left[e^{ik_I x} + 0.268e^{-ik_I x}\right]\left[e^{-ik_I x} + 0.268e^{ik_I x}\right]$$

$$= A^2\left[(1+0.268^2) + 2(0.268)\cos(k_I x)\right]$$

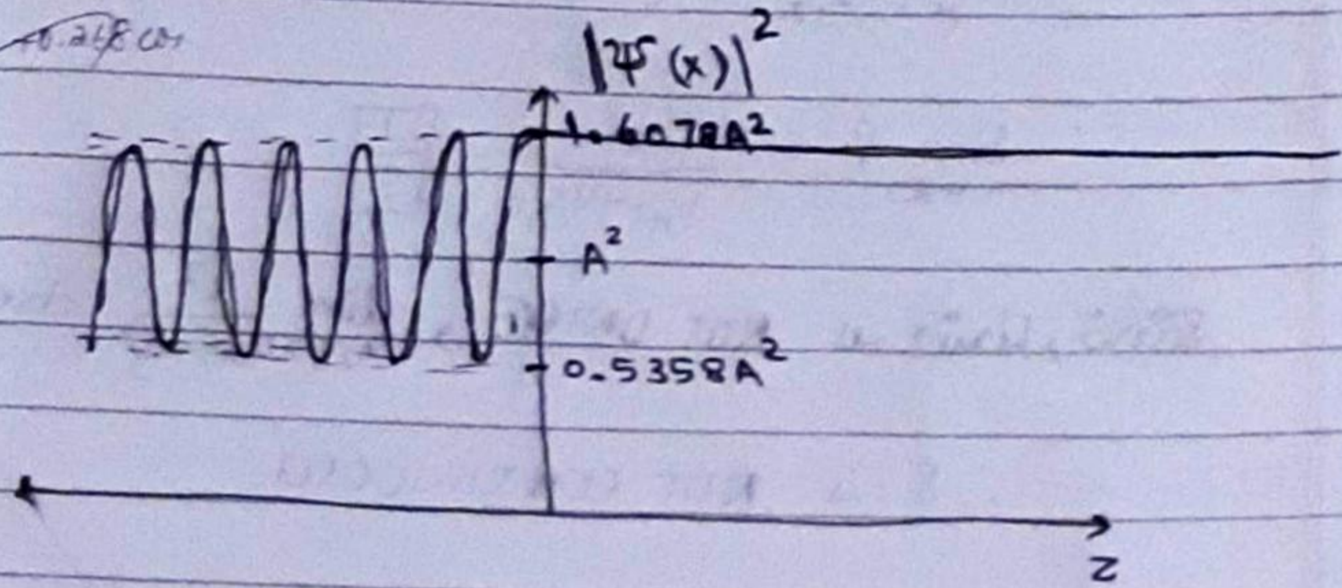
$$\max\{|\psi_I|^2\} = A^2(1.268)^2$$

$$\min\{|\psi_I|^2\} = A^2(0.732)^2$$

$$|\psi_{II}|^2 = A^2(1.268)^2 e^{ik_{II}x}e^{-ik_{II}x}$$

$$= A^2(1.268)^2$$

(b)



- at $\boxed{z=0}$ probability is max $= 1.6078A^2$
- at $\boxed{z = (2n+1)\frac{\pi}{k_{II}}}$ probability is min $= 0.5358A^2$.
  where $n \leq -1$ and $n \in \mathbb{Z}$.

- at $\boxed{z \geq 0}$ and $\boxed{z = (2n+1)\frac{\pi}{2k_{II}}}$ where $n \leq -1$ & $n \in \mathbb{Z}$,

probability $= 1.6078A^2$

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$10^{-31} \times 10^{8}$

10^{-23}

15

4.9764×10^{-11}

4. (a) $\lambda_{dB} = 4 \times 10^{-15}\,m$

$$p = \frac{h}{\lambda_{dB}} = \frac{6.635 \times 10^{-34}}{4 \times 10^{-15}}$$

$$= \boxed{1.6588 \times 10^{-19}}\ kgm/s$$

$$\therefore KE = c\sqrt{p^2 - m_0^2 c^2} - m_0 c^2$$

$$= (3 \times 10^{8})\sqrt{(2.7516 \times 10^{-38}) + (746.9289 \times 10^{-46})} - 81.99 \times 10^{-15}$$

$$\approx (3 \times 10^{8})(1.6588 \times 10^{-19}) - (81.99 \times 10^{-15})$$

$$\approx \boxed{4.984 \times 10^{-11}\ J}$$

(b) $KE = c\sqrt{p^2 + m_p^2 c^2} - m_p c^2$

$= (2.9979 \times 10^8)\sqrt{(2.7516 \times 10^{-38}) + (5.01 \times 10^{-19})^2} - 15.03 \times 10^{-11}$

$= (2.9979 \times 10^8)(5.27747 \times 10^{-19}) - 15.03 \times 10^{-11}$

$= 15.8213 \times 10^{-11} - 15.03 \times 10^{-11}$

$= \boxed{7.9133 \times 10^{-12}\ J}$

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$1 - \frac{v^2}{c^2} = v^2 \left(\frac{\lambda_{dB} m_p}{h}\right)^2$

5. (a) we know

$$\Delta E \, \Delta t = \frac{\hbar}{2}$$

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{6.635 \times 10^{-34}}{4 \times 10^{-9} \times \pi}$$

$$= \boxed{0.52801 \times 10^{-25} \text{ J}}$$

Also, $\Delta E = \left|\Delta\left(\frac{hc}{\lambda}\right)\right| = \frac{hc}{\lambda^2}\Delta\lambda$

$$\therefore \Delta\lambda = \frac{\lambda^2 \Delta E}{hc} = \frac{\lambda^2}{\cancel{h}c} \times \frac{\cancel{h}}{4\pi\Delta t}$$

$$= \frac{36 \times 10^{-14}}{2.9979 \times 10^8 \times 4 \times 10^{-9} \times 3.1415}$$

$$= \boxed{0.9556 \times 10^{-13} \text{ m}}$$

(b) $\psi(x) = Ae^{-\alpha x^2}$

$$\int_{-\infty}^{\infty} \psi^2(x)\,dx = 1 = \int_{-\infty}^{\infty} |A|^2 e^{-2\alpha x^2} dx$$

$$\cancel{-\frac{|A|^2}{2\alpha^2}\left(e^{-2\alpha^2 x^2}\Big|_{-\infty}^{\infty}\right) = 1}$$

$$\frac{\pi |A|^2}{\sqrt{2\alpha}} = 1$$

$$|A|^2 = \frac{\sqrt{2\alpha}}{\sqrt{\pi}}$$

$$\therefore \boxed{\psi(x) = \left(\frac{2\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2}}$$

6. $\psi(r) = \frac{A}{r} e^{ikr} \Rightarrow \psi^*(r) = \frac{A}{r} e^{-ikr}$

$$\cancel{j(r) = \frac{i\hbar}{2m}\left[\psi \frac{\partial \psi^*}{\partial r} - \psi^* \frac{\partial \psi}{\partial r}\right]}$$

$$\cancel{= \frac{i\hbar}{2m}\left[\frac{A}{r} e^{ikr} \ldots\right]}$$

$$\cancel{\frac{\partial \psi^*(r)}{\partial r} =}$$

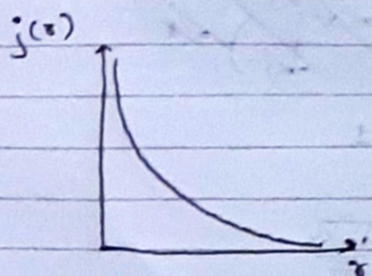
$$j(r) = \frac{i\hbar}{2m}\left[\psi \frac{\partial \psi^*}{\partial r} - \psi^* \frac{\partial \psi}{\partial r}\right]$$

$$= \frac{i\hbar}{2m}\left[\left(\frac{A}{r} e^{ikr}\right)\left(-A\frac{ik}{r} e^{-ikr} - \frac{A}{r^2} e^{-ikr}\right) - \left(\frac{A}{r} e^{-ikr}\right)\left(A\frac{ik}{r} e^{ikr} - \frac{A}{r^2} e^{-ikr}\right)\right]$$

$$= \frac{i\hbar}{2m}\left[\frac{-A^2 ik}{r^2} - \cancel{\frac{A^2}{r^3}} - \frac{A^2 ik}{r^2} + \cancel{\frac{A^2}{r^3}}\right]$$

$$= \frac{\hbar}{\cancel{2}m} \cdot \cancel{2}\frac{A^2 k}{r^2}$$

$$\boxed{= \left(\frac{\hbar A^2 k}{m}\right)\frac{1}{r^2}}$$



7. $V(x) = \alpha^2 x^2$

$\psi(x) = e^{-\sqrt{m\alpha^2/2\hbar^2}\,x^2} = e^{-kx^2}$ $\left[k = \sqrt{\frac{m\alpha^2}{2\hbar^2}}\right]$

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E(x)\psi(x)$$

$$\frac{-\hbar^2}{2m}\cancel{e^{-kx^2}}\left[4k^2x^2 - 2k\right] + \alpha^2x^2\cancel{e^{-kx^2}} = E \times \cancel{e^{-kx^2}}$$

$$E = \frac{-\hbar^2}{2m}\left[\frac{4m\alpha^2}{2\hbar^2}x^2 - 2\sqrt{\frac{2m\alpha^2}{4\hbar^2}}\right] + \alpha^2x^2$$

$$= -\cancel{\alpha^2x^2} + \frac{\hbar\alpha}{\sqrt{2m}} + \cancel{\alpha^2x^2}$$

$$= \cancel{\boxed{\frac{\hbar\alpha}{\sqrt{2m}} - \alpha^2x^2}} \qquad = \boxed{\frac{\hbar\alpha}{\sqrt{2m}}}$$

8. $\lambda_{dB} = 1.5 \times 10^{-12}\,m$

(i) $KE = c\sqrt{p^2 + m_0^2c^2} - m_0c^2$

$$= (3\times10^8)\sqrt{\left(\frac{6.635\times10^{-34}}{1.5\times10^{-12}}\right)^2 + (746.9289\times10^{-46})} - (81.99\times10^{-15})$$

$$= (3\times10^8)\sqrt{(19.566\times10^{-44}) + (7.469289\times10^{-44})} - (81.99\times10^{-15})$$

$= 15.598641\times10^{-14} - 81.99\times10^{-15}$

$= 73.99641\times10^{-15}\,J$

$= 46.24\times10^4\,eV = \boxed{462.477\ keV}$

(ii) $v_g = v = \dfrac{pc}{\sqrt{p^2 + m_0^2 c^2}}$

$= \dfrac{(4.473 \times 10^{-22})(2.9979 \times 10^8)}{5.1995 \times 10^{-22}}$

$= \boxed{2.9501 \times 10^8 \text{ m/s}}$

(iii) $v_p = \dfrac{c^2}{v} = \boxed{9.5243 \times 10^8 \text{ m/s}}$

9. Compton wavelength ($\lambda_c = h/m_0 c$) of an electron is the wavelength of a photon of energy = mass equivalent energy of the electron.

For $\lambda_c = \lambda_{dB}$,

$$\frac{\cancel{h}}{\cancel{m_0} c} = \frac{h}{\gamma \cancel{m_0} v}$$

$$\therefore \gamma v = c$$

$$\frac{v}{\sqrt{1 - v^2/c^2}} = c$$

$$v^2 = c^2 - v^2$$

$$\boxed{v = c/\sqrt{2} \text{ m/s}}$$

10. $A\psi(x) = \psi(x) + x$

$\therefore A[cf(x) + dg(x)] = [cf(x) + dg(x)] + x$

$= cf(x) + dg(x) + cx + dx + x[1 - c - d]$

$= c[f(x) + x] + d[g(x) + x] + x[1 - c - d]$

$= cAf(x) + dAg(x) + x[1 - c - d]$

$\neq cAf(x) + dAg(x)$

$\therefore$ $\boxed{A\psi(x) \text{ is NOT linear}}$

$$B\psi(x) = \left(\frac{d\psi}{dx}\right) + 2\psi(x)$$

$$\therefore B[cf(x) + dg(x)] = \cancel{\frac{d(f(x))}{dx} + 2f(x)}$$

$$= \frac{d[cf(x) + dg(x)]}{dx} + 2(cf(x) + dg(x))$$

$$= c \times \frac{df(x)}{dx} + 2cf(x) + d \times \frac{dg(x)}{dx} + 2dg(x)$$

$$= c\left[\frac{df}{dx} + 2f\right] + d\left[\frac{dg}{dx} + 2g\right]$$

$$= \boxed{cBf(x) + dBg(x)}$$

$\therefore$ $\boxed{B\psi(x) \text{ IS LINEAR}}$