EP 1108 ASSIGNMENT - 3

1. $\lambda_{threshlod} = 2300\,\text{Å} = 230\text{ nm}.$

$\therefore E_{threshold} = \frac{1242}{230} = \boxed{5.4\text{ eV}}$

$\therefore E_{reqd} - E_{threshold} = \text{Max Energy of } e^{-}\text{s}.$

$E_{reqd} - 5.4 = 1.5$

$\boxed{E_{reqd} = 6.9\text{eV}}$

$\therefore \lambda_{reqd} = \frac{1242}{6.9} = \boxed{180\text{nm}} = \boxed{1800\,\text{Å}}$

2. From Lorentz Eqns, we see

$$x' = \frac{1}{\sqrt{1-v^2/c^2}}(x - vt) \Rightarrow x = kx' + vt \quad \text{①} \quad \left[k = \sqrt{1-v^2/c^2}\right]$$

$$t' = \frac{1}{\sqrt{1-v^2/c^2}}(t - vx/c^2) \Rightarrow t = kt' + vx/c^2 \quad \text{②}$$

$$\therefore t = kt' + \frac{v}{c^2}(kx' + vt)$$

$$c^2 t = kc^2 t' + vkx' + v^2 t$$

$$(c^2 - v^2)t = k(c^2 t' + vx')$$

$$\therefore t = \frac{k(c^2 t' + vx')}{c^2 - v^2} = \frac{(ct' + \frac{v}{c}x')}{\sqrt{c^2 - v^2}}$$

$$= \boxed{\frac{\left(t' + \frac{vx'}{c^2}\right)}{\sqrt{1 - v^2/c^2}}}$$

$$x = kx' + vt$$

$$= kx' + v\left(kt' + \frac{vx}{c^2}\right)$$

$$x = kx' + vkt' + \frac{v^2 x}{c^2}$$

$$\left(1 - \frac{v^2}{c^2}\right)x = k(x' + vt')$$

$$x = \frac{\sqrt{1 - \frac{v^2}{c^2}}\,(x' + vt')}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\boxed{x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}}$$

3\. By Lorentz Transformations & Length contraction eqn.

We know $99 = 100\sqrt{1 - v^2/c^2}$

$$\therefore\ v^2 = c^2\left(1 - \left(\frac{99}{100}\right)^2\right)$$

$$v = c\sqrt{1 - \left(\frac{99}{100}\right)^2}$$

$$= 0.14107 \times c$$

$$= 0.4232 \times 10^8 \text{ m/s}$$

$$= \boxed{4.232 \times 10^7 \text{ m/s}} \quad \left[\text{Taking } c = 3\times10^8 \text{ m/s}\right]$$

4\. For $m_e = m_p(\text{rest})$: $\left[m_e = \frac{m_e(\text{rest})}{\sqrt{1 - v^2/c^2}}\right]$

$$\therefore\ \frac{m_e(\text{rest})}{\sqrt{1 - v^2/c^2}} = m_p(\text{rest})$$

$$\therefore v = c\sqrt{1 - \left(\frac{m_e(\text{rest})}{m_p(\text{rest})}\right)^2}$$

$$= c\sqrt{1 - (0.5 \times 10^{-3})^2}$$

$$= c\,(0.999999875)$$

$$= \boxed{2.999999625 \times 10^8 \text{ m/s}} \quad \left[\text{Taking } c = 3\times10^8 \text{ m/s}\right]$$

5. $KE = (m - m_0)c^2$ Rest Energy $= m_0 c^2$

$$\therefore (m - m_0)\cancel{c^2} = m_0 \cancel{c^2}$$

$$m = 2m_0 \quad \left[m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \right]$$

$$\frac{\cancel{m_0}}{\sqrt{1 - v^2/c^2}} = 2\cancel{m_0}$$

$$1 - v^2/c^2 = 1/4$$

$$v^2/c^2 = 3/4$$

$$\boxed{\begin{aligned} v &= \frac{\sqrt{3}}{2}c \\ &= 2.598 \times 10^8 \text{ m/s} \quad \left[\text{Taking } c = 3 \times 10^8 \text{ m/s} \right] \end{aligned}}$$

6. Total Energy $= mc^2 = \gamma m_0 c^2$

$\therefore \gamma m_0 c^2 = 10^5$ MeV

$m_0 c^2$ (rest energy) $= 10^2$ MeV

$\therefore \boxed{\gamma = 10^3}$

Let the speed of the muon be v.

$\therefore$ Distance as seen by muon $= 2\mu s \times v$

$\Rightarrow$ Distance as seen by earth $= \frac{2v \times 10^{-6}}{\cancel{\gamma}} \times \gamma$

$$= 2v \times 10^{-6} \times \gamma$$

$$= 2\gamma \times 10^{-6} \times c \frac{\sqrt{\gamma^2 - 1}}{\gamma}$$

$$= 2c \times 10^{-6} \times \sqrt{10^6 - 1}$$

$$= \boxed{599.9997 \text{ km}}$$

[Taking $c = 3 \times 10^8 \text{ m/s}$]