PROBABILITY ASSIGNMENT

1. (a) $P(\text{wearing seat belt}) = \dfrac{\text{Total "Yes"}}{\text{Total "Yes"} + \text{Total "No"}}$

$$= \frac{858}{858 + 228} = \frac{858}{1086}$$

$$= \boxed{0.7900552486}$$

(b) $P(\text{wearing seat belt in NE}) = \dfrac{148}{148+52} = 0.74$

$P(\text{wearing seat belt in MW}) = \dfrac{162}{162+54} = 0.75$

$P(\text{wearing seat belt in S}) = \dfrac{296}{296+74} = 0.8$

$P(\text{wearing seat belt in W}) = \dfrac{252}{252+48} = \boxed{0.84}$

MAXIMUM

∴ The West has the highest seat belt usage

(b) $0.7900552486 > 0.78$

∴ Dr. Jeffrey would be pleased with the results.

(d) [At end]

1. (d) Proportions of

$$NE = \frac{148 + 52}{1086} = 0.18416$$

$$MW = \frac{162 + 54}{858 + 228} = 0.19889$$

$$S = \frac{296 + 74}{1086} = 0.34069$$

$$W = \frac{252 + 48}{1086} = 0.27624$$

2 $P(\text{oil}) = P(HQ) + P(MQ)$

$= 0.5 + 0.2$

$= \boxed{0.7}$

[HQ - high qual. oil
MQ - med. qual. oil]

→ We need to find $P(\text{oil} \mid \text{soil})$

$$\Rightarrow P(\text{oil} \mid \text{soil}) = \frac{P(\text{oil} \cap \text{soil})}{P(\text{soil})}$$

$$= \frac{P(\text{soil} \cap HQ) + P(\text{soil} \cap MQ)}{P(\text{soil} \cap HQ) + P(\text{soil} \cap MQ) + P(\text{soil} \cap \text{no oil})}$$

$$= \frac{P(\text{soil} \mid HQ)P(HQ) + P(\text{soil} \mid MQ)P(MQ)}{P(\text{soil} \mid HQ)P(HQ) + P(\text{soil} \mid MQ)P(MQ) + P(\text{soil} \mid NO)P(\text{No oil})}$$

$$= \frac{(0.2)(0.5) + (0.8)(0.2)}{(0.2)(0.5) + (0.8)(0.2) + (0.2)(0.3)}$$

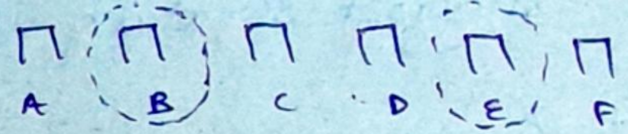
$= \boxed{0.8125}$

$\Rightarrow \boxed{P(\text{soil}) = 0.32}$

3. Assuming everyone observing were independent,

$$P(\text{NONE observed a cut}) = \underbrace{(0.9)^{22}}_{\text{none of the casual observers saw}} \underbrace{(0.5)^{1}}_{\text{the carefully studying guy dint observe}} \underbrace{(0.6)^{1}}_{\text{the one who drove car didnt see the cut}}$$

$$= \boxed{0.02954313}$$

4. 

Lets say the prizes lie behind doors B & E.

↳ The probability of choosing one of the right doors $= 2/6$

$= \boxed{1/3}$

$\therefore$ P(choosing wrong door) $= \boxed{2/3}$

↳ Now, lets say on the first choice, the player picks a correct door [B or E, say B] & asks to change. Let doors C & D be opened (wrong doors).

Out of the remaining doors [A, E, F], one of them is correct, which leaves us with a probability of $\frac{1}{3}$ of choosing a right door.

↳ Now, lets say on the first choice, the player selects a wrong door (say C) and asks to change. Let two other wrong doors (say A & D) be opened. Out of the remaining 3 doors (B, E, F), 2 of them (B, E) are correct. ∴ The probability of choosing the right door = $\frac{2}{3}$.

⇒ Hence, the total probability of choosing a right door if opted to change his first option =

P(right on first chance) P(correct on chance 2 | correct on chance 1)

+ P(wrong on chance 1) P(correct on chance 2 | wrong on chance 1)

$$= \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$$

$$= \boxed{\frac{5}{9}}$$

⇒ As an analyst, I would suggest switching their initial guess since $\frac{5}{9} > \frac{1}{3}$

↳ (if only one guess was made)

5. $P(\text{fever} | CP) = 0.47$

$P(\text{fever} | CN) = 0.08$.

[CP - Covid positive
CN - Covid -ve]

~~P(Covid)~~

$P(CP) = 0.005 \Rightarrow P(CN) = 1 - 0.005 = 0.995$

$$\therefore P(CP | \text{fever}) = \frac{P(CP \cap \text{fever})}{\cancel{P(CP)}\, P(\text{fever})}$$

$$= \frac{P(\text{fever}|CP)\,P(CP)}{P(\text{fever}|CP)\,P(CP) + P(\text{fever}|CN)\,P(CN)}$$

$$= \frac{(0.47)(0.005)}{(0.47)(0.005) + (0.08)(0.995)}$$

$$= \boxed{0.028676}$$

6. The expected demand in number of copies (x)

$$= \sum_{i=1}^{5} x P(x) \quad \text{, where } x = 50{,}000, \; 70{,}000, \; 90{,}000, \; 110{,}000, \; 130{,}000$$

$$= (5\times10^4)(0.1) + (7\times10^4)(0.25) + (9\times10^4)(0.4)$$

$$+ (11\times10^4)(0.2) + (13\times10^4)(0.05)$$

$$= 10^4 \left[0.5 + 1.75 + 3.6 + 2.2 + 0.65\right]$$

$$= 10^4 \times 8.7$$

$$= \boxed{87000 \text{ copies}}$$

7. $P(\text{Specialist } 1) = 0.3$ $\qquad$ $P(\text{incorrect} \mid Sp\,1) = 0.03$

$P(\text{Specialist } 2) = 0.45$ $\qquad$ $P(\text{incorrect} \mid Sp\,2) = 0.05$

$P(\text{Specialist } 3) = 0.25$ $\qquad$ $P(\text{incorrect} \mid Sp3) = 0.02$

$$P(\text{Specialist } 1 \mid \text{incorrect}) = \frac{P(Sp\,1 \cap inc)}{P(inc)}$$

$$= \cancel{P(Sp\,1 \cap inc)\,P(}$$

$$= \frac{P(inc \mid Sp\,1)\,P(Sp\,1)}{P(inc \mid Sp1)P(Sp1) + P(inc \mid Sp2)P(Sp2) + P(inc \mid Sp3)P(Sp3)}$$

$$= \frac{(0.03)(0.3)}{(0.03)(0.3) + (0.05)(0.45) + (0.02)(0.25)}$$

$$= \boxed{0.246575.}$$

8. This is a binomial distribution with non-conforming coils as the random variable [x]

Let P(non-conforming coil) be 'p'.

We know in BD,

$$\text{expectation values of } x = np$$

$$\therefore np = 5 \rightarrow (\text{Given})$$

$$50p = 5$$

$$\boxed{p = 0.1}$$

$\therefore P(x) = 0.1$

$\therefore$ P(Not a non conforming coil) $= 1 - 0.1$
$= 0.9$

$\Rightarrow \therefore$ The probability that not even 1 of the 50 coils are non-conforming $= (0.9)^{50}$

$$= \boxed{0.0051537}$$