

1.



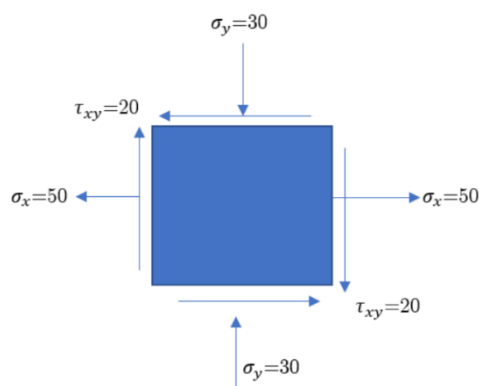
The normal stress on the cross-section(pq) is given by,

$$\begin{aligned}\sigma &= \frac{\text{Force}}{\text{Area}} = \frac{P \cos \phi}{A / \cos \phi} = \sigma_x (\cos \phi)^2 \\ &= (50 / (3.14 \times 10^{-6})) \cos^2 20^\circ \\ &= 14.0608 \text{ MPa}\end{aligned}$$

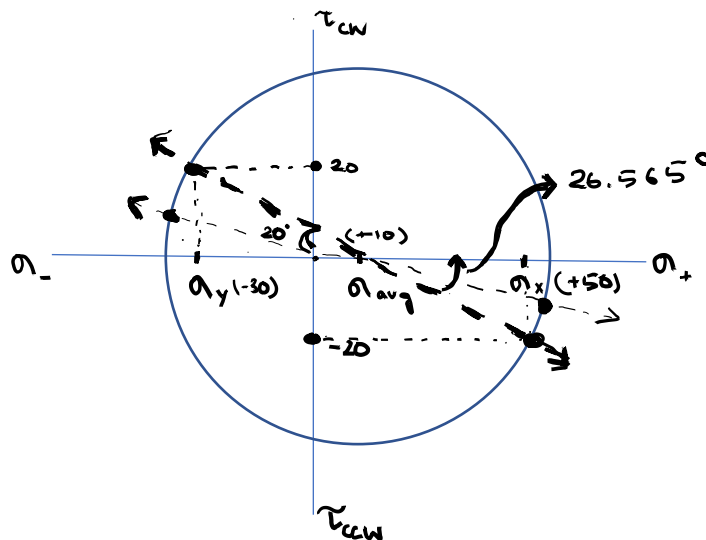
There the shear stress is given by

$$\begin{aligned}\tau &= \frac{P \sin \phi}{A / \cos \phi} = \sigma_x \sin \phi \cos \phi \\ &= (50 / (3.14 \times 10^{-4})) \cos 20^\circ \sin 20^\circ \\ &= 5.11773 \text{ MPa}\end{aligned}$$

2.



(a)



$$\begin{aligned}
 \text{(b) Maximum principal stress} \quad \sigma_a &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
 &= ((50-30)/2) + (\sqrt{((50+30)/2)^2 + 20^2}) \\
 &= 10+44.721 = \mathbf{54.721}
 \end{aligned}$$

$$\begin{aligned}
 \text{Minimum principal} \quad \sigma_b &= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{stress} \\
 &= 10 - 44.721 = \mathbf{-34.721}
 \end{aligned}$$

$$\text{Maximum shear stress} = \text{Radius of circle} = \mathbf{44.721}$$

$$\text{Also, } \tan 2\theta = 20/40 = 0.5$$

$$\begin{aligned}
 \therefore \theta &= \tan^{-1} 0.5/2 \\
 &= \mathbf{26.565/2} \\
 &= \mathbf{13.2825^\circ}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c) At } \phi = 10^\circ, \quad \sigma_{x'} &= \sigma_{\text{avg}} + R \cos(26.565 - 20) \text{ [From Mohr's Circle]} \\
 &= 10 + 44.4278 \\
 &= \mathbf{54.4278} \\
 \sigma_{y'} &= \sigma_{\text{avg}} - R \cos(26.565 - 20) \\
 &= \mathbf{-34.4278} \\
 \tau &= R \sin(26.565 - 20) \\
 &= 44.721 \times 0.11433 \\
 &= \mathbf{5.113}
 \end{aligned}$$

3.

$$\sigma = \begin{bmatrix} 10 & 20 & -50 \\ -30 & 44 & 0 \\ 72 & 28.8 & -5 \end{bmatrix}$$

$$\begin{aligned}
 \text{We know } \sigma_{hyd} &= \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} \\
 &= (10+44-5)/3
 \end{aligned}$$

$$= 16.333$$

Which can be written as $\sigma_{hyd} = \begin{pmatrix} 16.33 & 0 & 0 \\ 0 & 16.33 & 0 \\ 0 & 0 & 16.33 \end{pmatrix}$

$$\sigma_{ij} = \sigma_{hyd} + \sigma_{dev} \quad \text{Also,}$$

$$\sigma_{dev} = \begin{pmatrix} 10 & 20 & -50 \\ -30 & 44 & 0 \\ 72 & 22.8 & -5 \end{pmatrix} - \begin{pmatrix} 16.33 & 0 & 0 \\ 0 & 16.33 & 0 \\ 0 & 0 & 16.33 \end{pmatrix} = \begin{pmatrix} \frac{-633}{100} & 20 & -50 \\ -30 & \frac{2767}{100} & 0 \\ 72 & \frac{114}{5} & \frac{-2133}{100} \end{pmatrix}$$

4.

Isotropic Materials: Since $C_{ij} = C_{ji}$, the stiffness tensor becomes a symmetric matrix, hence reducing the number of independent components required in the C matrix from 4 to **2**.

Anisotropic Materials: $C_{\text{anisotropic}}$ has **3** independent elements. (Less symmetry than isotropic materials)

Orthotropic Materials: $C_{\text{orthotropic}}$ has **9** independent elements. (Less symmetry than anisotropic materials)

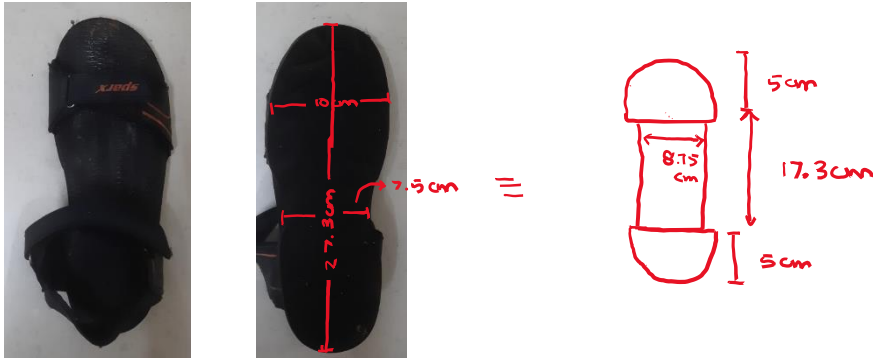
5.

(a) My weight = 54 kgwt. = 529.2N (Assuming $g = 9.8 \text{ ms}^{-2}$)

(b) The weight experienced by each slipper = $529.2/2$

$$= 264.6\text{N}$$

(c)



$$\begin{aligned}\text{Approximate area of slipper} &= (\pi \times 5^2 + 8.75 \times 17.3) \text{ cm}^2 \\ &= 229.9 \text{ cm}^2 = 0.02299 \text{ m}^2\end{aligned}$$

(d) The approximate Youngs Modulus of the material we assume it to be made of (tanned leather) = **51MPa**

$$\begin{aligned}\text{(e) Stress in the slippers} &= (\text{Weight per slipper}) / (\text{Area of slipper}) \\ &= 264.6 / 0.02299 \\ &= \mathbf{11.5094 \text{ kPa}}\end{aligned}$$