भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

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CH2120 Numerical Methods

Exam 4

December 2, 2021
11-12:00 PM
60 points

1. This is an open book exam.
2. Turn on your *live video feed* for the entire duration of the exam. Mute your microphones but unmute your speakers.
3. If you face technical issues during the exam, please contact your TA *immediately* by sending a SMS briefly describing the problem. Rejoin as soon as possible.
4. **This is a written exam. There are no coding sections.**
5. Typed answers will not be accepted. Write your answers using pen and paper.
6. Write the page number on every page.
7. Take pictures of your answers, compile the images into a single PDF file, and upload to Google Classroom under the *assignment titled Exam 4*.
8. *The deadline for uploading your final PDF is 12 PM. Late submissions will not be accepted.*

Good luck!

Consider the ODE

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

$$x \in [0,1]$$

with boundary conditions:

$$y(0) = 0$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = 3$$

$$\left(\frac{dy}{dx}\right)_{x=1} = \left(\frac{d^2y}{dx^2}\right)_{x=1}$$

1) Write the ODE as a set of first-order ODEs using appropriate substitutions:

$$u = \frac{dy}{dx}$$

$$w = \frac{d^2y}{dx^2}$$

   Also write the boundary conditions in terms of the new variables. ***[6 Points]***

2) Solve the system of ODEs written in *part (1)* by assuming that

$$u_0 = -10$$

   Here, u_0 represents the value of u at $x = 0$. Use the midpoint method with a step size of $h = 1$, and report the estimated values of y, u, and w at $x = 1$. ***[18 Points]***

3) Define an appropriate error, e, to account for the inaccuracy in the solution at the end point of the domain. Evaluate this error corresponding to $u_0 = -10$. ***[5 Points]***
4) Repeat *part (2)* by assuming that

$$u_0 = +10$$

   [9 Points]
5) Evaluate the error, e, corresponding to $u_0 = +10$. ***[5 Points]***
6) Starting with the two values of e evaluated in *part (3)* and *part (5)*, perform one iteration of the bisection method. Comment on whether the correct value of u_0 lies in $[-10,0]$ or in $[0, +10]$. ***[17 Points]***