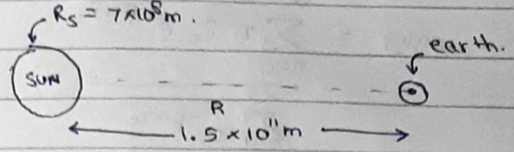
EP 1108 ASSIGNMENT-2

1.



~~Let I be the intensity of radiation~~

Let P be the energy per unit second radiated by the sun.

$$\therefore \frac{P}{4\pi\times(1.5\times10^{11})^2\ m^2} = 1.4\times10^3\ W/m^2$$

$$P = 9\pi\times10^{25}\times1.4\ W$$

$$= \frac{63\pi}{5}\times10^{25}\ W$$

$\therefore$ By Stephan-Boltzmann Law,

$$\frac{63}{5}\pi\times10^{25} = \sigma\times(4\pi)\times(7\times10^8)^2\ T^4,$$

where σ = stephans constant = $5.67\times10^{-8}\ W/m^2K^4$

T = temp. of sun

$$\therefore T^4 = \frac{\overset{9}{\cancel{63}}\cancel{\pi}\times10^{\cancel{24}}}{2\cancel{0}\cancel{\pi}\times\overset{7}{\cancel{49}}\times10^{\cancel{16}}\times5.67\times10^{\cancel{-8}}}\times10^{16}$$

$$= \frac{90{,}000}{2\times7\times5.67}\times10^{12}$$

$$\boxed{T = 5.8027\times10^3\ K \approx 5802.74\ K}$$

2. Planks Spectral Distribution Law :

$$\bar{\varepsilon}\text{(mean energy)} = \frac{\sum_{n=0}^{\infty} n\varepsilon_0 e^{-\beta n \varepsilon_0}}{\sum_{n=0}^{\infty} e^{-\beta n \varepsilon_0}}$$

$$= -\frac{d}{d\beta}\left[\ln\left(\sum_{n=0}^{\infty} e^{-\beta n \varepsilon_0}\right)\right]$$

$$= \boxed{\frac{\varepsilon_0}{e^{\beta\varepsilon_0}-1}}$$

$\sum e^{-\beta n \varepsilon_0}$

$e^{-n\varepsilon_0}$

We know

$$e(\lambda, T) = \frac{8\pi}{\lambda^4}\bar{\varepsilon}$$

$$= \frac{8\pi}{\lambda^4} \times \frac{\varepsilon_0}{e^{\beta\varepsilon_0}-1}$$

Now,

$$\varepsilon_0 = \frac{hc}{\lambda} \qquad \beta = \frac{1}{kT}$$

$$\therefore e(\lambda, T) = \frac{8\pi hc}{\lambda^5} \times \frac{1}{e^{\left(\frac{hc}{\lambda kT}\right)}-1}.$$

$$\boxed{\text{As } \lambda \to \infty,\ e^{\left(\frac{hc}{\lambda kT}\right)} - 1 \to \frac{hc}{\lambda kT}}$$

$$\therefore \lim_{\lambda\to\infty} e(\lambda, T) = \frac{8\pi \cancel{hc}}{\lambda^4} \times \frac{\cancel{\lambda} kT}{\cancel{hc}}$$

$$= \boxed{\frac{8\pi}{\lambda^4} kT}$$

This is Rayleigh-Jeans law which is obtained by simplification of Planks law at very long wavelengths

3. Planck's Theory for $\rho(\lambda, T)$:

$$\rho(\lambda, T) = \frac{8\pi hc}{\lambda^5} \times \frac{1}{e^{\left(\frac{hc}{\lambda kT}\right)} - 1}$$

For ρ to peak [OR, $R(\lambda, T)$ to peak],

$$\boxed{\frac{d\rho(\lambda, T)}{d\lambda} = 0}$$

$$8\pi hc\left[\frac{-5}{\lambda^6} \times \frac{1}{e^{\left(\frac{hc}{\lambda kT}\right)} - 1} + \frac{1}{\lambda^5} \times \frac{-1}{\left[e^{hc/\lambda kT} - 1\right]^2} \times e^{\frac{hc}{\lambda kT}} \times \frac{-hc}{\lambda^2 kT}\right] = 0$$

or

$$\boxed{\frac{5}{\cancel{\lambda^6}} \times \frac{1}{e^{\left(\frac{hc}{\lambda kT}\right)} - 1} = \frac{1}{\cancel{\lambda^5}} \times \frac{1}{\left[e^{hc/\lambda kT} - 1\right]^2} \times e^{\frac{hc}{\lambda kT}} \times \frac{hc}{\cancel{\lambda^2} kT}}$$

$$x = \frac{hc}{\lambda kT}$$

$$\frac{5}{\cancel{(e^x - 1)}} = \frac{1}{(e^x - 1)^2} \times e^x \times x$$

$$xe^x = 5(e^x - 1)$$

$$\boxed{x = 5(1 - e^{-x})}$$

$\therefore$

$$x = 4.965 \Rightarrow \frac{hc}{\lambda kT} = 4.965 \Rightarrow \boxed{\lambda T = \frac{hc}{4.965k} \text{ mK} = 2.898 \times 10^{-3} \text{ mK} = b}$$

Wien's Displacement Law

4.

Emissive Power of Barnard Star $= \sigma T^4$

$= 5.67 \times 10^{-8} \times 3^4 \times 10^{12}$ W/m²

$= \boxed{4.593 \times 10^6 \text{ W/m}^2}$

$R(\lambda, T)$ peaks according to Wiens Displacement Law

$\therefore \lambda_{peak} \times T = \underbrace{b}_{\text{wiens constant}}$

$\lambda_{peak} \times 3 \times 10^3 = 2.898 \times 10^{-3}$

$$\boxed{\begin{aligned}\lambda_{peak} &= 9.66 \times 10^{-7} \text{ m} \\ &= 966 \text{ nm.}\end{aligned}}$$