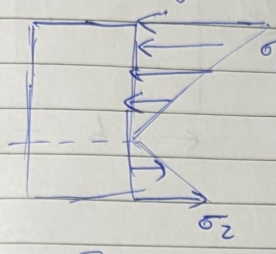
Date
Page

12/10/22

Neutral Axis

$\sigma_x = E\varepsilon_y = -yEK$



$\sigma_1 = -c_1 E K$
$\sigma_2 = c_2 E K$

$dF = \sigma\, dA$

$\int \sigma\, dA = 0$

$\int_A -yEK\, dA = 0 \rightarrow \int y\, dA = 0$

Find neutral surface or axis

Moment-Curvature Rel^n

$dM = y\sigma_x\, dA$

$\int y\,\sigma_x\, dA + M = 0$

$M = -\int y(-yEK)\, dA$

$= EK \int y^2 dA$ — Moment of inertia wrt neutral axis

$= EKI$

$K = \frac{M}{EI}$ — EI → Flexural Rigidity

Flexural Formula

$\sigma_x = -EKy$

$= -E\frac{My}{EI} = -\frac{My}{I}$

More I, reduce stress

$\sigma_1 = -\frac{M}{S_1} \quad y = c_1$

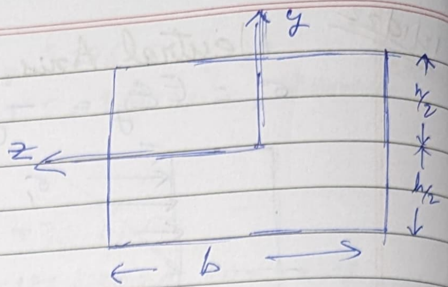
S - section moduli

$\sigma_2 = \frac{M}{S_2} \quad y = -c_2$

$I = \int y^2 dA$ — $b\,dy$

$= b\int_{-h/2}^{h/2} y^2\,dy$

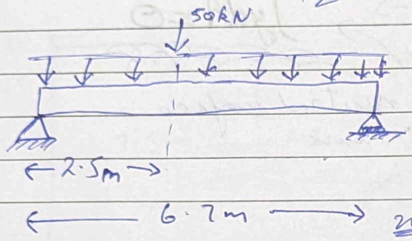
$= \dfrac{bh^3}{12}$



$C_1 = \dfrac{h}{2},\ C_2 = h/2$

$S = \dfrac{I}{c} = \dfrac{\frac{bh^3}{12}}{h/2} = \dfrac{bh^2}{6}$

Q)



$q = 22\,kN/m$

$h = 700\,mm$

$b = 220\,mm$

$\sigma_1 = ?$

$\sigma_2 = ?$

$R_A + R_B = 197.4$

$50\times2.5 + 11\times6.7^2 - R_B\times6.7 = 0$

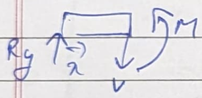
$R_B = \dfrac{50\times2.5 + 11\times6.7^2}{6.7}$

$= 92.36$

$R_A = 105.04$

$R_y = 197.4\ kN$

x_1



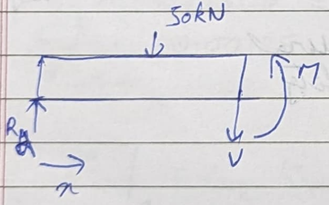
$V = R_A - 22x$

$= 105.04 - 22x$

$M = \cancel{R_y}\ V\cdot x + \int_0^x 22\,dx\cdot x$

$= 105.04x - 22x^2 + 11x^2$

$= 105.04x - 11x^2$



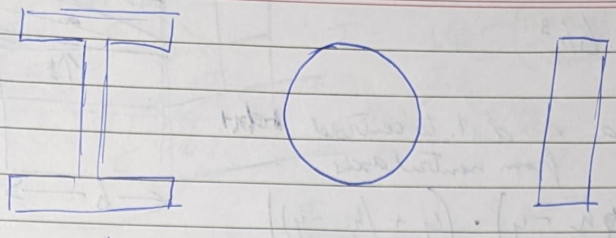
$V = R_A - 50 - 22x$

$= \cancel{105.04}\ 55.04 - 22x$

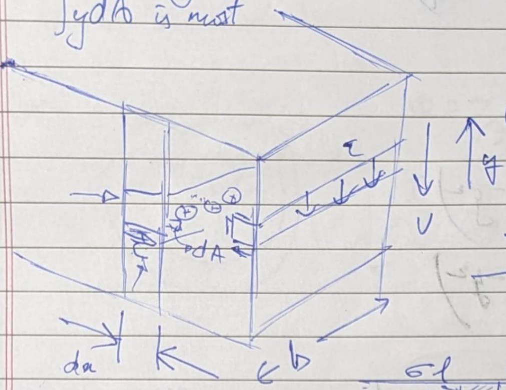
$M = V\cdot x + 50\times2.5 + 11x^2$

$= 105.04x - 11x^2 + 125$

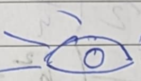
17/10/22



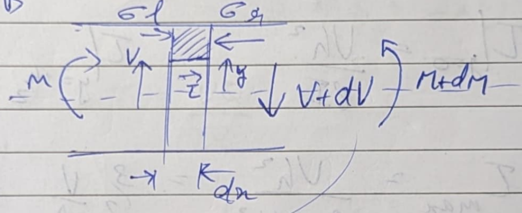
Most resistant to bending, as $\int y\,dA$ is most



Assumptions
① ~~Shear~~ $\tau \parallel$ y-axis
② τ is uniform along b


when integrating

$\tau(y)\,V$



$C_l = h/2$

$$F_l + F_\tau = F_r$$

$$F_l = \int_{y_1}^{h/2} \frac{My}{I}\, dA \qquad F_r = \int_{y_1}^{h/2} \frac{(M+dM)\,y}{I}\, dA$$

$$F_\tau = \tau\, b\, dx$$

$$F_r - F_l = \frac{dM}{I}\int y\,dA = \tau\, b\, dx$$

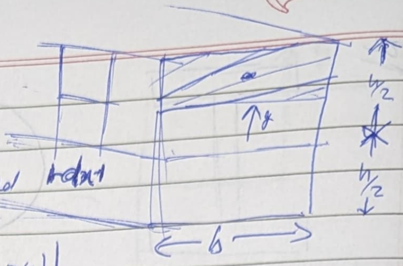
Shear Formula

$$\tau = \frac{dM}{dx}\,\frac{Q}{Ib} = \frac{VQ}{Ib} \qquad Q = \int y\,dA$$

19/10/22

$I = \frac{1}{12} bh^3$

Q = Area × dist. to centroid from neutral axis



$= b\left(\frac{h}{2} - y\right) \cdot \left(y + \frac{\left(\frac{h}{2} - y\right)}{2}\right)$

$= b\left(\frac{h}{2} - y\right)\left(\frac{y}{2} + \frac{h}{4}\right)$

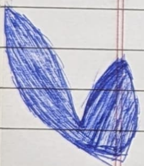
$= \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$

$\tau = \frac{V}{I\cancel{b}} \frac{\cancel{b}}{2}\left(\frac{h^2}{4} - y^2\right)$

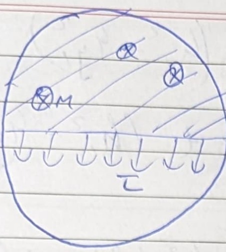
$= \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right)$

$\tau|_{y=0} = \frac{Vh^2}{8I}$ $\qquad \tau|_{y=\pm h/2} = 0$

$\tau_{max} = \frac{Vh^2}{8 \frac{1}{12} bh^2} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\tau_{avg}$

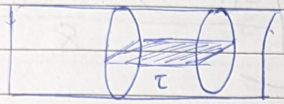


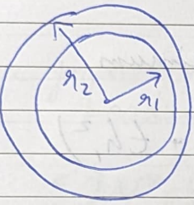
The normal stress/strain obtained assumed for case of pure bending, holds (to a very good extent) for beams w/ non-uniform shear force variation



$$\tau = \frac{VQ}{Ib} \quad Q = \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right) \quad I = \frac{\pi r^4}{4} \quad b \to 2r$$

$$\tau = \frac{V\, 2 r^{3}\, 2}{3 \frac{\pi r^4}{4} \cdot 2r} = \frac{4V}{3\pi r^2} = \frac{4}{3}\frac{V}{A}$$

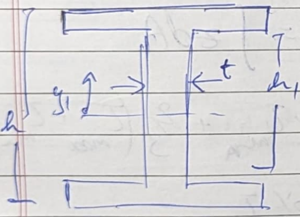




$$Q = \frac{2}{3}\left(r_2^3 - r_1^3\right)$$

$$I = \frac{\pi}{4}\left(r_2^4 - r_1^4\right)$$

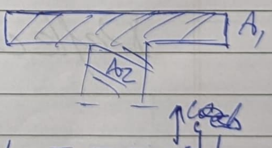
$$\tau = \frac{4}{3}\frac{V}{A}\left(\frac{r_1^2 + r_2^2 + r_1 r_2}{r_1^2 + r_2^2}\right) \quad A \to \pi\left(r_2^2 - r_1^2\right)$$



$$\tau = \frac{VQ}{It}$$

$$\frac{bh^3}{12} - \left(\frac{b-t}{2}\right)\frac{h_1^3}{6}$$

$$= \frac{bh^3}{12} - \frac{bh_1^3}{12} + \frac{th_1^3}{12}$$



$$Q_1 = b\left(\frac{h-h_1}{2}\right)\left(\frac{h_1}{2} + \frac{h-h_1}{4}\right)$$

$$Q_1 = b\left(\frac{h-h_1}{2}\right)\left(\frac{h+h_1}{4}\right) = \frac{b}{8}\left(h^2 - h_1^2\right)$$ — web

$$Q_2 = t\left(\frac{h_1}{2} - y_1\right)\left(y_1 + \frac{h_1}{4} - \frac{y_1}{2}\right)$$

$$Q = Q_1 + Q_2 \qquad Q_2 = \frac{t}{8}\left(h_1^2 - 4y_1^2\right)$$

$$\tau = \frac{V}{8It}\left[b\left(h^2 - h_1^2\right) + t\left(h_1^2 - 4y_1^2\right)\right]$$

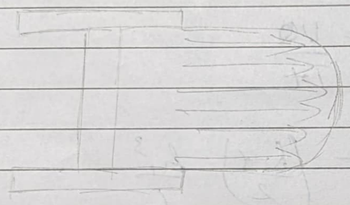
$$= \frac{V}{It}\left(Q_1 + Q_2\right) = \frac{V}{It}\left[\frac{b\left(h^2 - h_1^2\right) + t\left(h_1^2 - 4y_1^2\right)}{8}\right]$$

At $y_1 = \frac{h_1}{2}$ $\quad \tau =$ minimum

$y_1 = 0$ $\quad \tau =$ Maximum

$$\tau_{max} = \frac{V}{8It}\left(b\left(h^2 - h_1^2\right) + t h_1^2\right)$$

$$\tau_{min} = \frac{Vb}{8It}\left(h^2 - h_1^2\right)$$

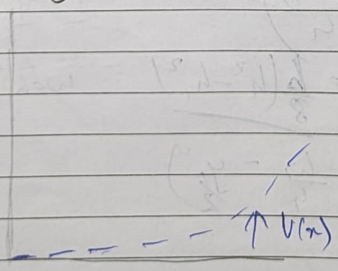


$$V = \int \tau \, dA$$

$$V = h_1 \times \tau_{min} + \frac{2}{3}\left(\tau_{max} - \tau_{min}\right) t h_1$$

$$\frac{V}{h_1 t} \pm 10\% \ \tau_{max}$$

Deflection of Beams



$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d\theta}{dx} \text{ (small angles)}$$

$\tan\theta = dv/dx = v'(x)$

Small deflection $\tan\theta = \theta$

$$\theta = v'(x)$$

$$\frac{d\theta}{dx} = V''(x)$$

$$K = \frac{1}{\rho} = V''(x)$$

$$= \frac{M}{EI}$$

$$M = EI\,V''(x)$$ Bending Moment Equation

$$M = EI(x)\,V''(x)$$

Shear force Eq^n = $(EI_x V(x))' = V$

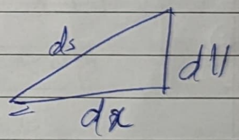
Loading Eq^n = $(EI_x V(x))'' = -q$

$$EIV'' = V \qquad EIV''' + q = 0$$

Note: If deformation isn't small

$$\frac{d\theta}{ds} = \frac{d}{ds}\tan^{-1}(V') = \frac{d\tan^{-1}(V')}{dx}\cdot\frac{dx}{ds}$$

$$= \frac{1}{1+(V')^2}V'' \cdot \frac{1}{(1+V'^2)^{1/2}}$$



$$, \frac{V''}{(1+V'^2)^{3/2}}$$

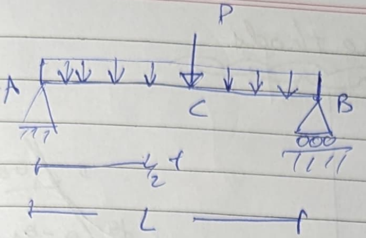
$$(ds)^2 = dx^2 + dV^2$$

$$\frac{ds}{dx} = (1+V'^2)^{1/2}$$

3/10/22

Look at Append. H

Q1



$$\delta_1 = \frac{5qL^4}{384EI}$$

$$\delta_2 = \frac{PL^3}{48EI}$$

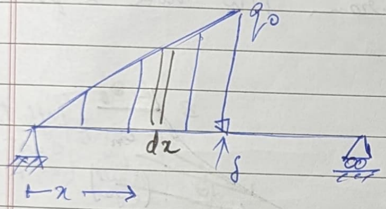
$$V_1 = \frac{-qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$\delta_C = \delta_1 + \delta_2 = \frac{5qL^4}{384EI} + \frac{PL^3}{48EI}$$

$$\theta_A = \theta_{A_1} + \theta_{A_2}$$

$$= \frac{qL^3}{24EI} + \frac{PL^2}{16EI}$$

Q2



$$\frac{2q_0x}{L}$$

$$\frac{2q_0 x\,dx \; x(3L^2 - 4x^2)}{48EI}$$

$$\delta_C = \frac{2q_0}{L\,48EI}\int_0^{L/2} x^2(3L^2 - 4x^2)\,dx$$

Superposition Principle

* ~~Super~~ Hooke's Law (Stress & Strain are linearly dependent)
* Deflections/rotations are small
* Deflection due to ~~loads~~ load(s) do not affect each other.

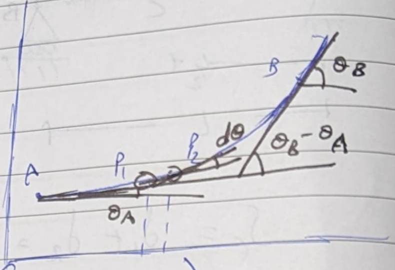
Moment - Area methods

$$K = \frac{M}{EI}$$

$$\frac{d\theta}{dx} = \frac{d(v')}{dx} = \frac{M}{EI}$$

$$d\theta = \frac{M}{EI}dx$$

$$\theta_{B/A} = \int_{x_A}^{x_B} \frac{M}{EI}\, dx$$



$$(\theta_B - \theta_A = \theta_{B/A})$$

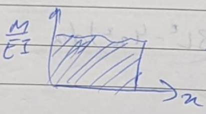
☆ First Moment - Area Theorem

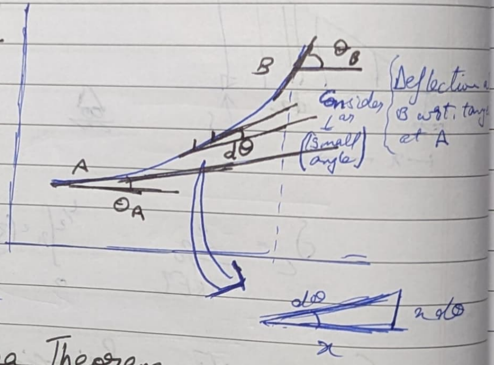
Angle $\theta_{B/A}$ between the tangents to the deflection curve at ~~any~~ pts. A & B is equal to area of M/EI diagram b/w those 2 pts.

Tangential deflection

Taken from end that is bendy (B)

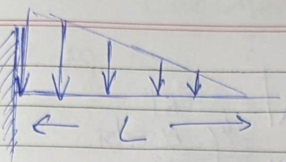
$$\int x \frac{M}{EI}\, dx$$





☆ Second Moment - Area Theorem

Tangential deviation at B is equal to first moment of area of M/EI diagram b/w A & B evaluated w.r.t. B



$$q = q_0\left(1 - \frac{x}{L}\right) \qquad EI\,v'''' = \cancel{0} = -q$$

$$v''' = -\frac{q_0}{EI}\int_0^x \left(1 - \frac{x}{L}\right) dx$$

$$= -\frac{q_0}{EI}\left(x - \frac{x^2}{2L}\right) + C_1$$

$$v''' = \frac{q_0 L}{EI}\left(1 - \frac{x}{L}\right)^2 + C_1$$

$x = L, \quad C_1 = 0$

$$v''' = \frac{q_0 L}{2EI}\left(1 - \frac{x}{L}\right)^2$$

$$v'' = -\frac{q_0 L^2}{6EI}\left(1 - \frac{x}{L}\right)^3 + C_2 \qquad x = L,\ C_2 = 0$$

$$v' = \frac{q_0 L^3}{24EI}\left(1 - \frac{x}{L}\right)^4 + C_3$$

$$v = -\frac{q_0 L^3}{120EI}\left(1 - \frac{x}{L}\right)^5 + C_3 x + C_4$$

$v(0) = 0$

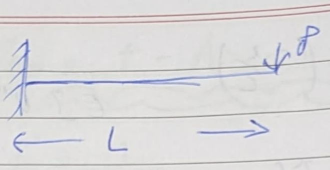
$v'(0) = 0$

$$C_4 = \frac{q_0 L}{120EI}$$

$$C_3 = \frac{-q_0 L^3}{24EI}$$

$$v = \frac{-q_0 L^4}{120EI}\left(1 - \frac{x}{L}\right)^5 - \frac{q_0 L^3 x}{24EI} + \frac{q_0 L}{120EI}$$

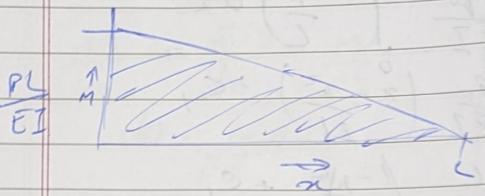
2/11/22



$\theta_A = 0$

$|\theta_{B/A}| = |\theta_B|$

$= \frac{PL}{EI} \cdot \frac{L}{2} = \frac{PL^2}{2EI}$



$t_{B/A} = \delta_B$

$= \frac{2L}{3} \frac{PL^2}{2EI} = \frac{PL^3}{3EI}$

Strain Energy - Bending

$\int_V \frac{\sigma \varepsilon}{2} dV$

$\int_0^L x \frac{P(L-x)}{EI}$ Px

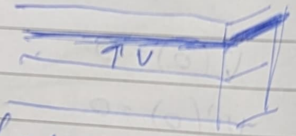
$\frac{PLx^2}{2} - \frac{Px^3}{3}$

$= \int \frac{\sigma^2}{2E} dV$

$\left(\sigma = \frac{My}{I}\right)$ Bend

$= \int \frac{M^2 y^2}{2EI^2} dA\, dV$

$= \int \frac{M^2}{2EI^2} \int y^2 dA\, dx$



$\int y^2 dA = I$

$\Rightarrow \int \frac{M^2}{2EI} dx$

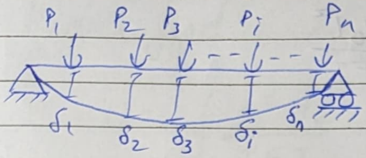
$= \frac{M^2 L}{2EI}$

$\rightarrow \int \frac{E^2 K^2 I^2}{2EI} dx$

$= \int \frac{EK^2 I}{2} dx$

Castigliano's Thm

Castligano's Thm



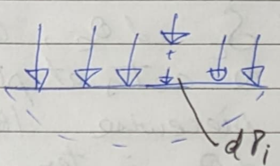
$U(P_1, P_2, P_3, \ldots P_i \ldots P_n)$

$P_i + dP_i$ Incrementally loading P_i

$dU = \frac{\partial U}{\partial P_i} dP_i$

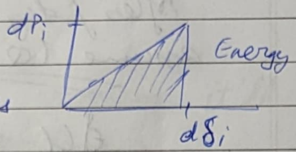
$\cancel{U} + \frac{\partial U}{\partial P_i} dP_i = \cancel{\frac{dP_i \, d\delta_i}{2}} + \cancel{U} + \frac{dP_i \, \delta_i}{2}$

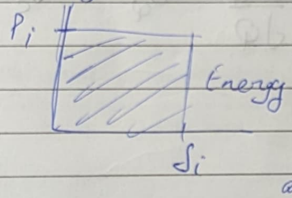
(Too small)



By incrementally increasing load by dP_i, we deflect by $d\delta_i$

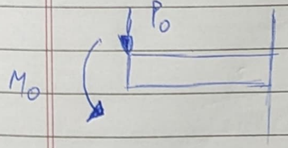
~~We now apply an incremental load dP_i that pushes beam down by δ_i~~





~~The system~~ We now add in the old system. The ~~load~~ energy U comes into eq.^{n}. Since that ~~[illegible]~~ P_i was already going down by dist. δ_i, the load dP_i moves down by δ_i along with.

$$\boxed{\frac{\partial U}{\partial P_i} = \delta_i}$$



$M = -M_0 - P_0 x$

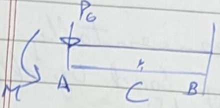
$U = \int_0^L \left(\frac{M_0^2 + P_0^2 x^2 + 2 M_0 P_0 x}{2EI} \right) dx$ $\qquad U = \int \frac{M^2}{2EI} dx$

$= \dfrac{M_0^2 L + \frac{P_0^2 L^3}{3} + M_0 P_0 L^2}{2EI}$

$$\delta_A = \frac{\partial U}{\partial P_0} = \frac{2P_0L^3}{6EI} + \frac{M_0L^2}{2EI}$$

$$\theta_A = \frac{M_0L}{EI} + \frac{P_0L^2}{2EI}$$

Dummy Load Method

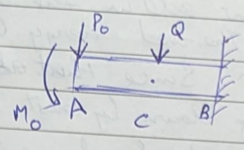


To find δ_C and θ_C, assume a force Q acting at pt. C. Solve the eq.ⁿ piecewise (A→C, C→B)

After finding δ_C and θ_C in terms of P_0, M and Q, assume (Q = 0)

$\frac{\partial U}{\partial P_i}$ becomes $\frac{\partial U}{\partial Q} = \delta_Q$ or δ_C

Q)



3/11/22 Modified Castigliano's Thm

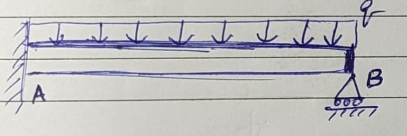
$U = \int_0^L \frac{M^2\,dx}{2EI}$ $\qquad \frac{\partial U}{\partial P_i} = \delta_i$

$\delta_i = \int_0^L \frac{\cancel{2}M}{\cancel{2}EI} \frac{\partial M}{\partial P_i}\,dx$

(i) Write M piecewise

(ii) $\frac{\partial M}{\partial P_i}$ piecewise

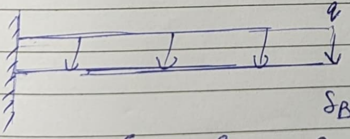
∴ When we overconstrain our problem, it'll become a statically indeterminate problem.



$\Sigma F_y = 0$

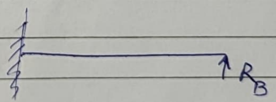
$\Sigma M = 0$

$\delta_B = 0$



δ_{B_1}

+



δ_{B_2}

$\delta = \delta_{B_1} - \delta_{B_2} = 0$

$\Rightarrow \delta_{B_1} = \delta_{B_2}$

$\delta_{B_1} = \frac{qL^4}{8EI}$ (downward) $\quad \delta_{B_2} = +\frac{R_B L^3}{3EI}$ (upward)

(-ve deflection w/ -ve force)

$\frac{qL^4}{8EI} = \frac{R_B L^3}{3EI}$

$\cancel{qL = \frac{8}{3} R_B}$

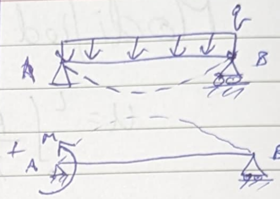
$R_B = \frac{3}{8} qL$

$R_A + R_B = qL$

$M_A + R_B L = \frac{qL^2}{2}$

$\Rightarrow R_A = qL - \frac{3}{8}qL = \frac{5}{8}qL$

$M_A = \frac{q}{2}L^2 - \frac{3}{8}qL^2 = \frac{qL^2}{8}$



$\theta_{A_1} = \theta_{A_2}$