Trajectory optimization Theory

- After we get way points from Path Planning, we need to use these way points to create a smooth trajectory which can be given to controller for reference

- Differential flatness property is used to create smooth trajectories in flat outputs $\rightarrow (x, y, z\ \psi)$

- Here trajectory optimization is done using 'min Snap Trajectory'

- we are assuming $x(t), y(t), z(t)$ & $\psi(t)$ to be polynomials in 't'

$$P(t) = P_n t^n + P_{n-1} t^{n-1} + \dots + P_2 t^2 + P_1 t + P_0$$

- Cost functional: $\min_{P(t)} \int_0^T (P^{(r)}(t))^2 . dt$

  - In our case we are min snap $\downarrow$ 4th deri of position

$$\therefore \min \int_0^T (\ddddot{x})^2 . dt$$

- we will use Euler - Lagrange method for optimization.
  - ↳ the order of polynomial is '7' in our case as per the Euler-lagrange method.

- In 2 waypoints between $t=0$, $t=T$ our trajectory will look like: $x(t) = c_7 t^7 + c_6 t^6 + c_5 t^5 + \cdots + c_1 t + c_0$

  our aim is to find $\underline{c_7, c_6, \ldots c_0 \text{ coeffs}}$.

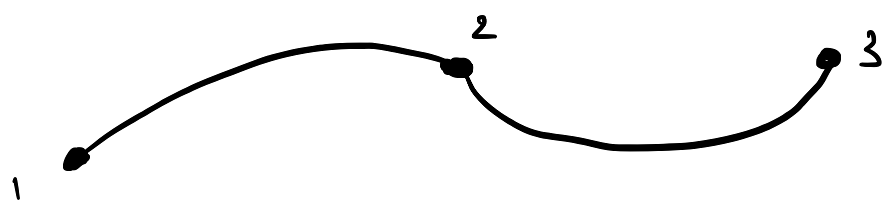
- Based on 4 start & end constraints for pos, vel, acc, jerk we can formulate 8 equations to find 8 unknowns.

$$A_{8\times 8} \, \underline{(\text{coeffs})_{8\times 1}} = b_{8\times 1}$$

- Ref link for more detailed explanation.

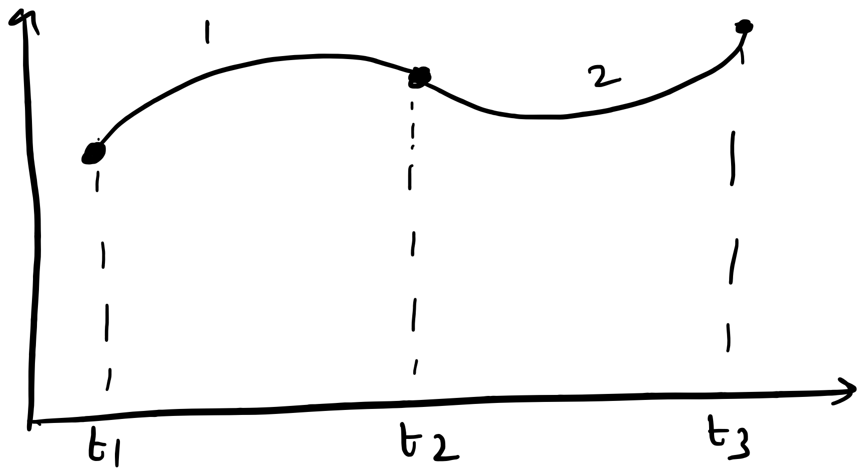
<u>multi - segment optimization</u>

- Lets take eg of 2 spline trajectory



- we need to perform this optimization between each set of waypoints (splines).

- Here an additional set of constraints is added



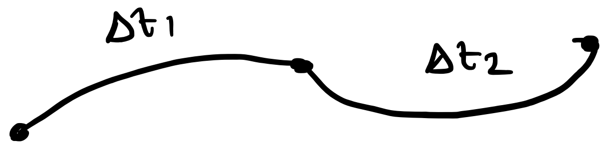
$$x_1(t_2) = x_2(t_2)$$
$$v_1(t_2) = v_2(t_2)$$
$$a_1(t_2) = a_2(t_2)$$
$$j_1(t_1) = j_2(t_2)$$

These set of continity constraints have to be imposed throughout the waypoints

Code Setup

- we will talk about how these constraints are structured in code

I] time per spline



- Based on set velocity we find dist required to travel b/w waypoints
- Due to this we dont need a time serial, we just keep record of Δt_i
- Hence, we can always do optimization b/w $t = 0 \rightarrow t = \Delta t_i$ for i^{th} spline

II) Constraints

$$A = \left[\begin{array}{cc} \overbrace{\phantom{[\text{start sp1, velocity}]}}^{\text{all constraints for spline 1}} & \overbrace{\phantom{[\text{end sp2, velocity}]}}^{\text{all constraints for spline 2}} \\ [\text{sp1}, t=0] & \\ & [\text{sp2}, t=0] \\ [\text{sp1}, t=T] & \\ & [\text{sp2}, t=T] \\ [\text{start sp1, velocity}] & \\ [\text{start sp1, acc}] & \\ [\text{start sp1, jerk}] & \\ & [\text{end sp2, velocity}] \\ & [\text{end sp2, acc}] \\ & [\text{end sp2, jerk}] \\ -[\text{pos at } t=\Delta t_1] & [\text{pos at } t=0] \\ -[\text{vel at } t=\Delta t_1] & [\text{vel at } t=0] \\ -[\text{acc at } t=\Delta t_1] & [\text{acc at } t=0] \\ -[\text{jerk at } t=\Delta t_1] & [\text{jerk at } t=0] \end{array}\right]$$

$$\left.\begin{array}{l} -[\text{pos at } t=\Delta t_1] \; [\text{pos at } t=0] \\ -[\text{vel at } t=\Delta t_1] \; [\text{vel at } t=0] \\ -[\text{acc at } t=\Delta t_1] \; [\text{acc at } t=0] \\ -[\text{jerk at } t=\Delta t_1] \; [\text{jerk at } t=0] \end{array}\right\} \text{continuity constraints.}$$

$$B = \begin{bmatrix} \text{waypoint 1 } (t=0) \\ \text{waypoint 2 } (t=0) \\ \text{wp 1 } (t=T) \\ \text{wp 2 } (t=T) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{\text{coeffs} = A^{-1} B}$$