Rule-based Optimal Control for Autonomous Driving

Team 6

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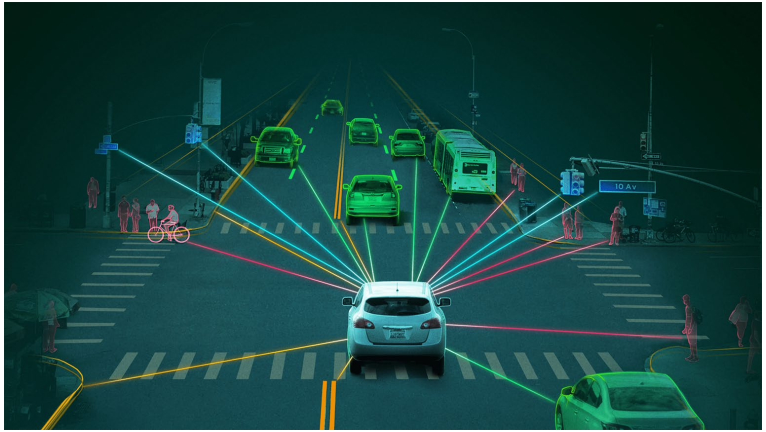
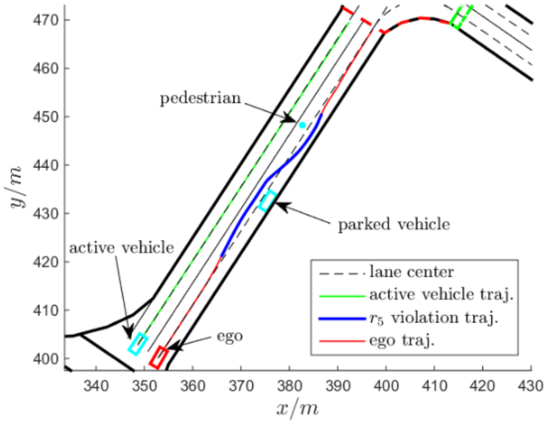
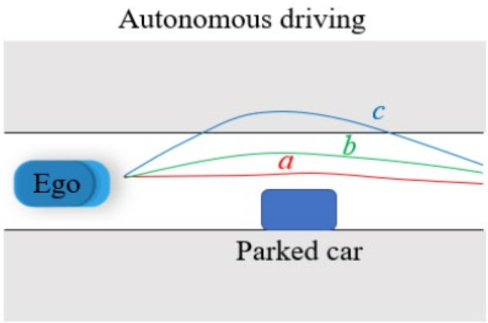


Introduction - Paper Proposal

- In this project we implemented an optimal control approach for AV to navigate on road.
- While driving on road, certain traffic rules and driving behaviour needs to be demonstrated.
- For example, an AV has to **avoid collisions** with other road users (high priority), **drive faster** than the minimum speed limit (low priority), and **maintain longitudinal clearance** with the lead car (medium priority).

  - Constraints are formulated as **Control Lyapunov Functions (CLF)** and **Control Barrier Functions (CBFs)**. We formulate iterative rule relaxation according to the priority on the rules





Control Lyapunov Functions

- Consider an affine control system, with following general form:

$$\dot{x} = f(x) + g(x)u$$

- **CLF is designed for reaching a target state (or set)**

- <u>CLF general form:</u> $\boxed{L_f V(x) + L_g V(x) u + \epsilon V(x) \leq \delta_e}$

where

L_f	Lie derivative along f
$V(x)$	Lyapunov Function
δ_e	CLF Slack Variable

- ϵ is a tuning constant. As ϵ gets bigger, the CLF constraint imposes stricter condition. It requires $V(x)$ to decay more quickly.

Control Barrier Functions

- CBF are designed for designed for avoiding an unsafe set
- CBF general form: $\boxed{L_f B(x) + L_g B(x) u + \gamma_i B(x) - \delta_i \geq 0}$
- HOCBF (2nd order): $\boxed{L_f^2 B(x) + L_f L_g B(x) u + \gamma_i B(x) - \delta_i \geq 0}$

where $B(x)$ Control Barrier Function

δ_i CBF Slack Variable

Problem Formulation

1. Dynamics Model:

Affine control system:

$$\dot{x} = f(x) + g(x)u$$

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \dot{\theta} \end{bmatrix}$$

2. Cost Function:

$$\min_{u,\delta_e,\delta_i} J = \int_0^{t_f} [c_1 v^2 + c_2\dot{\theta}^2 + c_3\delta_e^2 + c_4\delta_1^2 + c_5\delta_2^2 + c_6\delta_3^2]\, dt$$

v	Linear Velocity
ω	Angular Velocity
δ_e	CLF Slack Variable
δ_1	Pedestrian Distance CBF Slack Variable
δ_2	Lane-keeping CBF Slack Variable
δ_3	Minimum Velocity CBF Slack Variable

Problem Formulation

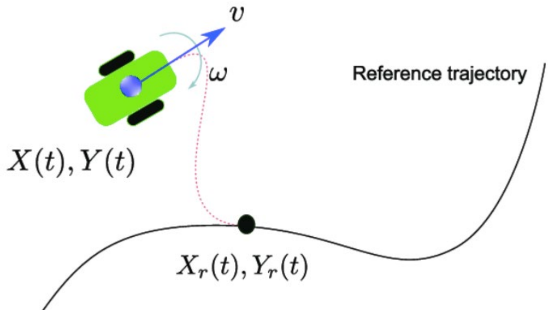
3. Regular state and control constraints :

$$v_{min} \leq v \leq v_{max} \quad \forall t \in [0, t_f]$$
$$\dot{\theta}_{min} \leq \dot{\theta} \leq \dot{\theta}_{max} \quad \forall t \in [0, t_f]$$

$$y_{min} \leq y(t) \leq y_{max} \quad \forall t \in [0, t_f]$$
$$X(t_f) = X_{goal}$$

4. CLF:

$$V(x) = \|y\|^2$$



Error:

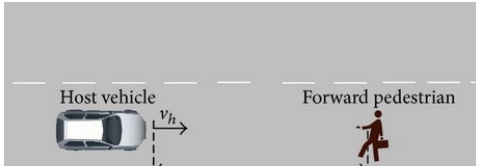
$$\|y\|^2 = (x - x_{ref})^2 + (y - y_{ref})^2 + (\theta - \theta_{ref})^2$$

CLF Constraint:

$$2v(x - x_{ref})\cos\theta + 2v(y - y_{ref})\sin\theta + 2(\theta - \theta_{ref})\dot{\theta} + \epsilon\|y\|^2 \leq \delta_e$$

Problem Formulation

5. CBFs

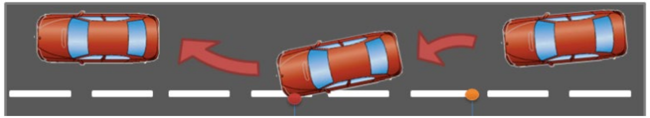


- **Pedestrian avoidance HOCBF:** $B(x) = (x - x_p)^2 + (y - y_p)^2 - r^2$

$$\Rightarrow \boxed{\gamma_1[(x - x_p)^2 + (y - y_p)^2 - r^2] + 2v(x - x_p)\cos\theta + 2v(y - y_p)\sin\theta - \delta_1 \geq 0}$$

- **Lane keeping CBF:** $B(x) = y - l$

$$\Rightarrow \boxed{\gamma_2(y - l) + v\cos\theta - \delta_2 \geq 0}$$

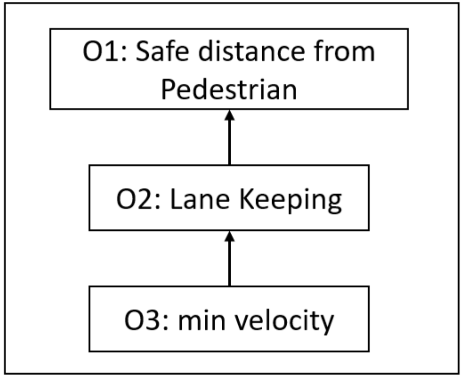


- **Minimum Velocity CBF:** $B(x) = v - v_{min}$

$$\Rightarrow \boxed{\gamma_3 v - \gamma_3 v_{min} - \delta_3 \geq 0}$$

Priority structure Algorithm - Iterative approach

The algorithm iterates through the power set created which specifies which CBFs are relaxed in a priority order.



Algorithm 1 Recursive relaxation algorithm

Input: System dynamics, initial condition, control bounds, state constraints, rules power set R, reference trajectory $\mathcal{X}$.

Output: Optimal ego trajectory.

```
m rules
k = 0
while k ≤ 2^m do
    Get S_k, k^th priority scenario of relaxed rules from R.
    Solve optimization problem with S_k
    if the above problem is feasible then
        Generate optimal trajectory X*.
    end if
end while
```

Power Set

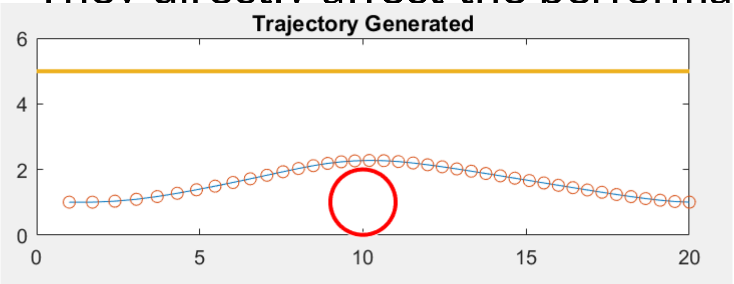
$$\{\{\emptyset\}, \{O_3\}, \{O_2\}, \{O_3, O_2\}, \{O_1\}, \{O_3, O_1\}, \{O_2, O_1\}, \{O_3, O_2, O_1\}\}$$

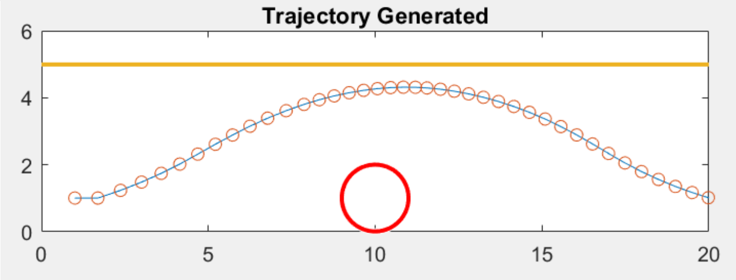
Critical Review of The Paper

- In the paper, optimization problem is solved at each time step using QP by fixing the state values whereas we used SQP method by optimizing the whole trajectory via fmincon in MATLAB, due to the existence of nonlinear constraints
- The dynamic model and control inputs we employ differ from those outlined in the paper
- The CLF and HOCBF equations are structured differently
- Our priority rule framework comprises 3 rules as opposed to the rules specified in the paper's rulebook

Tuning Parameters

- Selecting appropriate hyperparameters is very important.
  They directly affect the performance of the controller.





γ = 0.1

γ = 0.5

- The graph shows the effect of changing hyperparameter γ associated with the Pedestrian safe circle
- Inappropriate values of hyperparameters can also render an infeasible solution

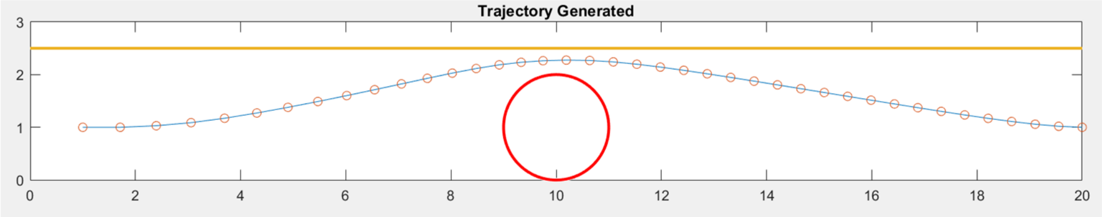
Results

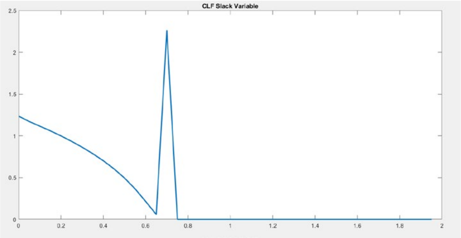
We are testing 4 scenarios here:

1. No CBF constraints are relaxed (corresponds to relaxing $\{\emptyset\}$)
2. Iteratively Lane keeping constraint is relaxed (corresponds to relaxing $\{O_2\}$)
3. Iteratively Pedestrian constraint is relaxed (corresponds to relaxing $\{O_1\}$)
4. Iteratively Lane keeping and Pedestrian constraints are relaxed (corresponds to relaxing $\{O_{1,}O_2\}$)

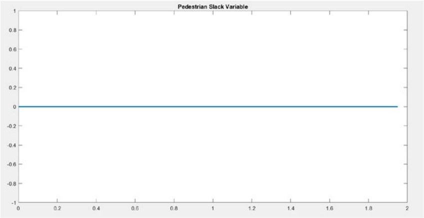
Scenario 1

- All constraints act as hard constraints, hence we can see the slack variables for all CBFs to be zero.
- The trajectory follows a path between the gap of lane boundary and pedestrian safe circle.

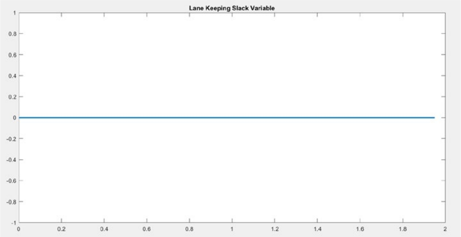




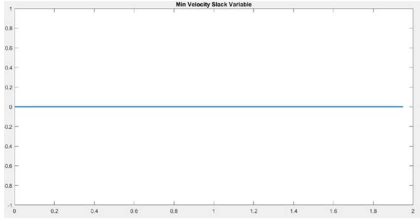
CLF Slack



Pedestrian CBF Slack



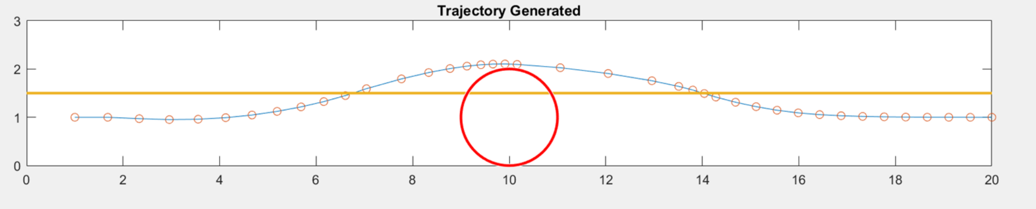
Lane-keeping CBF Slack

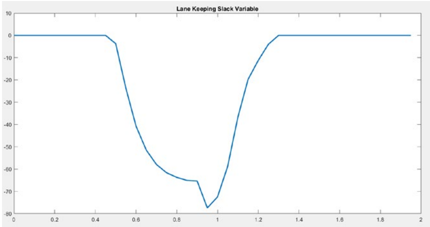


Minimum velocity CBF Slack

Scenario 2

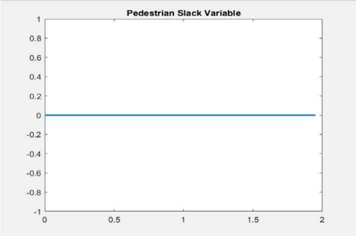
- Iterative power set finds optimal trajectory when lane keeping constraint is relaxed.
- Lane keeping constraint is violated without violating the pedestrian and min velocity constraint.
- The lane keeping slack variable has a non zero value, but only when constraint is violated.



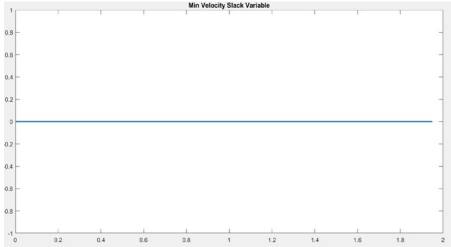


Lane-keeping CBF Slack

Lane keeping slack variable is negative, signifying constraint loosening.



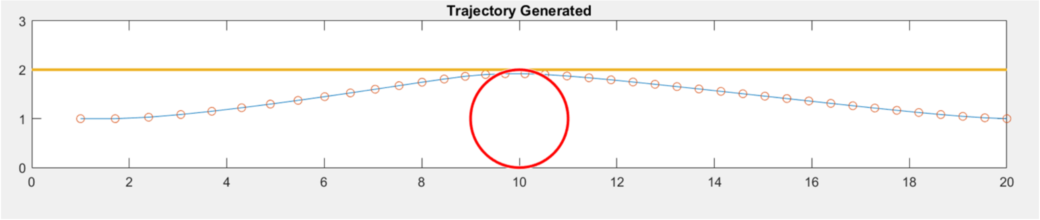
Pedestrian CBF Slack

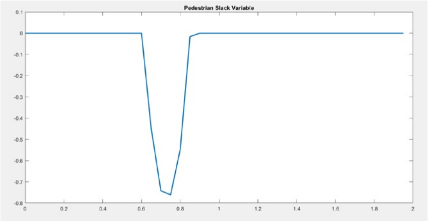


Minimum velocity CBF Slack

Scenario 3

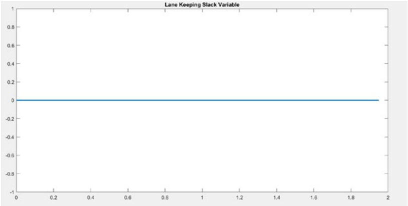
- Here the lane keeping is made a hard constraint for test case purposes.
- We can see that the trajectory violates the pedestrian safe circle, since there is no other feasible trajectory.
- However, it tries to stay as far away as possible from the pedestrian to minimize constraint violation.



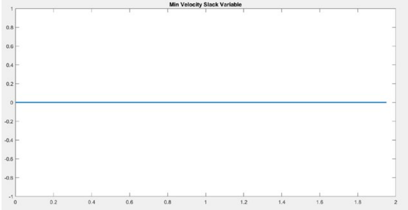


Pedestrian CBF Slack

Pedestrian distance slack variable is negative, signifying constraint loosening.



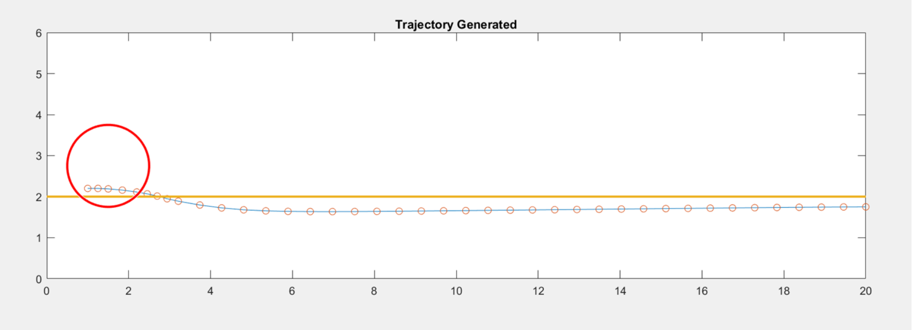
Lane-keeping CBF Slack

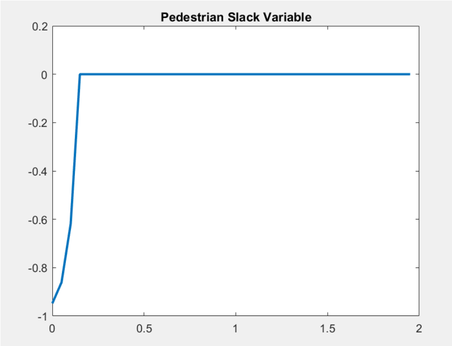


Minimum velocity CBF Slack

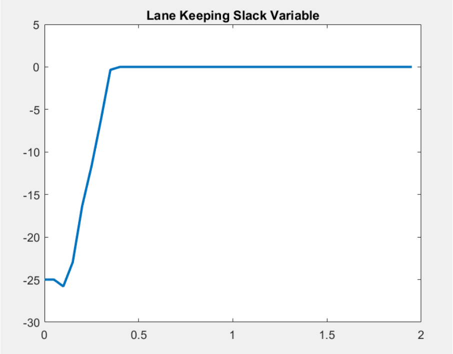
Scenario 4

- Here both lane keeping and pedestrian constraints are relaxed at the starting point.
- We can see that the trajectory tries to move out of the circle as fast as possible.
- However, it tries to exit the pedestrian circle and go inside the lane as soon as possible to minimize constraint violation.





Pedestrian CBF Slack



Lane-keeping CBF Slack

Thank you

References

1. https://researchleap.com/research-in-autonomous-driving-a-historic-bibliometric-view-of-the-research-development-in-autonomous-driving/
2. https://github.com/HybridRobotics/CBF-CLF-Helper
3. https://www.hindawi.com/journals/tswj/2014/218246/
4. https://www.researchgate.net/figure/Basic-idea-of-trajectory-tracking-in-a-differential-drive-mobile-robot_fig1_338722172
5. Xiao, W., Mehdipour, N., Collin, A., Bin-Nun, A. Y., Frazzoli, E., Tebbens, R. D., & Belta, C. (2021). Rule-based optimal control for autonomous driving. *Proceedings of the ACM/IEEE 12th International Conference on Cyber-Physical Systems*. https://doi.org/10.1145/3450267.3450542