MULTIPHYSICS PROBLEMS AND THEIR APPLICATIONS IN ENGINEERING

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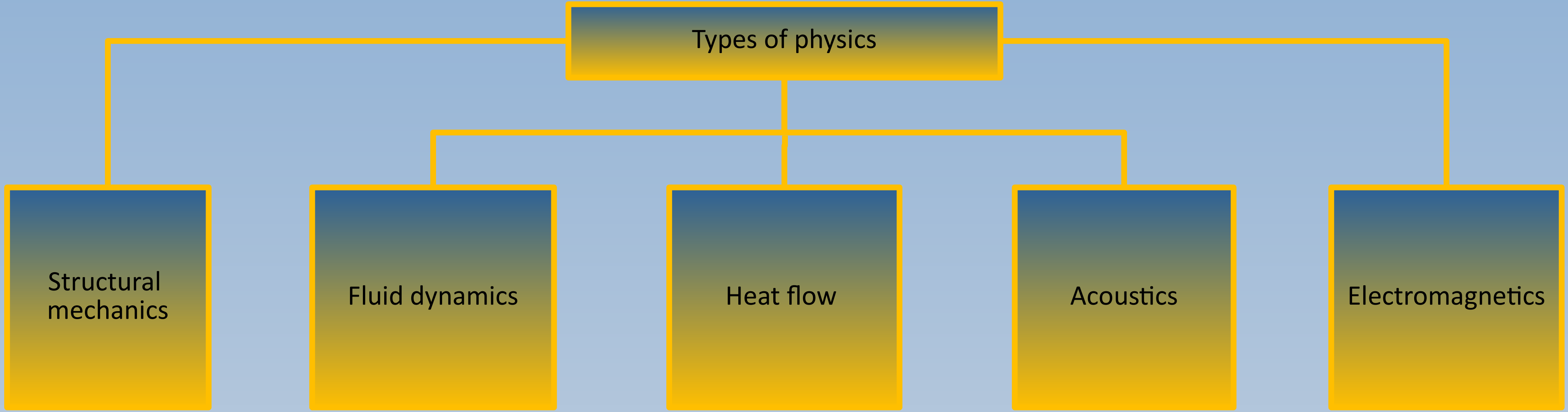
MOTIVATION AND INTRODUCTION

- The phenomena which occur in the world can be described using sets of physical laws. These physical laws have been implemented using computers since many years. Earlier, computing resources had limited capabilities, thus physical phenomena were observed in isolation. But nature is multiphysics and an isolated study can only give us an incomplete understanding.

- Thus, today we study multiphysics phenomena using experiments and computer simulations. This presentation aims to give an introduction to various multiphysics problems and their applications in engineering.



STRUCTURAL MECHANICS

- Governing equations for structural analysis-

A. Governing equation-

$$\rho \frac{d^2 \overrightarrow{u_L}}{d^2 t}\Bigg|_X = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i + \bar{f}_l \quad \text{in} \quad \Omega_t^S.$$

B. Strain-displacement equation-

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \quad \text{in } \Omega_t^S.$$

C. Constitutive equation-

$$\sigma_{ij} = c_{ijkl}\left(\varepsilon_{kl} - \frac{\alpha_{ij}\overleftarrow{\Delta T}}{T}\right) \quad \text{in} \quad \Omega_t^s.$$

D. Boundary conditions-

$$u_i = \overrightarrow{g_i} \qquad \text{on} \quad \Gamma_t^g$$

$$\sigma_{ji} n_j = \bar{h}_l \qquad \text{on} \quad \Gamma_t^h$$

- Equilibrium equation in the matrix form-

$$\boldsymbol{M}\ddot{\boldsymbol{d}} + \boldsymbol{Q}(\boldsymbol{u}) + \boldsymbol{K}_{ut}^S \Delta \boldsymbol{T} = \boldsymbol{F}^E + \overleftarrow{\boldsymbol{F}_f^s} + \overleftarrow{\boldsymbol{F}_{EM}^S}.$$

- The primary variable for structural analysis is displacement

FLUID DYNAMICS

- Equations for fluid flow-

A. Basic equations-

$$\frac{1}{B}\frac{\partial \vec{p}}{\partial t}\bigg|_{\chi} + \frac{1}{B}c_i\frac{\partial p}{\partial x_i} + \frac{\partial v_i}{\partial x_i} = 0 \quad \text{in} \quad \Omega_t^f$$

$$\rho\frac{\partial \vec{v}_i}{\partial t}\bigg|_{\chi} + \rho c_j\frac{\partial v_i}{\partial x_j} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i + \overleftarrow{f_i} \quad \text{in} \quad \Omega_t^f$$

$$\frac{\partial(\rho c_v \vec{T})}{\partial t}\bigg|_{\chi} + \frac{\partial}{\partial x_i}\left(\rho c_v T v_i - k\frac{\partial T}{\partial x_i}\right) = 2\mu D^2$$

$$+\frac{\partial v_i}{\partial x_i}\left(-p + \lambda\frac{\partial v_i}{\partial x_i}\right) + \overleftarrow{q^B} \quad \text{in} \quad \Omega_t^f .$$

$$D^2 = e_{ij}e_{ij} \qquad e_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}).$$

B. Constitutive equation-

$$\sigma_{ij} = -p\,\delta_{ij} + \frac{1}{2}\mu\left(v_{i,j} + v_{j,i}\right)$$

C. Boundary conditions-

$$v_i = \overleftarrow{g_i} \quad \text{on} \quad \Gamma_g^f$$

$$\sigma_{ji}n_j = \vec{h_i} \quad \text{on} \quad \Gamma_h^f$$

$$T = T^s \quad \text{on} \quad \Gamma_g^T$$

$$k \cdot n_i \partial T_i / \partial x_i = q^s \quad \text{on} \quad \Gamma_q^T$$

D. Initial conditions-

$$v_i(0) = {}^0v_i \quad \text{on} \quad \Omega^f$$

$$p(0) = {}^0p \quad \text{on} \quad \Omega^f$$

$$T(0) = {}^0T \quad \text{on} \quad \Omega^f$$

HEAT FLOW

- Governing equation-

$$\frac{\partial(\rho c_v T)}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho c_v T v_i - k\frac{\partial T}{\partial x_i}\right) = \overleftarrow{q^B},\ \text{in}\ \Omega_t^T$$

- Boundary conditions-

$$T = \overleftrightarrow{T^s} \quad \text{on} \quad \Gamma_g$$

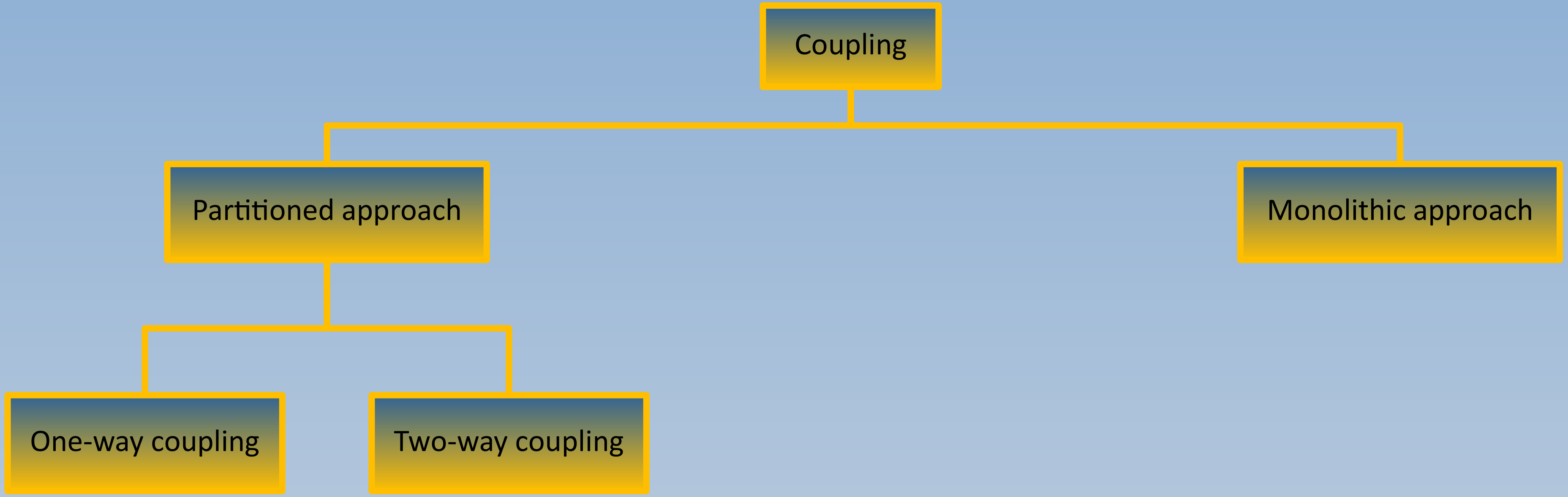
$$k \cdot \frac{(n_i \partial T)}{(\partial x_i)} = \overleftarrow{q^s} \quad \text{on} \quad \Gamma_{q1}$$

$$k \cdot \frac{(n_i\ \partial T)}{(\partial x_i)} = h_f(T_B - T_S) \quad \text{on} \quad \Gamma_{q2}$$

COUPLING SEVERAL PHYSICS PHENOMENA

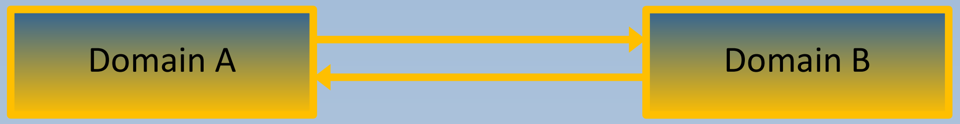
- When two different physical domains are meshed together, the nodal positions between the two physical meshes are coincident. Thus, the governing equations of two physical domains are solved jointly. This approach is called the monolithic approach.

- Unlike the monolithic approach, the partitioned approach solves two physics phases separately in sequential order using different meshing. Therefore, the nodal positions in the two physics phases do not match. Compared with the monolithic approach, the partitioned approach is very fast and efficient, although it requires much more convergences.



COUPLING SEVERAL PHYSICS PHENOMENA

- One-way coupling requires that quantities are sent from one domain to another, but not in the opposite direction.
- In two-way coupling, data is transferred frequently between the two domains in both directions. This procedure continues in an iterative process until the convergence is reached.
- The monolithic approach and two-way coupling are used for a coupling system with a high correlation between physics models. Therefore, they are called strong coupling. On the other hand, one-way coupling is applied for the coupling with low dependency between physical models, which is called weak coupling.



FLUID-STRUCTURE INTERACTION

Applications	
1	Abdominal aortic aneurysm
2	Arterial blood flow
3	Vibrating reeds
4	Peristaltic pump
5	Hydrodynamics of ships
6	Mitral valve
7	Ribbed helix lip seals
8	Wind turbine blades
9	Aircraft wings
10	MEMS devices
11	Sea floor energy harvesting
12	Artificial cilia

THERMAL STRESS COUPLING

- Modified heat equation-

$$\frac{\partial Q}{\partial t} = T_0 \beta^T \frac{\partial \varepsilon}{\partial t} + \rho C_V \frac{\partial \Delta T}{\partial t} - k\nabla^2 T.$$

- Coupled constitutive equation-

$$\boldsymbol{\varepsilon} = [\boldsymbol{D}]^{-1} \boldsymbol{\sigma} + \boldsymbol{\alpha}\Delta T,$$

$$S = \boldsymbol{\alpha}^T \boldsymbol{\sigma} + \frac{\rho C_P}{T_0} \Delta T.$$

$$\boldsymbol{\varepsilon} = \left[\varepsilon_x, \varepsilon_y, \varepsilon_z, \varepsilon_{xy}, \varepsilon_{yz}, \varepsilon_{xz}\right]^{\mathrm{T}}$$

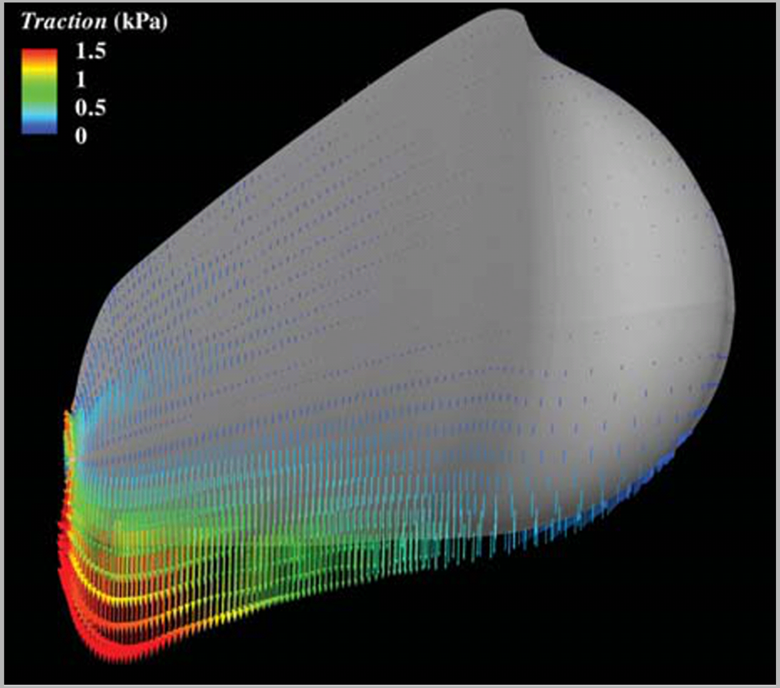
$$\boldsymbol{\sigma} = \left[\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}\right]^{\mathrm{T}}$$

$$[\mathrm{C}] = \text{elastic stiffness matrix}$$

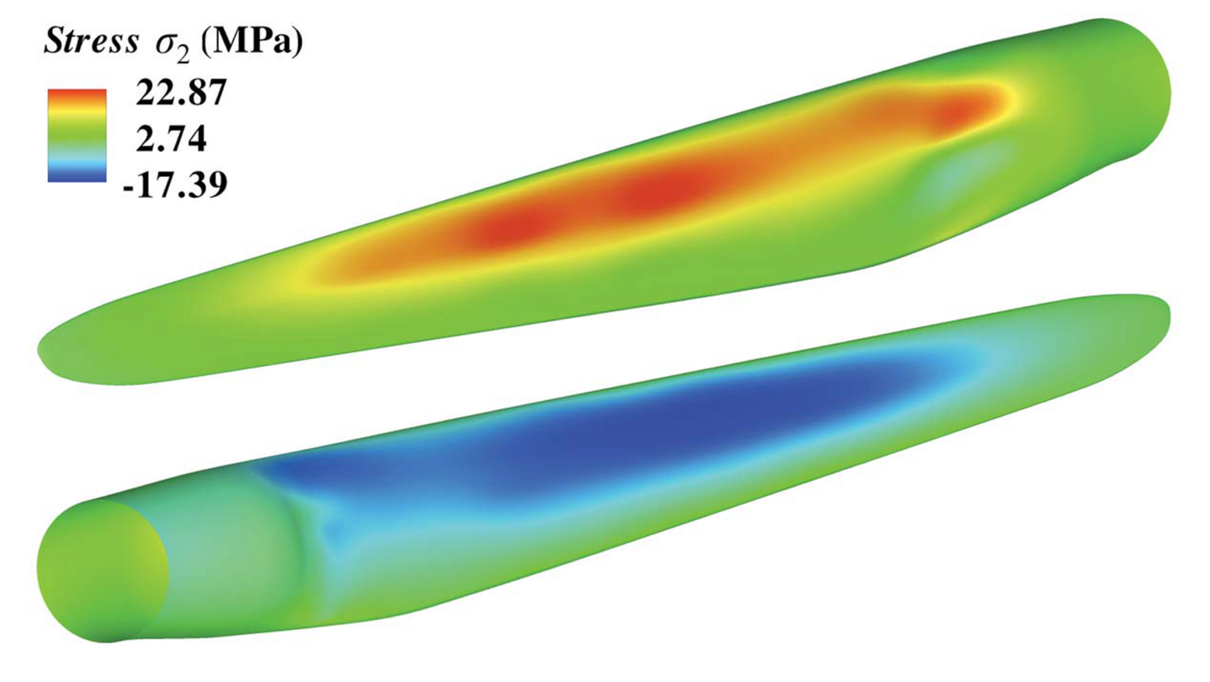
	Applications
1	Solar PV module
2	Globe valve
3	Shell and tube heat exchanger
4	Disc brakes
5	Nuclear reactors
6	Package reliability in electronics products
7	Manufacturing processes like additive manufacturing, soldering, welding, forging, hot stamping
8	Automobile and aircraft tires
9	Heat Generation in Biological Tissues under Cyclic Loading
10	Battery electric vehicles
11	Engine exhaust manifold
12	Turbine blades

FLUID-STRUCTURE INTERACTION

Case study 1: Wind turbine blade



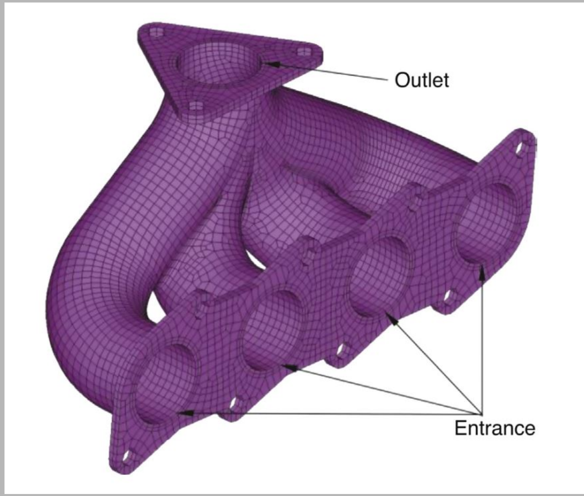
Fluid traction vectors acting on the blade



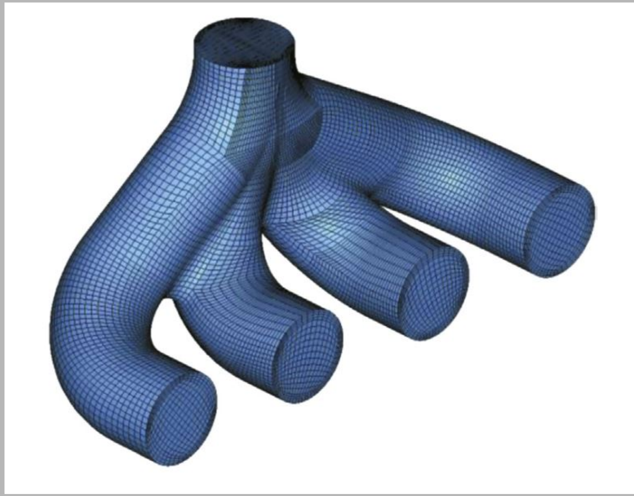
Normal stress distribution. TOP: Pressure side. BOTTOM: Suction side

STRUCTURAL-FLUID-THERMAL COUPLING

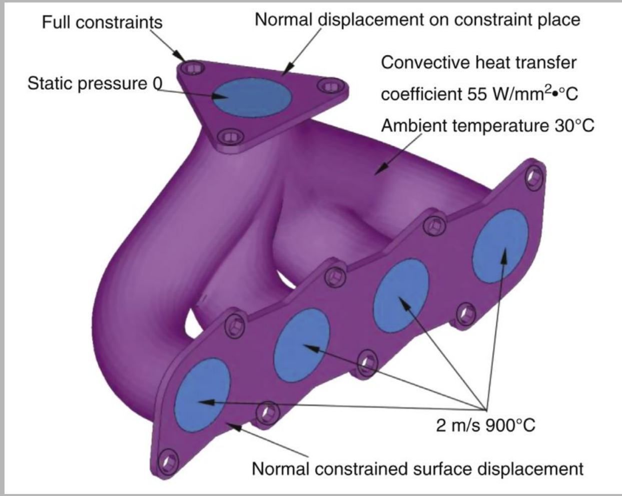
Case study 2: Engine exhaust manifold



Meshed model of
thermal-structural physics



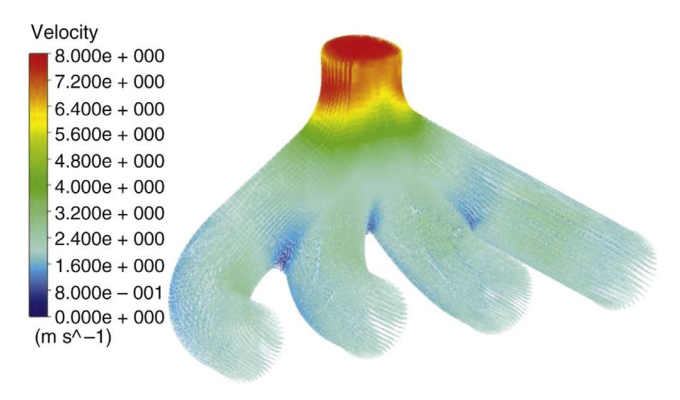
Meshed model of
fluid-thermal physics



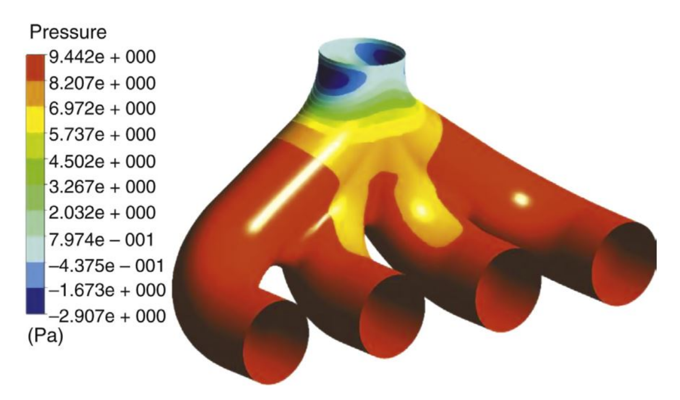
Loads and boundary conditions for combined
fluid-thermal-structural coupling

STRUCTURAL-FLUID-THERMAL COUPLING

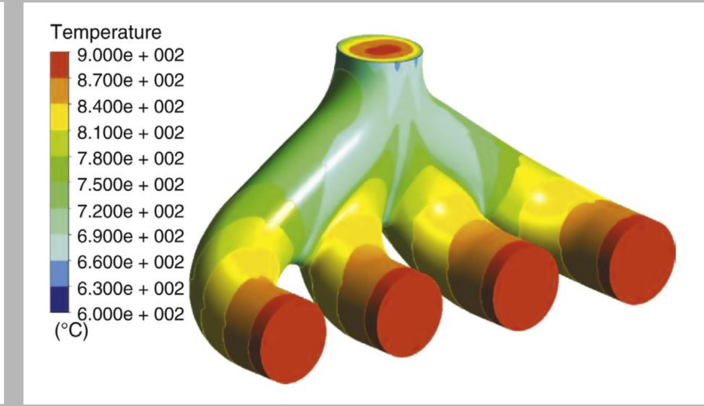
Case study 2: Engine exhaust manifold



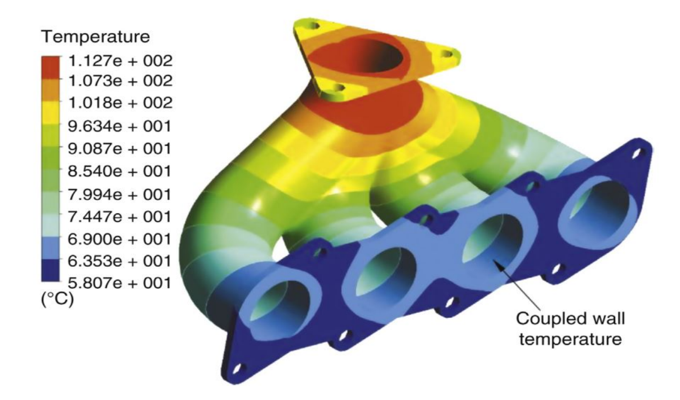
Gas flow velocity



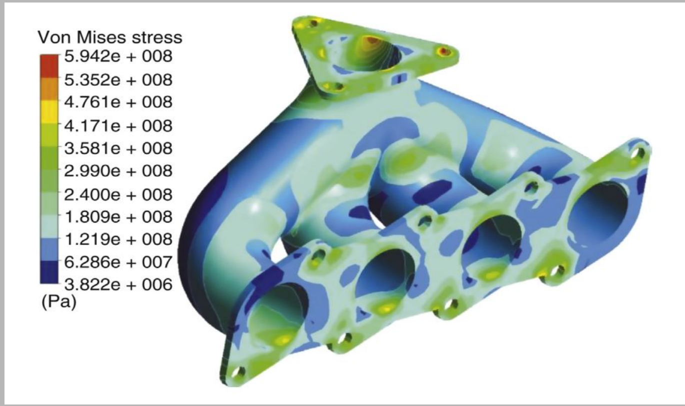
Inner surface gas pressure



Gas temperature



Temperature distribution



Stress distribution

Thank You