Unit IV FUNDAMENTAL OF FUZZY LOGIC

Definition: Fuzzy Set

A fuzzy set of set X is a pair, $\tilde{A} = \{(x, \mu_A(x)) \mid x \in X\}$, where μ_A: X→ [0,1] is a function which map all values in X to [0,1] which is also called as **membership function**. The value $\mu_A(x)$ is called grade of membership of x.

Example: Let Ã be the fuzzy set of "smart" students, where "smart" is fuzzy term.

$$\tilde{A} = \{(g_1, 0.4)(g_2, 0.5)(g_3, 1)(g_4, 0.9)(g_5, 0.8)\}$$

Here Ã indicates that the smartness of g_1 is 0.4 and so on

For Finite set, $X = \{x_1, x_2, x_3, \dots, x_n\}$, the fuzzy , $\tilde{A} = \{(x, \mu_A(x)) \mid x \in X\}$ for each $x \in X$ is often denoted by

$$\tilde{A} = \left\{\frac{\mu_X(x_1)}{x_1}, \frac{\mu_X(x_2)}{x_2}, \dots, \frac{\mu_X(x_n)}{x_n}\right\} \text{ or } \tilde{A} = \sum_{i=1}^{n} \frac{\mu_X(x_i)}{x_i}$$

Operation on Fuzzy Sets

Let $\tilde{A} = \{(x, \mu_A(x)) \mid x \in X\}$ and $\tilde{B} = \{(x, \mu_B(x)) \mid x \in X\}$

be any two fuzzy sets of $X = \{x_1, x_2, \dots x_n\}$

① Union :

$$\tilde{A} \cup \tilde{B} = \{(x, \mu_{A \cup B}(x)) \mid x \in X\}$$

where $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$

② Intersection

$$\tilde{A} \cap \tilde{B} = \{(x, \mu_{A \cap B}(x)) \mid x \in X\}$$

where $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$

③ Complement

$$\tilde{A}^c = \{(x, \mu_{A^c}(x)) \mid x \in X\}$$

where $\mu_{A^c}(x) = 1 - \mu_A(x)$

④ Equality

$\tilde{A} = \tilde{B}$ iff $\mu_A(x) = \mu_B(x)$ for each $x \in X$

⑤ Algebraic product of fuzzy sets

$$\tilde{A} \cdot \tilde{B} = \{(x, \mu_A(x) \cdot \mu_B(x)) \mid x \in X\}$$

⑥ Multiplication of fuzzy sets by a crisp number

$$d\tilde{A} = \{(x, d\,\mu_A(x)) \mid x \in X, d \in [0,1]\}$$

⑦ Power of fuzzy set

$$(\tilde{A})^P = \{(x, (\mu_A(x))^P) \mid x \in X, P \geq 0\}$$

⑧ Algebraic sum of fuzzy sets

$$\tilde{A} + \tilde{B} = \{(x, \mu_{A+B}(x)) \mid x \in X\}$$

where $\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$

⑨ Bounded sum of fuzzy sets

$$\tilde{A} \oplus \tilde{B} = \{(x, \mu_{A \oplus B}(x)) \mid x \in X\}$$

where $\mu_{A \oplus B}(x) = \min\{1, \mu_A(x) + \mu_B(x)\}$

(10) Algebraic difference

$$\tilde{A} - \tilde{B} = \{(x, \mu_{A-B}(x)) \mid x \in X\}$$

where
$$\mu_{A-B}(x) = \mu_{A \cap B^c}(x)$$
$$= \min\{\mu_A(x), \mu_{B^c}(x)\}$$
$$= \min\{\mu_A(x), 1 - \mu_B(x)\}$$

(11) Bounded Difference

$$\tilde{A} \ominus \tilde{B} = \{(x, \mu_{A \ominus B}(x)) \mid x \in X\}$$

where,
$$\mu_{A \ominus B}(x) = \max\{0, \mu_A(x) + \mu_B(x) - 1\}$$

EXAMPLES

Q.1 Let $\tilde{A} = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$

& $\tilde{B} = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.8), (x_4, 0.9)\}$

be any two fuzzy sets on set $X = \{x_1, x_2, x_3, x_4\}$

then find the following values

(1) $\tilde{A} \cup \tilde{B}$ (2) $\tilde{A} \cap \tilde{B}$ (3) $(\tilde{A})^c, (\tilde{B})^c$ (4) $\tilde{A} \cdot \tilde{B}$

(5) $0.7\tilde{B}$ (6) $(\tilde{A})^{10}$ (7) $\tilde{A} + \tilde{B}$ (8) $\tilde{A} \oplus \tilde{B}$

⑨ $\tilde{A} - \tilde{B}$ ⑩ $\tilde{A} \ominus \tilde{B}$

MEMBERSHIP FUNCTION

* **Universe of Discourse** : It is domain of the membership function.

Defn : It is a function that maps elements in the universe of discourse to a value in the interval set [0,1] with each element having degree of membership.

Mathematically : Let X be any set then a map

$\mu_A : X \rightarrow [0,1]$ is called membership function

Universe of Discourse

Note : Membership function provides smooth (or gradual) transition from region outside the interval to region inside the interval.

Ex.

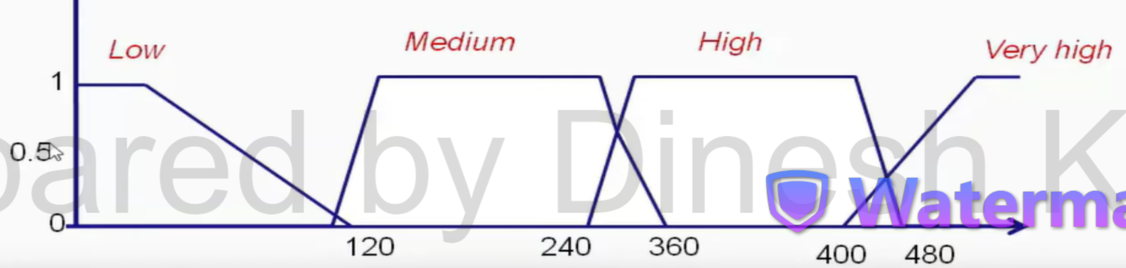
Consider a problem to identify the risk of cancer among a section of workforce employed in Lead industry based on no. of minutes a worker is exposed per day to the Lead processing unit.

So we identify 4 sets of risk of having cancer amongst the workers-

Low - [0 -120] ; Medium-[118- 360] ; High- [240-480] ;Very high=[400 or more]

1 → Less risk of cancer

0 → High risk of cancer



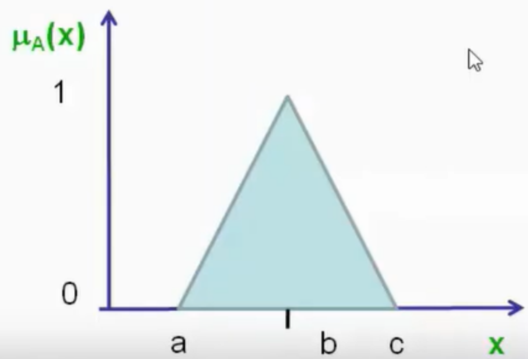
TYPES OF MEMBERSHIP FUNCTION

Prepared by Mr.Dinesh Kute

1. Traingular membership function
2. Trapezoidal membership function
3. Gaussian membership function

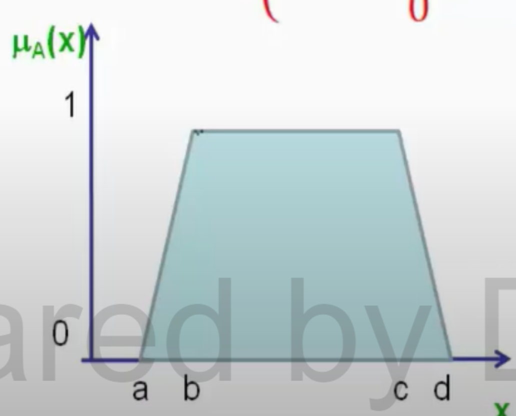
- Triangular: specified as

$$triangle(x{:}\,a,b,c) = \begin{cases} 0 & x<a \\ (x-a)/(b-a) & a\le x\le b \\ c-x/c-b & b\le x\le c \\ 0 & x>c \end{cases}$$



- Trapezoidal: specified as

$$= \begin{cases} 0 & x<a \\ x-a)/(b-a) & a\le x<b \\ 1 & b\le x<c \\ d-x/d-c & c\le x<d \\ 0 & x>d \end{cases}$$

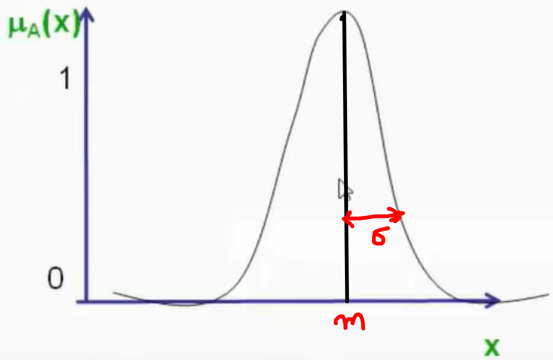


- Gaussian: specified as

$$gaussian(x{:}\,m,\sigma) = e^{\left(\frac{(x-m)^2}{\sigma^2}\right)}$$

m = mean

σ = standard deviation.



FEATURES OF MEMBERSHIP FUNCTION

① core : set of $x \in X$ for which $\mu_A(x) = 1$

OR $\{x \in X \mid \mu_A(x) = 1\}$

② support : set of $x \in X$ for which $\mu_A(x) > 0$

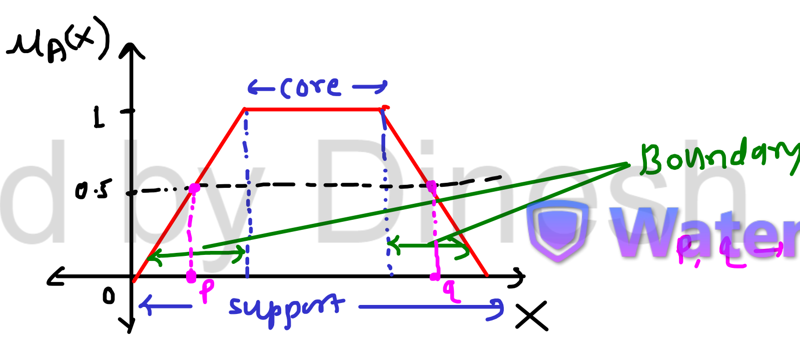
OR $\{x \in X \mid \mu_A(x) > 0\}$

③ crossover : set of $x \in X$ for which $\mu_A(x) = 0.5$

OR $\{x \in X \mid \mu_A(x) = 0.5\}$

④ Boundary : set of $x \in X$ for which $0 < \mu_A(x) < 1$

* Graphical representation of features of membership function.

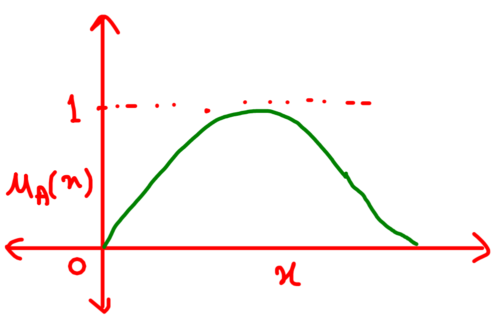


a) CONVEX FUZZY SET

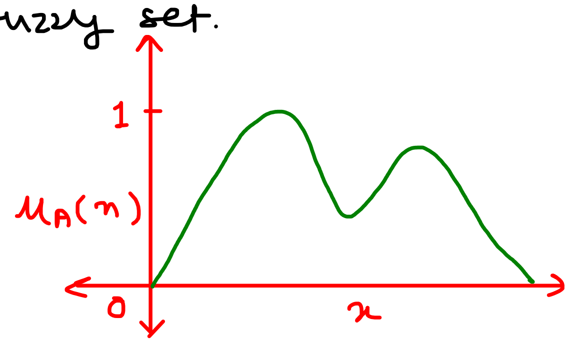
Defⁿ: A fuzzy set $\tilde{A}$ on X is said to be convex if and only if $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$ for all $x_1, x_2 \in X$ and $\lambda \in [0,1]$

Note: It is described by membership function whose membership values are strictly monotonically increasing or monotonically decreasing or initially monotonically increasing and then monotonically decreasing.

* Non convex fuzzy set: A fuzzy set which is not convex is called non convex fuzzy set.



convex fuzzy set



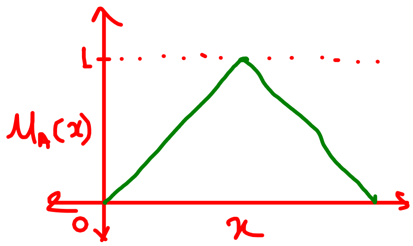
Non convex fuzzy set.

b) NORMAL FUZZY SETS

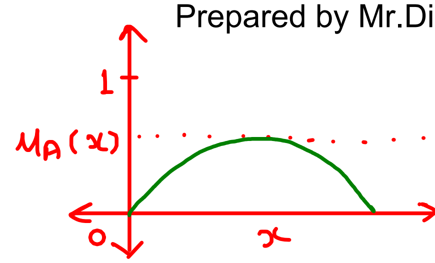
Defⁿ: A fuzzy set $\tilde{A}$ is normal fuzzy set if there exist at one element whose membership value is one

i.e. there exist one $x \in X$ for which $\mu_A(x) = 1$

* subnormal fuzzy set : A fuzzy set is not normal is called subnormal fuzzy set



"Normal fuzzy set"



Not normal fuzzy set.
"subnormal fuzzy set"

ALPHA CUT (α-cut)

Def^n: The α-cut or α-level or cut worthy set of fuzzy set $\tilde{A}$ of set X is the following crisp set given by

$$\tilde{A}_\alpha = \{ x \in X \mid \mu_A(x) \geq \alpha , \alpha \in [0,1] \}$$

STRONG ALPHA CUT

Def^n: The α-cut or α-level or cut worthy set of fuzzy set $\tilde{A}$ of set X is the following crisp set given by

$$\tilde{A}_\alpha = \{ x \in X \mid \mu_A(x) > \alpha , \alpha \in [0,1] \}$$

HEIGHT OF FUZZY SET

Def^n: The height of fuzzy set $\tilde{A}$ is the largest membership value of element.

i.e. $h(\tilde{A}) = \max \{ \mu_A(x) \mid x \in X \}$

* Examples on Alpha cut and Strong Alpha cut

Q.1 If $\tilde{A} = \{(1, 0.2), (2, 0.4), (3, 0.6), (4, 0.8), (5, 1)$
$(6, 0.8), (7, 0.6), (8, 0.4), (9, 0.2)\}$

then find (i) α cut of 0.4 $(A_{0.4})$

(ii) Strong α cut of 0.6 $(A^{+}_{0.6})$

Soln : (i)

$$A_{0.4} = \{x \in A \mid \mu_A(x) \geq 0.4\}$$
$$= \{2, 3, 4, 5, 6, 7, 8\} \quad \text{—— } 1$$

(ii)

$$A^{+}_{0.6} = \{x \in A \mid \mu_A(x) > 0.6\}$$
$$= \{4, 5, 6\}$$

* Example on core, support, height & boundary of Fuzzy set

Q.1 If $\tilde{A} = \{(a, 0), (b, 0.4), (c, 0.8), (d, 1), (e, 1),$
$(f, 0.8), (g, 0.4), (h, 0)\}$

then find core $(\tilde{A})$, support $(\tilde{A})$, height $(\tilde{A})$,
boundary $(\tilde{A})$ and crossover $(\tilde{A})$

Soln : core $(\tilde{A}) = \{x \in A \mid \mu_A(x) = 1\}$
$= \{d, e\}$

$$\text{support}(\tilde{A}) = \{x \in A \mid \mu_A(x) > 0\}$$

$$= \{b, c, d, e, f, g\}$$

$$\text{height}(\tilde{A}) = \max\{\mu_A(x) \mid x \in A\}$$

$$= 1 \quad (\because \text{largest membership value})$$

$$\text{boundary}(\tilde{A}) = \{x \in A \mid 0 < \mu_A(x) < 1\}$$

$$= \{b, c, f, g\}$$

$$\text{crossover}(\tilde{A}) = \{x \in A \mid \mu_A(x) = 0.5\}$$

$$= \{\phi\}$$

Prepared by Mr.Dinesh Kute

There are 3 types of cardinality of fuzzy set

a) scalar cardinality

b) Relative cardinality

c) fuzzy cardinality

a) SCALAR CARDINALITY

For a fuzzy set $\tilde{A}$ defined on finite set X, it's scalar cardinality is denoted by $|\tilde{A}|$ and defined as

$$|\tilde{A}| = \sum_{x \in X} \mu_A(x)$$

b) RELATIVE CARDINALITY

For a fuzzy set $\tilde{A}$ defined on finite set X, it's Relative cardinality is denoted by $\|\tilde{A}\|$ and defined as

$$\|\tilde{A}\| = \frac{|\tilde{A}|}{|X|} = \frac{\sum_{x \in X} \mu_A(x)}{|X|}$$

c) FUZZY CARDINALITY

For a fuzzy set $\tilde{A}$ defined on finite set X, it's Fuzzy cardinality is denoted by $|\tilde{A}|_F$ and defined as

$$|\tilde{A}|_F = \{ (|\tilde{A}_\alpha|, \alpha) \mid \text{for all } \alpha \}$$

EXAMPLE

Q. Find the scalar,relative and fuzzy cardinality for each of the following fuzzy sets

* Examples on Fuzzy cardinality.

Q.1 If $\tilde{A} = \{(x_1, 0), (x_2, 0.3), (x_3, 0.6), (x_4, 0.9), (x_5, 1), (x_6, 0.9), (x_7, 0.6), (x_8, 0.3), (x_9, 0)\}$

then find scalar, relative and fuzzy cardinality of $\tilde{A}$.

Soln (1) scalar cardinality:

$$|\tilde{A}| = \sum_{x \in A} \mu_A(x)$$

$$= 0 + 0.3 + 0.6 + 0.9 + 1 + 0.9 + 0.6 + 0.3 + 0$$

$$= 4.6$$

(2) Relative cardinality

$$||\tilde{A}|| = \frac{\sum_{x \in A} \mu_A(x)}{|X|} = \frac{|\tilde{A}|}{|X|} = \frac{4.6}{9} = 0.51$$

(3) Fuzzy cardinality:

$$|\tilde{A}_\alpha| = \{(|\tilde{A}_\alpha|, \alpha) \mid \text{for all } \alpha\}$$

$\alpha = 0, 0.3, 0.6, 0.9, 1$

$\tilde{A}_0 = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\} \Rightarrow |\tilde{A}_0| = 9$

$\tilde{A}_{0.3} = \{x_2, x_3, x_4, x_5, x_6, x_7, x_8\} \Rightarrow |\tilde{A}_{0.3}| = 7$

$$\tilde{A}_{0.6} = \{x_3, x_4, x_5, x_6, x_7\} \Rightarrow |\tilde{A}_{0.6}| = 5$$

$$\tilde{A}_{0.9} = \{x_4, x_5, x_6\} \Rightarrow |\tilde{A}_{0.9}| = 3$$

$$\tilde{A}_1 = \{x_5\} \Rightarrow |\tilde{A}_1| = 1$$

$$\therefore \quad |\tilde{A}|_F = \{(9, 0), (7, 0.3), (5, 0.6), (3, 0.9), (1, 1)\}$$

X	A_1	A_2	A_3	A_4	A_5
Age	Infant	Kid	Young	Adult	Senior
5	0	1	0	0	0
15	0	0.3	0.2	0	0
25	0	0	0.8	0.7	0
35	0	0	1	0.9	0
45	0	0	0.6	1	0
55	0	0	0.4	1	0.8
65	0	0	0.1	1	0.9

FUZZY RELATION

Def^n: A fuzzy relation is a mapping from cartesian product of fuzzy sets to [0,1]

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets defined on set X and Y respectively then fuzzy relation $\tilde{R}$ is defined as

$$\tilde{R} = \tilde{A} \times \tilde{B}$$

where relation $\tilde{R}$ has membership function,

$\mu_R : \tilde{A} \times \tilde{B} \rightarrow [0,1]$ defined as

$$\mu_R(x, y) = \min \{\mu_A(x), \mu_B(y)\}$$

* Fuzzy relation represensation using matrix

Let $\tilde{A} = \{(a_1, \mu_A(a_1)), (a_2, \mu_A(a_2)), \ldots (a_n, \mu_A(a_n))\}$

$\tilde{B} = \{(b_1, \mu_A(b_1)), (b_2, \mu_A(b_2)), \ldots (b_m, \mu_A(b_m))\}$

then $\tilde{R} = \tilde{A} \times \tilde{B}$

$$= \begin{bmatrix} \mu_R(a_1, b_1) & \mu_R(a_1, b_2) & \cdots & \mu_R(a_1, b_m) \\ \mu_R(a_2, b_1) & \mu_R(a_2, b_2) & \cdots & \mu_R(a_2, b_m) \\ \vdots & & & \\ \mu_R(a_n, b_1) & \mu_R(a_n, b_2) & \cdots & \mu_R(a_n, b_m) \end{bmatrix}$$

* Example :

Q.1 Find fuzzy relation matrix of $\tilde{A} \times \tilde{B}$,

Given $\tilde{A} = \{(a_1, 0.2), (a_2, 0.7), (a_3, 0.4)\}$

$\tilde{B} = \{(b_1, 0.5), (b_2, 0.6)\}$

Soln : $\tilde{R} = \tilde{A} \times \tilde{B}$

and $\mu_R(x, y) = \min\{\mu_A(x), \mu_R(y)\}$

$$\therefore \tilde{R} = \tilde{A} \times \tilde{B} = \begin{bmatrix} \mu_R(a_1, b_1) & \mu_R(a_1, b_2) \\ \mu_R(a_2, b_1) & \mu_R(a_2, b_2) \\ \mu_R(a_3, b_1) & \mu_R(a_3, b_2) \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 0.2 \\ 0.5 & 0.6 \\ 0.4 & 0.4 \end{bmatrix}$$

OPERATIONS ON FUZZY RELATION

Let $\tilde{R}$ and $\tilde{S}$ be two fuzzy relation matrices then

① Union

$$\tilde{R} \cup \tilde{S} = \begin{bmatrix} \mu_{R\cup S}(x_1, y_1) & \mu_{R\cup S}(x_1, y_2) & \cdots & \mu_{R\cup S}(x_1, y_n) \\ \mu_{R\cup S}(x_2, y_1) & \mu_{R\cup S}(x_2, y_2) & \cdots & \mu_{R\cup S}(x_2, y_n) \\ \vdots & & & \vdots \\ \mu_{R\cup S}(x_m, y_1) & \mu_{R\cup S}(x_m, y_2) & \cdots & \mu_{R\cup S}(x_m, y_n) \end{bmatrix}$$

where $\mu_{R\cup S}(x_i, y_i) = \max\{\mu_R(x_i, y_i), \mu_S(x_i, y_i)\}$

② Intersection

$$\mu_{R \cap S}(x_i, y_i) = \min\{\mu_R(x_i, y_i), \mu_S(x_i, y_i)\}$$

③ Complement

$$\mu_{R^c}(x_i, y_i) = 1 - \mu_R(x_i, y_i)$$

④ Projection

$$\text{Projection of } \tilde{R} \text{ on } x_K = \max\{\mu_R(x_K, y_i) \mid \forall i\}$$

OR

$$\text{Projection of } \tilde{R} \text{ on } y_K = \max\{\mu_R(x_i, y_K) \mid \forall i\}$$

* Examples : Let $\tilde{R}$ and $\tilde{S}$ be two relation matrices defined by

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.9 & 1.0 & 0.7 & 0.8 \end{bmatrix} \end{array} \qquad \tilde{S} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.4 & 0.0 & 0.9 & 0.6 \\ 0.9 & 0.4 & 0.5 & 0.7 \\ 0.3 & 0.0 & 0.8 & 0.5 \end{bmatrix} \end{array}$$

then find

ⓐ $\tilde{R} \cup \tilde{S}$

ⓑ $\tilde{R} \cap \tilde{S}$

ⓒ Projection of $\tilde{R}$ on x_1, x_2, x_3

ⓓ Projection of $\tilde{S}$ on y_1, y_2, y_3.

Soln ①

$$\tilde{R} \cup \tilde{S} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.9 & 0.7 \\ 0.9 & 0.8 & 0.5 & 0.7 \\ 0.9 & 1 & 0.8 & 0.8 \end{bmatrix} \end{array}$$

② $$\tilde{R} \cap \tilde{S} = \begin{array}{c|cccc} & y_1 & y_2 & y_3 & y_4 \\ \hline x_1 & 0.4 & 0 & 0.1 & 0.6 \\ x_2 & 0 & 0.4 & 0 & 0 \\ x_3 & 0.3 & 0 & 0.7 & 0.5 \end{array}$$

③ Projection of $\tilde{R}$ on $x_1 = \max\{0.8, 0.1, 0.1, 0.7\}$

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0.7 \\ 0.0 & 0.8 & 0.0 & 0.0 \\ 0.9 & 1.0 & 0.7 & 0.8 \end{bmatrix} \end{array} \qquad \tilde{S} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.4 & 0.0 & 0.9 & 0.6 \\ 0.9 & 0.4 & 0.5 & 0.7 \\ 0.3 & 0.0 & 0.8 & 0.5 \end{bmatrix} \end{array}$$

$= 0.8$

$x_2 = \max\{0, 0.8, 0, 0\} = 0.8$

$x_3 = \max\{0.9, 1, 0.7, 0.8\} = 1$

④ Projection of $\tilde{S}$ on $y_1 = \max\{0.4, 0.9, 0.3\} = 0.9$

$y_2 = \max\{0, 0.4, 0\} = 0.4$

$y_3 = \max\{0.9, 0.5, 0.8\} = 0.9$

$y_4 = \max\{0.6, 0.7, 0.5\} = 0.7$

Def^n: If $\tilde{R}$ is fuzzy relation on $X \times Y$ and $\tilde{S}$ is fuzzy relation on $Y \times Z$ then fuzzy composition $\tilde{T}$ is a fuzzy relation on $X \times Z$ and defined as

(A) Fuzzy max-min composition

$$\tilde{T} = \{ ((x,z), \mu_T(x,z)) \mid (x,z) \in X \times Z \}$$

where $\mu_T(x,z) = \max_{y \in Y} \{ \min (\mu_R(x,y), \mu_S(y,z)) \}$

OR $\tilde{T} = \begin{array}{c|c} & z_1 \quad z_2 \quad \cdots \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \end{matrix} & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array}$

* Example:

Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, and $Z = \{z_1, z_2, z_3\}$

consider the following fuzzy relations

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{matrix} y_1 & y_2 \end{matrix} \\ \begin{bmatrix} 0.7 & 0.6 \\ 0.8 & 0.3 \end{bmatrix} \end{array}, \quad \tilde{S} = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{matrix} z_1 & z_2 & z_3 \end{matrix} \\ \begin{bmatrix} 0.8 & 0.5 & 0.4 \\ 0.1 & 0.6 & 0.7 \end{bmatrix} \end{array}$$

Find fuzzy composition $\tilde{T} = \tilde{R} \circ \tilde{S}$ using fuzzy max-min composition.

Soln: $\tilde{R} : X \to Y$ and $\tilde{S} : Y \to Z$

$$\therefore \tilde{T} = \tilde{R} \circ \tilde{S} : X \to Z$$

$$\therefore \tilde{T} = \tilde{R} \circ \tilde{S} = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline x_1 & 0.7 & 0.6 & 0.6 \\ x_2 & 0.8 & 0.5 & 0.4 \end{array}$$

$$\therefore \mu_T(x_1, z_1) = \max\{\min\{\underline{0.7}, 0.8\}, \min\{0.6, \underline{0.1}\}\} = \underline{\underline{0.7}}$$

$$\mu_T(x_1, z_2) = \max\{\min\{0.7, \underline{0.5}\}, \min\{0.6, \underline{0.6}\} = \underline{\underline{0.6}}$$

$$\mu_T(x_1, z_3) = \max\{\min\{0.7, \underline{0.4}\}, \min\{\underline{0.6}, 0.7\} = \underline{\underline{0.6}}$$

$$\mu_T(x_2, z_1) = \max\{\min\{0.8, \underline{0.8}\}, \min\{0.3, \underline{0.1}\} = \underline{\underline{0.8}}$$

$$\mu_T(x_2, z_2) = \max\{\min\{0.8, \underline{0.5}\}, \min\{\underline{0.3}, 0.6\} = \underline{\underline{0.5}}$$

$$\mu_T(x_2, z_3) = \max\{\min\{0.8, \underline{0.4}\}, \min\{\underline{0.3}, 0.7\} = \underline{\underline{0.4}}$$

(B) Fuzzy Max product composition :

$$\tilde{T} = \{ ((x,z), \mu_T(x,z)) \mid (x,z) \in X \times Z \}$$

Where $\mu_T(x,z) = \max\limits_{y \in Y} \{ \mu_R(x,y) \cdot \mu_S(y,z) \}$

OR

$$\tilde{T} = \begin{array}{c|ccc} & z_1 & z_2 & \cdots \\ \hline x_1 & & & \\ x_2 & & & \\ \vdots & & & \end{array}$$

* **Example :**

Let $X = \{x_1, x_2\}$, $Y = \{y_1, y_2\}$, and $Z = \{z_1, z_2, z_3\}$

consider the following fuzzy relations

$$\tilde{R} = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.7 & 0.6 \\ x_2 & 0.8 & 0.3 \end{array}, \quad \tilde{S} = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline y_1 & 0.8 & 0.5 & 0.4 \\ y_2 & 0.1 & 0.6 & 0.7 \end{array}$$

Find Fuzzy composition $\tilde{T} = \tilde{R} \circ \tilde{S}$ using fuzzy max-Product composition.

Soln: $\tilde{R} : X \to Y$ & $\tilde{S} : Y \to Z$

$\therefore \tilde{R} \circ \tilde{S} : X \to Z$

$$\therefore \tilde{T} = \tilde{R} \circ \tilde{S} = \begin{array}{c c} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{bmatrix} 0.56 & 0.36 & 0.42 \\ 0.64 & 0.4 & 0.32 \end{bmatrix} \end{array}$$

$$\therefore \mu_T(x_1, z_1) = \max\{0.7 \times 0.8,\ 0.6 \times 0.1\}$$
$$= \max\{0.56,\ 0.06\}$$
$$= \underline{\underline{0.56}}$$

$$\mu_T(x_1, z_2) = \max\{0.7 \times 0.5,\ 0.6 \times 0.6\}$$
$$= \max\{0.35,\ 0.36\} = \underline{\underline{0.36}}$$

$$\mu_T(x_1, z_3) = \max\{0.7 \times 0.4,\ 0.6 \times 0.7\}$$
$$= \max\{0.28,\ 0.42\}$$
$$= \underline{\underline{0.42}}$$

$$\mu_T(x_2, z_1) = \max\{0.8 \times 0.8,\ 0.3 \times 0.1\}$$
$$= \max\{0.64,\ 0.03\}$$
$$= \underline{\underline{0.64}}$$

$$\mu_T(x_2, z_2) = \max\{0.8 \times 0.5,\ 0.3 \times 0.6\}$$
$$= \max\{0.4,\ 0.18\} = \underline{\underline{0.4}}$$

$$\mu_T(x_2, z_3) = \max\{0.8 \times 0.4, 0.3 \times 0.7\}$$

$$= \max\{0.32, 0.21\} = \underline{\underline{0.32}}$$

*** Example :**

If $\tilde{R}$ and $\tilde{S}$ are fuzzy relations define on the fuzzy sets, $\tilde{A} = \{(x_1, 0.1), (x_2, 0.6), (x_3, 0.7), (x_4, 0)\}$, $\tilde{B} = \{(y_1, 0.2), (y_2, 0.4), (y_3, 0.1)\}$ and $\tilde{C} = \{(z_1, 0.5), (z_2, 0.1)\}$ such that $\tilde{R} = \tilde{A} \times \tilde{B}$ and $\tilde{S} = \tilde{B} \times \tilde{C}$. Find fuzzy composition $\tilde{T} = \tilde{R} \circ \tilde{S}$ using Max- Min fuzzy composition method	07

Soln : $\tilde{R} = \tilde{A} \times \tilde{B} : X \to Y$

$$\therefore \tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.4 & 0.1 \\ 0.2 & 0.4 & 0.1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$\tilde{S} : \tilde{B} \times \tilde{C} : Y \to Z$

$$\therefore \tilde{S} = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.1 \\ 0.4 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \end{array}$$

$\therefore$ By Max – min

$\therefore \tilde{T} = \tilde{R} \circ \tilde{S} : X \to Z$

$$\therefore \tilde{T} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cc} z_1 & z_2 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 \\ 0.4 & 0.1 \\ 0.4 & 0.1 \\ 0 & 0 \end{bmatrix} \end{array}$$

$$\mu_T(x_1, z_1) = \max\{\min\{\underline{\underline{0.1}}, 0.2\}, \min\{\underline{\underline{0.1}}, 0.4\}, \min\{\underline{0.1}, 0.1\}\} = \underline{\underline{0.1}}$$

$$\mu_T(x_1, z_2) = \max\{\min\{\underline{\underline{0.1}}, 0.1\}, \min\{\underline{\underline{0.1}}, 0.1\}, \min\{\underline{0.1}, 0.1\}\} = \underline{\underline{0.1}}$$

$$\mu_T(x_2, z_1) = \max\{\min\{0.2, 0.2\}, \min\{0.4, 0.4\}, \min\{0.1, 0.1\}\} = \underline{\underline{0.4}}$$

$$\mu_T(x_2, z_2) = \max\{\min\{0.2, \underline{\underline{0.1}}\}, \min\{0.4, \underline{\underline{0.1}}\}, \min\{0.1, \underline{\underline{0.1}}\}\} = \underline{\underline{0.1}}$$

$$\mu_T(x_3, z_1) = \max\{\min\{0.2, 0.2\}, \min\{0.4, 0.4\}, \min\{0.1, 0.1\}\} = \underline{\underline{0.4}}$$

$$\mu_T(x_3, z_2) = \max\{\min\{0.2, \underline{\underline{0.1}}\}, \min\{0.4, \underline{\underline{0.1}}\}, \min\{0.1, \underline{\underline{0.1}}\}\} = \underline{\underline{0.1}}$$

$$\mu_T(x_4, z_1) = \max\{\min\{0, 0.2\}, \min\{0, 0.4\}, \min\{0, 0.1\}\} = 0$$

$$\mu_T(x_4, z_2) = \max\{\min\{0, 0.1\}, \min\{0, 0.1\}, \min\{0, 0.1\}\} = \underline{\underline{0}}$$

DEFUZZIFICATION

① Fuzzification converts the crisp input (binary) into fuzzy value.

② In general, fuzzy result generated can not be used in an application.

③ Any controller can only understand the crisp output. So it is necessary to convert fuzzy output into crisp value.

④ 'Defuzzification' converts fuzzy output into crisp value.

⑤ There is no systematic procedure for choosing a good defuzzification strategy. Selection of defuzzification procedure depends on the properties of the application.

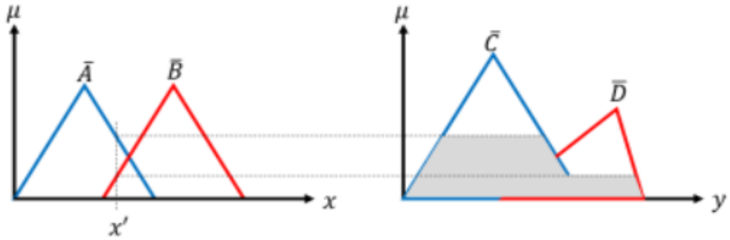
RULE BASE

Consider the following two rules in the fuzzy rule base.

R_1: If x is A then y is C

R_2: If x is B then y is D

A pictorial representation of the above rule base is shown in the following figures



What is the **crisp output** for an input say x' ?

DEFUZZIFICATION METHODS

① Lamda cut method (α - cut method)

$$\tilde{A}_\alpha = \{ x \in X \mid \mu_A(x) \geq \alpha , \alpha \in (0,1] \}$$

*** Example :**

① $\tilde{A} = \{ (x_1, 1), (x_2, 0.5), (x_3, 0.3), (x_4, 0.4) \}$

Find $\tilde{A}_{0.5}$, $\tilde{A}_{0.4}$

② $\tilde{R} = \begin{bmatrix} 0.9 & 0 & 0.2 \\ 1 & 0.5 & 0 \\ 0.3 & 1 & 0 \end{bmatrix}$

Find $\tilde{R}_{0.8}$, $\tilde{R}_{0.3}$

Solution : $\tilde{R}_{0.8} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $\tilde{R}_{0.3} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

*** Properties of α-cut**

Let $\tilde{A}$ and $\tilde{B}$ be any two fuzzy sets defined on same universe of discourse (X) then

ⓐ $(\tilde{A} \cup \tilde{B})_\alpha = \tilde{A}_\alpha \cup \tilde{B}_\alpha$

ⓑ $(\tilde{A} \cap \tilde{B})_\alpha = \tilde{A}_\alpha \cap \tilde{B}_\alpha$

(c) $(\tilde{A}^c)_\alpha \neq (\tilde{A}_\alpha)^c$

(d) for any $\alpha \geq \beta$ implies $\tilde{A}_\alpha \subseteq \tilde{A}_\beta$

(2) Maxima Methods :

Maxima methods are quite simple but not as trivial as lambda cut methods. Maxima methods relies on the position of maximum membership of element at particular position in fuzzy set.

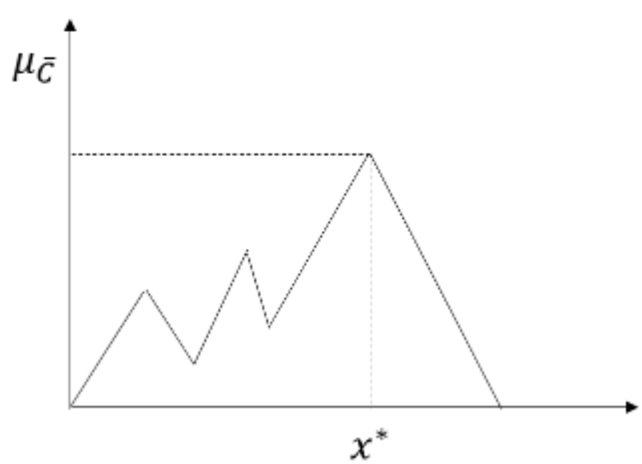
The set of methods under maxima methods we will be discussing here are:

- Height method
- First of maxima (FoM)
- Last of maxima (LoM)
- Mean of maxima (MoM)

Height method:

This method is based on **Max-membership principle**, and defined as follows.

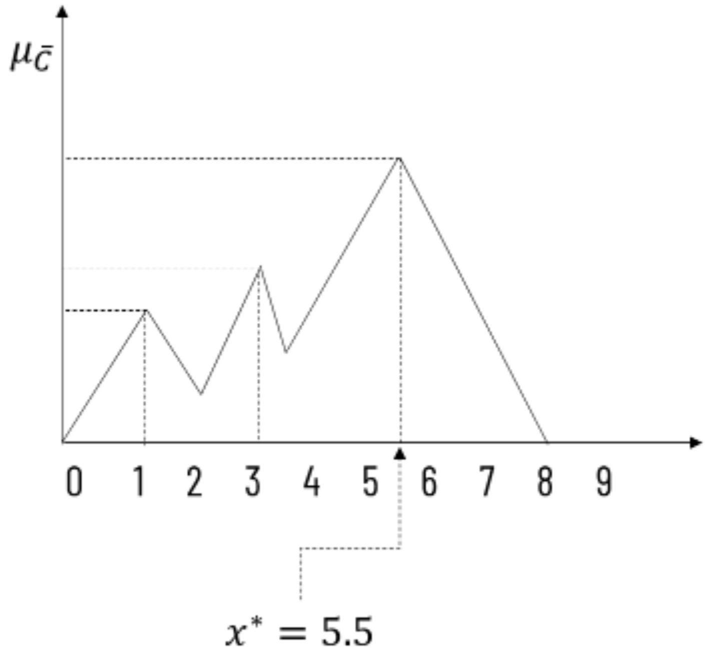
$$\mu_{\underline{C}}(x^*) \geq \mu_{\underline{C}}(x), \forall x \in X$$



Height method

Note: This method is applicable when **height is unique**.

Example:

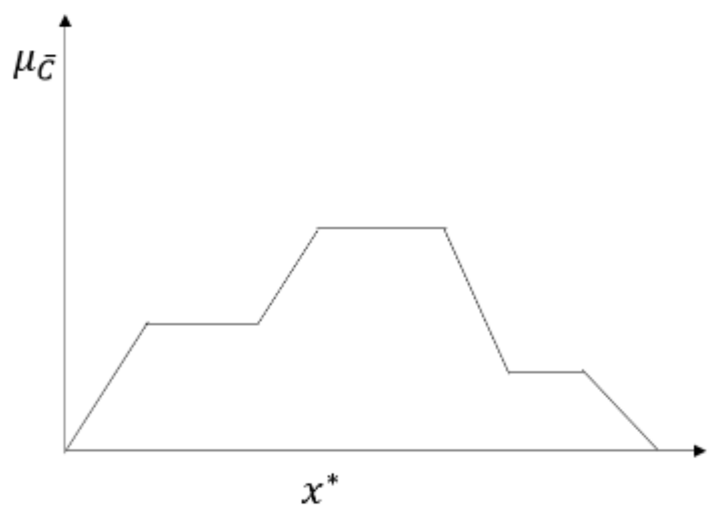


Example of height method

First of Maxima (FoM) method:

Determine the smallest value of the domain with maximized membership degree

FoM = First of Maxima: $x^* = \min\{ x \mid \boldsymbol{\mu}_{\underline{C}}(x) = h(\underline{C}) \}$

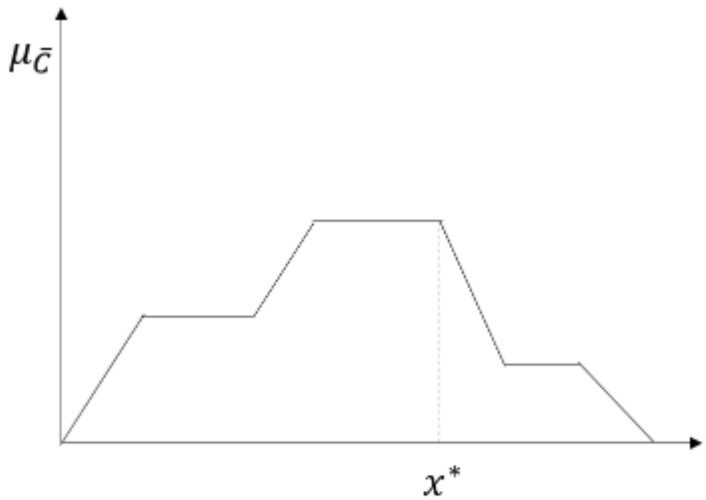


First of maxima

Last of Maxima (LoM) method:

Determine the largest value of the domain with maximized membership degree

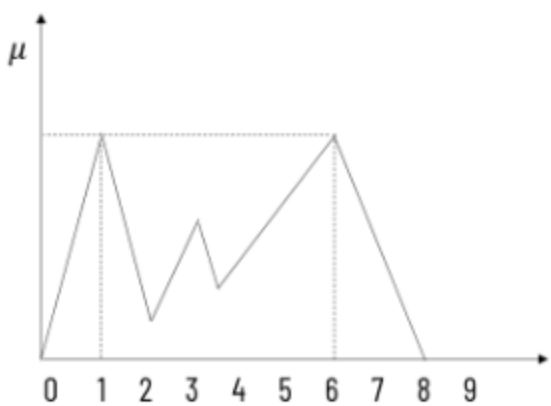
LoM = Last of Maxima: $x^* = \max\{ x \mid \mu_{\underline{C}}(x) = h(\underline{C}) \}$



Last of maxima

Example: First of Maxima and Last of Maxima

Find the defuzzification value for given fuzzy set



First of Maxima: $x^* = 1$

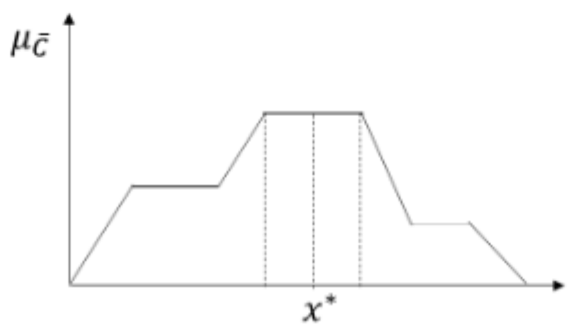
Last of Maxima: $x^* = 6$

Middle of Maxima (MoM) method:

In order to find middle of maxima, we have to find the "middle" of elements with maximum membership value

$$x^* = \frac{\sum_{x_i \in M}(x_i)}{|M|}$$

Where, M ={ x_i | $\mu_{\underline{C}}(x_i)$ = h($\underline{C}$) }, Or M is the set of points having highest membership value

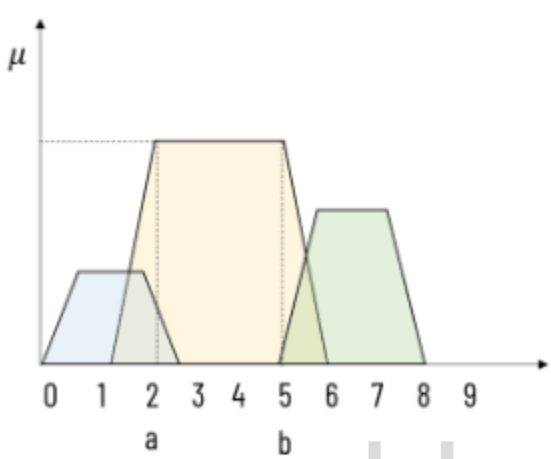


Middle of maxima

Note: This method is applicable to **symmetric functions** only

Example: Middle of maxima

Find the deffizified value for given fuzzy set using middle of maxima method:



x* = (a + b) / 2

x* = (2 + 5) / 2

x* = 3.5

(B) Weighted Average Method

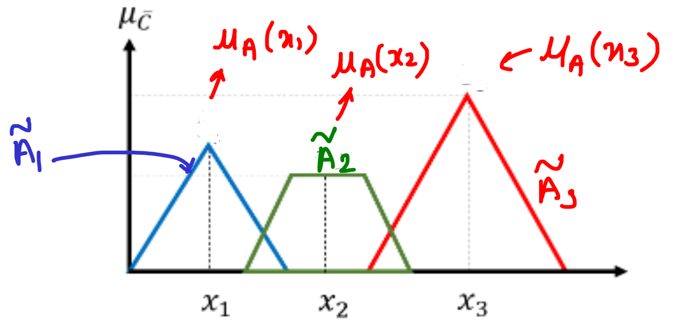
① It is one of the simplest and widely used defuzzification technique

② This method is also called "sugeno defuzzification" method

③ Formed by weighting each functions in the output by it's respective maximum membership value

④ The crisp value according to this method is

$$x^* = \frac{\sum_i \mu_{A_i}(x_i) \cdot x_i}{\sum_i \mu_{A_i}(x_i)}$$



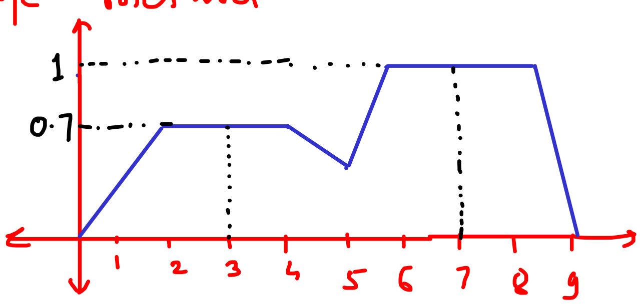
Aggregated fuzzy output

⑤ Less computationally intensive

$$x^* = \frac{x_1 \mu_{A_1}(x_1) + x_2 \mu_{A_2}(x_2) + x_3 \mu_{A_3}(x_3)}{\mu_{A_1}(x_1) + \mu_{A_2}(x_2) + \mu_{A_3}(x_3)}$$

* Example:

Find defuzzified value of given fuzzy output using weighted average method.



Solution:

$$x^* = \frac{(3 \times 0.7) + (7 \times 1)}{0.7 + 1}$$

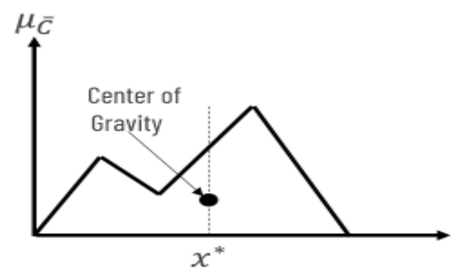
$$x^* = 5.941$$

(c) Center of Gravity (CoG)

① It is most prevalent and physically appealing method from all the defuzzification methods.

② The basic principle in CoG method is find point x^* where a vertical line would slice the aggregate into two equal masses

③ It is defined as



Defuzzification using CoG method

$$x^* = \frac{\sum_i \mu_A(x_i) \cdot x_i}{\sum_i \mu_A(x_i)}$$

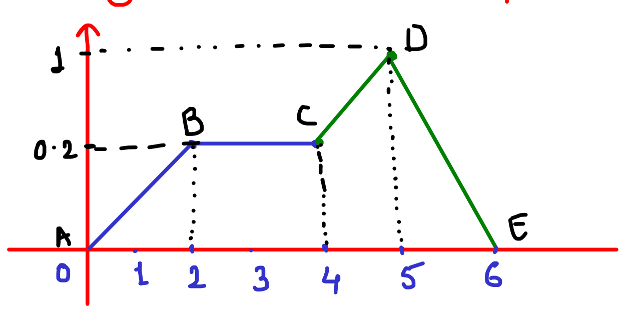
(for discrete values)

$$x^* = \frac{\int \mu_A(x)\, x\, dx}{\int \mu_A(x)\, dx}$$

(for continuous values)

* **Example:**

Find crisp value corresponding to following fuzzy output using center of gravity method



Solution: (a) for line AB

$A(0,0)$, $B(2, 0.2)$

Eqⁿ of line is

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\therefore \frac{y-0}{x-0} = \frac{0.2-0}{2-0} \Rightarrow \boxed{y = 0.1x}$$

(b) Eqⁿ of line BC

$$y = 0.2$$

(c) Eqⁿ of CD

$C(4, 0.2)$, $D(5,1)$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{y-0.2}{x-4} = \frac{1-0.2}{5-4}$$

$$\Rightarrow \frac{y-0.2}{x-4} = \frac{0.8}{1} \Rightarrow y-0.2 = 0.8x-3.2$$

$$\boxed{\therefore y = 0.8x-3}$$

(d) Eqⁿ of line DE

$D(5,1)$ and $E(6,0)$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{y-1}{x-5} = \frac{0-1}{6-5}$$

$$\Rightarrow \frac{y-1}{x-5} = \frac{-1}{1} \Rightarrow y-1 = -x+5$$

$$\boxed{y = -x+6}$$

$$\therefore x^* = \frac{\int \mu_A(x)\, x\, dx}{\int \mu_A(x)\, dx}$$

$$= \frac{\int_A^B \mu_A(x)\, x\,dx + \int_B^C \mu_A(x)\, x\,dx + \int_C^D \mu_A(x)\, x\,dx + \int_D^E \mu_A(x)\, x\,dx}{\int_A^B \mu_A(x)\,dx + \int_B^C \mu_A(x)\,dx + \int_C^D \mu_A(x)\,dx + \int_D^E \mu_A(x)\,dx}$$

$$= \frac{\int_0^2 (0.1x)\, x\,dx \quad \int_2^4 (0.2)\, x\,dx + \int_4^5 (0.8x-3)\, x\,dx + \int_5^6 (-x+6)\, x\,dx}{\int_0^2 0.1x\,dx + \int_2^4 0.2\,dx + \int_4^5 (0.8x-3)\,dx + \int_5^6 (-x+6)\,dx}$$

$$= \frac{\frac{4}{15} + \frac{6}{5} + \frac{83}{30} + \frac{8}{3}}{\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{1}{2}} = \frac{69}{17} = \underline{\underline{4.059}}$$

(D) Centre of Sum (CoS)

In this method crisp value is calculated using area of individual traingle rather than overlapped region in centre of Gravity.

crisp value using centre of sum (CoS) is

$$x^* = \frac{A_1 x_1 + A_2 x_2 + \cdots A_n x_n}{A_1 + A_2 + \cdots + A_n}$$

where x_i = centre of Largest area of fuzzy set $\tilde{A}_i$

A_i = area of the fuzzy set $\tilde{A}_i$

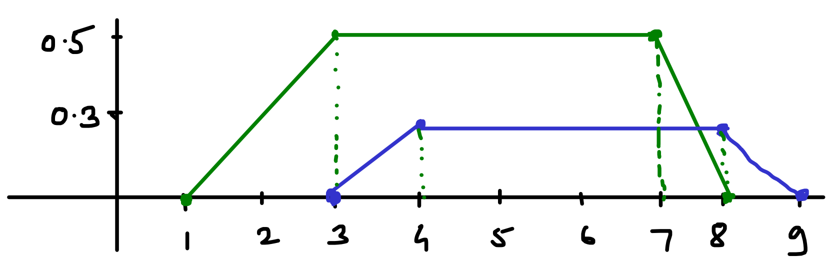
* Example :

Find crisp value corresponding to following fuzzy output using center of sum method

$\tilde{A}_1 = \{(1,0), (3, 0.5), (7, 0.5), (8,0)\}$

$\tilde{A}_2 = \{(3,0), (4, 0.3), (8, 0.3), (9,0)\}$

Soln



$$x^* = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

Where, A_1 = Area of fuzzy set $\tilde{A}_1$

$= \frac{1}{2}(2 \times 0.5) + (4 \times 0.5) + \frac{1}{2}(1 \times 0.5)$

$= 2.75$

$x_1 =$ centre of largest area of fuzzy set $\tilde{A}_1$

$= \frac{1}{2}(7+3) = 5$

$A_2 =$ Area of fuzzy set $\tilde{A}_2$

$= \frac{1}{2}(1 \times 0.3) + (4 \times 0.3) + \frac{1}{2}(1 \times 0.3)$

$= 1.5$

$x_2 =$ centre of largest area of fuzzy set $\tilde{A}_2$

$= \frac{1}{2}(4+8) = 6$

$$\therefore \quad x^* = \frac{(5 \times 2.75) + (6 \times 1.5)}{2.75 + 1.5} = \underline{\underline{5.353}}$$

UNIT V FUZZY LOGIC CONTROL

UNIVERSAL FUZZY SET

Def^n: If X is any set then universal fuzzy set of X is denoted by $\tilde{X}$ and defined as

$$\tilde{X} = \{ (x, 1) \mid x \in X \}$$

Example: Let $X = \{ a_1, a_2, a_3, a_4 \}$ then

$$\tilde{X} = \{ (a_1, 1), (a_2, 1), (a_3, 1), (a_4, 1) \}$$

FUZZY "IF THEN" RULE

It is also called as Fuzzy implication, Fuzzy Rule or Fuzzy conditional statement.

The standard fuzzy if then rule is given below,

(a) If x is $\tilde{A}$ then y is $\tilde{B}$

i.e. $\tilde{R} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{Y})$, $\tilde{Y}$ - Universal fuzzy set

(b) If x is $\tilde{A}$ then y is $\tilde{B}$ else y is $\tilde{C}$

i.e. $\tilde{R} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{C})$

* Example

Let $X = \{ a, b, c, d \}$ and $Y = \{ 1, 2, 3, 4 \}$ and

$$\tilde{A} = \{ (a, 0), (b, 0.8), (c, 0.6), (d, 1) \}$$

$$\tilde{B} = \{ (1, 0.2), (2, 1), (3, 0.8), (4, 0) \}$$

$\tilde{C} = \{(1,0), (2,0.4), (3,1), (4,0.8)\}$

Determine the implication relations

(a) If x is $\tilde{A}$ then Y is $\tilde{B}$

(b) If x is $\tilde{A}$ then Y is $\tilde{B}$ else Y is $\tilde{C}$

Soln : (a) If x is $\tilde{A}$ then Y is $\tilde{B}$

$\therefore \tilde{R} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{Y})$, $\tilde{Y}$ - universal fuzzy set of Y

Where $\tilde{Y} = \{(1,1), (2,1), (3,1), (4,1)\}$

$\tilde{A}^c = \{(a,1), (b,0.2), (c,0.4), (d,0)\}$

$$\tilde{A} \times \tilde{B} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

$$\tilde{A}^c \times \tilde{Y} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\tilde{R} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{Y}) = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.4 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

(b) If x in $\tilde{A}$ then y in $\tilde{B}$ else y in $\tilde{C}$

$\tilde{A} = \{(a, 0), (b, 0.8), (c, 0.6), (d, 1)\}$

$\tilde{B} = \{(1, 0.2), (2, 1), (3, 0.8), (4, 0)\}$

$\tilde{C} = \{(1, 0), (2, 0.4), (3, 1), (4, 0.8)\}$

$\tilde{R} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{C})$

$\tilde{A}^c = \{(a, 1), (b, 0.2), (c, 0.4), (d, 0)\}$

$$\tilde{A} \times \tilde{B} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

$$\tilde{A}^c \times \tilde{C} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0.4 & 1 & 0.8 \\ 0 & 0.2 & 0.2 & 0.8 \\ 0 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Now, $\tilde{R} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{C})$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0.4 & 1 & 0.8 \\ 0.2 & 0.8 & 0.8 & 0.8 \\ 0.2 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

b) Let $X=\{a,b,c,d\}$ & $Y=\{1,2,3,4\}$ be any two sets. If $\widetilde{A}$ & $\widetilde{B}$ are the fuzzy sets defined on the X & Y respectively defined by,

$\widetilde{A}=\{(a,0),(b,0.7),(c,0.6),(d,1)\}$

$\widetilde{B}=\{(1,0.2),(2,0.9),(3,0.5),(4,0)\}$ then find following fuzzy relation

"If X is $\widetilde{A}$ then Y is $\widetilde{B}$"

Solⁿ: If x is $\widetilde{A}$ then Y is $\widetilde{B}$

$\widetilde{R} = (\widetilde{A}\times\widetilde{B}) \cup (\widetilde{A}^c \times Y)$, Y - Universal set of Y

$\widetilde{A} = \{(a,0), (b,0.7), (c,0.6), (d,1)\}$

$\widetilde{B} = \{(1,0.2), (2,0.9), (3,0.5), (4,0)\}$

$\widetilde{Y} = \{(1,1), (2,1), (3,1), (4,1)\}$

$\widetilde{A}^c = \{(a,1), (b,0.3), (c,0.4), (d,0)\}$

$$\widetilde{A}\times\widetilde{B} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.7 & 0.5 & 0 \\ 0.2 & 0.6 & 0.5 & 0 \\ 0.2 & 0.9 & 0.5 & 0 \end{bmatrix} \end{array}$$

$$\widetilde{A}^c\times\widetilde{Y} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.3 & 0.3 & 0.3 & 0.3 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\therefore \widetilde{R} = (\widetilde{A}\times\widetilde{B}) \cup (\widetilde{A}^c\times\widetilde{Y}) = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.3 & 0.7 & 0.5 & 0.3 \\ 0.4 & 0.6 & 0.5 & 0.4 \\ 0.2 & 0.9 & 0.5 & 0 \end{bmatrix} \end{array}$$

Fuzzy Inference System

Mamdani and Sugeneo fuzzy inference system

Fuzzy Inference System – Concept

Fuzzy inference is the process of mapping from a given input to an output using fuzzy logic.

What is the Fuzzy Inference System(FIS)?

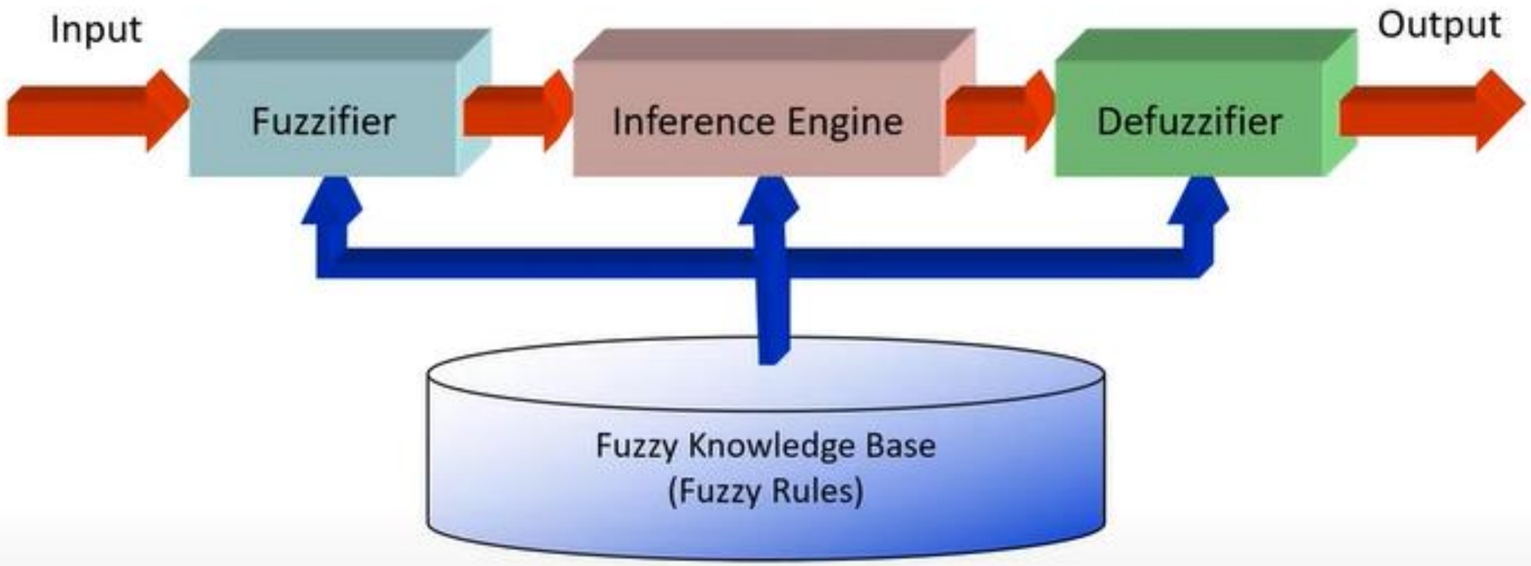
A nonlinear mapping that derives its output based on fuzzy reasoning and set of fuzzy if then rules.

Fuzzy inference systems have been successfully applied in various fields such as automatic control, data classification, decision analysis, expert systems, computer vision, etc.

Fuzzy Logic Toolbox™ software supports two types of fuzzy inference systems:

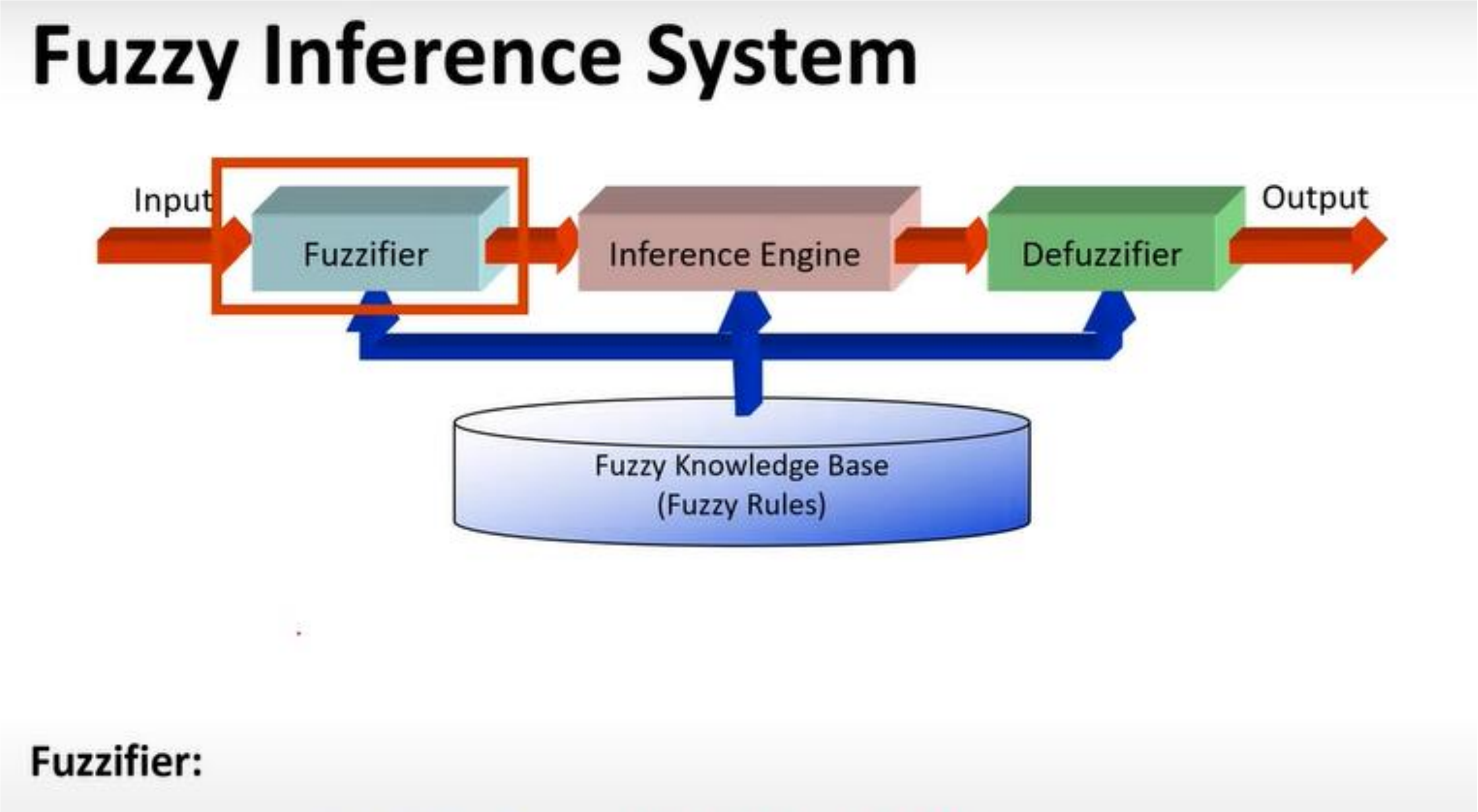
- Mamdani systems
- Sugeno systems

Architecture of Fuzzy Inference System



Architecture of Fuzzy Inference System

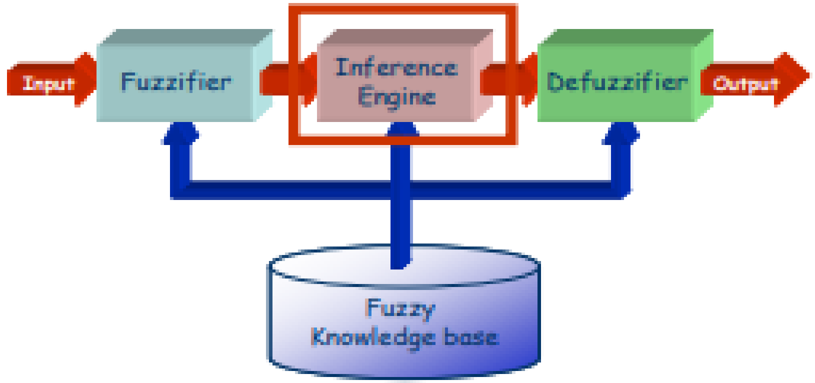
Architecture of Fuzzy Inference System



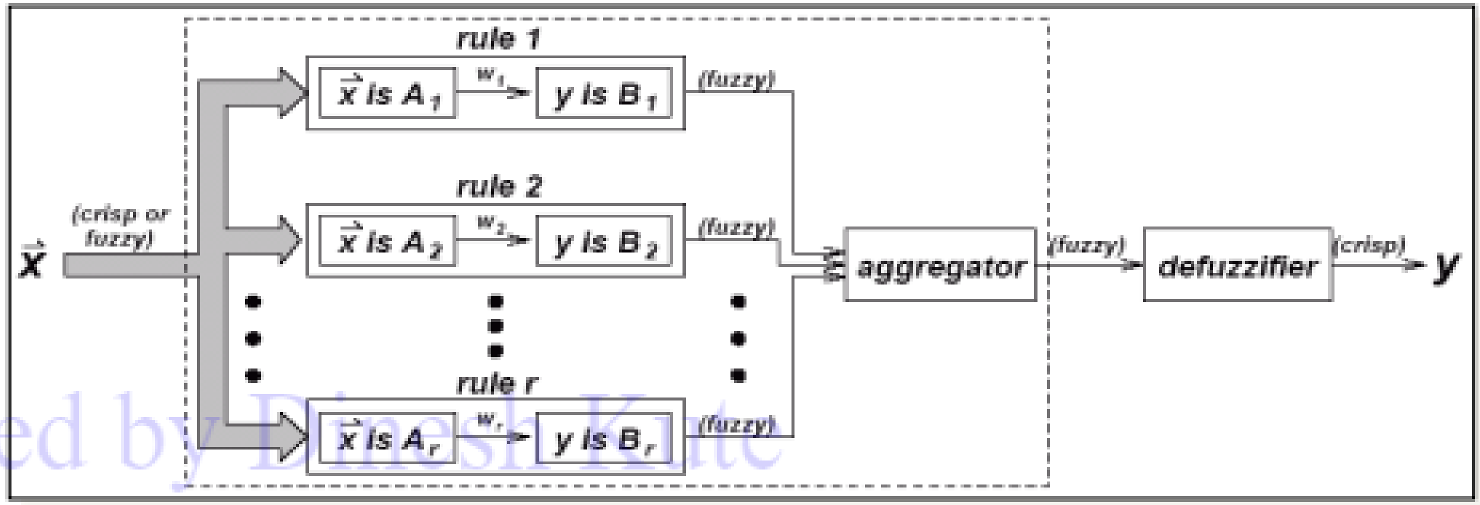
Covert **crisp input** to **fuzzy input (or linguistic variable)**

Architecture of Fuzzy Inference System

Inference Engine

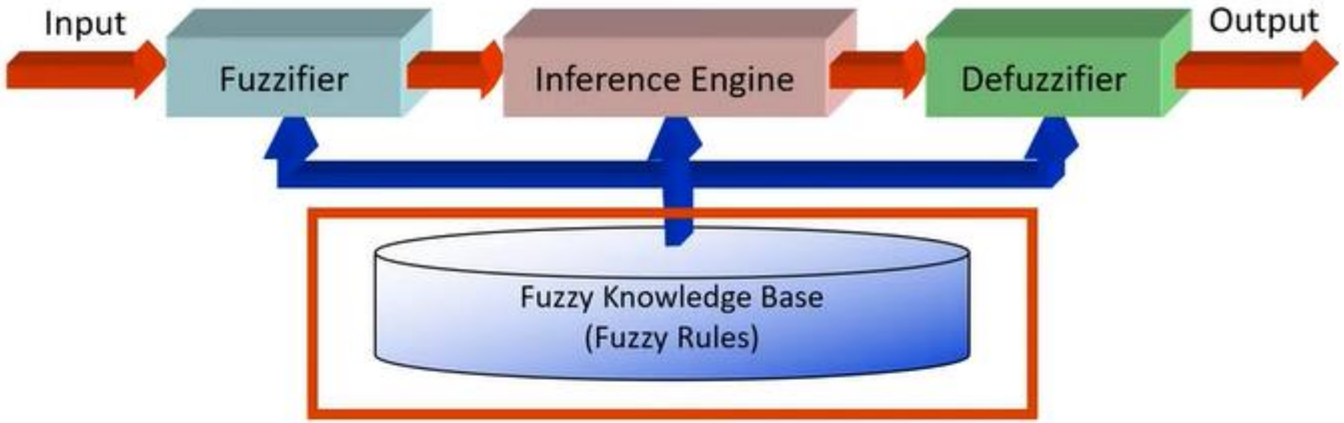


Using If-Then type fuzzy rules converts the fuzzy input to the fuzzy output.



Architecture of Fuzzy Inference System

Fuzzy Inference System



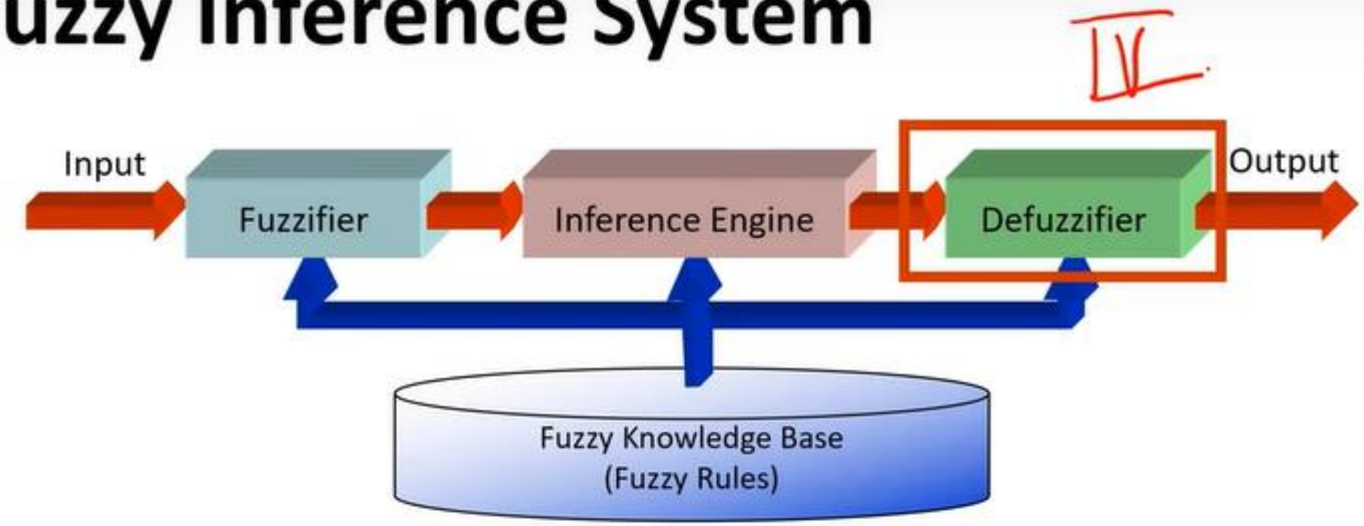
Fuzzy Knowledge Base:

The rule base referred to as the knowledge base.

- A rule base contains a number of fuzzy IF-THEN rules;
- A database which defines the membership functions of the fuzzy sets used the fuzzy rules

Architecture of Fuzzy Inference System

Fuzzy Inference System



Defuzzifier:

- It converts the fuzzy output of the inference engine to crisp.
- Here are some commonly used defuzzification methods are as follows:
  - Weighted average method
  - Center of gravity method
  - Mean of maximum (MOM)
  - Smallest of maximum (SOM)
  - Largest of maximum (LOM)

- center of sum method

Sugeno fuzzy inference system

Sugeno fuzzy inference, also referred to as Takagi-Sugeno-Kang fuzzy inference, uses *singleton* output membership functions that are either constant or a linear function of the input values.

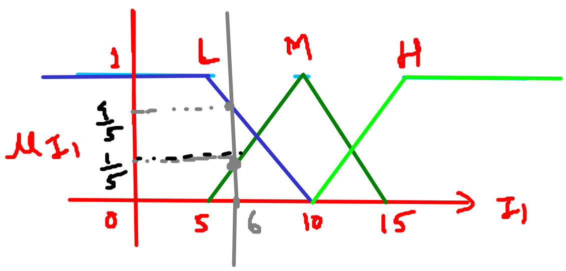
The defuzzification process for a Sugeno system is more computationally efficient compared to that of a Mamdani system, since it uses a weighted average or weighted sum of a few data points rather than compute a centroid of a two-dimensional area.

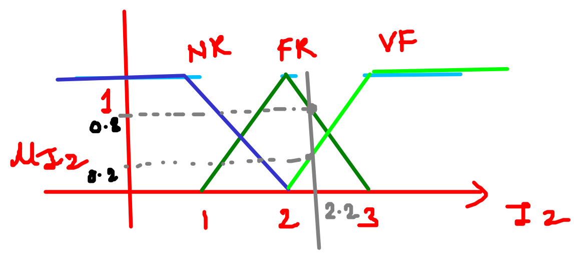
Difference Between Mamdani and Sugeno Fuzzy Inference System:

Mamdani FIS	Sugeno FIS
• Output membership function is present	• No output membership function is present
• The output of surface is discontinuous	• The output of surface is continuous
• Distribution of output	• Non distribution of output, only Mathematical combination of the output and the rules strength
• Through defuzzification of rules consequent of crisp result is obtained	• No defuzzification here. Using weighted average of the rules of consequent crisp result is obtained
• Expressive power and interpretable rule consequent	• Here is loss of interpretability
• Mamdani FIS possess less flexibility in the system design	• Sugeno FIS possess more flexibility in the system design
• It has more accuracy in security evaluation block cipher algorithm	• It has less accuracy in security evaluation block cipher algorithm
• It is using in MISO (Multiple Input and Single Output) and MIMO (Multiple Input and Multiple Output) systems	• It is using only in MISO (Multiple Input and Single Output) systems
• Mamdani inference system is well suited to human input	• Sugeno inference system is well suited to mathematically analysis
• Application: Medical Diagnosis System	• Application: To keep track of the change in aircraft performance with altitude

EXAMPLE ON SUGENO INFERENCE SYSTEM

Example: Find the output of the following fuzzy model for input $I_1 = 6$ and $I_2 = 2.2$ using sugeno inference system





with the following fuzzy rules

Rule 1 : If I_1 is L and I_2 is FR then $y = I_1 + 2I_2$

2 : I_1 L I_2 VF then $y = I_1 + 3I_2$

3 I_1 M I_2 FR then $y = 2I_1 + 2I_2$

4 I_1 M I_2 VF then $y = 2I_1 + 3I_2$

Soln

$$\mu_m(x) = \frac{x-5}{10-5} = \frac{x-5}{5} \quad \therefore \mu_m(6) = \frac{1}{5}$$

$$\mu_L(x) = \frac{10-x}{10-5} = \frac{10-x}{5} \quad \therefore \mu_L(6) = \frac{4}{5}$$

I_1

$$\mu_{VF}(x) = \frac{x-2}{3-2} = \frac{x-2}{1} \quad \therefore \mu_{VF}(2.2) = 0.2$$

$$\mu_{FR}(x) = \frac{3-x}{3-2} = \frac{3-x}{1} \quad \therefore \mu_{FR}(2.2) = 0.8$$

I_2

Rule (i)	w^i	y^i
1	$\mu_L(6) \times \mu_{FR}(2.2)$ $= 0.64$	$y_1 = I_1 + 2I_2$ $= 6 + 2(2.2) = 10.4$
2	$\mu_L(6) \times \mu_{VF}(2.2)$ $= 0.16$	$y_2 = I_1 + 3I_2 = 12.6$
3	$\mu_m(6) \times \mu_{FR}(2.2)$ $= 0.16$	$y_3 = 2I_1 + 2I_2 = 16.4$
4	$\mu_m(6) \times \mu_{VF}(2.2)$ $= 0.04$	$y_4 = 2I_1 + 3I_2$ $= 18.6$

By sugeno inference system

$$\text{crisp output} = y = \frac{\sum w^i y^i}{\sum w^i}$$

$$\therefore y = \frac{(0.64 \times 10.4) + (0.16 \times 12.6) + (0.16 \times 16.4) + (0.04 \times 18.6)}{0.64 + 0.16 + 0.16 + 0.04}$$

$$= \underline{\underline{12.04}}$$

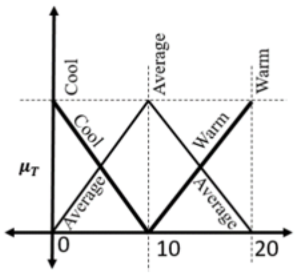
MAMDANI INFERENCE SYSTEM

- In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination.

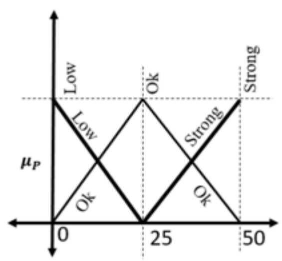
- He applied a set of fuzzy rules supplied by experienced human operators.

EXAMPLES

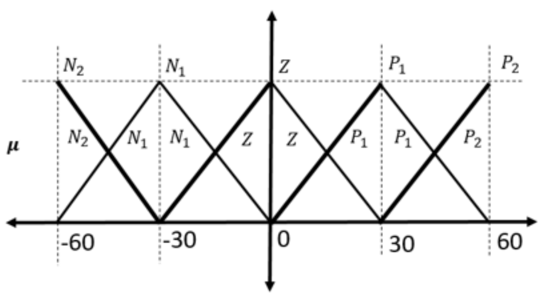
Consider the design of a fuzzy controller for a steam turbine. Assume the input of the fuzzy controller as temperature and pressure with 3 descriptors and output will be throttle setting of a steam turbine with 5 descriptors given below,



Temperature (T)



Pressure (P)



Throttle Setting of Steam Turbine

N_2 : Large Negative
N_1 : Small Negative
Z : Zero
P_1 : Small Positive
P_2 : Large Positive

Find throttle position of the turbine for temperature = 8 & pressure= 40 using Mamdani Inference System and defuzzification method 'middle of Maxima' with following fuzzy Rules,

	Low	Ok	Strong
Cool	P_2	Z	N_2
Average	P_2	Z	N_1
Warm	P_1	N_2	N_1

Soln : step I : Identify input and output variables and decide descriptor for the same.

Descriptors for Input variable

Temperature

1. Cool
2. Average
3. Warm

Pressure

1. Low
2. OK
3. Strong

Descriptors for the output variables

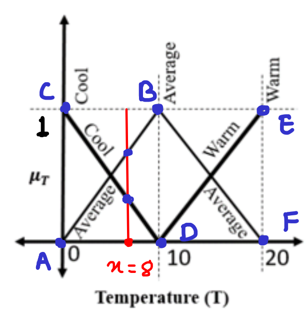
Throttle setting for steam turbine

1. N_2 = large negative
2. N_1 = small negative
3. Z = zero
4. P_1 = small positive
5. P_2 = large positive

step II : Find membership value of given input variables.

case I : When temperature = 8

$$\boxed{\mu_{CD}(x) = \mu_{cool}(x) = \frac{10-x}{10}}$$



Eq^n of line CD :

$C(0,1)$ and $D(10,0)$

$x_1 \ y_1 \qquad x_2 \ y_2$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\therefore \frac{y-1}{x-0} = \frac{0-1}{10-0} \Rightarrow y-1 = \frac{-x}{10} \Rightarrow y = 1-\frac{x}{10} = \frac{10-x}{10}$$

$$\mu_{cool}(8) = \frac{10-8}{10} = \frac{2}{10} = \frac{1}{5} \qquad \boxed{\therefore \mu_{cool}(8) = \frac{1}{5}}$$

$$\boxed{\mu_{AB}(x) = \mu_{Average}(x) = \frac{x}{10}}$$

Eqⁿ of line AB:

$$A(0, 0) \text{ and } B(10, 1)$$

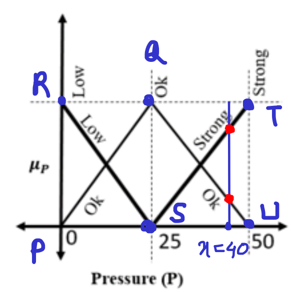
$$x_1 \;\; y_1 \qquad\qquad x_2 \;\; y_2$$

$$\frac{y-0}{x-0} = \frac{1-0}{10-0} \Rightarrow y = \frac{x}{10}$$

$$\therefore \mu_{Average}(8) = \frac{8}{10} = \frac{4}{5} \qquad \boxed{\therefore \mu_{Average}(8) = \frac{4}{5}}$$

Case II : When pressure = 40

$$\boxed{\mu_{QU} = \mu_{OK}(x) = \frac{50-x}{25}}$$



Eqⁿ of line QU :

$$Q(25, 1) \text{ and } U(50, 0)$$

$$x_1 \;\; y_1 \qquad\qquad x_2 \;\; y_2$$

$$\therefore \frac{y-1}{x-25} = \frac{0-1}{50-25} \Rightarrow y-1 = \frac{-(x-25)}{25}$$

$$\Rightarrow y = 1 - \frac{x-25}{25}$$

$$\Rightarrow y = \frac{50-x}{25}$$

$$\therefore \mu_{OK}(40) = \frac{50-40}{25} = \frac{10}{25} = \frac{2}{5} \qquad \boxed{\therefore \mu_{OK}(40) = \frac{2}{5}}$$

$$\boxed{\mu_{ST}(x) = \mu_{strong}(x) = \frac{x-25}{25}}$$

Eqn of line ST :

$S(25, 0)$ and $T(50, 1)$

$x_1 \; y_1 \qquad x_2 \; y_2$

$$\frac{y-0}{x-25} = \frac{1-0}{50-25} \Rightarrow y = \frac{x-25}{25}$$

$$\therefore \mu_{strong}(40) = \frac{40-25}{25} = \frac{15}{25} = \frac{3}{5}$$

$$\boxed{\therefore \mu_{strong}(40) = \frac{3}{5}}$$

Step III : Fuzzy rule evaluation

When temperature = 8 and pressure = 40

$\because \mu_{cool}(8) = \frac{1}{5}$ $\quad \mu_{OK}(40) = \frac{2}{5}$

$\mu_{Average}(8) = \frac{4}{5}$ $\quad \mu_{strong}(40) = \frac{3}{5}$

Pressure

Temp.	Low	Ok	Strong
Cool	P_2	Z	N_2
Average	P_2	Z	N_1
Warm	P_1	N_2	N_1

Rule 1 : Temp is cool and Pressure is OK

$$R_1 = \min\left\{\frac{1}{5}, \frac{2}{5}\right\} = \min\{0.2, 0.4\} = 0.2$$

Rule 2 : Temp is cool and Pressure is strong

$$R_2 = \min\left\{\frac{1}{5}, \frac{3}{5}\right\} = \min\{0.2, 0.6\} = 0.2$$

Rule 3 : Temp is Average and Pressure is OK

$$R_3 = \min\left\{\frac{4}{5}, \frac{2}{5}\right\} = \min\{0.8, 0.4\} = 0.4$$

Rule 4 : Temp is Average and Pressure is Strong

$$R_4 = \min\left\{\frac{4}{5}, \frac{3}{5}\right\} = \min\{0.8, 0.6\} = \underline{0.6}$$

Step IV : Defuzzification method

By Middle of maxima method

Throttle position of the Turbine $= \max\{R_1, R_2, R_3, R_4\}$

$$= \max\{0.2, 0.2, 0.4, 0.6\}$$

$$= \underline{0.6}\ \left(= \frac{3}{5}\right)$$

Hence it is corresponding Rule 4 where temp is Average and Pressure is strong

$$\therefore \mu_{N_1}(x) = 0.6$$

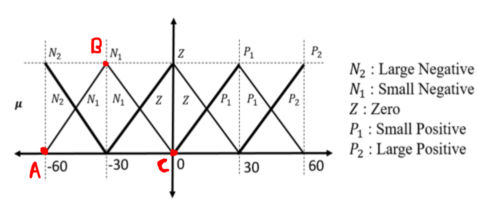
Pressure

Temp	Low	Ok	Strong
Cool	P_2	Z	N_2
Average	P_2	Z	$\boxed{N_1}$
Warm	P_1	N_2	N_1

* Membership function of N_1

ⓐ Eq^n of line AB :

A(−60, 0) and B(−30, 1)
x_1 y_1 x_2 y_2



Throttle Setting of Steam Turbine

$$\therefore \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore \frac{y - 0}{x + 60} = \frac{1 - 0}{-30 + 60}$$

$$\therefore \quad y = \frac{x+60}{30}$$

$$\boxed{\therefore \mu_{AB}(x) = \frac{x+60}{30}}$$

ⓑ Eqn of line BC :

$B(-30, 1)$ and $C(0, 0)$

$x_1 \ y_1 \qquad x_2 \ y_2$

$$\therefore \quad \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\therefore \quad \frac{y-1}{x+30} = \frac{0-1}{0+30}$$

$$\therefore \quad y-1 = -\frac{x+30}{30}$$

$$\therefore \quad y = 1 - \frac{(x+30)}{30}$$

$$\therefore \quad y = -\frac{x}{30}$$

$$\boxed{\therefore \quad \mu_{BC}(x) = -\frac{x}{30}}$$

$\therefore \quad 0.6 = \mu_{AB}(x)$ and $0.6 = \mu_{BC}(x)$

$0.6 = \frac{x+60}{30}$ and $0.6 = \frac{-x}{30}$

$\Rightarrow \boxed{x = -42}$ and $x = -18$

$\therefore$ Throttle setting of Turbine $= \frac{-42-18}{2} = \underline{\underline{-30}}$

* Examples

Design a controller to determine the wash time of a domestic washing machine. Assume the input is dirt and grease on cloths. Use three descriptors for input variables and five descriptors for output variables wash time as below

Dirt = { SD, MD, LD } , Grease = { NG, MG, LG }

SD = Small dirt

MD = Medium dirt

LD = Large dirt

NG = No grease

MG = Medium grease

LG = Large grease

Wash Time = { VS, S, M, L, VL }

VS = very short

S = short

M = Medium

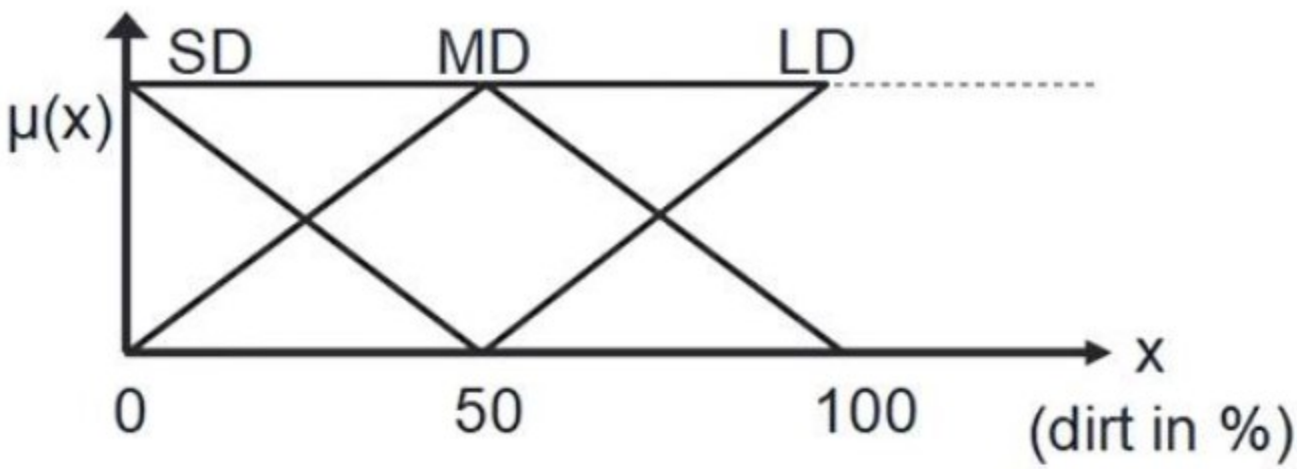
L = Large

VL = Very Large.

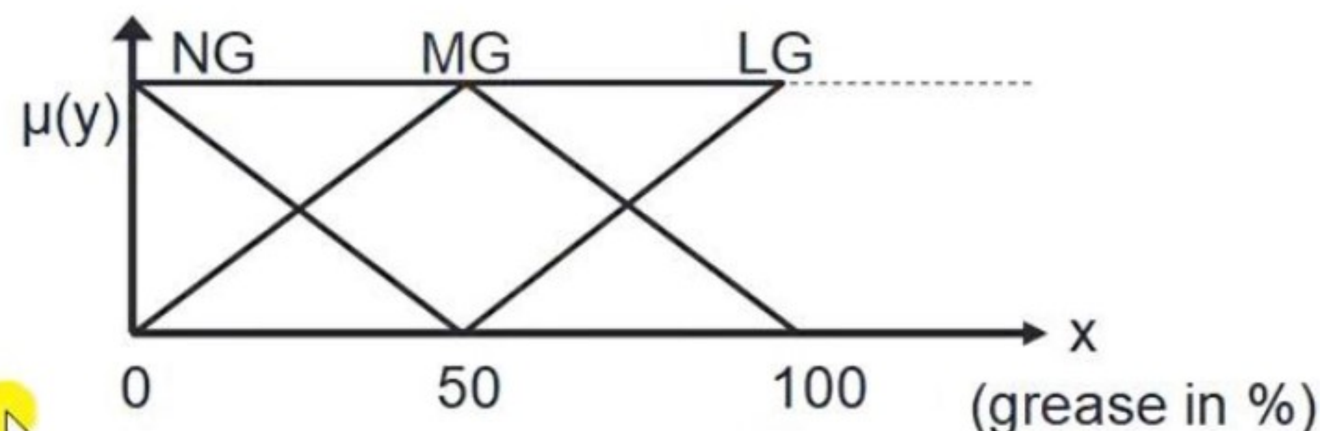
Find the wash time of washing machine for dirt = 60 % and grease = 70 % using defuzzification method - middle of maxima and fuzzy rules given below,

dirt \ grease	NG	MG	LG
SD	VS	M	L
MD	S	M	L
LD	M	L	VL

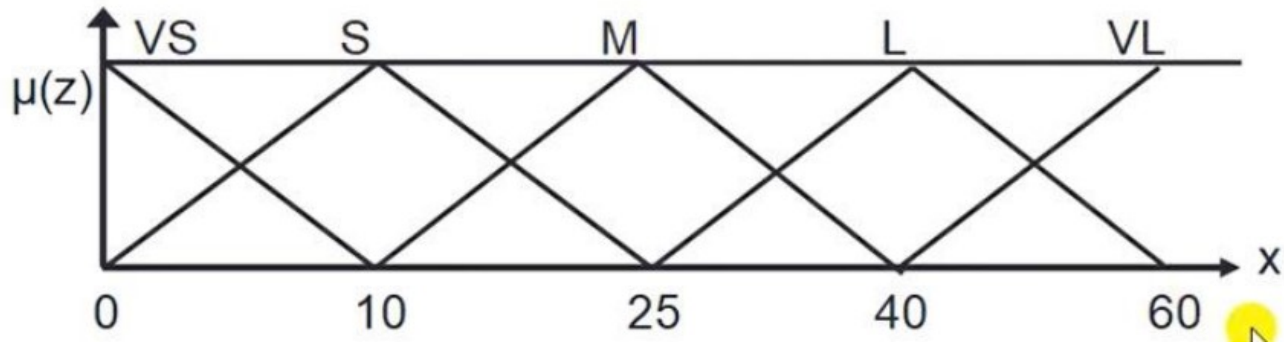
(1) Membership function for dirt:



(2) Membership function for grease:



(3) Membership function for Wash time:



Sol^n : Step I : Identify input and output variables and decide descriptor for the same.

Dirt = { SD, MD, LD } , Grease = { NG, MG, LG }

SD = Small dirt

MD = Medium dirt

LD = Large dirt

NG = No grease

MG = Medium grease

LG = Large grease

Wash Time = { VS, S, M, L, VL }

VS = very short

S = short

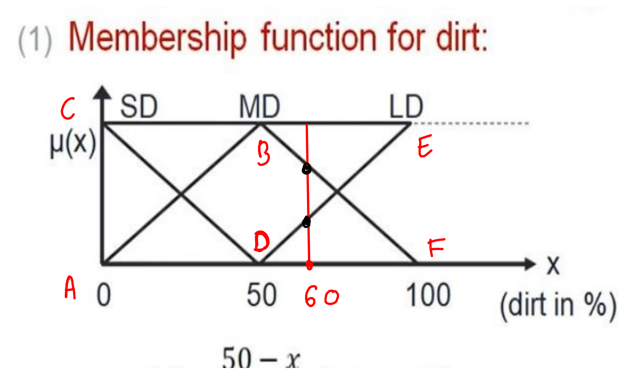
M = Medium

L = Large

VL = Very Large.

Step II : Find membership value of given input variables.

When dirt = 60%



Eq^n of line DE:

D(50, 0) and E(100, 1)

$x_1 \; y_1 \qquad x_2 \; y_2$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{y-0}{x-50} = \frac{1-0}{100-50} \Rightarrow y = \frac{x-50}{50}$$

$$\therefore \mu_{DE}(x) = \mu_{LD}(x) = \frac{x-50}{50}$$

$$\therefore \mu_{LD}(60) = \frac{60-50}{50} = \frac{1}{5} = \underline{\underline{0.2}} \qquad \therefore \mu_{LD}(60) = \underline{\underline{0.2}}$$

Eq^n of line BF:

B(50, 1) and F(100, 0)

$x_1 \; y_1 \qquad x_2 \; y_2$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{y-1}{x-50} = \frac{0-1}{100-50} \Rightarrow y-1 = \frac{-(x-50)}{50}$$

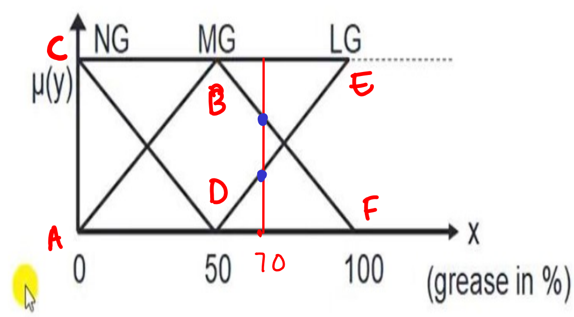
$$\therefore y = 1 - \frac{x-50}{50} = \frac{100-x}{50}$$

$$\therefore \mu_{BF}(x) = \mu_{MD}(x) = \frac{100-x}{50}$$

$$\therefore \mu_{MD}(60) = \frac{100-60}{50} = \frac{4}{5} = 0.8 \Rightarrow \mu_{MD}(60) = \underline{\underline{0.8}}$$

When grease = 70 %.

(2) Membership function for grease:



Eqn of line DE:

D (50, 0) and E(100, 1)

$x_1 \; y_1 \qquad x_2 \; y_2$

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow \frac{y - 0}{x - 50} = \frac{1 - 0}{100 - 50} \Rightarrow y = \frac{x - 50}{50}$$

$$\therefore \mu_{DE}(x) = \mu_{LG}(x) = \frac{x - 50}{50}$$

$$\therefore \mu_{LG}(70) = \frac{70 - 50}{50} = \frac{2}{5} = 0.4 \qquad \therefore \mu_{LG}(70) = \underline{\underline{0.4}}$$

Eqn of line BF

B(50, 1) and F(100, 0)

$x_1 \; y_1 \qquad x_2 \; y_2$

$$\therefore \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{y - 1}{x - 50} = \frac{0 - 1}{100 - 50} \Rightarrow y - 1 = \frac{-(x - 50)}{50}$$

$$\therefore y = 1 - \frac{x - 50}{50} \Rightarrow y = \frac{100 - x}{50}$$

$$\therefore \mu_{BF}(x) = \mu_{MG}(x) = \frac{100 - x}{50}$$

$$\therefore \mu_{MG}(70) = \frac{100 - 70}{50} = \frac{3}{5} = \underline{\underline{0.6}} \qquad \therefore \mu_{MG}(70) = \underline{\underline{0.6}}$$

Step III : Fuzzy rule evaluation

When dirt = 60% and grease = 70%

$\mu_{LD}(60) = 0.2$ $\mu_{LG}(70) = 0.4$

$\mu_{MD}(60) = 0.8$ $\mu_{MG}(70) = 0.6$

grease

dirt	NG	MG	LG
SD	VS	M	L
MD	S	M	L
LD	M	L	VL

Rule 1 : Dirt is MD and Grease is MG

$$R_1 = \min\{0.8, 0.6\} = \underline{\underline{0.6}}$$

Rule 2 : Dirt is MD and Grease is LG

$$R_2 = \min\{0.8, 0.4\} = \underline{\underline{0.4}}$$

Rule 3: Dirt is LD and Grease is MG

$$R_3 = \{0.2, 0.6\} = 0.2$$

Rule 4 : Dirt is LD and Grease is LG

$$R_4 = \{0.2, 0.4\} = 0.2$$

Step IV : Defuzzification method

By middle of maxima

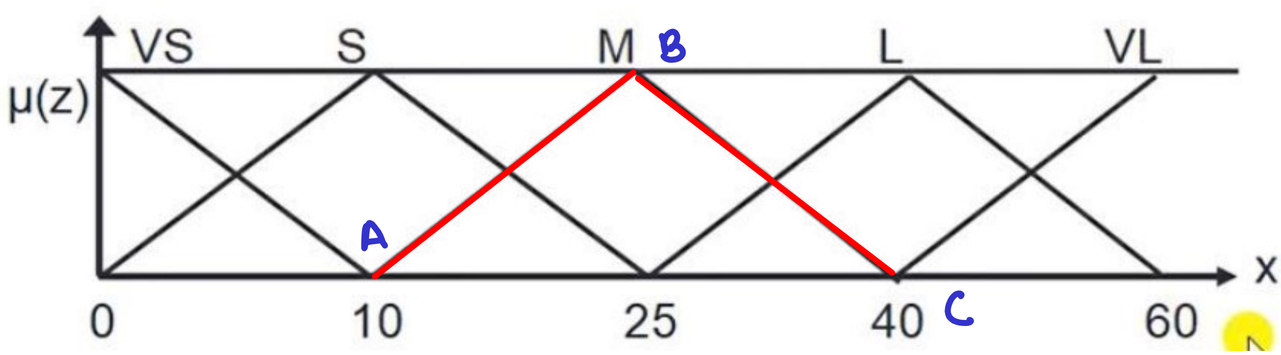
Wash Time position $= \max\{0.6, 0.4, 0.2, 0.2\}$

$= 0.6$

It correspond to Rule 1 where Dirt is MD and Grease is MG

$\therefore$ Hence output wash time is M

(3) Membership function for Wash time:



$\therefore \mu_m(x) = 0.6$

Eqn of line AB :

A(10,0) and B(25,1)

$\therefore \quad \frac{y-0}{x-10} = \frac{1-0}{25-10} \Rightarrow y = \frac{x-10}{15}$

$\therefore \quad \mu_{AB}(x) = \frac{x-10}{15}$

Eq^n of line BC

B(25, 1) and C(40, 0)

$\therefore \quad \frac{y-1}{x-25} = \frac{0-1}{40-25} \Rightarrow y-1 = \frac{-(x-25)}{15}$

$\therefore \quad y = 1 - \frac{x-25}{15} = \frac{40-x}{15}$

$\therefore \quad \mu_{BC}(x) = \frac{40-x}{15}$

Since $\mu_{M}(x) = 0.6$

$\therefore \quad \mu_{AB}(x) = 0.6$ and $\mu_{BC}(x) = 0.6$

$\therefore \quad \frac{x-10}{15} = 0.6$ $\qquad \therefore \quad \frac{40-x}{15} = 0.6$

$\therefore \quad x = 19$ and $x = 31$

$\therefore$ Wash time $= \frac{19+31}{2} = \underline{\underline{25}}$ min

```
%PERSONAL DETAILS

%MATLAB PROGRAMMING ASSIGNMENT II

%ROLL NUMBER: SYCO01

%PRN: 1212121212

%NAME: DINESH BABAN KUTE
```

```
%Triangular Membership Function

%Syntax: y = trimf(x,params)
```

```
%Example
```

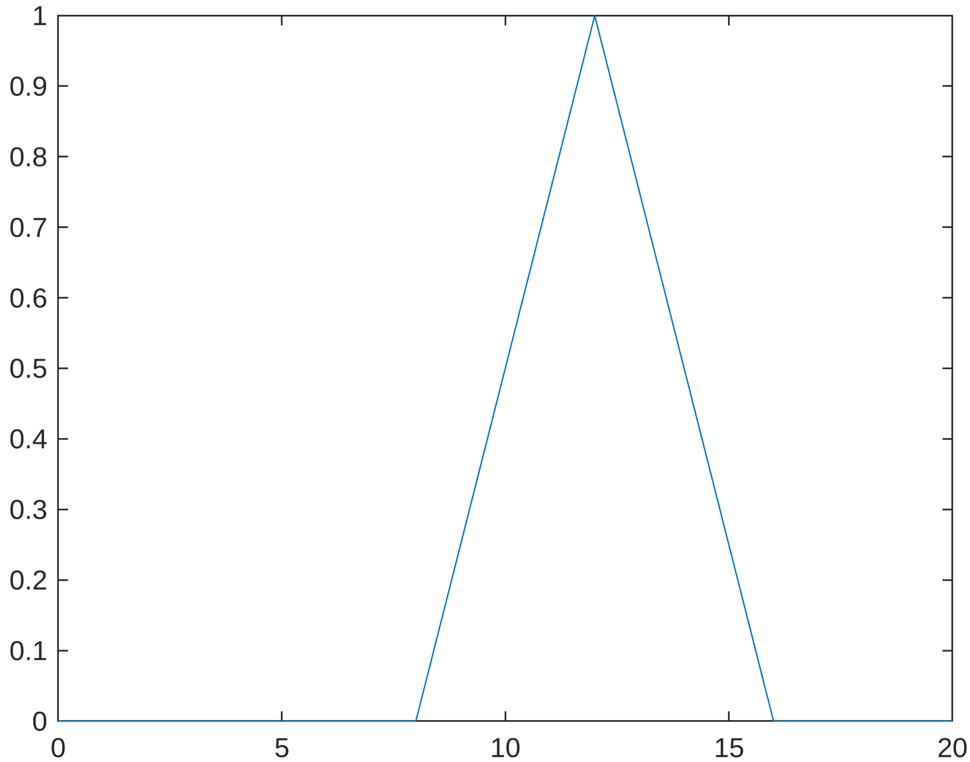
```
x1=0:0.2:20
```

```
x1 = 1×101
         0    0.2000    0.4000    0.6000    0.8000    1.0000    1.2000    1.4000 ···
```

```
y1=trimf(x,[8 12 16])
```

```
y1 = 1×101
         0         0         0         0         0         0         0         0 ···
```

```
plot(x,y)
```




```
%Trapezoidal Membership Function

%Syntax: y = trapmf(x,params)
```

```
%Example
```

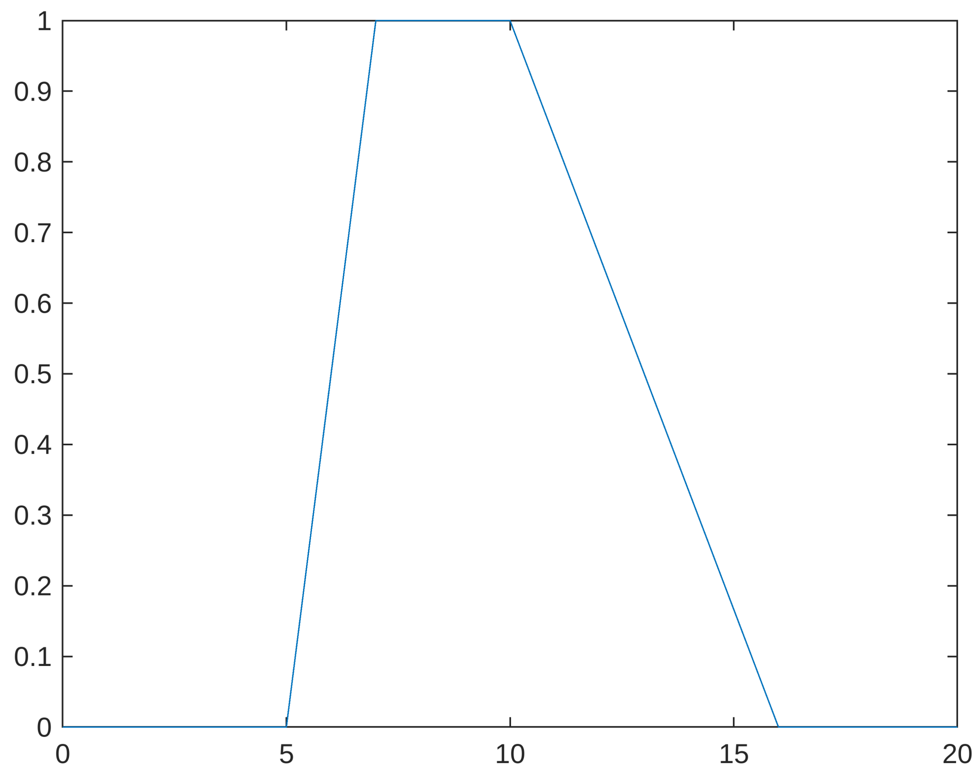
```
x2=0:0.2:20
```

```
x2 = 1×101
         0    0.2000    0.4000    0.6000    0.8000    1.0000    1.2000    1.4000 ···
```

```
y2 = trapmf(x,[5 7 10 16])
```

```
y2 = 1×101
         0         0         0         0         0         0         0         0 ···
```

```
plot(x2,y2)
```



```
%Sigmoidal Membership Function

%Syntax: y = sigmf(x,params)
```

```
%Example
```

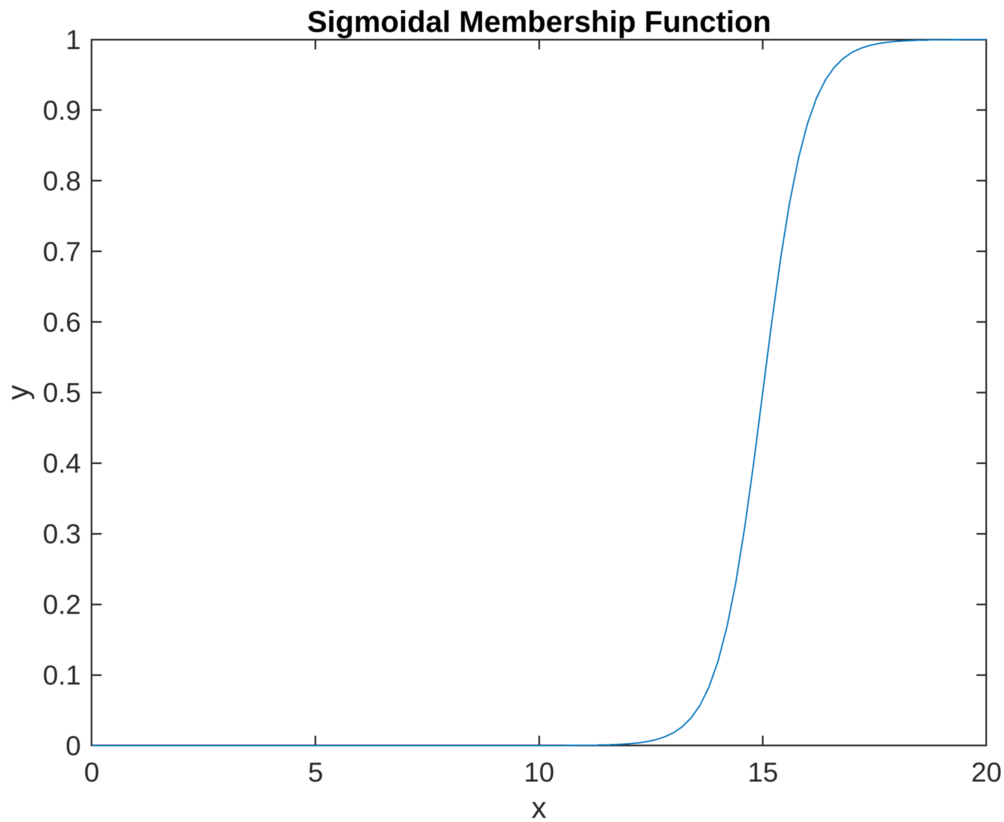
```
x3=0:0.2:20
```

```
x3 = 1×101
         0    0.2000    0.4000    0.6000    0.8000    1.0000    1.2000    1.4000 ···
```

```
y3 = sigmf(x,[2 15])
```

```
y3 = 1×101
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000 ···
```

```
plot(x3,y3)
```




```
%Gaussian Membership Function

%Syntax: y = gaussmf(x,params)
```

```
%Example
```

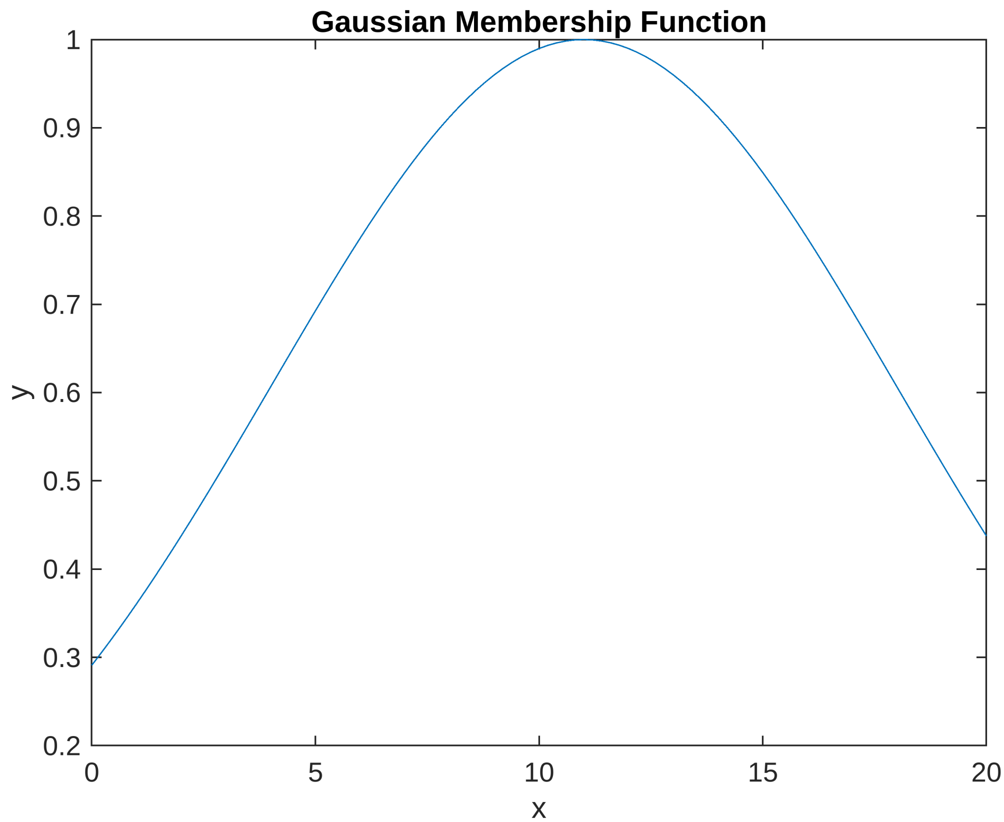
```
x4=0:0.2:20
```

```
x4 = 1×101
         0    0.2000    0.4000    0.6000    0.8000    1.0000    1.2000    1.4000 ⋯
```

```
y4 = gaussmf(x,[7 11])
```

```
y4 = 1×101
    0.2909    0.3042    0.3177    0.3317    0.3459    0.3604    0.3753    0.3905 ⋯
```

```
plot(x4,y4)
```



Gaussian Membership Function
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
y
0
5
10
15
20
x

```
%PERSONAL DETAILS

%MATLAB PROGRAMMING ASSIGNMENT II

%ROLL NUMBER: SYCO01

%PRN: 1212121212

%NAME: DINESH BABAN KUTE
```

```
%Defuzzification Methods
```

```
% 1) Center of Gravity / Centroid
```

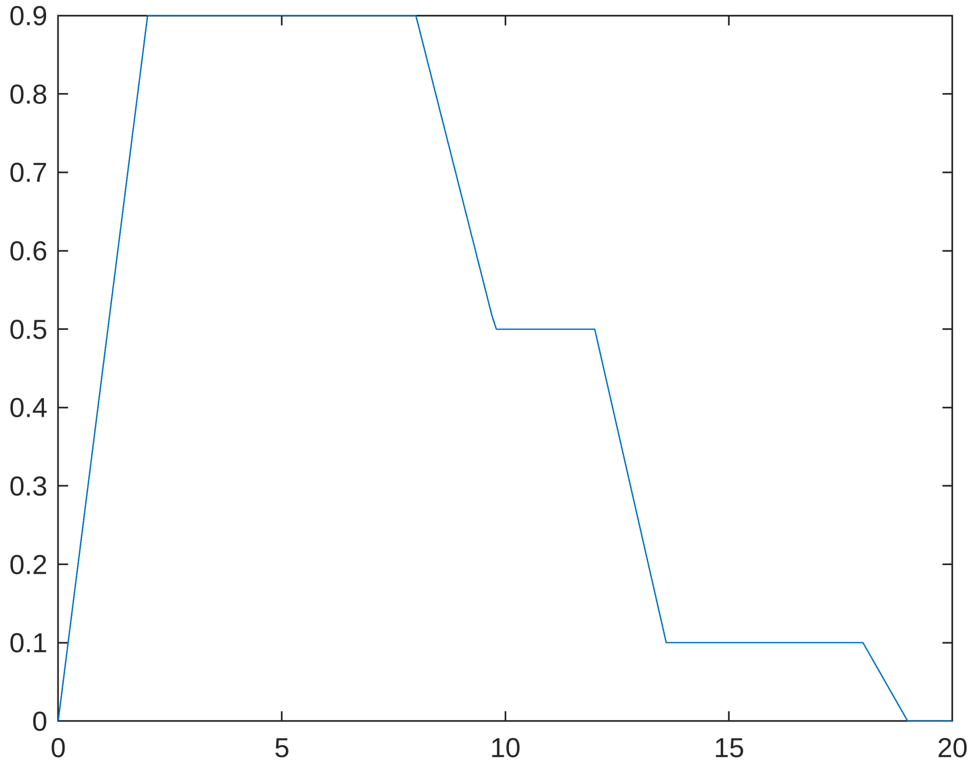
```
x = 0:0.1:20
```

```
x = 1×201
         0    0.1000    0.2000    0.3000    0.4000    0.5000    0.6000    0.7000 ···
```

```
mf1 = trapmf(x,[0 2 8 12]);
mf2 = trapmf(x,[5 7 12 14]);
mf3 = trapmf(x,[12 13 18 19]);
mf = max(0.5*mf2,max(0.9*mf1,0.1*mf3))
```

```
mf = 1×201
         0    0.0450    0.0900    0.1350    0.1800    0.2250    0.2700    0.3150 ···
```

```
plot(x,mf)
```




```
xCentroid = defuzz(x,mf,'centroid')
```

```
xCentroid = 6.7719
```

```
% 2)Middle of Maxima (MOM)
```

```
xMOM = defuzz(x,mf,'mom')
```

```
xMOM = 5
```

```
% 3) First of Maxima / Smallest of Maxima (SOM)
```

```
xSOM = defuzz(x,mf,'som')
```

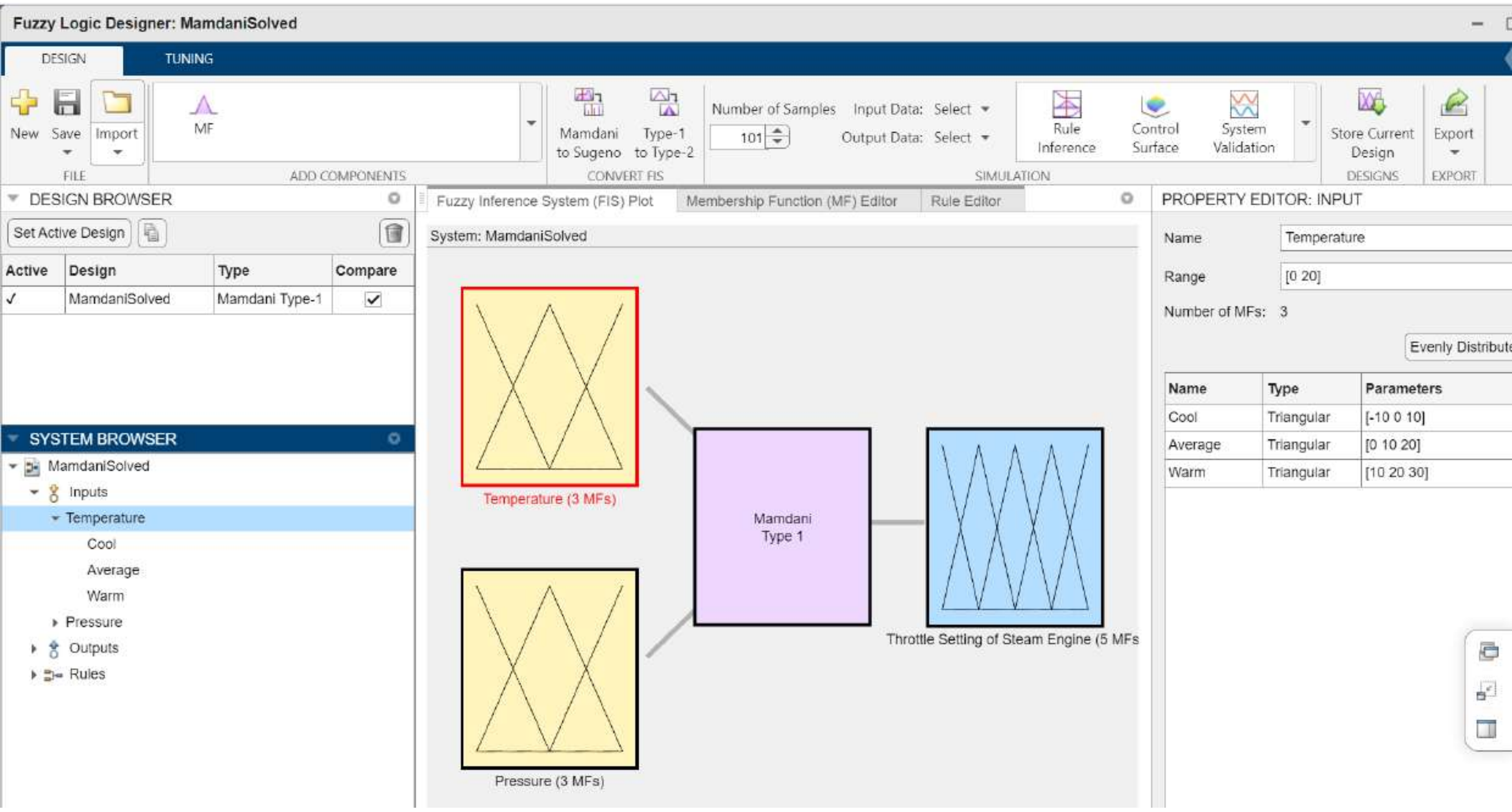
```
xSOM = 2
```

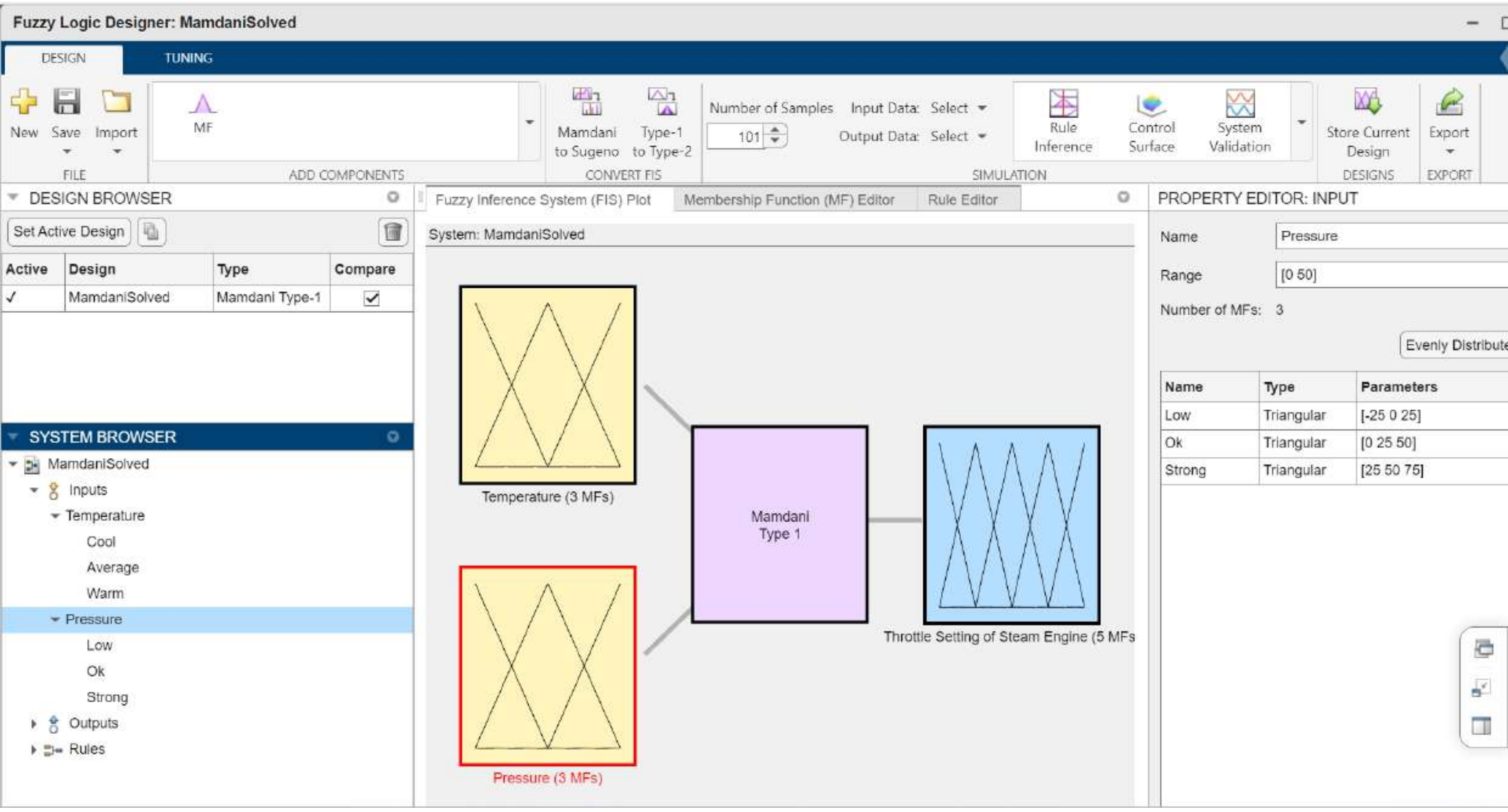
```
% 4) Last of Maxima / Largest of Maxima (LOM)
```

```
xLOM = defuzz(x,mf,'lom')
```

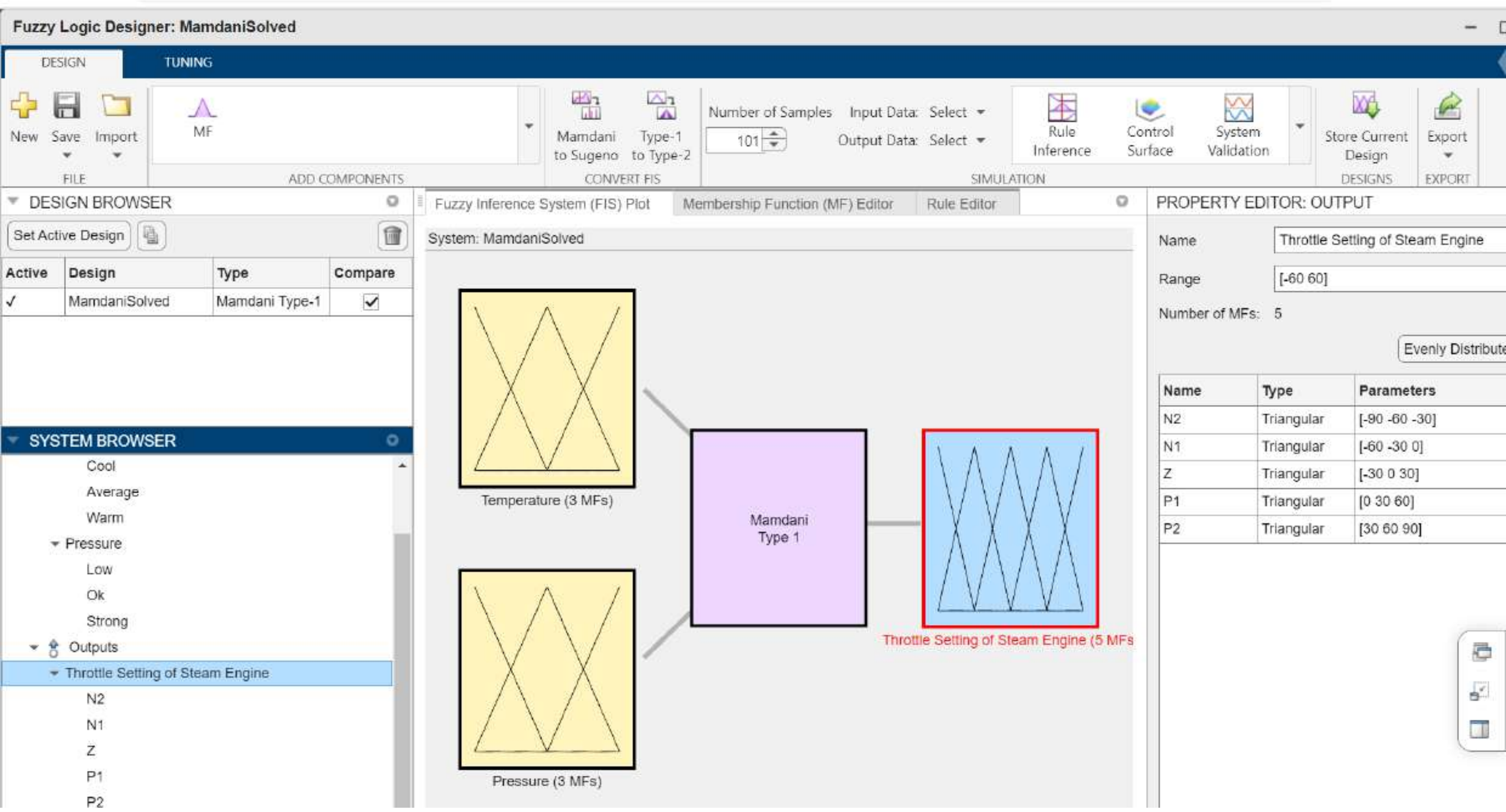
```
xLOM = 8
```

MAMDANI INFERENCE SYSTEM USING MATLAB

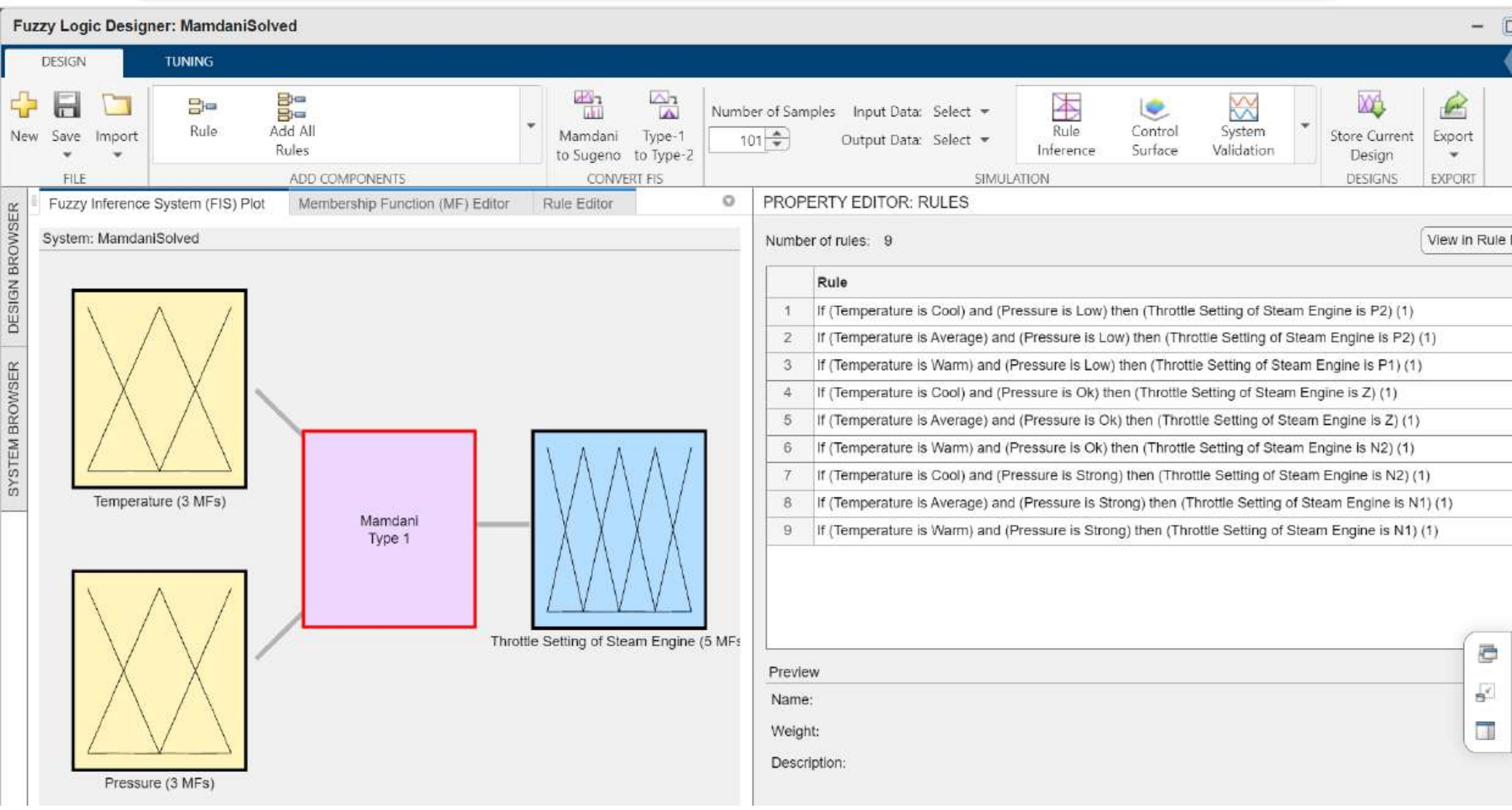


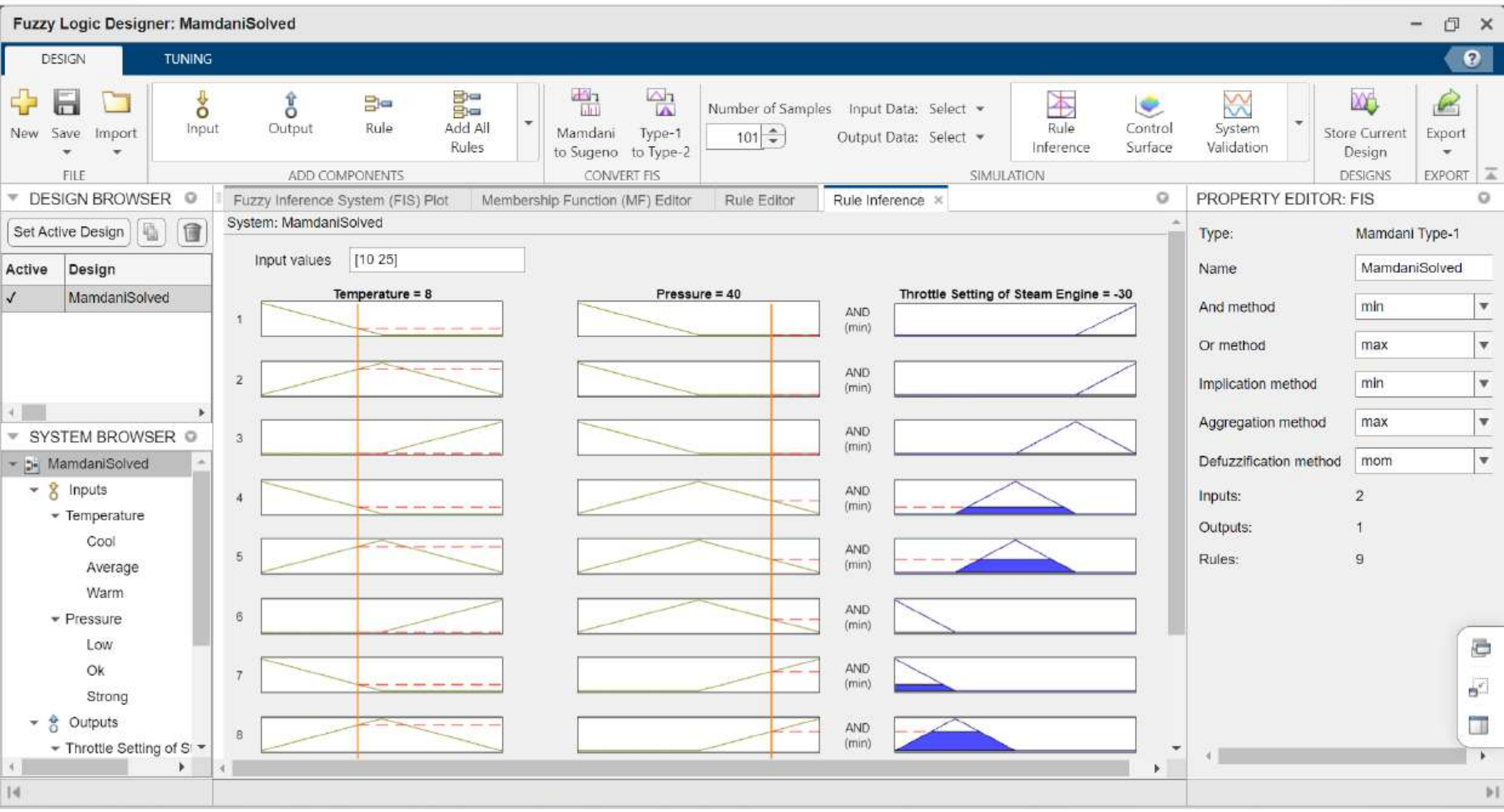


Fuzzy Logic Designer: MamdaniSolved
DESIGN
TUNING
New
Save
Import
FILE
MF
ADD COMPONENTS
Mamdani to Sugeno
Type-1 to Type-2
CONVERT FIS
Number of Samples
101
Input Data: Select
Output Data: Select
Rule Inference
Control Surface
System Validation
SIMULATION
Store Current Design
DESIGNS
Export
EXPORT
DESIGN BROWSER
Set Active Design
Active
Design
Type
Compare
MamdaniSolved
Mamdani Type-1
SYSTEM BROWSER
MamdaniSolved
Inputs
Temperature
Cool
Average
Warm
Pressure
Low
Ok
Strong
Outputs
Rules
Fuzzy Inference System (FIS) Plot
Membership Function (MF) Editor
Rule Editor
System: MamdaniSolved
Temperature (3 MFs)
Mamdani Type 1
Throttle Setting of Steam Engine (5 MFs
Pressure (3 MFs)
PROPERTY EDITOR: INPUT
Name
Pressure
Range
[0 50]
Number of MFs: 3
Evenly Distribute
Name
Type
Parameters
Low
Triangular
[-25 0 25]
Ok
Triangular
[0 25 50]
Strong
Triangular
[25 50 75]



Fuzzy Logic Designer: MamdaniSolved
DESIGN
TUNING
New
Save
Import
FILE
MF
ADD COMPONENTS
Mamdani to Sugeno
Type-1 to Type-2
CONVERT FIS
Number of Samples
101
Input Data: Select
Output Data: Select
Rule Inference
Control Surface
System Validation
SIMULATION
Store Current Design
DESIGNS
Export
EXPORT
DESIGN BROWSER
Set Active Design
Active
Design
Type
Compare
MamdaniSolved
Mamdani Type-1
SYSTEM BROWSER
Cool
Average
Warm
Pressure
Low
Ok
Strong
Outputs
Throttle Setting of Steam Engine
N2
N1
Z
P1
P2
Fuzzy Inference System (FIS) Plot
Membership Function (MF) Editor
Rule Editor
System: MamdaniSolved
Temperature (3 MFs)
Mamdani Type 1
Throttle Setting of Steam Engine (5 MFs
Pressure (3 MFs)
PROPERTY EDITOR: OUTPUT
Name
Throttle Setting of Steam Engine
Range
[-60 60]
Number of MFs: 5
Evenly Distribute
Name
Type
Parameters
N2
Triangular
[-90 -60 -30]
N1
Triangular
[-60 -30 0]
Z
Triangular
[-30 0 30]
P1
Triangular
[0 30 60]
P2
Triangular
[30 60 90]


Fuzzy Logic Designer: MamdaniSolved
DESIGN
TUNING
New
Save
Import
FILE
Rule
Add All Rules
ADD COMPONENTS
Mamdani to Sugeno
Type-1 to Type-2
CONVERT FIS
Number of Samples
101
Input Data: Select
Output Data: Select
Rule Inference
Control Surface
System Validation
SIMULATION
Store Current Design
DESIGNS
Export
EXPORT
DESIGN BROWSER
SYSTEM BROWSER
Fuzzy Inference System (FIS) Plot
Membership Function (MF) Editor
Rule Editor
System: MamdaniSolved
Temperature (3 MFs)
Pressure (3 MFs)
Mamdani Type 1
Throttle Setting of Steam Engine (5 MFs
PROPERTY EDITOR: RULES
Number of rules: 9
View in Rule
Rule
1 If (Temperature is Cool) and (Pressure is Low) then (Throttle Setting of Steam Engine is P2) (1)
2 If (Temperature is Average) and (Pressure is Low) then (Throttle Setting of Steam Engine is P2) (1)
3 If (Temperature is Warm) and (Pressure is Low) then (Throttle Setting of Steam Engine is P1) (1)
4 If (Temperature is Cool) and (Pressure is Ok) then (Throttle Setting of Steam Engine is Z) (1)
5 If (Temperature is Average) and (Pressure is Ok) then (Throttle Setting of Steam Engine is Z) (1)
6 If (Temperature is Warm) and (Pressure is Ok) then (Throttle Setting of Steam Engine is N2) (1)
7 If (Temperature is Cool) and (Pressure is Strong) then (Throttle Setting of Steam Engine is N2) (1)
8 If (Temperature is Average) and (Pressure is Strong) then (Throttle Setting of Steam Engine is N1) (1)
9 If (Temperature is Warm) and (Pressure is Strong) then (Throttle Setting of Steam Engine is N1) (1)
Preview
Name:
Weight:
Description:



Fuzzy Logic Designer: MamdaniSolved
DESIGN
TUNING
New
Save
Import
FILE
Input
Output
Rule
Add All Rules
ADD COMPONENTS
Mamdani to Sugeno
Type-1 to Type-2
CONVERT FIS
Number of Samples
101
Input Data: Select
Output Data: Select
Rule Inference
Control Surface
System Validation
SIMULATION
Store Current Design
DESIGNS
Export
EXPORT
DESIGN BROWSER
Set Active Design
Active
Design
MamdaniSolved
SYSTEM BROWSER
MamdaniSolved
Inputs
Temperature
Cool
Average
Warm
Pressure
Low
Ok
Strong
Outputs
Throttle Setting of S
Fuzzy Inference System (FIS) Plot
Membership Function (MF) Editor
Rule Editor
Rule Inference
System: MamdaniSolved
Input values
[10 25]
Temperature = 8
Pressure = 40
Throttle Setting of Steam Engine = -30
AND (min)
PROPERTY EDITOR: FIS
Type: Mamdani Type-1
Name MamdaniSolved
And method min
Or method max
Implication method min
Aggregation method max
Defuzzification method mom
Inputs: 2
Outputs: 1
Rules: 9