UNIT V FUZZY LOGIC CONTROL

UNIVERSAL FUZZY SET

Def^n: If X is any set then universal fuzzy set of X is denoted by $\tilde{X}$ and defined as

$$\tilde{X} = \{ (x, 1) \mid x \in X \}$$

Example: Let $X = \{ a_1, a_2, a_3, a_4 \}$ then

$$\tilde{X} = \{ (a_1, 1), (a_2, 1), (a_3, 1), (a_4, 1) \}$$

FUZZY "IF THEN" RULE

It is also called as Fuzzy implication, Fuzzy Rule or Fuzzy conditional statement.

The standard fuzzy if then rule is given below,

(a) If x is $\tilde{A}$ then y is $\tilde{B}$

i.e. $\tilde{R} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{Y})$, $\tilde{Y}$ - Universal fuzzy set

(b) If x is $\tilde{A}$ then y is $\tilde{B}$ else y is $\tilde{C}$

i.e. $\tilde{R} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{C})$

* Example

Let $X = \{ a, b, c, d \}$ and $Y = \{ 1, 2, 3, 4 \}$ and

$$\tilde{A} = \{ (a, 0), (b, 0.8), (c, 0.6), (d, 1) \}$$

$$\tilde{B} = \{ (1, 0.2), (2, 1), (3, 0.8), (4, 0) \}$$

$\tilde{C} = \{ (1, 0), (2, 0.4), (3, 1), (4, 0.8) \}$

Determine the implication relations

(a) If x is $\tilde{A}$ then Y is $\tilde{B}$

(b) If x is $\tilde{A}$ then Y is $\tilde{B}$ else Y is $\tilde{C}$

Soln : (a) If x is $\tilde{A}$ then Y is $\tilde{B}$

$\therefore \tilde{R} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{Y})$, $\tilde{Y}$ - universal fuzzy set of Y

Where $\tilde{Y} = \{ (1,1), (2,1), (3,1), (4,1) \}$

$\tilde{A}^c = \{ (a, 1), (b, 0.2), (c, 0.4), (d, 0) \}$

$$\tilde{A} \times \tilde{B} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline a & 0 & 0 & 0 & 0 \\ b & 0.2 & 0.8 & 0.8 & 0 \\ c & 0.2 & 0.6 & 0.6 & 0 \\ d & 0.2 & 1 & 0.8 & 0 \end{array}$$

$$\tilde{A}^c \times \tilde{Y} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline a & 1 & 1 & 1 & 1 \\ b & 0.2 & 0.2 & 0.2 & 0.2 \\ c & 0.4 & 0.4 & 0.4 & 0.4 \\ d & 0 & 0 & 0 & 0 \end{array}$$

$$\tilde{R} = (\tilde{A} \times \tilde{B}) \cup (\tilde{A}^c \times \tilde{Y}) = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline a & 1 & 1 & 1 & 1 \\ b & 0.2 & 0.8 & 0.8 & 0.2 \\ c & 0.4 & 0.6 & 0.6 & 0.4 \\ d & 0.2 & 1 & 0.8 & 0 \end{array}$$

(b) If x in $\tilde{A}$ then y in $\tilde{B}$ else y in $\tilde{C}$

$$\tilde{A} = \{(a,0),\ (b,0.8),\ (c,0.6),\ (d,1)\}$$

$$\tilde{B} = \{(1,0.2),\ (2,1),\ (3,0.8),\ (4,0)\}$$

$$\tilde{C} = \{(1,0),\ (2,0.4),\ (3,1),\ (4,0.8)\}$$

$$\tilde{R} = (\tilde{A}\times\tilde{B}) \cup (\tilde{A}^c \times \tilde{C})$$

$$\tilde{A}^c = \{(a,1),\ (b,0.2),\ (c,0.4),\ (d,0)\}$$

$$\tilde{A}\times\tilde{B} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

$$\tilde{A}^c\times\tilde{C} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 1 & 0.8 \\ 0 & 0.2 & 0.2 & 0.8 \\ 0 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Now, $\tilde{R} = (\tilde{A}\times\tilde{B}) \cup (\tilde{A}^c\times\tilde{C})$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 1 & 0.8 \\ 0.2 & 0.8 & 0.8 & 0.8 \\ 0.2 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1 & 0.8 & 0 \end{bmatrix} \end{array}$$

b) Let $X=\{a,b,c,d\}$ & $Y=\{1,2,3,4\}$ be any two sets. If $\tilde{A}$ & $\tilde{B}$ are the fuzzy sets defined on the X & Y respectively defined by,

$\tilde{A}=\{(a,0),(b,0.7),(c,0.6),(d,1)\}$

$\tilde{B}=\{(1,0.2),(2,0.9),(3,0.5),(4,0)\}$ then find following fuzzy relation

"If X is $\tilde{A}$ then Y is $\tilde{B}$"

Soln: If x is $\tilde{A}$ then Y is $\tilde{B}$

$\tilde{R}=(\tilde{A}\times\tilde{B})\cup(\tilde{A}^c\times Y)$, Y - Universal set of Y

$\tilde{A}=\{(a,0),(b,0.7),(c,0.6),(d,1)\}$

$\tilde{B}=\{(1,0.2),(2,0.9),(3,0.5),(4,0)\}$

$\tilde{Y}=\{(1,1),(2,1),(3,1),(4,1)\}$

$\tilde{A}^c=\{(a,1),(b,0.3),(c,0.4),(d,0)\}$

$$\tilde{A}\times\tilde{B}=\begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}a\\b\\c\\d\end{matrix} & \begin{bmatrix}0 & 0 & 0 & 0\\ 0.2 & 0.7 & 0.5 & 0\\ 0.2 & 0.6 & 0.5 & 0\\ 0.2 & 0.9 & 0.5 & 0\end{bmatrix}\end{array}$$

$$\tilde{A}^c\times\tilde{Y}=\begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}a\\b\\c\\d\end{matrix} & \begin{bmatrix}1 & 1 & 1 & 1\\ 0.3 & 0.3 & 0.3 & 0.3\\ 0.4 & 0.4 & 0.4 & 0.4\\ 0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

$$\therefore\ \tilde{R}=(\tilde{A}\times\tilde{B})\cup(\tilde{A}^c\times\tilde{Y})=\begin{array}{c|c} & \begin{matrix}1 & 2 & 3 & 4\end{matrix} \\ \begin{matrix}a\\b\\c\\d\end{matrix} & \begin{bmatrix}1 & 1 & 1 & 1\\ 0.3 & 0.7 & 0.5 & 0.3\\ 0.4 & 0.6 & 0.5 & 0.4\\ 0.2 & 0.9 & 0.5 & 0\end{bmatrix}\end{array}$$

Fuzzy Inference System

Mamdani and Sugeneo fuzzy inference system

Fuzzy Inference System – Concept

Fuzzy inference is the process of mapping from a given input to an output using fuzzy logic.

What is the Fuzzy Inference System(FIS)?

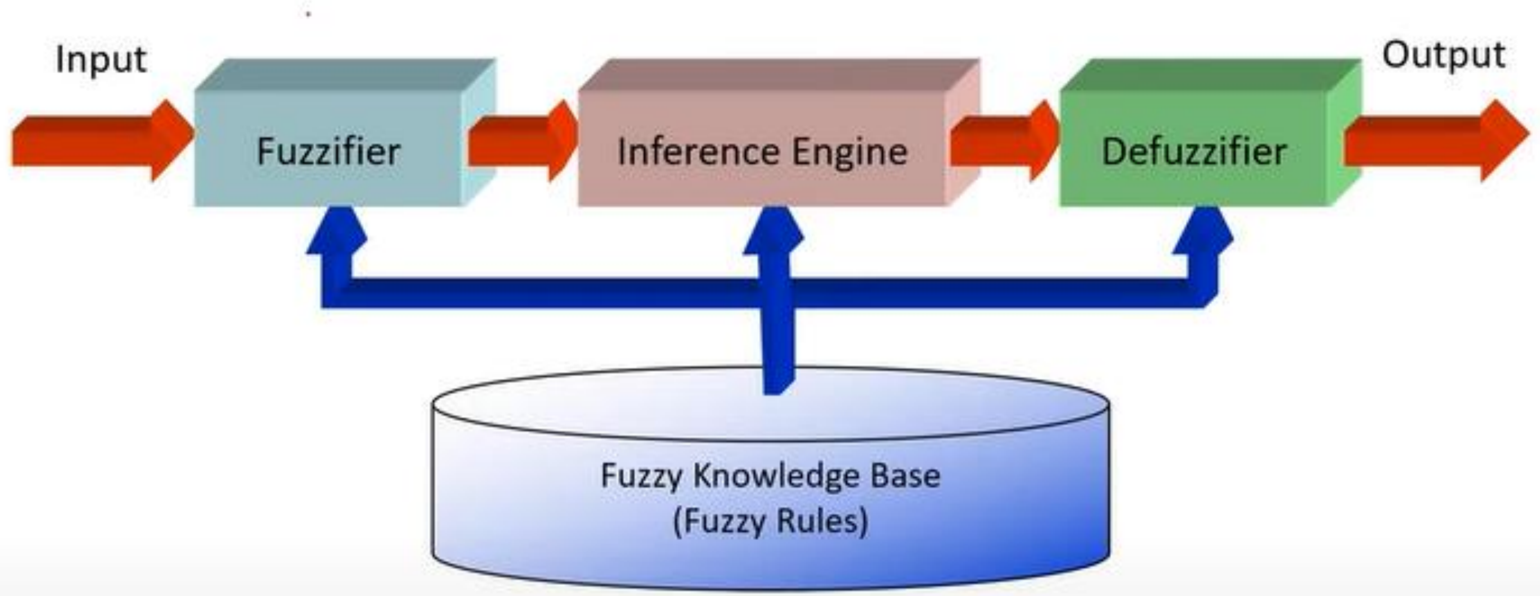
A nonlinear mapping that derives its output based on fuzzy reasoning and set of fuzzy if then rules.

Fuzzy inference systems have been successfully applied in various fields such as automatic control, data classification, decision analysis, expert systems, computer vision, etc.

Fuzzy Logic Toolbox™ software supports two types of fuzzy inference systems:

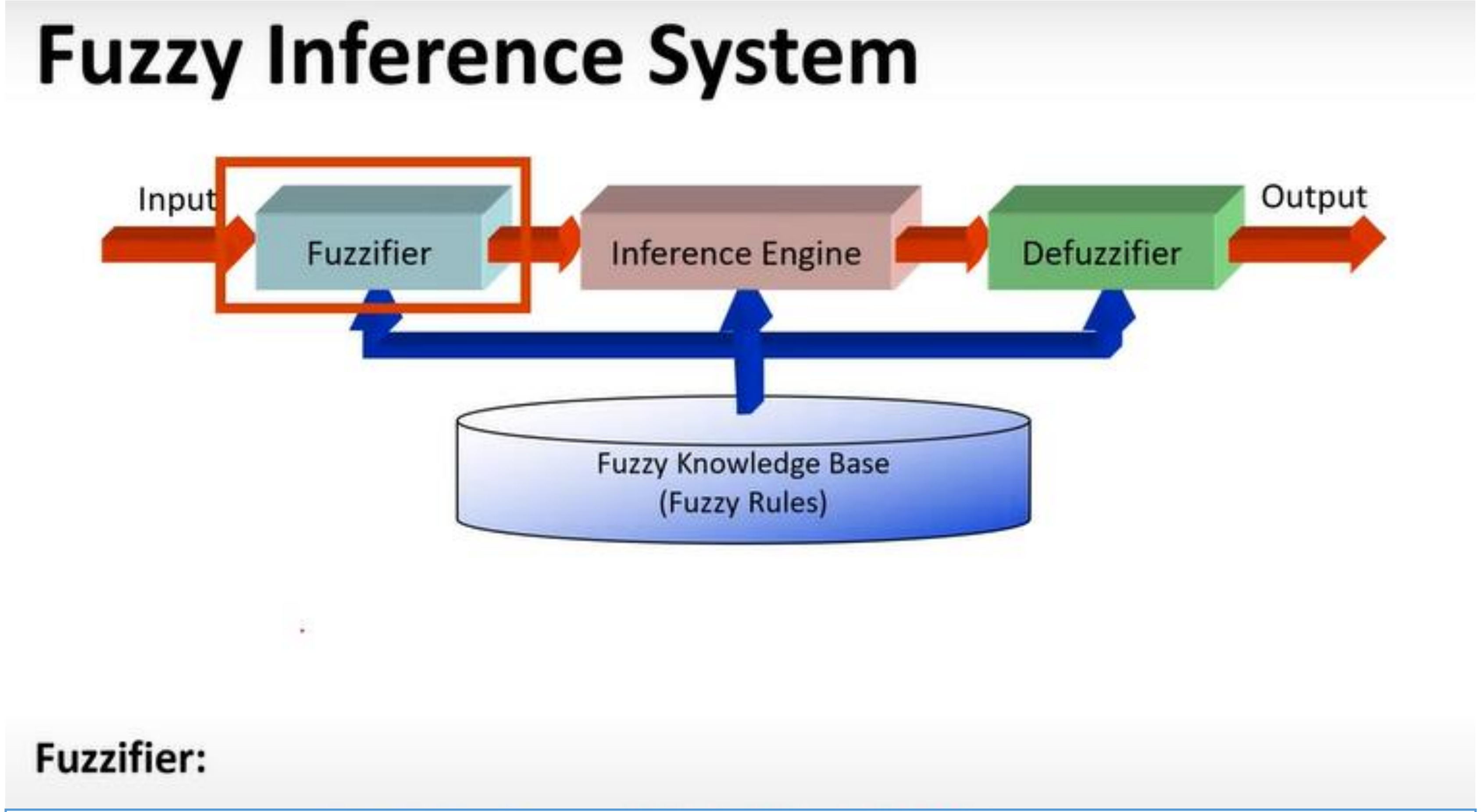
- Mamdani systems
- Sugeno systems

Architecture of Fuzzy Inference System



Architecture of Fuzzy Inference System

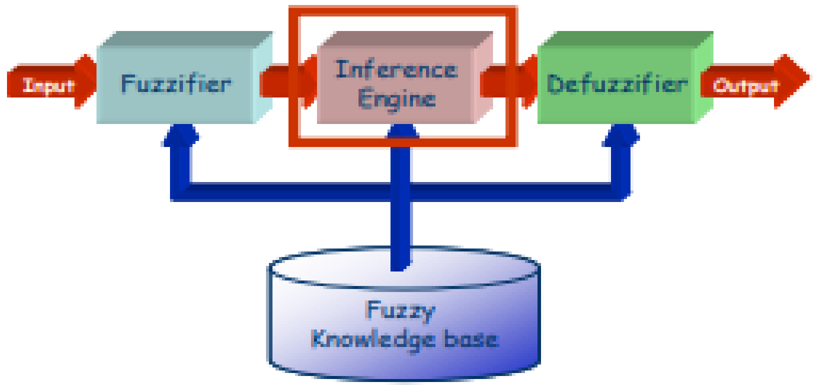
Architecture of Fuzzy Inference System



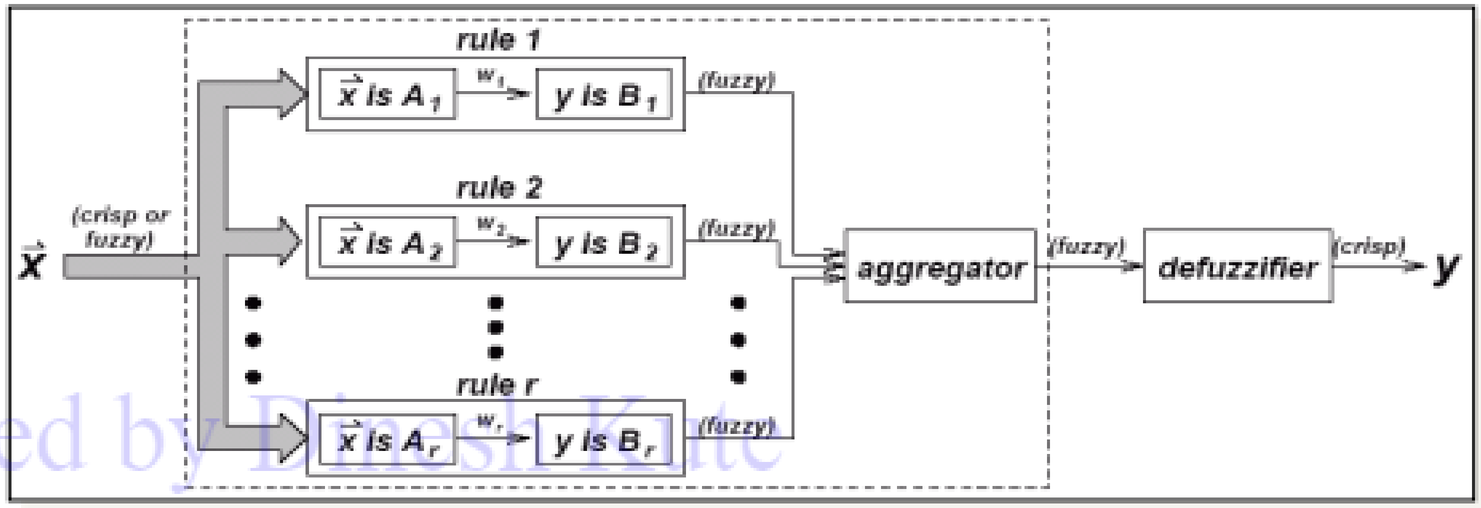
Covert **crisp input** to **fuzzy input (or linguistic variable)**

Architecture of Fuzzy Inference System

Inference Engine

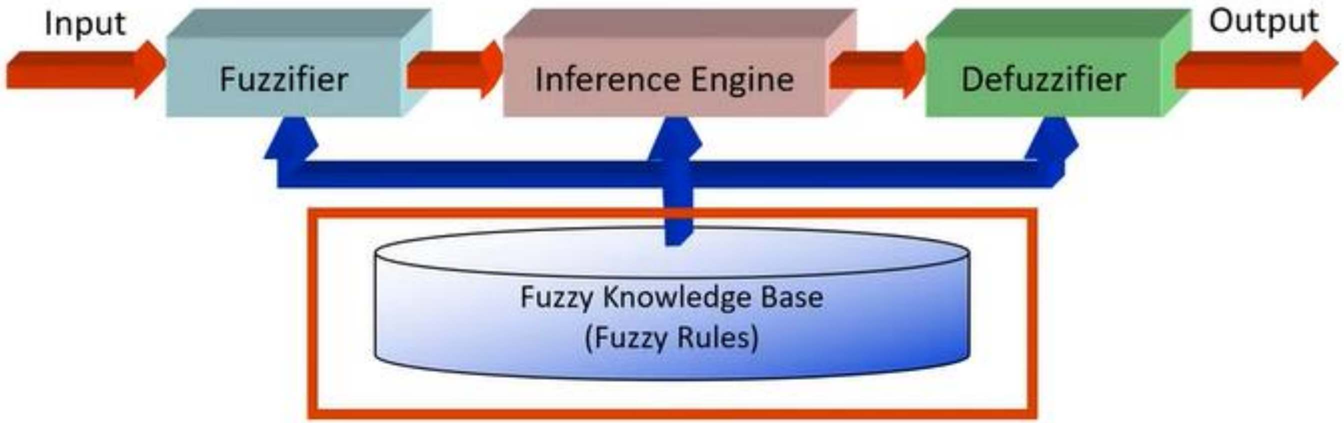


Using If-Then type fuzzy rules converts the fuzzy input to the fuzzy output.



Architecture of Fuzzy Inference System

Fuzzy Inference System



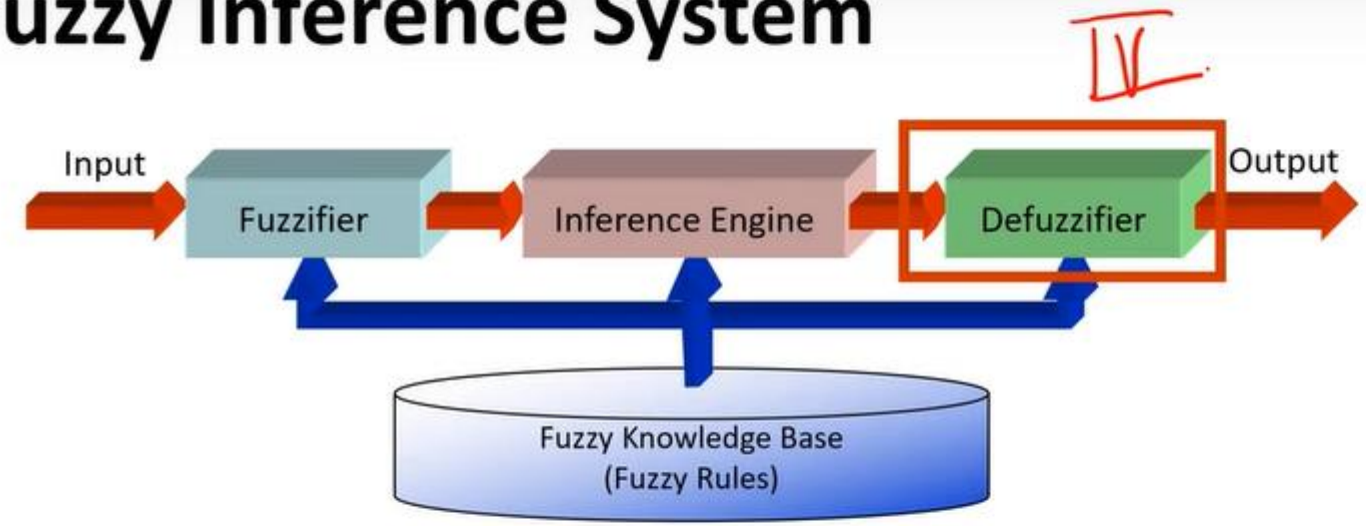
Fuzzy Knowledge Base:

The rule base referred to as the knowledge base.

- A rule base contains a number of fuzzy IF-THEN rules;
- A database which defines the membership functions of the fuzzy sets used the fuzzy rules

Architecture of Fuzzy Inference System

Fuzzy Inference System



Defuzzifier:

- It converts the fuzzy output of the inference engine to crisp.
- Here are some commonly used defuzzification methods are as follows:
  - Weighted average method
  - Center of gravity method
  - Mean of maximum (MOM)
  - Smallest of maximum (SOM)
  - Largest of maximum (LOM)

- center of sum method

Sugeno fuzzy inference system

Sugeno fuzzy inference, also referred to as Takagi-Sugeno-Kang fuzzy inference, uses *singleton* output membership functions that are either constant or a linear function of the input values.

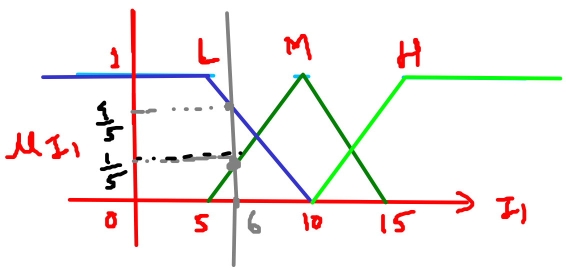
The defuzzification process for a Sugeno system is more computationally efficient compared to that of a Mamdani system, since it uses a weighted average or weighted sum of a few data points rather than compute a centroid of a two-dimensional area.

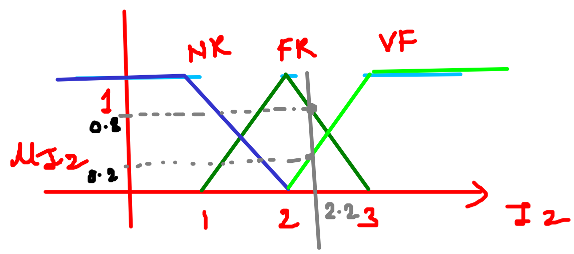
Difference Between Mamdani and Sugeno Fuzzy Inference System:

Mamdani FIS	Sugeno FIS
• Output membership function is present	• No output membership function is present
• The output of surface is discontinuous	• The output of surface is continuous
• Distribution of output	• Non distribution of output, only Mathematical combination of the output and the rules strength
• Through defuzzification of rules consequent of crisp result is obtained	• No defuzzification here. Using weighted average of the rules of consequent crisp result is obtained
• Expressive power and interpretable rule consequent	• Here is loss of interpretability
• Mamdani FIS possess less flexibility in the system design	• Sugeno FIS possess more flexibility in the system design
• It has more accuracy in security evaluation block cipher algorithm	• It has less accuracy in security evaluation block cipher algorithm
• It is using in MISO (Multiple Input and Single Output) and MIMO (Multiple Input and Multiple Output) systems	• It is using only in MISO (Multiple Input and Single Output) systems
• Mamdani inference system is well suited to human input	• Sugeno inference system is well suited to mathematically analysis
• Application: Medical Diagnosis System	• Application: To keep track of the change in aircraft performance with altitude

EXAMPLE ON SUGENO INFERENCE SYSTEM

Example: Find the output of the following fuzzy model for input $I_1 = 6$ and $I_2 = 2.2$ using sugeno inference system





with the following fuzzy rules

Rule 1 : IF I_1 is L and I_2 is FR then $y = I_1 + 2I_2$

2 : I_1 L I_2 VF then $y = I_1 + 3I_2$

3 I_1 M I_2 FR then $y = 2I_1 + 2I_2$

4 I_1 M I_2 VF then $y = 2I_1 + 3I_2$

Soln

$$\mu_M(x) = \frac{x-5}{10-5} = \frac{x-5}{5} \quad \therefore \mu_m(6) = \frac{1}{5}$$

$$\mu_L(x) = \frac{10-x}{10-5} = \frac{10-x}{5} \quad \therefore \mu_L(6) = \frac{4}{5}$$

I_1

$$\mu_{VF}(x) = \frac{x-2}{3-2} = \frac{x-2}{1} \quad \therefore \mu_{VF}(2.2) = 0.2$$

$$\mu_{FR}(x) = \frac{3-x}{3-2} = \frac{3-x}{1} \quad \therefore \mu_{FR}(2.2) = 0.8$$

I_2

Rule (i)	w^i	y^i
1	$\mu_L(6) \times \mu_{FR}(2.2)$ $= 0.64$	$y_1 = I_1 + 2I_2$ $= 6 + 2(2.2) = 10.4$
2	$\mu_L(6) \times \mu_{VF}(2.2)$ $= 0.16$	$y_2 = I_1 + 3I_2 = 12.6$
3	$\mu_m(6) \times \mu_{FR}(2.2)$ $= 0.16$	$y_3 = 2I_1 + 2I_2 = 16.4$
4	$\mu_m(6) \times \mu_{VF}(2.2)$ $= 0.04$	$y_4 = 2I_1 + 3I_2$ $= 18.6$

By sugeno inference system

$$\text{crisp output} = y = \frac{\sum w^i y^i}{\sum w^i}$$

$$\therefore y = \frac{(0.64 \times 10.4) + (0.16 \times 12.6) + (0.16 \times 16.4) + (0.04 \times 18.6)}{0.64 + 0.16 + 0.16 + 0.04}$$

$$= \underline{\underline{12.04}}$$

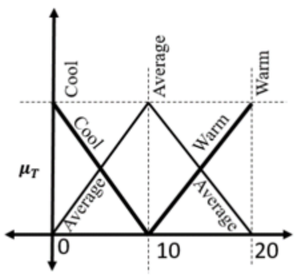
MAMDANI INFERENCE SYSTEM

- In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination.

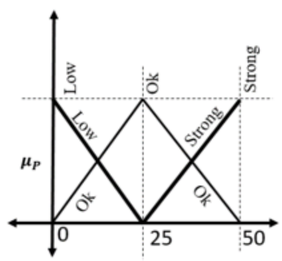
- He applied a set of fuzzy rules supplied by experienced human operators.

EXAMPLES

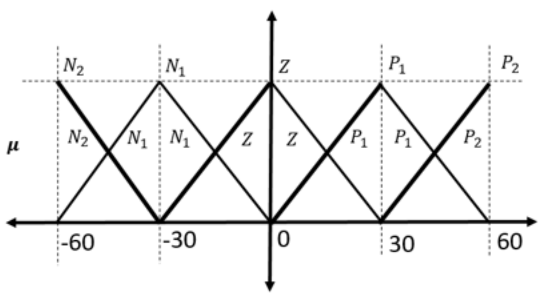
Consider the design of a fuzzy controller for a steam turbine. Assume the input of the fuzzy controller as temperature and pressure with 3 descriptors and output will be throttle setting of a steam turbine with 5 descriptors given below,



Temperature (T)



Pressure (P)



Throttle Setting of Steam Turbine

N_2 : Large Negative
N_1 : Small Negative
Z : Zero
P_1 : Small Positive
P_2 : Large Positive

Find throttle position of the turbine for temperature = 8 & pressure= 40 using Mamdani Inference System and defuzzification method 'middle of Maxima' with following fuzzy Rules,

	Low	Ok	Strong
Cool	P_2	Z	N_2
Average	P_2	Z	N_1
Warm	P_1	N_2	N_1

Soln : Step I : Identify input and output variables and decide descriptor for the same.

Descriptors for Input variable

Temperature
1. Cool
2. Average
3. Warm

Pressure
1. Low
2. OK
3. Strong

Descriptors for the output variables

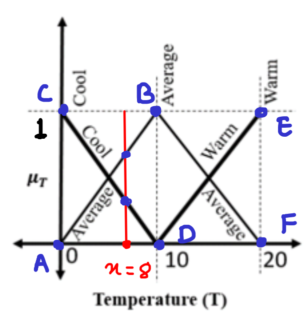
Throttle setting for steam turbine

1. N_2 = large negative
2. N_1 = small negative
3. Z = zero
4. P_1 = small positive
5. P_2 = large positive

Step II : Find membership value of given input variables.

Case I : When temperature = 8

$$\boxed{\mu_{CD}(x) = \mu_{cool}(x) = \frac{10 - x}{10}}$$



Eqⁿ of line CD :

$C(0, 1)$ and $D(10, 0)$
$x_1 \; y_1 \qquad\qquad x_2 \; y_2$

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore \frac{y - 1}{x - 0} = \frac{0 - 1}{10 - 0} \Rightarrow y - 1 = \frac{-x}{10} \Rightarrow y = 1 - \frac{x}{10} = \frac{10 - x}{10}$$

$$\mu_{cool}(8) = \frac{10-8}{10} = \frac{2}{10} = \underline{\underline{\frac{1}{5}}}$$

$$\boxed{\therefore \mu_{cool}(8) = \frac{1}{5}}$$

$$\boxed{\mu_{AB}(x) = \mu_{Average}(x) = \frac{x}{10}}$$

Eqn of line AB :

$$A(\underset{x_1}{0}, \underset{y_1}{0}) \quad \text{and} \quad B(\underset{x_2}{10}, \underset{y_2}{1})$$

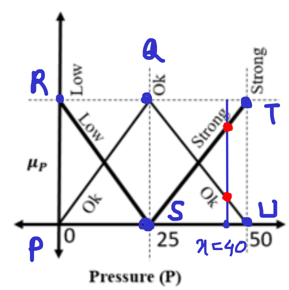
$$\frac{y-0}{x-0} = \frac{1-0}{10-0} \Rightarrow y = \frac{x}{10}$$

$$\therefore \mu_{Average}(8) = \frac{8}{10} = \underline{\underline{\frac{4}{5}}}$$

$$\boxed{\therefore \mu_{Average}(8) = \frac{4}{5}}$$

<u>Case II</u> : When pressure = 40

$$\boxed{\mu_{QU} = \mu_{OK}(x) = \frac{50-x}{25}}$$



Eqn of line QU :

$$Q(\underset{x_1}{25}, \underset{y_1}{1}) \quad \text{and} \quad U(\underset{x_2}{50}, \underset{y_2}{0})$$

$$\therefore \frac{y-1}{x-25} = \frac{0-1}{50-25} \Rightarrow y-1 = \frac{-(x-25)}{25}$$

$$\Rightarrow y = 1 - \frac{x-25}{25}$$

$$\Rightarrow y = \frac{50-x}{25}$$

$$\therefore \mu_{OK}(40) = \frac{50-40}{25} = \frac{10}{25} = \underline{\underline{\frac{2}{5}}}$$

$$\boxed{\therefore \mu_{OK}(40) = \frac{2}{5}}$$

$$\boxed{\mu_{ST}(x) = \mu_{strong}(x) = \frac{x-25}{25}}$$

Eqn of line ST :

S(25, 0) and T(50, 1)

$x_1 \; y_1 \qquad x_2 \; y_2$

$$\frac{y-0}{x-25} = \frac{1-0}{50-25} \Rightarrow y = \frac{x-25}{25}$$

$$\therefore \mu_{strong}(40) = \frac{40-25}{25} = \frac{15}{25} = \frac{3}{5} \qquad \boxed{\therefore \mu_{strong}(40) = \frac{3}{5}}$$

Step III : Fuzzy rule evaluation

When temperature = 8 and pressure = 40

$\because \mu_{cool}(8) = \frac{1}{5}$ $\qquad \mu_{OK}(40) = \frac{2}{5}$

$\mu_{Average}(8) = \frac{4}{5}$ $\qquad \mu_{strong}(40) = \frac{3}{5}$

Pressure

Temp.	Low	Ok	Strong
Cool	P_2	Z	N_2
Average	P_2	Z	N_1
Warm	P_1	N_2	N_1

Rule 1 : Temp is cool and Pressure is OK

$$R_1 = \min\left\{\frac{1}{5}, \frac{2}{5}\right\} = \min\{0.2, 0.4\} = 0.2$$

Rule 2 : Temp is cool and Pressure is strong

$$R_2 = \min\left\{\frac{1}{5}, \frac{3}{5}\right\} = \min\{0.2, 0.6\} = 0.2$$

Rule 3 : Temp is Average and Pressure is OK

$$R_3 = \min\left\{\frac{4}{5}, \frac{2}{5}\right\} = \min\{0.8, 0.4\} = 0.4$$

<u>Rule 4</u> : Temp is Average and Pressure is Strong

$$R_4 = \min\left\{\frac{4}{5}, \frac{3}{5}\right\} = \min\{0.8, 0.6\} = \underline{0.6}$$

<u>Step IV</u> : Defuzzification method

By Middle of maxima method

Throttle position of the Turbine $= \max\{R_1, R_2, R_3, R_4\}$

$= \max\{0.2, 0.2, 0.4, 0.6\}$

$= \underline{0.6} \left(= \frac{3}{5} \right)$

Hence it is corresponding <u>Rule 4</u> where temp is Average and Pressure is strong

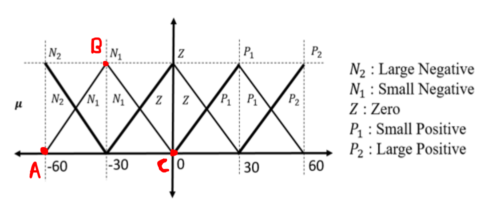
$\therefore \mu_{N_1}(x) = 0.6$

Temp. \ Pressure	Low	Ok	Strong
Cool	P_2	Z	N_2
Average	P_2	Z	$\boxed{N_1}$
Warm	P_1	N_2	N_1

* Membership function of N_1

ⓐ Eq^n of line AB :

$A(-60, 0)$ and $B(-30, 1)$
$\quad x_1 \; y_1 \qquad\qquad x_2 \; y_2$



Throttle Setting of Steam Turbine

$$\therefore \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore \frac{y - 0}{x + 60} = \frac{1 - 0}{-30 + 60}$$

$$\therefore \quad y = \frac{x+60}{30}$$

$$\boxed{\therefore \mu_{AB}(x) = \frac{x+60}{30}}$$

ⓑ Eqn of line BC :

$B(-30, 1)$ and $C(0, 0)$

$\quad\; x_1 \;\; y_1 \qquad\qquad\qquad x_2 \;\; y_2$

$$\therefore \quad \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\therefore \quad \frac{y-1}{x+30} = \frac{0-1}{0+30}$$

$$\therefore \quad y-1 = -\frac{x+30}{30}$$

$$\therefore \quad y = 1 - \frac{(x+30)}{30}$$

$$\therefore \quad y = -\frac{x}{30}$$

$$\boxed{\therefore \quad \mu_{BC}(x) = -\frac{x}{30}}$$

$\therefore \quad 0.6 = \mu_{AB}(x) \quad$ and $\quad 0.6 = \mu_{BC}(x)$

$0.6 = \dfrac{x+60}{30} \quad$ and $\quad 0.6 = \dfrac{-x}{30}$

$\Rightarrow \boxed{x = -42} \quad$ and $\quad x = -18$

$\therefore$ Throttle setting of Turbine $= \dfrac{-42-18}{2} = \underline{\underline{-30}}$

* Examples

Design a controller to determine the wash time of a domestic washing machine. Assume the input is dirt and grease on cloths. Use three descriptors for input variables and five descriptors for output variables wash time as below

Dirt = { SD, MD, LD } , Grease = { NG, MG, LG }

SD = Small dirt

MD = Medium dirt

LD = Large dirt

NG = No grease

MG = Medium grease

LG = Large grease

Wash Time = { VS, S, M, L, VL }

VS = very short

S = short

M = Medium

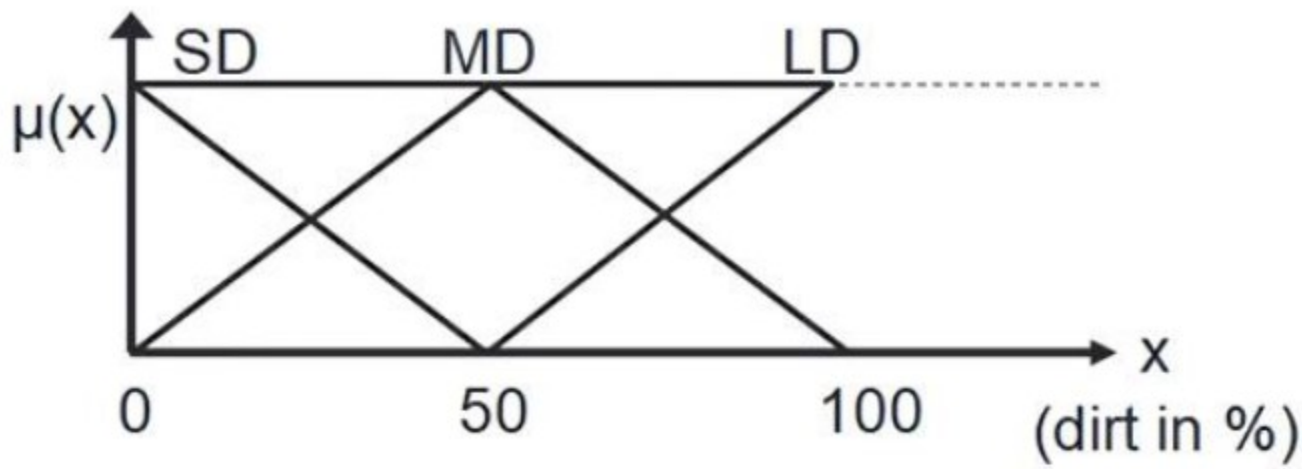
L = Large

VL = Very Large.

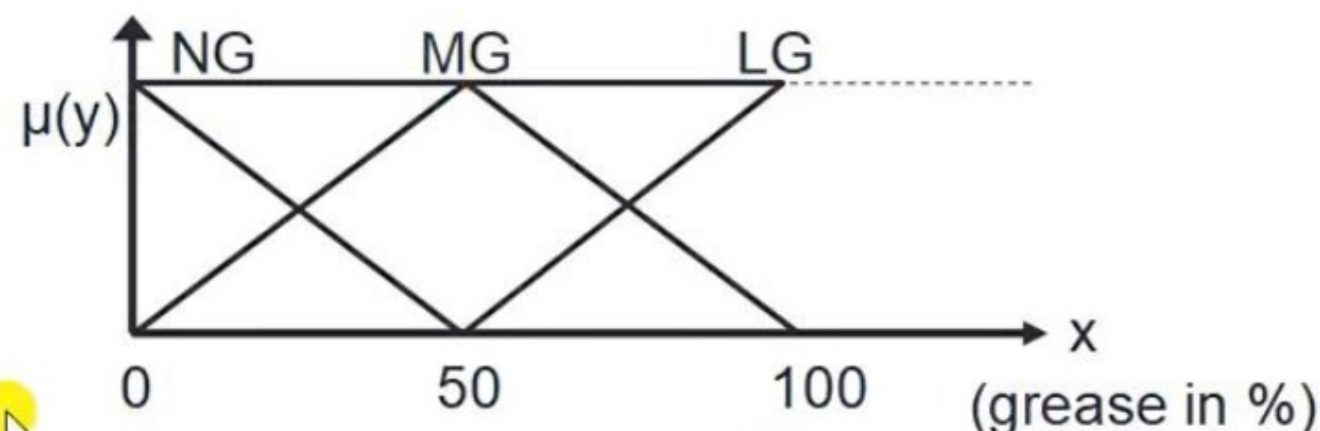
Find the wash time of washing machine for dirt = 60 % and grease = 70 % using defuzzification method - middle of maxima and fuzzy rules given below, grease

dirt		NG	MG	LG
	SD	VS	M	L
	MD	S	M	L
	LD	M	L	VL

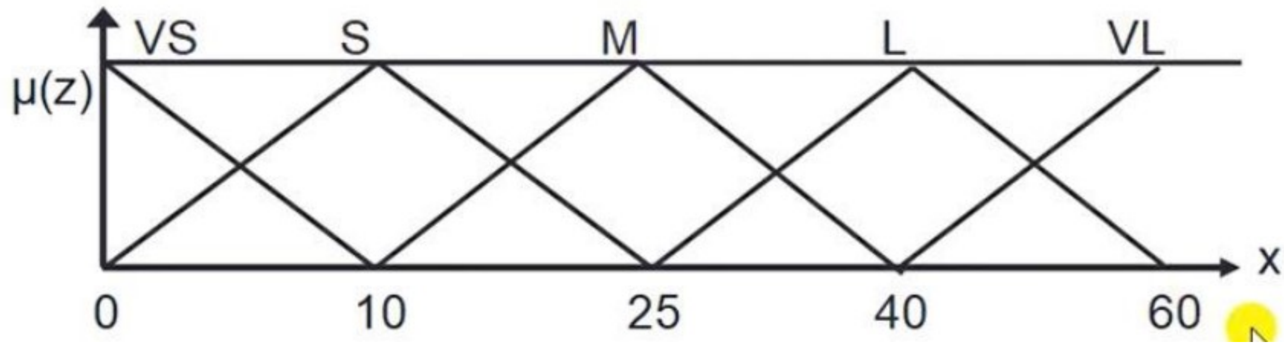
(1) Membership function for dirt:



(2) Membership function for grease:



(3) Membership function for Wash time:



Sol^n : Step I : Identify input and output variables and decide descriptor for the same.

Dirt = { SD, MD, LD } , Grease = { NG, MG, LG }

SD = Small dirt

MD = Medium dirt

LD = Large dirt

NG = No grease

MG = Medium grease

LG = Large grease

Wash Time = { VS, S, M, L, VL }

VS = very short

S = short

M = Medium

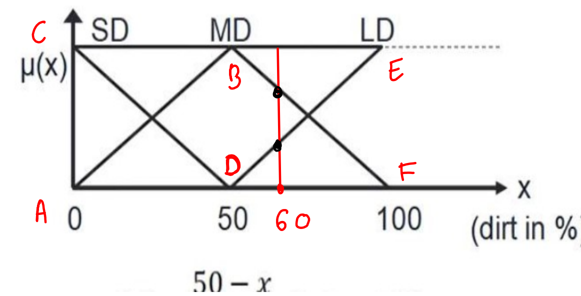
L = Large

VL = Very Large.

Step II : Find membership value of given input variables.

When dirt = 60%

(1) Membership function for dirt:



$\underline{\text{Eq}^n \text{ of line DE :}}$

$D(50, 0)$ and $E(100, 1)$

$\;\;\;\;x_1\;y_1 \qquad\qquad x_2\;y_2$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{y-0}{x-50} = \frac{1-0}{100-50} \Rightarrow y = \frac{x-50}{50}$$

$$\therefore \mu_{DE}(x) = \mu_{LD}(x) = \frac{x-50}{50}$$

$$\therefore \mu_{LD}(60) = \frac{60-50}{50} = \frac{1}{5} = \underline{\underline{0.2}} \qquad \therefore \mu_{LD}(60) = \underline{\underline{0.2}}$$

$\underline{\text{Eq}^n \text{ of line BF :}}$

$B(50, 1)$ and $F(100, 0)$

$\;\;\;\;x_1\;y_1 \qquad\qquad x_2\;y_2$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{y-1}{x-50} = \frac{0-1}{100-50} \Rightarrow y-1 = \frac{-(x-50)}{50}$$

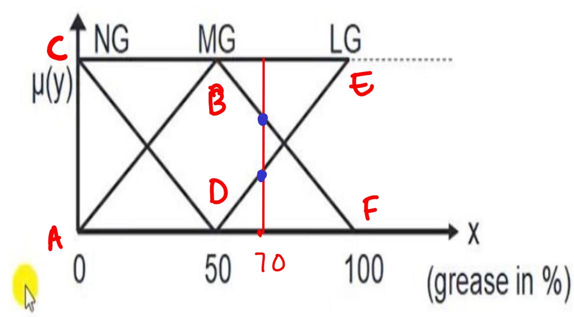
$$\therefore y = 1 - \frac{x-50}{50} = \frac{100-x}{50}$$

$$\therefore \mu_{BF}(x) = \mu_{MD}(x) = \frac{100-x}{50}$$

$$\therefore \mu_{MD}(60) = \frac{100-60}{50} = \frac{4}{5} = 0.8 \Rightarrow \mu_{MD}(60) = \underline{\underline{0.8}}$$

When grease = 70 %.

(2) Membership function for grease:



Eqn of line DE:

D (50, 0) and E(100, 1)

$x_1 \quad y_1 \qquad\qquad x_2 \quad y_2$

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-0}{x-50} = \frac{1-0}{100-50} \Rightarrow y = \frac{x-50}{50}$$

$$\therefore \mu_{DE}(x) = \mu_{LG}(x) = \frac{x-50}{50}$$

$$\therefore \mu_{LG}(70) = \frac{70-50}{50} = \frac{2}{5} = 0.4 \qquad \therefore \mu_{LG}(70) = \underline{\underline{0.4}}$$

Eqn of line BF

B(50, 1) and F(100, 0)

$x_1 \quad y_1 \qquad\qquad x_2 \quad y_2$

$$\therefore \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \Rightarrow \frac{y-1}{x-50} = \frac{0-1}{100-50} \Rightarrow y-1 = \frac{-(x-50)}{50}$$

$$\therefore y = 1 - \frac{x-50}{50} \Rightarrow y = \frac{100-x}{50}$$

$$\therefore \mu_{BF}(x) = \mu_{MG}(x) = \frac{100-x}{50}$$

$$\therefore \mu_{MG}(70) = \frac{100-70}{50} = \frac{3}{5} = \underline{\underline{0.6}} \qquad \therefore \mu_{MG}(70) = \underline{\underline{0.6}}$$

Step III : Fuzzy rule evaluation

When dirt = 60% and grease = 70%

$\mu_{LD}(60) = 0.2$ $\quad\quad$ $\mu_{LG}(70) = 0.4$

$\mu_{MD}(60) = 0.8$ $\quad\quad$ $\mu_{MG}(70) = 0.6$

grease

dirt	NG	MG	LG
SD	VS	M	L
MD	S	M	L
LD	M	L	VL

<u>Rule 1</u> : Dirt is MD and Grease is MG

$$R_1 = \min\{0.8, 0.6\} = \underline{\underline{0.6}}$$

<u>Rule 2</u> : Dirt is MD and Grease is LG

$$R_2 = \min\{0.8, 0.4\} = \underline{\underline{0.4}}$$

<u>Rule 3</u>: Dirt is LD and Grease is MG

$$R_3 = \{0.2, 0.6\} = 0.2$$

Rule 4 : Dirt is LD and Grease is LG

$$R_4 = \{0.2, 0.4\} = 0.2$$

Step IV : Defuzzification method

By middle of maxima

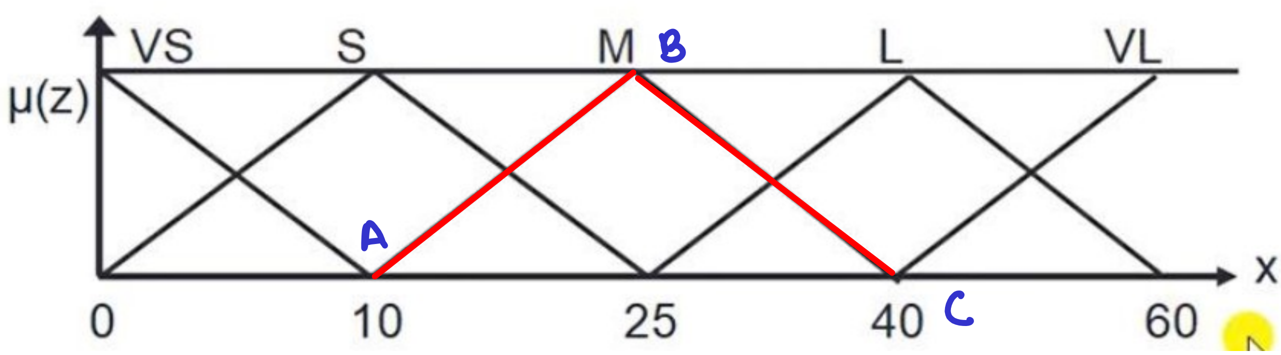
Wash Time position $= \max\{0.6, 0.4, 0.2, 0.2\}$

$= 0.6$

It correspond to Rule 1 where Dirt is MD and Grease is MG

$\therefore$ Hence output wash time is $\underline{\underline{M}}$

(3) Membership function for Wash time:



$$\therefore \mu_M(x) = 0.6$$

Eqn of line AB:

A(10,0) and B(25,1)

$$\therefore \quad \frac{y-0}{x-10} = \frac{1-0}{25-10} \Rightarrow y = \frac{x-10}{15}$$

$$\therefore \quad \mu_{AB}(x) = \frac{x-10}{15}$$

Eq^n of line BC

B(25,1) and C(40,0)

$$\therefore \quad \frac{y-1}{x-25} = \frac{0-1}{40-25} \Rightarrow y-1 = \frac{-(x-25)}{15}$$

$$\therefore \quad y = 1 - \frac{x-25}{15} = \frac{40-x}{15}$$

$$\therefore \quad \mu_{BC}(x) = \frac{40-x}{15}$$

Since $\mu_{M}(x) = 0.6$

$$\therefore \quad \mu_{AB}(x) = 0.6 \quad \text{and} \quad \mu_{BC}(x) = 0.6$$

$$\therefore \quad \frac{x-10}{15} = 0.6 \qquad \therefore \quad \frac{40-x}{15} = 0.6$$

$$\therefore \quad x = 19 \quad \text{and} \quad x = 31$$

$$\therefore \quad \text{Wash time} = \frac{19+31}{2} = \underline{\underline{25}} \text{ min}$$