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1. Conjecture:

Every natural number, except the two numbers, {1,6} , can be represented as the sum of natural (non - zero) perfect powers of distinct prime numbers.

2. Primary Analysis:

For natural number n :

$$n = (p1)^a + (p2)^b + (p3)^c \dots\dots$$

Where, p1,p2, p3..... are distinct prime numbers.

And, a, b, c..... are natural numbers(non-zero) numbers.

I propose that such representations are possible for all natural numbers except the two natural numbers {1,6}.

$$n \in N - \{1, 6\}$$

$$1 = *$$

$$2 = (2)^1$$

$$3 = (3)^1$$

$$4 = (2)^2$$

$$6 = *$$

$$7 = (7)^1$$

$$8 = (2)^3$$

$$9 = (3)^2$$

$$10 = (3)^1 + (7)^1$$

$$11 = (11)^1$$

$$12 = (5)^1 + (7)^1$$

$$13 = (13)^1$$

$$14 = (2)^1 + (5)^1 + (7)^1$$

$$15 = (2)^1 + (13)^1$$

$$16 = (2)^4$$

**No such representations are possible for the two numbers {1,6}.*

Refer to appendix 1 for further representations.

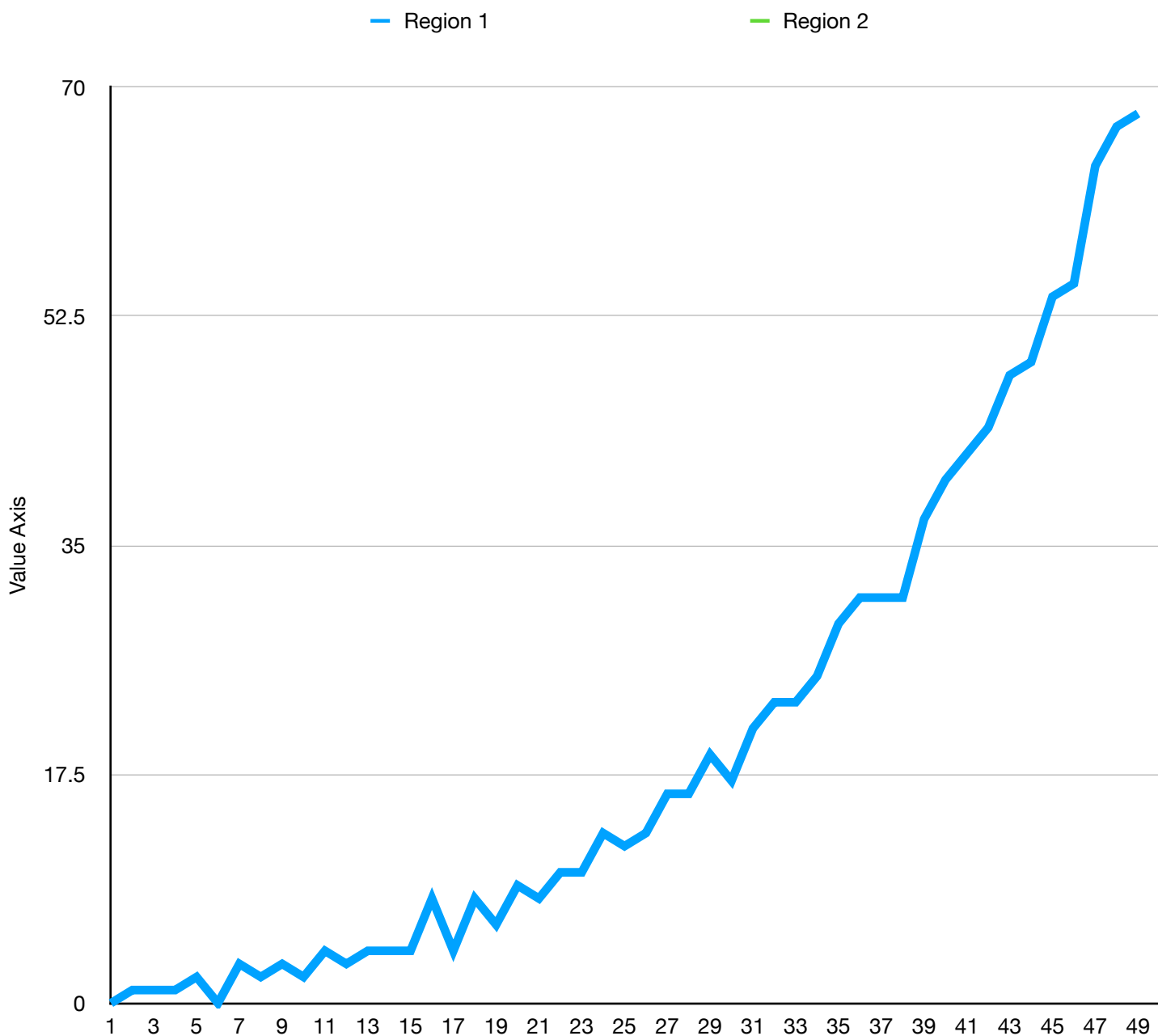
There are some numbers which may be represented in more than one way. For instance:

$$8 = (2)^3 \qquad 8 = (3)^1 + (5)^1$$

$$10 = (3)^1 + 7^1. \qquad 10 = (2)^1 + (3)^1 + (5)^1$$

Here, the vertical axis represents number of representations for the number at corresponding horizontal axis.

In the graph, only two values touch the horizontal axis. These are {1, 6}. The general trend observed is that the higher the numbers get, more are the number of representations for each. As we go on to find



representations for larger numbers, the process becomes easier due to increase in the number of prime numbers less than or equal to n .

However, this is not enough to prove this conjecture.

3. Similarities with Goldbach's conjecture

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states that every even whole number greater than 2 is the sum of two prime numbers.

Most Goldbach representations can be accommodated as representations in my conjecture. The only Goldbach representations that can be excluded for a given number while accommodating into my conjecture are ones where the same prime number is used twice.

For instance, Goldbach's conjecture represents 6 as

$$6 = 3 + 3$$

However, this representation can't be used in my conjecture as repetition of primes isn't allowed.

There however many Goldbach representations that can be used. Such as:

$$12=5+7$$

$$18=7+11$$

The limitation here would be that Goldbach Conjecture is focused on representations of even numbers only. However, since lots of research has been done on Goldbach Conjecture, there must be lots of representations that can be accommodated in my conjecture. This can and will ease the burden on calculations while verifying this conjecture.

4. Author

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5. Appendix 1

List below shows one of the representations for the first 50 natural numbers. The number is represented by the expression or the sum of items in the tuple.

$17 = (2)^3 + (3)^2$
 $18 = (2)^2 + (3)^2 + (5)^1$
 $19 = (2)^4 + 3$
 $20 = (3)^2 + 11$
 $21 = (2)^2 + 17$
 $22 = (2)^4 + 5$
 $23 = (23)^1$
 $24 = 11 + 13$
 $25 = (0, 0, 5, 7, 0, 13, 0, 0, 0)$
 $26 = (0, 0, 0, 7, 0, 0, 0, 19, 0)$
 $27 = (0, 3, 0, 0, 11, 13, 0, 0, 0)$
 $28 = (0, 0, 0, 0, 11, 0, 17, 0, 0)$
 $29 = (0, 0, 0, 0, 0, 0, 0, 0, 29)$
 $30 = (0, 0, 0, 0, 0, 13, 17, 0, 0, 0)$
 $31 = (0, 0, 0, 0, 0, 0, 0, 0, 0, 31)$
 $32 = (0, 0, 0, 0, 0, 13, 0, 19, 0, 0, 0)$
 $33 = (0, 0, 5, 0, 11, 0, 17, 0, 0, 0, 0)$
 $34 = (0, 0, 0, 0, 11, 0, 0, 0, 23, 0, 0)$
 $35 = (0, 0, 0, 7, 11, 0, 17, 0, 0, 0, 0)$
 $36 = (0, 0, 0, 0, 0, 0, 17, 19, 0, 0, 0)$
 $37 = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 37)$
 $38 = (0, 0, 0, 7, 0, 0, 0, 0, 0, 0, 31, 0)$
 $39 = (0, 0, 0, 7, 0, 13, 0, 19, 0, 0, 0, 0)$
 $40 = (0, 0, 0, 0, 0, 0, 17, 0, 23, 0, 0, 0)$
 $41 = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 41)$
 $42 = (0, 0, 0, 0, 0, 0, 0, 19, 23, 0, 0, 0, 0)$
 $43 = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 43)$
 $44 = (0, 0, 0, 0, 0, 13, 0, 0, 0, 0, 31, 0, 0, 0)$
 $45 = (0, 0, 5, 0, 0, 0, 17, 0, 23, 0, 0, 0, 0, 0)$
 $46 = (0, 0, 0, 0, 0, 0, 17, 0, 0, 29, 0, 0, 0, 0)$
 $47 = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 47)$
 $48 = (0, 0, 0, 0, 0, 0, 0, 19, 0, 29, 0, 0, 0, 0, 0)$
 $49 = (0, 0, 0, 0, 0, 13, 17, 19, 0, 0, 0, 0, 0, 0, 0)$
 $50 = (0, 0, 0, 0, 0, 0, 0, 19, 0, 0, 31, 0, 0, 0, 0)$

No. Of different representations possible for a given number:

1 - 0
2 - 1
3 - 1
4 - 1
5 - 2
6 - 0
7 - 3
8 - 2
9 - 3
10 - 2
11 - 4
12 - 3
13 - 4
14 - 4
15 - 4
16 - 8
17 - 4
18 - 8
19 - 6
20 - 9
21 - 8
22 - 10
23 - 10
24 - 13
25 - 12
26 - 13
27 - 16
28 - 16
29 - 19
30 - 17
31 - 21
32 - 23
33 - 23
34 - 25
35 - 29
36 - 31
37 - 31
38 - 31
39 - 37
40 - 40
41 - 42
42 - 44
43 - 48
44 - 49
45 - 54
46 - 55
47 - 64
48 - 67
49 - 68

